ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II**, **III**, and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The population of Austin, Texas from 1850 to 2010 is summarized in the table below.

Year	1850	1870	1890	1910	1930	1950	1970	1990	2010
Population	629	4428	14,575	29,860	53,120	132,459	251,808	494,290	790,390

Over which period of time was the average rate of change in population the greatest?

(1) 1850 to 1910

(2) 1990 to 2010

(3) 1950 to 1970

(4) 1890 to 1970

2 Which expression is *not* equivalent to $36x^6 - 25y^4$?

(1) $6^2(x^3)^2 - 5^2(y^2)^2$

(3) $(6x^6 - 5y^4)(6x^6 + 5y^4)$

(2) $(6x^3 - 5y^2)(6x^3 + 5y^2)$

(4) $(3 \cdot 2x^3 - 5y^2)(3 \cdot 2x^3 + 5y^2)$

3 What are the zeros of $s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$?

(1) $\{-3, -2, 5\}$

(3) $\{-3, -2, 3, 5\}$

(2) $\{-2, 3, 5\}$

(4) $\{-5, -3, 2, 3\}$

4 If θ is an angle in standard position whose terminal side passes through the point $(-2, -3)$, what is the numerical value of $\tan \theta$?

(1) $\dfrac{2}{3}$

(3) $-\dfrac{2}{\sqrt{13}}$

(2) $\dfrac{3}{2}$

(4) $-\dfrac{3}{\sqrt{13}}$

5 The average monthly temperature, $T(m)$, in degrees Fahrenheit, over

a 12 month period, can be modeled by $T(m) = -23 \cos\left(\dfrac{\pi}{6}m\right) + 56$,

where m is in months. What is the range of temperatures, in degrees Fahrenheit, of this function?

(1) $[-23, 23]$ \qquad\qquad (3) $[-23, 56]$

(2) $[33, 79]$ \qquad\qquad (4) $[-79, 33]$

6 Which expression is an equivalent form of $a\sqrt[5]{a^4}$?

(1) a \qquad\qquad (3) $a^{\frac{9}{4}}$

(2) $a^{\frac{9}{5}}$ \qquad\qquad (4) $a^{\frac{1}{5}}$

7 The expression $3i(ai - 6i^2)$ is equivalent to

(1) $3a + 18i$ (3) $-3a + 18i$

(2) $3a - 18i$ (4) $-3a - 18i$

8 Which equation best represents the graph below?
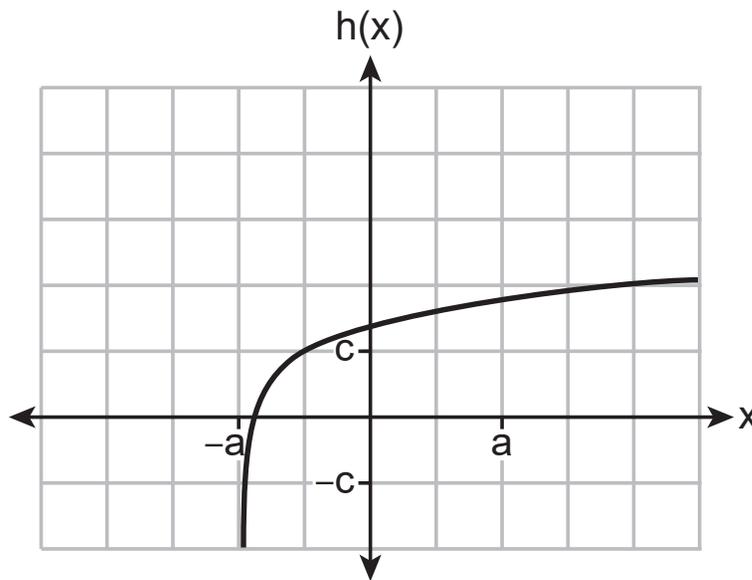
(1) $h(x) = \log(x + a) + c$ (3) $h(x) = \log(x + a) - c$
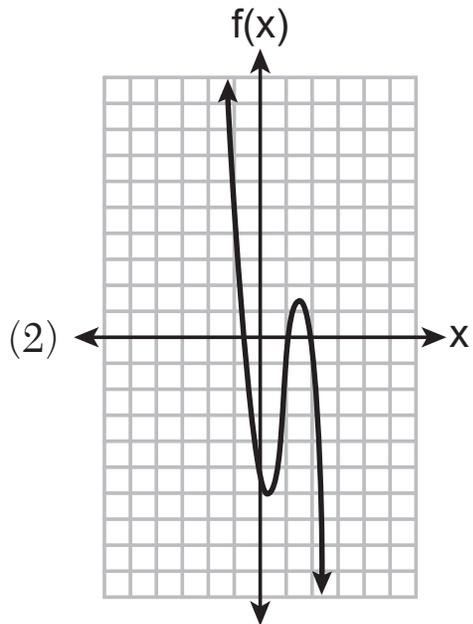
(2) $h(x) = \log(x - a) + c$ (4) $h(x) = \log(x - a) - c$

