

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Tuesday, June 20, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.



This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice ...**

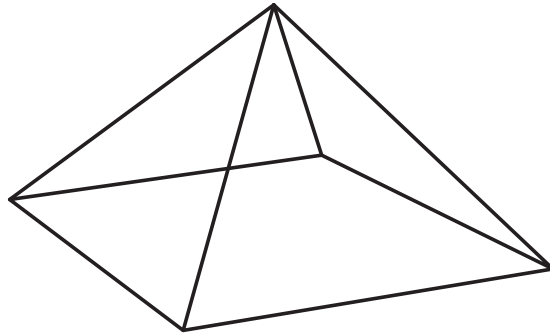
**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

- 1 A square pyramid is intersected by a plane passing through the vertex and perpendicular to the base.



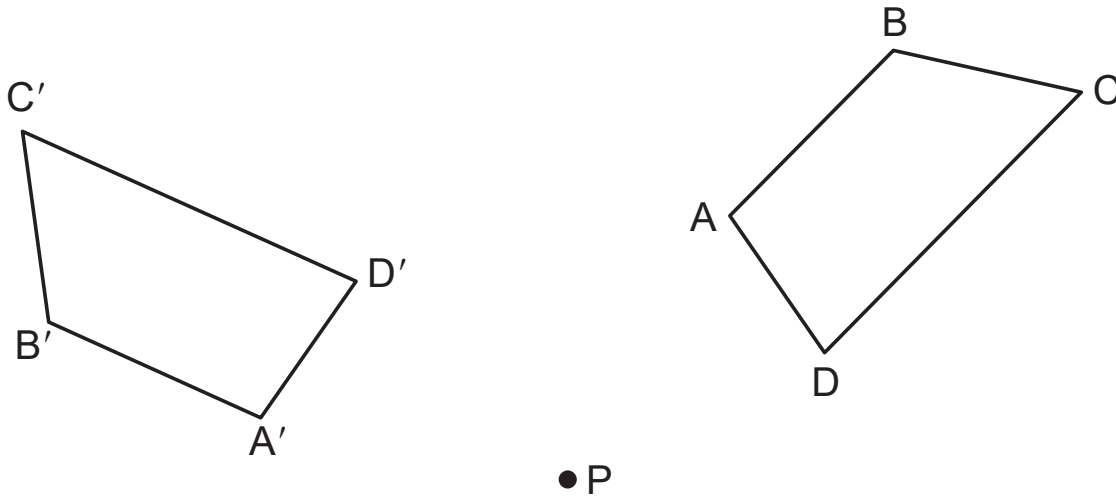
Use this space for  
computations.

Which two-dimensional shape describes this cross section?

- |              |               |
|--------------|---------------|
| (1) square   | (3) pentagon  |
| (2) triangle | (4) rectangle |

Use this space for  
computations.

- 2 Trapezoid  $ABCD$  is drawn such that  $\overline{AB} \parallel \overline{DC}$ .  
Trapezoid  $A'B'C'D'$  is the image of trapezoid  $ABCD$  after a rotation  
of  $110^\circ$  counterclockwise about point  $P$ .

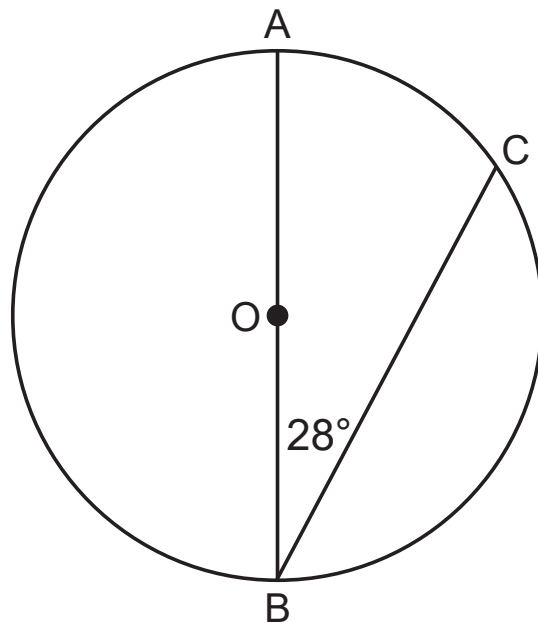


Which statement is always true?

- (1)  $\angle A \cong \angle D'$                       (3)  $\overline{A'B'} \parallel \overline{D'C'}$   
(2)  $\overline{AC} \cong \overline{B'D'}$                       (4)  $\overline{B'A'} \cong \overline{C'D'}$



- 5 In the diagram below of circle  $O$ , diameter  $\overline{AOB}$  and chord  $\overline{CB}$  are drawn, and  $m\angle B = 28^\circ$ .



Use this space for  
computations.

What is  $m\widehat{BC}$ ?

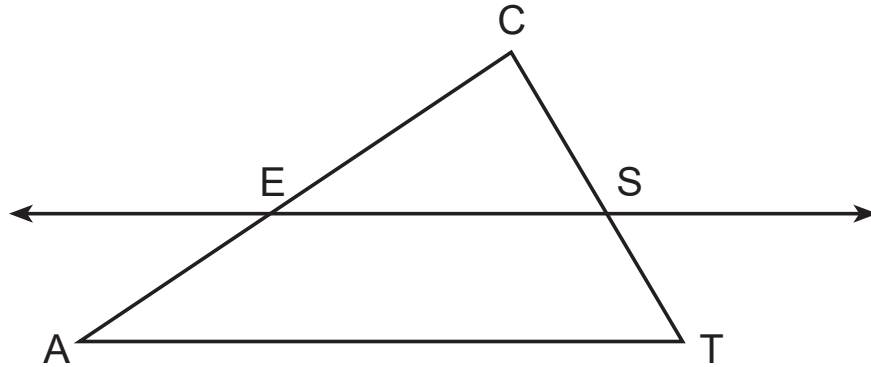
- (1)  $56^\circ$                       (3)  $152^\circ$   
(2)  $124^\circ$                     (4)  $166^\circ$





Use this space for  
computations.

- 7 In the diagram below of  $\triangle ACT$ ,  $\overline{ES}$  is drawn parallel to  $\overline{AT}$  such that  $E$  is on  $\overline{CA}$  and  $S$  is on  $\overline{CT}$ .

