New York State Testing Program

Regents Examination in Algebra II (Common Core)

Selected Questions with Annotations

June 2016

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Selected Questions with Annotations

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. In Spring 2014, New York State administered the first set of Regents Exams designed to assess student performance in accordance with the instructional shifts and the rigor demanded by the Common Core State Standards (CCSS). To aid in the transition to new tests, New York State released a number of resources including sample questions, test blueprints and specifications, and criteria for writing test questions. These resources can be found at http://www.engageny.org/resource/regents-exams.

New York State administered the first Algebra II (Common Core) Regents Exam in June 2016 and is now annotating a portion of the questions from this test available for review and use. These annotated questions will help students, families, educators, and the public better understand how the test has changed to assess the instructional shifts demanded by the Common Core and to assess the rigor required to ensure that all students are on track to college and career readiness.

Annotated Questions Are Teaching Tools

The annotated questions are intended to help students, families, educators, and the public understand how the Common Core is different. The annotated questions demonstrate the way the Common Core should drive instruction and how tests have changed to better assess student performance in accordance with the instructional shifts demanded by the Common Core. They are also intended to help educators identify how the rigor of the Regents Examinations can inform classroom instruction and local assessment. The annotations will indicate common student misunderstandings related to content clusters; educators should use these to help inform unit and lesson planning. In some cases, the annotations may offer insight into particular instructional elements (conceptual thinking, mathematical modeling) that align to the Common Core that may be used in curricular design. It should not be assumed, however, that a particular cluster will be measured with identical items in future assessments.

The annotated questions include both multiple-choice and constructed-response questions. With each multiple-choice question annotated, a commentary is available to demonstrate why the question measures the intended cluster. The rationales describe why the correct answer is correct and why the wrong answer choices are plausible but incorrect, based on common misconceptions or common procedural errors. While these rationales speak to a possible and likely reason for the selection of the incorrect option by the student, these rationales do not contain definitive statements as to why the student chose the incorrect option, or what we can infer about the knowledge and skills of the student based on the student's selection of an incorrect response. These multiple-choice questions are designed to assess student proficiency, not to diagnose specific misconceptions/errors with each and every incorrect option.

For each constructed-response question, there is a commentary describing how the question measures the intended cluster, plus sample student responses representing possible student errors or misconceptions at each possible score point.

The annotated questions do not represent the full spectrum of standards assessed on the State test, nor do they represent the full spectrum of how the Common Core should be taught and assessed in the classroom. Specific criteria for writing test questions as well as test information are available at http://www.engageny.org/resource/regents-exams.

Understanding Math Annotated Questions

All questions on the Regents Exam in Algebra II (Common Core) are designed to measure the Common Core Learning Standards identified by the PARCC Model Content Framework for Algebra II. More information about the relationship between the New York State Testing Program and PARCC can be found here: http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf.

Multiple Choice

Multiple-choice questions will primarily be used to assess procedural fluency and conceptual understanding. Multiple-choice questions measure the Standards for Mathematical Content and may incorporate Standards for Mathematical Practices and real-world applications. Some multiple-choice questions require students to complete multiple steps. Likewise, questions may measure more than one cluster, drawing on the simultaneous application of multiple skills and concepts. Within answer choices, distractors will all be based on plausible missteps.

Constructed Response

Constructed-response questions will require students to show a deep understanding of mathematical procedures, concepts, and applications. The Regents Examination in Algebra II (Common Core) contains 2-, 4-, and 6-credit constructed-response questions.

2-credit constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, 2-credit constructed-response questions may involve multiple steps, the application of multiple mathematics skills, and real-world applications. These questions may ask students to explain or justify their solutions and/or show their process of problem solving.

4-credit and 6-credit constructed-response questions require students to show their work in completing more extensive problems that may involve multiple tasks and concepts. Students will be asked to make sense of mathematical and real-world problems in order to demonstrate procedural and conceptual understanding. For 6-credit constructed-response questions, students will analyze, interpret, and/or create mathematical models of real-world situations to solve multi-step problems that connect multiple major clusters or a major cluster to supporting or additional content.

#1

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

(1) $\dfrac{1}{\left(\sqrt[d]{3b}\right)^2}$

(2) $\left(\sqrt{3b}\right)^d$

(3) $\dfrac{1}{\sqrt{3b^d}}$

(4) $\left(\sqrt[d]{3b}\right)^2$

Measured CCLS Cluster: N-RN.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within N-RN.A because it requires the student to extend the properties of exponents to rational exponents. The student must rewrite an expression involving rational exponents into an equivalent radical form. This question requires the student to employ Mathematical Practice 7(Look for and make use of structure), because the student must shift perspective from one form to another.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common student errors made when a student has to rewrite rational exponents in radical form. Choosing the correct solution requires students to know how to manipulate rational exponents.

Answer Choice: (1) This response is incorrect and represents an incorrect equivalent expression. The student may have understood the fractional exponent, but did not understand that the reciprocal should only be written when the exponent is negative.

Answer Choice: (2) This response is incorrect. The expression represents the fractional exponent applied with the numerator as the root and the denominator as the power instead of the numerator as the power and the denominator as the root.

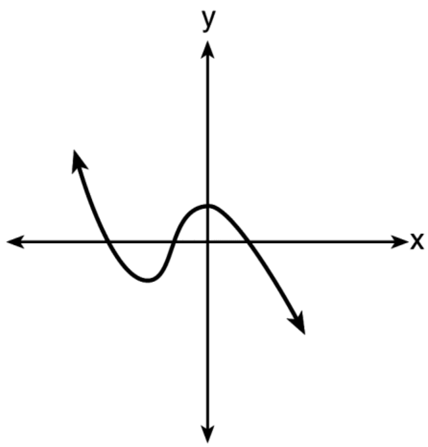
Answer Choice: (3) This response is incorrect. The student response is a combination of choice (1) and choice (2). The student used a reciprocal and made the denominator of the exponent the power instead of the root.

Answer Choice: (4) This response is correct and represents an equivalent expression. A student who selects this choice understands how to apply a fractional exponent by using the numerator as the power and the denominator as the root.

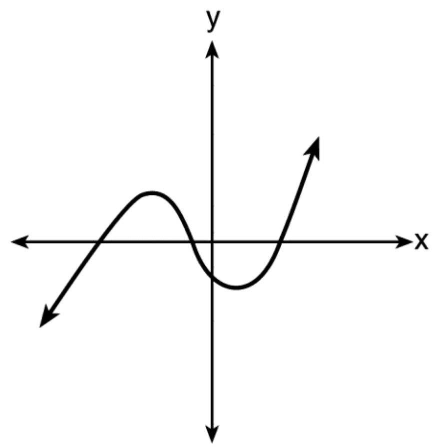
#4

4 Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



(1)



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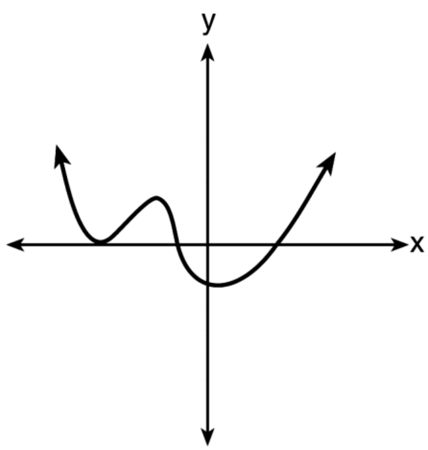

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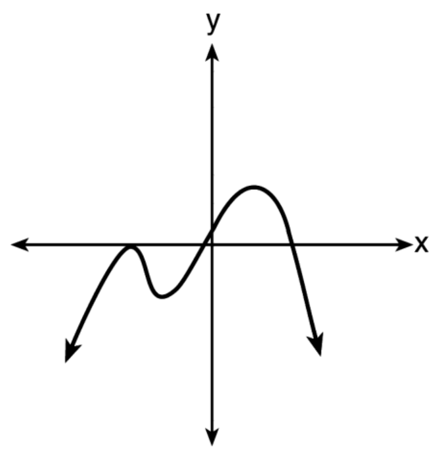


(4)

Measured CCLS Cluster: F-IF.C

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because it requires the student to analyze functions using different representations. The student must understand how to identify the graph of a polynomial function given zeros and end behavior.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to identify zeros and end behavior. Choosing the correct solution requires students to know how to determine the characteristics of a function from a graph. The graph's characteristics include three real zeros, where the left-end behavior is decreasing and the right-end behavior is increasing. Compare with question 36, which also assesses F-IF.C.

Answer Choice: (1) This response is incorrect. Although the graph shows three real zeros, the end behavior is opposite to the given characteristics. For this function, the left-end behavior is increasing and the right-end behavior is decreasing.

Answer Choice: (2) This response is incorrect. Although the graph shows three real zeros and correct right-end behavior, the left-end behavior is incorrect because it is increasing.

Answer Choice: (3) This response is correct, and a student who selects this response understands zeros and end behavior. The graph shows three real zeros, and has end behavior matching the given end behavior.

Answer Choice: (4) This response is incorrect. Although the graph shows three real zeros and correct left-end behavior, the right-end behavior is incorrect because it is decreasing.

#6

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

(1) $\{0, \pm 3, 4\}$

(2) $\{0, 3, 4\}$

(3) $\{0, \pm 3, -4\}$

(4) $\{0, 3, -4\}$

Measured CCLS Cluster: A-APR.B

Key: 1

Commentary: This question measures the knowledge and skills described by the standards within A-APR.B because it requires the student to understand the relationship between zeros and factors of polynomials. The student must identify zeros of a quartic polynomial either by factoring or by constructing a graph of the function. This question requires the student to employ Mathematical Practice 5 (Use appropriate tools strategically) if the student chooses to analyze the graph of the functions using a graphing calculator, or Mathematical Practice 7 (Look for and make use of structure) if the student chooses to factor the polynomial.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common student errors made when finding the zeros of a factorable polynomial function. Choosing the correct solution requires students to know how to rewrite a polynomial expression using factor by grouping, and solving for the zeros or analyzing key features of the graph of the function.

Answer Choice: (1) This response is correct. The student understands how to find the zeros for a higher degree factorable polynomial function.

$$0 = x^2(x^2 - 9) - 4x(x^2 - 9)$$
$$0 = (x^2 - 4x)(x^2 - 9)$$
$$0 = x(x - 4)(x + 3)(x - 3)$$
$$x = 0, 4, -3, 3$$

Answer Choice: (2) This response is incorrect. The student found the positive square root of 9, but failed to find the negative square root of 9.

Answer Choice: (3) This response is incorrect. The student may have factored incorrectly to get $x + 4$ and $x = -4$ or factored correctly to get $x - 4$, but concluded $x = -4$. Using a graphical approach, the student may have found an x-intercept at 4, but interpreted the zero as -4.

Answer Choice: (4) This response is incorrect. The student failed to find the negative square root of 9 and incorrectly found $x = -4$ possibly due to a factoring or graphing error.

#9

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

(1) 6 (3) 68

(2) 48 (4) 95

Measured CCLS Cluster: S-ID.A

Key: 2

Commentary: The question measures the knowledge and skills described by the standards within S-ID.A because it requires the student to use the mean and standard deviation to estimate population percentages. The student must use a statistical function of a calculator or use knowledge of common normal curve percentages and z-scores to obtain a correct percent. This question requires students to employ Mathematical Practice 4 (Model with mathematics) as students apply mathematics to solve a problem arising in everyday life.

Rationale: Choices (1), (3), (4) are plausible but incorrect. They represent common student errors made when a student has to determine population percentages, with or without using technology. Choosing the correct solution requires students to know how to compute a population percentage using the normal cumulative density function on a graphing calculator or by finding the area associated with z-scores on a normal curve.

Answer Choice: (1) This response is incorrect. The student subtracted the mean of 64 from the value 69.5 and rounded. No connection to population percentages was made.

Answer Choice: (2) This response is correct and represents the percentage of women whose heights are between 64 and 69.5 inches to the nearest whole percent. The student understands how to find the population percentage using a graphing calculator or using knowledge of common normal curve percentages.

Answer Choice: (3) This response is incorrect. By the empirical rule, 68% of data lie within one standard deviation of the mean. The student found the percentage of women whose heights are between 61.25 and 66.75 inches rather than between 64 and 69.5 inches.

Answer Choice: (4) This response is incorrect. By the empirical rule, 95% of data lie within two standard deviations of the mean. The student found the percentage of women whose heights are between 58.5 and 69.5 inches rather than between 64 and 69.5 inches.

