New York State Testing Program

Regents Examination in Geometry (Common Core)

Selected Questions with Annotations

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THE STATE EDUCATION DEPARTMENT /
THE UNIVERSITY OF THE STATE OF NEW YORK / ALBANY, NY 12234

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With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. In Spring 2014, New York State administered the first set of Regents Exams designed to assess student performance in accordance with the instructional shifts and the rigor demanded by the Common Core State Standards (CCSS). To aid in the transition to new tests, New York State released a number of resources including sample questions, test blueprints and specifications, and criteria for writing test questions. These resources can be found at http://www.engageny.org/resource/regents-exams.

New York State administered the first Geometry (Common Core) Regents Exam in June 2015 and is now annotating a portion of the questions from this tests available for review and use. These annotated questions will help students, families, educators, and the public better understand how the test has changed to assess the instructional shifts demanded by the Common Core and to assess the rigor required to ensure that all students are on track to college and career readiness.

Annotated Questions Are Teaching Tools

The annotated questions are intended to help students, families, educators, and the public understand how the Common Core is different. The annotated questions will demonstrate the way the Common Core should drive instruction and how tests have changed to better assess student performance in accordance with the instructional shifts demanded by the Common Core. They are also intended to help educators identify how the rigor of the Regents Examinations can inform classroom instruction and local assessment. The annotations will indicate common student misunderstandings related to content clusters; educators should use these to help inform unit and lesson planning. In some cases, the annotations may offer insight into particular instructional elements (conceptual thinking, mathematical modeling) that align to the Common Core that may be used in curricular design. It should not be assumed, however, that a particular cluster will be measured with identical items in future assessments.

The annotated questions include both multiple-choice and constructed-response questions. With each multiple-choice question annotated, a commentary will be available to demonstrate why the question measures the intended cluster. The rationales describe why the wrong answer choices are plausible but incorrect and are based in common misconceptions or common procedural errors and why the correct answer is correct. While these rationales speak to a possible and likely reason for the selection of the incorrect option by the student, these rationales do not contain definitive statements as to why the student chose the incorrect option or what we can infer about knowledge and skills of the student based on the students selection of an incorrect response. These multiple-choice questions are designed to assess student proficiency, not to diagnose specific misconceptions/errors with each and every incorrect option.

For each constructed-response question, there will be a commentary describing how the question measures the intended cluster, and sample student responses representing possible student errors or misconceptions at each possible score point.

The annotated questions do not represent the full spectrum of standards assessed on the State test, nor do they represent the full spectrum of how the Common Core should be taught and assessed in the classroom. Specific criteria for writing test questions as well as test information are available at http://www.engageny.org/resource/regents-exams.

Understanding Math Annotated Questions

All questions on the Regents Exam in Geometry (Common Core) are designed to measure the Common Core Learning Standards identified by the PARCC Model Content Framework for Geometry. More information about the relationship between the New York State Testing Program and PARCC can be found here: http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf.

Multiple Choice

Multiple-choice questions will primarily be used to assess procedural fluency and conceptual understanding. Multiple-choice questions measure the Standards for Mathematical Content and may incorporate Standards for Mathematical Practices and real-world applications. Some multiple-choice questions require students to complete multiple steps. Likewise, questions may measure more than one cluster, drawing on the simultaneous application of multiple skills and concepts. Within answer choices, distractors will all be based on plausible missteps.

Constructed Response

Constructed-response questions will require students to show a deep understanding of mathematical procedures, concepts, and applications, as well as demonstrating geometric concepts through constructions. The Regents Examination in Geometry (Common Core) contains 2-, 4-, and 6-credit constructed-response questions.
2-credit constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, 2-credit constructed-response questions may involve multiple steps, the application of multiple mathematics skills, and real-world applications. These questions may ask students to explain or justify their solutions and/or show their process of problem solving.
Constructed-response questions that are worth 4 credits require students to show their work in completing more extensive problems which may involve multiple tasks and concepts. Students will need to reason abstractly by constructing viable arguments to explain, justify, and/or prove geometric relationships in order to demonstrate conceptual understanding. Students will also need to reason quantitatively when solving real-world modeling problems.

There are two 6-credit constructed-response questions on the Regents Examination in Geometry (Common Core). One 6-credit question requires students to develop multi-step, extended logical arguments and proofs involving major content, and one 6-credit question requires students to use modeling to solve real-world problems.

#7

7 A shipping container is in the shape of a right rectangular prism with a length of 12 feet, a width of 8.5 feet, and a height of 4 feet. The container is completely filled with contents that weigh, on average, 0.25 pound per cubic foot. What is the weight, in pounds, of the contents in the container?

(1) 1,632
(2) 408
(3) 102
(4) 92

Measured CCLS Cluster: G-MG.A

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-MG.A because it requires the student to use density and volume in a modeling context. The shipping container is modeled by a rectangular prism; the student must determine its volume and use its density to find the weight of the contents in the container.

Rationale: Choices (1), (2), and (4) are plausible but incorrect and represent errors in writing or interpreting an expression based on a context involving density. Choosing the correct solution requires students to know how to set up correct expressions based on the context and perform computations accurately. Compare with question 35, which also assesses G-MG.A.

Answer Choice: (1) 1,632. This response is incorrect. The student may have misunderstood the relationship between the volume of an object and its density and divided the volume of the rectangular prism by the density.

Answer Choice: (2) 408. This response is incorrect. The student may have confused the concepts of volume and weight, calculating the volume only instead of using the density to determine the weight of the container.

Answer Choice: (3) 102. This response is the correct weight of the container. The weight of the container is calculated by multiplying the dimensions to determine the volume, then multiplying by the density.

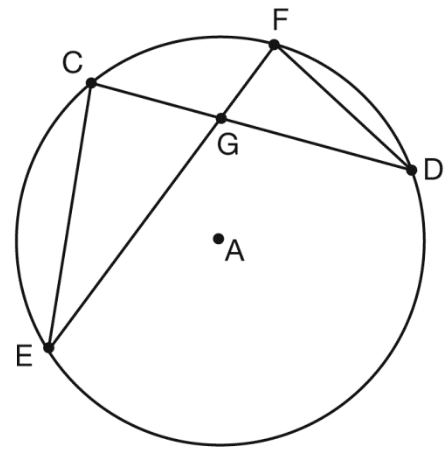
$12 \times 8.5 \times 4 = 408$

$408 \times 0.25 = 102$

Answer Choice: (4) 92. This response is incorrect. The student may have confused the concepts of surface area and volume, calculating the surface area of the container, then multiplying this quantity by the density.

#8

8 In the diagram of circle A shown below, chords $\overline{CD}$ and $\overline{EF}$ intersect at G, and chords $\overline{CE}$ and $\overline{FD}$ are drawn.



Which statement is *not* always true?

(1) $\overline{CG} \cong \overline{FG}$

(2) $\angle CEG \cong \angle FDG$

(3) $\dfrac{CE}{EG} = \dfrac{FD}{DG}$

(4) $\triangle CEG \sim \triangle FDG$

Measured CCLS Cluster: G-SRT.B

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.B because it requires the student to apply similarity criteria to reason about geometric relationships. The student must conclude that the two triangles are similar because they have two pairs of congruent angles and therefore the corresponding sides of the similar triangles are proportional. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another domain, Circles (G-C), which includes angles inscribed in circles.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when recognizing geometric relationships between triangles, angles, and segments in a circle and using triangle similarity to reason about those relationships. Choosing the correct solution requires students to know how to analyze a geometric diagram and apply the AA criterion for triangle similarity. Compare with questions 11, 15, and 31, which also assess G-SRT.B.

Answer Choice: (1) $\overline{CG} \cong \overline{FG}$. This response is correct because it is a statement that is not always true; line segment CG is not always congruent to line segment FG. These segments are only congruent when

$\triangle CEG \cong \triangle FDG$. A student that selects this response understands that the diagram implies that $\triangle CEG$ and $\triangle FDG$ are similar, but not necessarily congruent.

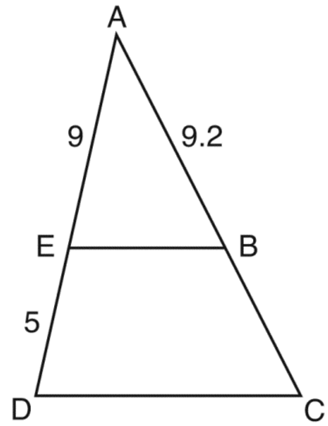
Answer Choice: (2) $\angle CEG \cong \angle FDG$. This response is incorrect because it is always true that $\angle CEG \cong \angle FDG$ because $\angle CEG$ and $\angle FDG$ are inscribed angles that intercept the same arc, CF. The student may not have recognized the angles as angles inscribed in the circle.

Answer Choice: (3) $\frac{CE}{EG} = \frac{FD}{DG}$. This response is incorrect because it is always true that $\frac{CE}{EG} = \frac{FD}{DG}$ because $\triangle CEG \sim \triangle FDG$ and corresponding sides of similar triangles are proportional. The student may have confused the concepts of congruence and similarity or made an error in defining the correspondence between $\triangle CEG$ and $\triangle FDG$.

Answer Choice: (4) $\triangle CEG \sim \triangle FDG$. This response is incorrect because it is always true that $\triangle CEG \sim \triangle FDG$; the triangles can be shown to satisfy the AA similarity criterion. The pairs of angles that can be used for the AA similarity criteria are the inscribed angles CEF and FDC because they intercept the same arc CF, inscribed angles ECD and DFE because they intercept the same arc, ED, and the vertical angles CGE and FGD because vertical angles are always congruent. The student may not have understood that the AA similarity criterion would apply to this situation or made an error in defining the correspondence between $\triangle CEG$ and $\triangle FDG$.

#11

11 In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.



What is the length of $\overline{AC}$, to the *nearest tenth*?

(1) 5.1
(2) 5.2
(3) 14.3
(4) 14.4

Measured CCLS Cluster: G-SRT.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.B because it requires the student to apply similarity criteria to solve a geometric problem. The student must analyze the given diagram and reason that the triangles are similar by the AA similarity criterion, then use the fact that corresponding sides of similar triangles are proportional to find the length of $\overline{AC}$.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when working with the concept of triangle similarity and applying similarity criteria to solve a geometric problem. Choosing the correct solution requires students to know how to analyze a geometric diagram and apply the AA criterion for similarity. Compare with questions 8, 15, and 31, which also assess G-SRT.B.

Answer Choice: (1) 5.1. This response is incorrect and does not represent the length of $\overline{AC}$. The student found the length of $\overline{BC}$, but did not add it to the length of $\overline{AB}$ to find the length of $\overline{AC}$.

Answer Choice: (2) 5.2. This response is incorrect and does not represent the length of $\overline{AC}$. The student may have attempted to find the length of $\overline{BC}$ instead of $\overline{AC}$ by assuming the difference between $\overline{AB}$ and $\overline{BC}$ had to be the same as the difference between $\overline{AE}$ and $\overline{AD}$.

Answer Choice: (3) 14.3. This response is correct and is the length of $\overline{AC}$. This length is determined by recognizing that $\triangle ABE$ is similar to $\triangle ACD$. The student reasons that triangle ABE and triangle ACD are similar using by using the fact that parallel lines form corresponding congruent angles and/or using the reflexive angle A. The student uses the similar triangles to write an equation for the length of $\overline{AC}$.

$$\frac{9}{9.2} = \frac{9+5}{x}$$

$$9x = 128.8$$

$$x = 14.3111\ldots$$

$$x \approx 14.3$$

Answer Choice: (4) 14.4. This response is incorrect and does not represent the length of $\overline{AC}$. The student may have attempted to find the length of $\overline{BC}$ instead of $\overline{AC}$ by assuming the difference between $\overline{AB}$ and $\overline{BC}$ had to be the same as the difference between $\overline{AE}$ and $\overline{AD}$. The student then added it to the length of $\overline{AB}$ to find the length of $\overline{AC}$.

#13

13 Quadrilateral $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

(1) $\overline{AC}$ and $\overline{BD}$ bisect each other.

(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$

(3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

(4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

Measured CCLS Cluster: G-CO.C

Key: (4)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.C because it requires the student to reason using the theorems involving the diagonals and the sides of a quadrilateral that would prove it a parallelogram.

Rationale: Choices (1), (2), and (3) are plausible but incorrect. They represent common student errors made when working with parallelograms and indicate a limited understanding of how to reason about theorems of parallelograms. Choosing the correct solution requires students to know how to reason using theorems involving parallelograms. Compare with questions 17, 26, 32, and 33, which also assess G-CO.C.

Answer Choice: (1) $\overline{AC}$ and $\overline{BD}$ bisect each other. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if the diagonals of a quadrilateral bisect each other, then it is a parallelogram. The student may have assumed that this theorem applied only to a larger subset of quadrilaterals than the parallelograms, such as trapezoids.

Answer Choice: (2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram. The student may have concluded that a quadrilateral whose opposite sides are congruent is a rectangle, without also reasoning that a rectangle is a parallelogram.

Answer Choice: (3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. This response is incorrect because this information *is* sufficient to prove that $ABCD$ is a parallelogram; if a quadrilateral has one pair of opposite sides that are both congruent and parallel, then it is a parallelogram. The student may have assumed that information about only one pair of sides would not be sufficient to determine a parallelogram.

Answer Choice: (4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. This response is correct because this information is *not* sufficient to prove that $ABCD$ is a parallelogram. A quadrilateral that has one pair of opposite sides congruent and the other pair of opposite sides parallel may not be a parallelogram. A student who selects this response understands how to reason about parallelograms using given information.

#14

14 The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

(1) center (0,3) and radius 4

(2) center (0,−3) and radius 4

(3) center (0,3) and radius 16

(4) center (0,−3) and radius 16

Measured CCLS Cluster: G-GPE.A

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within G-GPE.A because it requires the student to complete the square to rewrite the given equation representing a circle and identify the coordinates of its center and the length of its radius. The question also requires the student to employ Mathematical Practice 7 (Look for and make use of structure) because the student must notice and use the structure of the equation to rewrite it and determine properties of the circle.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when completing the square and identifying the coordinates of the center and length of the radius from its equation. Choosing the correct solution requires students to know how to complete the square and identify the radius and the coordinates of the center.

Answer Choice: (1) center $(0,3)$ and radius 4. This response is incorrect and does not show the center for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square to rewrite the equation of the circle and identified the radius by taking the square root of 16, but incorrectly interpreted the coordinates of the center as $(0,3)$.

Answer Choice: (2) center $(0, -3)$ and radius 4. This response is correct and shows the center and radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. A student who selects this response understands how to complete the square and identify the coordinates of the center and length of the radius.

$x^2 + y^2 + 6y = 7$

$x^2 + y^2 + 6y + 9 = 7 + 9$

$x^2 + (y + 3)^2 = 16$

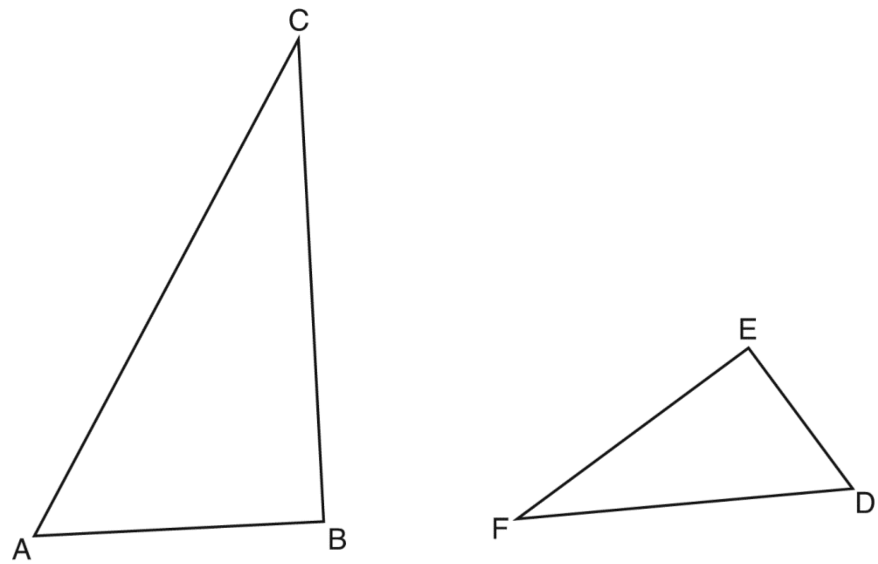
Center $(0, -3)$; radius 4

Answer Choice: (3) center $(0,3)$ and radius 16. This response is incorrect and does not show the center and radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square to rewrite the equation of the circle, but made an error in interpreting the coordinates of the center, while also not taking the square root of the constant, 16, to find the length of the radius.

