ALGEBRA
II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Friday, June 21, 2019 — 1:15 to 4:15 p.m., only

Student Name: ____

School Name: ____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 A sociologist reviews randomly selected surveillance videos from a public park over a period of several years and records the amount of time people spent on a smartphone. The statistical procedure the sociologist used is called

(1) a census
(2) an experiment
(3) an observational study
(4) a sample survey

2 Which statement(s) are true for all real numbers?

I $(x - y)^2 = x^2 + y^2$
II $(x + y)^3 = x^3 + 3xy + y^3$

(1) I, only
(2) II, only
(3) I and II
(4) neither I nor II

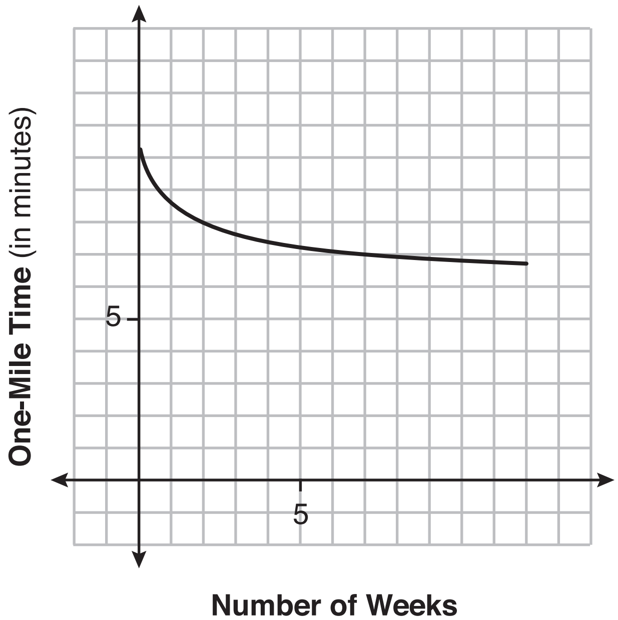
3 What is the solution set of the following system of equations?

$$y = 3x + 6$$
$$y = (x + 4)^2 - 10$$

(1) $\{(-5,-9)\}$
(2) $\{(5,21)\}$
(3) $\{(0,6),(-5,-9)\}$
(4) $\{(0,6),(5,21)\}$

Use this space for computations.

4 Irma initially ran one mile in over ten minutes. She then began a training program to reduce her one-mile time. She recorded her one-mile time once a week for twelve consecutive weeks, as modeled in the graph below.



Which statement regarding Irma's one-mile training program is correct?

(1) Her one-mile speed increased as the number of weeks increased.

(2) Her one-mile speed decreased as the number of weeks increased.

(3) If the trend continues, she will run under a six-minute mile by week thirteen.

(4) She reduced her one-mile time the most between weeks ten and twelve.

5 A 7-year lease for office space states that the annual rent is $85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?

(1) $42,809.63

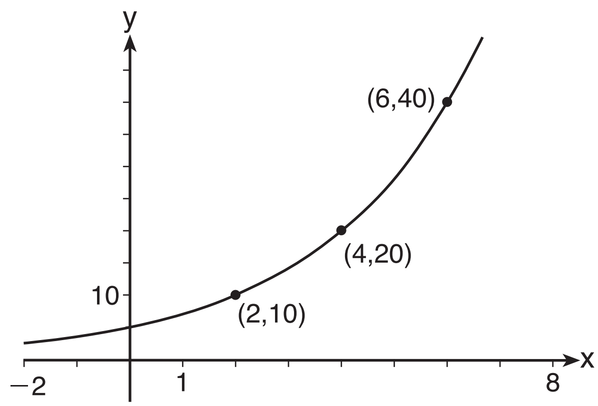
(2) $90,425.53

(3) $595,000.00

(4) $713,476.20

Use this space for computations.

6 The graph of $y = f(x)$ is shown below.



Which expression defines $f(x)$?