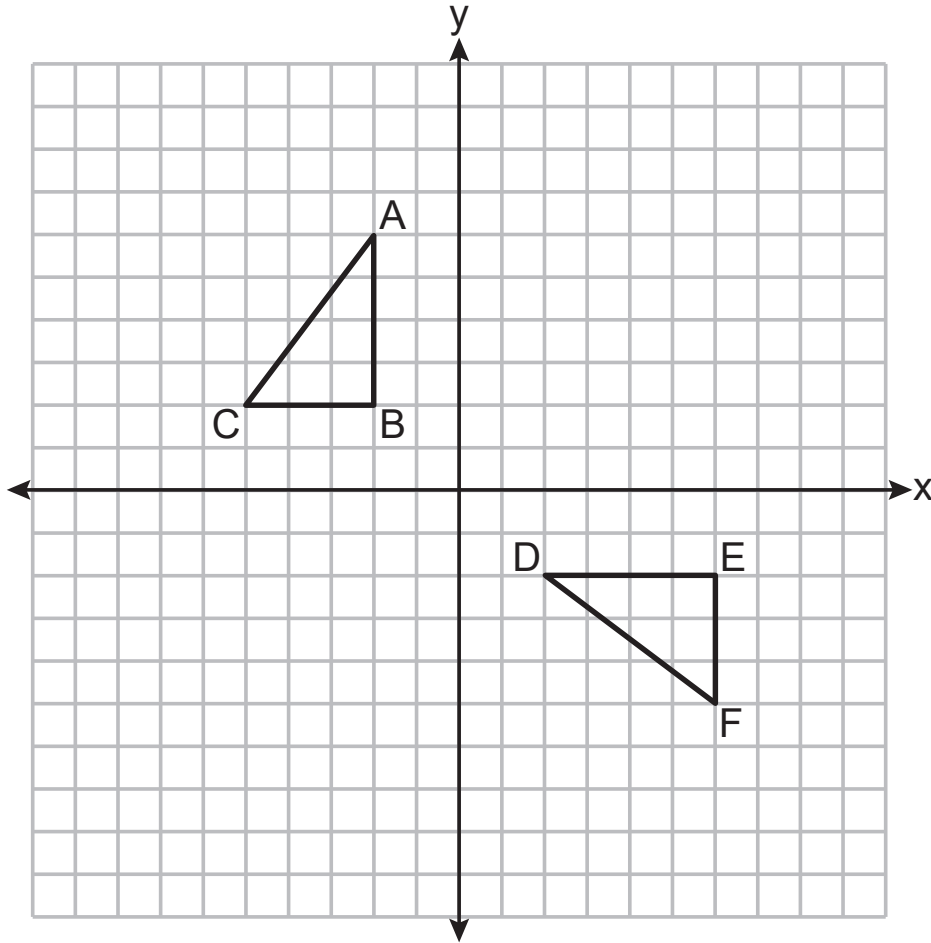


Which statement is always true?

- (1)  $\frac{CE}{CA} = \frac{CS}{ST}$                       (3)  $\frac{CE}{EA} = \frac{CS}{ST}$   
(2)  $\frac{CE}{ES} = \frac{EA}{AT}$                       (4)  $\frac{CE}{ST} = \frac{EA}{CS}$

8 On the set of axes below, congruent triangles  $ABC$  and  $DEF$  are drawn.

Use this space for  
computations.



Question 8 is continued on the next page.

### Question 8 continued

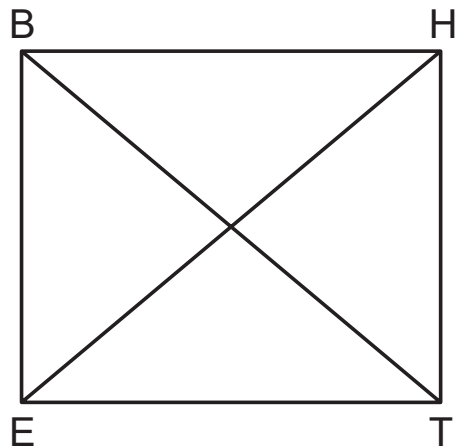
Use this space for  
computations.

Which sequence of transformations maps  $\triangle ABC$  onto  $\triangle DEF$ ?

- (1) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 8 units to the right.
  - (2) A counterclockwise rotation of 90 degrees about the origin, followed by a reflection over the  $y$ -axis.
  - (3) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 4 units down.
  - (4) A clockwise rotation of 90 degrees about the origin, followed by a reflection over the  $x$ -axis.
- 9** An equation of circle  $M$  is  $x^2 + y^2 + 6x - 2y + 1 = 0$ . What are the coordinates of the center and the length of the radius of circle  $M$ ?
- (1) center  $(3, -1)$  and radius 9
  - (2) center  $(3, -1)$  and radius 3
  - (3) center  $(-3, 1)$  and radius 9
  - (4) center  $(-3, 1)$  and radius 3

10 Parallelogram  $BETH$ , with diagonals  $\overline{BT}$  and  $\overline{HE}$ , is drawn below.

Use this space for  
computations.



Which additional statement is sufficient to prove that  $BETH$  is a rectangle?

(1)  $\overline{BT} \perp \overline{HE}$

(3)  $\overline{BT} \cong \overline{HE}$

(2)  $\overline{BE} \parallel \overline{HT}$

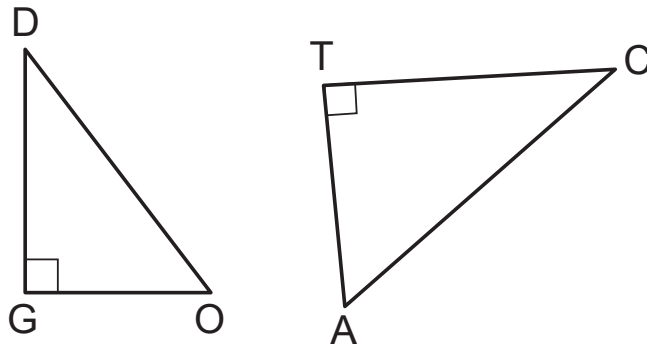
(4)  $\overline{BE} \cong \overline{ET}$

Use this space for  
computations.

11 A gardener wants to buy enough mulch to cover a rectangular garden that is 3 feet by 10 feet. One bag contains 2 cubic feet of mulch and costs \$3.66. How much will the minimum number of bags cost to cover the garden with mulch 3 inches deep?

- (1) \$3.66                      (3) \$14.64  
(2) \$10.98                    (4) \$29.28