Answer Choice: (4) center $(0, -3)$ and radius 16. This response is incorrect and does not show the radius for the circle represented by the equation $x^2 + y^2 + 6y = 7$. The student may have completed the square and identified the coordinates of the center, but made an error by not taking the square root of 16 to find the length of the radius.

#15

15 Triangles ABC and DEF are drawn below.



If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

(1) $\angle CAB \cong \angle DEF$

(2) $\frac{AB}{CB} = \frac{FE}{DE}$

(3) $\triangle ABC \sim \triangle DEF$

(4) $\frac{AB}{DE} = \frac{FE}{CB}$

Measured CCLS Cluster: G-SRT.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.B because it requires the student to apply triangle similarity criteria to reach a conclusion about geometric relationships. Specifically, the student must reason about the similarity of two triangles by applying the SAS similarity criterion.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when working with triangle similarity criteria and how to apply triangle similarity criteria. Choosing the correct solution requires students to know how to analyze a diagram and apply similarity criteria to reach a conclusion. Compare with questions 8, 11, and 31, which also assess G-SRT.B.

Answer Choice: (1) $\angle CAB \cong \angle DEF$. This response is incorrect because the given information does not imply that $\angle CAB$ and $\angle DEF$ must be congruent. The student may have assumed that because the triangles are similar, any pair of angles would be congruent.

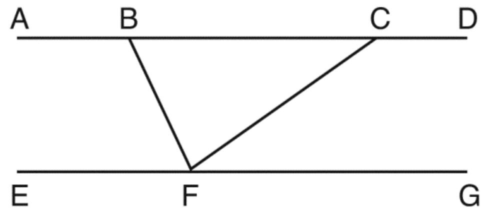
Answer Choice: (2) $\frac{AB}{CB} = \frac{FE}{DE}$. This response is incorrect because the given information does not imply the proportional relationship illustrated by this equation. The student may have concluded that the triangles are similar, and recognized a proportional relationship between sides, but incorrectly identified the relationship of the corresponding sides in the proportion.

Answer Choice: (3) $\triangle ABC \sim \triangle DEF$. This response is correct and shows a correct conclusion from the given information. The student understands that because the two pairs of corresponding sides are proportional and the included angles are congruent, the triangles are similar by the SAS similarity criterion. A student who selects this response understands how to apply triangle similarity criteria.

Answer Choice: (4) $\frac{AB}{DE} = \frac{FE}{CB}$. This response is incorrect because the given information does not imply the proportional relationship illustrated by this equation. The student may have concluded that the triangles are similar, and recognized a proportional relationship between sides, but incorrectly identified the relationship of the corresponding sides in the proportion.

#17

17 Steve drew line segments $ABCD$, EFG, BF, and CF as shown in the diagram below. Scalene $\triangle BFC$ is formed.



Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

(1) $\angle CFG \cong \angle FCB$

(2) $\angle ABF \cong \angle BFC$

(3) $\angle EFB \cong \angle CFB$

(4) $\angle CBF \cong \angle GFC$

Measured CCLS Cluster: G-CO.C

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.C because it requires the student to reason about lines and angles by identifying congruent alternate interior angles to prove that lines are parallel. Additionally, the item requires the student to employ Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others) because the student must identify evidence that will support the claim that two lines are parallel.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with lines and angles and indicate a limited understanding of how to reason about lines and angles. Choosing the correct solution requires students to identify the correct angle pairs needed for the parallel lines to be proven. Compare with questions 13, 26, 32, and 33, which also assess G-CO.C.

Answer Choice: (1) $\angle CFG \cong \angle FCB$. This response is correct and is valid evidence that will support the claim that the lines are parallel. The student identified that transversal $\overline{CF}$ intersects $\overline{ABCD}$ and $\overline{AEG}$ to form the alternate interior angles CFG and FCB, and reasoned that that when segments are intersected by a transversal such that the alternate interior angles are congruent, the segments are parallel. A student who selects this response understands how to reason about lines and angles.

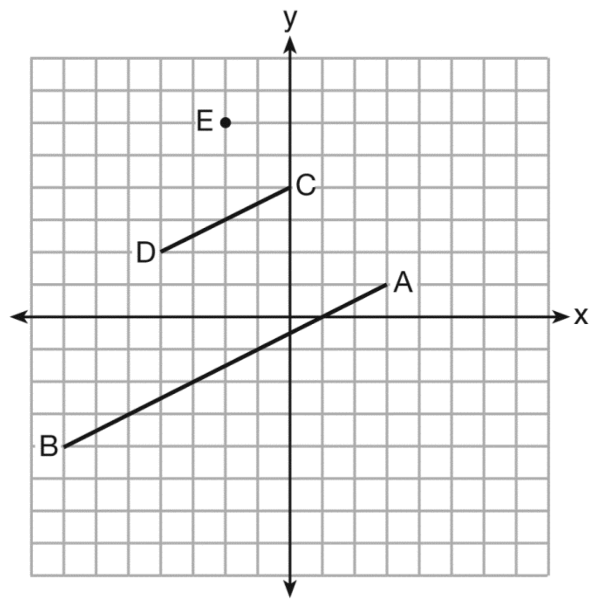
Answer Choice: (2) $\angle ABF \cong \angle BFC$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as alternate interior angles, reasoning then that because they are congruent, the lines would be parallel.

Answer Choice: (3) $\angle EFB \cong \angle CFB$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as alternate interior angles, reasoning then that because they are congruent, the lines would be parallel.

Answer Choice: (4) $\angle CBF \cong \angle GFC$. This response is incorrect and is not evidence that will support the claim that the lines are parallel. The student may have mistakenly identified these angles as corresponding angles, reasoning then that because they are congruent, the lines would be parallel.

#18

18 In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



Which ratio is equal to the scale factor k of the dilation?

(1) $\frac{EC}{EA}$

(2) $\frac{BA}{EA}$

(3) $\frac{EA}{BA}$

(4) $\frac{EA}{EC}$

Measured CCLS Cluster: G-SRT.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.A because it requires the student to use similarity transformations to reason about the effect of a dilation on a line segment. Specifically, the student must understand that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. The student may also understand that the line segments joining the center of dilation and the corresponding endpoints of the given line segment and its image forms two similar triangles.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with dilations in the plane. Students who select these responses may not understand the effect of a dilation on the length of a segment. Choosing the correct solution requires students to know that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. Compare with question 22, which also assesses G-SRT.A.

Answer Choice: (1) $\frac{EC}{EA}$. This response is correct and is the scale factor of the dilation. The student correctly chose the scale factor by identifying a ratio between a dimension of the image and the corresponding dimension of its pre-image. A student who selects this response understands the effect of a dilation on the length of a segment.

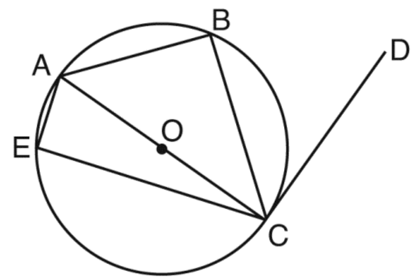
Answer Choice: (2) $\frac{BA}{EA}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that the ratio of any two distances in a diagram would be equal to the scale factor of the dilation, or lacked a general understanding of how figures are dilated.

Answer Choice: (3) $\frac{EA}{BA}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that the ratio of any two distances in a diagram would be equal to the scale factor of the dilation, or lacked a general understanding of how figures are dilated.

Answer Choice: (4) $\frac{EA}{EC}$. This response is incorrect and is not the scale factor of the dilation. The student may have incorrectly assumed that $\overline{AB}$ was the image of $\overline{CD}$, or lacked a general understanding of how figures are dilated.

#20

20 In circle O shown below, diameter $\overline{AC}$ is perpendicular to $\overline{CD}$ at point C, and chords $\overline{AB}$, $\overline{BC}$, $\overline{AE}$, and $\overline{CE}$ are drawn.



Which statement is *not* always true?

(1) $\angle ACB \cong \angle BCD$

(2) $\angle ABC \cong \angle ACD$

(3) $\angle BAC \cong \angle DCB$

(4) $\angle CBA \cong \angle AEC$

Measured CCLS Cluster: G-C.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-C.A because it requires the student to apply theorems to circles. The student must reason using theorems about inscribed angles, angles formed by a chord and a tangent, and other circle relationships to determine the statement that is not always true.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with relationships in circles and applying theorems to circles. Choosing the correct solution requires that students be able to reason using relationships between angles and segments in circles.

Answer Choice: (1) $\angle ACB \cong \angle BCD$. This response is correct because it is *not* always true. The angles *ACB* and *BCD* are not always congruent since the arc intercepted by the inscribed angle ACB and the arc intercepted by the angle BCD, formed by the intersection of the chord BC and the tangent CD, are not always congruent. A student who selects this response understands how to apply theorems to circles.

Answer Choice: (2) $\angle ABC \cong \angle ACD$. This response is incorrect because it *is* always true. The inscribed $\angle ABC$ intercepts a semicircle, therefore $\angle ABC$ is a right angle. It is given that $\overline{AC}$ and $\overline{CD}$ are perpendicular, therefore $\angle ACD$ is a right angle. The student may not have used these relationships in the circle to reason that $\angle ABC$ is a right angle.

Answer Choice: (3) $\angle BAC \cong \angle DCB$. This response is incorrect because it *is* always true. The inscribed $\angle BAC$ and the angle formed by the intersection of chord $\overline{BC}$ and tangent $\overline{CD}$ intercept the same arc BC, therefore these angles are congruent because both their measures are half the measure of the intercepted arc. The student may not have used these relationships in the circle to reason that these angles are congruent.

Answer Choice: (4) $\angle CBA \cong \angle AEC$. This response is incorrect because it *is* always true. The inscribed angles *CBA* and *AEC* both intercept a semicircle, and are congruent because the measures of inscribed angles are half the measure of the intercepted arc and the arc, measure of all semicircles are equal. The student may not have used these relationships in the circle to reason that these angles are congruent.

#22

22 The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

(1) $2x + 3y = 5$ (3) $3x + 2y = 5$

(2) $2x - 3y = 5$ (4) $3x - 2y = 5$

Measured CCLS Cluster: G-SRT.A

Key: (1)

Commentary: This question measures the knowledge and skills described by the standards within G-SRT.A because it requires the student to use similarity transformations to reason about lines. The student must reason that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of the dilation. The question is also an example of the instructional shift of coherence, since the student must draw on understandings from another domain, Expressing Geometric Properties with Equations (G-GPE), including work with coordinates and the slope criteria for parallel lines.

Rationale: Choices (2), (3), and (4) are plausible but incorrect. They represent common student errors made when working with dilated lines represented by equations. Choosing the correct solution requires that students be able to reason about how a dilation affects the equation of a line. Compare with question 18, which also assesses G-SRT.A.

Answer Choice: (1) $2x + 3y = 5$. This response is correct because it represents a line parallel to the given line. Knowing that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of dilation, the student found the slope of the line and chose the line with the same slope as the given line. When the lines $3y = -2x + 8$ and $2x + 3y = 5$ are rewritten in the form $y = mx + b$, the slopes are equal. Since both lines have the same slope but different *y*-intercepts, they are parallel. A student who selects this response understands that the image of a dilated line is always parallel to its pre-image when the line does not pass through the center of the dilation.

$3y = -2x + 8$

$y = -\frac{2}{3}x + \frac{8}{3}$

$\text{slope} = -\frac{2}{3}$

$2x + 3y = 5$

$3y = -2x + 5$

$y = -\frac{2}{3}x + \frac{5}{3}$

$\text{slope} = -\frac{2}{3}$

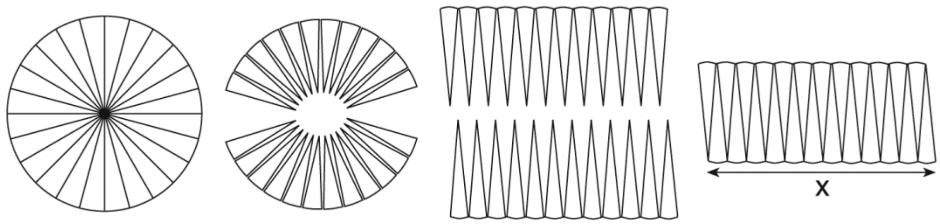
Answer Choice: (2) $2x - 3y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope of $\frac{2}{3}$, which is the opposite of the slope of the given line. The student may have incorrectly concluded that parallel lines have opposite slopes or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

Answer Choice: (3) $3x + 2y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope of $-\frac{3}{2}$, which is the reciprocal of the slope of the given line. The student may have incorrectly concluded that parallel lines have reciprocal slopes or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

Answer Choice: (4) $3x - 2y = 5$. This response is incorrect because it represents a line that is not parallel to the given line. The line given by this equation has a slope that is the negative reciprocal of the slope of the given line. The student may have incorrectly concluded that the image of a dilated line is perpendicular to its pre-image or the student may have incorrectly rewritten the equation to have a slope that is equal to the slope of the given line.

#23

23 A circle with a radius of 5 was divided into 24 congruent sectors. The sectors were then rearranged, as shown in the diagram below.



To the *nearest integer*, the value of x is

(1) 31 (3) 12

(2) 16 (4) 10

Measured CCLS Cluster: G-GMD.A

Key: (2)

Commentary: This question measures the knowledge and skills described by the standards within G-GMD.A because it requires the student to analyze an informal argument for the area of a circle. The diagram shows a circle decomposed into congruent sectors, which are reassembled to form a figure that approximates a parallelogram with a base equal to approximately half the circumference of the circle and a height equal to the radius of the circle. The student must reason using the informal argument and knowledge of the circle area formula to determine an approximate length of the base of the parallelogram-like figure.

Rationale: Choices (1), (3), and (4) are plausible but incorrect. They represent common student errors made when working with the area of a circle and indicate a limited understanding of an informal argument for the area of a circle. Choosing the correct solution requires that students be able to reason using an informal argument for the area of a circle.

Answer Choice: (1) 31. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that the value of x would be equivalent to the circumference of the circle.

Answer Choice: (2) 16. This response is correct and represents an approximate length of the base of the figure. The area of the circle can be determined using the formula $A = \pi r^2$; once this is found, the student must recognize that the area of the parallelogram-like figure is the same as area of the circle, $5x = 25\pi$. A student who selects this response understands an informal argument for the area of a circle.

$A = \pi r^2$

$5x = 25\pi$

$x = 5\pi$

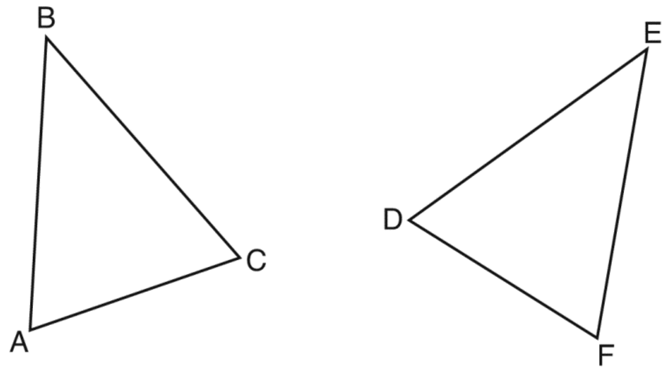
$x \approx 16$

Answer Choice: (3) 12. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that because the circle was divided into 24 congruent sectors and the base of the figure accounts for half of these sectors, that the base is 12.

Answer Choice: (4) 10. This response is incorrect and does not represent an approximate length of the base of the figure. The student may have assumed that the parallelogram-like figure would have a base length that is equal to the length of the diameter of the circle.

#24

24 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?



(1) $AB = DE$ and $BC = EF$

(2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

(3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$.

(4) There is a sequence of rigid motions that maps point A onto point D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$.

Measured CCLS Cluster: G-CO.B

Key: (3)

Commentary: This question measures the knowledge and skills described by the standards within G-CO.B because it requires the student to use rigid motions to reason about the congruence of triangles. The diagram shows two triangles; the student must determine which evidence, including information about corresponding angles and sides and also various rigid motions, is sufficient to prove the triangles are congruent.

Rationale: Choices (1), (2), and (4) are plausible but incorrect. They represent common student errors made when reasoning about the congruence of triangles. Choosing the correct solution requires that students be able to carefully reason using triangle congruence criteria and the definition of congruence in terms of rigid motions. Compare with question 30, which also assesses G-CO.B.

Answer Choice: (1) $AB = DE$ and $BC = EF$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. Two pairs of corresponding sides of equal measure do not meet the congruence criteria for congruent triangles. The student may have a misconception about the criteria needed to conclude the triangles are congruent.