(1) $2x$

(2) $5(2^x)$

(3) $5\left(2^{\frac{x}{2}}\right)$

(4) $5(2^{2x})$

7 Given $P(x) = x^3 - 3x^2 - 2x + 4$, which statement is true?

(1) $(x - 1)$ is a factor because $P(-1) = 2$.

(2) $(x + 1)$ is a factor because $P(-1) = 2$.

(3) $(x + 1)$ is a factor because $P(1) = 0$.

(4) $(x - 1)$ is a factor because $P(1) = 0$.

8 For $x \geq 0$, which equation is *false*?

(1) $\left(x^{\frac{3}{2}}\right)^2 = \sqrt[4]{x^3}$

(2) $(x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$

(3) $\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} = \sqrt[4]{x^3}$

(4) $\left(x^{\frac{2}{3}}\right)^2 = \sqrt[3]{x^4}$

Use this space for computations.

9 What is the inverse of the function $y = 4x + 5$?

(1) $x = \frac{1}{4}y - \frac{5}{4}$

(2) $y = \frac{1}{4}x - \frac{5}{4}$

(3) $y = 4x - 5$

(4) $y = \frac{1}{4x + 5}$

10 Which situation could be modeled using a geometric sequence?

(1) A cell phone company charges \$30.00 per month for 2 gigabytes of data and \$12.50 for each additional gigabyte of data.

(2) The temperature in your car is 79°. You lower the temperature of your air conditioning by 2° every 3 minutes in order to find a comfortable temperature.

(3) David's parents have set a limit of 50 minutes per week that he may play online games during the school year. However, they will increase his time by 5% per week for the next ten weeks.

(4) Sarah has \$100.00 in her piggy bank and saves an additional \$15.00 each week.

11 The completely factored form of $n^4 - 9n^2 + 4n^3 - 36n - 12n^2 + 108$ is

(1) $(n^2 - 9)(n + 6)(n - 2)$

(2) $(n + 3)(n - 3)(n + 6)(n - 2)$

(3) $(n - 3)(n - 3)(n + 6)(n - 2)$

(4) $(n + 3)(n - 3)(n - 6)(n + 2)$

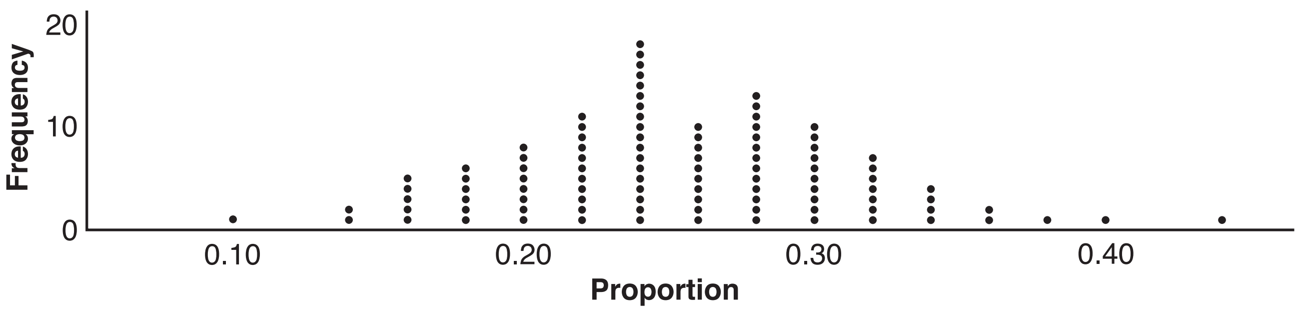
Use this space for computations.

12 What is the solution when the equation $wx^2 + w = 0$ is solved for x, where w is a positive integer?

(1) -1 (3) 6

(2) 0 (4) $\pm i$

13 A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data?

(1) (0.194, 0.314) (3) (-0.448, 0.568)

(2) (0.134, 0.374) (4) (0.254, 0.374)

Use this space for computations.

