

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, August 16, 2023 — 12:30 to 3:30 p.m.

MODEL RESPONSE SET

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Question 25

25 Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

$$\begin{aligned} &x^2(2x-3) - 9(2x-3) \\ &(x^2-9)(2x-3) \\ &(x+3)(x-3)(2x-3) \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 25

25 Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

$$\begin{array}{l} x^2(2x^2 - 3x) + (-18x + 27) \\ x^2(2x-3) \quad | \quad -9(2x-3) \\ (2x-3)(x^2-9) \\ (2x-3)(x-3)(x+3) \end{array}$$

Score 2: The student gave a complete and correct response.

Question 25

25 Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

$$\begin{aligned} & 2x^3 - 3x^2 - 18x + 27 \\ &= x^2(2x-3) - 9(2x-3) \\ &= (x^2-9)(2x-3) \end{aligned}$$

Score 1: The student did not factor completely.

Question 25

25 Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

$$\begin{array}{l} 2x^3 - 3x^2 - 18x + 27 \\ x^2(2x - 3) - 9(2x - 3) \end{array}$$

$$\begin{array}{l} (x^2 - 9)(2x - 3) \\ \boxed{(x^2 + 3)(x^2 - 3)(2x - 3)} \end{array}$$

Score 1: The student made one factoring error.

Question 25

25 Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

$$2x^3 - 3x^2 - 18x + 27$$
$$x(2x^2 - 3) - 3(6x - 9) = 0$$
$$x + 3(2x^2 - 3)(6x + 9) = 0$$
$$(\quad) (\quad)$$

Score 0: The student did not show enough relevant course-level work to receive any credit.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$x^2 + 8x - 5 = 8x - 4$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Score 2: The student gave a complete and correct response.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$\begin{array}{r} x^2 + 8x - 5 = 8x - 4 \\ -5x + 4 - 8x + 4 \end{array}$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$\{1, -1\}$$

Score 2: The student gave a complete and correct response.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$\begin{array}{r} 8x - 4 = x^2 + 8x - 5 \\ -8x + 5 \quad -8x + 5 \end{array}$$

$$\sqrt{1} = \sqrt{x^2}$$

$$(x = 1)$$

Score 1: The student did not indicate $x = -1$.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$8x - 4 = x^2 + 8x - 5$$

$$x^2 + 8x - 8x - 5 + 4 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1)$$

$$x^2 + x - x - 1$$

$$x=1 \quad x=-1$$

Score 1: The student rejected a correct solution.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$y = 1^2 + 8(1) - 5 = 1 + 8 - 5 = \boxed{4}$$

$$y = 8(1) - 4 = 8 - 4 = \boxed{4}$$

$$x = 1$$

Score 0 : The student did not solve algebraically and only stated one correct solution.

Question 26

26 Algebraically determine the values of x that satisfy the system of equations shown below:

$$y = x^2 + 8x - 5$$

$$y = 8x - 4$$

$$x^2 + 8x - 5 = 8x - 4$$

$$x^2 - 5 = 4$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = 3$$

Score 0: The student made a transcription error writing -4 as 4 and did not state two solutions.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

$$\frac{3x^2 + 5x + 8 = 0}{3} \quad \frac{0}{3}$$

$$x^2 + \frac{5}{3}x + \frac{8}{3} = 0$$

$$x^2 + \frac{5}{3}x = -\frac{8}{3} = \frac{-96}{36}$$

$$x^2 + \frac{10}{6}x + \frac{25}{36} = \frac{-71}{36}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{-71}{36}$$

$$x + \frac{5}{6} = \pm \sqrt{\frac{-71}{36}}$$

$$x + \frac{5}{6} = \pm i \frac{\sqrt{71}}{6}$$

$$x = \frac{-5}{6} \pm \left(\frac{\sqrt{71}}{6}\right)i$$

$$x \in \left\{ \frac{-5}{6} - \left(\frac{\sqrt{71}}{6}\right)i, \frac{-5}{6} + \left(\frac{\sqrt{71}}{6}\right)i \right\}$$

Score 2: The student gave a complete and correct response.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

$$3x^2 + 5x + 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(8)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{-71}}{6}$$

$$x = \frac{-5}{6} \pm \frac{i\sqrt{71}}{6}$$

Score 2: The student gave a complete and correct response.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(8)}}{2(3)} = \frac{-5 \pm \sqrt{-71}}{6} = \frac{-5 \pm 71i}{6}$$



$$\frac{-5}{6} \pm \frac{71i}{6}$$

Score 1: The student eliminated the radical.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(8)}}{2(3)} \quad \frac{-5 \pm \sqrt{25 - 96}}{6} \quad \frac{-5 \pm \sqrt{-71}}{6}$$

$$\frac{-5 \pm \sqrt{71}i}{6}$$

$$x_1 = \frac{-5 + \sqrt{71}i}{6}$$

$$x_2 = \frac{-5 - \sqrt{71}i}{6}$$

Score 1: The student did not express the answer in $a + bi$ form.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

$$\frac{-b \pm \sqrt{(-b)^2 - 4ac}}{2a}$$
$$\frac{-(5) \pm \sqrt{(5)^2 - 4(3)(8)}}{2(3)} = \frac{-5 \pm \sqrt{-71}}{6}$$

↘

$$\frac{-5 \pm 71i}{6}$$

Score 0: The student eliminated the radical and did not express the answer in $a + bi$ form.

Question 27

27 Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

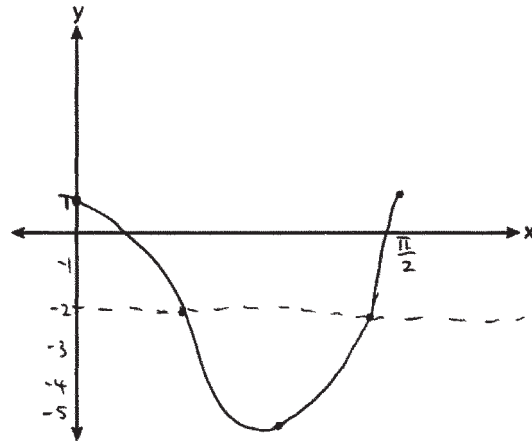
$$\frac{-5 \pm \sqrt{5^2 - 4(3)(8)}}{2(3)}$$

$$\frac{-5 \pm \sqrt{-71}}{6}$$

Score 0: The student did not simplify the radical and did not express the answer in $a + bi$ form.

Question 28

28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

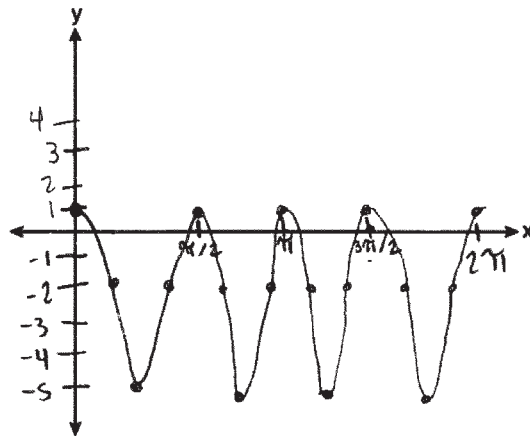


Score 2: The student gave a complete and correct response.

Question 28

28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

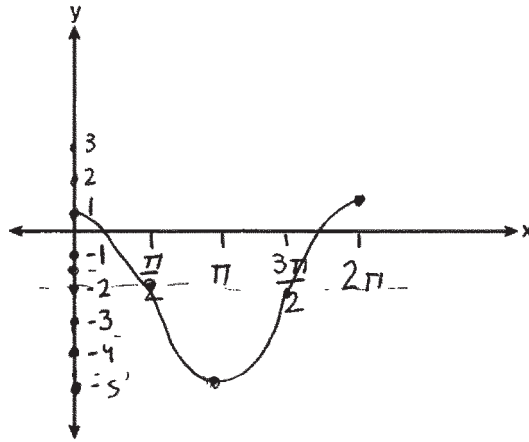
$$y = 3\cos 4x - 2$$



Score 2: The student gave a complete and correct response.

Question 28

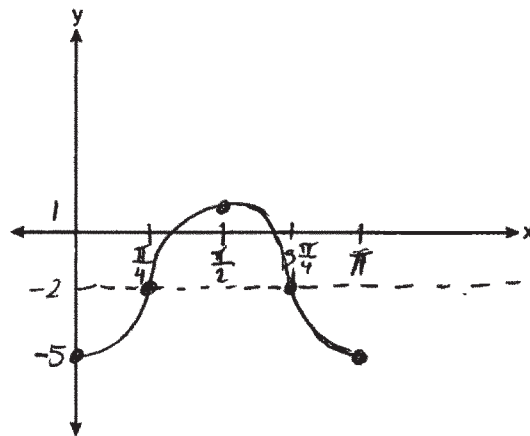
28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.



Score 1: The student used an incorrect period.