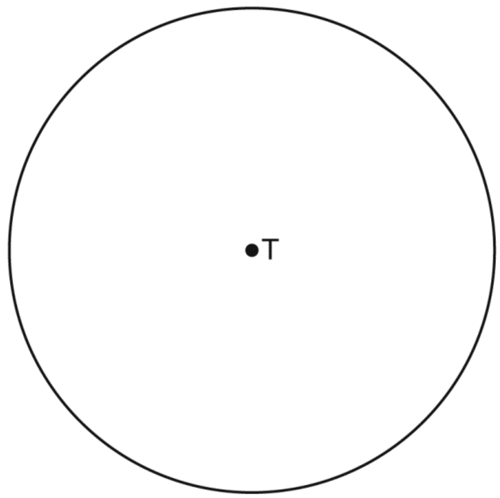
Answer Choice: (2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. Two or more pairs of congruent corresponding angles do not meet the congruence criteria for congruent triangles. The student may have confused the criteria for triangle congruence with the criteria for triangle similarity.

Answer Choice: (3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$. This response is correct because it is sufficient evidence to prove the triangles congruent. If there exists a rigid motion that maps all three sides of one triangle onto three corresponding sides of another triangle, then the triangles are congruent. A student who selects this response understands how to reason about the congruence of triangles.

Answer Choice: (4) There is a sequence of rigid motions that maps point A onto point D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$. This response is incorrect and is insufficient evidence to prove that the triangles are congruent. There is one pair of corresponding congruent sides and one pair of corresponding congruent angles, but it is still possible that one triangle is not mapped onto the other. The student may have incorrectly assumed that mapping point A onto point D will result in $\angle A \cong \angle D$.

#25

25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



Measured CCLS Cluster: G-CO.D

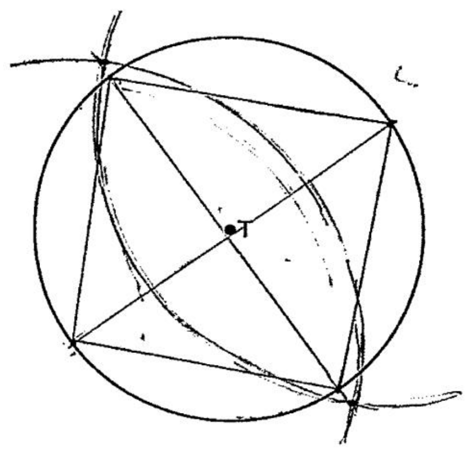
Commentary: The question measures the knowledge and skills described by the standards within G-CO.D because it requires the student to construct a square inscribed in a circle.

Rationale: This question requires students to construct a square inscribed in a circle. As indicated in the rubric, a correct response requires a correct construction showing all appropriate arcs.

Sample student responses and scores appear on the following pages.

Question 25

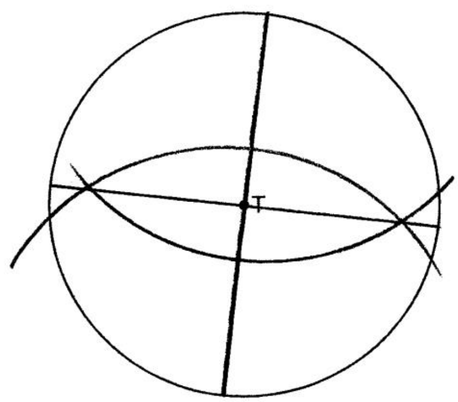
25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



Score 2: The student drew a correct construction showing all appropriate construction marks and the square was drawn.

Question 25

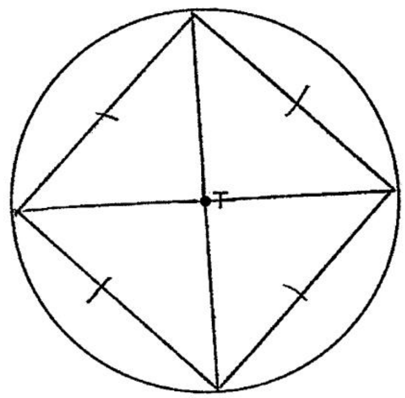
25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



Score 1: The student drew a correct construction showing all appropriate construction marks, but the square was not drawn.

Question 25

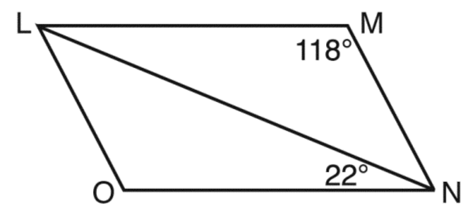
25 Use a compass and straightedge to construct an inscribed square in circle T shown below. [Leave all construction marks.]



Score 0: The student made a drawing that is not a construction.

#26:

26 The diagram below shows parallelogram $LMNO$ with diagonal $\overline{LN}$, $m\angle M = 118°$, and $m\angle LNO = 22°$.



Explain why $m\angle NLO$ is 40 degrees.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason using theorems about parallelograms and triangles to explain the measure of the noted angle. Theorems that students might use include: opposite angles of a parallelogram are congruent, consecutive angles of a parallelogram are supplementary, and/or angles of a triangle add up to 180 degrees. Additionally, the item requires the student to employ Mathematical Practice 3 because the student must identify and explain evidence that will support the claim that $\angle NLO$ measures 40 degrees.

Rationale: This question requires students to explain the measure of the noted angle in a diagram. One possible line of reasoning could be that since $\mathrm{m}\angle M = 118°$ and opposite angles of a parallelogram are congruent, $\mathrm{m}\angle O = 118°$. Then, the angles of triangle LNO add up to $180°$ so $\mathrm{m}\angle NLO + \mathrm{m}\angle LNO + \mathrm{m}\angle O = 180°$, therefore $\mathrm{m}\angle NLO + 22° + 118° = 180°$ and $\mathrm{m}\angle NLO = 40°$.

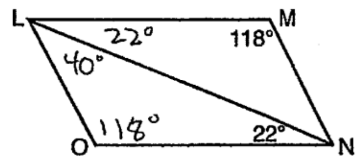
Another line of reasoning is that since opposite sides of a parallelogram are parallel, then the alternate interior angles LNO and NLM are congruent and therefore $\mathrm{m}\angle NLM = 22°$. Then, consecutive angles M and MLO of parallelogram $LMNO$ are supplementary and therefore $\mathrm{m}\angle M + \mathrm{m}\angle NLM + \mathrm{m}\angle NLO = 180°$. So, $118° + 22° + \mathrm{m}\angle NLO = 180°$ and $\mathrm{m}\angle NLO = 40°$.

Compare with items 13, 17, 32, and 33, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 26

26 The diagram below shows parallelogram *LMNO* with diagonal $\overline{LN}$, m$\angle M$ = 118°, and m$\angle LNO$ = 22°.



Explain why m$\angle NLO$ is 40 degrees.

$\angle LON$ is 118° b/c opposite ∡'s of a ▱ are $\cong$.

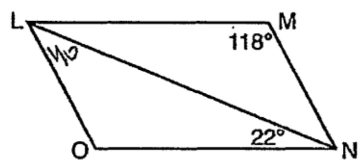
~~A~~ △'s ∡ measures add up to 180°.

118+22 = 140 so $\angle NLO$ must be 40°,

Score 2: The student has a complete and correct response.

Question 26

26 The diagram below shows parallelogram *LMNO* with diagonal $\overline{LN}$, m$\angle M$ = 118°, and m$\angle LNO$ = 22°.



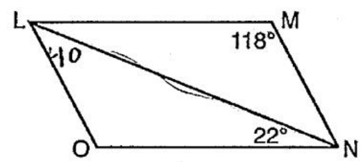
Explain why m$\angle NLO$ is 40 degrees.

becaus if you add 118 and 22 you Get 140 and every triangle equals 180 so you subtract 140 from 180 to get 40.

Score 1: The student gave an incomplete explanation, because a geometric relationship between 118° and 22° was not established.

Question 26

26 The diagram below shows parallelogram $LMNO$ with diagonal $\overline{LN}$, $m\angle M = 118°$, and $m\angle LNO = 22°$.



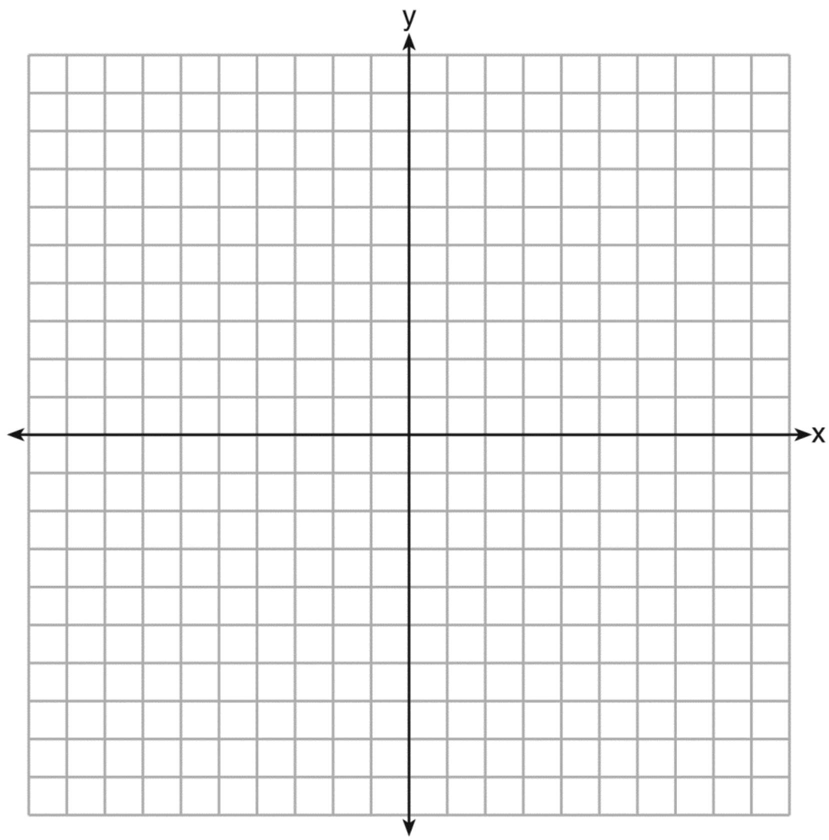
Explain why $m\angle NLO$ is 40 degrees.

because ∡M & ∡N are complemantary angles so when you add them up and equal it to 180 you get 140 then subtract that from 180 and you get 40°

Score 0: The student gave a completely incorrect explanation.

#27:

27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP{:}PB$ is 2:3.
[The use of the set of axes below is optional.]



Measured CCLS Cluster: G-GPE.B

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.B because the student is required to use coordinates to apply understanding of geometric figures. The student uses coordinates to determine the location of a point dividing a directed line segment in the given ratio.

Rationale: This question requires students to find the coordinates of a point on a line segment that divides the line segment into a given ratio, optionally using the provided set of axes. The student who uses the set of axes must graph the line segment and divide it into five congruent parts. This can be done by dividing both the vertical change and the horizontal change between points A and B by 5, resulting in vertical increments of 1 and horizontal increments of 2; the segment can be divided into five equal parts by starting at point A and repeatedly moving right two units, then up one unit to mark the segment. The student will see that the line segment has been divided into five equal parts. The point that divides the line segment, such that $AP{:}PB$ is 2:3, is $(-2,-3)$.

A second method to solve the problem involves the same principle, but uses numerical coordinates. Two-fifths of the horizontal and vertical distance between A and B is added to the x- and y-coordinates of A, respectively:

$x = -6 + \frac{2}{5}(4 - -6)$ $\qquad$ $y = -5 + \frac{2}{5}(0 - -5)$

$x = -6 + \frac{2}{5}(10)$ $\qquad$ $y = -5 + \frac{2}{5}(5)$

$x = -6 + 4$ $\qquad$ $y = -5 + 2$

$x = -2$ $\qquad$ $y = -3$

$$(-2, -3)$$

Compare with question 36, which also assesses G-GPE.B.

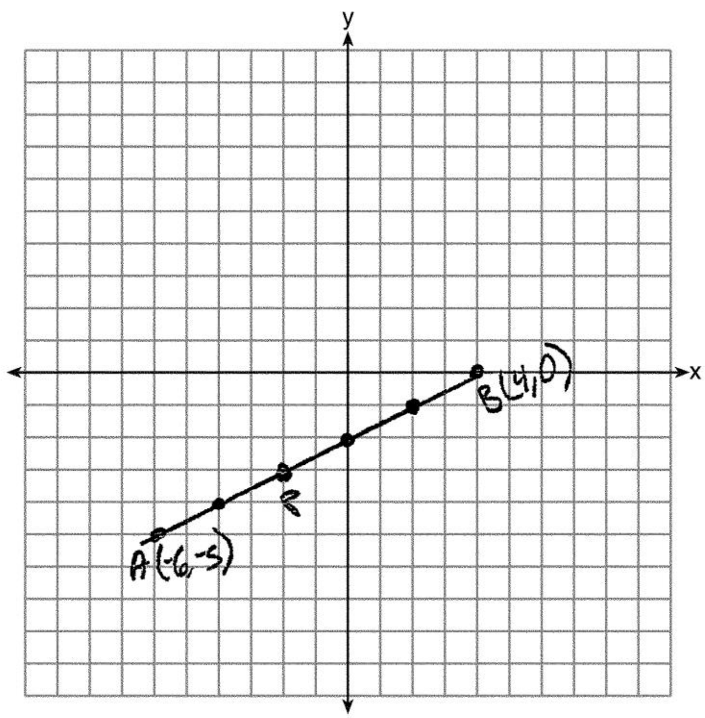
Sample student responses and scores appear on the following pages.

Question 27

27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP{:}PB$ is 2:3.
[The use of the set of axes below is optional.]

$$d_{\overline{AB}} = \sqrt{(x-x)^2+(y-y)^2}$$
$$= \sqrt{(-6-4)^2+(-5-0)^2}$$
$$= \sqrt{(-10)^2+(-5)^2}$$
$$= \sqrt{100+25}$$
$$= \sqrt{125}$$
$$= \sqrt{25}\cdot\sqrt{5}$$
$$= 5\sqrt{5}$$

$P(-2,-3)$



Score 2: The student has a complete and correct response. The student showed correct work that was not necessary.

Question 27

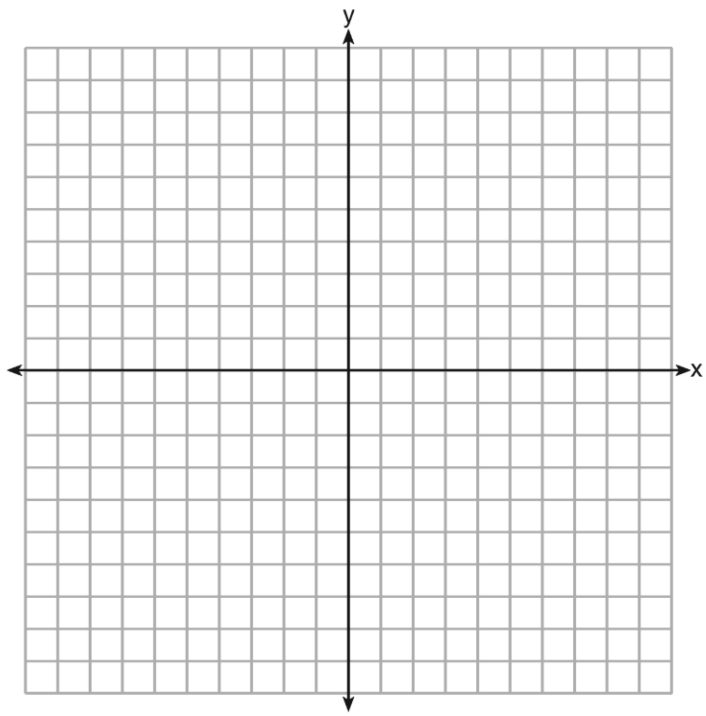
27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is 2:3.
[The use of the set of axes below is optional.]

$$P\left(-6+\frac{2}{3}(10), -5+\frac{2}{3}(5)\right)$$

$$P\left(-6+\frac{20}{3}, -5+\frac{10}{3}\right)$$

$$P\left(-6+6\tfrac{2}{3}, -5+3\tfrac{1}{3}\right)$$

$$P\left(\tfrac{2}{3}, -1\tfrac{2}{3}\right)$$



Score 1: The student made an error by multiplying by $\frac{2}{3}$ instead of $\frac{2}{5}$.