12 In the diagram below,  $\triangle DOG \sim \triangle CAT$ , where  $\angle G$  and  $\angle T$  are right angles.

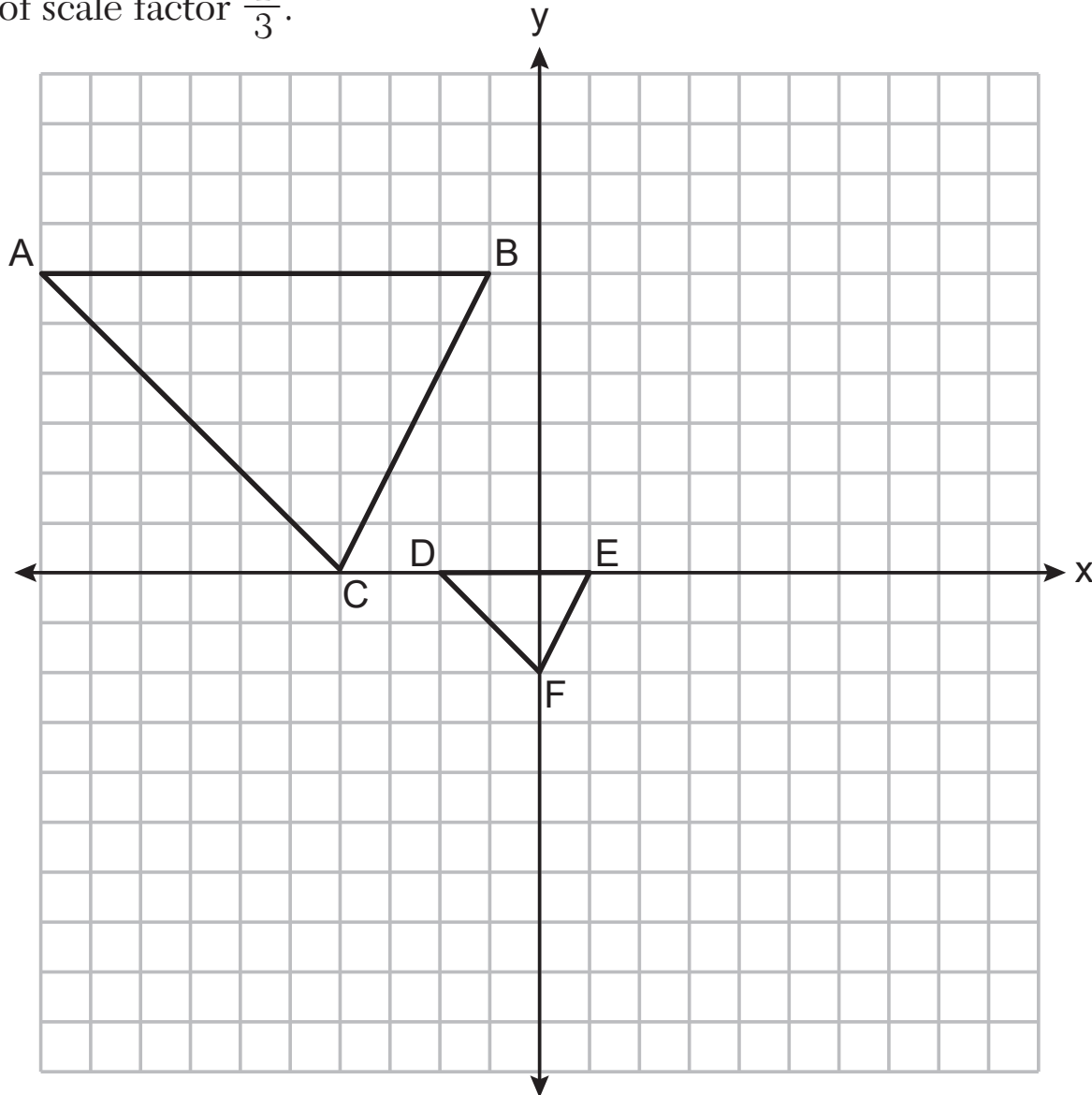


Which expression is always equivalent to  $\sin D$ ?

- (1)  $\cos A$                       (3)  $\tan A$   
(2)  $\sin A$                       (4)  $\cos C$

- 13 On the set of axes below,  $\triangle DEF$  is the image of  $\triangle ABC$  after a dilation of scale factor  $\frac{1}{3}$ .

Use this space for computations.

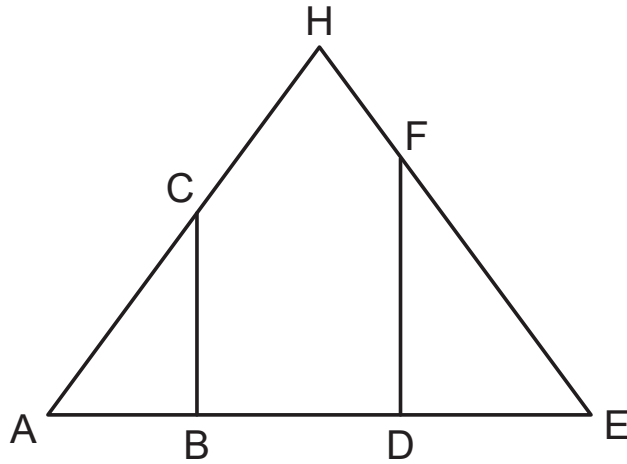


The center of dilation is at

- (1)  $(0,0)$                       (3)  $(0,-2)$   
(2)  $(2,-3)$                     (4)  $(-4,0)$

- 14 In the diagram below of isosceles triangle  $AHE$  with the vertex angle at  $H$ ,  $\overline{CB} \perp \overline{AE}$  and  $\overline{FD} \perp \overline{AE}$ .

Use this space for computations.



Which statement is always true?

- (1)  $\frac{AH}{AC} = \frac{EH}{EF}$                       (3)  $\frac{AB}{ED} = \frac{CB}{FE}$   
(2)  $\frac{AC}{EF} = \frac{AB}{ED}$                       (4)  $\frac{AD}{AB} = \frac{BE}{DE}$

- 15 Rectangle  $ABCD$  has two vertices at coordinates  $A(-1,-3)$  and  $B(6,5)$ . The slope of  $\overline{BC}$  is

- (1)  $-\frac{7}{8}$                                       (3)  $-\frac{8}{7}$   
(2)  $\frac{7}{8}$                                       (4)  $\frac{8}{7}$

**16** In right triangle  $ABC$ ,  $m\angle A = 90^\circ$ ,  $m\angle B = 18^\circ$ , and  $AC = 8$ .

To the *nearest tenth*, the length of  $\overline{BC}$  is

- (1) 2.5
- (2) 8.4
- (3) 24.6
- (4) 25.9

**Use this space for  
computations.**

**17** The measure of one of the base angles of an isosceles triangle is  $42^\circ$ .

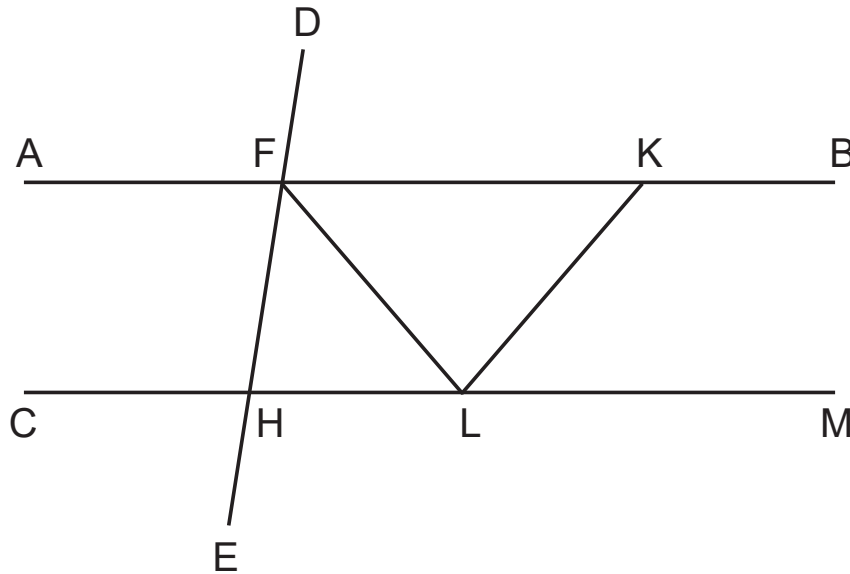
The measure of an exterior angle at the vertex of the triangle is

- (1)  $42^\circ$
- (2)  $84^\circ$
- (3)  $96^\circ$
- (4)  $138^\circ$



18 In the diagram below,  $\overline{AFKB} \parallel \overline{CHLM}$ ,  $\overline{FH} \cong \overline{LH}$ ,  $\overline{FL} \cong \overline{KL}$ , and  $\overline{LF}$  bisects  $\angle HFK$ .