Question 28

28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

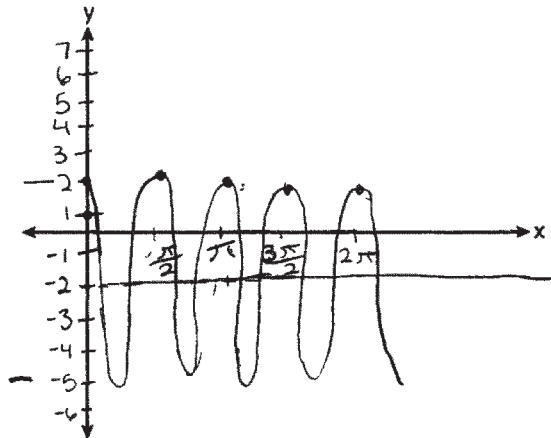


Score 1: The student graphed an acceptable negative cosine function, but has an incorrect period.

Question 28

28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

Starts at max
↓
how long it takes to make one wave

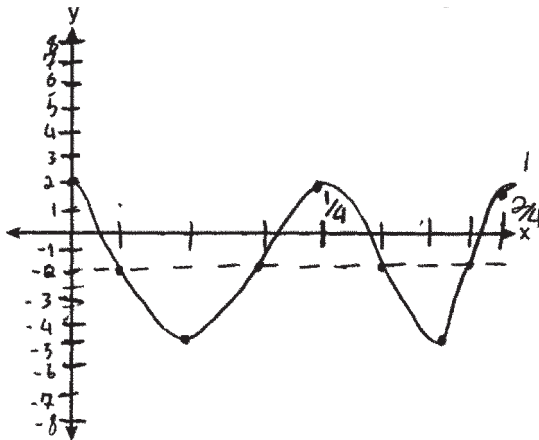


Score 1: The student graphed an incorrect maximum value.

Question 28

28 On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

$$\frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}$$



Score 0: The student graphed an incorrect maximum value and an incorrect period.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$(5xi^3 - 4i)(5xi^3 - 4i)$$

$$25x^2i^6 - 40xi^4 + 16i^2$$

$$\boxed{-25x^2 - 40x - 16}$$

Score 2: The student gave a complete and correct response.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$\begin{aligned} & (5xi^3 - 4i)^2 \\ & (5xi^3 - 4i)(5xi^3 - 4i) \\ & 25x^2 \overset{i^6}{i^2 \cdot i^2} - 20xi^4 - 20xi^4 + 16i^2 \\ & \begin{array}{ccc} i^2 \cdot i^2 \cdot i^2 & i^2 \cdot i^2 & i^2 = -1 \\ -1 \cdot -1 \cdot -1 & -1 \cdot -1 & \\ 1 \cdot -1 & 1 & \\ -1 & & \end{array} \\ & -25x^2 - 20x - 20x - 16 \\ & \boxed{-25x^2 - 40x - 16} \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$\begin{array}{r|l} 5xi^3 & -4i \\ \hline 5xi^3 & 25x^2i^6 & -20xi^4 \\ \hline -4i & -20xi^4 & -16i^2 \end{array} \quad (-25x^2 - 40x + 16)$$

Score 1: The student made one computational error.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$(5xi^3 - 4i)(5xi^3 - 4i)$$
$$25x^2i^6 - 25xi^4 - 25xi^4 + 16i^2$$

$$\boxed{-25x^2 - 50x - 16}$$

Score 1: The student made one computational error.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$\begin{array}{r} 5xi^3 - 4i \\ \begin{array}{|c|c|} \hline 5xi^3 & -4i \\ \hline 25x^2i^6 & -20xi^4 \\ \hline -20xi^4 & 16i^2 \\ \hline \end{array} \\ 25x^2i^6 - 20xi^4 - 20xi^4 + 16i^2 \\ 25x^2i^6 - 20xi^4 - 20xi^4 + 16 \\ \hline -15x^4i^8 - 16 \end{array}$$

Score 0: The student made multiple computational errors.

Question 29

29 Given i is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

$$25x^2(-1) - 16(-1)$$

$$-25x^2 + 16$$

$$-9x^2$$

$$(x-3)(x+3)$$

Score 0: The student made multiple errors.

Question 30

30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.

To determine the equation of the directrix, you would graph the parabola and the focus, then find the distance from the vertex to the focus and the directrix is equal distance from the vertex in the opposite direction.

Score 2: The student gave a complete and correct response.

Question 30

30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.

I found
directrix by
graphing the
parabola and
plotting the vertex
and focus. The I
found the space between
vertex and focus and
went in opposite direction
to find the directrix.
Distance between vertex and focus
is same as the distance between
vertex and directrix.

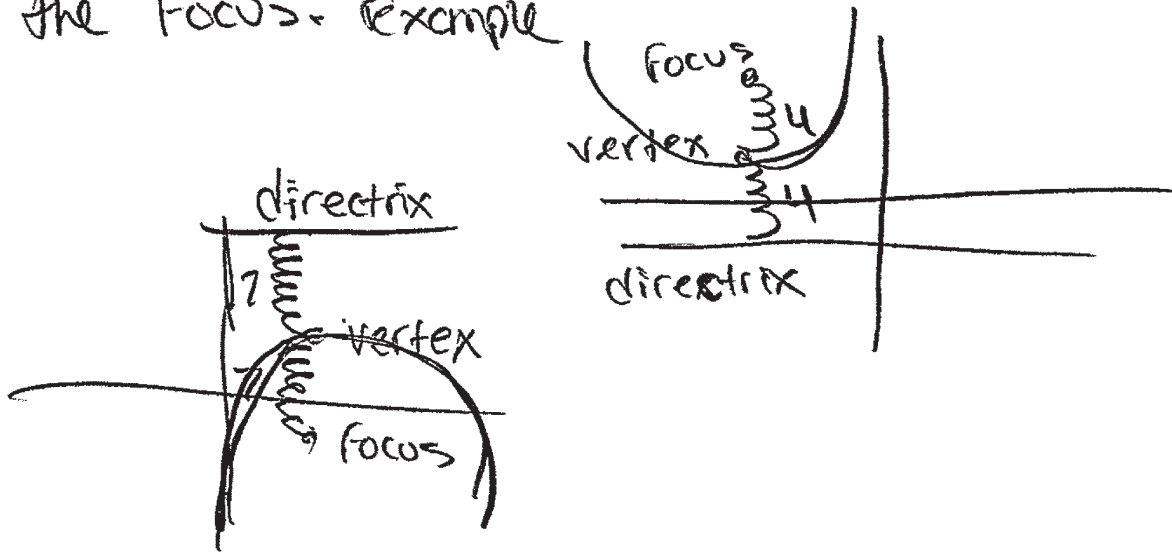
directrix
is
 $y = 6$

Score 2: The student gave a complete and correct response.

Question 30

30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.

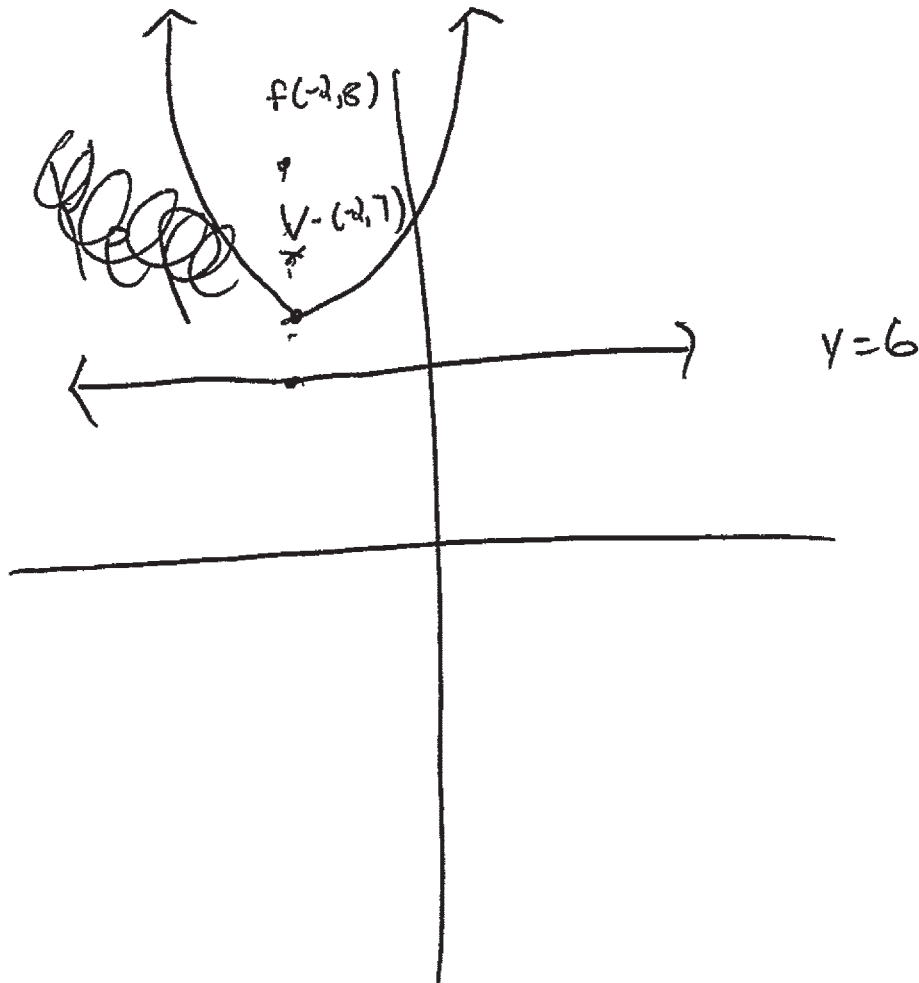
First you plot the vertex and the focus. when that is done you count how many spaces away they are from each other then you count away from the focus. Example



Score 1: The student wrote an incomplete explanation.