14 Selected values for the functions f and g are shown in the tables below.

x	f(x)
−3.12	−4.88
0	−6
1.23	−4.77
8.52	2.53
9.01	3.01

x	g(x)
−2.01	−1.01
0	0.58
8.52	2.53
13.11	3.01
16.52	3.29

A solution to the equation $f(x) = g(x)$ is

(1) 0
(2) 2.53
(3) 3.01
(4) 8.52

15 The expression $6 - (3x - 2i)^2$ is equivalent to

(1) $-9x^2 + 12xi + 10$
(2) $9x^2 - 12xi + 2$
(3) $-9x^2 + 10$
(4) $-9x^2 + 12xi - 4i + 6$

16 A number, minus twenty times its reciprocal, equals eight. The number is

(1) 10 or -2
(2) 10 or 2
(3) -10 or -2
(4) -10 or 2

Use this space for computations.

17 A savings account, S, has an initial value of \$50. The account grows at a 2% interest rate compounded n times per year, t, according to the function below.

$$S(t) = 50\left(1 + \frac{.02}{n}\right)^{nt}$$

Which statement about the account is correct?

(1) As the value of n increases, the amount of interest per year decreases.

(2) As the value of n increases, the value of the account approaches the function $S(t) = 50e^{0.02t}$.

(3) As the value of n decreases to one, the amount of interest per year increases.

(4) As the value of n decreases to one, the value of the account approaches the function $S(t) = 50(1 - 0.02)^t$.

18 There are 400 students in the senior class at Oak Creek High School. All of these students took the SAT. The distribution of their SAT scores is approximately normal. The number of students who scored within 2 standard deviations of the mean is approximately

(1) 75 (3) 300

(2) 95 (4) 380

19 The solution set for the equation $b = \sqrt{2b^2 - 64}$ is

(1) $\{-8\}$ (3) $\{\pm 8\}$

(2) $\{8\}$ (4) $\{\ \}$

Use this space for computations.

20 Which table best represents an exponential relationship?

(1)

x	y
1	8
2	4
3	2
4	1
5	$\frac{1}{2}$

(2)

x	y
8	0
4	1
0	2
–4	3
–8	4

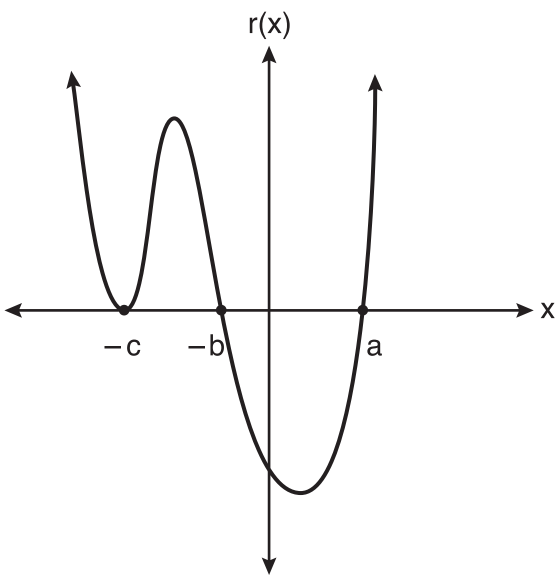
(3)

x	y
0	0
1	1
2	4
3	9
4	16

(4)

x	y
1	1
2	8
3	27
4	64
5	125

21 A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

(1) $r(x) = (x - a)(x + b)(x + c)$

(2) $r(x) = (x + a)(x - b)(x - c)^2$

(3) $r(x) = (x + a)(x - b)(x - c)$

(4) $r(x) = (x - a)(x + b)(x + c)^2$

Use this space for computations.

22 The temperature, in degrees Fahrenheit, in Times Square during a day in August can be predicted by the function $T(x) = 8\sin(0.3x - 3) + 74$, where x is the number of hours after midnight. According to this model, the predicted temperature, to the *nearest degree* Fahrenheit, at 7 P.M. is

(1) 68 (3) 77

(2) 74 (4) 81

23 Consider the system of equations below:

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= -19 \\ -x + 4y - z &= 17 \end{aligned}$$

Which number is *not* the value of any variable in the solution of the system?