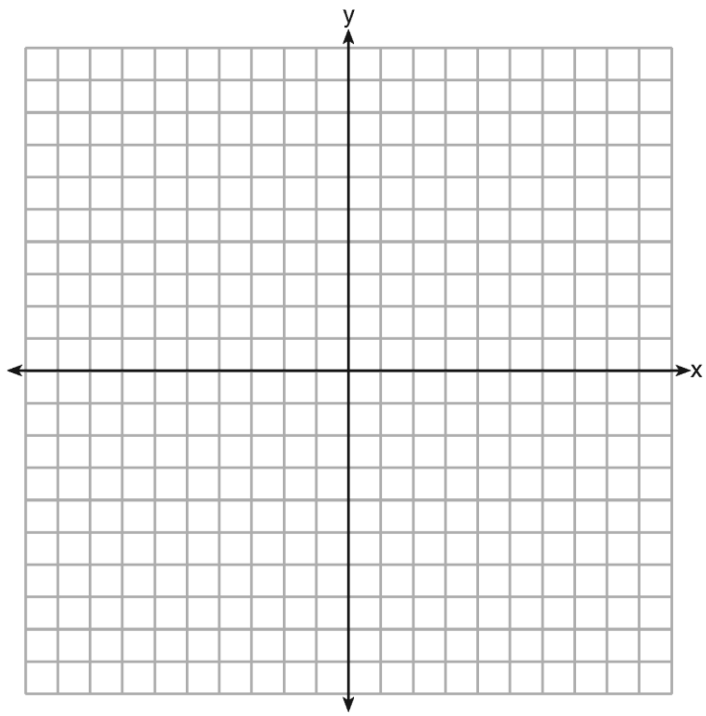
Question 27

27 The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP{:}PB$ is 2:3.
[The use of the set of axes below is optional.]

$$\frac{-6+4}{2}, \frac{-5+0}{2}$$

$$-\frac{2}{2}, -\frac{5}{2}$$

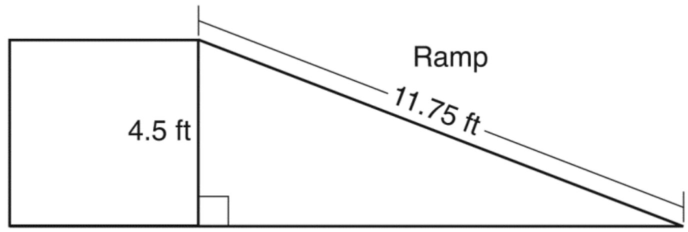
$$\left(-1, -\frac{5}{2}\right)$$



Score 0: The student's use of the midpoint formula was irrelevant to the question.

#28:

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

Measured CCLS Cluster: G-SRT.C

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.C because the student is required to apply understanding of relationships between angles and sides of right triangles using trigonometry. The question also requires the student to employ Mathematical Practice 4 (Model with mathematics), because the student must use right triangle trigonometry to solve a real-world problem.

Rationale: This question instructs the student to determine and state the angle of elevation a ramp makes with the ground given the length of the ramp and the height the ramp will reach. The student must determine which trigonometric ratio is appropriate for finding the angle of elevation. Although there are multiple methods of solving this problem, the most direct method is to use arcsine to find the angle of elevation, since the two given side lengths of the right triangle are the side opposite the angle of elevation and the hypotenuse of the right triangle.

Let x be the angle of elevation of the ramp.

$$\sin x = \frac{4.5}{11.75}$$

$$x = \arcsin\left(\frac{4.5}{11.75}\right)$$

$$x = 22.51831413$$

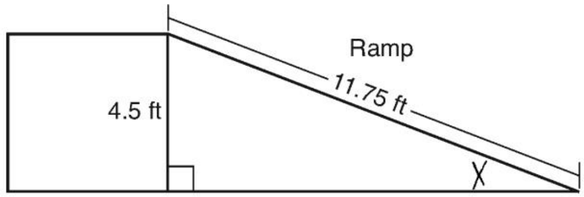
$$x = 23$$

Compare with question 34, which also assesses G-SRT.C.

Sample student responses and scores appear on the following pages.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$X = \sin^{-1}\left(\frac{4.5}{11.75}\right)$$

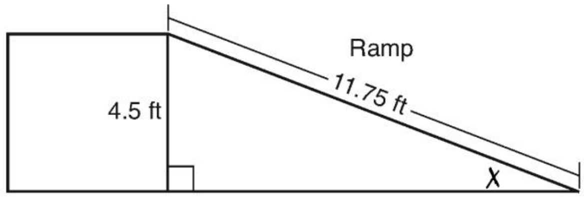
$$X = 22.518$$

$$X = 23^{\circ}$$

Score 2: The student has a complete and correct response.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$\text{Tan } x = \frac{4.5}{11.75}$$

$$x = \tan^{-1} \frac{4.5}{11.75}$$

$$x = 20.9557767306$$

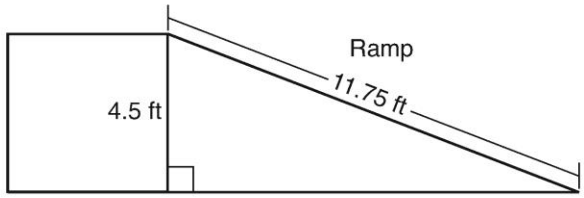
$$x = 21$$

21

Score 1: The student made an error by using the wrong trigonometric function, but found an appropriate angle of elevation.

Question 28

28 The diagram below shows a ramp connecting the ground to a loading platform 4.5 feet above the ground. The ramp measures 11.75 feet from the ground to the top of the loading platform.



Determine and state, to the *nearest degree*, the angle of elevation formed by the ramp and the ground.

$$a^2 + b^2 = c^2$$

$$(4.5)^2 + b^2 = (11.75)^2$$

$$20.25 + b^2 = 138.0625$$

$$-20.25 \qquad -20.25$$

$$\sqrt{b^2} = \sqrt{117.8125}$$

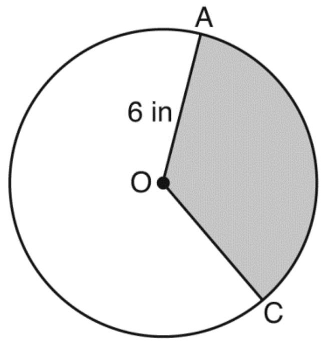
$$b = 10.854146673$$

$$\boxed{11^\circ}$$

Score 0: The student had a completely incorrect response.

#29

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



Measured CCLS Cluster: G-C.B

Commentary: The question measures the knowledge and skills described by the standards within G-C.B because the student is required to apply their understanding of equations that relate radii, arc length, and/or areas of sectors and circles. Specifically, given the area of a sector and radius of the circle, the student must determine the central angle that defines the sector. The angle may be determined in degrees or radians, since it is not specified in the question stem.

Rationale:

$A = \pi r^2$

$A = \pi 6^2$

$A = 36\pi$

Let x represent m$\angle AOC$

$\frac{12\pi}{36\pi} = \frac{x}{360}$

$36\pi x = 4320\pi$

$x = 120°$

OR

$A = \pi r^2$

$A = \pi 6^2$

$A = 36\pi$

Let x represent m$\angle AOC$

$\frac{12\pi}{36\pi} = \frac{x}{2\pi}$

$36\pi x = 24(\pi)(\pi)$

$x = \frac{24(\pi)(\pi)}{36\pi}$

$x = \frac{2\pi}{3}$ radians

OR

$A = \pi r^2$

$A = \pi 6^2$

$A = 36\pi$

$C = \pi d$

$C = 12\pi$

Let S represent m$\widehat{AC}$

$\frac{12\pi}{36\pi} = \frac{S}{12\pi}$

$36\pi S = 144(\pi)(\pi)$

$S = 4\pi$

Let θ represent m$\angle AOC$

$S = \theta r$

$4\pi = \theta(6)$

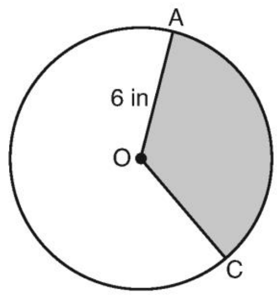
$\theta = \frac{4\pi}{6}$

$\theta = \frac{2\pi}{3}$ radians

Sample student responses and scores appear on the following pages.

Question 29

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



$A = \pi r^2$

$= 6^2 \cdot \pi$

$= 36\pi$

$\frac{12\pi}{36\pi} = \frac{1}{3}$

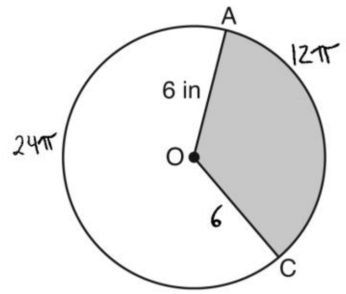
$\frac{1}{3} \cdot 360$

120°

Score 2: The student has a complete and correct response.

Question 29

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



$A = \pi r^2$

$A = \pi 6^2$

$A = 36\pi$

$36\pi - 12\pi = 24\pi$

$\frac{24\cancel{\pi}}{36\cancel{\pi}} = \frac{x}{360}$

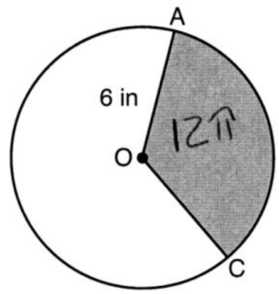
$8640 = 36x$

$240° = x$

Score 1: The student made an error by finding the central angle for the unshaded sector.

Question 29

29 In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m$\angle AOC$.



$$\text{Sector} = \theta r$$

$$\frac{12\pi}{6} = \frac{\theta(6)}{6}$$

$$2\pi = \theta$$

$$m\angle AOC = 2\pi$$

Score 0: The student had a completely incorrect response.

#30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Measured CCLS Cluster: G-CO.B

Commentary: The question measures the knowledge and skills described by the standards within G-CO.B because the student is required to use rigid motions to reason about the congruence of geometric figures. Additionally, the item requires the student to employ Mathematical Practice 3 because the student must identify and explain evidence that will support the claim that the triangles are congruent.

Rationale: This question asks students to explain why a triangle is congruent to its image after a reflection. Since a reflection is a rigid motion and all rigid motions will map one triangle onto the other and preserve side lengths, then the triangle and its image are congruent.

Compare with question 24, which also assesses G-CO.B.

Sample student responses and scores appear on the following pages.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Reflections are rigid motions and Rigid motions ~~or~~ keep distances the same. So $\overline{AB} \cong \overline{A'B'}$ and $\overline{BC} \cong \overline{B'C'}$ and $\overline{AC} \cong \overline{A'C'}$, so Δ's $\cong$ SSS

Score 2: The student has a complete and correct response.

Question 30

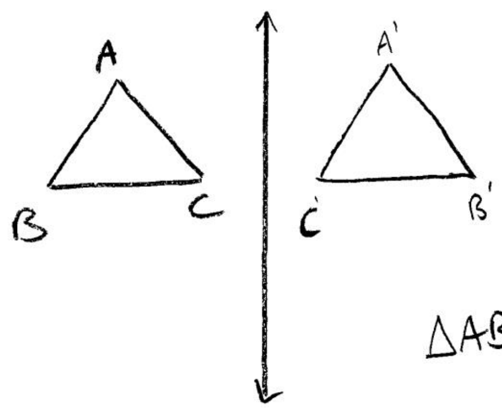
30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.

Because reflections are rigid motions.

Score 1: The student wrote an incomplete explanation.

Question 30

30 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $A'B'C'$.



$\triangle ABC \cong \triangle A'B'C'$

Score 0: The student did not provide an explanation.

#31

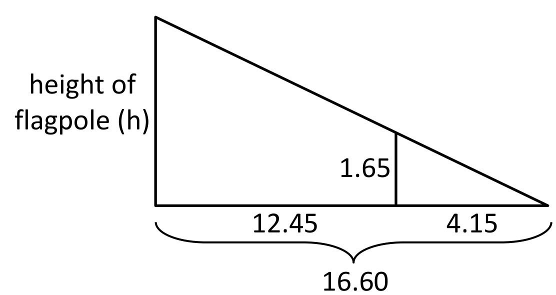
31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*

Measured CCLS Cluster: G-SRT.B

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.B because the student is required to apply similarity criteria to geometric figures to solve problems. The student must recognize that the situation can be modeled with two triangles that are similar by the AA similarity criteria, then apply that corresponding sides of similar triangles are proportional to solve the problem. The question is also an example of the instructional shift of coherence, as the student may draw on understandings from another cluster, G-SRT.C, in using right triangle trigonometry to find the height of the flagpole. The question also requires the student to employ Mathematical Practice 4, because the student must model with triangles to solve a real-world problem.

Rationale: This question asks students to find the height of a flagpole using its shadow length, the height of Tim, and his shadow length. This scenario can be modeled using two triangles which are similar by AA. Once similar triangles are determined, the height of the flagpole can be found using the corresponding sides of the similar triangles in a proportion.

$$\frac{16.60}{h} = \frac{4.15}{1.65}$$
$$4.15h = 27.39$$
$$h = 6.6$$

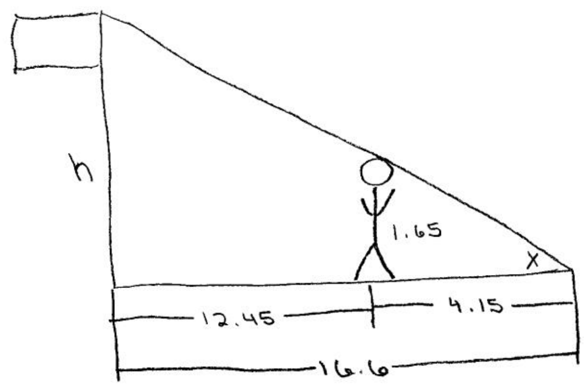


Compare with questions 8 and 11, which also assess G-SRT.B.

Sample student responses and scores appear on the following pages.

Question 31

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*



$$\tan x = \frac{1.65}{4.15}$$

$$x = 21.7$$

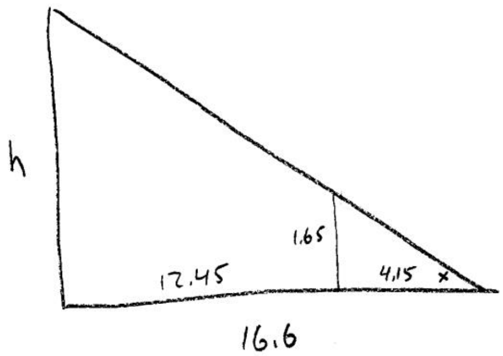
$$\tan 21.7 = \frac{h}{16.6}$$

$$h = 6.6$$

Score 2: The student has a complete and correct response.

Question 31

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*



$$\sin x = \frac{1.65}{4.15}$$

$$x = \sin^{-1}\left(\frac{1.65}{4.15}\right)$$

$$x = 23.427626509$$

$$16.6 \cdot \sin(23.427626509) = \frac{h}{16.6} \cdot 16.6$$

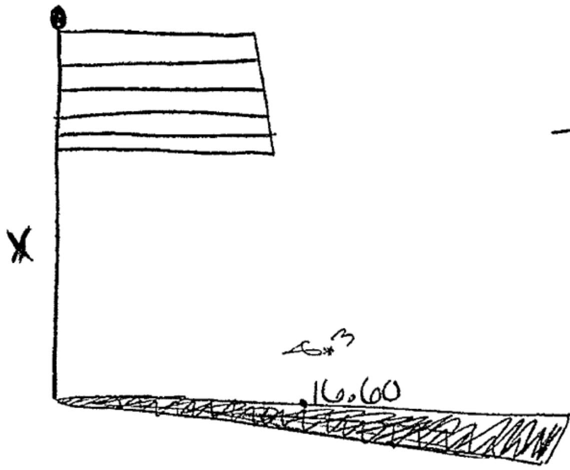
$$6.6 = h$$

$$\boxed{6.6\text{meters}}$$

Score 1: The student made an error using the incorrect trigonometric function, and found an incorrect angle measure for x. The student made the same error in finding the height.