Question 30

30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2, 7)$ and focus $(-2, 8)$. Use this information to explain how to determine the equation of the directrix.



Score 1: The student wrote a correct equation of the directrix, but gave no explanation.

Question 30

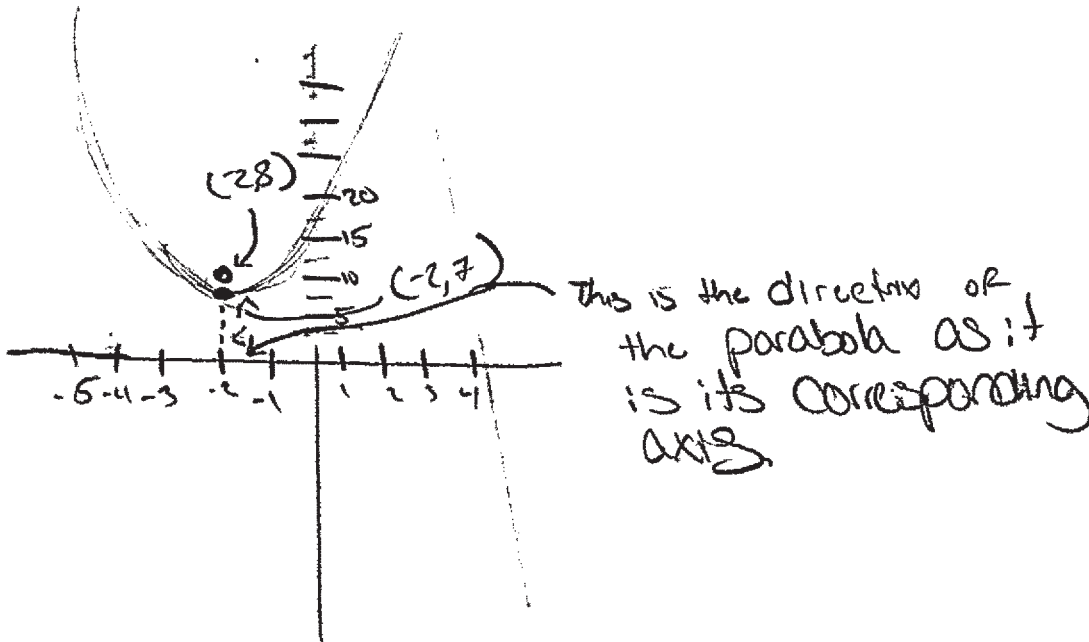
30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.

$$\text{directrix} = y = 6$$

Score 0: The student did not write an explanation and showed no work to find $y = 6$.

Question 30

30 Consider the parabola given by $y = \frac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.



Score 0: The student wrote an incorrect explanation.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

$$\begin{aligned} & x(x^3)^{\frac{1}{2}} \\ \frac{x \cdot x^{\frac{3}{2}}}{(x^5)^{\frac{1}{3}}} &= \frac{x^{\frac{5}{2}}}{x^{\frac{5}{3}}} \\ & x^{\frac{5}{6}} \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

$$x\sqrt{x^3} = x\left(x^{\frac{3}{2}}\right) = x^{\frac{5}{2}}$$

$$\frac{15}{6} - \frac{10}{6} = \frac{5}{6}$$

$$\sqrt[3]{x^5} = x^{\frac{5}{3}}$$

$$\frac{x^{\frac{5}{2}}}{x^{\frac{5}{3}}} = x^{\left(\frac{5}{2} - \frac{5}{3}\right)}$$

$$x^{\frac{5}{6}}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

$$\frac{x(x^3)^2}{(x^3)^5} = \frac{x(x^6)}{x^{15}} = \frac{x^7}{x^{15}} = x^{7-15} = x^{-8} = \frac{1}{x^8}$$

Score 1: The student made an error converting from radical form to rational exponents, but then followed through correctly.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

$$\frac{x(x^{\frac{3}{2}})}{x^{\frac{5}{3}}} = \frac{x^{\frac{5}{2}}}{x^{\frac{5}{3}}} = x^{\frac{5}{2}} \cdot x^{\frac{3}{5}} = x^{\frac{31}{10}}$$

Score 1: The student made an error dividing.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

7

$$\frac{x}{\sqrt[3]{x^2}}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 31

31 Write $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

$$\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}} \cdot \frac{\sqrt{x^4}}{\sqrt{x^4}} = \frac{x^2}{\sqrt{x^8}} = x^{-6}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 32

32 A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where P represents the number of fruit flies after t days. What is the average rate of change of the population, rounded to the nearest hundredth, over the interval $[0, 10.5]$? Include appropriate units in your answer.

t	$P(t)$
0	10
10.5	123.01

$$\frac{123.01 - 10}{10.5 - 0} = 10.76$$

The average rate of change in the population over the interval $[0, 10.5]$ is 10.76 fruit flies per day.

Score 2: The student gave a complete and correct response.

Question 32

32 A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where P represents the number of fruit flies after t days. What is the average rate of change of the population, rounded to the nearest hundredth, over the interval $[0, 10.5]$? Include appropriate units in your answer.

$$\begin{array}{l} (0, 10) \\ x_2 \quad y_2 \\ (10.5, 123.01) \\ x_1 \quad y_1 \end{array}$$

$$\frac{10 - 123.01}{0 - 10.5}$$

10.76 Fruitflies
per day

Score 2: The student gave a complete and correct response.

Question 32

32 A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where P represents the number of fruit flies after t days. What is the average rate of change of the population, rounded to the nearest hundredth, over the interval $[0, 10.5]$? Include appropriate units in your answer.

$$P = 10(1.27)^0$$
$$\boxed{P = 10}$$

$$P = 10(1.27)^{10.5}$$
$$\boxed{P = 123.0096181}$$

$$\frac{\Delta y}{\Delta x}$$

$$\frac{123.0096181 - 10}{10.5 - 0}$$

$$\frac{113.0096181}{10.5} = 10.76282$$

~~✗~~ $\boxed{\text{average rate} = 10.76 \text{ of change}}$

Score 1: The student did not include units.

Question 32

32 A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where P represents the number of fruit flies after t days. What is the average rate of change of the population, rounded to the nearest hundredth, over the interval $[0, 10.5]$? Include appropriate units in your answer.

$$0 = 10$$
$$P = 10(1.27)^{10.5}$$
$$P \approx 123.01$$

$$\text{A.R.O.C.} \quad \frac{123.01 - 10}{10.5 - 0} = 10.76$$

$$10.8 \text{ flies/day}$$

Score 1: The student made a rounding error.

Question 32

32 A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where P represents the number of fruit flies after t days. What is the average rate of change of the population, rounded to the nearest hundredth, over the interval $[0, 10.5]$? Include appropriate units in your answer.

$$(0, 10) \quad (10.5, 123.01)$$

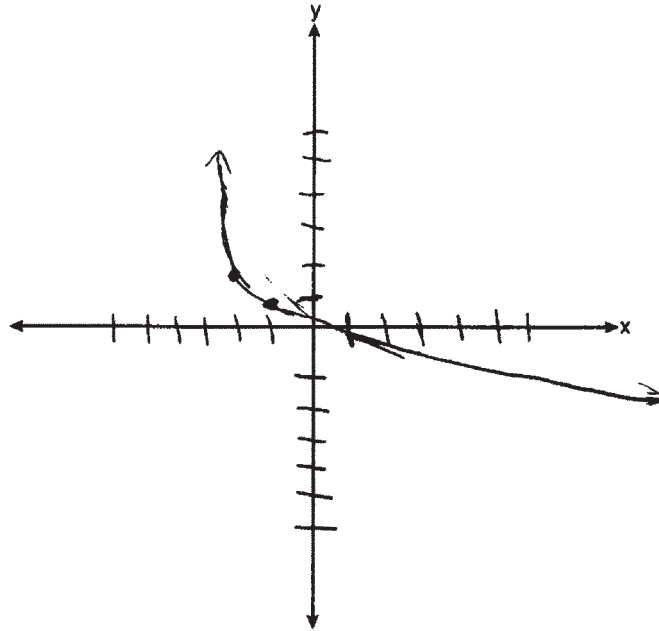
$$\frac{x_2 - x_1}{y_2 - y_1} = \frac{10.5 - 0}{123.01 - 10}$$

$$\frac{10.5}{113.01} = .09 \text{ increase per day}$$

Score 0: The student made an error finding the average rate of change and did not include correct units.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