(1) -1 (3) 3

(2) 2 (4) -4

24 Camryn puts \$400 into a savings account that earns 6% annually. The amount in her account can be modeled by $C(t) = 400(1.06)^t$ where t is the time in years. Which expression best approximates the amount of money in her account using a weekly growth rate?

(1) $400(1.001153846)^t$ (3) $400(1.001153846)^{52t}$

(2) $400(1.001121184)^t$ (4) $400(1.001121184)^{52t}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

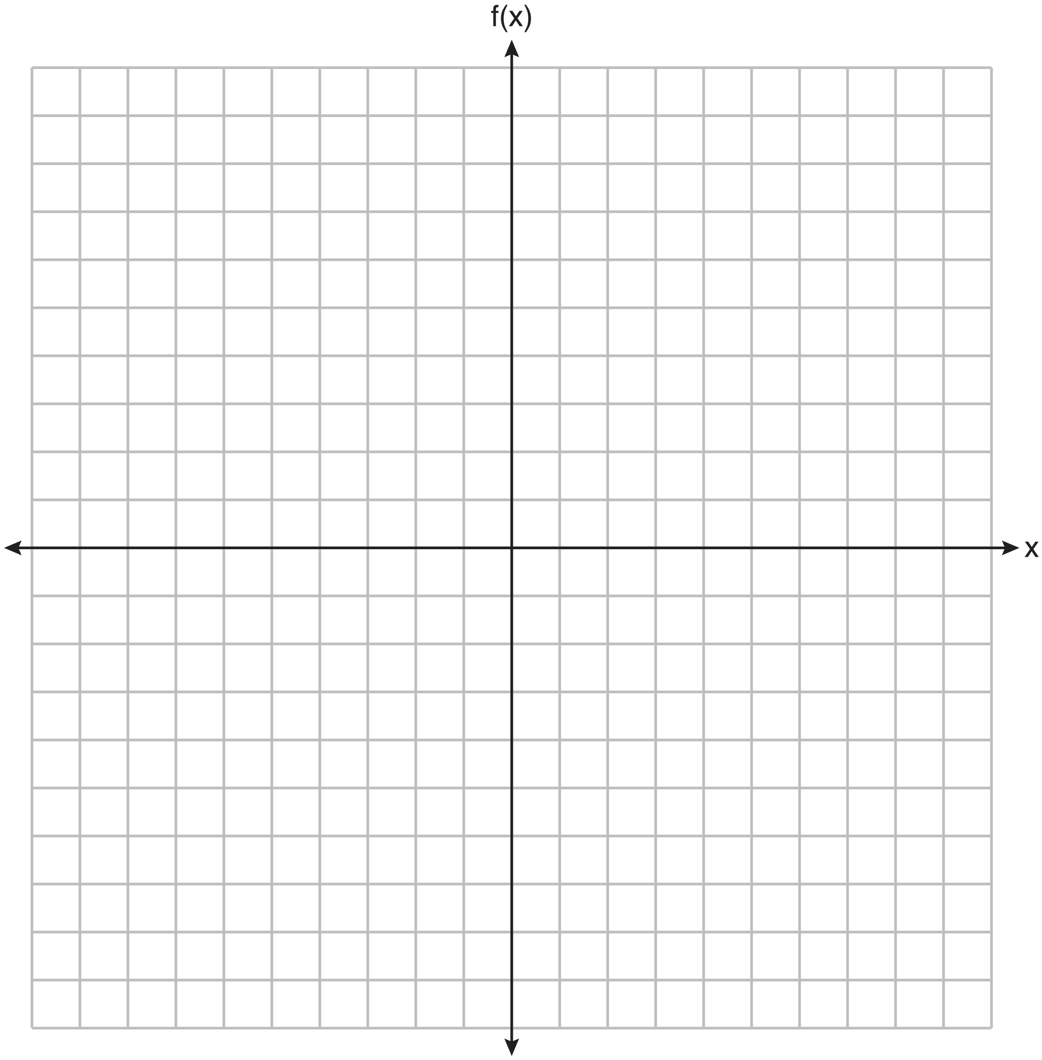
Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

Interpret what this means in the context of the problem.

