The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

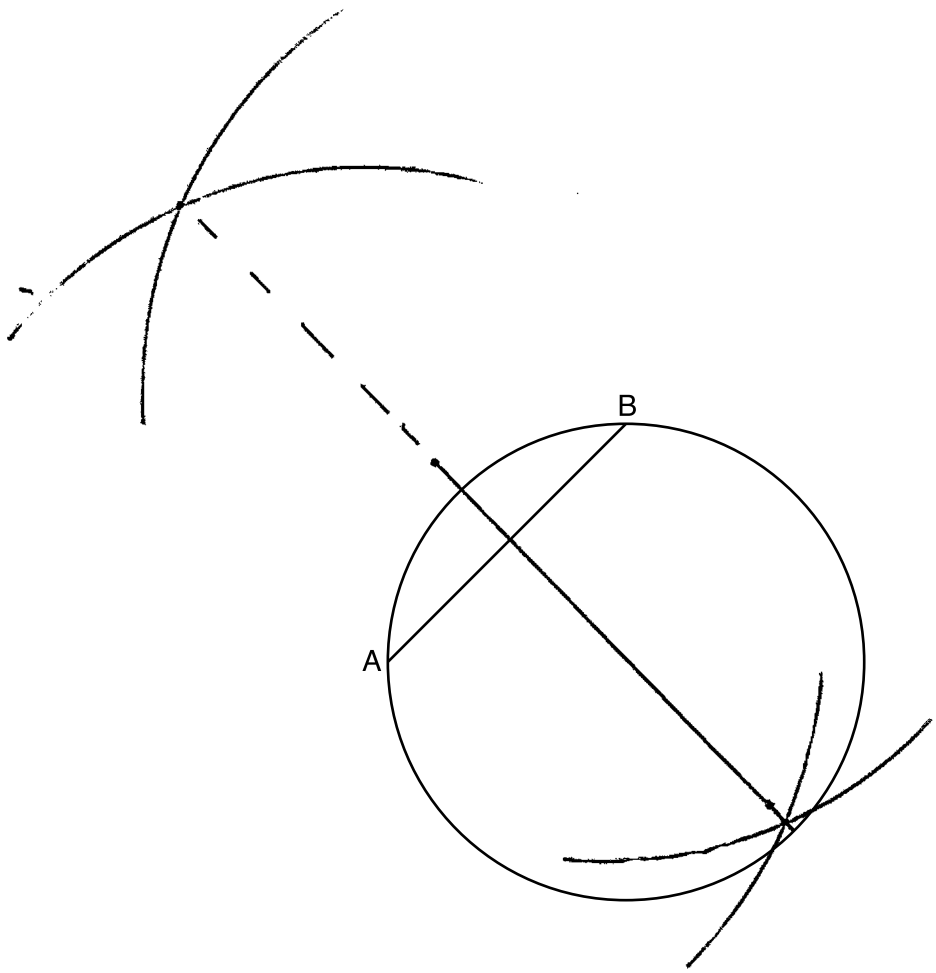
**Friday,** August 17, 2018 — 12:30 to 3:30 p.m.

## MODEL RESPONSE SET

Table of Contents

## Question 25

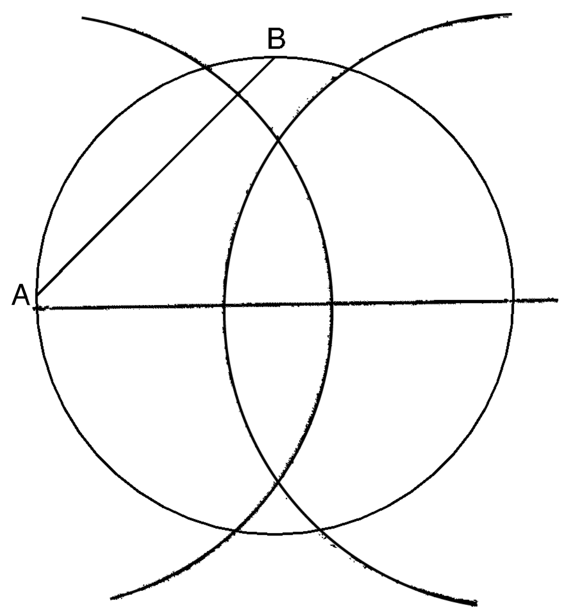
**25** In the circle below, $\overline{AB}$ is a chord. Using a compass and straightedge, construct a diameter of the circle. [Leave all construction marks.]




**Score 2:** The student gave a complete and correct response.

## Question 25

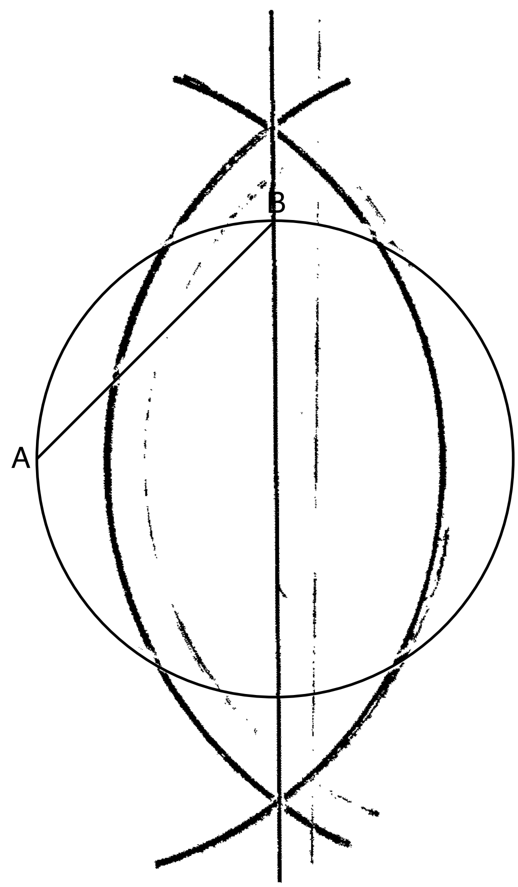
**25** In the circle below, $\overline{AB}$ is a chord. Using a compass and straightedge, construct a diameter of the circle. [Leave all construction marks.]




**Score 1:** The student drew appropriate arcs for a chord other than $\overline{AB}$, but did not draw the diameter.

## Question 25

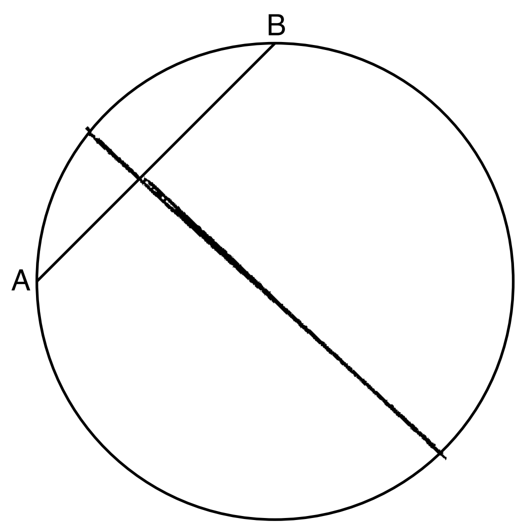
**25** In the circle below, $\overline{AB}$ is a chord. Using a compass and straightedge, construct a diameter of the circle. [Leave all construction marks.]




**Score 1:** The student drew an appropriate construction, but the endpoint of the chord used is missing.

## Question 25

**25** In the circle below, $\overline{AB}$ is a chord. Using a compass and straightedge, construct a diameter of the circle. [Leave all construction marks.]



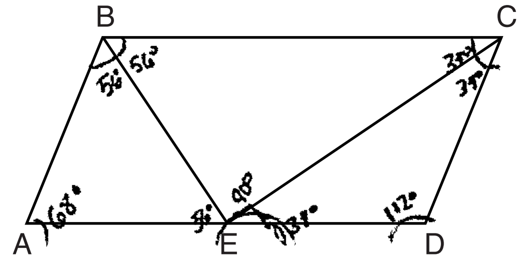

**Score 0:** The student gave a completely incorrect response.

## Question 26

**26** In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at $E$, a point on $\overline{AD}$.

① splits angle in half

② opposite angles are equal!!




$180 - 68 = 112$

$112 \div 2 = 56$

If m$\angle A$ = 68°, determine and state m$\angle BEC$.

③

$112 + 34 = 146$

$56 + 34 = 90$

$68 \div 2 = 34$

④

$180 - 146 = 34$

$180 - 90 = 90$

$\boxed{m\angle BEC = 90°}$

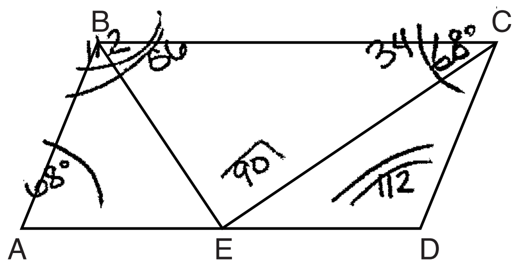
* Numbered all of the steps

Find out all the parts and subtract them from 180°. (Final step)

**Score 2:** The student gave a complete and correct response.

## Question 26

**26** In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at $E$, a point on $\overline{AD}$.




If m$\angle A$ = 68°, determine and state m$\angle BEC$.

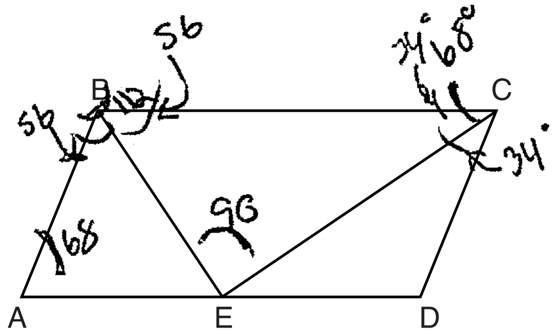
$360 - 68(2) = \frac{224}{2} = 112$

$\boxed{m\angle BEC = 90°}$

**Score 2:** The student gave a complete and correct response.

## Question 26

**26** In parallelogram *ABCD* shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at *E*, a point on $\overline{AD}$.