As $x \rightarrow -3$
 $y \rightarrow \infty$

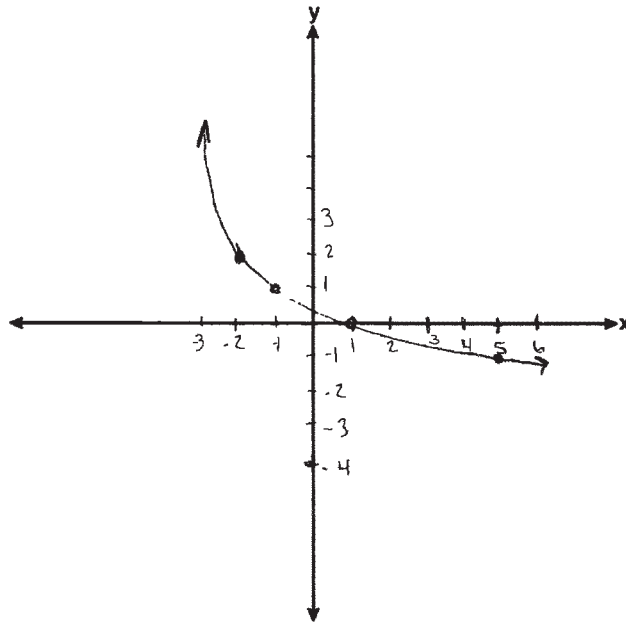
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

As $x \rightarrow \infty$
 $y \rightarrow -\infty$

Score 4: The student gave a complete and correct response.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

as x approaches -3 , $p(x)$ increases infinitely

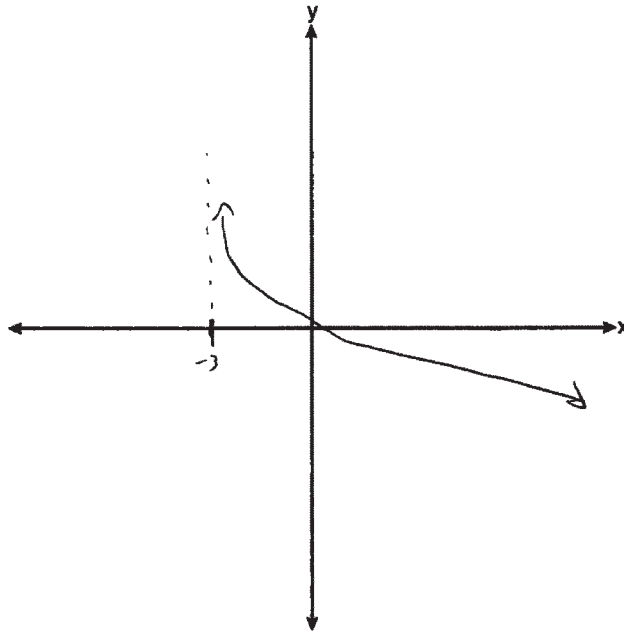
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

as x approaches ∞ , $p(x)$ approaches $-\infty$

Score 4: The student gave a complete and correct response.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

as $x \rightarrow -3$, $p(x) \rightarrow$

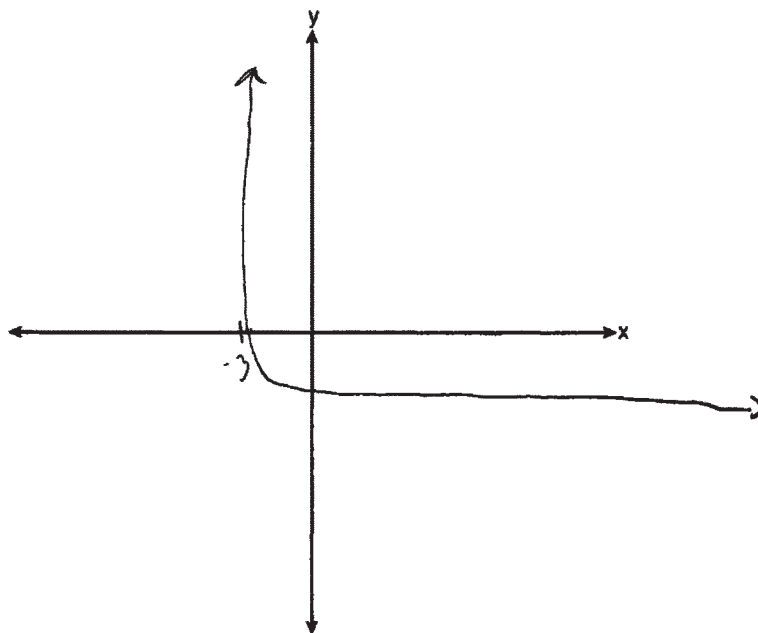
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

as $x \rightarrow \infty$, $p(x) \rightarrow -\infty$

Score 3: The student did not state the end behavior as $x \rightarrow -3$.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

The value continues to increase.

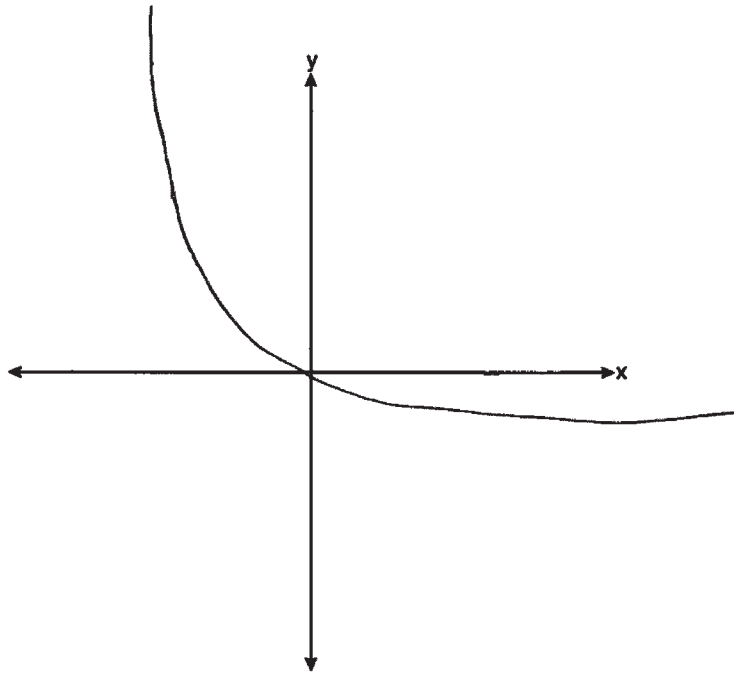
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

The value continues to decrease

Score 3: The student made one graphing error.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

As x decreases to -3 , the y -value increases

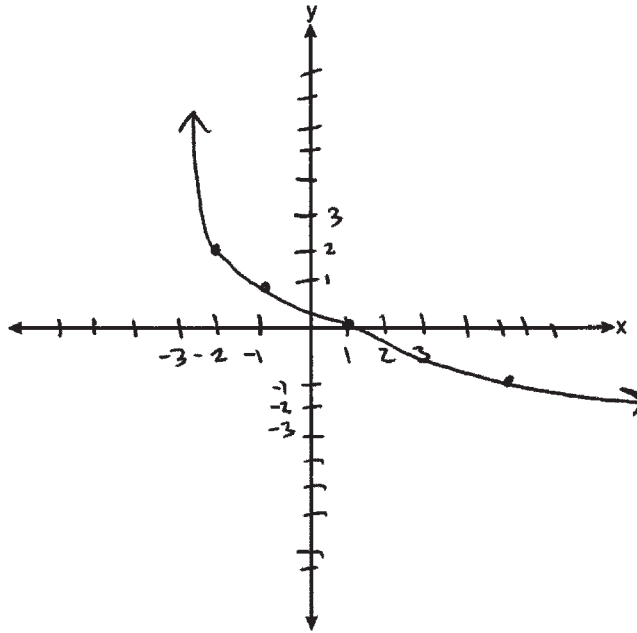
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

As x increases to ∞ , the y -value decreases

Score 2: The student only received credit for the descriptions.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



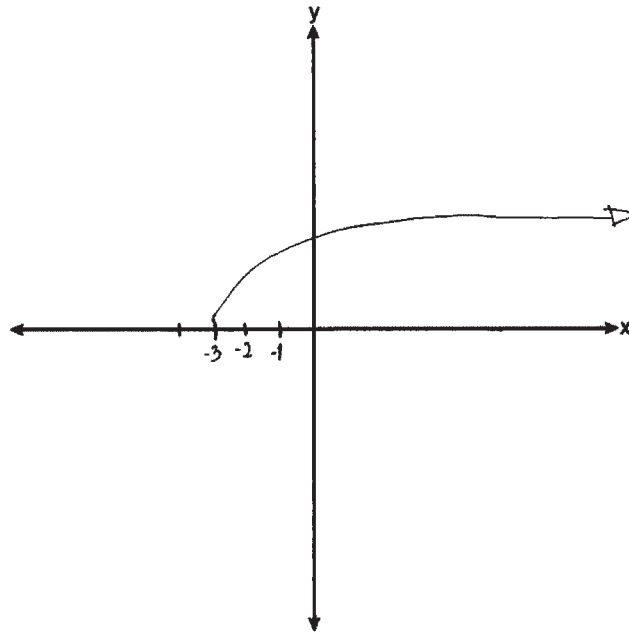
Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

Score 2: The student sketched a correct graph.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

the end of $p(x)$ as $x \rightarrow -3$ is Error
this means that graph, don't have
a exactly value.