Question 31

31 A flagpole casts a shadow 16.60 meters long. Tim stands at a distance of 12.45 meters from the base of the flagpole, such that the end of Tim's shadow meets the end of the flagpole's shadow. If Tim is 1.65 meters tall, determine and state the height of the flagpole to the *nearest tenth of a meter.*



$$\frac{x}{1.65} = \frac{12.45}{16.60}$$

$$\frac{20.5425}{16.60} = \frac{\cancel{16.60}x}{\cancel{16.60}}$$

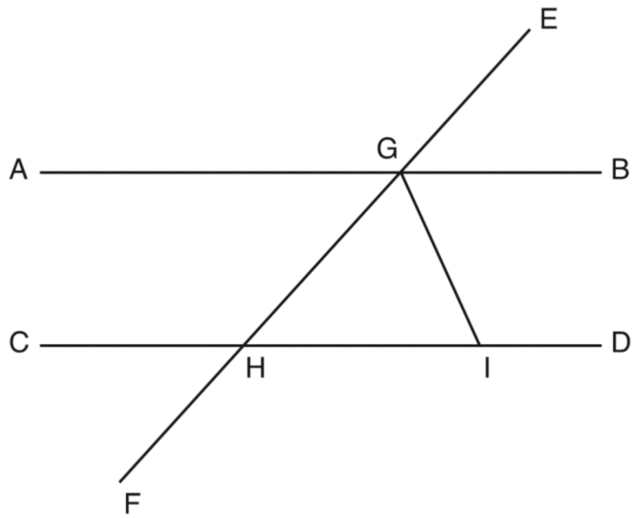
$$1.2375 = x$$

Height of Flagpole = 1.2 meters long

Score 0: The student did not subtract 12.45 from 16.60. The student also wrote an incorrect proportion.

#32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50°$ and $m\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason about lines and angles. The student must use theorems (e.g., linear pairs form supplementary angles, base angles of an isosceles triangle are equal, or if two lines are cut by a transversal and the corresponding angles are equal, the two lines are parallel) in order to explain why $\overline{AB}||\overline{CD}$. Additionally, the item requires the student to employ Mathematical Practice 3, because the student must identify and explain evidence that will support the claim.

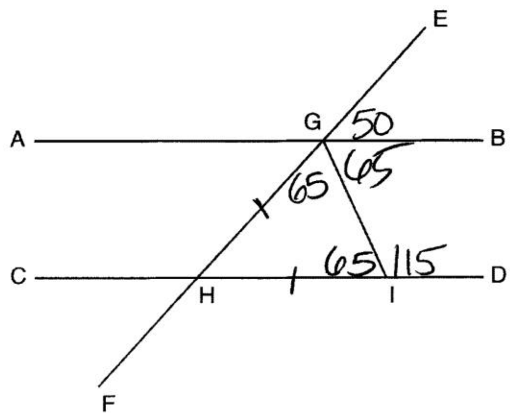
Rationale: This question asks students to explain why two lines are parallel given a diagram and two angle measures in the diagram. Since $\angle DIG$ and $\angle HIG$ are supplementary, then $m\angle HIG = 65°$ because $m\angle DIG = 115°$. Triangle GHI is an isosceles triangle because $\overline{GH} \cong \overline{IH}$. The base angles of the isosceles triangle are equal, so $m\angle HGI = 65°$. The angles of triangle GHI add to $180°$, so $m\angle GHI = 50°$. Since the lines AB and CD are cut by a transversal and corresponding angles EGB and GHI are equal, then $\overline{AB} \parallel \overline{CD}$.

Compare with questions 13, 17, 26, and 33, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



If m$\angle EGB$ = 50° and m$\angle DIG$ = 115°, explain why $\overline{AB} \parallel \overline{CD}$.

m∡GIH = 65 linear pairs are supplementary

m∡HGI = 65 - Base angles of an isosceles triangle are equal

m∡EGB + m∡BGI + m∡HGI = 180

50 + m∡BGI + 65 = 180

115 + m∡BGI = 180

−115 −115

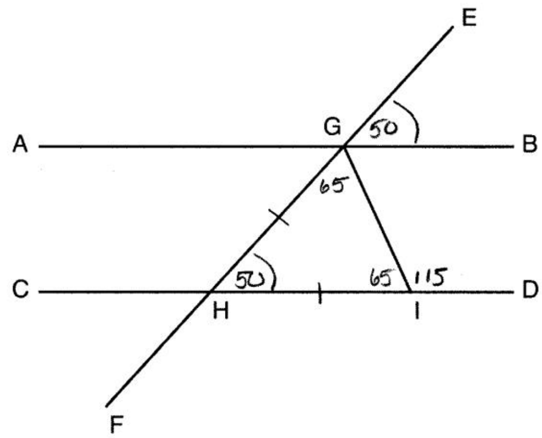
m∡BGI = 65

∡BGI and ∡DIG are same-side interior ∡'s, and since they are supplementary, $\overline{AB} \parallel \overline{CD}$.

Score 4: The student has a complete and correct response.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



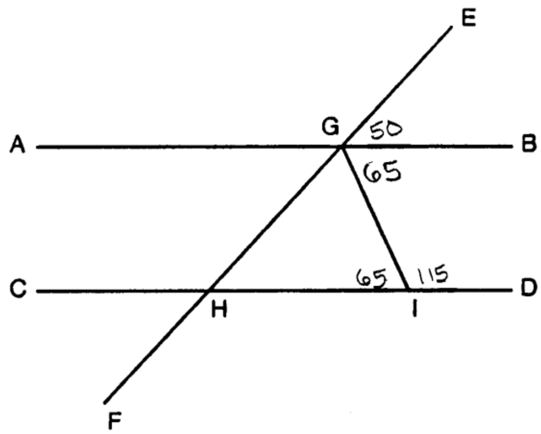
If $m\angle EGB = 50°$ and $m\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

<DIG is supplementary to <HIG, so m< HIG = 65.
<HIG = < HGI because angles opposite equal sides are equal.
The sum of angles of a triangle is 180° so <GHI is 50.
So, $\overline{AB} \parallel \overline{CD}$.

Score 3: The student stated correct angle measures with explanations, but did not explain why $\overline{AB} \parallel \overline{CD}$.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



If m$\angle EGB$ = 50° and m$\angle DIG$ = 115°, explain why $\overline{AB} \parallel \overline{CD}$.

$m\angle DIG + m\angle HIG = 180$ supplementary

$m\angle HIG \cong m\angle BGI$ alternate interior

$m\angle BGI + mDIG = 180$ same side interior

$65 + 115 = 180$

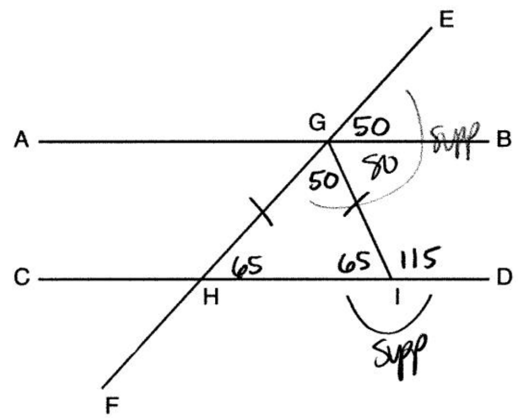
$180 = 180$

lines parallel when same side interior angles add up to 180.

Score 2: The student made one conceptual error using alternate interior angles of parallel lines to prove the same lines parallel.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.

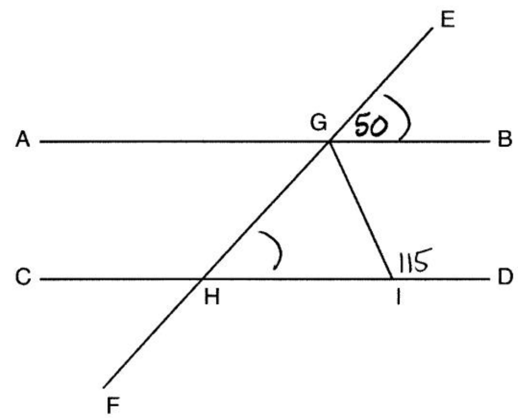


If m$\angle EGB = 50°$ and m$\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

Score 1: The student found appropriate angle measures based on a mislabeled diagram, and the explanation was missing.

Question 32

32 In the diagram below, $\overline{EF}$ intersects $\overline{AB}$ and $\overline{CD}$ at G and H, respectively, and $\overline{GI}$ is drawn such that $\overline{GH} \cong \overline{IH}$.



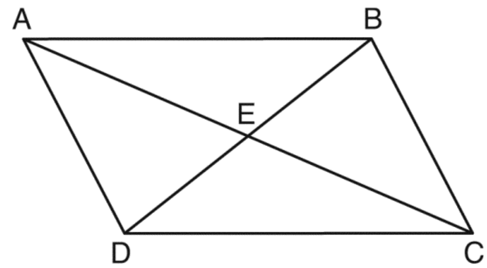
If m$\angle EGB = 50°$ and m$\angle DIG = 115°$, explain why $\overline{AB} \parallel \overline{CD}$.

Corresponding $\angle$s are $\cong$,
so AB$\parallel$ $\overline{CD}$.

Score 0: The student did not show enough work on which to base the explanation.

#33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Measured CCLS Cluster: G-CO.C

Commentary: The question measures the knowledge and skills described by the standards within G-CO.C because the student is required to reason about lines, angles, triangles and parallelograms. The student must construct a proof using theorems about these figures (e.g., the diagonals of a parallelogram bisect each other, parallel lines cut by a transversal form alternate interior angles, or vertical angles are congruent) to prove two triangles are congruent. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another cluster, G-CO.A, in describing the rigid motion that will map one triangle onto the other.

Rationale: This question asks students to prove triangles are congruent given a parallelogram with both diagonals drawn. The student must construct a proof using facts about parallelograms and parallel lines. An example is as follows:

$AD = BC$ because opposite sides of a parallelogram are equal in length. Additionally, $BE = DE$ and $CE = EA$ because the diagonals of a parallelogram bisect each other. Therefore, $\triangle AED \cong \triangle CEB$ by the SSS criterion.

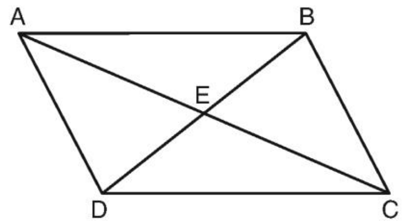
For the second part, the student must describe any valid single transformation that would map $\triangle AED$ onto $\triangle CEB$. An example of this is a rotation 180 degrees about point E.

Compare with questions 13, 17, 26, and 32, which also assess G-CO.C.

Sample student responses and scores appear on the following pages.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statment	Reason
1. Quad ABCD is a parallelogm	1. given
2. $\overline{AD} \cong \overline{CB}$	2. opposite sides of parallelogram are congruent
3. $\overline{AC}$ and $\overline{DB}$ intersect at E	3. given
4. $\angle AED \cong \angle CEB$	4. vertical angles are congruent
5. $\overline{BC} \parallel \overline{DA}$	5 def. of ▱
6. $\angle DBC \cong \angle BDA$	6. alt. interior angles are $\cong$
7. $\triangle AED \cong \triangle CEB$	7. AAS $\cong$ AAS

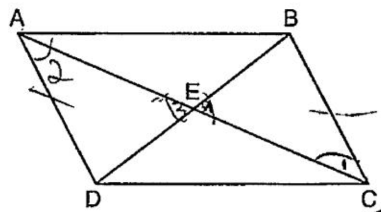
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation of $\triangle AED$ around point E of $180°$

Score 4: The student has a complete and correct proof, and a correct rigid motion is described.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

AAS

Statements	Reasons
1. Quadrilateral ABCD is a ▱ with diagonals $\overline{AC}$ + $\overline{DB}$ intersecting at E	1. GN
2. AD ≅ BC	2. All opposite sides of a ▱ are ≅
3. AD ∥ BC	3. Opposite sides of a ▱ are ∥
4. ∠1 ≅ ∠2	4. If 2 ∥ lines are cut by a transversal, the alternate interior ∠'s are ≅.
5. ∠3 ≅ ∠4	5. Vertical ∠'s are ≅
6. △AED ≅ △BEC	6. AAS ≅ AAS

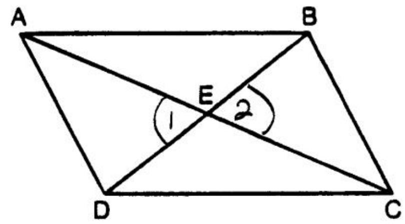
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Reflection

Score 3: The student wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

In a parallelogram, the diagonals bisect each other, so $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. $\angle 1 \cong \angle 2$. So $\triangle AED \cong \triangle CEB$ by SAS.

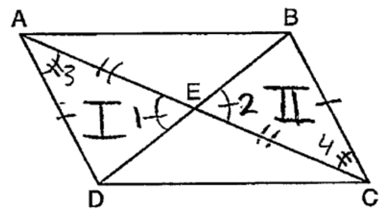
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

$180°$ rotation

Score 2: The student was missing the reason $\angle 1 \cong \angle 2$ and wrote an incomplete description of the rigid motion.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

Statement	Reason
1. ABCD is a Parallelogram $\overline{AC}$ + $\overline{BD}$ intersect at E	1. Given
2. ∡2 ≅ ∡1	2. verticle ∡s are ≅
3. ∡3 ≅ ∡4	3. Opposite inverse are ≅
4. $\overline{AE}$ ≅ $\overline{EC}$	4. Diagonals of a parallelogram are ≅
5. ΔI ≅ ΔII	5. ASA

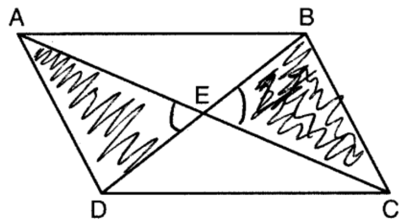
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotation 180°

Score 1: The student had some correct statements about the proof. The description of the rigid motion was incomplete.

Question 33

33 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



Prove: $\triangle AED \cong \triangle CEB$

1) parallelogram ABCD diagonals $\overline{AC}$ and $\overline{BC}$ intersecting at E   1) given

Prove: $\triangle AED \cong \triangle CEB$

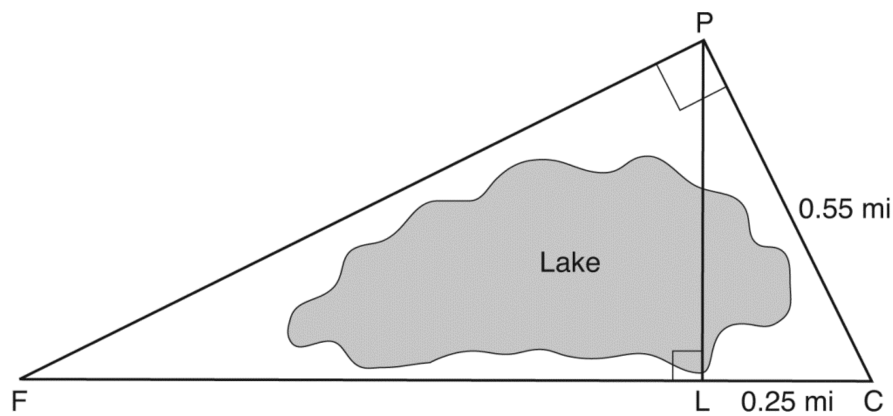
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Rotate $180°$

Score 0: The student wrote only the "given" information, and an incomplete description of the rigid motion.

#34

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Measured CCLS Cluster: G-SRT.C

Commentary: The question measures the knowledge and skills described by the standards within G-SRT.C because the student is required to apply understanding of relationships between angles and sides in right triangles. Specifically, the student must use the Pythagorean Theorem to determine the distance between the park ranger station and the lifeguard chair. The question is also an example of the instructional shift of coherence, as the student must draw on understandings from another cluster, G-SRT.B, in using similarity to respond to Gerald's claim that the distance from the first aid station to the campground is greater than 1.5 miles. In doing so, the question also requires the student to employ Mathematical Practice 4, because the student must model with triangles to solve a real-world problem.

Rationale:
The distance between the park ranger station and the lifeguard chair can be determined using the Pythagorean Theorem:

$(0.25)^2 + (PL)^2 = (0.55)^2$

$(PL)^2 = 0.3025 - 0.0625$

$(PL)^2 = 0.24$

$PL \approx 0.49$

To determine the distance from the first aid station to the campground, the student employed the understanding of similar right triangles by solving an appropriate proportion of corresponding sides.

$$\frac{FC}{0.55} = \frac{0.55}{0.25}$$

$$0.25(FC) = 0.3025$$

$$FC = 1.21$$

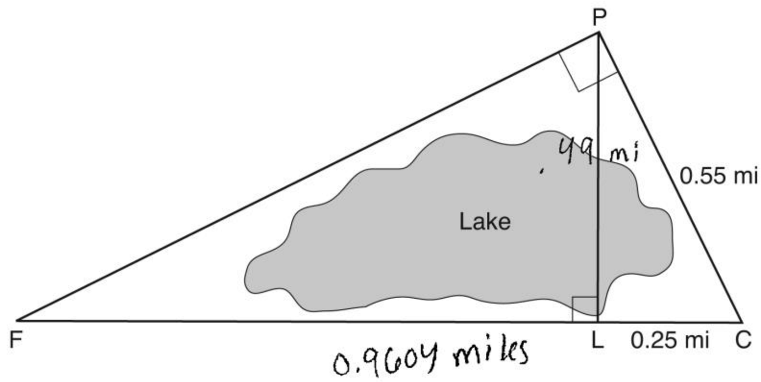
The total distance of 1.21 is less than 1.5, so Gerald is incorrect.