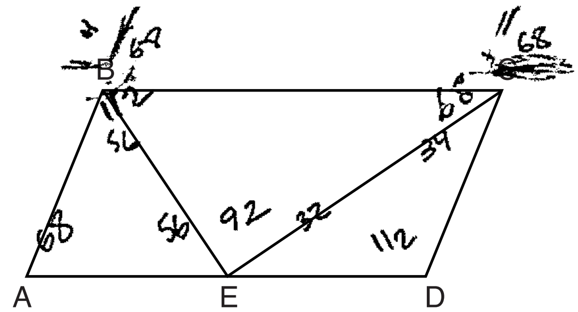



If m$\angle A$ = 68°, determine and state m$\angle BEC$.

| Statements | Reasons |
| --- | --- |
| ① $\overline{BE}$ & $CE$ bisect $\angle ABC$ & $\angle DCB$<br>$\angle A = 68°$<br>ABCD is a parallelogram | ① Given |
| ② $\angle ABC = 112°$ | ② adjacent ∠'s in a parallelogram are supp. |
| ③ $\angle DCD = 68°$ | ③ opposite ∠'s in a parallelogram are ≅ |
| ④ $\angle EBC = 56°$<br>$\angle ECB = 34°$ | ④ Angle bisectors cut ∠'s in half |
| ⑤ $\angle BEC = 90$ | ⑤ all angles in a triangle add to 180 |

**Score 2:** The student gave a complete and correct response.

**26** In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at $E$, a point on $\overline{AD}$.




$180 - 68 = 112$

If m$\angle A$ = 68°, determine and state m$\angle BEC$.

$\boxed{m\angle BEC = 92^\circ}$

$112 \div 2 = 56$

$180 - 112 = 68$

$180 - 148 = 32$

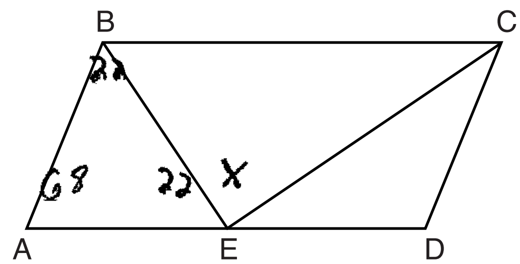
$32 + 56 = 88$

$180 - 88 = 92$

**Score 1:** The student made one computational error in determining m$\angle CED$.

## Question 26

**26** In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at $E$, a point on $\overline{AD}$.




If m$\angle A$ = 68°, determine and state m$\angle BEC$.

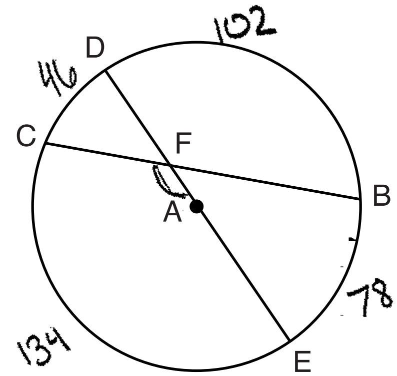
Consectutive angles are suppimentory

180 - 68 = 112

**Score 0:** The student gave a completely incorrect response.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.




If $m\overset{\frown}{CD} = 46°$ and $m\overset{\frown}{DB} = 102°$, what is $m\angle CFE$?

$$\frac{134+102}{2} = m\angle CFE$$

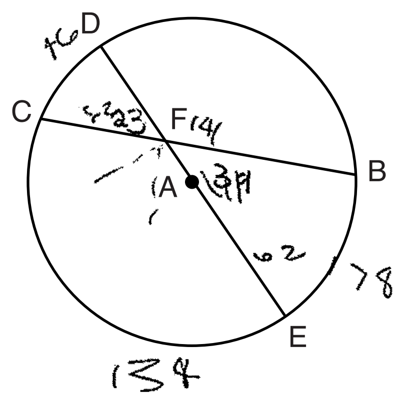
$$118 = m\angle CFE$$

Int. Vertical ∠s have a measure = ½ sum intercepted arcs.

**Score 2:** The student gave a complete and correct response.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.




If m$\overset{\frown}{CD}$ = 46° and m$\overset{\frown}{DB}$ = 102°, what is m∠$CFE$?

$(46 + 78)/2 = 62$

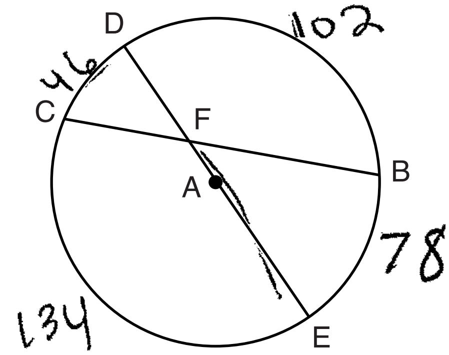
$180 - 62 = 118$

118°

**Score 2:** The student gave a complete and correct response.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.




If m$\overset{\frown}{CD}$ = 46° and m$\overset{\frown}{DB}$ = 102°, what is m$\angle CFE$?

$180 - 102 = 78$

$180 - 46 = 134$

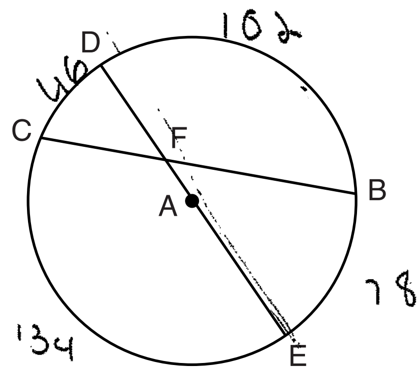
$\frac{134}{2} = 67$

$\boxed{m\angle CFE = 67°}$

**Score 1:** The student made an error by taking half of $\overset{\frown}{CE}$ to find m$\angle CFE$.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.




$$\begin{array}{r} 102 \\ +132 \\ \hline 234 \\ \hline 2 \end{array}$$

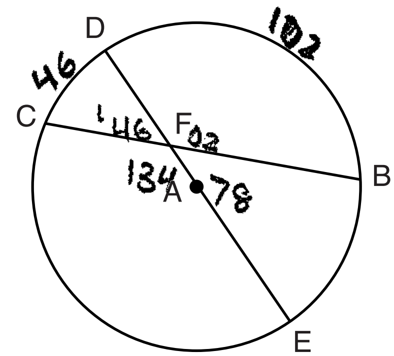
If $m\widehat{CD} = 46°$ and $m\widehat{DB} = 102°$, what is $m\angle CFE$?

$\boxed{117°}$

**Score 1:** The student made a transcription error on $\widehat{CE}$.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.



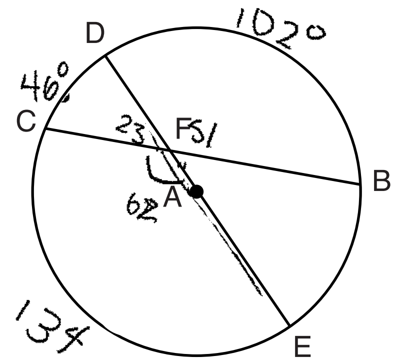

If $m\widehat{CD} = 46°$ and $m\widehat{DB} = 102°$, what is $m\angle CFE$?

$m\angle CFE = 134°$

**Score 1:** The student made an error in thinking $\angle CFE$ is a central angle.

## Question 27

**27** In circle $A$ below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at $F$.



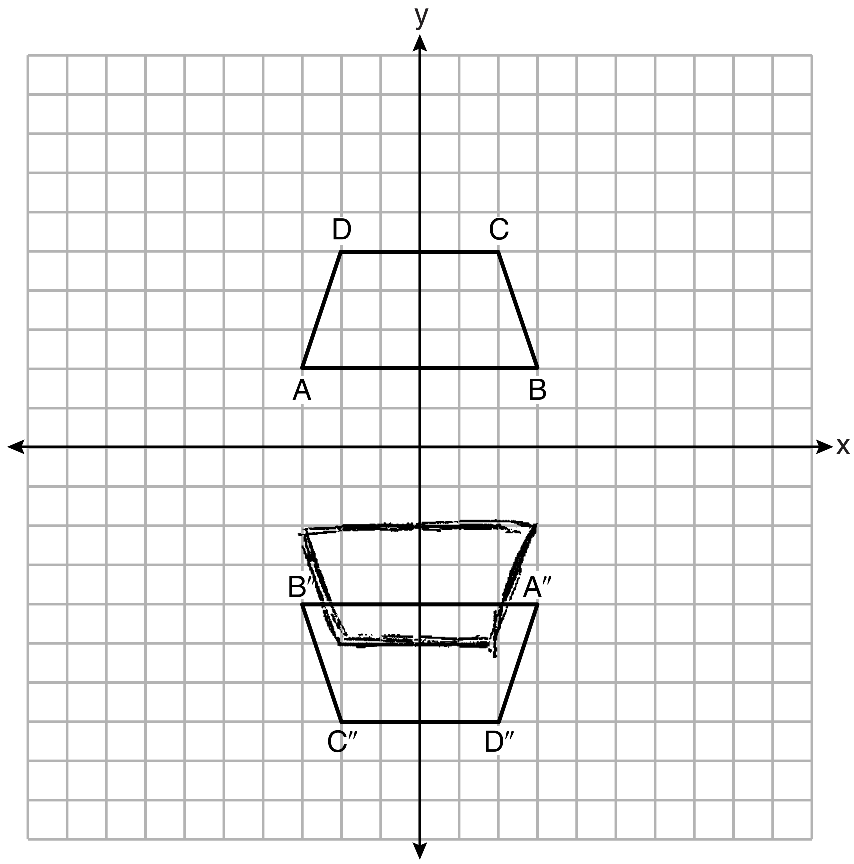

If $m\overset{\frown}{CD} = 46°$ and $m\overset{\frown}{DB} = 102°$, what is $m\angle CFE$?