#10

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

Measured CCLS Cluster: F-BF.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within F-BF.A because it requires the student to determine a scenario modeled by a recursive process. This question requires the student to employ Mathematical Practice 4 (Model with mathematics) as the student will interpret mathematical results within a given context and reflect on whether the results make sense.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to know which scenario models a geometric sequence expressed recursively. Choosing the correct solution requires students to recognize a geometric sequence with an initial term of 3000 and subsequent terms decreasing by 20%.

Answer Choice: (1) This response is incorrect. The scenario represents a model whose seats are increasing as an arithmetic sequence with a common difference of 80.

Answer Choice: (2) This response is incorrect. Although the scenario represents a decreasing number of seats, it is an arithmetic sequence with a common difference of -80.

Answer Choice: (3) This response is incorrect. The scenario represents a model where an account is growing as an increasing geometric sequence with a common ratio of 1.80.

Answer Choice: (4) This response is correct. The student understands that the scenario represents a decreasing geometric sequence with a common ratio of 0.80.

#11

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

Measured CCLS Cluster: S-CP.A

Key: 1

Commentary: The question measures the knowledge and skills described by the standards within S-CP.A because it requires the student to understand independence and conditional probability. This question also requires the student to employ Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others) because the student can make conjectures, build a logical progression of statements, and reason inductively about data.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common errors made when a student has to determine that if $P(A|B) = P(A)$, then events A and B are independent.

Answer Choice: (1) This response is correct. The student understands the relationship between conditional probability and independence.

$$P(Rain|Sean\ pitches) = \frac{P(Rain \cap Sean\ pitches)}{P(Sean\ pitches)}$$

$$0.4 = \frac{P(Rain \cap Sean\ pitches)}{0.5}$$

$$P(Rain \cap Sean\ pitches) = 0.2$$

Since $P(Rain) \cdot P(Sean\ pitches) = 0.2$, the two events are independent.

Another method is to show: $P(Rain|Sean\ pitches) = P(Rain)$

$$0.4 = 0.4$$

Answer Choice: (2) This response is incorrect. There is no evidence that the probability of rain impacts the probability that Sean pitches.

Answer Choice: (3) This response is incorrect. If the events were mutually exclusive, then the 2 events could not occur at the same time. Since Sean could pitch in the rain, $P(Rain \cap Sean\ pitches) = 0.2$, the events are not mutually exclusive.

Answer Choice: (4) This response is incorrect. If the events were complements, the probabilities would add to 1. Additionally, complements are mutually exclusive.

#13

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150	(3) 10
(2) 70	(4) 0

Measured CCLS Cluster: F-IF.B

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-IF.B because it requires the student to interpret functions that arise in applications. This question requires the student to employ Mathematical Practice 7 (Look for and make use of structure), because the student recognizes the parameters of a sinusoidal function that relate to the characteristics of the model.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine the minimum height for a trigonometric function by subtracting the amplitude (70) from the vertical shift (80).

Answer Choice: (1) This response is incorrect. The student found the maximum value by adding the amplitude to the vertical shift.

Answer Choice: (2) This response is incorrect. The student only found the amplitude and did not understand how to use it to get the minimum height.

Answer Choice: (3) This response is correct. The student who selects this response has an understanding of how to determine the minimum height knowing the amplitude and the vertical shift.

Answer Choice: (4) This response is incorrect. The student did not take into account the vertical shift or amplitude.

#17

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

(1) $-\frac{3}{5}$

(3) $\frac{3}{5}$

(2) $-\frac{3}{4}$

(4) $\frac{4}{5}$

Measured CCLS Cluster: F-TF.A

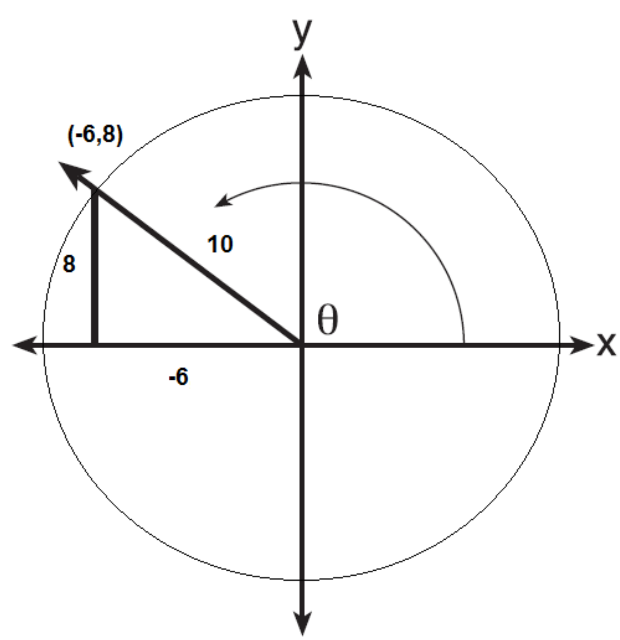
Key: 1

Commentary: The question measures the knowledge and skills described by the standards within F-TF.A because it requires the student to extend the domain of trigonometric functions by using the unit circle. This question requires the student to employ Mathematical Practice 2 (Reason abstractly and quantitatively) because the student contextualizes their knowledge of right triangle trigonometry and applies that knowledge within the unit circle.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common errors made when a student has to determine the value of a trigonometric function when extending the domain to any quadrant.

Answer Choice: (1) This response is correct. The student who selects this response understands how to find the value of the cosine of an angle in Quadrant II.

$$\cos\theta = -\frac{6}{10} = -\frac{3}{5}$$



Answer Choice: (2) This response is incorrect. Although the student understood the value should be negative, the cotangent of the angle was found.

Answer Choice: (3) This response is incorrect. The student does not understand that the cosine of an angle in Quadrant II is a negative value.

Answer Choice: (4) This response is incorrect. The student may have found the sine of the angle or may have used the given *y*-coordinate as the *x*-coordinate in Quadrant I.

#19

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

Measured CCLS Cluster: A-SSE.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.A because it requires the student to use the structure of an equation to identify ways to rewrite it. This question requires the student to employ Mathematical Practice 7 (Look for and make use of structure) because the student must recognize an opportunity to write the original equation as a circle in center-radius form.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to determine how to write an equivalent equation using the completing the square process. Compare with item 31, which also assesses A-SSE.A.

Answer Choice: (1) This response is incorrect. When completing the square, the student did not add the distributed values of 36 and 324 to the right side.

$$4x^2 - 24x + 4y^2 + 72y = 76$$
$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76$$

Answer Choice: (2) This response is incorrect. The student added 36 and 9 to the right side rather than 36 and 324.

Answer Choice: (3) This response is incorrect. The student added 9 and 81 to the right side by failing to distribute the 4 to each term.

Answer Choice: (4) This response is correct. The student who selects this response has an understanding of structure and the process of completing the square.

$$4x^2 - 24x + 4y^2 + 72y = 76$$
$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324$$
$$4(x - 3)^2 + 4(y + 9)^2 = 436$$

#21

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

Measured CCLS Cluster: A-SSE.B

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.B because it requires the student to model an exponential growth rate as an expression. The student must write an equivalent monthly exponential expression from a yearly exponential growth rate. This item requires skills associated with both Mathematical Practice 4 (Model with mathematics) and Mathematical Practice 7 (Look for and make use of structure).

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine how to use the properties of exponents to transform an exponential expression. Compare with item 34, which also assesses A-SSE.B.

Answer Choice: (1) This response is incorrect. The student applied the annual rate to the commonly used form of $(1+r)^n$.

Answer Choice: (2) This response is incorrect. The student applied the annual rate to the commonly used form of $(1+r)^n$ and stated the reciprocal of the correct exponent.

Answer Choice: (3) This response is correct. The student who selected this response understands how to convert from an annual rate to a monthly rate, and correctly expressed and applied the new exponent.

$$(1+\ 0.0525)^{\frac{m}{12}}$$
$$\left((1.0525)^{\frac{1}{12}}\right)^m$$
$$\approx (1.00427)^m$$

Answer Choice: (4) This response is incorrect. The student converted to a monthly rate but left the exponent expressed in years instead of months.

#23

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

Measured CCLS Cluster: F-BF.A

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-BF.A because it requires the student to write a geometric sequence recursively. The student must recognize the difference between recursive and an explicit formulas, and arithmetic and geometric sequences.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine a recursive model for a geometric sequence. Choosing the correct solution requires the student to develop a recursive formula generating the four given values. Compare with item 10, which also assesses F-BF.A.

Answer Choice: (1) This response is incorrect. An explicit geometric sequence is expressed rather than a recursive geometric sequence.

Answer Choice: (2) This response is incorrect. The student found a sequence in which the difference grows exponentially.

Answer Choice: (3) This response is correct. The student who selects this response understands how to model a geometric sequence recursively by stating an initial term and a formula for all subsequent terms.

Answer Choice: (4) This response is incorrect. Although the initial term is correctly represented, subsequent terms are incorrectly represented as an arithmetic sequence.

#24

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$ (3) $V = 120 \sin(60\pi t)$

(2) $V = 120 \sin(60t)$ (4) $V = 120 \sin(120\pi t)$

Measured CCLS Cluster: F-TF.B

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within F-TF.B because it requires the student to model periodic phenomena with trigonometric functions. This question requires the students to employ Mathematical Practice 4 (Model with Mathematics) because the student must choose a sine function that models voltage given the maximum voltage and the number of cycles per time period.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to model periodic phenomena with specified amplitude and period.

Answer Choice: (1) This response is incorrect. The student lacks a coefficient of t, and the function therefore does not complete the required cycles per second.

Answer Choice: (2) This response is incorrect. The student simply used 60 as the coefficient of x. As a result, this function completes 60 cycles in 2π seconds, rather than in one second.

Answer Choice: (3) This response is incorrect. The student likely solved $Period = \frac{\pi}{B}$, omitting the 2 in the numerator.

Answer Choice: (4) This response is correct. The student who selects this response has an understanding of how to determine the period and amplitude of a trigonometric model.

$$Period = \frac{1}{60}$$
$$\frac{1}{60} = \frac{2\pi}{B}$$
$$B = 120\pi$$
$$V = 120sin(120\pi t)$$

#25

$$\textbf{25} \text{ Solve for } x: \quad \frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$$

Measured CCLS Cluster: A-REI.A

Commentary: The question measures the knowledge and skills described by the standards within A-REI.A because it requires the student to solve a simple rational equation. Additionally, the item requires the student to employ Mathematical Practice 1 because the student must make sense of a problem and persevere in solving it.

Rationale: This question requires students to solve a simple rational equation by eliminating the denominator by either multiplying by the LCD or finding a common denominator, and setting numerators equal to each other.

$$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$$

$$\left(\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}\right)3x$$

$$3 - x = -1$$

$$x = 4$$

Sample student responses and scores appear on the following pages.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{3}{3}\left(\frac{1}{x}\right)\left(-\frac{1}{3}\right)\frac{x}{x} = -\frac{1}{3x}$$

$$\frac{3}{3x} - \frac{1x}{3x} = \frac{-1}{3x}$$

$$3 - 1x = -1$$
$$-3 \qquad\quad -3$$

$$\frac{-1x}{-1} = \frac{-4}{-1}$$

$$x = 4$$

Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{(3)1}{(3)x} - \frac{(x)1}{(x)3} = \frac{-1}{3x}$$

$$\frac{3}{3x} - \frac{x}{3x} = \frac{-1}{3x}$$

$$\frac{3-x}{3x} = \frac{-1}{3}x$$

Score 1: The student only found a common denominator and combined like terms.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{1}{x} - \frac{1}{3} = \frac{1}{3x}$$

$$\frac{0}{x-3} \times \frac{1}{3x}$$

$$0 = x - 3$$

$$x = 3$$

Score 0: The student made an error combining the fractions, and also made a transcription error by omitting the negative.

#26:

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

Measured CCLS Cluster: S-IC.B

Commentary: The question measures the knowledge and skills described by the standards within S-IC.B because the requirements of a controlled experiment by describing how a control group can be created to examine the effect of including an ingredient in toothpaste. Additionally, the item requires the student to employ Mathematical Practice 6 because the student must examine claims and make explicit use of definitions.

Rationale: This question requires students to describe a controlled experiment. Two critical aspects of this controlled experiment involve random assignment and the use of a control group. Experiments generally do not require a randomly selected group of individuals, but rather volunteers who are available at the time of study. However, random assignment of the volunteers to groups is critical to reduce bias. Additionally, a control group is required so that ingredient X is the sole differentiator in the outcomes of the experiment, rather than potential confounding variables that could influence results.

Compare with item 35, which also assesses S-IC.B.