Compare with question 28, which also assesses G-SRT.C.

Sample student responses and scores appear on the following pages.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$a^2+b^2=c^2$

$a^2+0.0625=0.3025$

$a^2=0.24$

$a=.489897..$

the distance is 0.49 miles

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Altitude = $\frac{x}{h}=\frac{h}{y}$

$\frac{0.25}{0.49}=\frac{0.49}{y}$

$0.25y=0.2401$

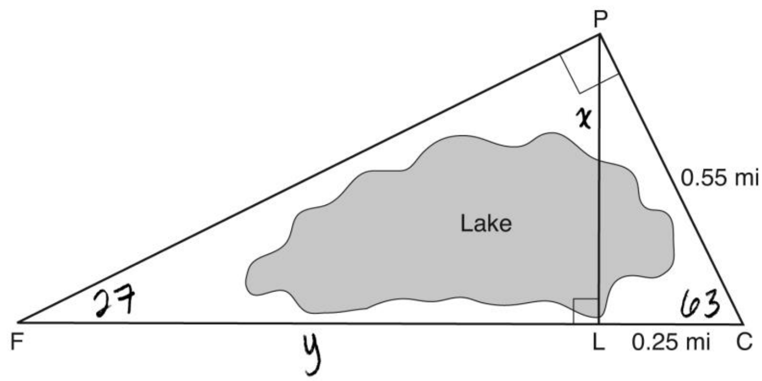
$y=0.9604$

NO, the distance from F to L is 0.9604 miles. When added to the distance from L to C, it's only around 1.2 miles, not 1.5 miles.

Score 4: The student has a complete and correct response.

Question 34

34 In the diagram below, the line of sight from the park ranger station, *P*, to the lifeguard chair, *L*, on the beach of a lake is perpendicular to the path joining the campground, *C*, and the first aid station, *F*. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$\cos C = \frac{.25}{.55}$

$\angle C = 63^\circ$

$\tan 63 = \frac{x}{.25}$

$x = .4906$

$x = .49$

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

$180 - 90 - 63 = 27$

$\tan 27 = \frac{.49}{y}$

$y = .96$

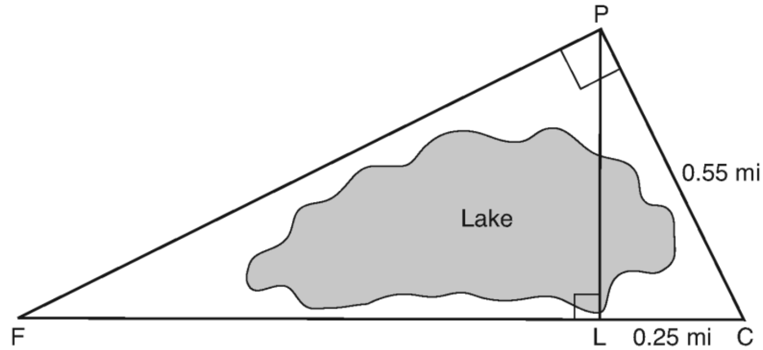
$+ .25$

1.21

Score 3: The student did not state if Gerald is correct.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$\frac{0.55}{FC} = \frac{0.25}{0.55}$

$0.25\ FC = 0.3025$

$FC = 1.21$

$\frac{x}{0.96} = \frac{0.25}{x}$

$x^2 = 0.24$

$x = 0.5^5$

Distance between P and L = 0.5 mi.

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

let x be FC

No because it is 1.21 miles. $\frac{0.55}{x} = \frac{0.25}{0.55}$

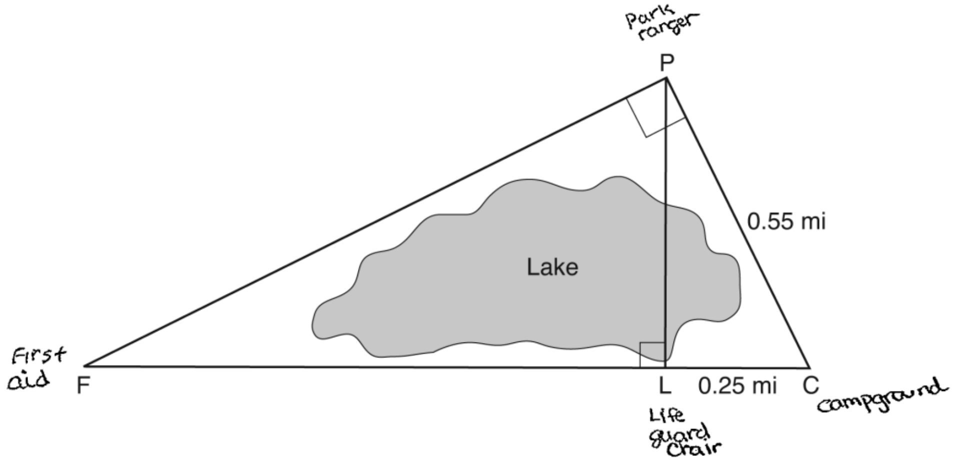
$0.25\ FC = 0.3025$

$FC = 1.21$

Score 2: The student made one computational error and one rounding error in finding the distance between the park ranger station and the lifeguard chair.

Question 34

34 In the diagram below, the line of sight from the park ranger station, *P*, to the lifeguard chair, *L*, on the beach of a lake is perpendicular to the path joining the campground, *C*, and the first aid station, *F*. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$(0.25)^2 + b^2 = (0.55)^2$

$\cancel{0.0625} + b^2 = .3025$

$-\,\cancel{.0625} \qquad -.0625$

$b^2 = \sqrt{0.24}$

$b = 0.4898979486$

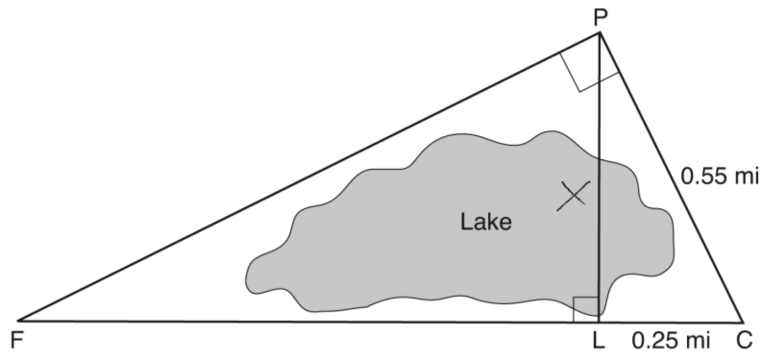
$\boxed{\text{Distance} \approx 0.5 \text{ miles}}$

Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Score 1: The student made one rounding error, and no further correct work was shown.

Question 34

34 In the diagram below, the line of sight from the park ranger station, P, to the lifeguard chair, L, on the beach of a lake is perpendicular to the path joining the campground, C, and the first aid station, F. The campground is 0.25 mile from the lifeguard chair. The straight paths from both the campground and first aid station to the park ranger station are perpendicular.



If the path from the park ranger station to the campground is 0.55 mile, determine and state, to the *nearest hundredth of a mile*, the distance between the park ranger station and the lifeguard chair.

$$.25^2 + .55^2 = x^2$$

$$.0625 + .3025 = x^2$$

$$.365 = x^2$$

$$\boxed{.6 = x}$$

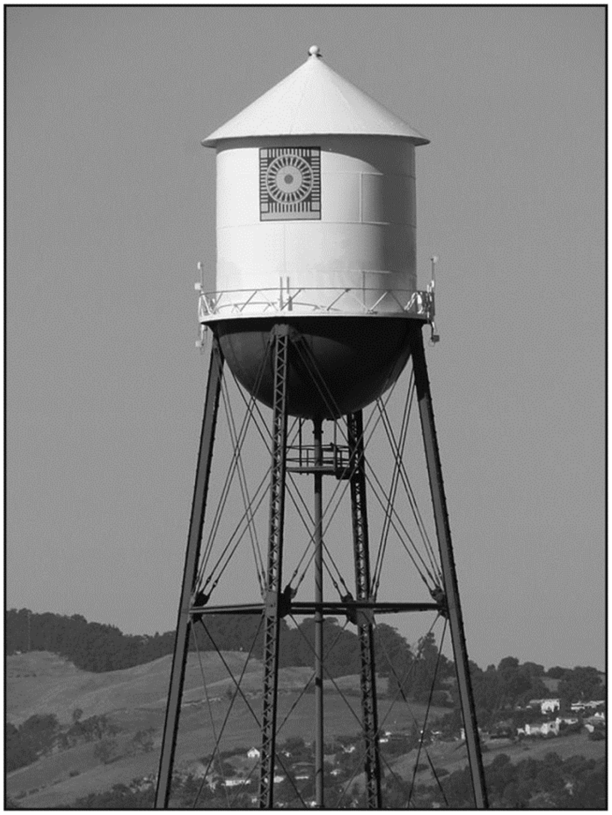
Gerald believes the distance from the first aid station to the campground is at least 1.5 miles. Is Gerald correct? Justify your answer.

Yes - its far away.

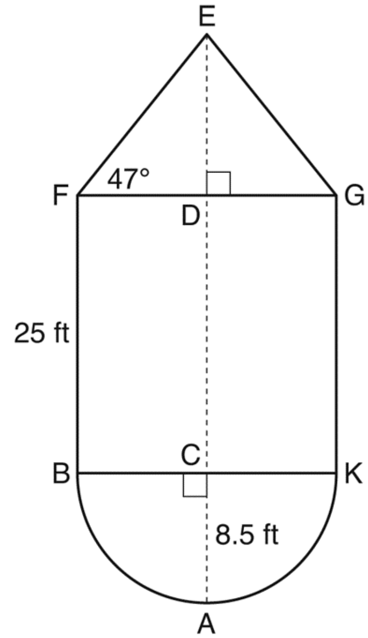
Score 0: The student had a completely incorrect response.

#35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: http://en.wikipedia.org



If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

Measured CCLS Cluster: G-MG.A

Commentary: The question measures the knowledge and skills described by the standards within G-MG.A because the student is required to use volume and density to solve a real-world problem. The student must find the volume of a compound figure composed of a cone, a cylinder, and a hemisphere. Then the student must apply the concept of density to the volume to find the solution to the problem. In doing so, the question also requires the student to employ Mathematical Practice 4, because the student must model with geometric figures to solve a real-world problem.

Rationale: This question asks students to find the volume of the water tower and determine if the water tower can be filled to 85% capacity without exceeding the weight limit. The sum volumes of a cone $\left(V = \frac{1}{3}\pi r^2 h\right)$, a cylinder ($V = \pi r^2 h$), and a hemisphere $\left(V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3\right)$ will be used to find the total volume of the water tower. A key piece is using right triangle trigonometry to find the height of the cone. The angle of incline of the cone and its radius can be used with the tangent to find the height of the cone.

<u>Height of the cone is h</u>	<u>Volume of cone</u>	<u>Volume of cylinder</u>	<u>Volume of hemisphere</u>
$\tan 47° = \frac{h}{8.5}$	$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$	$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$
$h = 8.5(\tan 47°)$	$V = \frac{1}{3}\pi 8.5^2 \cdot 9.11513$	$V = \pi 8.5^2 \cdot 25$	$V = \frac{1}{2} \cdot \frac{4}{3}\pi 8.5^3$
$h = 9.11513$	$V = 689.6509461$	$V = 5674.501731$	$V = 1286.220392$

Total volume of the water tower is 7650 cubic feet.

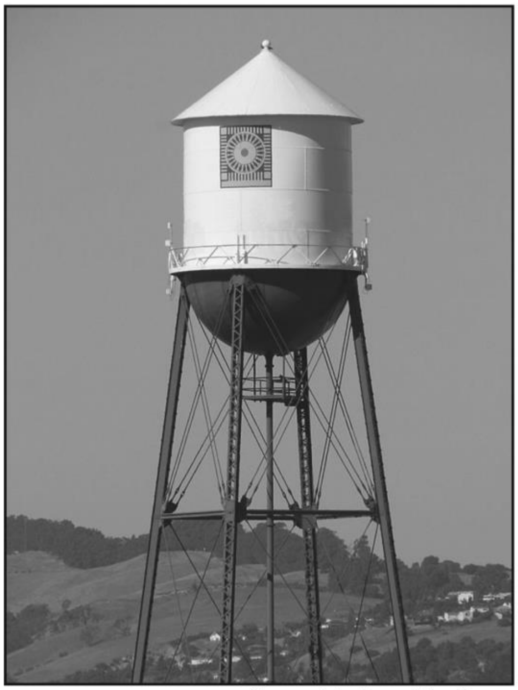
The water tower can hold a maximum of 400,000 pounds of water. If the tower were filled to 85% capacity and water weighs 62.4 pounds per cubic foot then, $62.4(7650)(0.85) = 405{,}756$ pounds of water. So, the water tower cannot be filled to 85% capacity because it would exceed the weight limit by 5,756 pounds.

Compare with question 7, which also assesses G-MG.A.

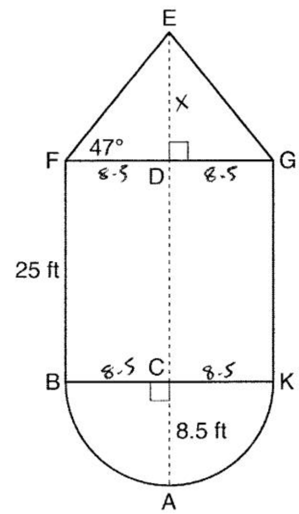
Sample student responses and scores appear on the following pages.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



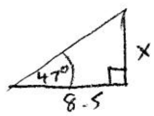
Source: http://en.wikipedia.org



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.



$\tan 47° = \frac{x}{8.5}$

$x = 8.5 \tan 47°$

$x = 9.11513$

Volume cone

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (8.5)^2(9.11513)$

$V = 689.65125$

Volume cylinder

$V = \pi r^2 h$

$V = \pi (8.5)^2(25)$

$V = 5674.50173$

Volume Hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2}{3}\pi(8.5)^3$

$= 1286.22039$

$V = 689.65125 + 5674.50173 + 1286.22039 = 7650.37337$

$= \boxed{7650 \text{ ft}^3}$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$7650 \times 62.4 = 477{,}360$ lbs

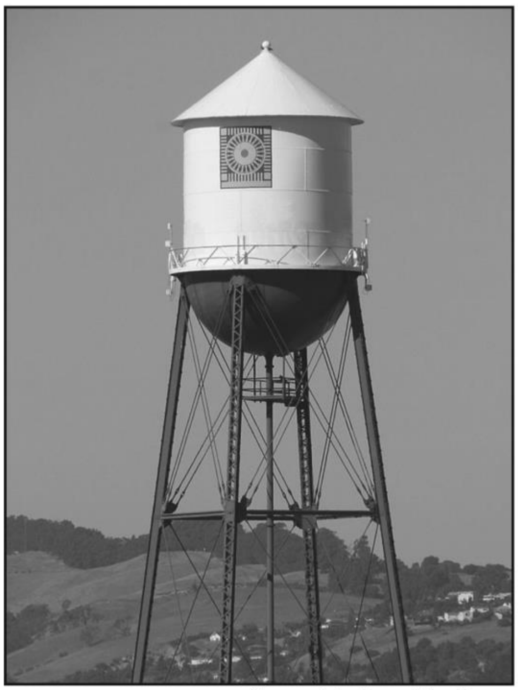
$.85 \times 477{,}360 = \boxed{405{,}756 \text{ lbs}}$

No – the weight would exceed 400,000 lbs

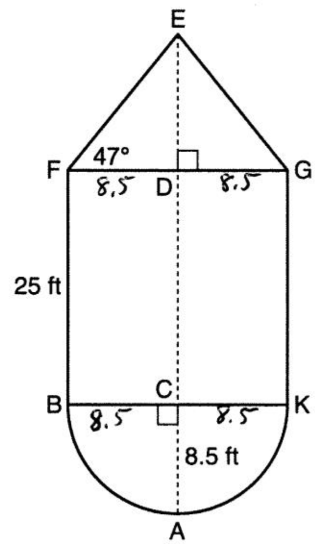
Score 6: The student had a complete and correct response.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



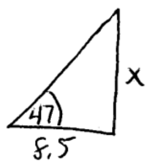
Source: http://en.wikipedia.org



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.