m∠CFE is 62°

**Score 0:** The student did not show enough correct relevant work to receive any credit.

## Question 28

**28** Trapezoids $ABCD$ and $A''B''C''D''$ are graphed on the set of axes below.



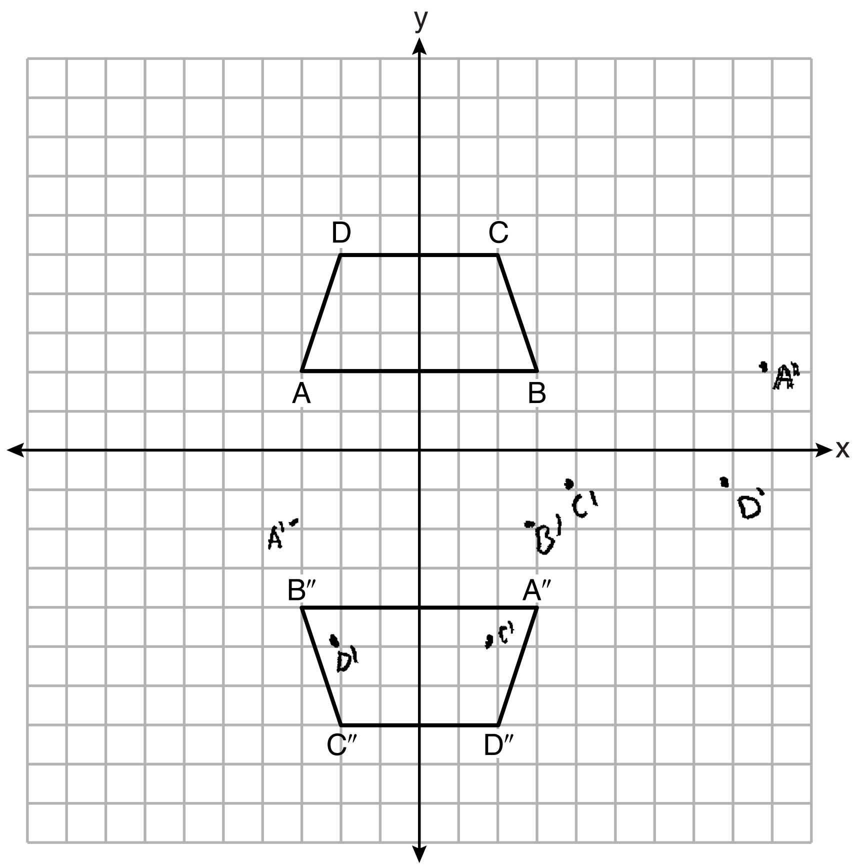

Describe a sequence of transformations that maps trapezoid $ABCD$ onto trapezoid $A''B''C''D''$.

① A rotation of 180° on the origin
② A translation of two units down

**Score 2:** The student gave a complete and correct response.

## Question 28

**28** Trapezoids *ABCD* and *A″B″C″D″* are graphed on the set of axes below.




Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A″B″C″D″*.

$R_{180°}$ at point B

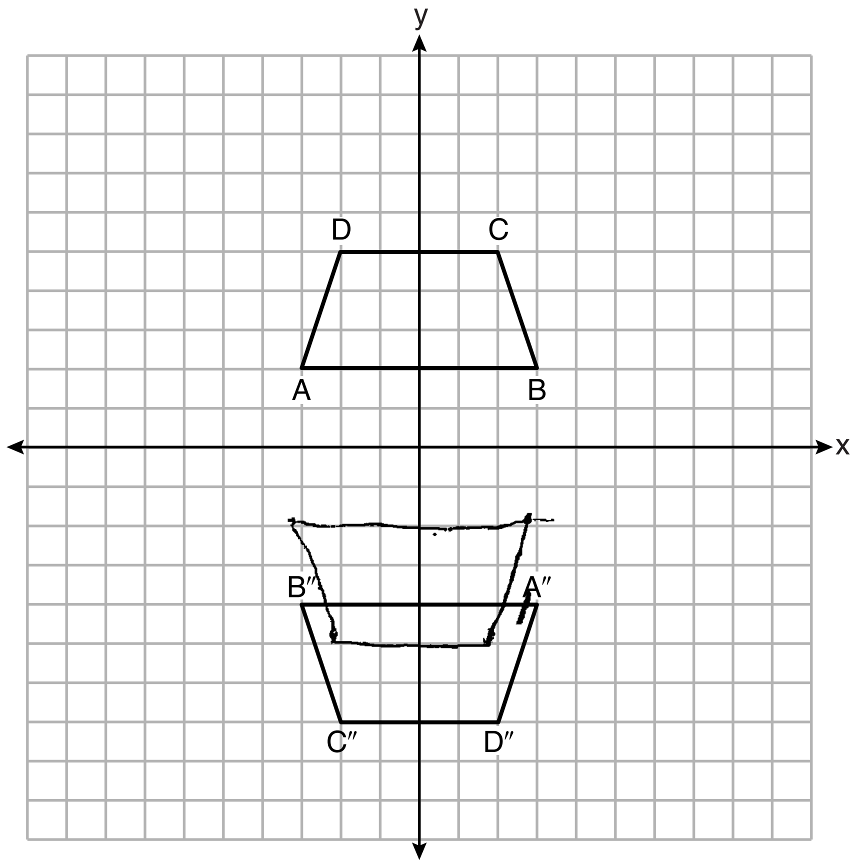
Translation of 6 units down &

6 units left

**Score 2:** The student gave a complete and correct response.

## Question 28

**28** Trapezoids $ABCD$ and $A''B''C''D''$ are graphed on the set of axes below.



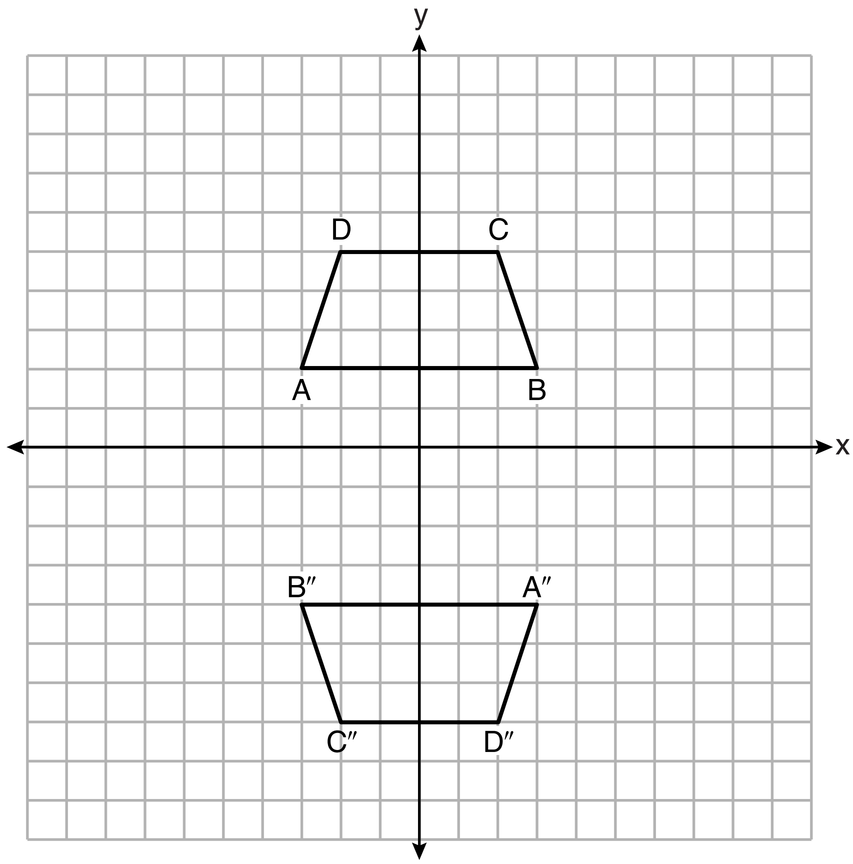

Describe a sequence of transformations that maps trapezoid $ABCD$ onto trapezoid $A''B''C''D''$.

1. reflection over x -axis
2. translation down 2 units
3. reflection over y-axis

**Score 2:** The student gave a complete and correct response.

## Question 28

**28** Trapezoids *ABCD* and *A″B″C″D″* are graphed on the set of axes below.



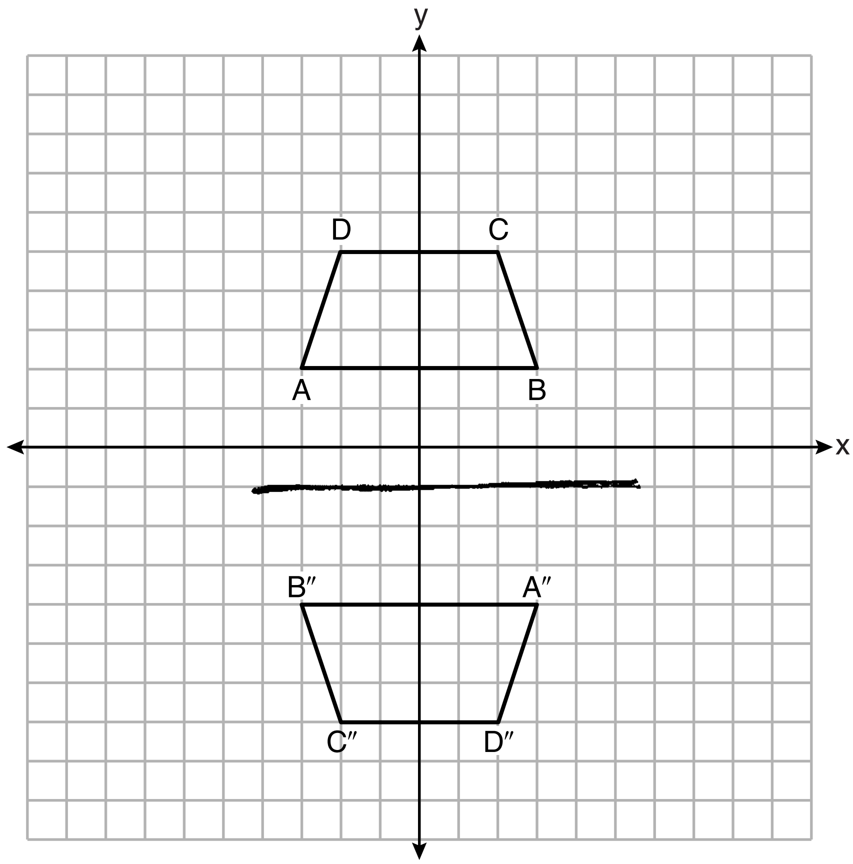

Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A″B″C″D″*.

A reflection of ABCD over the y-axis.
Another reflestion over the x-axis.
Translation down 1 unit.