9 Which function has the characteristic as $x \to -\infty$, $f(x) \to -\infty$?

(1)

(3) $f(x) = 5(4)^{-x}$

(2)

(4) $f(x) = -\log_5(-x)$

10 The expression $(x^2 + 3)^2 - 2(x^2 + 3) - 24$ is equivalent to

(1) $(x^2 + 9)(x^2 - 1)$

(3) $x^4 - 2x^2 - 21$

(2) $(x^2 - 3)(x^2 + 7)$

(4) $x^4 + 4x^2 - 9$

11 What is the solution for the system of equations below?

$$x + y + z = 2$$
$$x - 2y - z = -4$$
$$x - 9y + z = -18$$

(1) $(-2,2,2)$

(3) $(0,2,0)$

(2) $(-2,-2,6)$

(4) $(0,2,4)$

12 The roots of the equation $x^2 - 4x = -13$ are

(1) $2 \pm 3i$

(3) $2 \pm \sqrt{17}$

(2) $2 \pm 6i$

(4) $2 \pm i\sqrt{13}$

13 Which expression is equivalent to $\dfrac{2x^3 + 2x - 7}{2x + 4}$?

(1) $x^2 - 2x + 5 - \dfrac{27}{2x + 4}$

(3) $x^2 + 2x + 5 + \dfrac{13}{2x + 4}$

(2) $x^2 - 1 - \dfrac{3}{2x + 4}$

(4) $x^2 + 2x - 3 + \dfrac{5}{2x + 4}$

14 A popular celebrity tracks the number of people, in thousands, who have followed her on social media since January 1, 2015. A summary of the data she recorded is shown in the table below:

Number of Months Since January 2015	2	11	16	20	27	35	47	50	52
Number of Social Media Followers (thousands)	3.1	7.5	29.7	49.7	200.3	680.3	5200.3	8109.3	12,107.1

The celebrity uses an exponential regression equation to model the data. According to the model, about how many followers did she have on June 1, 2018?

(1) 13,000,000

(3) 1,850,000

(2) 5,420,000

(4) 790,000

15 Luminescence is the emission of light that is not caused by heat. A luminescent substance decays according to the function below.

$$I = I_0 e^{3\left(-\frac{t}{0.6}\right)}$$

This function can be best approximated by

(1) $I = I_0 e^{\left(-\frac{t}{0.18}\right)}$

(2) $I = I_0 e^{5t}$

(3) $I = I_0(0.0067)^t$

(4) $I = I_0(0.0497)^{0.6t}$

16 The heights of the students at Central High School can be modeled by a normal distribution with a mean of 68.1 and a standard deviation of 3.4 inches. According to this model, approximately what percent of the students would have a height less than 60 inches or greater than 75 inches?