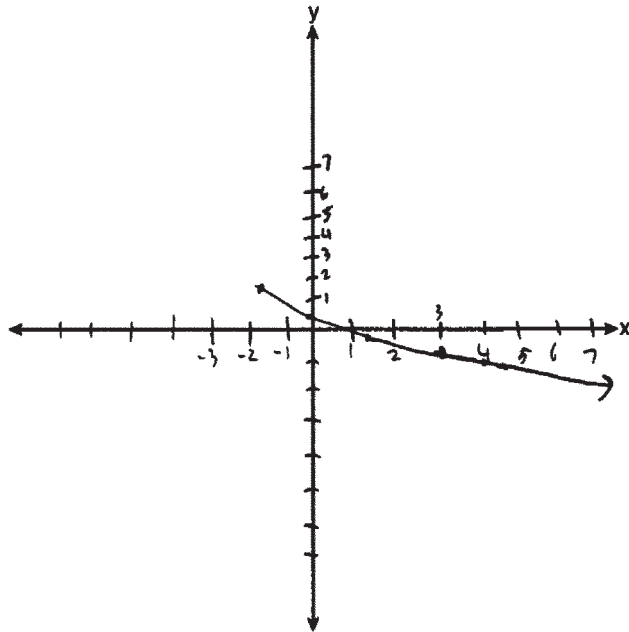
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

the end of $p(x)$ as $x \rightarrow \infty$ is
infinity

Score 1: The student correctly described the end behavior of the graph they sketched as $x \rightarrow \infty$.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

$[-3, \infty)$

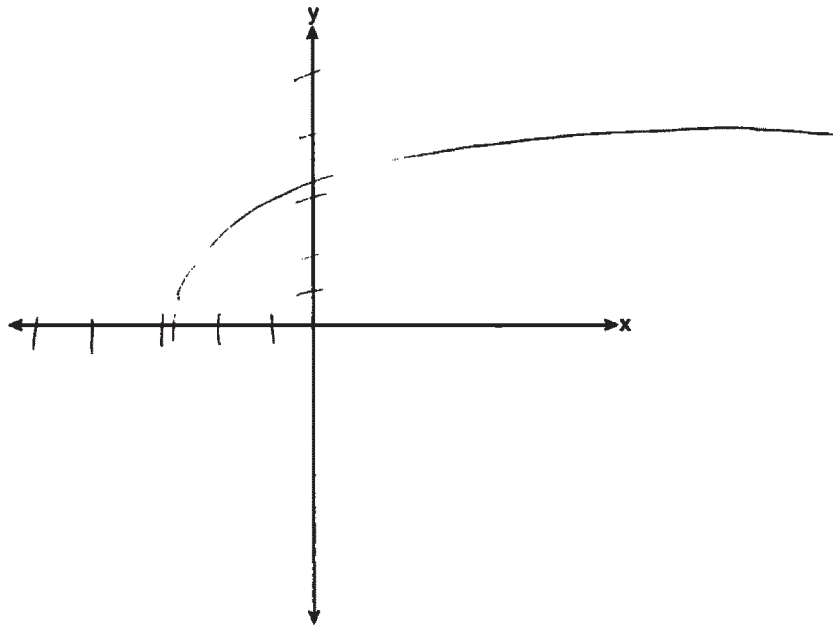
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

$[-\infty, \infty)$

Score 1: The student made one graphing error and received no credit for the descriptions.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

It will never go past -3

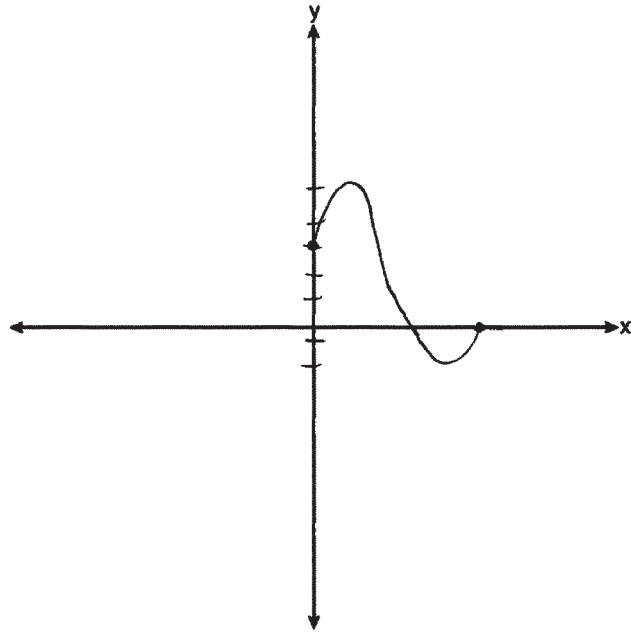
Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

It will go on forever

Score 0: The student did not show enough correct work to receive any credit.

Question 33

33 Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.



Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

It will be opposite
of +3

Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

It will continue
going

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$\begin{aligned}\frac{1}{x-6} + \frac{x}{x-2} &= \frac{4}{x^2 - 8x + 12} \\ \frac{1(x-2)}{x-6} + \frac{x(x-6)}{x-2} &= \frac{4}{(x-2)(x-6)} \\ \frac{x-2}{(x-6)(x-2)} + \frac{x^2-6x}{(x-2)(x-6)(x-2)} &= \frac{4}{(x-2)(x-6)} \\ x^2 - 5x - 2 &= 4 \\ x^2 - 5x - 6 &= 0 \\ (x-6)(x+1) &= 0 \\ \cancel{x=6} \quad \boxed{x=-1}\end{aligned}$$

Score 4: The student gave complete and correct response.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$\frac{1}{x-6} + \frac{x}{x-2} - \frac{4}{x^2 - 8x + 12} = 0$$

$$\frac{1}{x-6} + \frac{x}{x-2} - \frac{4}{x^2 - 7x - 6x + 12} = 0$$

$$\frac{1}{x-6} + \frac{x}{x-2} - \frac{4}{x(x-2) - 6(x-2)}$$

$$\frac{1}{x-6} + \frac{x}{x-2} - \frac{4}{(x-2)(x-6)}$$

$$\frac{x-2 + x(x-6) - 4}{(x-2)(x-6)} = \frac{-5x - 6 + x^2}{(x-2)(x-6)} = 0$$

$$\frac{x^2 + x - 6x - 6}{(x-2)(x-6)} = 0$$

$$\frac{(x+1)(x-6)}{(x-2)(x-6)} = 0$$

$$\frac{x+1}{x-2} \quad x+1=0$$

$$\boxed{-1 = x}$$

Score 4: The student gave complete and correct response.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$x^2 - 8x + 12$$
$$(x-2)(x-6)$$

$$\cdot (x-2) + x(x-6) = 4$$

$$x-2 + x^2 - 6x = 4$$

$$x^2 - 6x + x - 2 - 4 = 0$$

$$x^2 - 5x - 6 = 0$$

$$(x+1)(x-6) = 0$$

$$x = -1 \checkmark$$

$$x = 6$$

$$\frac{1}{-1-6} + \frac{-1}{-3} = \frac{4}{1+8+12}$$

$$\frac{2}{-3} \frac{1}{-7} + \frac{1}{3} = \frac{4}{21}$$

$$\frac{-3}{21} + \frac{7}{21} = \frac{4}{21}$$

$$\frac{4}{21} = \frac{4}{21}$$

$$\frac{1}{6-6} = \frac{1}{0} \times \text{reject}$$

$$\boxed{-1}$$

Score 4: The student gave complete and correct response.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$$

$$\frac{x-2+x^2-6x}{\cancel{(x-6)}\cancel{(x-2)}} = \frac{4}{\frac{x^2-8x+12}{\cancel{(x-6)}\cancel{(x-2)}}}$$

$$x^2 - 5x - 2 = 4$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$\boxed{\begin{array}{l} x=6 \\ x=-1 \end{array}}$$

Score 3: The student failed to reject $x = 6$.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$\begin{aligned}\frac{1}{x-6} + \frac{x}{x-2} &= \frac{4}{(x-6)(x-2)} \\ (x-2) + x(x-6) &= 4 \\ x^2 - 6x + x - 2 &= 4 \\ x^2 - 5x - 6 &= 0 \\ (x-6)(x+1) &= 0 \\ x &= 6, -1\end{aligned}$$

Score 2: The student wrote a correct quadratic equation in standard form.