**Score 1:** The student made one error in stating the translation of down one instead of down two.

## Question 28

**28** Trapezoids *ABCD* and *A″B″C″D″* are graphed on the set of axes below.



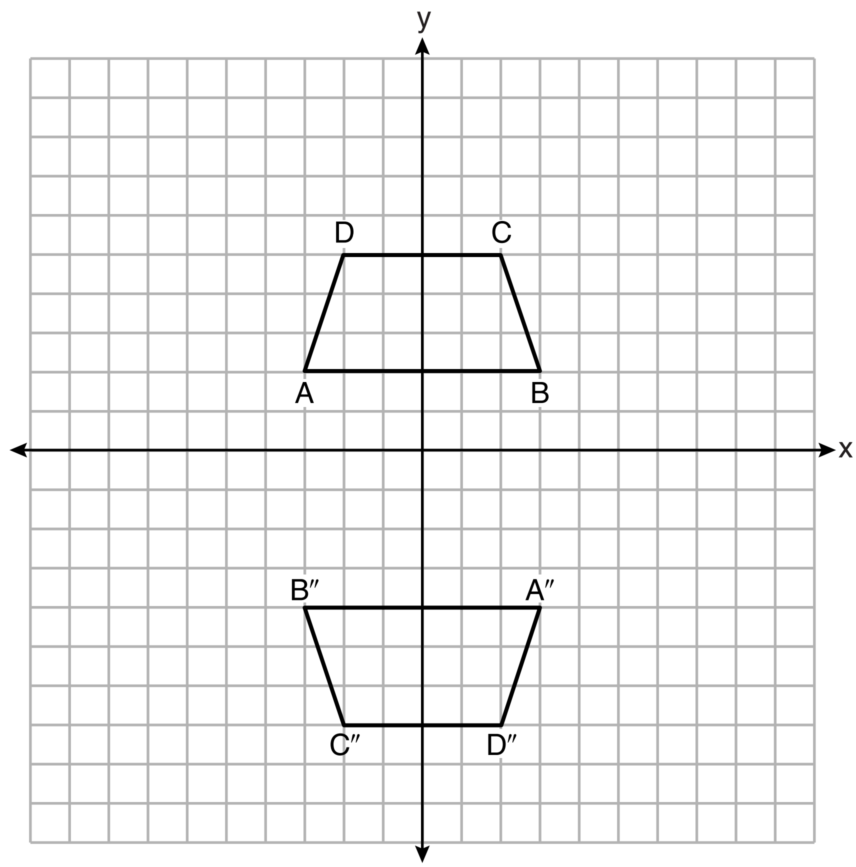

Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A″B″C″D″*.

A reflection over (y=-1) maps
ABCD onto A''B'C'D', since its Rigid
motion only orientation changes

**Score 1:** The student made an error by mapping trapezoid *ABCD* onto trapezoid *B″A″D″C″*.

## Question 28

**28** Trapezoids *ABCD* and *A″B″C″D″* are graphed on the set of axes below.



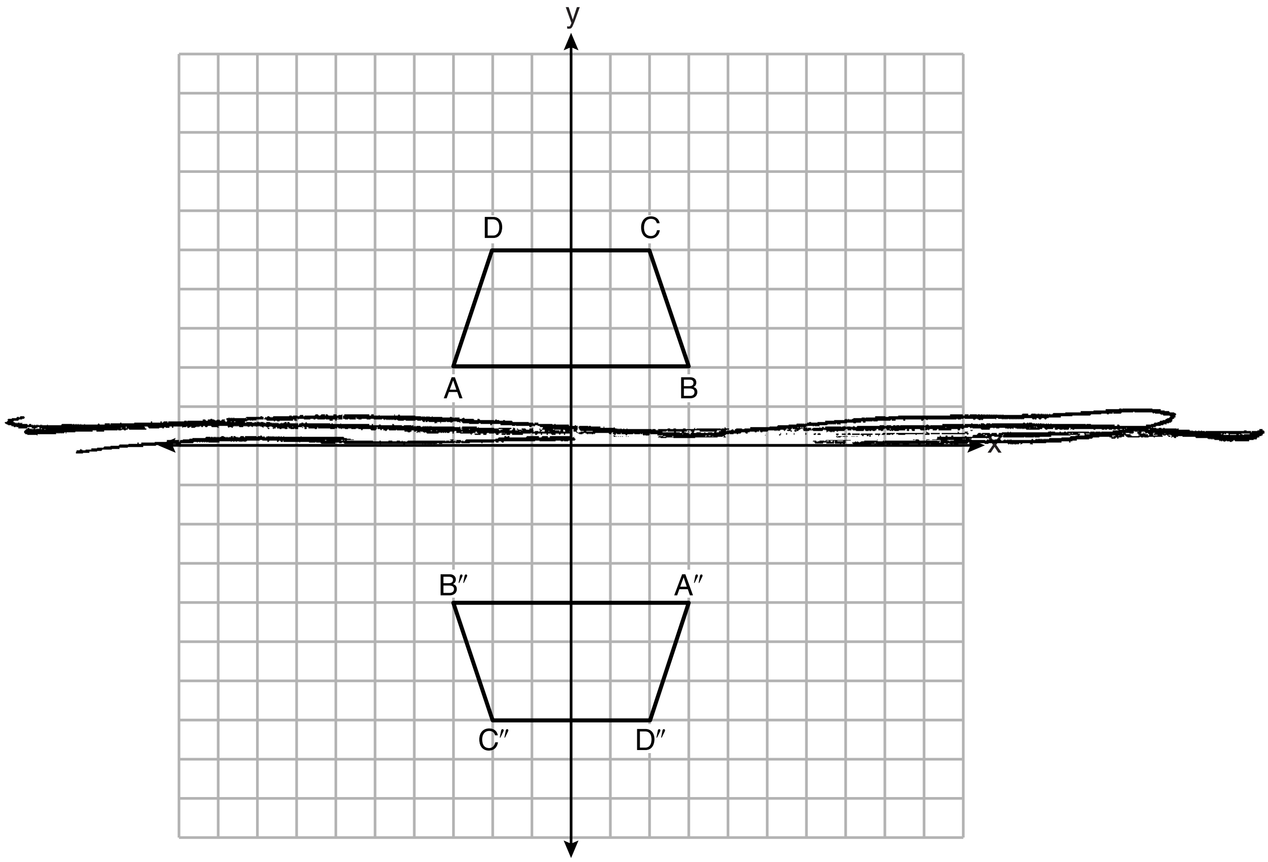

Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A″B″C″D″*.

A reflection over the x axis followed by a translation down 2.

**Score 1:** The student made an error by mapping trapezoid *ABCD* onto trapezoid *B″A″D″C″*.

## Question 28

**28** Trapezoids *ABCD* and *A″B″C″D″* are graphed on the set of axes below.



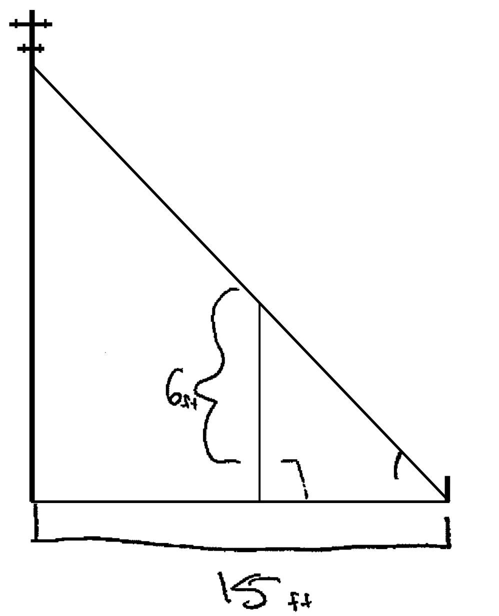

Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A″B″C″D″*.

The sequence of transformation that maps the trapezoid is a ~~R~~ Reflection over the x-axis.

**Score 0:** The student gave a completely incorrect response.

## Question 29

**29** In the model below, a support wire for a telephone pole is attached to the pole and anchored to a stake in the ground 15 feet from the base of the telephone pole. Jamal places a 6-foot wooden pole under the support wire parallel to the telephone pole, such that one end of the pole is on the ground and the top of the pole is touching the support wire. He measures the distance between the bottom of the pole and the stake in the ground.



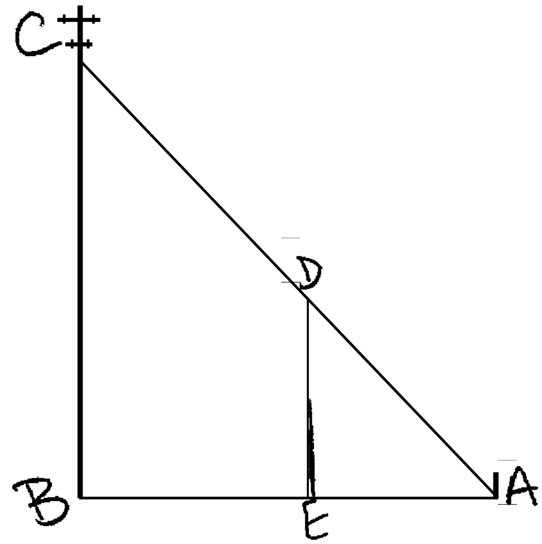

Jamal says he can approximate how high the support wire attaches to the telephone pole by using similar triangles. Explain why the triangles are similar.

The triangles are similar because they have the same angle measurements. They both share the angle that the stake makes with the ground and the ground and pole and ground and Jamal make right angles. Due to this they are similar by AA ≅ AA.

**Score 2:** The student gave a complete and correct response.

## Question 29

**29** In the model below, a support wire for a telephone pole is attached to the pole and anchored to a stake in the ground 15 feet from the base of the telephone pole. Jamal places a 6-foot wooden pole under the support wire parallel to the telephone pole, such that one end of the pole is on the ground and the top of the pole is touching the support wire. He measures the distance between the bottom of the pole and the stake in the ground.