(1) 0.86%

(2) 1.26%

(3) 2.12%

(4) 2.98%

17 Marissa and Sydney are trying to determine if there is enough interest in their school to put on a senior musical. They randomly surveyed 100 members of the senior class and 43% of them said they would be interested in being in a senior musical. Marissa and Sydney then conducted a simulation of 500 more surveys, each of 100 seniors, assuming that 43% of the senior class would be interested in being in the musical. The output of the simulation is shown below.
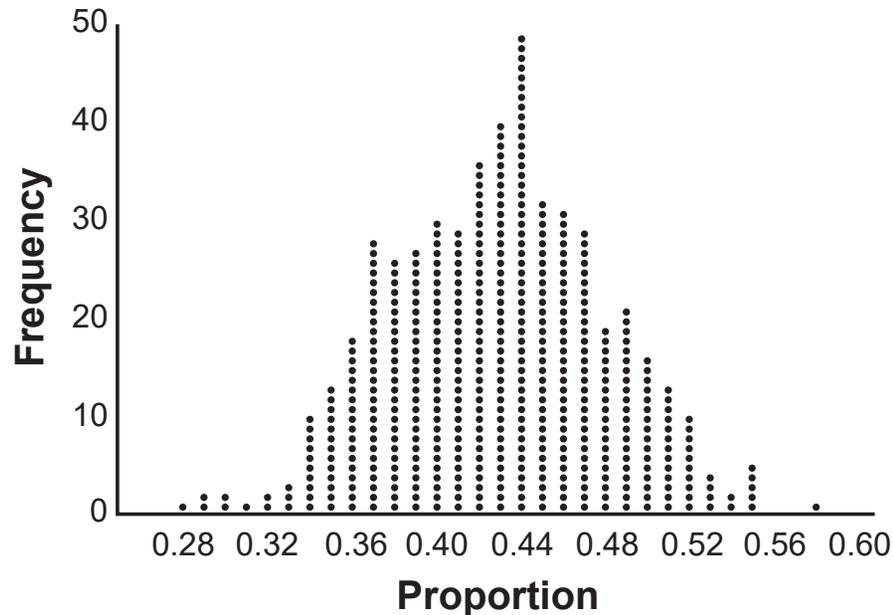
The standard deviation of the simulation is closest to

(1) 0.02   (3) 0.09

(2) 0.05   (4) 0.43

18 For $f(x) = \cos x$, which statement is true?

(1) $2f(x)$ and $f(2x)$ are even functions.

(2) $f(2x)$ and $f(x) + 2$ are odd functions.

(3) $2f(x)$ and $f\left(x + \dfrac{\pi}{2}\right)$ are odd functions.

(4) $f(x) + 2$ is an odd function and $f\left(x + \dfrac{\pi}{2}\right)$ is an even function.

19 The solution set of $\dfrac{x + 3}{x - 5} + \dfrac{6}{x + 2} = \dfrac{6 + 10x}{(x - 5)(x + 2)}$ is

(1) $\{-6\}$ (3) $\{-6,5\}$

(2) $\{5\}$ (4) $\{-5,6\}$

20 Given x and y are positive, which expressions are equivalent to $\dfrac{x^3}{y}$?

I. $\left(\dfrac{y}{x^3}\right)^{-1}$

II. $\sqrt[3]{x^9}\,(y^{-1})$

III. $\dfrac{x^6\sqrt[4]{y^8}}{x^3 y^3}$

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

21 Given the inverse function $f^{-1}(x) = \dfrac{2}{3}x + \dfrac{1}{6}$, which function represents $f(x)$?

(1) $f(x) = -\dfrac{2}{3}x + \dfrac{1}{6}$

(2) $f(x) = -\dfrac{3}{2}x + \dfrac{1}{4}$

(3) $f(x) = \dfrac{3}{2}x - \dfrac{1}{4}$

(4) $f(x) = \dfrac{3}{2}x - \dfrac{1}{6}$

22 How many equations below are identities?

- $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$
- $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$
- $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