Question 34

34 Solve for x algebraically: $\frac{x^2-1}{x^2} \cdot \frac{1}{x-6} + \frac{x-6}{x-2} = \frac{4}{x^2-8x+12} \cdot \frac{4}{(x-6)(x-2)}$

$$\frac{(x-2) + \cancel{x(x-6)}}{(x-2)(x-6)(x-6)(x-2)} = \frac{4}{(x-6)(x-2)}$$

$$(x-2) + x(x-6) = 4$$

$$(x-2) + x^2 - 6x = 4$$

$$x + x^2 - 6x = 6$$

$$\cancel{x^3} - 6x = 6 - x^3$$

$$x = 1 - x^3$$

Score 1: The student gave a correct equation cleared of fractions.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$$
$$\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{(x-6)(x-2)}$$
$$\frac{1(x-2)}{x-6} + \frac{x(x-6)}{x-2} = \frac{4(x-6)(x-2)}{(x-6)(x-2)}$$
$$1 \cdot x = 4$$
$$x = 3$$

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 Solve for x algebraically: $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$

$$(x-2)(x-6)$$

$$(x-2)\left(\frac{1}{x-6}\right) + \left(\frac{x}{x-2}\right) \cdot \frac{(x-6)}{(x-2)(x-6)} = \frac{4}{(x-2)(x-6)}$$

$$\frac{\cancel{x-2}}{(x-2)(x-6)} + \frac{x^2-6x}{(x-2)(x-6)} = \frac{4(\cancel{x-2})(\cancel{x-6})}{(\cancel{x-2})(\cancel{x-6})}$$

$$(x-6) + \frac{x^2-6x}{(x-2)(x-6)} = 4$$

$$(x-6) + \frac{x(\cancel{x-6})}{(x-2)(\cancel{x-6})} = 4$$

$$(x-6) + \frac{x}{(x-2)} = 4 \quad (x-2)$$

$$x-6+x = 4x-8$$

$$2x-6 = 4x-8$$

$$\begin{array}{r} -2x \\ -6 \\ \hline \end{array} \quad \begin{array}{r} -4x \\ -8 \\ \hline \end{array}$$

$$\begin{array}{r} -6 = 2x-8 \\ +8 \quad +8 \\ \hline \end{array}$$

$$\frac{2}{2} = \frac{2x}{2}$$

$$\boxed{x=1}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$2x + 4y - 3z = 12 \rightarrow$$

$$3x - 2y + 2z = -9$$

$$-x + y - 3z = 0$$

$$y - 3z = x$$

$$3(y - 3z) - 2y + 2z = -9$$

$$3y - 9z - 2y + 2z$$

$$y - 7z = -9$$

$$2((2z - 9) - 3z) + 4(7z - 9) - 3z = 12$$

$$\boxed{4z} - 18 - 6z + \boxed{28z} - 36 - 3z = 12$$

$$33z - 54 = 12$$

$$\begin{array}{r} +54 \\ 33z = 66 \\ \hline 33 \quad 33 \end{array}$$

$$\boxed{z = 2}$$

$$y = 7z - 9$$

$$y = 7(2) - 9$$

$$\boxed{y = 5}$$

$$-x + 5 - 3(2) = 0$$

$$-x - 1 = 6$$

$$\begin{array}{r} -x = 7 \\ \hline -1 \quad -1 \end{array}$$

$$\boxed{x = -7}$$

Score 4: The student gave a complete and correct response.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$\begin{array}{r} 2x + 4y - 3z = 12 \\ 2(3x - 2y + 2z) = (9)2 \\ \hline 2x + 4y - 3z = 12 \\ + 6x - 4y + 4z = 18 \\ \hline 8x + z = 30 \end{array}$$

$$\begin{array}{r} 2x + 4y - 3z = 12 \\ 3x - 2y + 2z = -9 \\ -x + y - 3z = 0 \end{array}$$

$$\begin{array}{r} 3x - 2y + 2z = -9 \\ 2(-x + y - 3z) = (0)2 \\ \hline 3x - 2y + 2z = -9 \\ -2x + 2y - 6z = 0 \\ \hline x - 4z = -9 \end{array}$$

$$\begin{array}{r} 4 \\ \left. \begin{array}{l} 8x + z = -6 \\ x - 4z = -9 \end{array} \right\} \\ \hline (-7) - 4z = -9 \\ 2z = -8 \\ \frac{2z}{2} = \frac{-8}{2} \\ \boxed{z = -4} \end{array}$$

$$\begin{array}{r} 3(2x + 4y - 3z) = 3(-24) \\ = 6x + 12y - 9z = -72 \\ \hline 6x - 4y + 4z = -18 \\ \hline 33y - 13z = -54 \\ \frac{33y}{33} = \frac{-54}{33} \\ \boxed{y = -1.636} \\ \boxed{z = 2} \\ \boxed{y = -5} \end{array}$$

$$\begin{array}{r} 7(11) + y = 3(2) = 0 \\ 1 + y - 6 = 0 \\ \hline 5 + y = 0 \\ -5 \quad -5 \\ \boxed{y = -5} \end{array}$$

Score 3: The student made one computational error when solving for y .

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$\begin{array}{l} \textcircled{1} \quad 2x + 4y - 3z = 12 \\ \textcircled{2} \quad 3x - 2y + 2z = -9 \\ \textcircled{3} \quad -x + y - 3z = 0 \end{array}$$

$$\begin{array}{l} \textcircled{1} \quad 2x + 4y - 3z = 12 \\ \textcircled{2} \quad (3x - 2y + 2z = -9) \\ \quad \quad 8x + z = -6 \end{array}$$

$$\begin{array}{l} 9(-.5) + z = -9 \\ -4 + z = -9 \\ +4 \quad +4 \end{array}$$

$$z = -5$$

$$\begin{array}{l} \textcircled{2} \quad 3x + 2y + 2z = -9 \\ \textcircled{3} \quad 2(-x + y - 3z = 0) \end{array}$$

$$-x - 4z = -9$$

$$\begin{array}{l} 2(-.5) + 4y - 3(-5) = 12 \\ 1 + 4y - 15 = 12 \end{array}$$

$$\begin{array}{l} 4(8x + z = -6) \\ -x - 4z = -9 \end{array}$$

$$\begin{array}{l} 4y - 14 = 12 \\ +14 \quad +14 \end{array}$$

$$\frac{30x}{30} = \frac{-15}{30}$$

$$\frac{4x = 26}{4 \quad 4}$$

$$x = -.5$$

$$y = 6.5$$

Score 2: The student made multiple computational errors.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$x = -1$$

$$y =$$

$$z =$$

$$2x + 4y - 3z = 12$$

$$3x - 2y + 2z = -9$$

$$-x + y - 3z = 0$$

~~Handwritten work showing a system of equations with a circled 'x' in the first equation and various scribbles and corrections.~~

$$2x + 4y - 3z = 12$$

$$3x - 2y + 2z = -9$$

$$2(-x + y - 3z = 0) \quad | \quad 4(8x + 2 = -6)$$

$$-2x + 2y - 6z = 0 \quad | \quad x - 4z = -9$$

$$3x - 2y + 2z = -9 \quad | \quad 32x + 4z = -24$$

$$x - 4z = -9 \quad | \quad \frac{33x}{33} = \frac{-33}{33} = -1$$

Score 2: The student correctly found $x = -1$.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$2x + 4y - 3z = 12$$

$$3x - 2y + 2z = -9$$

$$-x + y - 3z = 0$$

ref([A])

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{l} x = -1 \\ y = 5 \\ z = 2 \end{array}$$

$$2(-1) + 4(5) - 3(2) = 12$$

$$-2 + 20 - 6 = 12$$

$$12 = 12 \quad \checkmark$$

Score 2: The student used a method other than algebraic.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$2x + 4y - 3z = 12$$

$$3x - 2y + 2z = -9$$

$$-4(-x + y - 3z = 0)$$

$$\begin{array}{r} 2x + 4y - 3z = 12 \\ 6x - 4y + 4z = -18 \end{array}$$

$$8x + z = -6$$

8c

$$z = -12.184$$

$$2x + 4y - 3z = 12$$

$$4x - 2y + 12z = 0$$

$$6x + 4z = 12$$

$$a(8x + z = -6)$$

$$6x + 4z = 12$$

$$-72x - 4z = -63$$

$$\begin{array}{r} -66x = -51 \\ \hline -66 \quad \quad -66 \end{array}$$

$$x = .773$$

$$2(.773) + 4y - 3(-12.184) = 12$$

$$1.544 + 4y + 36.552 = 12$$

no answer

Score 1: The student wrote a correct system in two variables.