26 Algebraically solve for x:

$$\frac{7}{2x} - \frac{2}{x + 1} = \frac{1}{4}$$

27 Graph $f(x) = \log_2(x + 6)$ on the set of axes below.



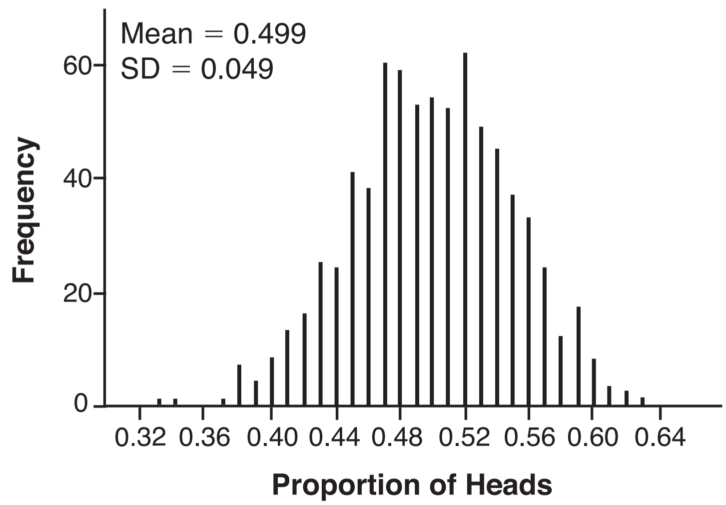
28 Given $\tan \theta = \frac{7}{24}$, and θ terminates in Quadrant III, determine the value of $\cos \theta$.

29 Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

30 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Factor completely over the set of integers: $16x^4 - 81$

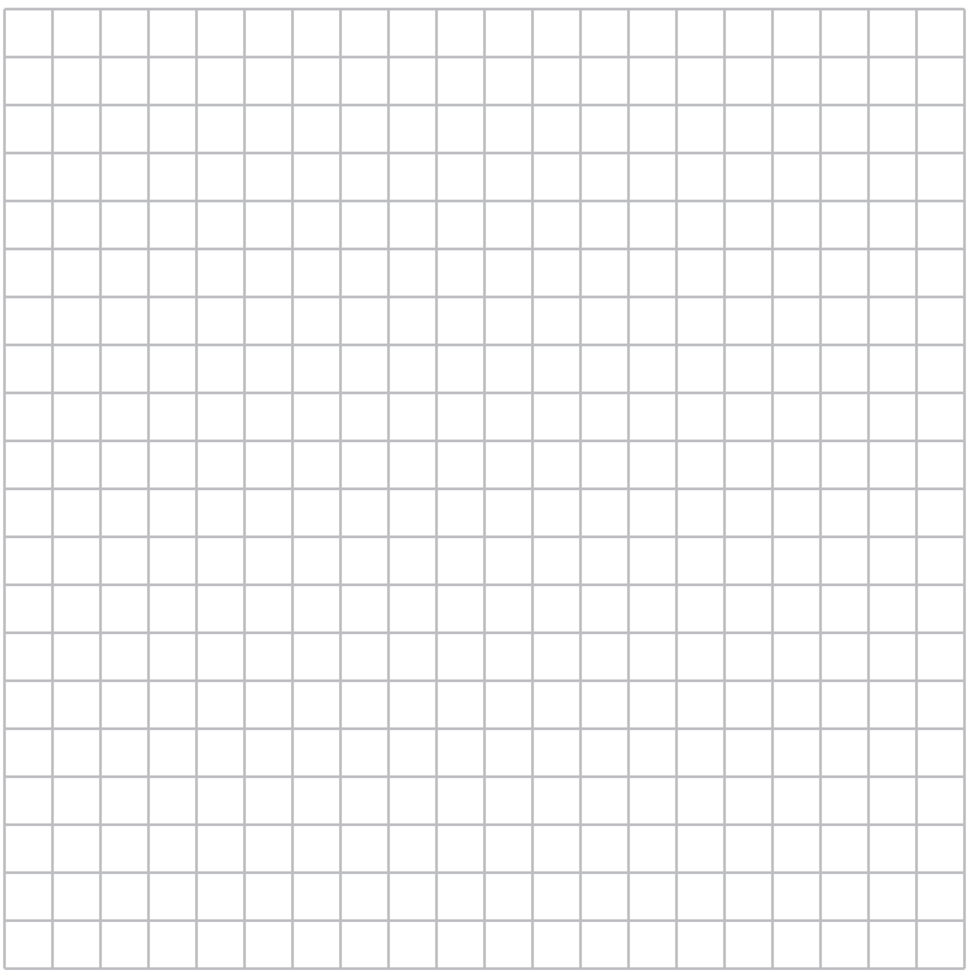
Sara graphed the polynomial $y = 16x^4 - 81$ and stated "All the roots of $y = 16x^4 - 81$ are real." Is Sara correct? Explain your reasoning.