Sample student responses and scores appear on the following pages.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

I would collect two groups of individuals that are of equal age and sex to ensure accuracy and eliminate any other variables that can have an effect. I would use a large group of people say 40 in each. Then, I would give one random group an equal amount of toothpaste with the ingredient, where as the other random group will recieve toothpaste with no ingredient. It will be given in the morning at the same time. By the end of the day at the same time for a week, I will record the results to determine the impact of the ingredient

Score 2: The student gave a complete and correct response.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

One group of people will use the version with ingredient X and another will use the toothpaste without. Compare the results.

Score 1: The student wrote an incomplete description by omitting the random assignment of two groups.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

A controlled experiment can be used by distributing products with the ingredients to a group, while giving the control group to ~~[illegible]~~ a different group of people.

Score 0: The student's response lacked random assignment and had an insufficient explanation of a control group.

#27:

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

Measured CCLS Cluster: A-APR.B

Commentary: The question measures the knowledge and skills described by the standards within A-APR.B because the student is required to apply understanding of the relationship between zeros and factors of polynomials. The student might directly apply the remainder theorem or use long or synthetic division with an appropriate explanation.

Rationale: This question instructs the student to determine whether $(x - 5)$ is a factor of the given polynomial. One possible method is to substitute 5 into the polynomial and observe the value is not zero, along with an explanation using the remainder theorem. Another possible method is to apply synthetic or long division and calculate the remainder, along with noting the nonzero remainder implies that $x - 5$ is not a factor.

Sample student responses and scores appear on the following pages.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10 \stackrel{?}{=} 0$

$250 - 100 - 35 - 10 \stackrel{?}{=} 0$

$105 \neq 0$

$x-5$ is not a factor of $2x^3-4x^2-7x-10$.

If $x-5$ is a factor of $2x^3-4x^2-7x-10$, then when $2x^3-4x^2-7x-10$ and 5 is substituted for x, the value of $2x^3-4x^2-7x-10$ should be 0.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$$2(-5)^3 - 4(-5)^2 - 7(-5) - 10 = 0$$

$$-325 \neq 0$$

x-5 is not a factor because when you use the remainder theorem the remainder is -325 not 0.

Score 1: The student made one error by substituting -5 instead of 5.

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

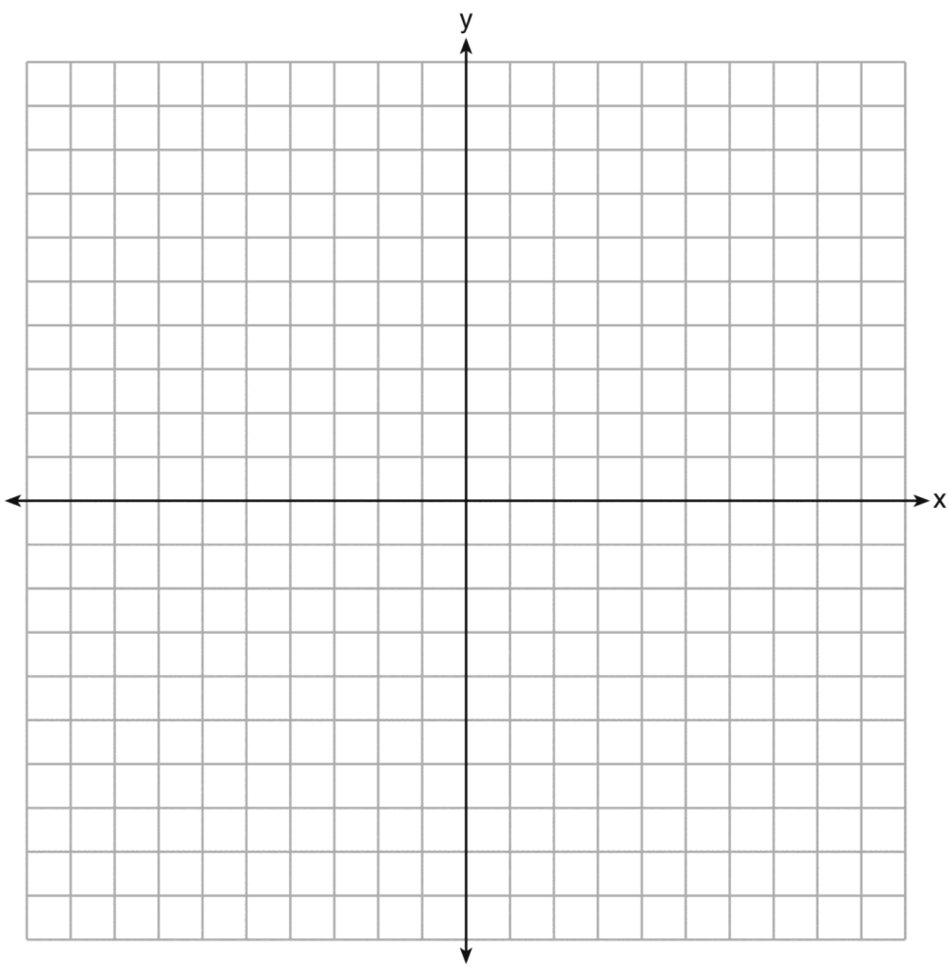
$$
\begin{array}{r}
2x^2 - 6x + 23 \\
x - 5 \overline{\big) 2x^3 - 4x^2 - 7x - 10} \\
2x^3 - 10x^2 \qquad\qquad \\
6x^2 - 7x - 10 \\
6x^2 - 30x \qquad \\
23x - 10
\end{array}
$$

No

Score 0: The student made multiple errors dividing and did not provide the explanation.

#28:

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point $(0,2)$.



Measured CCLS Cluster: F-IF.C

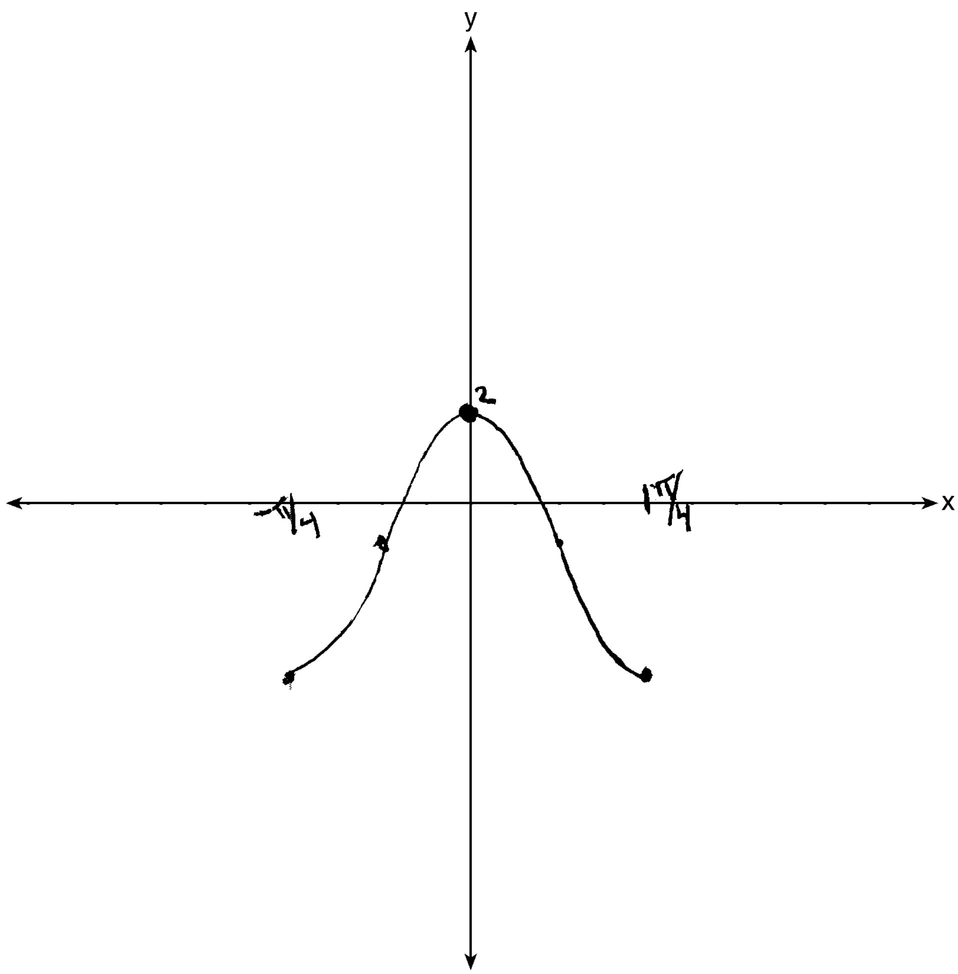
Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because the student is required to graph a trigonometric function given a description of its properties. Additionally, the item requires the student to employ Mathematical Practice 6 because the student must label the axes and sketch multiple features of the graph.

Rationale: This question requires the student to graph a cosine function given its period, amplitude, midline and the point $(0,2)$. The student may create the cosine function $y = 3\cos(4x) - 1$ from the given characteristics and graph one cycle with assistance from a graphing calculator. The amplitude 3 represents the distance between the highest point on the graph and the midline, the period $\frac{\pi}{2}$ represents the length of one complete cycle of the function, and the midline represents the equation of the horizontal line halfway between the maximum and minimum values. One cycle of the function can be completed on either side of the y-axis as long as it passes through the point $(0,2)$.

Sample student responses and scores appear on the following pages.

Question 28

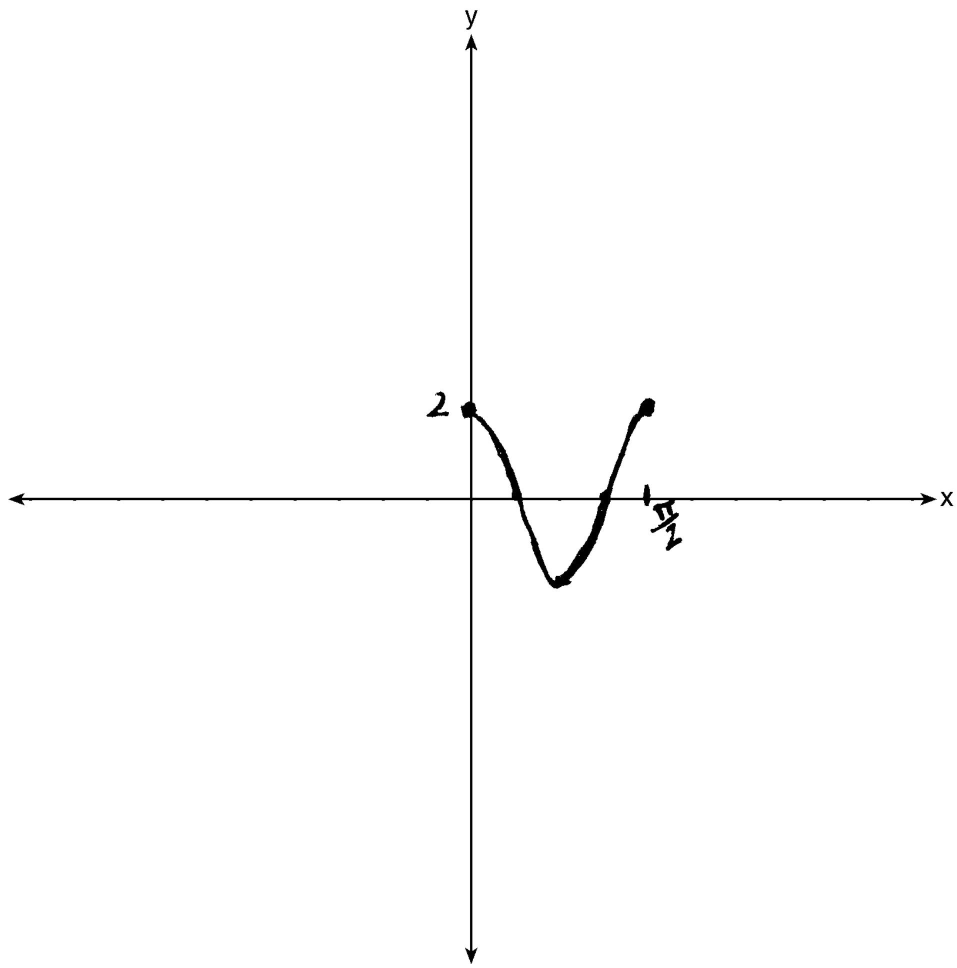
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