Question 35

35 Solve the following system of equations algebraically for x , y , and z .

$$2x + 4y - 3z = 12$$

$$3x - 2y + 2z = -9$$

$$-x + y - 3z = 0$$

$$\begin{array}{r} 2x + 4y - 3z = 12 \\ 6x - 4y + 4z = -18 \\ \hline 8x + z = -6 \end{array}$$

$$\begin{array}{r} 3x - 2y + 2z = -9 \\ -2x + 2z = 0 \\ \hline 5x - 4z = -9 \end{array}$$

$$\begin{array}{r} 5x - 4z = -9 \\ 8x + z = -6 \\ \hline (9x - 3z = -15) \cdot (-1) \end{array}$$

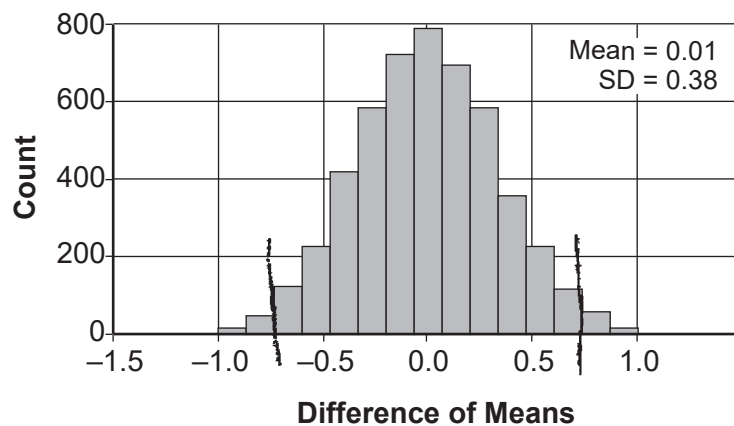
$$\begin{array}{r} -9x + 3z = 15 \\ -x + y - 3z = 0 \\ \hline y = 15 \end{array}$$

$$\boxed{x=2} \quad \boxed{y=8} \quad \boxed{z=2}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

$$\begin{aligned} \bar{x} \pm 2\sigma \\ 0.01 \pm 2(0.38) & \quad [-.75, .77] \\ 0.01 \pm .76 \end{aligned}$$

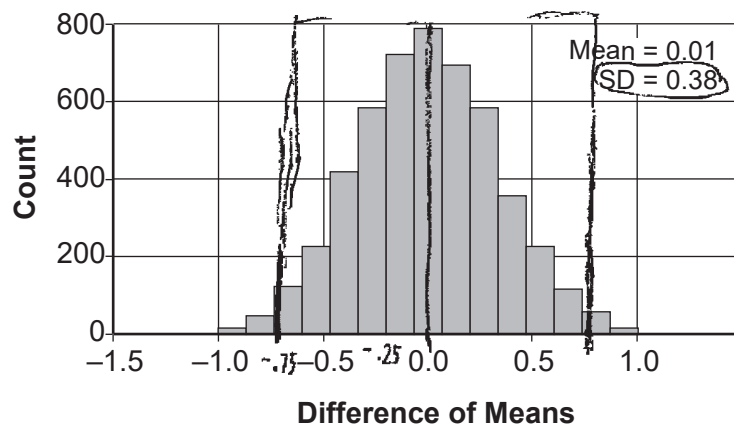
Does the interval indicate that the difference between the classes' grades is significant? Explain.

"0.6" is within the interval so it is usual, so its not significant

Score 4: The student gave a complete and correct response.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

-0.75 to 0.77

$$\begin{aligned} 0.01 + 0.76 &= 0.77 \\ 0.01 - 0.76 &= -0.75 \end{aligned}$$

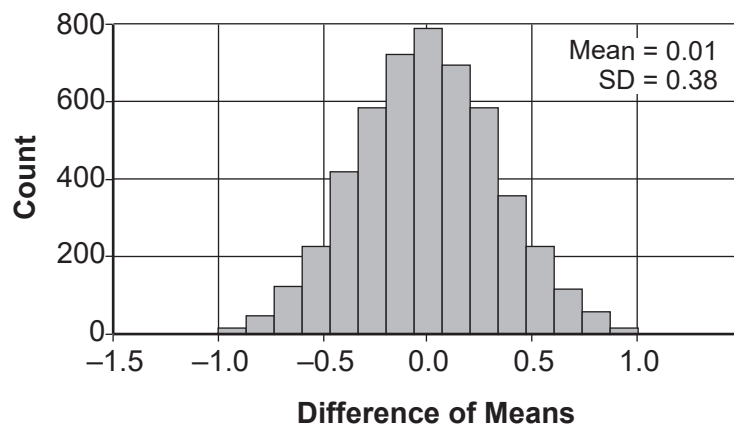
Does the interval indicate that the difference between the classes' grades is significant? Explain.

No, it's usual because 0.6 is in the 95% C.I. of -0.75 to 0.77.

Score 4: The student gave a complete and correct response.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

$-.75 - .77$

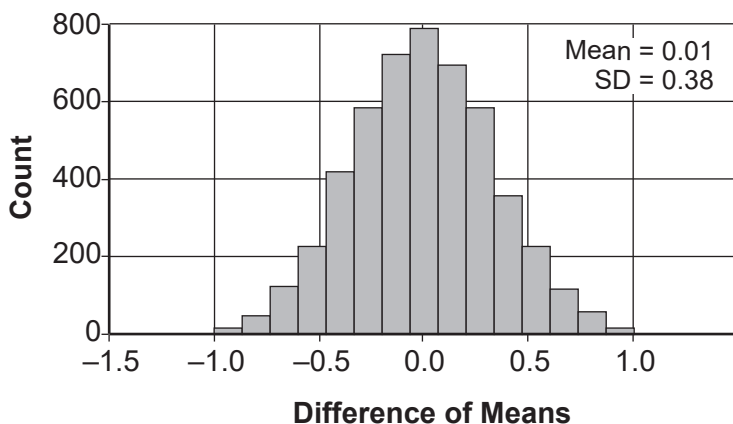
Does the interval indicate that the difference between the classes' grades is significant? Explain.

No, because .6 is within two standard deviations.

Score 3: The student did not show work to find the interval.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

$$\begin{aligned} 95\% &= \text{within } 2\text{SDs} \\ .01 + .38 + .38 \\ .01 - .38 - .38 \\ &(-.8, .8) \end{aligned}$$

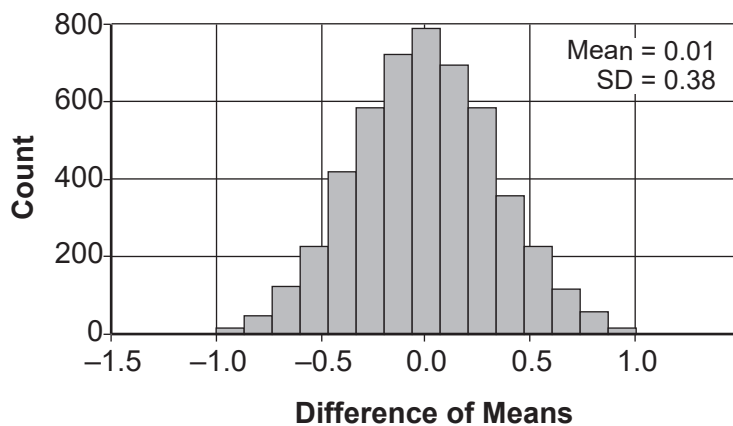
Does the interval indicate that the difference between the classes' grades is significant? Explain.

The difference between class grades is not significant, as .6 falls in the interval which contains 95% of the data.

Score 3: The student made a rounding error.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

$$\begin{aligned} & \bar{x} \pm MOE \\ & .01 \pm 2(.38) \\ & .01 \pm .76 \\ & \swarrow \quad \searrow \\ & .01 - .76 \quad .01 + .76 \end{aligned}$$

(-.75, .77)

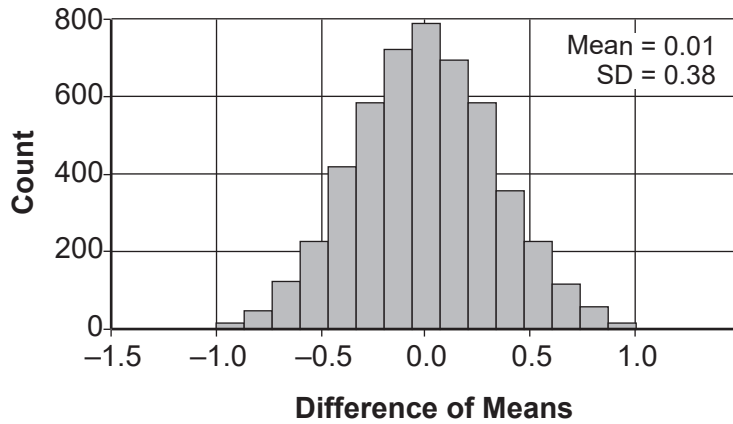
Does the interval indicate that the difference between the classes' grades is significant? Explain.

The difference of .6 lies within the interval therefore it is significant

Score 2: The student only received credit for the correct interval.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

~~MOE =~~
 $CI = \bar{x} \pm 2\sigma$
 $CI = 0.01 \pm 2(0.38)$

+ → 0.77
 - → -0.75

[-0.75, 0.77]

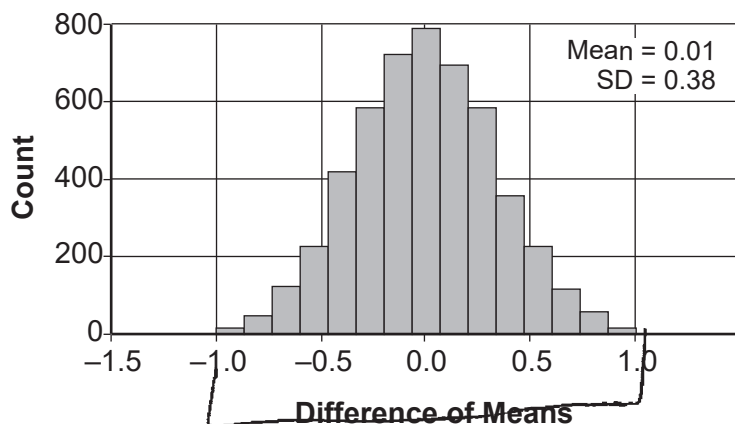
Does the interval indicate that the difference between the classes' grades is significant? Explain.