(1) 1 (3) 3

(2) 2 (4) 0

23 If the focus of a parabola is $(0, 6)$ and the directrix is $y = 4$, what is an equation for the parabola?

(1) $y^2 = 4(x - 5)$ (3) $y^2 = 8(x - 5)$

(2) $x^2 = 4(y - 5)$ (4) $x^2 = 8(y - 6)$

24 John and Margaret deposit $500 into a savings account for their son on his first birthday. They continue to make a deposit of $500 on the child's birthday, with the last deposit being made on the child's 21st birthday. If the account pays 4% annual interest, which equation represents the amount of money in the account after the last deposit is made?

(1) $S_{21} = 500(1.04)^{21}$

(3) $S_{21} = 500(1.04)^{20} + 500$

(2) $S_{21} = \dfrac{500(1 - 1.04^{21})}{1 - 1.04}$

(4) $S_{21} = \dfrac{500(1 - 0.04^{21})}{1 - 1.04}$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.   [16]

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Work space for question 25 is continued on the next page.

Question 25 continued

26 Determine the solution of $\sqrt{3x + 7} = x - 1$ algebraically.

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

Question 27 continued

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a.

Question 28 continued

29 Write a recursive formula for the sequence 189, 63, 21, 7, … .

Question 29 continued

30 Solve algebraically for x to the *nearest thousandth*:

$$2e^{0.49x} = 15$$

Question 30 continued

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

Question 31 continued

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.

Mean = 0.852
SD = 0.029
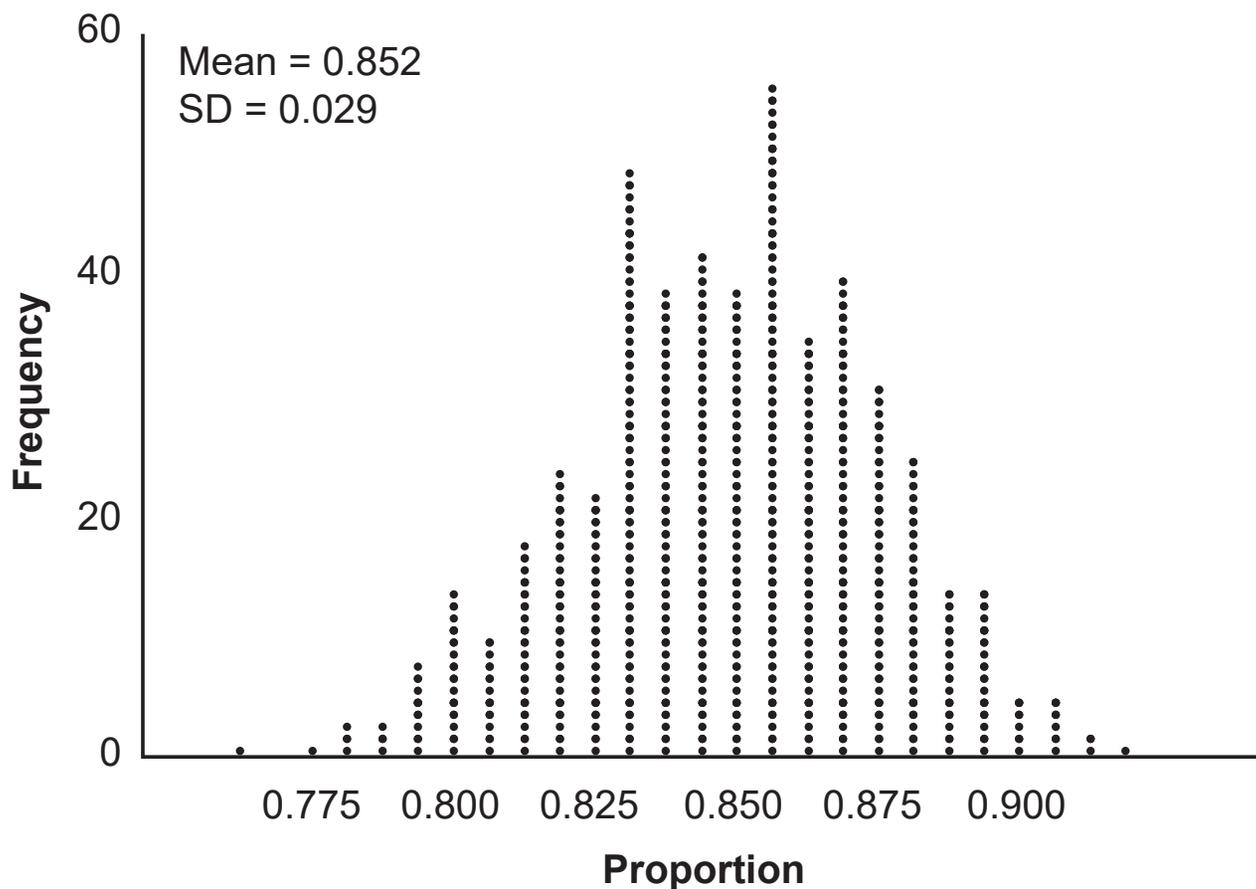
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