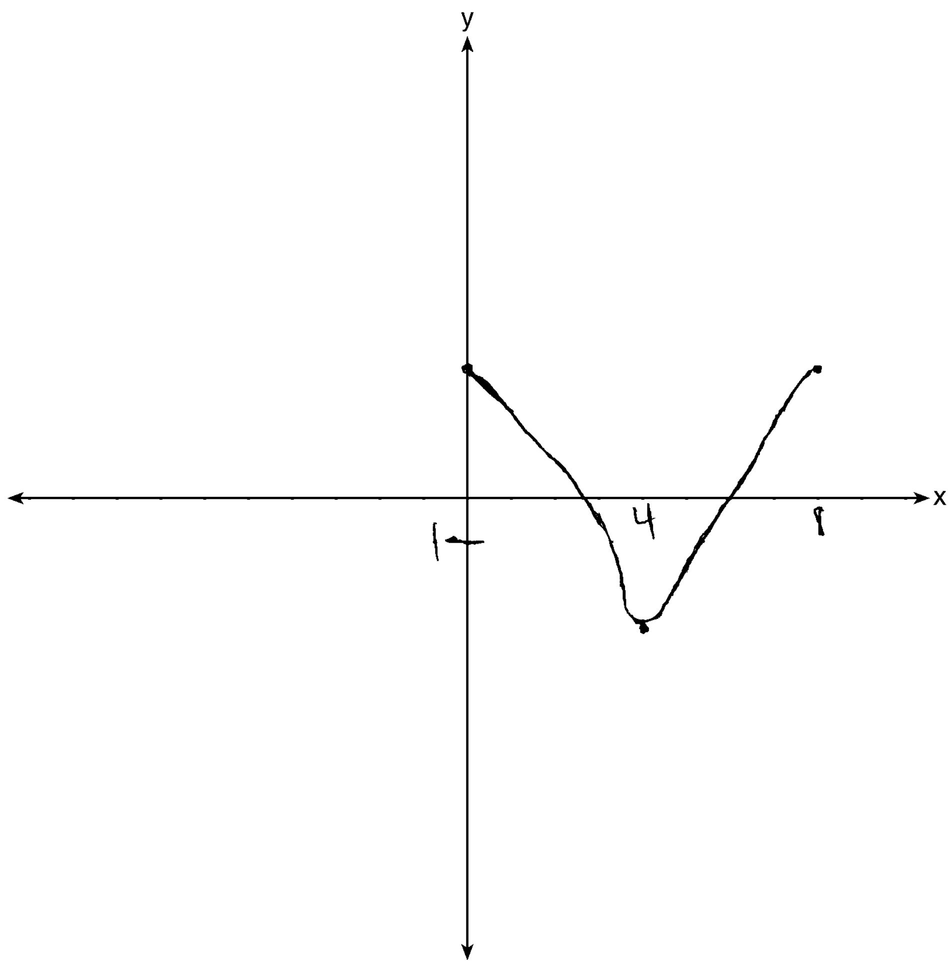
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student correctly graphed one cycle of a cosine function passing through (0,2) with period $\frac{\pi}{2}$, but used an incorrect amplitude that affected the midline.

Question 28

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 0: The student made multiple errors.

#29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

Measured CCLS Cluster: S-CP.B

Commentary: The question measures the knowledge and skills described by the standards within S-CP.B because the student is required to apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and interpret the answer in terms of the model. Additionally, the item requires the student to employ Mathematical Practice 4 because the student must be able to identify important quantities in a practical situation and map those relationships using such tools as diagrams.

Rationale: The question requires students to apply the Addition Rule as follows:

$$P(S \text{ or } M) = P(S) + P(M) - P(S \text{ and } M)$$

$$\frac{974}{1376} = \frac{649}{1376} + \frac{433}{1376} - P(S \text{ and } M)$$

$$\frac{974}{1376} = \frac{1082}{1376} - P(S \text{ and } M)$$

$$\frac{108}{1376} = P(S \text{ and } M)$$

Sample student responses and scores appear on the following pages.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$649 + 433$

$1082 - 974$

$\frac{108}{1376}$

Score 2: The student gave a complete and correct response.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$$\frac{649}{1376} + \frac{433}{1376}$$

$$\begin{array}{r} 649 \\ +433 \\ \hline 1082 \end{array}$$

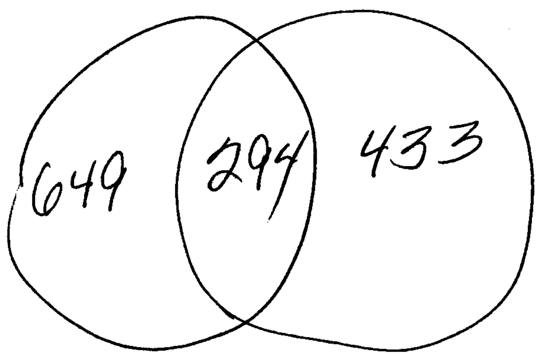
$$\frac{1082}{1376}$$

Score 1: The student made an error by not subtracting from $\frac{974}{1376}$.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$$\begin{array}{r} 1376 \\ -1082 \\ \hline 294 \end{array}$$



$$\frac{294}{974}$$

Score 0: The student made multiple errors.

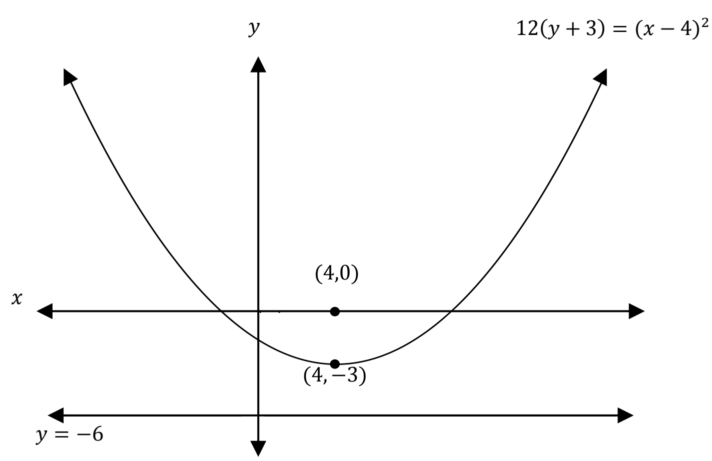
#30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Measured CCLS Cluster: G-GPE.A

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.A because the student is required to determine the focus of a parabola from its equation and its directrix. Additionally, the item requires the student to employ Mathematical Practice 7 because of the form of the equation provided.

Rationale: This question requires the student to find the vertex and use it to determine the focus. A parabola is the locus of points equidistant from a point (focus) and a line (directrix). The vertex can be determined simply from the original form of the equation as $(4, -3)$. Since the vertex is the minimum point on the given parabola, and any point on the parabola must be equidistant from the focus and directrix, the point $(4,0)$ can be determined as the focus.



Sample student responses and scores appear on the following pages.

Question 30

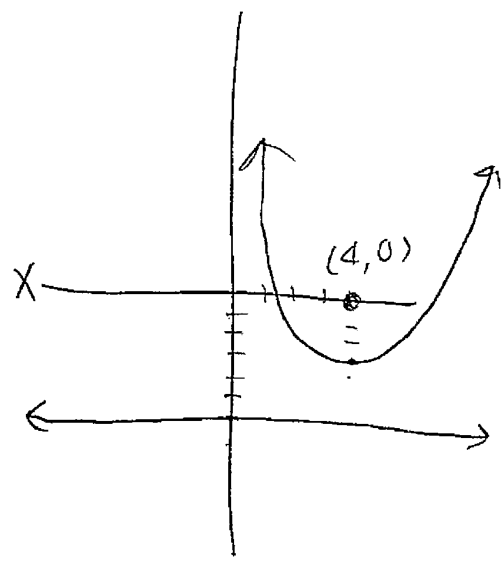
30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = x^2 - 8x + 16$
$-36 \qquad\qquad -36$
$12y = x^2 - 8x - 20$
$y = \frac{x^2}{12} - \frac{2x}{3} - \frac{5}{3}$

$x = \frac{-b}{2a} = \frac{-\frac{2}{3}}{24} = \frac{2}{72} = \frac{1}{}$

$(x-4)^2 = \underset{\uparrow\atop 4(3)}{12}(y+3)$

$\boxed{(4, 0)}$



Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Vertex = $(4, -3)$

since directrix is $y = -6$

need to add 6 to y in vertex

Focus = $(4, 3)$

Score 1: The student misused the directrix.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$$12y + 36 = x^2 - 8x + 16$$

$$\frac{\cancel{12}y}{\cancel{12}} = \frac{x^2 - 8x - 20}{12}$$

$$y = \frac{x^2 - 8x - 20}{12}$$

Focus: $(4, 0)$

$(-8, 0)$

Score 0: The student stated a partially correct answer that was obtained by an incorrect procedure.

#31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

Measured CCLS Cluster: A-SSE.A

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.A because the student is asked to use the structure of an expression to identify ways to rewrite it, while proving that two expressions equal each other independently. Additionally, the item requires the student to employ Mathematical Practice 7 by asking the student to look closely to discern a pattern or structure. Students can see complicated things, such as algebraic expressions as single objects or as being composed of several objects.

Rationale: This question requires a student to prove two expressions are equal. By noticing the structure of the first expression, a student could obtain the following proof:

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

Compare with item 19, which also assesses A-SSE.A.

Sample student responses and scores appear on the following pages.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} \overset{?}{=} 1 + \frac{1}{x^3+8}$$

$$= (1)\left(\frac{x^3+8}{x^3+8}\right) + \frac{1}{x^3+8}$$

$$= \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\cancel{\frac{x^3+9}{x^3+8} = \frac{(x^3+8)}{(x^3+8)1} + \frac{1}{x^3+8}}$$

$$x^3+9 = \cancel{x^3}+8+1$$
$$-x^3 \quad\quad -x^3$$

$$9 = 8+1$$

$$\boxed{9=9}$$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

let $x=2$

$$\frac{2^3+9}{2^3+8} = \frac{8+9}{8+8} = \frac{17}{16}$$

$$1+\frac{1}{2^3+8} = 1+\frac{1}{8+8} = \frac{16}{16}+\frac{1}{16}$$

$$= \frac{17}{16}$$

$$\frac{17}{16} = \frac{17}{16}$$

Score 0: The student used an incorrect procedure by substituting a single value in for x.

#32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

Measured CCLS Cluster: F-LE.A

Commentary: The question measures the knowledge and skills described by the standards within F-LE.A because the student is required to construct an exponential model to approximate the annual growth rate. The item requires the student to employ Mathematical Practice 4 because the student has to identify important qualities and create an equation.

Rationale: This question asks students to approximate an annual growth rate, which can be accomplished with an exponential model as shown below:

$$135000 = 100000x^5$$
$$1.35 = x^5$$
$$1.0618 = x$$

$$1.0618 - 1 = .0618$$
$$6.18 \rightarrow 6\%$$

Sample student responses and scores appear on the following pages.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$$\frac{135{,}000}{100{,}000} = \frac{100{,}000(1+x)^5}{100{,}000}$$

$$1.35 = (1+x)^5$$

$$\sqrt[5]{1.35} = \sqrt[5]{(1+x)^5}$$

$$1.0618587 59 = 1 + x$$
$$-1 \qquad\qquad -1$$

$$.061858759 = x$$

$$x = \boxed{6\%}$$

Score 2: The student gave a complete and correct response.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$$A = A_0 e^{k(t-t_0)} + B_0$$

$$A_t = A_0 (1+r)^t$$

$$135{,}000 = 100{,}000(1+r)^5$$

$$\frac{27}{20} = (1+r)^5$$

$$\sqrt[5]{\frac{27}{20}} = 1+r$$

Score 1: The student wrote an incomplete solution.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$135000 = 100000(1+r)^5$

$35000 = (1+r)^5$

$1.54 = 5\log(1+r)$

$.31 = \log(1+r)$

$10^{.31} = 10^{\log(1+r)}$

$2.042 = 1+r$

$1.042 = r$

Score 0: The student made an error by subtracting 100,000 and did not state a percentage.

#33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

Measured CCLS Cluster: A-REI.C

Commentary: The question measures the knowledge and skills described by the standards within A-REI.C because the student is required to solve a system consisting of a linear equation and a quadratic equation in two variables algebraically.

Rationale: The question requires students to solve a system consisting of a linear equation and a quadratic equation. Below, the linear equation is solved for y and substituted into the quadratic equation.

$$2x + 2y = 10$$
$$y = -x + 5$$

$$(x - 3)^2 + (-x + 5 + 2)^2 = 16$$
$$x^2 - 6x + 9 + x^2 - 14x + 49 = 16$$
$$2x^2 - 20x + 42 = 0$$
$$x^2 - 10x + 21 = 0$$
$$(x - 7)(x - 3) = 0$$
$$x = 7 \text{ or } x = 3$$

Substitute the x values into the linear equation to solve for the y-values of each solutions.

$$(7, -2), (3, 2)$$

Sample student responses and scores appear on the following pages.