The interval indicates that the difference between the classes is not significant because the difference is not less than 0.05.

Score 2: The student only received credit for the correct interval.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.



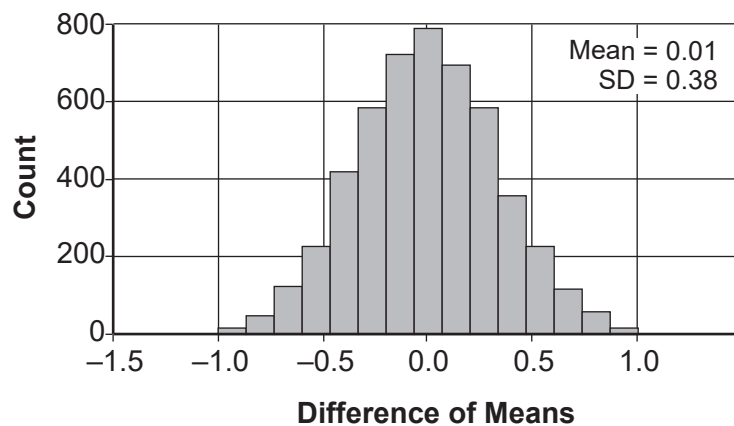
Does the interval indicate that the difference between the classes' grades is significant? Explain.

yes because the data changes.

Score 1: The student received one credit for the first part.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

20

$(.77, .75)$

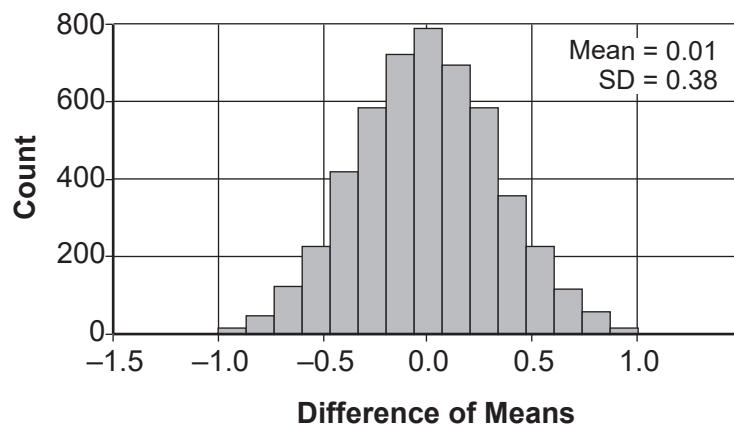
Does the interval indicate that the difference between the classes' grades is significant? Explain.

No, because the interval is very small which means the grades were very close together.

Score 0: The student did not show enough correct work to receive any credit.

Question 36

36 Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.



what

Determine an interval containing the middle 95% of the simulation results. Round your answer to the nearest hundredth.

$$\mu = 0.01$$
$$\sigma = 0.38$$

$$-.8 \text{ to } .8$$

Does the interval indicate that the difference between the classes' grades is significant? Explain.

No because 95% of the data lies between $\pm .8$ points in a grade difference.

Score 0: The student did not show enough correct work to receive any credit. A negative response was indicated, but the explanation was incorrect.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 \left(1 + \frac{.042}{4}\right)^{4t}$$

$$B(t) = 8000 e^{.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

Abby by \$828.63	$8000 \left(1 + \frac{.042}{4}\right)^{4(18)} = 11970.89992$
	$8000 e^{.039(18)} = \underline{\underline{-16142.27394}}$

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$24000 = 8000 e^{.039t}$$

$$3 = e^{.039t}$$

$$\frac{\ln 3}{.039} = \frac{.039t}{.039} \text{ the}$$

$t = 28.2$

Score 6: The student gave a complete and correct response.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 \left(1 + \frac{0.042}{4}\right)^{4t}$$

$$B(t) = 8000 e^{0.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 8000 (1.0105)^{72} = 16970.90$$

$$B(18) = 8000 e^{0.039(18)} = 16142.27$$

$$\boxed{\$828.63}$$

(Abby)

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$\underline{24000}$$

$$24000 = 8000 e^{0.039t}$$

$$\ln(3) = \ln(e^{0.039t})$$

$$\ln(3) = 0.039t$$

$$t = \frac{\ln(3)}{0.039} = 28.2 \text{ years}$$

Score 6: The student gave a complete and correct response.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8,000 \left(1 + \frac{0.042}{4}\right)^{4t}$$

$$B(t) = 8,000 e^{0.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 8,000 (1.0105)^{72} = 16970.90$$

$$B(18) = 8,000 e^{0.039(18)} = 1108.78$$

Abby will have more money after 18 yrs.
The Difference is 15862.12

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$24,000 = 8,000 e^{0.039(t)}$$

$$3 = e^{0.039(t)}$$

$$\log_e 3 = 0.039(t)$$

$$1.0986 = 0.039(t)$$

28.2 yrs

Score 5: The student made a computational error in finding $B(18)$.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

Abby

$$A(t) = 8000 \left(1 + \frac{0.042}{4}\right)^{4t}$$
$$B(t) = 8000 e^{.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 16970.10$$

Abby will

$$B(18) = 16142.27$$

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$\frac{24000}{8000} = \frac{8000 e^{.039t}}{8000}$$
$$\ln 3 = t e^{.039t}$$

28 years

Score 4: The student failed to determine the difference between accounts and made a rounding error in the third part.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 \left(1 + \frac{.042}{4}\right)^{4t}$$
$$B(t) = 8000 e^{.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 8000(1.0105)^{4(18)} = 16970.90$$
$$B(18) = 8000e^{.039(18)} = 16142.27$$

Abby will have more.
\$828.63 difference

$$\begin{array}{r} 16970.90 \\ - 16142.27 \\ \hline 828.63 \end{array}$$

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

Score 4: The student earned credit for the first two parts.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 \left(1 + \frac{.042}{4}\right)^{4t}$$

$$B(t) = 8,000 e^{.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 8000 \left(1 + \frac{.042}{4}\right)^{4(18)} = \$16,970.90$$

$$B(18) = 8000 e^{.039(18)} = \$16,192.27$$

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$\frac{48426.82}{8,000} = 8,000 e^{.039(2x)}$$

$$6.05 = e^{.039x}$$

$$\frac{\ln 6.05}{\ln e} = \frac{.039x}{\ln e}$$

$$\frac{1.8}{.039} = x$$

$$x = 46 \text{ yrs}$$

Score 3: The student failed to determine the difference between accounts, and made two errors in the third part.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000(1 + .042)^t$$

$$B(t) = 8000(1 + .039)^t$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$8000(1 + .042)^{18}$$

\$16776.79

$$8000(1 + .039)^{18}$$

\$15928.38

\$848.48

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

$$\frac{24000}{8000} = \frac{8000(1 + .039)^t}{8000}$$

$$3 = 1.039^t$$

$$t = \log_{1.039}(3)$$

$$t = 26.71534947 \text{ years}$$

Score 2: The student earned one point for determining the difference in the account and made one rounding error in the third part.

Question 37

- 37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 \left(1 + \frac{0.042}{4}\right)^{t \left(\frac{1}{4}\right)}$$
$$B(t) = 8000 \left(1 + \frac{0.039}{4}\right)^{t \left(\frac{1}{4}\right)}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(18) = 8000 \left(1 + \frac{0.042}{4}\right)^{\frac{1}{4}(18)} = 16090.99$$
$$B(18) = 8000 \left(1 + \frac{0.039}{4}\right)^{\frac{1}{4}(18)} = 15360.32$$

16090.99 - 15360.32

difference is

730.67

Abby will have more money

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

Score 2: The student received credit only for the second part.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$A(t) = 8000 + (.042)^{1/4}t$$

$$B(t) = 8000(e)^{.039t}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

$$A(t) = 8000 + (.042)^{1/4}(18) = 8019.34$$

$$B(t) = 8000(e)^{.039t} = 10142.27 \quad \text{WR that's not right}$$

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

Score 1: The student received credit for $B(t)$.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$P e^{rt}$ $8000e^{.039(12)t}$
1277438

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

Score 0: The student did not show enough correct work to satisfy the criteria for 1 credit.

Question 37

37 The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years.

$$8000 = (1.42)^t$$

$$8000 = (1.42)^{\frac{t}{4}}$$

$$8000 (1.36)^{\frac{t}{2}}$$

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

Abbi

~~$$8000 = (1.42)^t$$~~

$$8000 = (1.42)^{\frac{18}{4}}$$

38760

Brett

~~$$8000 = (1.36)^{\frac{t}{2}}$$~~

$$8000 = (1.36)^{\frac{18}{2}}$$

16680

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

84 years

Score 0: The student did not show enough correct work to receive any credit.