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after t years.

Determine algebraically, to the *nearest year*, how long it will take for $\frac{1}{10}$ of this substance to remain.

35 Determine an equation for the parabola with focus $(4, -1)$ and directrix $y = -5$. (Use of the grid below is optional.)



36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire.

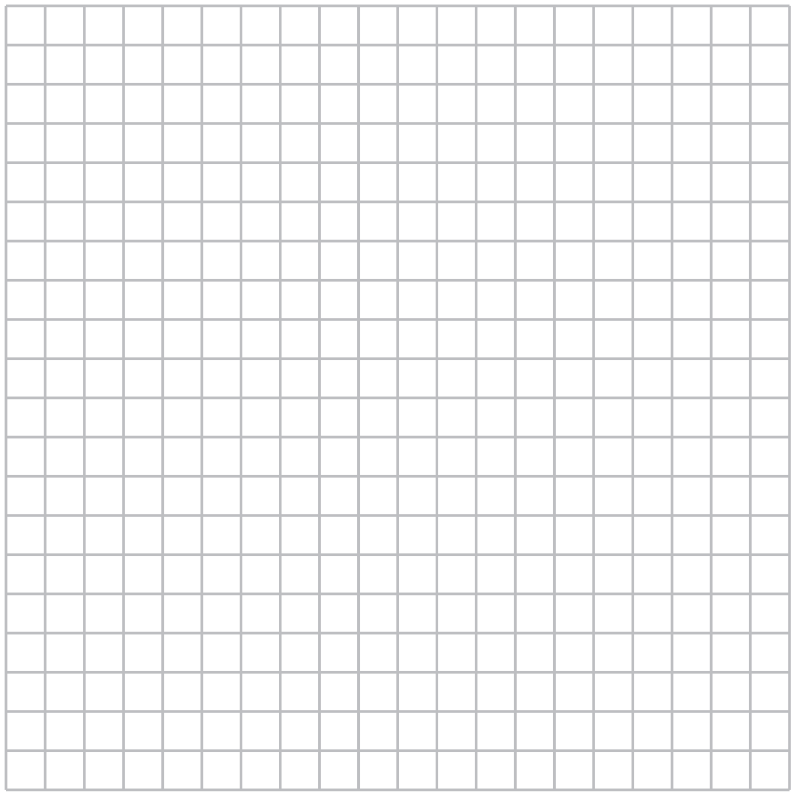
Determine the period of $f(t)$.

Interpret what the period represents in this context.

Question 37 is continued on the next page.

Question 37 continued

On the grid below, graph *at least one* cycle of $f(t)$ that includes the y-intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

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High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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