Question 33

33 Solve the system of equations shown below algebraically.

$$\begin{cases}(x-3)^2+(y+2)^2=16 & ① \\ 2x+2y=10 & ②\end{cases}$$

② $\div$ $2x+2y=10$

$2y=10-2x$

$y=5-x$ ③

put ③ to ① : $(x-3)^2+(5-x+2)^2=16$

$x^2-6x+9+49-14x+x^2=16$

$2x^2-20x+58=16$

$2x^2-20x+42=0$

$x^2-10x+21=0$

$\begin{matrix}1 & -3\\1 & -7\end{matrix}$

$(x-3)(x-7)=0$

$x_1=3 \quad x_2=7$

$y_1=2 \quad y_2=-2$

$2\times3+2y=16$

$2y=4$

$y=2$

$2\times7+2y=10$

$2y=-4$

$y=-2$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2(x+y)=10$

$x+y=5$

$y=5-x$

$x=5-y$

~~$y=5-x$~~

~~$y=5-10$~~

~~$y=-5$~~

~~$(x-3)^2+(y+2)^2=16$~~

~~$(x-3)^2+(5-x+2)^2=16$~~

~~$(x-3)^2+(-x+7)^2=16$~~

~~$(x^2-6x+9)+(x^2-7x-7x+49)^2=16$~~

~~$(x^2-6x+9)+(x^2-14x+49)=16$~~

~~$2x^2-20x+58=16$~~

~~$x^2-20x+100=0$~~

~~$(x-10)^2=0$~~

~~$(x-10)(x-10)=0$~~

~~$x=10$~~

$(-x+7)^2$

$(-x+7)(-x+7)$

$(5-y-3)^2$

$(-y+2)^2+(y+2)^2=16$

$(y^2-4y+4)+(y^2+4y+4)=16$

$2y^2+8=16$

$2y^2=8$

$y^2=4$

$\boxed{y=2}$

$2x+2(2)=10$

$2x+4=10$

$2x=6$

$\boxed{x=3}$

Score 3: The student found only one correct solution of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2y = 10-2x$

$y = 5-x$

$\cancel{(x-3)^2+(5-x)+2)^2=16}$

$(x-3)^2+(y+2)^2=16$

$x^2-3x-3x+9+y^2+2y+2y+4=16$

$x^2-6x+9+y^2+4y+4=16$

$x^2-6x+y^2+4y+13=16$

$y^2=-x^2+6x-4y+3$

$(5-x)^2=-x^2+6x-4(5-x)+3$

$5-x \quad 5-x$

$\cancel{x^2 5-5x-5x+25}$

$x^2-10x+25=-x^2+6x-20+4x+3$

$2x^2+45=10x+3$

$2x^2-10x+42=0$

$x^2-5x+21=0$

$x=\frac{5\pm\sqrt{25-4(1)(21)}}{2}$

$x=\frac{5\pm i\sqrt{59}}{2}$

Score 2: The student made a transcription error by losing a $-10x$, and did not find y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2x=10-2y$

$x=\frac{10-2y}{2}$

$x=5-y$

$((5-y)-3)^2+(y+2)^2=16$

$(5-y)(5-y)-9+(y+2)^2=16$

$25\cancel{-5y}\cancel{-5y}+\cancel{y^2}-9+\cancel{y^2}\cancel{+4y}+4=16$

$2y^2-6y+20=16$

$2y^2-6y+4=0$

$2(y^2-3y+2)=0$

$2(y-2)(y-1)=0$

$y-2=0$

$y=2$

$y-1=0$

$y=1$

$2x+2(2)=10$

$2x+4=10$

$2x=6$

$x=3$

$2x+2=10$

$2x=8$

$x=4$

Check:

$(3-3)^2+(2+2)^2=16$

$(0^2)+(4)^2=16$

$16=16$ ✓

$(3-3)^2+(1+2)^2=16$

$(0^2)+(3^2)=16$

$9\neq16$

$(4-3)^2+(2+2)^2=16$

$(1)^2+(4^2)=16$

$17\neq16$

$(4-3)^2+(1+2)^2=16$

$(1)^2+(3)^2=16$

$1+9=16$

$10\neq16$

$\boxed{\text{answer}= y=2,\ x=3}$

Score 1: The student made a conceptual error squaring the first term and did not express both ordered pairs.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x + 2y = 5$
$y = 5 + x$

$(x-3)^2 + (5+x+2)^2 = 16$
$x^2 - 6x + 9 + 49 + 14x + x^2 = 16$
$2x^2 + 8x + 42 = 0$
$x^2 + 4x + 21 = 0$
$(x+7)(x+3) = 0$
$x = 7 \quad | \quad x = 3$

Score 0: The student made several errors and did not find the y-values.

#34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

Measured CCLS Cluster: A-SSE.B

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.B because the student must be able derive the formula for the sum of a finite geometric series and use the formula to solve a problem. This item requires students to employ Mathematical Practice 4 because the student must be able to identify important quantities in a practical situation and map those relationships using a formula.

Rationale: This question asks the student to write a geometric series formula and evaluate at $n = 15$ years.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04}$$

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = 660{,}778.39$$

Compare with item 21, which also assesses A-SSE.B.

Sample student responses and scores appear on the following pages.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = \frac{33{,}000 - 33{,}000(1.80)}{-.04}$$

$$S_{15} = \frac{-26{,}431.14}{-.04}$$

$$S_{15} = 660{,}778.39$$

Score 4: The student gave a complete and correct response.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$\boxed{S_n = \$ {-26432.18}}$$

Score 3: The student failed to use parentheses when entering the expression into the calculator.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$Sn = \frac{33{,}000 - 33{,}000(0.04)^n}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(0.04)^{15}}{1 - 0.04}$$

$$\frac{33000}{0.96}$$

$$34375$$

Score 2: The student made a conceptual error interpreting the 4% increase.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000 - (1.04)^N}{1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000 - (1.04)^{15}}{1.04}$$

$$= \frac{33000 + 1.80094}{1.04}$$

$$= \$31732.50$$

Score 1: The student made a computational error in the second part, having received no credit for the first part.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$Sn = \frac{a_1 - a_1 r^n}{1 - r} \qquad S_n = \frac{a_1 - a_1^{(0.04)(t)}}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33{,}000^{(0.04)(15)}}{.96} =$$

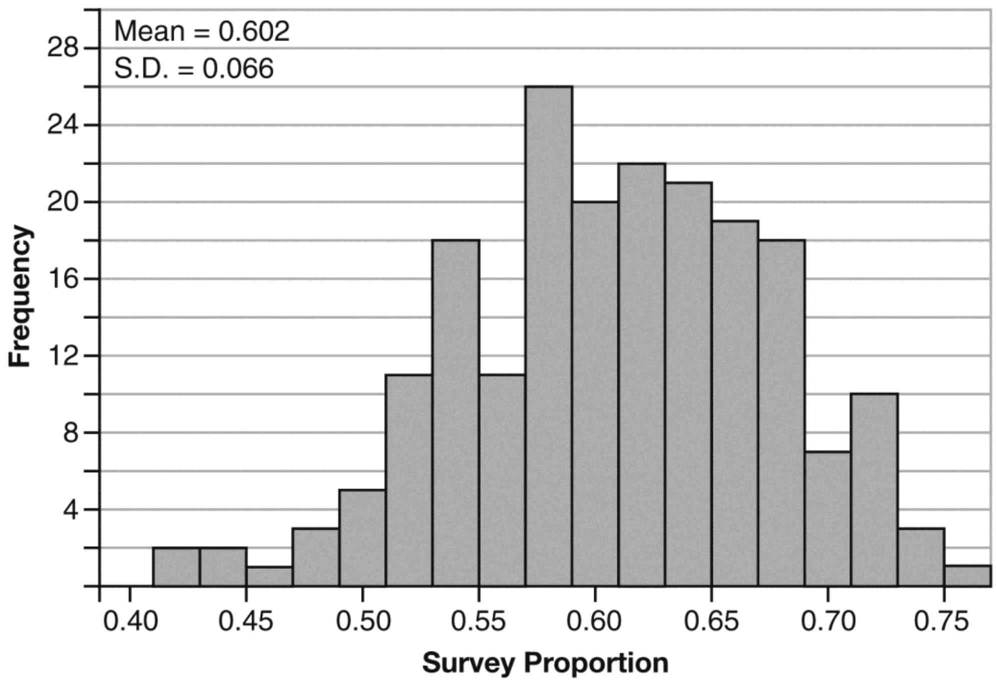
$$\boxed{S_n \approx \$535}$$

Score 0: The student made multiple errors.

#35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

Measured CCLS Cluster: S-IC.B

Commentary: The question measures the knowledge and skills described by the standards within S-IC.B because students use data from a sample survey to estimate a population mean or proportion. Students are given a graph of data, with the mean and standard deviation already computed, and asked to determine a plausible interval containing a middle 95% of the data. Additionally, the item requires the student to employ Mathematical Practice 6 because the student examines claims and makes explicit use of definitions.

Rationale: This question requires students to determine a 95% interval given the results of a normal simulation in a graph and related mean and standard deviations. Students are also asked to explain the statistical evidence of a 50%-50% split by referencing the interval that was calculated. About 95% of the simulated proportions lie within two standard deviations of the mean. This produces the interval 0.60 ± 0.13 or (0.47, 0.73). The reason the prom committee may be concerned is because 0.50 falls within the interval, making a 50%-50% split vote a possibility.

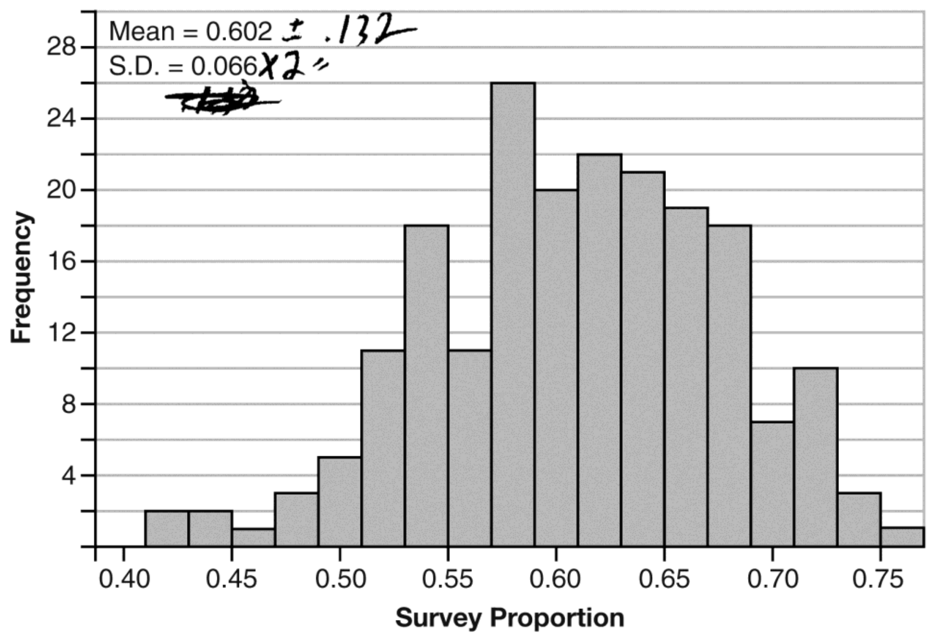
Compare with item 26, which also assesses S-IC.B.

Sample student responses and scores appear on the following pages.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

0.47 - 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

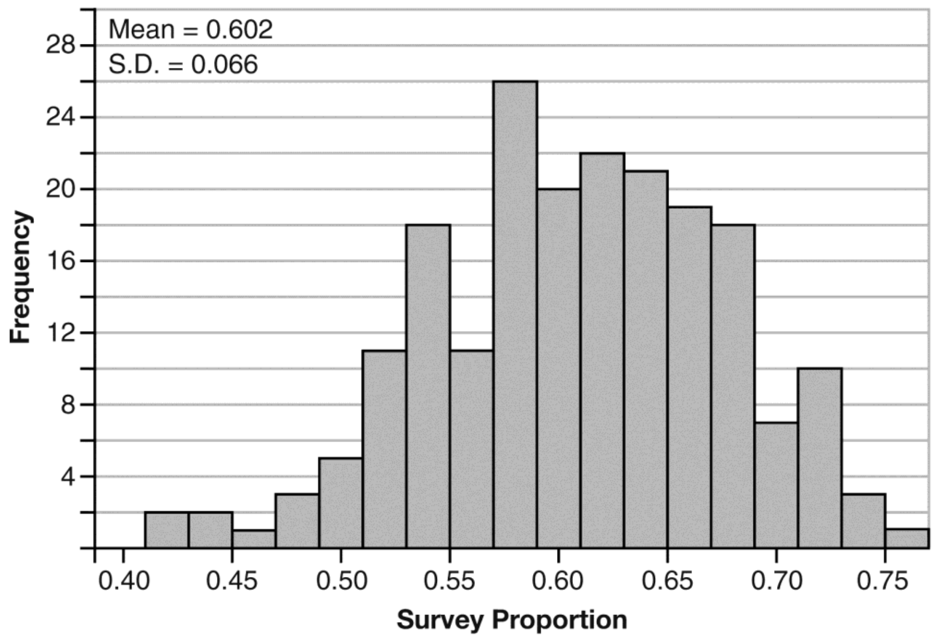
.50 is within this interval so its possible to get a split vote.