Work space for question 32 is continued on the next page.

Question 32 continued

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6. Write an equation for $p(x)$.

Question 33 continued

Sketch $y = p(x)$ on the set of axes below.
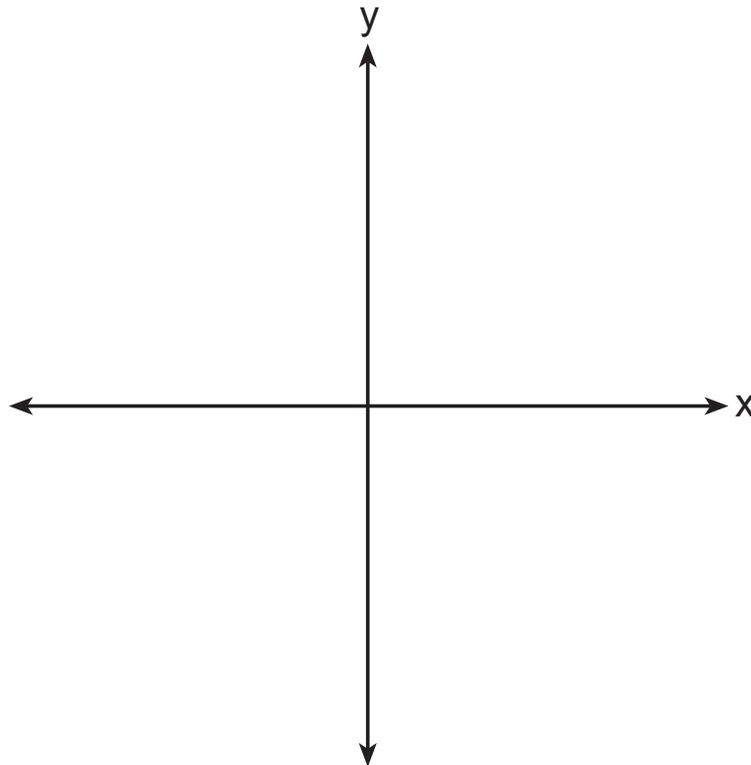
34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	**Patron**
Method of Donation	**Phone calls**	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