Use this space for computations.



Which statement is always true?

- (1)  $2(m\angle HLF) = m\angle CHE$       (3)  $m\angle AFD = m\angle BKL$   
 (2)  $2(m\angle FLK) = m\angle LKB$       (4)  $m\angle DFK = m\angle KLF$

**Use this space for  
computations.**

**19** The line whose equation is  $6x + 3y = 3$  is dilated by a scale factor of 2 centered at the point  $(0,0)$ . An equation of its image is

(1)  $y = -2x + 1$

(3)  $y = -4x + 1$

(2)  $y = -2x + 2$

(4)  $y = -4x + 2$

**20** Which figure will *not* carry onto itself after a 120-degree rotation about its center?

(1) equilateral triangle

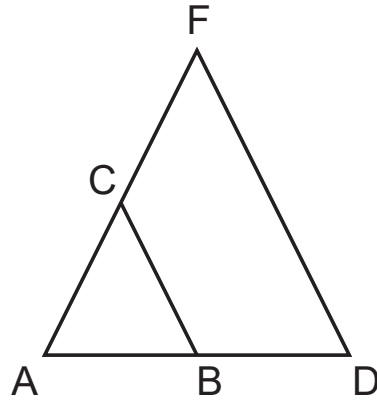
(3) regular octagon

(2) regular hexagon

(4) regular nonagon

21 Triangle  $ADF$  is drawn and  $\overline{BC} \parallel \overline{DF}$ .

Use this space for  
computations.



Which statement must be true?

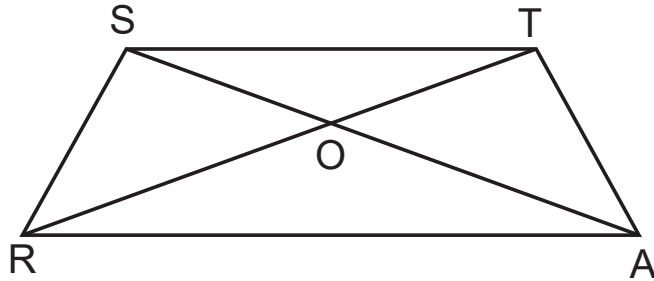
- (1)  $\frac{AB}{BC} = \frac{BD}{DF}$                       (3)  $AB:AD = AC:CF$   
(2)  $BC = \frac{1}{2}DF$                       (4)  $\angle ACB \cong \angle AFD$

22 In  $\triangle ABC$ ,  $M$  is the midpoint of  $\overline{AB}$  and  $N$  is the midpoint of  $\overline{AC}$ .  
If  $MN = x + 13$  and  $BC = 5x - 1$ , what is the length of  $\overline{MN}$ ?

- (1) 3.5                      (3) 16.5  
(2) 9                      (4) 22

Use this space for  
computations.

- 23 In the diagram below of isosceles trapezoid  $STAR$ , diagonals  $\overline{AS}$  and  $\overline{RT}$  intersect at  $O$  and  $\overline{ST} \parallel \overline{RA}$ , with nonparallel sides  $\overline{SR}$  and  $\overline{TA}$ .



Which pair of triangles are *not* always similar?

- (1)  $\triangle STO$  and  $\triangle ARO$                       (3)  $\triangle SRA$  and  $\triangle ATS$   
(2)  $\triangle SOR$  and  $\triangle TOA$                       (4)  $\triangle SRT$  and  $\triangle TAS$
- 24 The endpoints of  $\overline{AB}$  are  $A(0,4)$  and  $B(-4,6)$ . Which equation of a line represents the perpendicular bisector of  $\overline{AB}$ ?

- (1)  $y = -\frac{1}{2}x + 4$                       (3)  $y = 2x + 8$   
(2)  $y = -2x + 1$                       (4)  $y = 2x + 9$
-

**GO RIGHT ON TO THE NEXT PAGE ⇒**

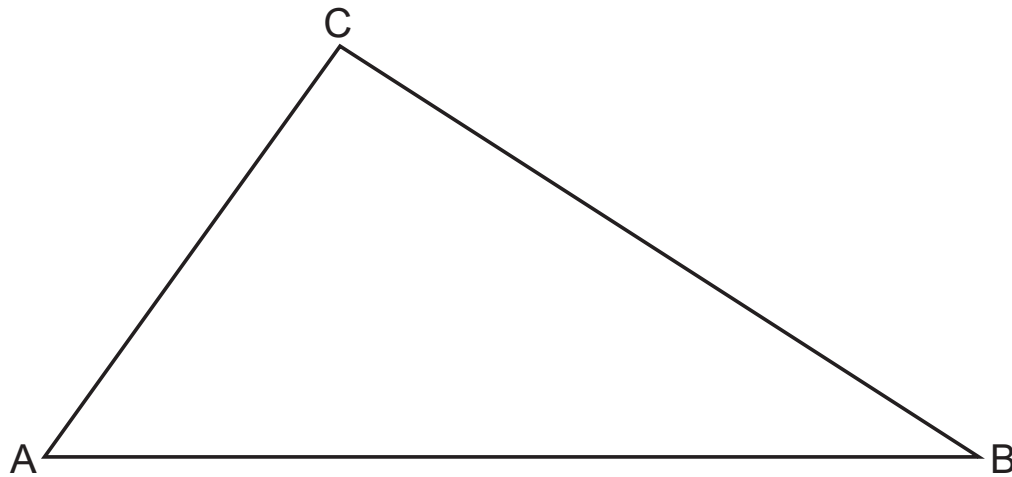
## Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