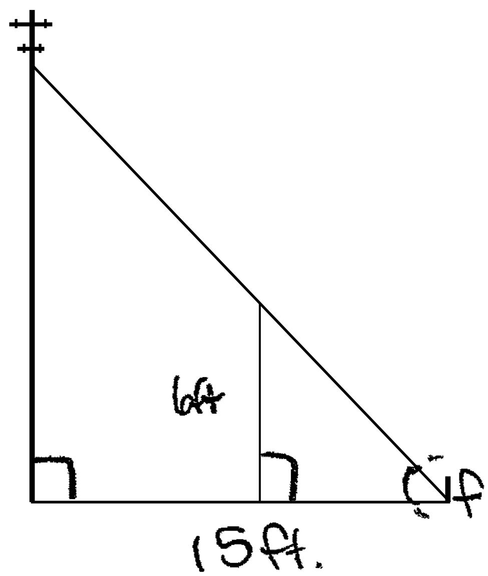

Jamal says he can approximate how high the support wire attaches to the telephone pole by using similar triangles. Explain why the triangles are similar.

These triangles are similar because of AA criteron. $\angle DAE \cong \angle CAB$ because they share that angle. $\angle DEA \cong \angle CBA$ because they are both right angles

**Score 2:** The student gave a complete and correct response.

## Question 29

**29** In the model below, a support wire for a telephone pole is attached to the pole and anchored to a stake in the ground 15 feet from the base of the telephone pole. Jamal places a 6-foot wooden pole under the support wire parallel to the telephone pole, such that one end of the pole is on the ground and the top of the pole is touching the support wire. He measures the distance between the bottom of the pole and the stake in the ground.



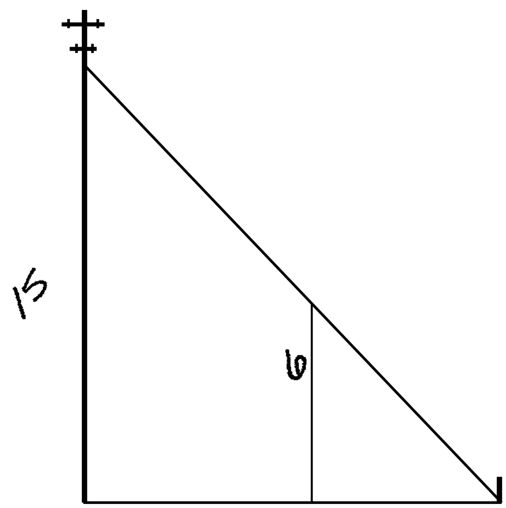

Jamal says he can approximate how high the support wire attaches to the telephone pole by using similar triangles. Explain why the triangles are similar.

They both share <f and both have right angles.

**Score 1:** The student wrote an incomplete explanation not connecting the angles to the similar triangles.

## Question 29

**29** In the model below, a support wire for a telephone pole is attached to the pole and anchored to a stake in the ground 15 feet from the base of the telephone pole. Jamal places a 6-foot wooden pole under the support wire parallel to the telephone pole, such that one end of the pole is on the ground and the top of the pole is touching the support wire. He measures the distance between the bottom of the pole and the stake in the ground.




Jamal says he can approximate how high the support wire attaches to the telephone pole by using similar triangles. Explain why the triangles are similar.

The triangles are similar because they both have right angles which makes them right triangles, all right triangles are similar.

**Score 0:** The student gave a completely incorrect response.

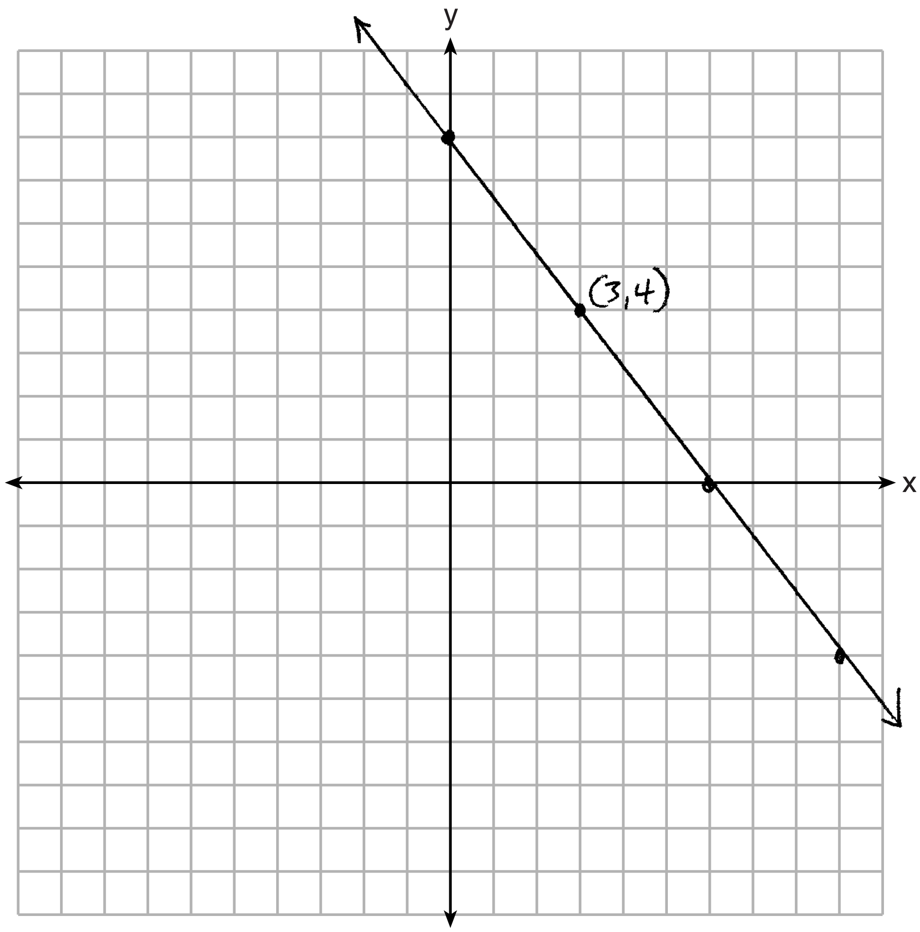
## Question 30

**30** Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

[The use of the set of axes below is optional.]

$$4x + 3y = 24$$
$$-4x \qquad -4x$$
$$\frac{3y}{3} = \frac{-4x + 24}{3}$$
$$y = -\frac{4}{3}x + 8$$

Aliyah is not correct. When the center of dilation is on the line, the equation remains the same. $y = -\frac{4}{3}x + 8$




**Score 2:** The student gave a complete and correct response.

## Question 30

**30** Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

[The use of the set of axes below is optional.]

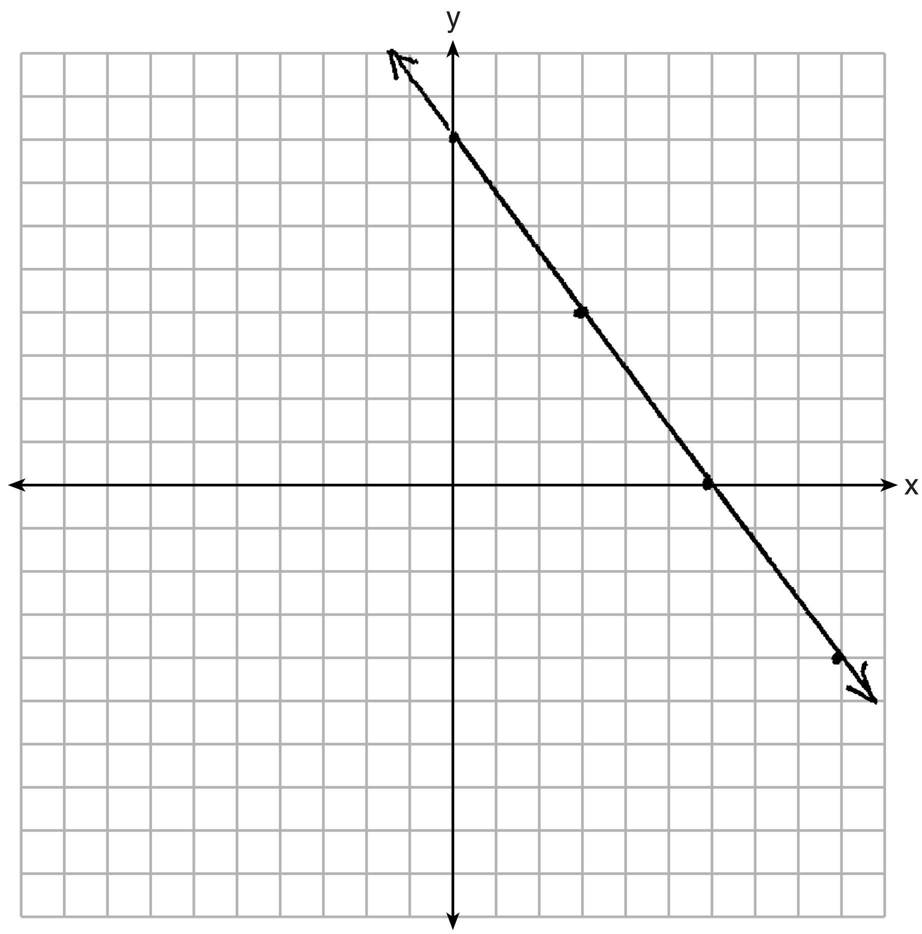
$4x + 3y = 24$

$3y - 4x = 24$

$y = \frac{4}{3}x + 8$

$y = -\frac{4}{3}x + 16$

Aliyah is not correct because the equation of the dilated line should be $y = -\frac{4}{3}x + 8$ to be correct.