Question 35 continued

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.
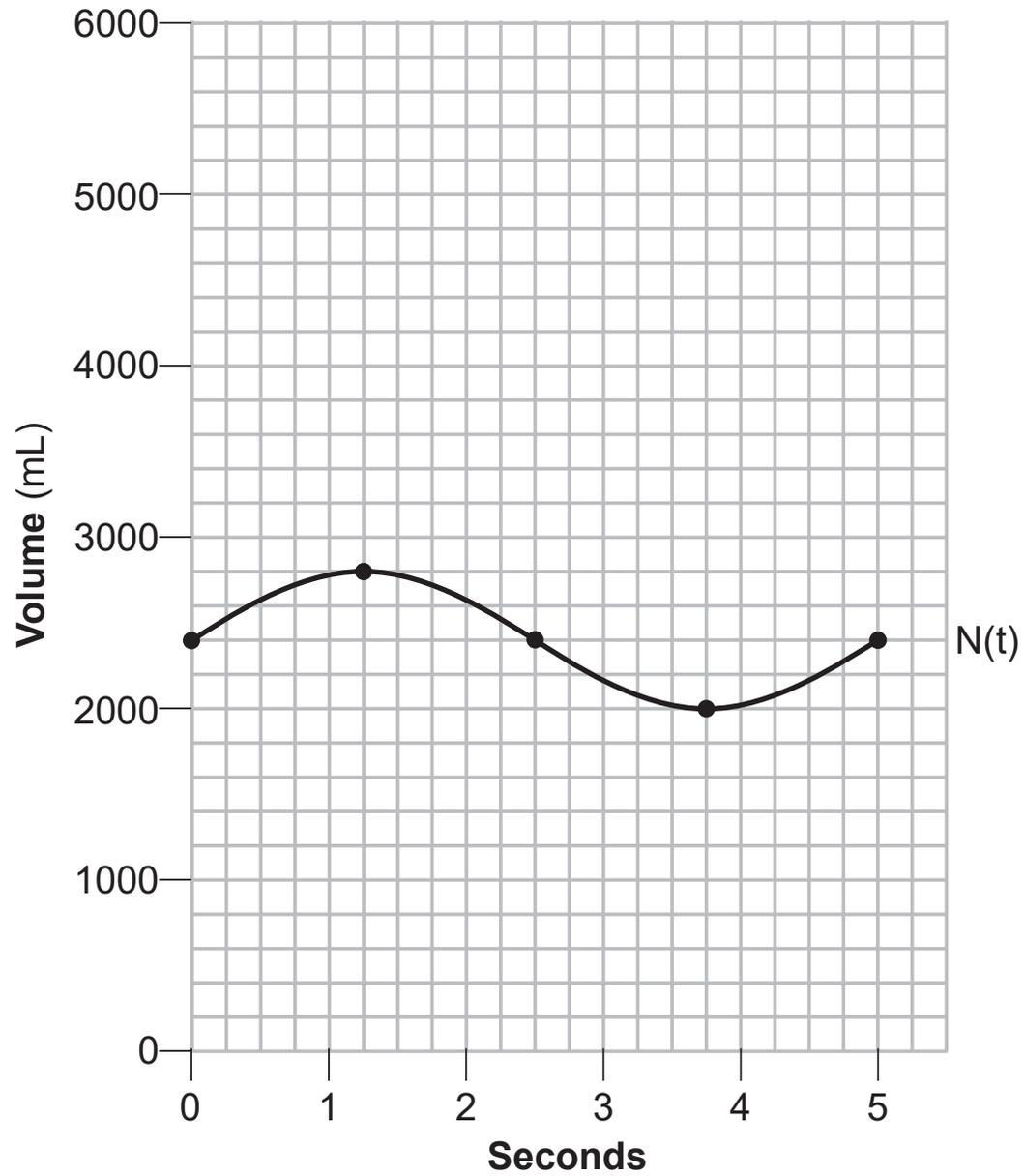
Question 36 continued