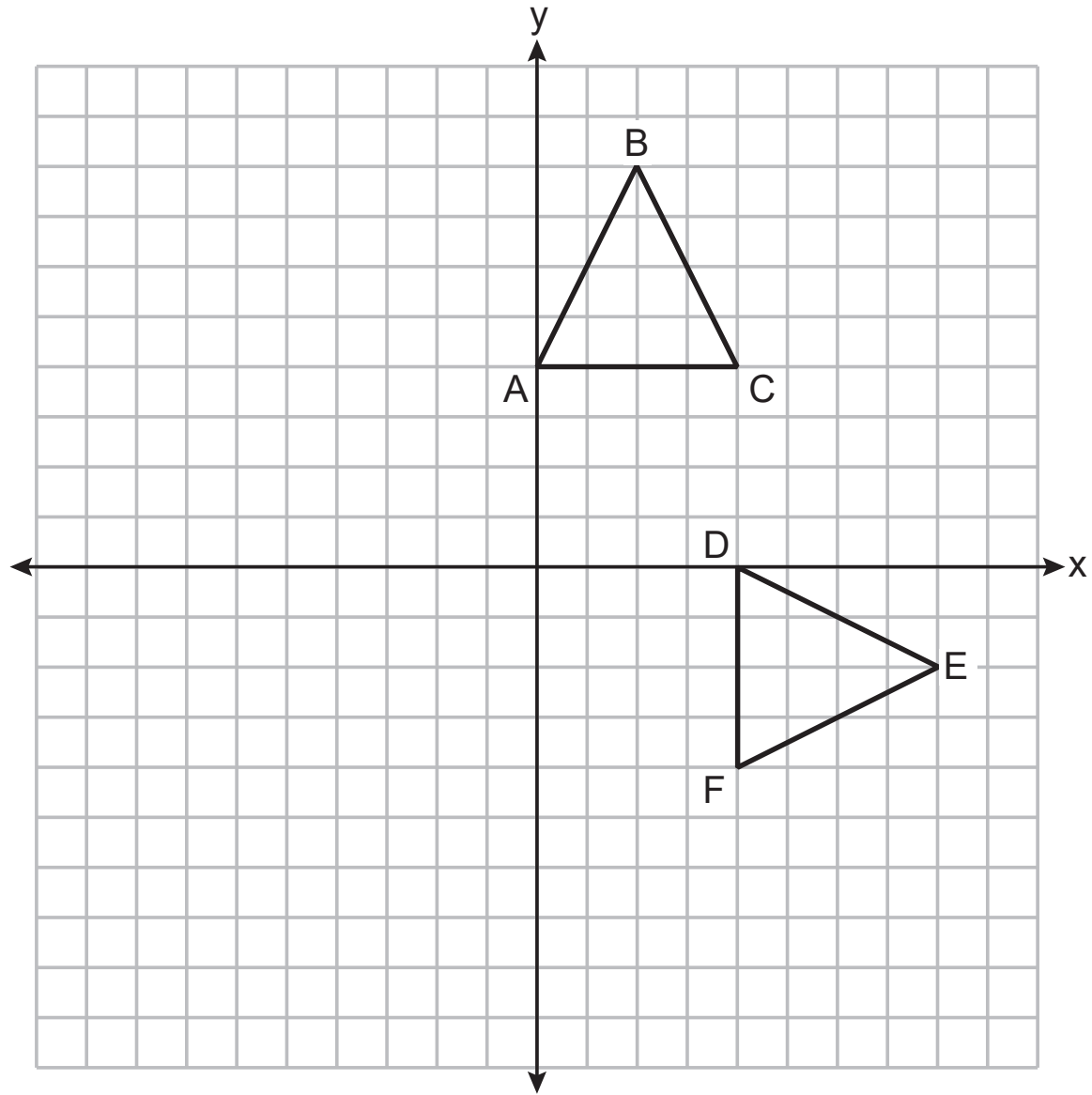
- 25** In  $\triangle ABC$  on the next page, use a compass and straightedge to construct the altitude from  $C$  to  $\overline{AB}$ .  
[Leave all construction marks.]

Question 25 is continued on the next page.

**Question 25 continued**



**26** Triangles  $ABC$  and  $DEF$  are graphed on the set of axes below.



**Question 26 is continued on the next page.**



**Question 26 continued**

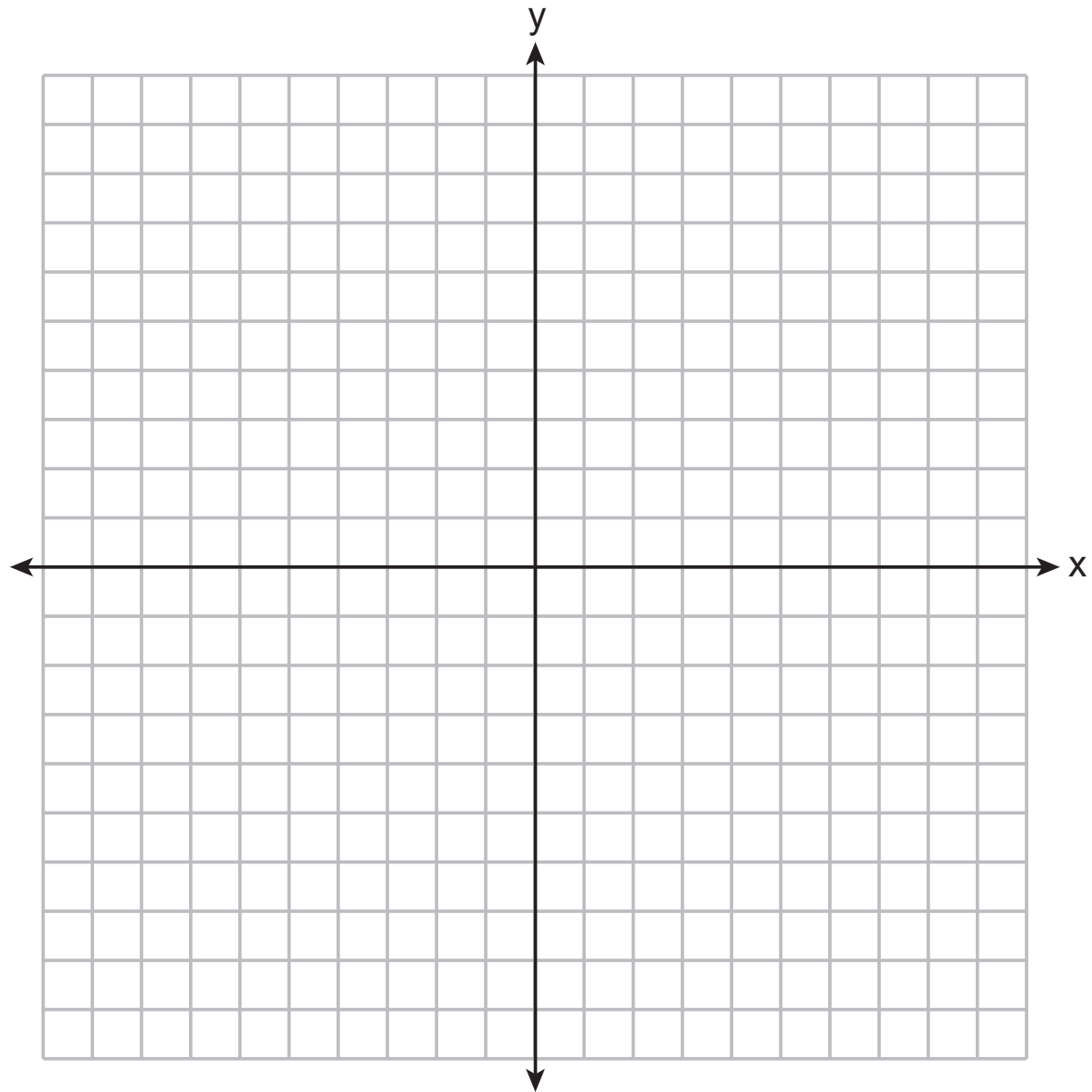
Describe a sequence of transformations that maps  $\triangle ABC$  onto  $\triangle DEF$ .

**27** Line segment  $PQ$  has endpoints  $P(-5,1)$  and  $Q(5,6)$ , and point  $R$  is on  $\overline{PQ}$ . Determine and state the coordinates of  $R$ , such that  $PR:RQ = 2:3$ .

[The use of the set of axes on the next page is optional.]