**Score 1:** The student wrote an incomplete explanation.

## Question 30

**30** Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

$\frac{3y = -4x = 24}{3}$

$\boxed{y = -4/3x + 8}$

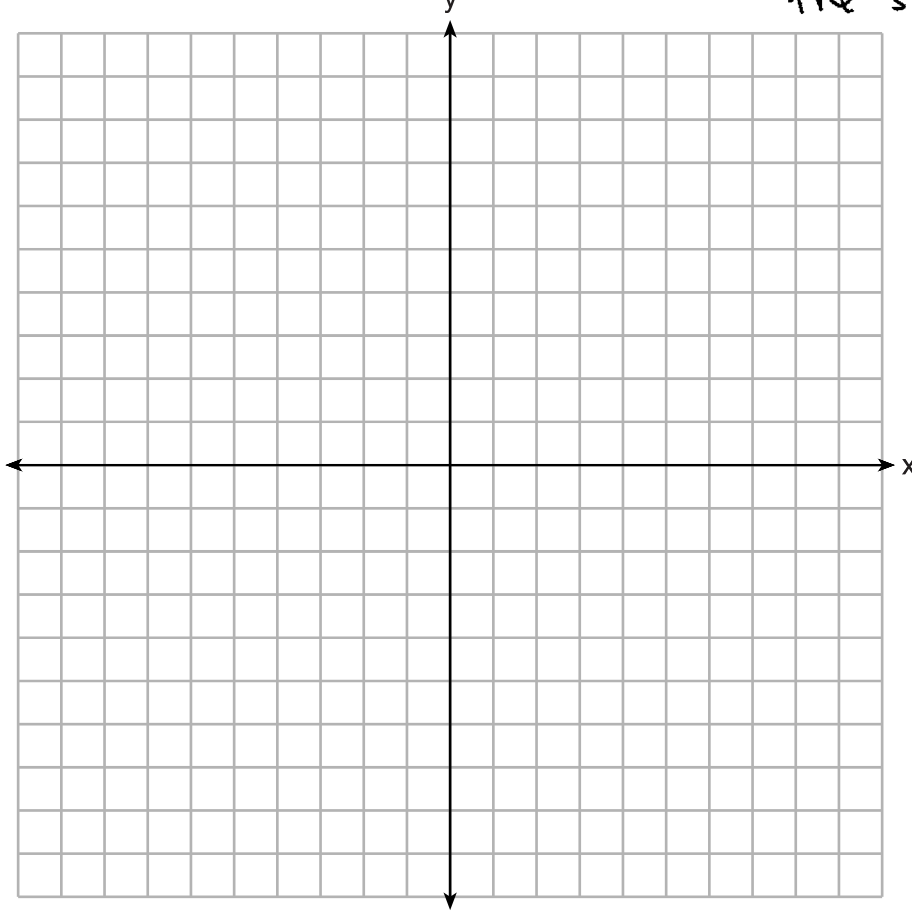
[The use of the set of axes below is optional.]

$4 = -4/3\,(3) + 8$

$4 = -4 + 8$

$4 = 4$

No, because it should stay the same as the lines are connected, they lie on the same line




**Score 1:** The student wrote a partially correct explanation.

## Question 30

**30** Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

[The use of the set of axes below is optional.]

Yes, she's correct because (3,4) multiplied by 2, (6,4) lies on the new, dilated line, $y=-\frac{4}{3}x+16$.

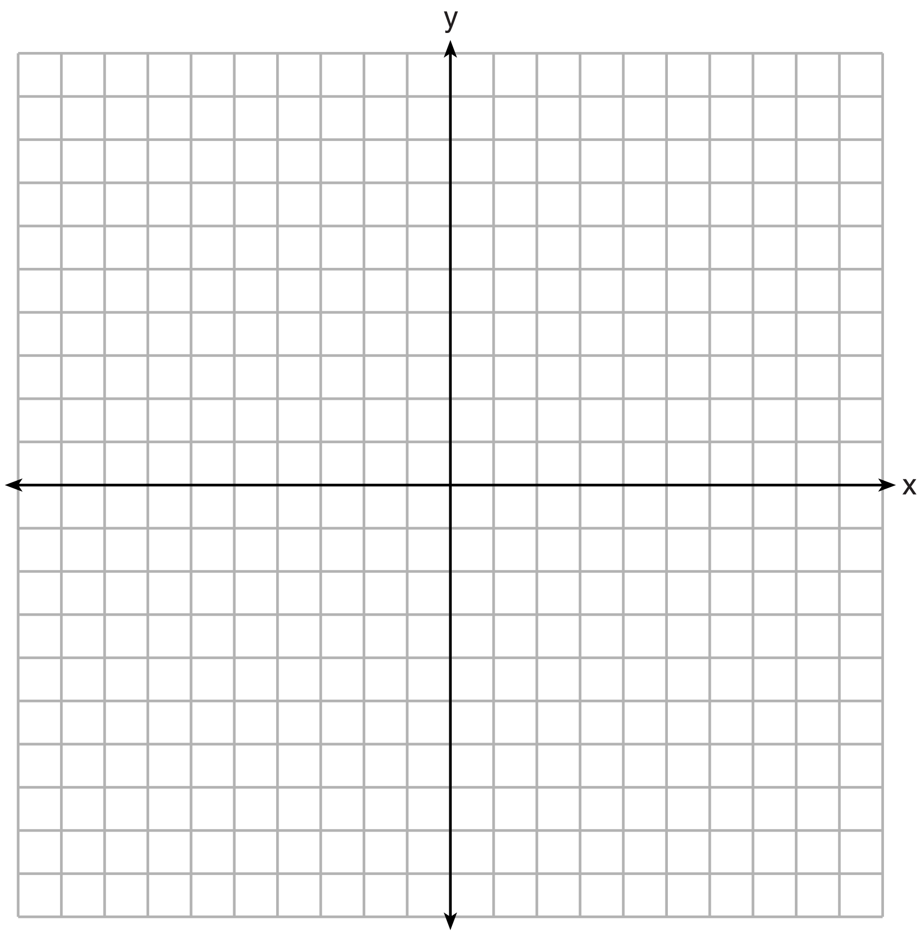
$\cancel{4x}+3y=24$
$\cancel{-4x} \qquad -4x$

$\frac{\cancel{3}y}{\cancel{3}}=\frac{-4x}{3}+\frac{24}{3}$

$y=-\frac{4}{3}x+8$

$\times 2$

$y=-\frac{4}{3}x+16$



**Score 0:** The student did not show enough correct relevant work to receive any credit.

## Question 30

**30** Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

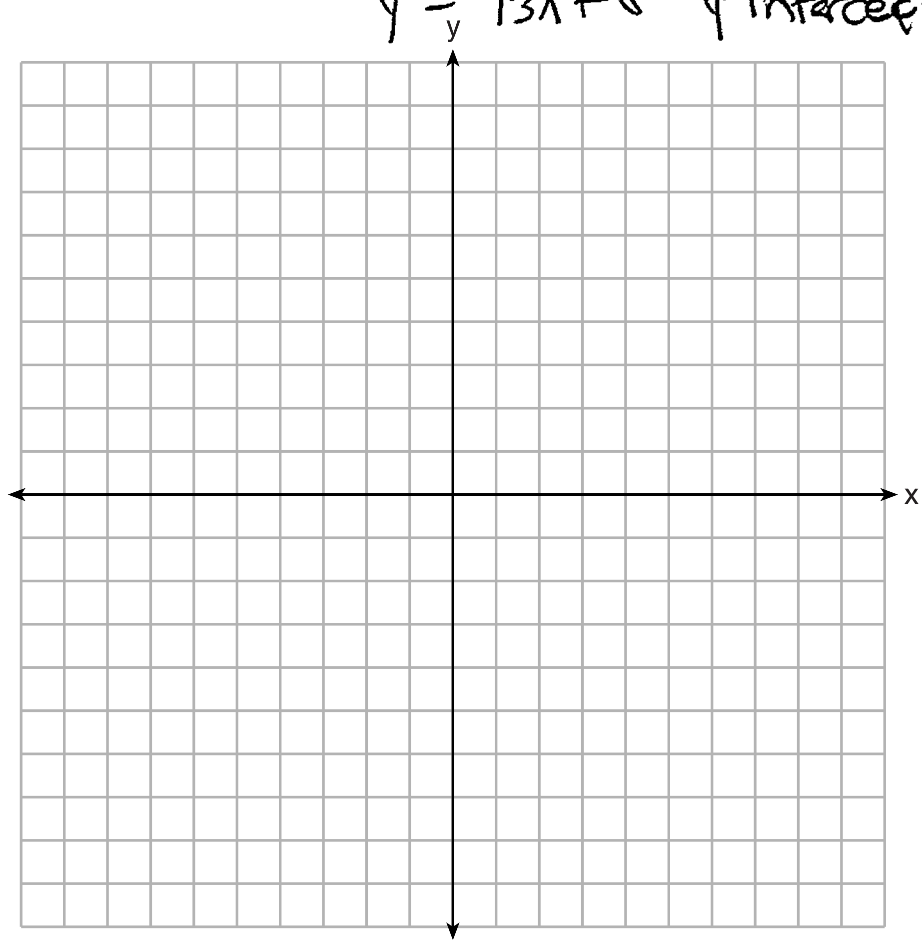
[The use of the set of axes below is optional.]

$\cancel{4x} + 3y = 24$
$\cancel{-4x} \quad -4x$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{4x}{3} + \frac{24}{3}$

$y = -4/3x + 8$

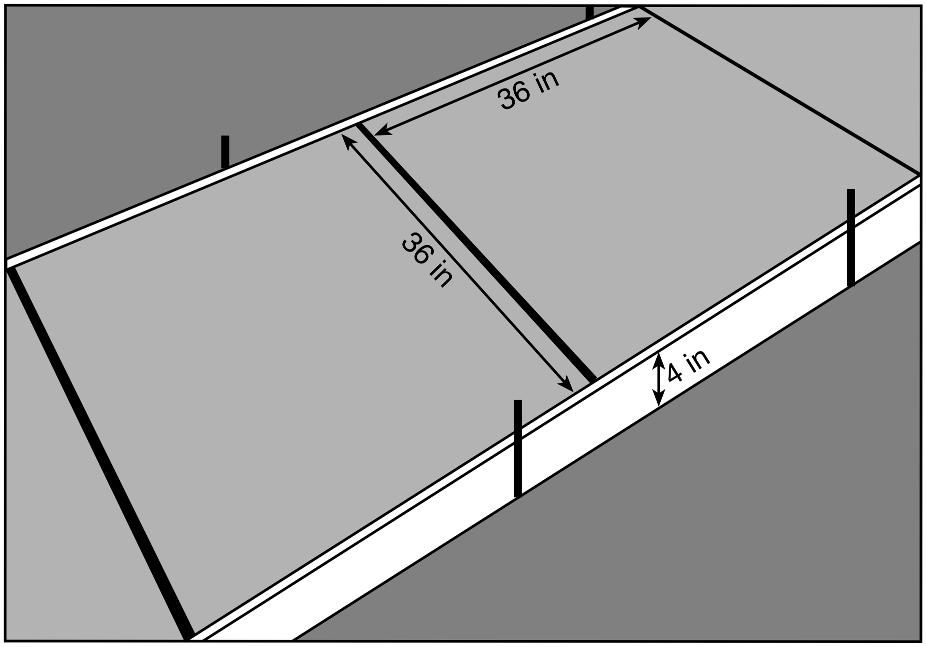
Yes, b/c when dialating a line slope never changes only y-intercept




**Score 0:** The student gave a completely incorrect response.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for \$3.25 per cubic foot.