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 + 2 \cdot 0.066 = 0.734$

$0.602 - 2 \cdot 0.066 = 0.47$

0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

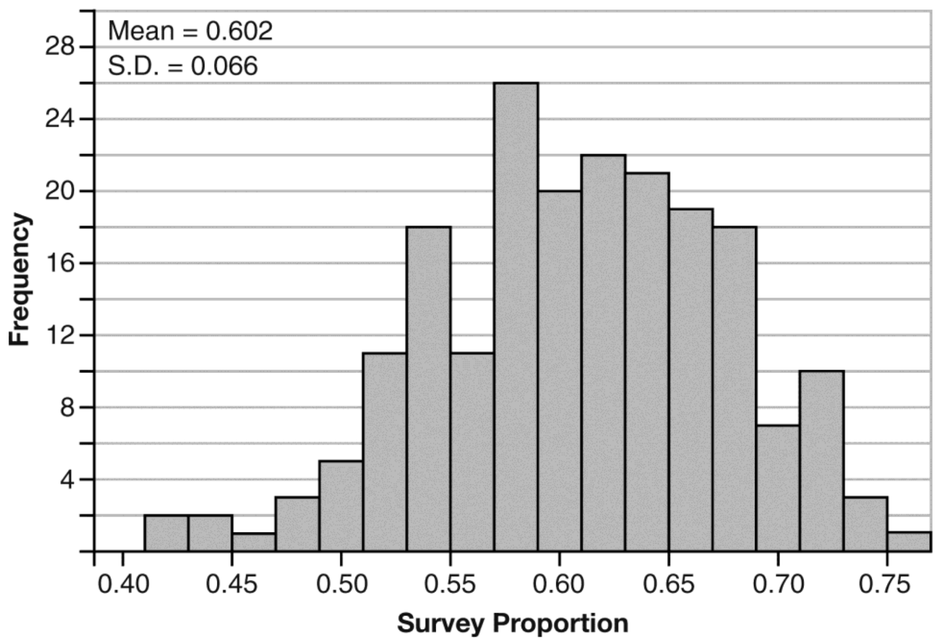
It is not a concern since .50 falls within the interval.

Score 3: The student determined a correct interval, but provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 \pm 2 \cdot 0.066$ → .47, .734

.47 to .73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

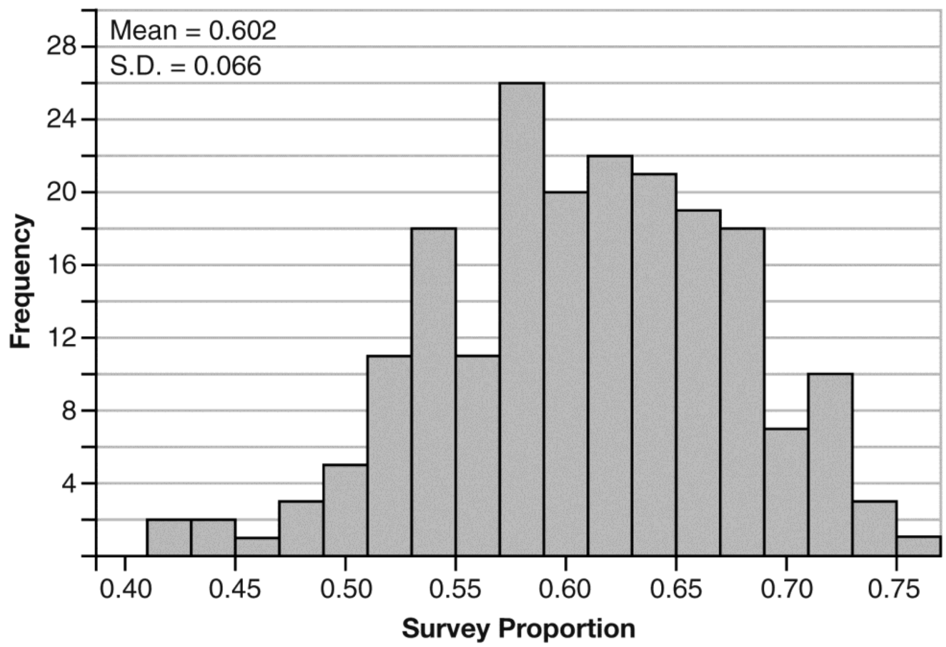
The graph shows 0.50

Score 2: The student gave no statistical explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$$\begin{array}{r} .066 \\ +2 \cdot .602 \\ \hline 1.27 \end{array} \qquad \begin{array}{r} 0.66 \\ -2 \cdot .602 \\ \hline -1.138 \end{array}$$

-1.138 to 1.27

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

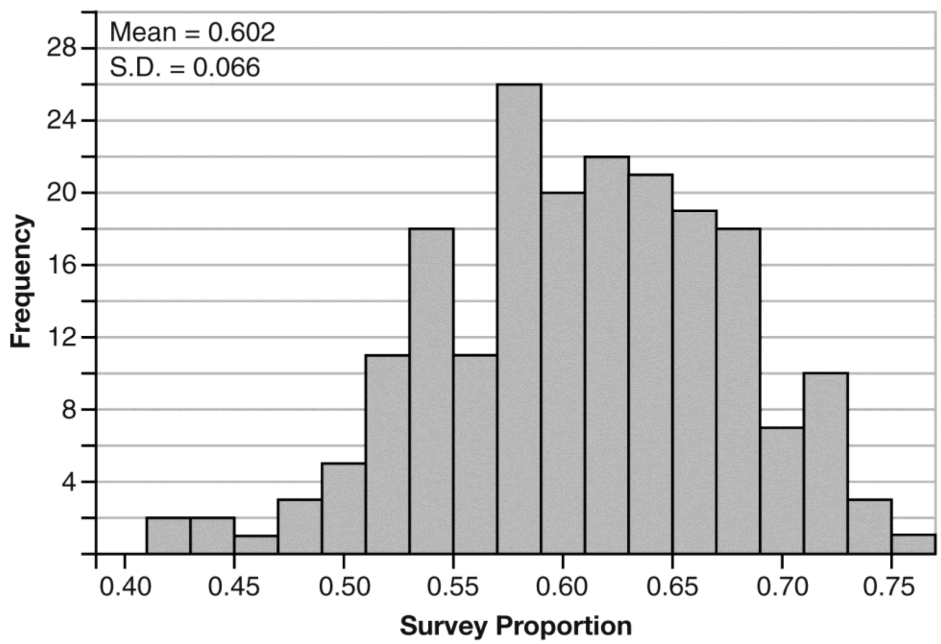
Since 60% of the students voted and 50% is close to 60%, they could be concerned that 50% is possible because voting can vary.

Score 1: The student used the standard deviation as the center and rounded incorrectly. The student gave an incomplete explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$.602 + .132 = .734 \quad (.602, .734)$

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.5 is in the interval.

Score 0: The student made multiple conceptual and computational errors.

#36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$$g(x) = 4x^3 - 5x^2 + 3$$

Measured CCLS Cluster: F-IF.C

Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because the student must analyze two functions, one of which is represented numerically in a table and the other is represented algebraically. The item requires the student to employ Mathematical Practice 2 and Mathematical Practice 3 because the student has to compare average rates of change and draw a correct conclusion.

Rationale: This question asks students to determine which function has a greater average rate of change over the interval $[-2,4]$. The average rate of change for f is 13.125 and the average rate of change for g is 38. A student could also argue that the change in y values over the same interval $[-2,4]$ is greater for g, which implies a greater average rate of change.

Compare with item 4, which also assesses F-IF.C.

Sample student responses and scores appear on the following pages.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -32-20+3$
$= -49$

$g(4) = 179$

$\frac{80-1.25}{4-(-2)} = \frac{78.75}{6}$

$= 13.125$

$= \frac{105}{8}$

$\frac{179-(-49)}{4-(-2)}$

$= \frac{228}{6}$

$= 38$

g(x) b/c its rate of change over [-2, 4] is 38, and f(x)'s rate of change is 13.125.

Score 4: The student gave a complete and correct response.

Question 36

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

X x	Y f(x)
−4	0.3125
−3	0.625
x_1 −2	Y_1 1.25
−1	2.5
0	5
1	10
2	20
3	40
x_2 4	Y_2 80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

X	Y
−2	−49
−1	−26
0	3
1	2
2	15
3	66
4	179

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{80 - 1.25}{4 - (-2)}$$

$$\frac{78.75}{7} = 11.25 \text{ rate}$$

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{179 - (-49)}{4 - (-2)}$$

$$\frac{228}{7} = 32.57142857 \text{ rate}$$

$g(x) = 4x^3 - 5x^2 + 3$ has the greater average rate of change on the interval [-2, 4]

Score 3: The student made a computational error when calculating the denominators.

Question 36

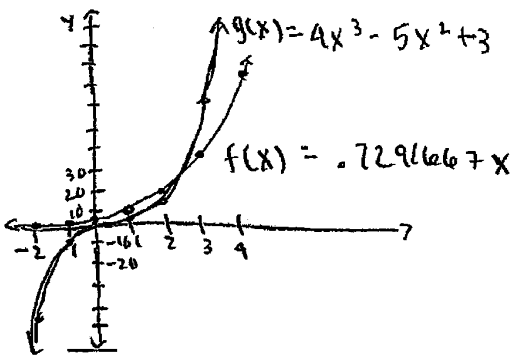
36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

x	g(x)
−2	−49
−1	−6
0	3
1	2
2	15
3	66
4	179

Function $g(x) = 4x^3 - 5x^2 + 3$ has a greater average rate of change in the interval $[-2, 4]$. This is because when both functions are graphed, g(x)'s slope is steeper than f(x)'s.



$f(x) = .7291667x^3 + 1.27976x^2 + 1.80059x + 5$

Score 2: The student made a conceptual error by creating an appropriate model for $f(x)$, but wrote an appropriate explanation for that model.

Question 36

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

78.75

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -8(4) - 5(4) + 3$

$= -32 - 20 + 3$

$= -52 + 3$

$= -49$

$g(4) = 4(64) - 5(16) + 3$

$= 252 - 80 + 3$

$= 169$

$\boxed{169 > 78.75}$

$g(x) = 4x^3 - 5x^2 + 3$ has a greater change.

Score 1: The student made an error finding the average rates of change by not dividing by Δx, and made one computational error.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$$g(x) = 4x^3 - 5x^2 + 3$$

↑ thes function has a greater ~~av~~ average change on the interval [-2,4] because this function is a geometric sequence which doubles its x-values.

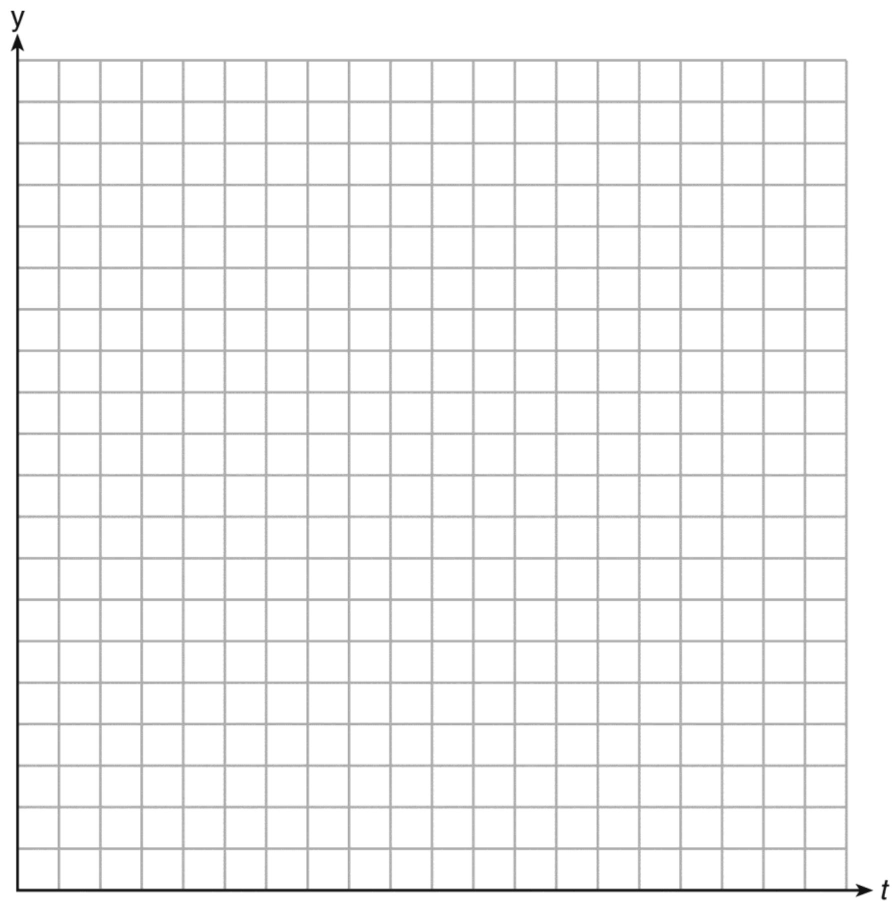
Score 0: The student did not calculate an average rate of change and wrote an irrelevant explanation.