**The set of axes for question 27 is on the next page.**

**Question 27 continued**



**28** A circle has a radius of 6.4 inches. Determine and state, to the *nearest square inch*, the area of a sector whose arc measures  $80^\circ$ .

**Work space for question 28 is continued on the next page.**

**Question 28 continued**

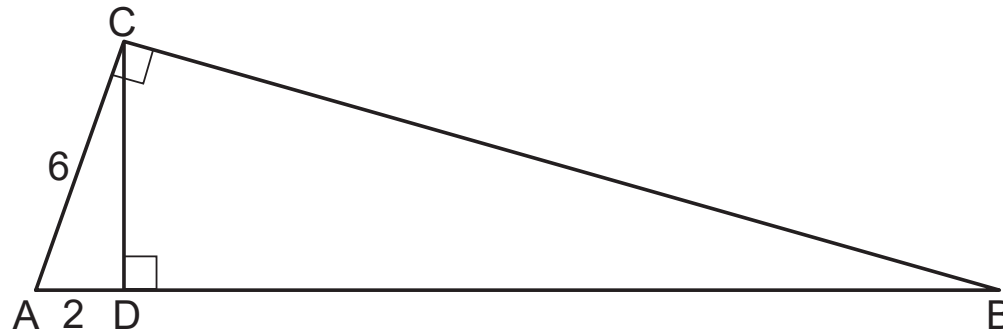
**29** A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.

[Leave your answer in terms of  $\pi$ .]

**Work space for question 29 is continued on the next page.**

**Question 29 continued**

- 30** In the diagram below of right triangle  $ACB$ , altitude  $\overline{CD}$  is drawn to hypotenuse  $\overline{AB}$ ,  $AD = 2$  and  $AC = 6$ .



Determine and state the length of  $\overline{AB}$ .

Work space for question 30 is continued on the next page.



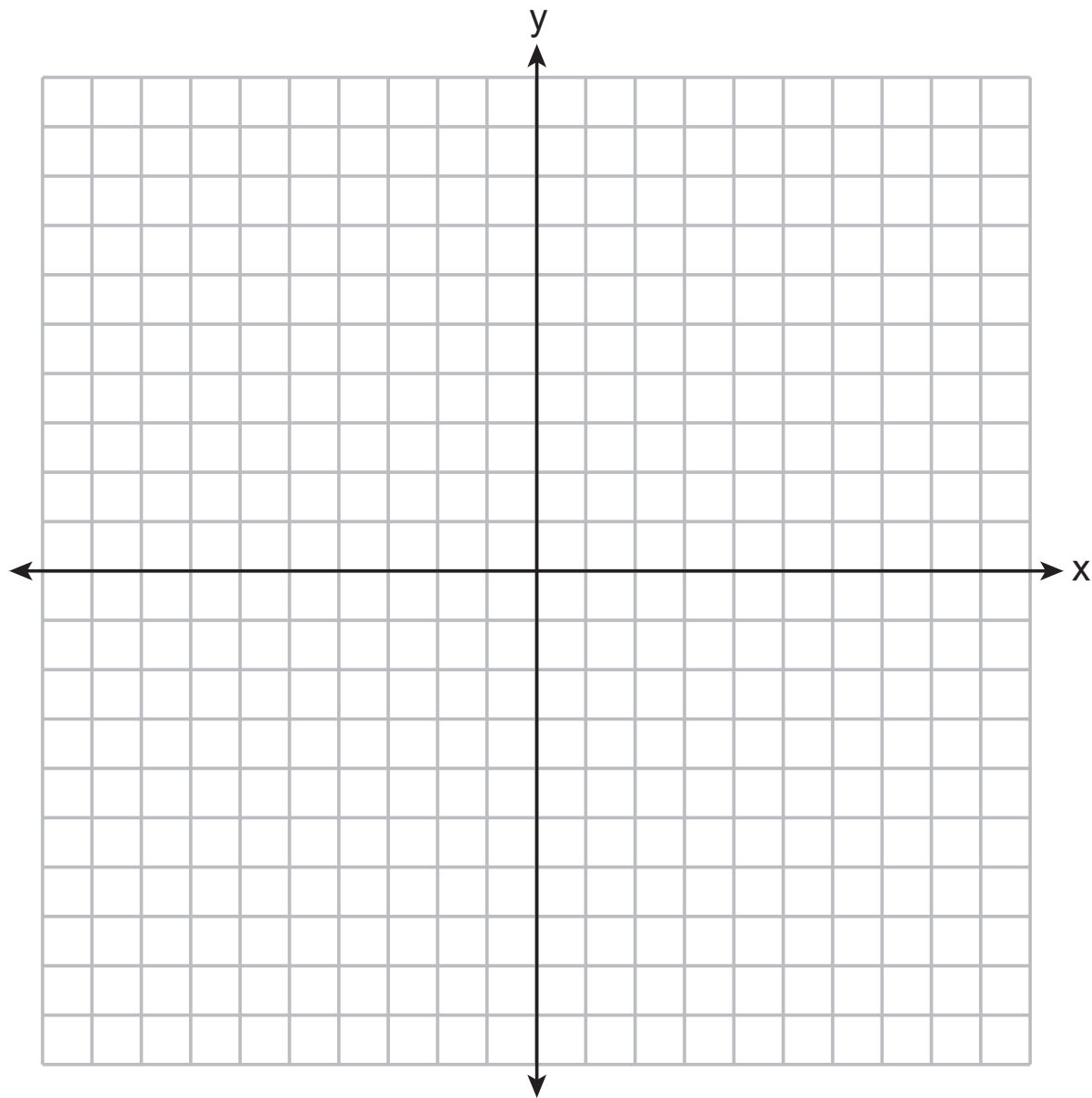
**Question 30 continued**

**31** Triangle  $RST$  has vertices with coordinates  $R(-3, -2)$ ,  $S(3, 2)$  and  $T(4, -4)$ . Determine and state an equation of the line parallel to  $\overline{RT}$  that passes through point  $S$ .

[The use of the set of axes on the next page is optional.]

**The set of axes for question 31 is on the next page.**

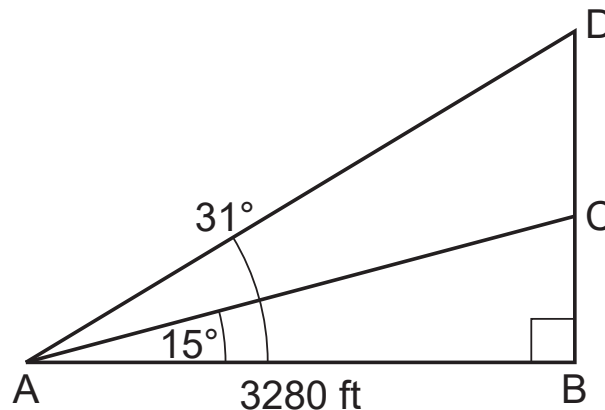
**Question 31 continued**



### Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area  $A$ , 3280 feet away from launch pad  $B$ . After launch, the rocket was sighted at  $C$  with an angle of elevation of  $15^\circ$ . The rocket was later sighted at  $D$  with an angle of elevation of  $31^\circ$ .



Question 32 is continued on the next page.