$\tan 47 = \frac{x}{8.5}$

$x = 9.115$

$V = \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{1}{2}(\frac{4}{3}\pi r^3)$

$V = \frac{1}{3}(3.14)(8.5)^2(9.115) + 3.14(8.5)^2(25) + \frac{1}{2}\cdot\frac{4}{3}(3.14)(8.5)^3$

$= 689.2914917 + 5671.625 + 1285.568333$

$= 7646.484825$

$V = 7646$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$7646(62.4) = 477{,}110.4$ pounds

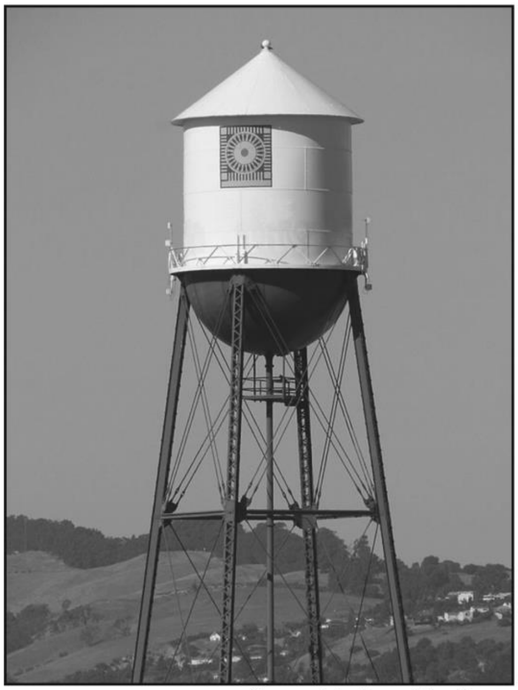
$477{,}110.4(.85) = 405{,}543.84$ pounds

No because it would exceed 400,000 pounds

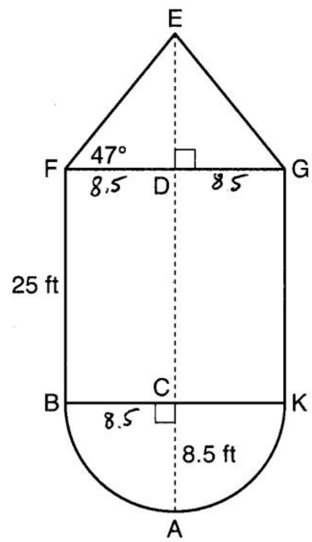
Score 5: The student used 3.14 instead of π to calculate the volume.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.

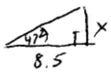


Source: http://en.wikipedia.org



Question 35 is continued on the next page.

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.



$\tan 47 = x/8.5$

$x = 8.5 \tan 47$

$x = 9.1151$

$\boxed{x = 9.1}$

Volume Hemisphere

$V = 1/2\ (4/3\ \pi r^3)$

$= 2/3\ \pi\ (8.5)^3$

$V = 1286.22039$

Volume Cylinder

$V = \pi r^2 h$

$V = \pi\ (8.5^2)(25)$

$V = 5674.5017$

Volume Cone

$V = 1/3\ \pi r^2 h$

$V = 1/3\ \pi\ (8.5)^2\ (9.1)$

$V = 688.5062$

$V = 1286.2704 + 5674.5017 + 688.5062$

$= 7649.2183$

$V = \boxed{7649\ ft^3}$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

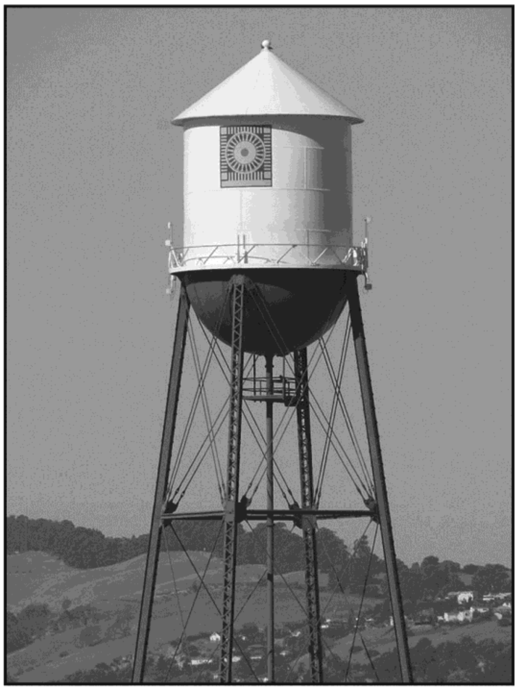
$7649 \times 62.4 = 477{,}297.6$ lbs.

$477{,}297.6 \times .85 = \boxed{405{,}702.96}$ lbs.

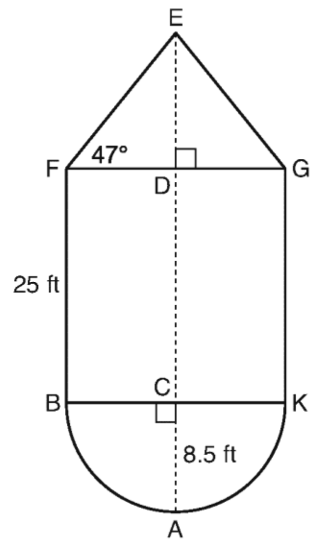
Score 4: The student rounded early with $x = 9.1$, and did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



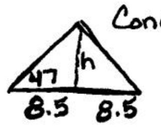
Source: http://en.wikipedia.org



Question 35 is continued on the next page.

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

Cone

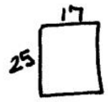


$\text{Tan } 47° = \frac{h}{8.5} = 9.12$

$\frac{1}{3}\pi(8.5)^2(9.12)$

219.64π +

cylinder



$\pi(8.5)^2(25)$

1806.25π +

½ Circle

$\frac{1}{2}\left[\pi(8.5)^2\right]$

36.125π

2062.015π

6478.01

6478

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$(.85)(6478) = 5506.3$

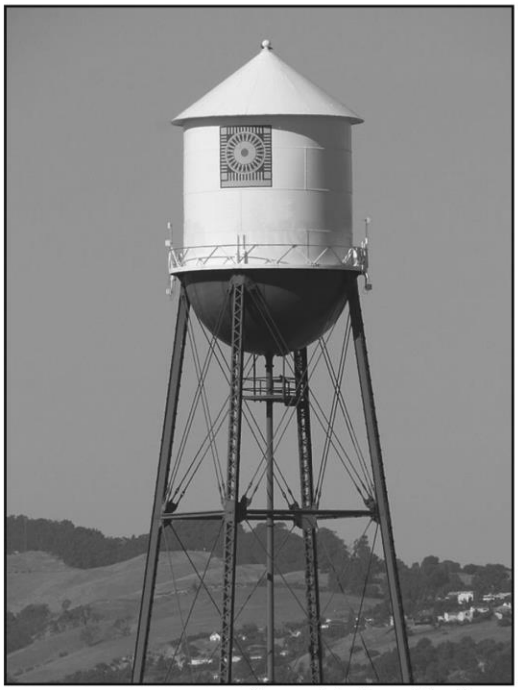
$(5506.3)(62.4) = 343,593.12$

yes because less than 400,000

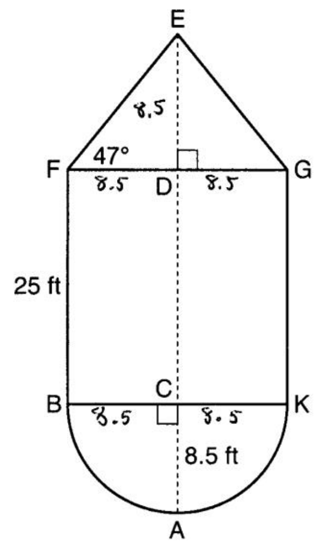
Score 3: The student made one conceptual error by finding the area of half of a circle instead of the volume of a hemisphere. The height of the cone was rounded incorrectly. The student used the answer from the first part to answer the second part appropriately.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: http://en.wikipedia.org



Question 35 is continued on the next page.

Question 35 continued

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

$V = \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{4}{3}\pi r^2$

$V = \frac{1}{3}\pi (8.5)^2(8.5) + \pi (8.5)^2 (25) + \frac{4}{3}\pi (8.5)^2$

$V = \frac{1}{3}\pi (614.125) + \pi (1806.25) + \frac{4}{3}\pi (72.25)$

$V = 643.1101961 + 5674.501731 + 302.6\cancel{?}4 00923$

$V = 6620.252019$

6620

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

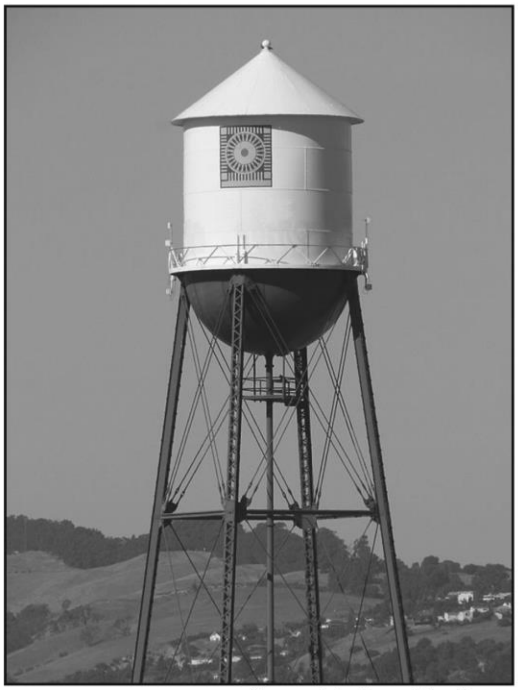
$6620(.85) = 5627$

$5627(62.4) = 351,124.8$

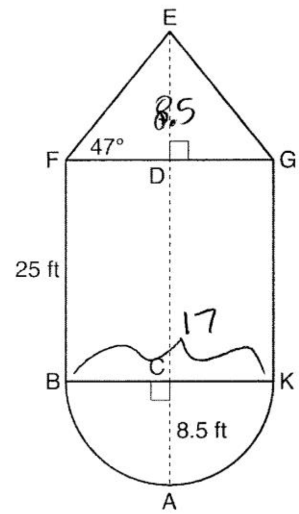
Score 2: The student made one conceptual error by using 8.5 for the height of the cone, and made an error by not dividing the volume of the sphere by 2. The student did not state if the water tower can be filled to 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: http://en.wikipedia.org



Question 35 is continued on the next page.

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

Cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (8.5)^2(8.5)$

$V = 643.1$

Cylinder

$\pi r^2 h$

$\pi(8.5)^2(33.5)$

7603.8

$V = 8247$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

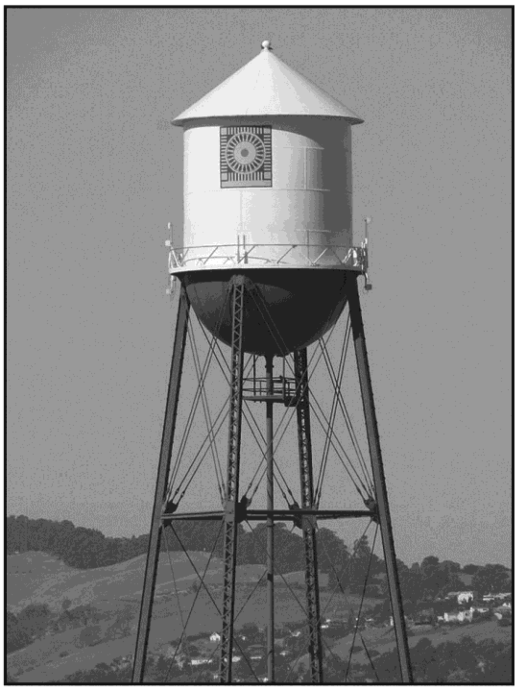
$8247 \times 62.4 = 514{,}612.8$ lbs

NO

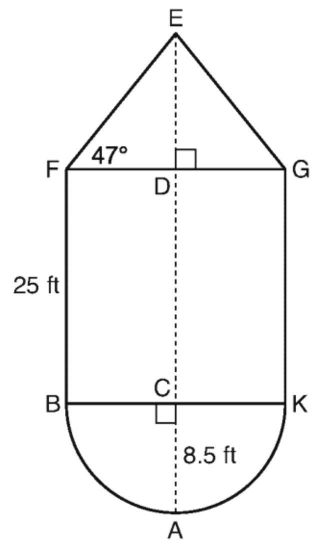
Score 1: The student made two conceptual errors in finding the volume of the water tower and one computational error by not multiplying by 85%.

Question 35

35 The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.



Source: http://en.wikipedia.org



Question 35 is continued on the next page.

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower.

17π

8.5

25

8.5

$(42)(17)^2\pi$

$12138\pi = 38132.65$

The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

$(400000)(.85) = 340000$

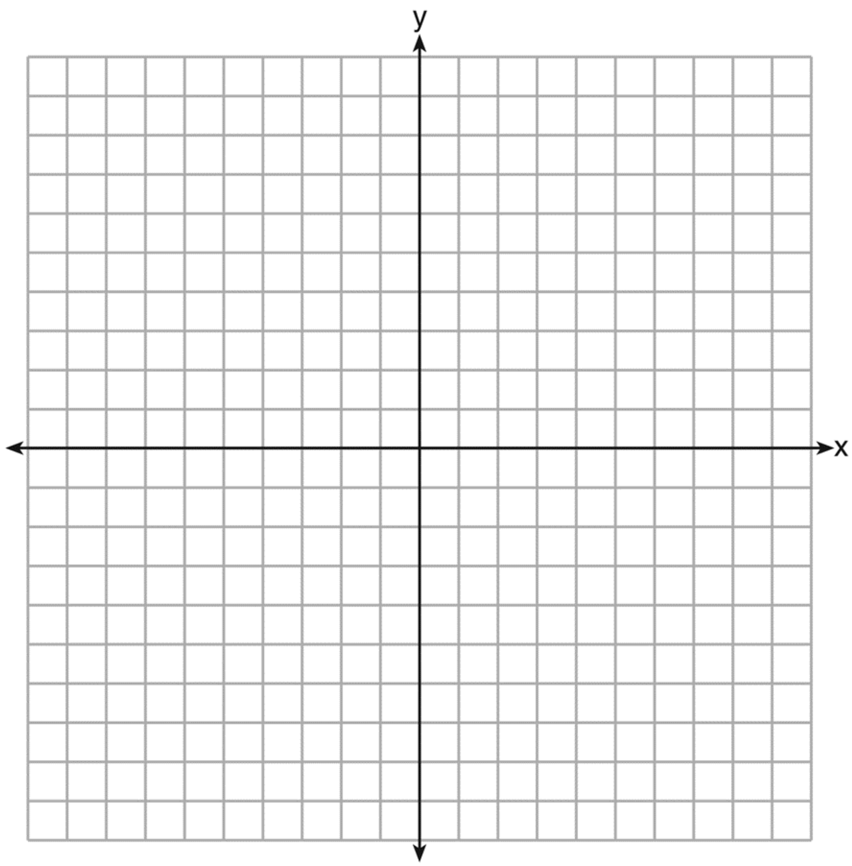
Score 0: The student had a completely incorrect response.

#36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

Prove that your quadrilateral $RSTP$ is a rectangle.
[The use of the set of axes below is optional.]



Measured CCLS Cluster: G-GPE.B

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.B because the student is required to use coordinates to apply understanding of geometric figures. The student must use coordinates to prove a triangle is a right triangle and then determine the coordinates of a fourth point such that the three vertices of the right triangle and the fourth point are the four points of a rectangle. The student must prove this quadrilateral is a rectangle. In doing so, the question also requires the student to employ Mathematical Practice 3, because the student must construct a complete line of reasoning to prove an assertion.

Rationale: This question asks students to prove that the given triangle is a right triangle. The student must also determine the coordinates of a fourth point such that the three vertices of the right triangle and the fourth point are the four points of a rectangle and prove it is a rectangle. These parts can be accomplished several different ways such as using slopes, distances, and/or the midpoints of the diagonals.

The slope of $\overline{SR}$ is $\frac{3}{5}$.

The slope of $\overline{ST}$ is $-\frac{10}{6} = -\frac{5}{3}$.