#37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

Graph each function on the set of axes below.



To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

Measured CCLS Cluster: A-REI.D

Commentary: The question measures the knowledge and skills described by the standards within A-REI.D because the student must represent, graph, and solve exponential equations. Knowledge and skills within clusters A-CED.A and F-LE.A are also assessed as students must create exponential equations and possibly solve an exponential equation algebraically. The item requires the student to employ Mathematical Practice 4 and 5 because the student uses a graphing calculator to graph exponential functions and solve a problem arising in everyday life, society, or the workplace.

Rationale: In this question, the student creates functions $A(t) = 800e^{-0.347t}$ and $B(t) = 400e^{-0.231t}$ by substituting given values into the general form of a given exponential equation. The student then graphs these functions using an appropriate scale. Using the graph, the student determines where the functions intersect since the t-value of this intersection point represents the nearest hour when the amount of the drug remaining in patient *B* begins to exceed that of patient *A*. Finally, the student solves an exponential equation graphically or using logarithms to determine when it is safe for patient *A* to take another dosage.

Sample student responses and scores appear on the following pages.

Question 37

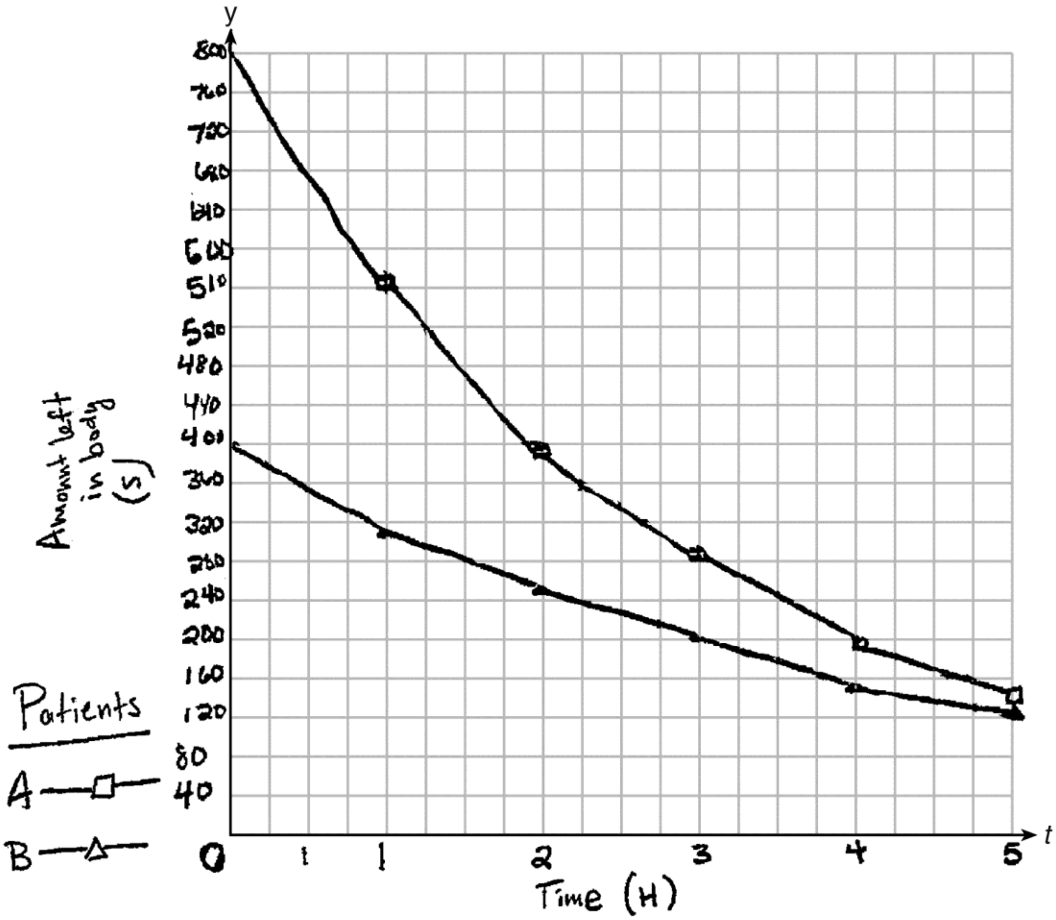
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

P_A $A(t) = 800\text{mg}(e)^{-.347t}$

P_B $B(t) = 400\text{mg}(e)^{-.231t}$

Graph each function on the set of axes below.



Score 6: The student gave a complete and correct response.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 6 because when the equation is solved the amount left in B is 100 while A has about 99.7.

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$800(.15) = 120\text{mg}$

$H_5 = 141$

$\frac{120}{800} = \frac{800\text{mg}}{800}(e)^{-.347(t)}$

$.15 = (e)^{-.347(t)}$

$\log_e .15 = \log_e e^{-.347(t)}$

$\frac{-1.897}{-.347} = \frac{-.347(t)}{-.347}$

$5.5\text{hrs} = t$

Question 37

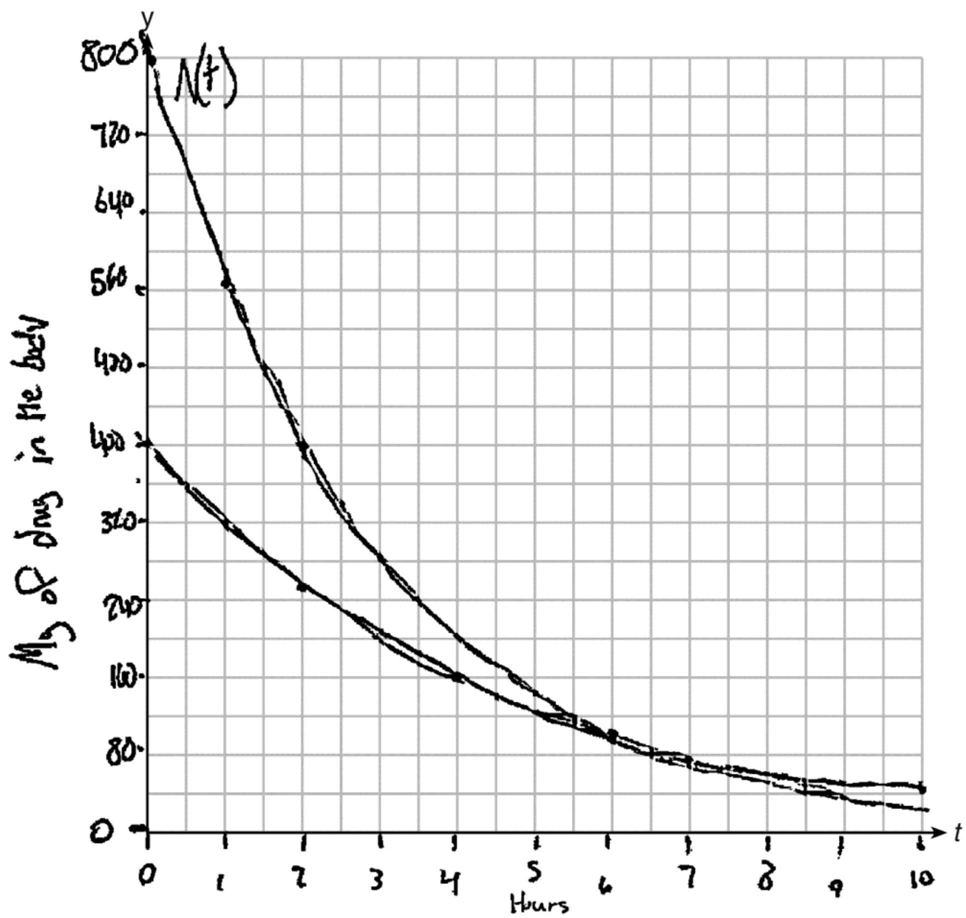
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$N(t) = 800(e)^{-.347t}$

$400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 5: The student did not indicate which function models which patient.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 6

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$\frac{120 = 800(e)^{-.347t}}{800}$$

$$.15 = e^{-.347t}$$

$$\log_e .15 = -.347t$$

$$\frac{\ln .15 = -.347t}{-.347}$$

$$5.5 = t$$

5.5 hours

Question 37

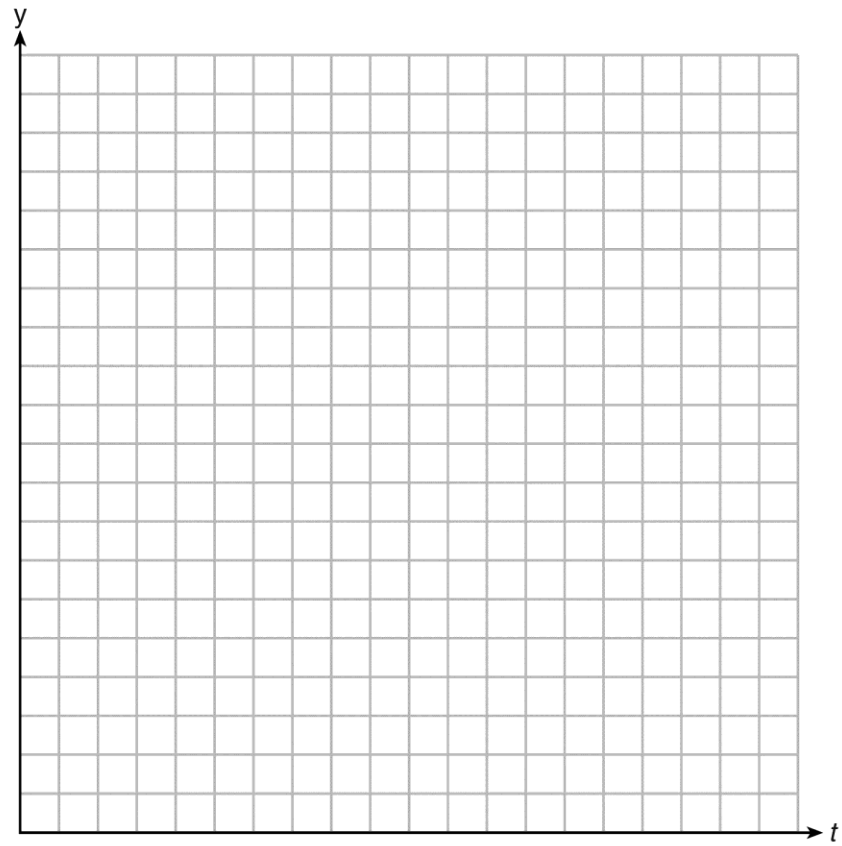
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800mg(e)^{-.347t}$

$B(t) = 400mg(e)^{-.231t}$

Graph each function on the set of axes below.



Score 4: The student did not graph either function.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

After 6 hours. I see this after graphing both functions on my calculator and looking at the table. I could then see that at 6 hours Patient a would have 99.74 mg of drug, while patient b would have 100.03. This is probably because, despite starting with more drug, Patient A's decay rate is also greater.

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$\frac{15}{100} = \frac{x}{800}$$

$$\frac{15 \cdot 800}{100} = \frac{\cancel{100}x}{\cancel{100}}$$

$$120 = x$$

120 mg left

Using my graphing calculator...

I know that Patient A would have to wait approximately 5.5 hours or 5 hours and 30 minutes.