**Question 32 continued**

Determine and state, to the *nearest foot*, the distance the rocket traveled between the two sightings,  $C$  and  $D$ .

**33** A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm. A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm.

Determine and state the volume of the small can and the volume of the large container to the *nearest cubic centimeter*.

**Question 33 is continued on the next page.**

**Question 33 continued**

What is the minimum number of small cans that must be opened to fill the large container?  
Justify your answer.

**34** Parallelogram  $MATH$  has vertices  $M(-7, -2)$ ,  $A(0, 4)$ ,  $T(9, 2)$ , and  $H(2, -4)$ .

Prove that parallelogram  $MATH$  is a rhombus.

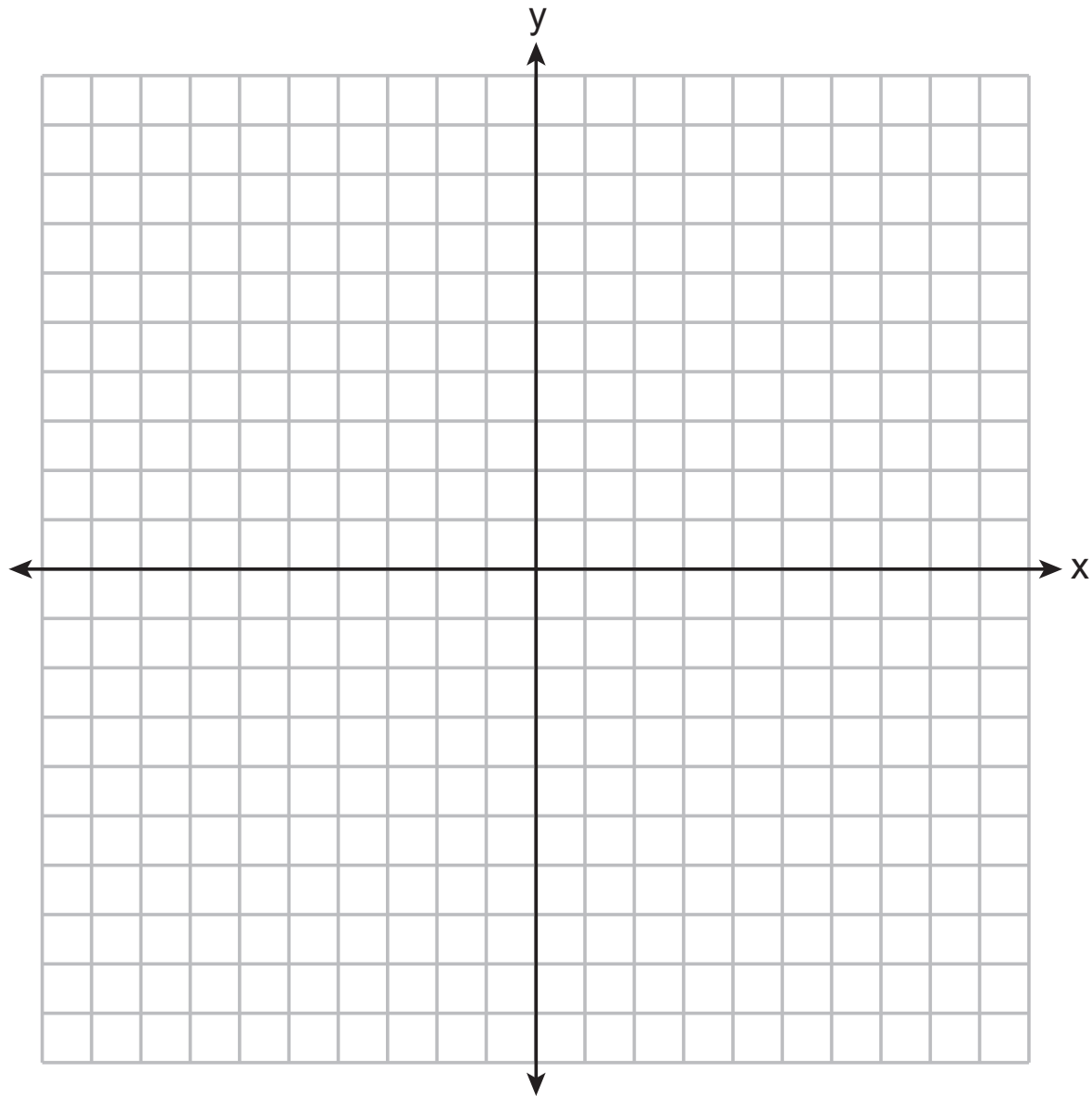
[The use of the set of axes on the next page is optional.]

Determine and state the area of  $MATH$ .

**The set of axes for question 34 is on the next page.**



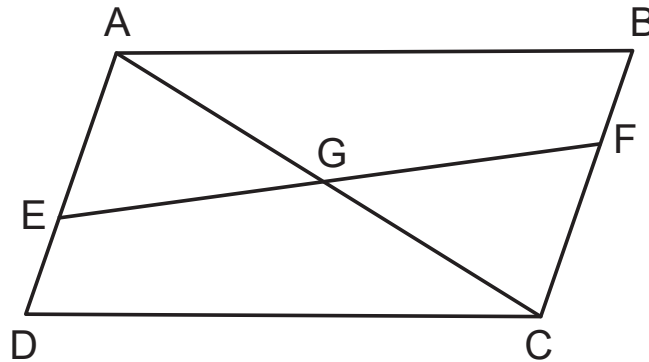
**Question 34 continued**



### Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral  $ABCD$ ,  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AB} \parallel \overline{CD}$ , diagonal  $\overline{AC}$  intersects  $\overline{EF}$  at  $G$ , and  $\overline{DE} \cong \overline{BF}$