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

37 The volume of air in an average lung during breathing can be modeled by the graph on the next page.
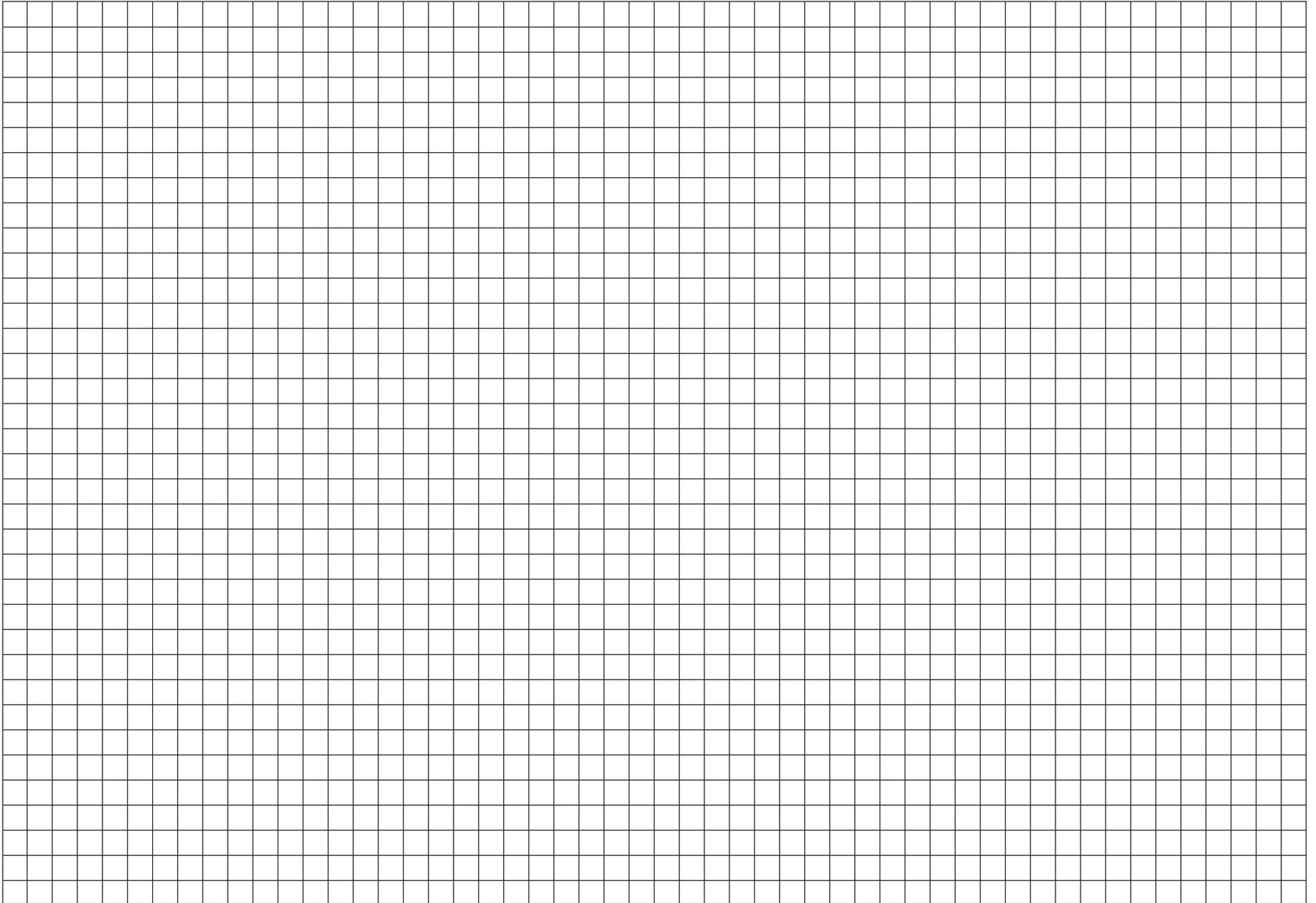
Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.