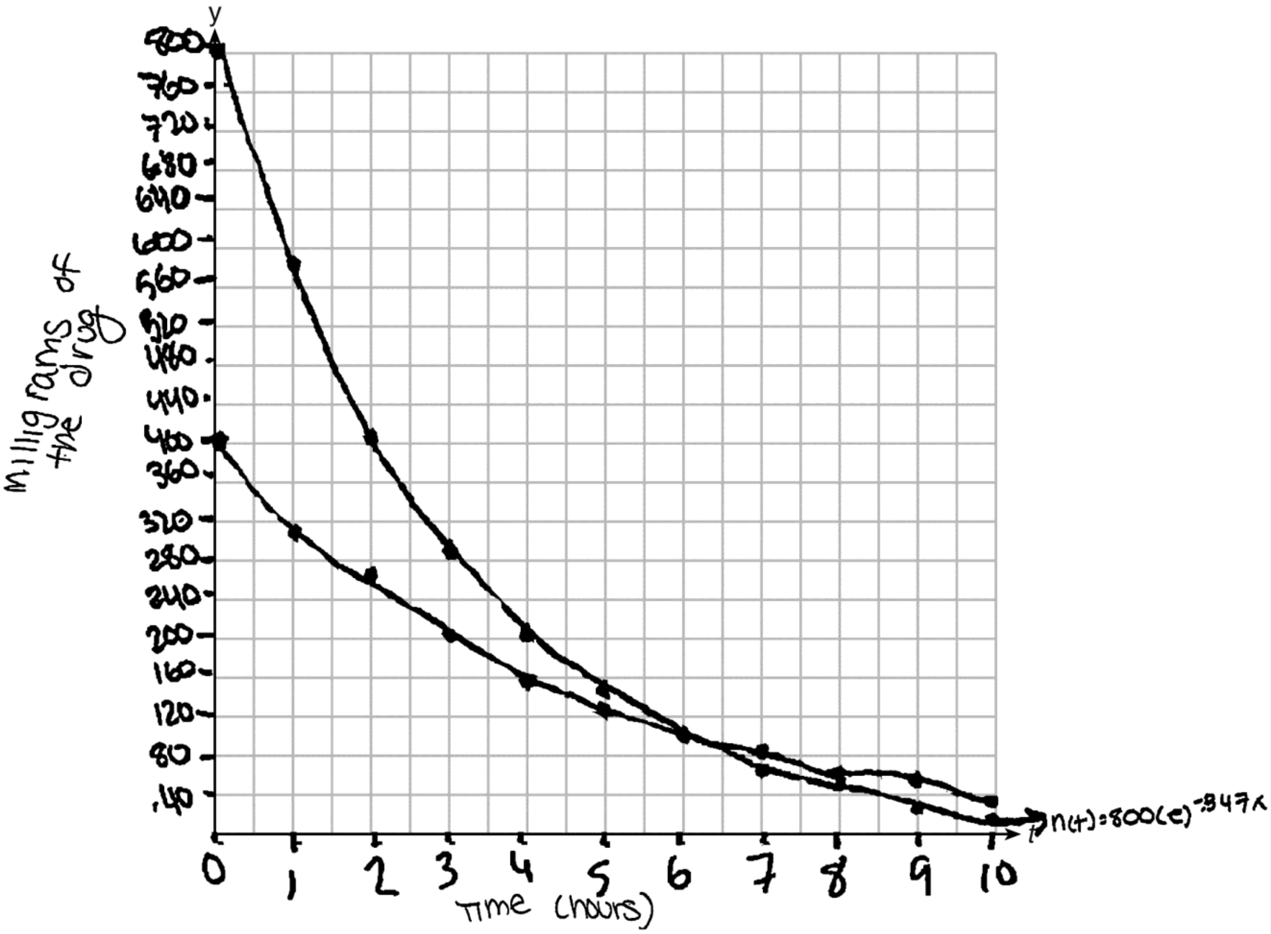
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347x}$

$400(e)^{-.281x}$

Graph each function on the set of axes below.



Score 3: The student drew a correct graph and gave a correct answer of 5.5 hours.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 7

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

5.5 hours

Question 37

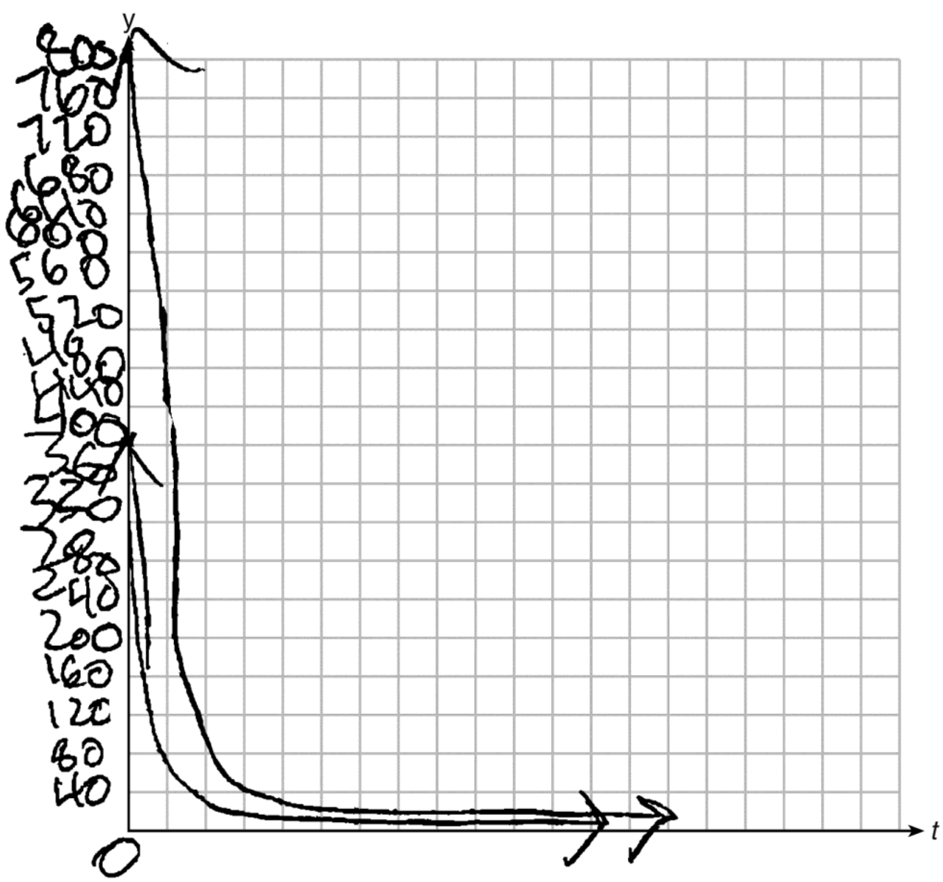
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-0.347t}$

$400(e)^{-0.231t}$

Graph each function on the set of axes below.



Score 2: The student correctly identified 6 hours and 5.5 hours.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

6 hours because that is what N(t) = when you plug 6 in for t

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

5.5 hours

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

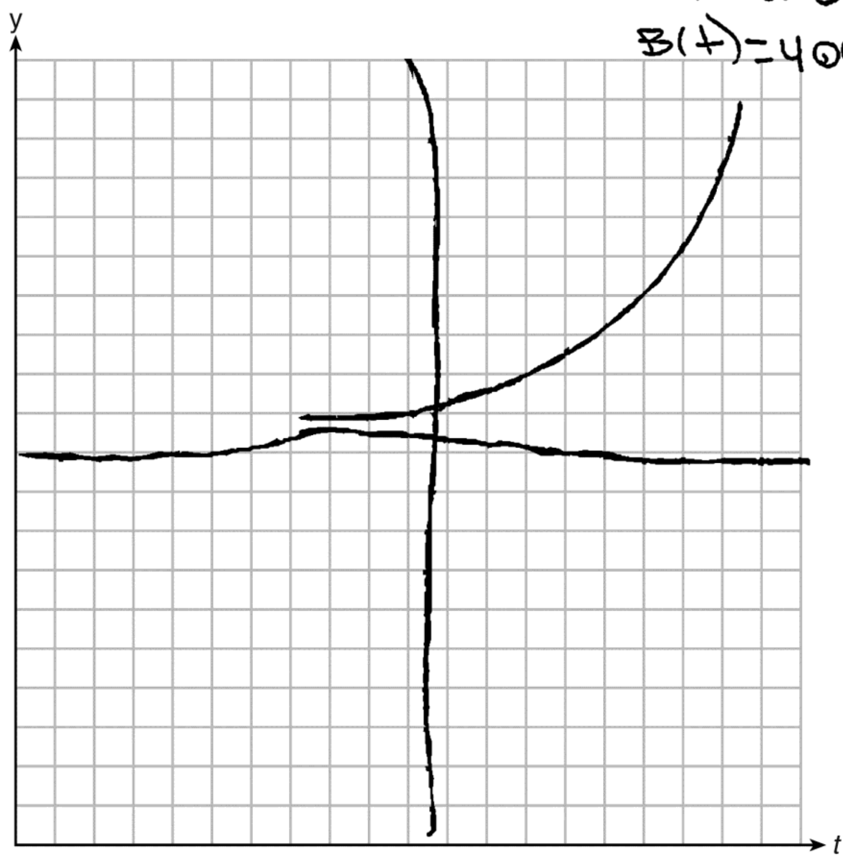
Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

~~800~~ A

$A(t)=800(e)^{-.347t}$

$B(t)=400(e)^{.231t}$

Graph each function on the set of axes below.



Score 1: The student created and labeled correct functions.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$.15 = 800(e)^{-.347t}$$

$$.15 = 800(e)^{-} .347 \ln t$$

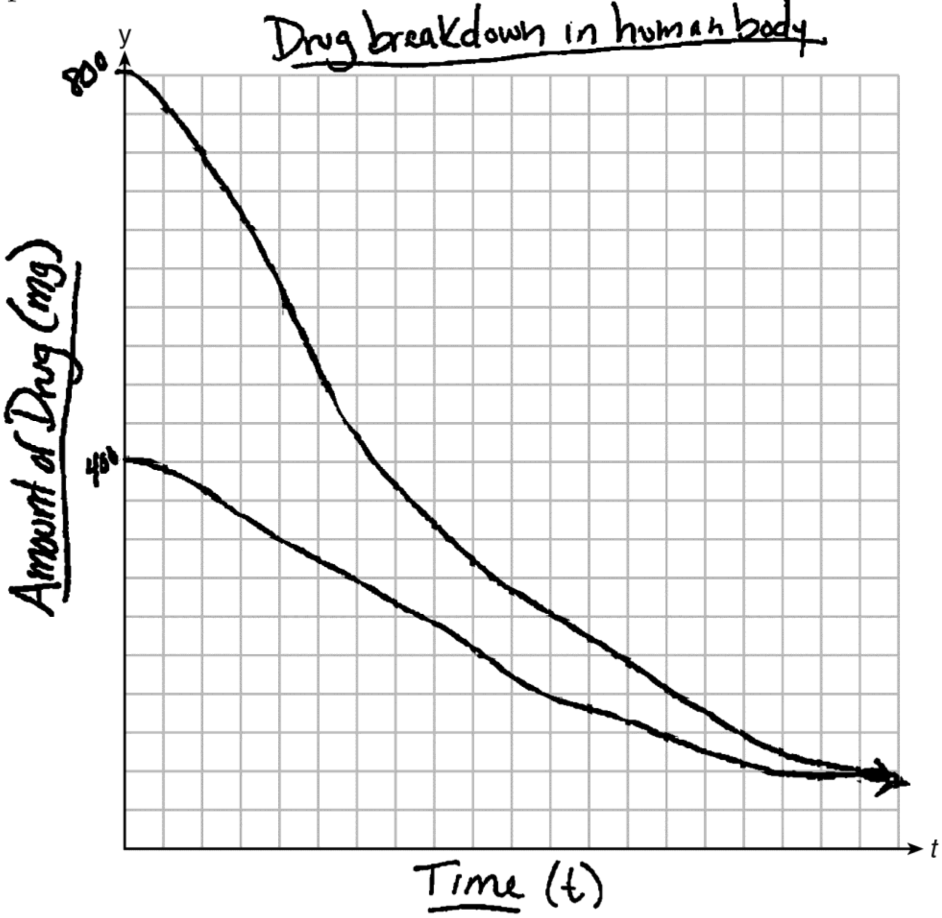
Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800\,(e)^{-.347t}$
400

Graph each function on the set of axes below.



Score 0: The student did not complete enough correct work in any part to receive credit.

Question 37

To the *nearest hour,* t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 5

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient A will have to wait to take another 800 milligram dose of the drug.

$$120 = 800(e)^{-.347t}$$