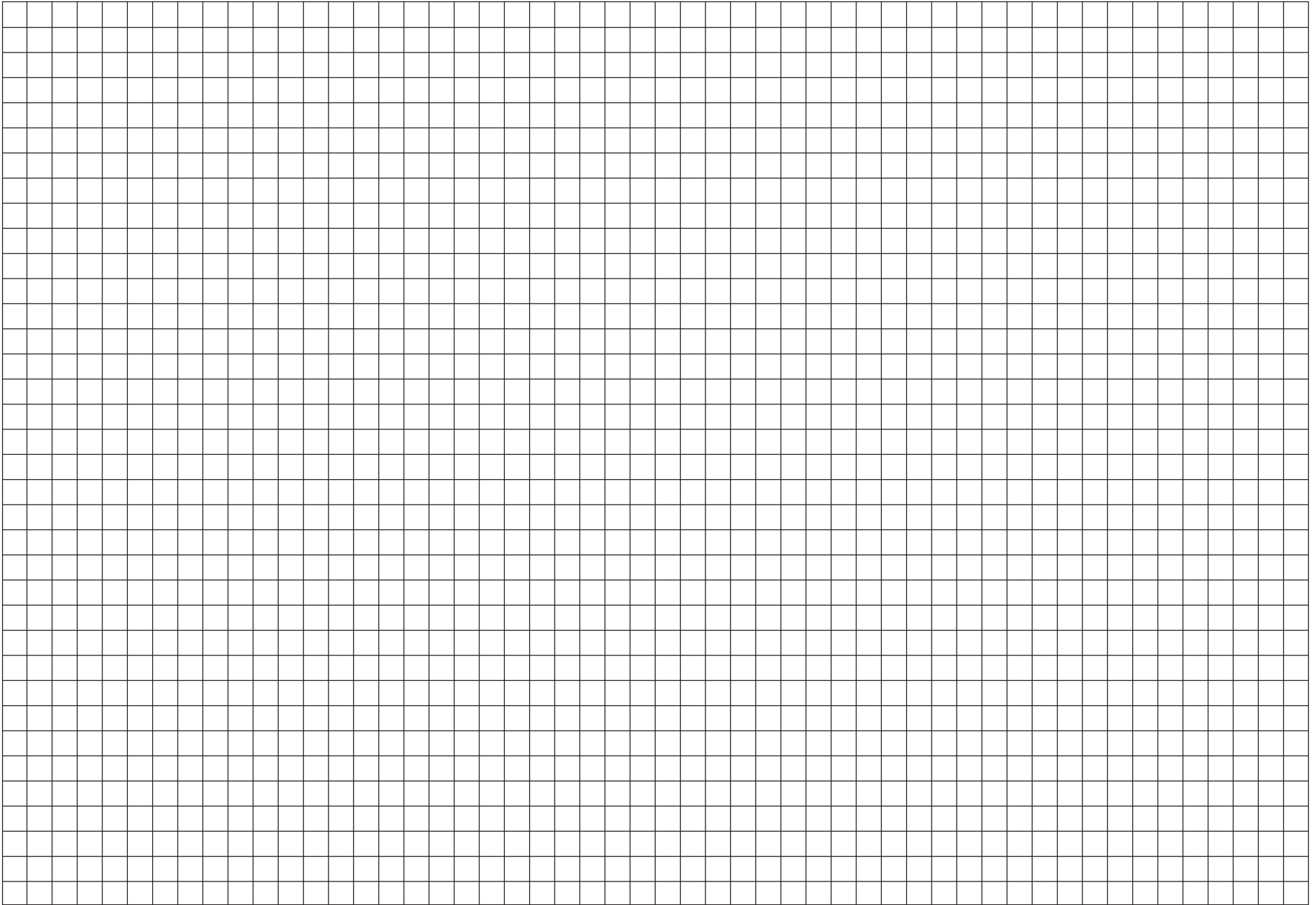
Prove:  $G$  is the midpoint of  $\overline{EF}$

Work space for question 35 is continued on the next page.

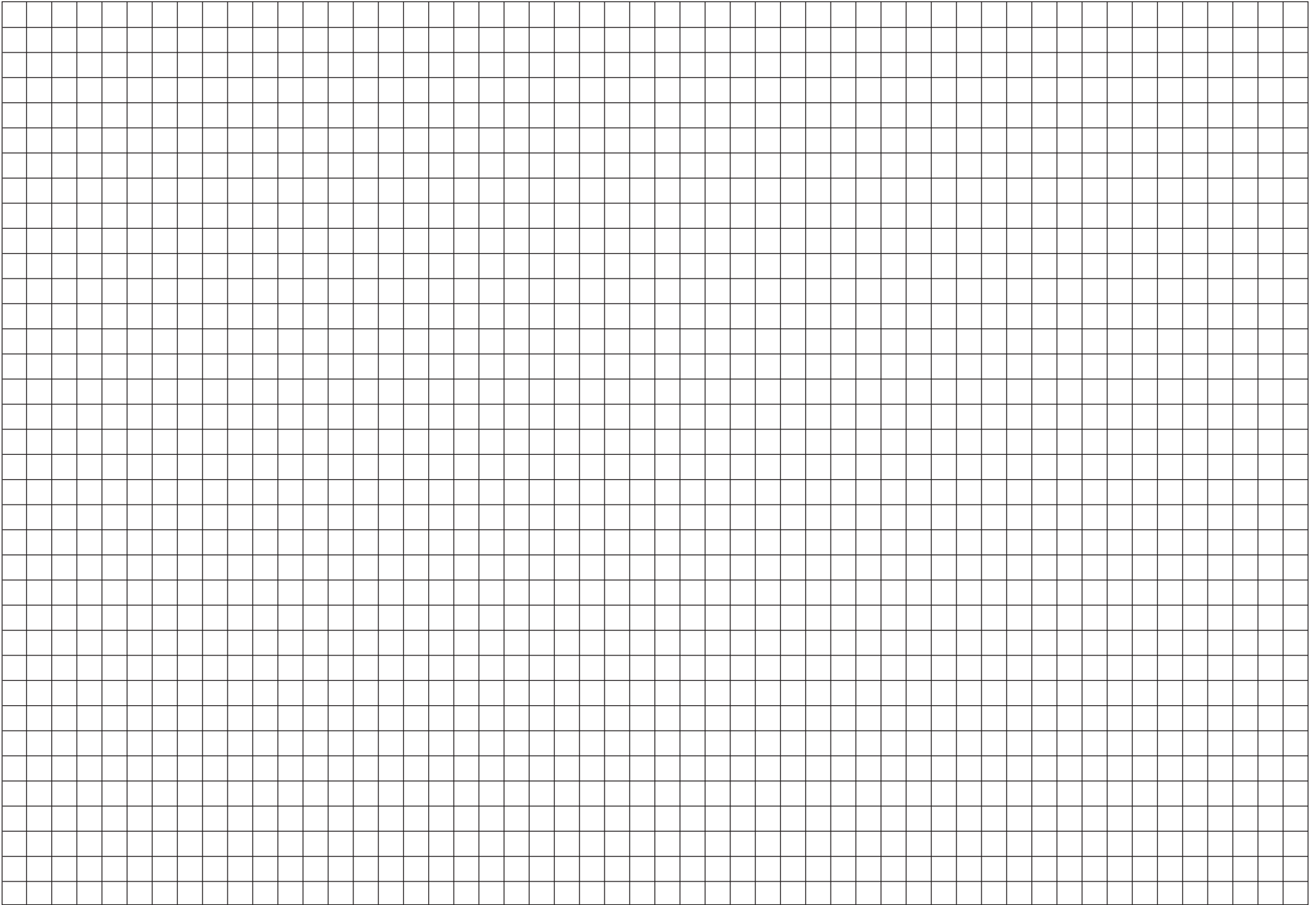
**Question 35 continued**



Scrap Graph Paper — this sheet will *not* be scored.



Scrap Graph Paper — this sheet will *not* be scored.



## High School Math Reference Sheet

1 inch = 2.54 centimeters

1 meter = 39.37 inches

1 mile = 5280 feet

1 mile = 1760 yards

1 mile = 1.609 kilometers

1 kilometer = 0.62 mile

1 pound = 16 ounces

1 pound = 0.454 kilogram

1 kilogram = 2.2 pounds

1 ton = 2000 pounds

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

1 gallon = 3.785 liters

1 liter = 0.264 gallon

1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

**The Reference Sheet is continued on the next page.**

## Reference Sheet — concluded

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3} \pi r^3$
Cone	$V = \frac{1}{3} \pi r^2 h$
Pyramid	$V = \frac{1}{3} B h$

Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$