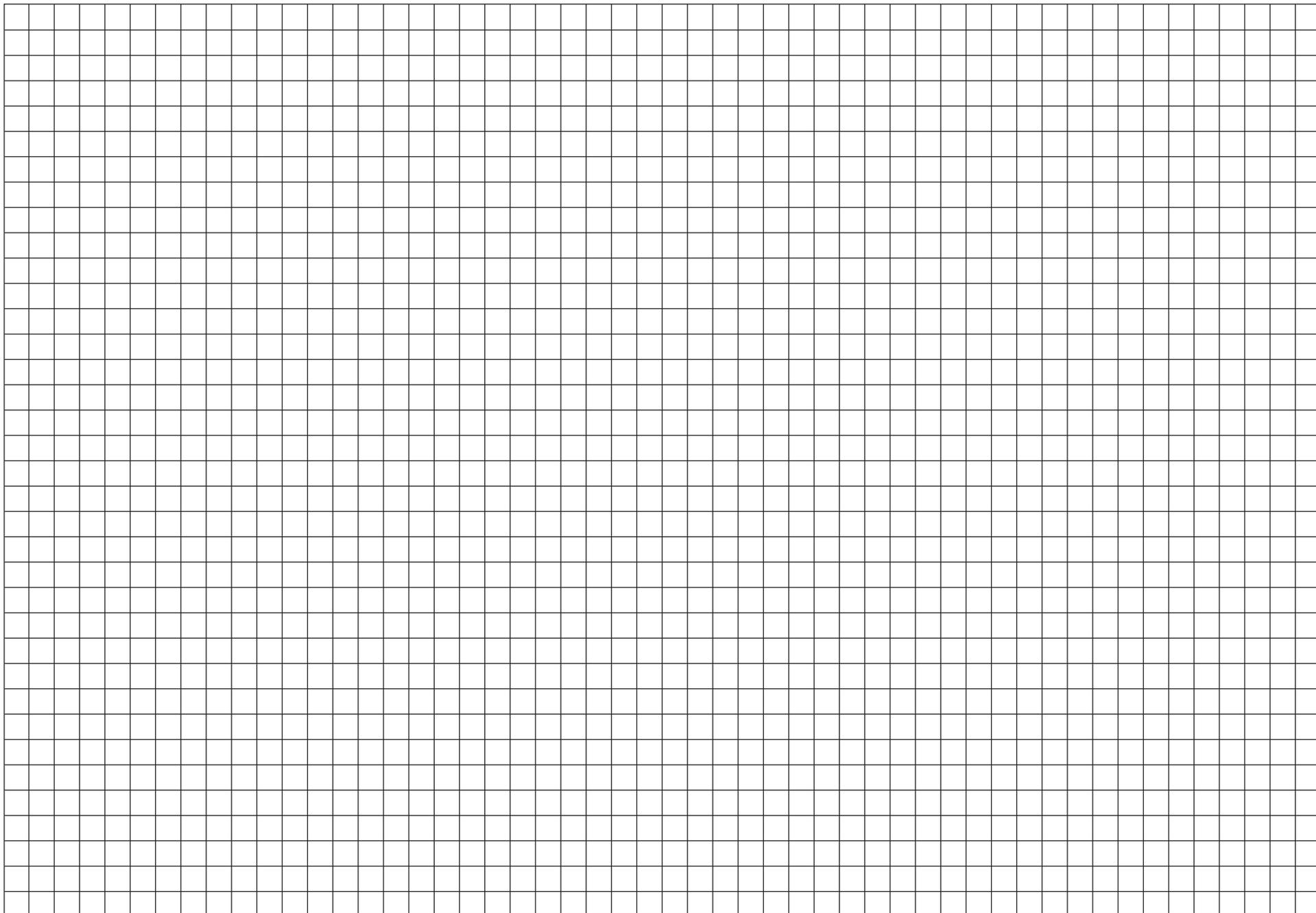
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Reference Sheet — concluded

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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