How much money will it cost Ian to replace the two concrete sections?

$V = l \cdot w \cdot h$

$V = 36 \times 36 \times 4$

$V = 5184 \text{ in}^3$

$\times 2$

$V = 10{,}368 \text{ in}^3$

$\frac{10{,}368}{12^3} = 6$

$V = 6 \text{ ft}^3$

$\$3.25$

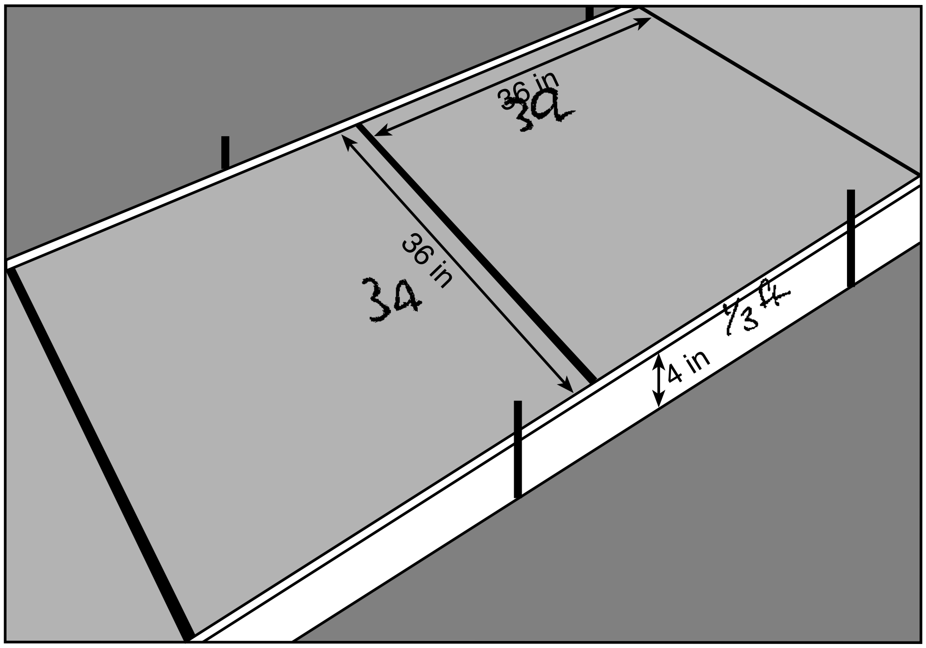
$\times\ 6$

$\boxed{\text{Cost} = \$19.50}$

**Score 2:** The student gave a complete and correct response.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.




$\frac{36}{12} = 3$

$\frac{4}{12} = 1/3$

How much money will it cost Ian to replace the two concrete sections?

$V = lwh$
$V = 3 \cdot 3 \cdot 1/3$
$V = 3 ft^3$

$3 \cdot 3.25 = 9.75$

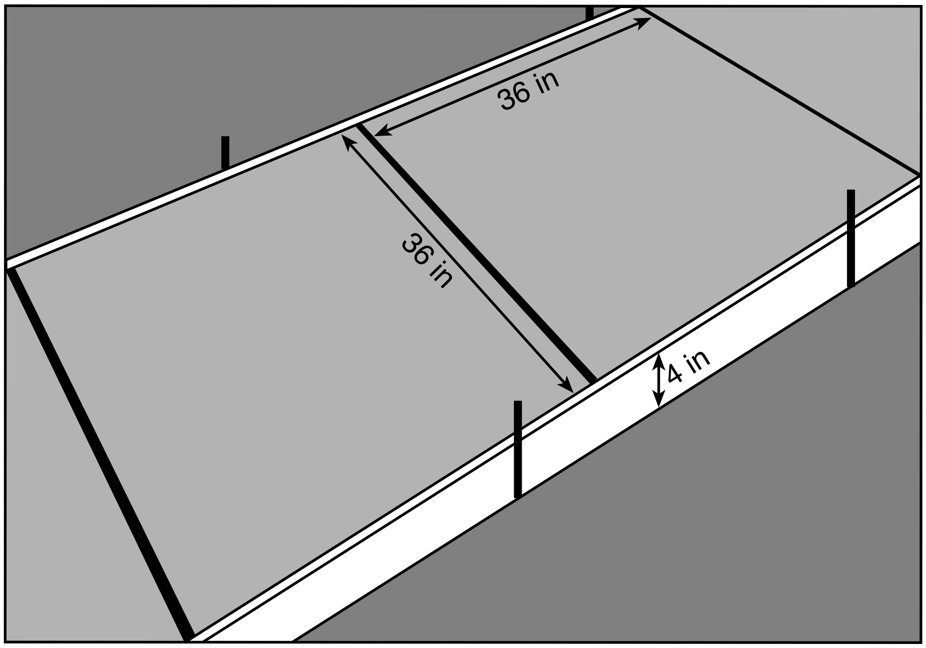
$9.75 \times 2 = 19.5$

It will cost $19.50 to replace the 2 concrete stations

**Score 2:** The student gave a complete and correct response.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.




How much money will it cost Ian to replace the two concrete sections?

$V = 36 \cdot 4 \cdot 36 = 5184$

$2(5184) = 10,368$

$10,368(3.25) = 33,696$

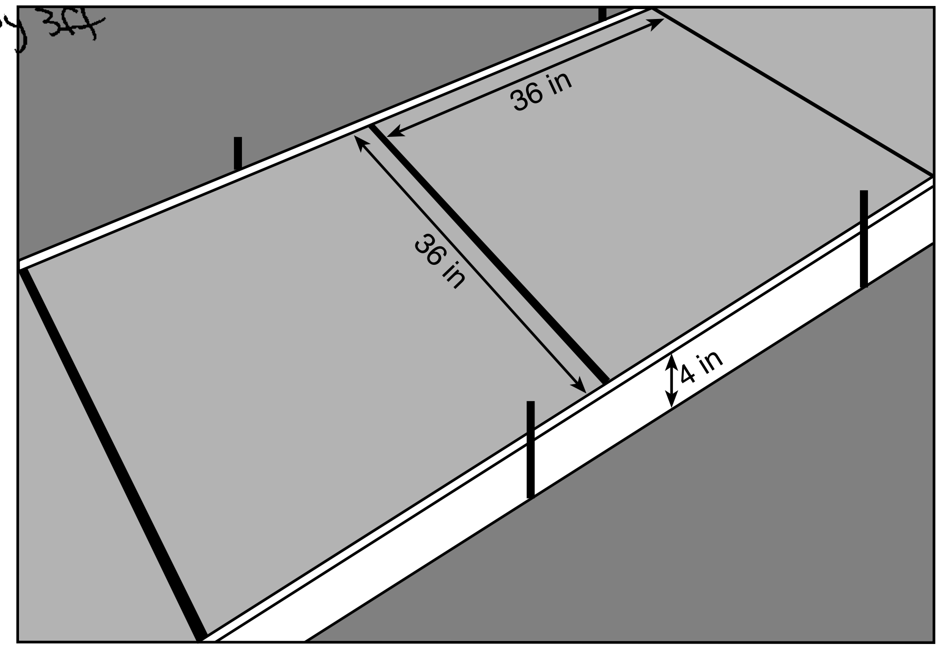
It will cost $33,696.00.

**Score 1:** The student did not convert from inches to feet.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.

3ft by 3ft




How much money will it cost Ian to replace the two concrete sections?

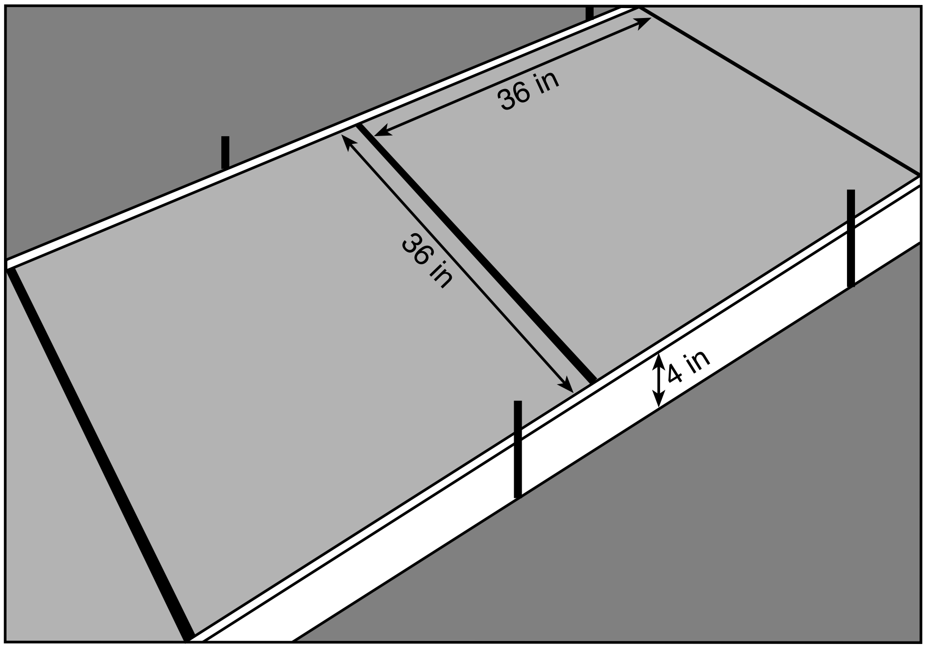
$$\begin{array}{r} \$3.25 \\ \times \quad 6 \\ \hline 19.50 \end{array}$$

$\boxed{\$19.50}$

**Score 1:** The student did not show appropriate work when showing the volume of a concrete section.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.




How much money will it cost Ian to replace the two concrete sections?