The slopes of sides $\overline{SR}$ and $\overline{ST}$ are negative reciprocals, therefore $\overline{SR} \perp \overline{ST}$. Since perpendicular lines form right angles, then $\angle S$ is a right angle. Since $\triangle RST$ has a right angle, $\triangle RST$ is a right triangle.

The coordinates of point P that make $RSTP$ a rectangle are $(0,9)$.

The slope of $\overline{TP}$ is $\frac{3}{5}$.

The slope of $\overline{RP}$ is $-\frac{10}{6} = -\frac{5}{3}$.

Using $P(0,9)$, both pairs of opposite sides of $RSTP$ have the same slope, so $\overline{TP} \parallel \overline{RS}$ and $\overline{PR} \parallel \overline{ST}$. Since both pairs of opposite sides of $RSTP$ are parallel, $RSTP$ is a parallelogram. Since $RSTP$ is a parallelogram and has a right angle at vertex S, then $RSTP$ must be a rectangle.

Compare with question 27, which also assesses G-GPE.B.

Sample student responses and scores appear on the following pages.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$m_{\overline{RS}} = \frac{3}{5}$

$m_{\overline{ST}} = \frac{-10}{6} = \frac{-5}{3}$

Therefore the slopes of $\overline{RS}$ and $\overline{ST}$ are negative reciprocals and so $\overline{RS} \perp \overline{ST}$. Since the segments are $\perp$, $\angle S$ is a rt $\angle$.

$\therefore$ $\triangle RST$ is a rt $\triangle$ because it has 1 rt $\angle$.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

(0,9)

Question 36 is continued on the next page.

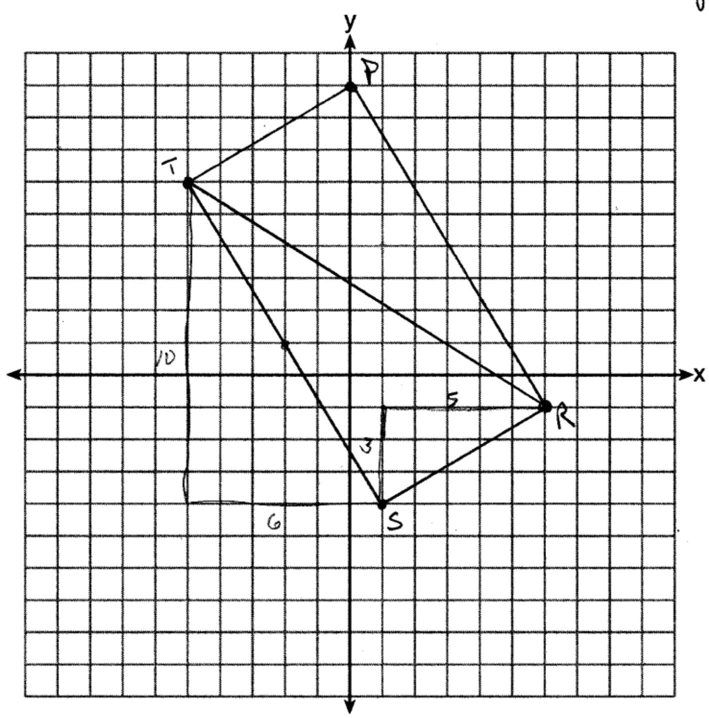
Question 36 continued

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

$$\left.\begin{array}{l} m_{\overline{RS}} = \frac{3}{5} \\ m_{\overline{PT}} = \frac{3}{5} \end{array}\right\} \therefore \overline{RS} \parallel \overline{PT}$$

$$\left.\begin{array}{l} m_{\overline{ST}} = \frac{-10}{6} = \frac{-5}{3} \\ m_{\overline{RP}} = -\frac{10}{6} = \frac{-5}{3} \end{array}\right\} \therefore \overline{ST} \parallel \overline{RP}$$

Since RSTP is a quadrilateral with both pairs of opposite sides $\parallel$ and one rt $\measuredangle$ at S, it must be a rectangle.



Score 6: The student has a complete and correct response.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

Slopes

$\overline{TS} = -\frac{10}{6} = \frac{-5}{3}$

$\overline{SR} = 3/5$

$\overline{TS} \perp \overline{SR}$ because their slopes are negative reciprocals of each other. $\angle S$ is a right $\angle$ because $\perp$ lines form rt. $\angle$'s. ΔRST is a right Δ because it has 1 right $\angle$.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

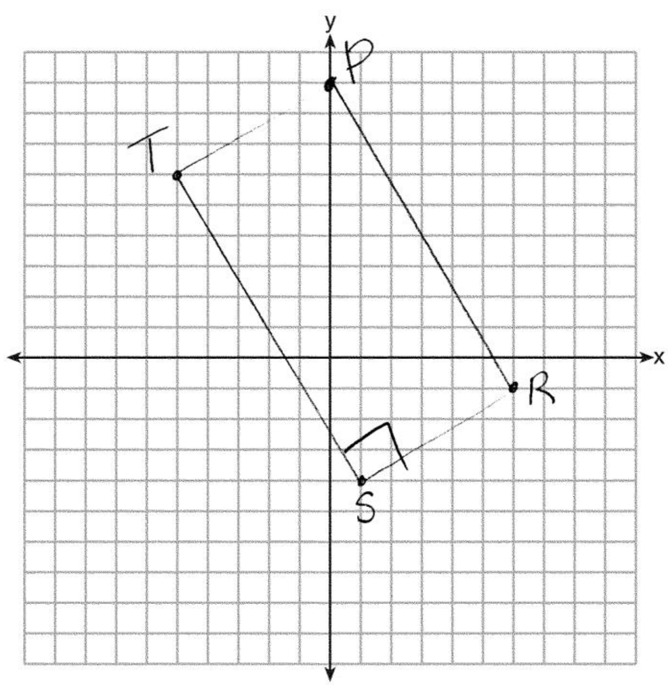
P(0,9)

Question 36 is continued on the next page.

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

$m_{TP} = 3/5$
$m_{\overline{SR}} = 3/5$
$m_{TS} = -5/3$
$m_{PR} = -5/3$

Opposite sides are parallel because they have the same slope. RSTP is a parallelogram because opposite sides are parallel.



Score 5: The student proved *RSTP* is a parallelogram, but did not have a concluding statement proving *RSTP* is a rectangle.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

Slope $\overline{RS} = \frac{3}{5}$ Slope $\overline{TS} = \frac{-10}{6} = \frac{-5}{3}$

$\overline{RS} \perp \overline{TS}$ since they have negative reciprocal slopes

There fore ∡S is a right ∡

Since △RST contains a right ∡, it is a right △.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

P (0,9)

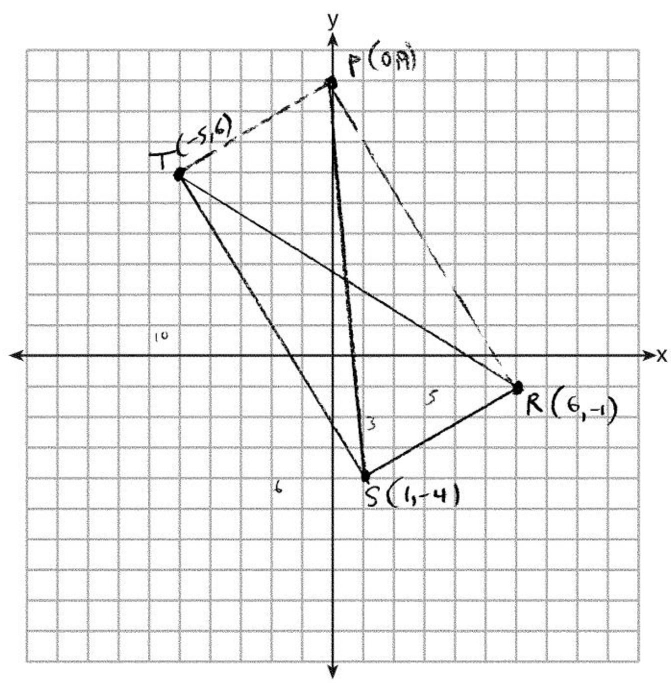
Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

Length $\overline{RT}$	Length $\overline{PS}$
$\sqrt{7^2+11^2}$	$\sqrt{13^2+1^2}$
$\sqrt{49+121}$	$\sqrt{169+1}$
$RT = \sqrt{170}$	$PS = \sqrt{170}$

Since the diagonals of RSTP are $\cong$, then it is a rectangle.



Score 4: The student made one conceptual error when proving the rectangle, because no work is shown to prove that *RSTP* is a parallelogram.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

sides are $\perp$ because their slopes are negative reciprocals

$$\begin{cases} m(TS) = \frac{10}{-6} = -\frac{5}{3} \\ m(SR) = \frac{3}{5} \end{cases}$$

$\perp$ lines form right angles

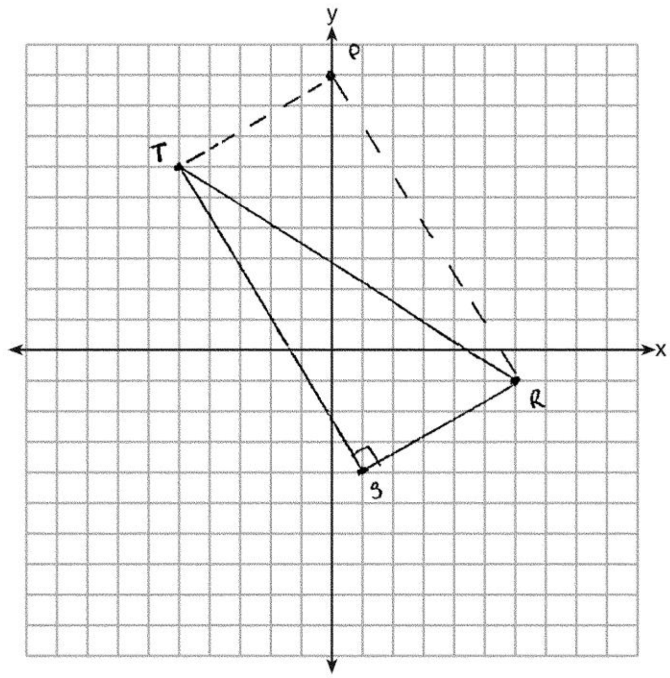
$\triangle$ RST is a right $\triangle$ because it has a right $\angle$.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

P (0,9)

Question 36 is continued on the next page.

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]



Score 3: The student correctly proved the right triangle and stated the coordinates of *P*, but no further correct work was shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$R(6,-1)$
$S(1,-4)$
$T(-5,6)$

$d_{RS} = \sqrt{(6-1)^2+(-1+4)^2} = \sqrt{25+9} = \sqrt{34}$

$d_{ST} = \sqrt{(1+5)^2+(-4-6)^2} = \sqrt{36+100} = \sqrt{136}$

$d_{RT} = \sqrt{(6+5)^2+(-1-6)^2} = \sqrt{121+49} = \sqrt{170}$

$(RS)^2 + (ST)^2 \stackrel{?}{=} (RT)^2$

$(\sqrt{34})^2 + (\sqrt{136})^2 \;\Big|\; (\sqrt{170})^2$

$34 + 136$

$170 = 170$

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

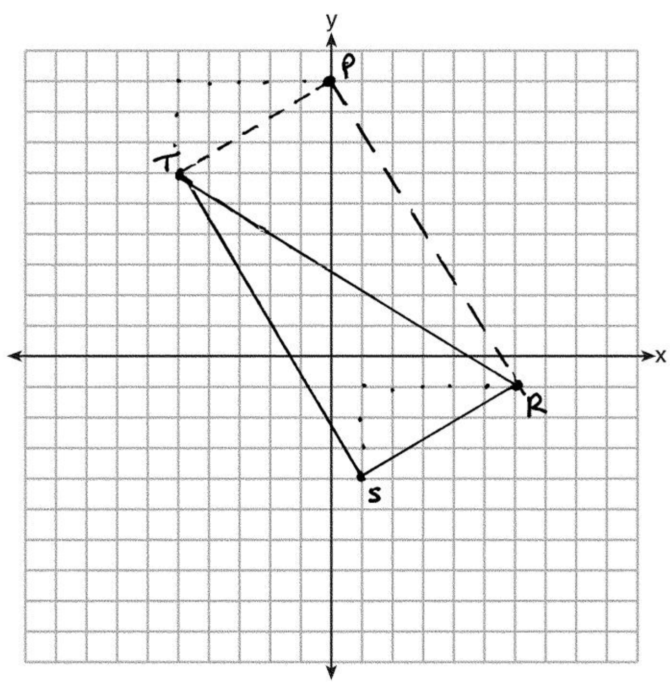
$(0,9)$

Question 36 is continued on the next page.

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

∡P is Rt ∡

RSTP is Rectangle because opposite ∡'s are Right ∡'s



Score 2: The student was missing a concluding statement when proving the right triangle, and the coordinates of *P* were correctly stated, but no further correct work is shown.

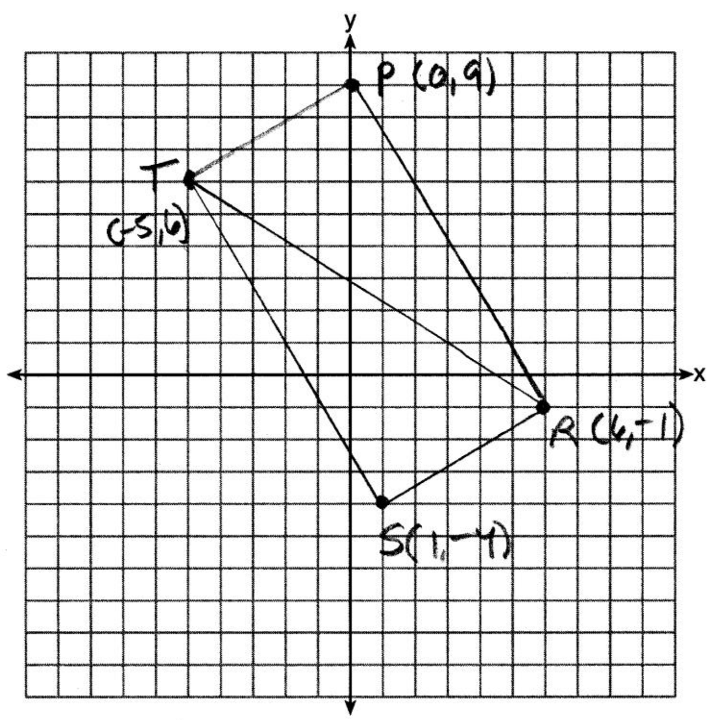
36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$.
Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

(0,9)

Question 36 is continued on the next page.

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]



Score 1: The student graphed point *P* correctly and stated its coordinates. No further work was shown.

Question 36

36 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle.
[The use of the set of axes on the next page is optional.]

$\triangle$RST is a right $\triangle$ because $\measuredangle$ S is a right angle.

State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle.

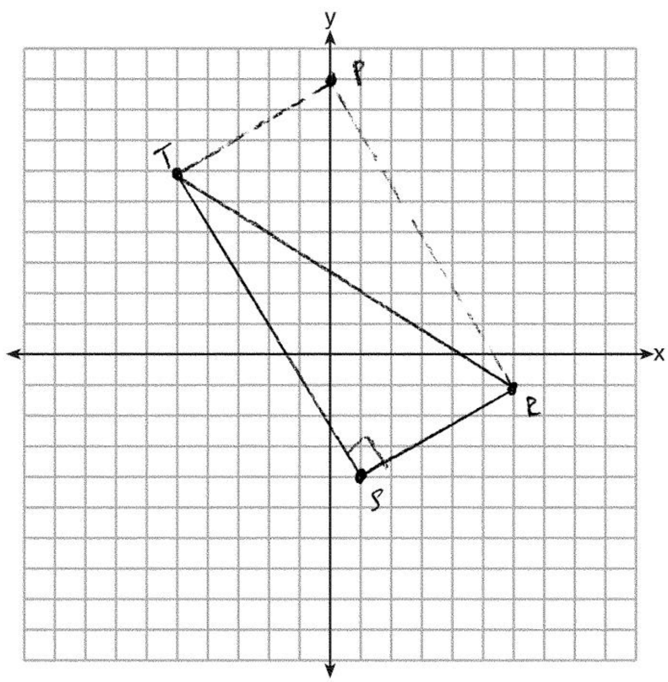
0,9

Question 36 is continued on the next page.

Question 36 continued

Prove that your quadrilateral *RSTP* is a rectangle.
[The use of the set of axes below is optional.]

RSTP is a rectangle because it has a right ∡.



Score 0: The student had no work to justify the statements, and the parentheses are missing on the coordinates of *P*.