$l \cdot w \cdot h$

$36 \cdot 36 \cdot 4 = \frac{5184 \text{ inches}^3}{12} = 432 \text{ feet}^3$

$\cdot\ 2$

$864 \text{ feet}^3$

$864 \cdot 3.25$

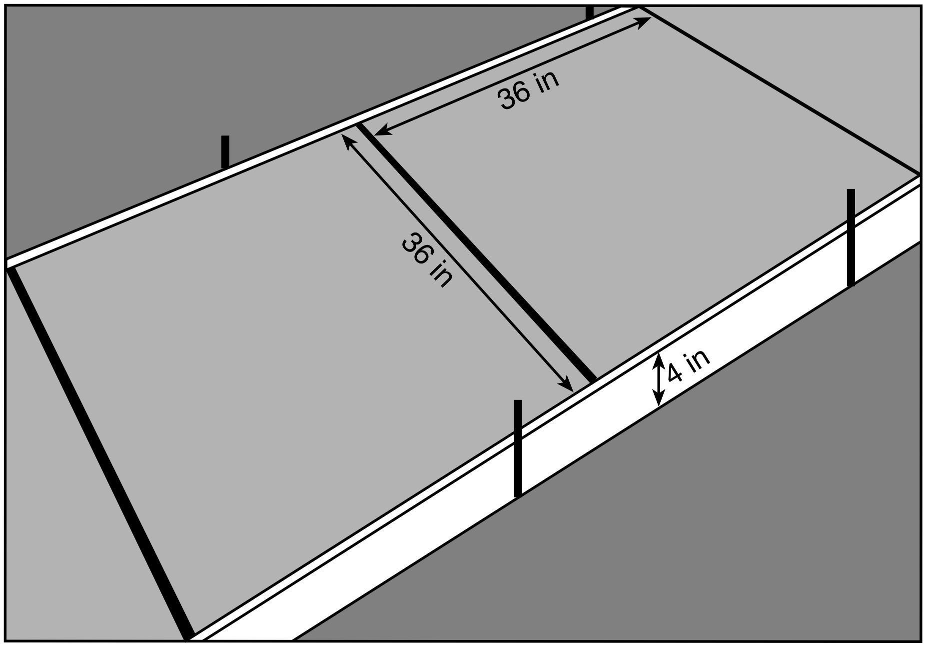
\$2808

**Score 1:** The student did not correctly convert from cubic inches to cubic feet.

## Question 31

**31** Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.

$$\begin{array}{r} 36 \\ \times 36 \\ \hline 1296 \\ \times \quad 4 \\ \hline 5184 \end{array}$$




$$\begin{array}{r} 432 \\ 12\overline{)5184} \end{array}$$

$$\begin{array}{r} 432 \\ \times 3.25 \\ \hline 1404 \end{array}$$

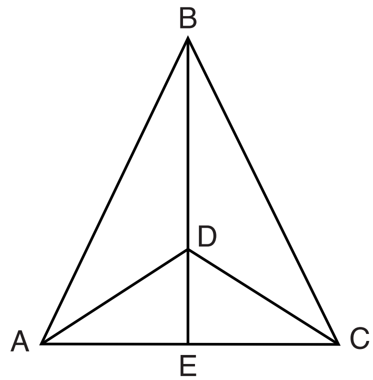
How much money will it cost Ian to replace the two concrete sections?

$\boxed{\$1404}$

**Score 0:** The student gave a completely incorrect response.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$
Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$




Fill in the missing statement and reasons below.

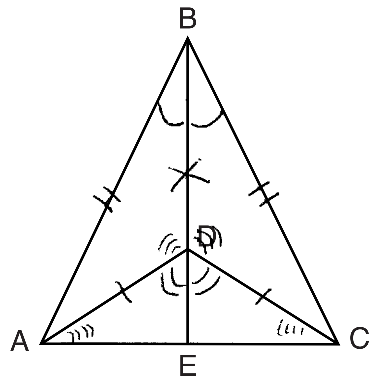
| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Reflexive |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle BDA \cong \angle BDC$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) CPCTC |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) Since $\triangle ADC$ has 2 $\cong$ sides it is isosceles. Isosceles $\triangle$'s have $\cong$ base $\measuredangle$'s so $\measuredangle DAE \cong \measuredangle DCA$. $\triangle ADE \cong \triangle CDE$ by ASA. By CPCTC, $\overline{AE} \cong \overline{CE}$ and $\measuredangle DEA \cong \measuredangle DEC$. If ~~a lines~~ 2 intersecting lines form a linear pair of $\cong$ $\measuredangle$'s then the lines are $\perp$ making $\overline{BDE}$ the $\perp$ bisector of AC. |

**Score 4:** The student gave a complete and correct response.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$

Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$




Fill in the missing statement and reasons below.

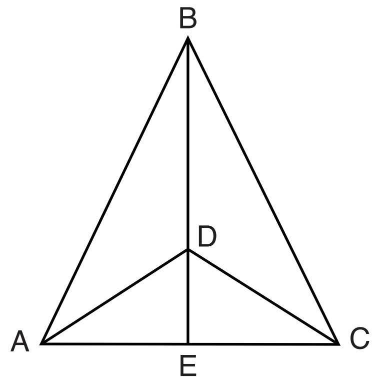
| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Reflexive |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle ADB \cong \angle CDB$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) CPCTC |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) Since E is in the middle ~~it~~ |

**Score 3** The student only wrote three correct reasons.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$

Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$




Fill in the missing statement and reasons below.

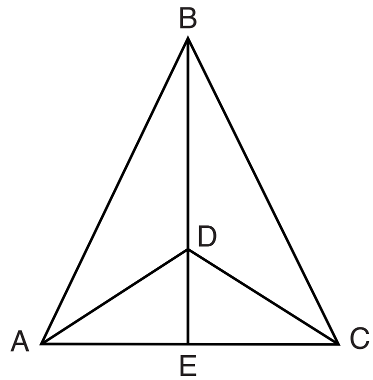
| Statements | Reasons |
| --- | --- |
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Reflexive prop. |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle ADE \cong \angle CDE$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) If points B and D are equidistant from the endpoints of $\overline{AC}$ then B and D are on the $\perp$ bisector of $\overline{AC}$. |

**Score 2:** The student only wrote two correct reasons.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$

Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$



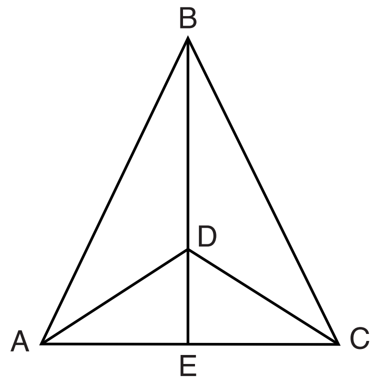

Fill in the missing statement and reasons below.

| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Reflexive |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle D \cong \angle D$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) CPCTC |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) IDK |

**Score 2:** The student only wrote two correct reasons.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$
Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$

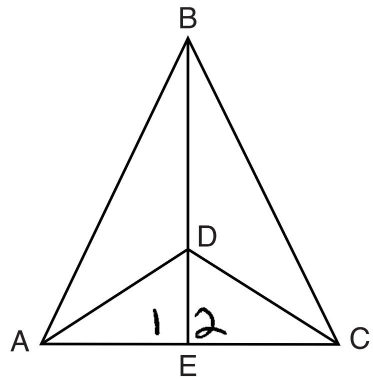



Fill in the missing statement and reasons below.

| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Symmetric Property |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle BDE \cong \angle BDE$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) CPCTC |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) YES IT IS !! |

**Score 1:** The student only wrote one correct reason.

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$
Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$



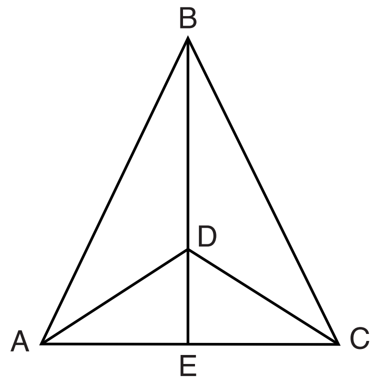

Fill in the missing statement and reasons below.

| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) Reflexive |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle 1 \cong \angle 2$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) CTPCT |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) Definition of perpendicular bisector |

**Score 1:** The student only wrote one correct reason.

## Question 32

**32** Given: $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$
Prove: $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$



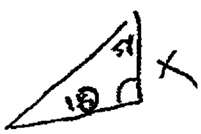

Fill in the missing statement and reasons below.

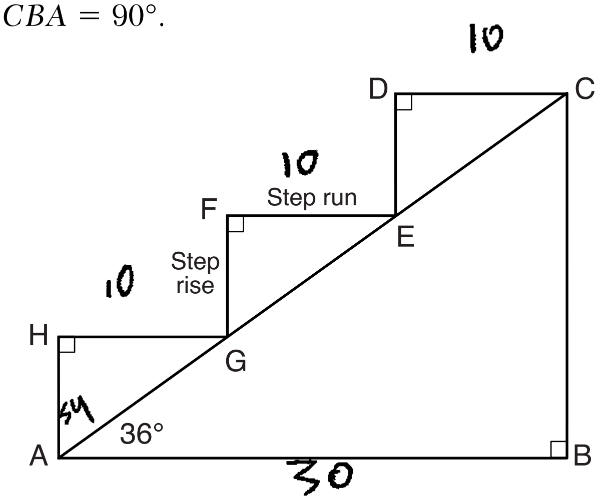
| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$, $\overline{AEC}$, $\overline{BDE}$ with $\angle ABE \cong \angle CBE$ and $\angle ADE \cong \angle CDE$ | (1) Given |
| (2) $\overline{BD} \cong \overline{BD}$ | (2) def of congruent |
| (3) $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary. | (3) Linear pairs of angles are supplementary. |
| (4) $\angle BDA + \angle ADE = 180^0$ | (4) Supplements of congruent angles are congruent. |
| (5) $\triangle ABD \cong \triangle CBD$ | (5) ASA |
| (6) $\overline{AD} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$ | (6) Transitive property |
| (7) $\overline{BDE}$ is the perpendicular bisector of $\overline{AC}$. | (7) because it is cut in half |

**Score 0:** The student gave a completely incorrect response.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

$\tan 54 = \frac{10}{x}$

$10/\tan 54 =$ 7.3 inches

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$(10/\tan(54)) \times 3 = CB$

$21.79627584 = CB$

$\uparrow^2 + 30^2 = AC^2$

$\sqrt{1375.077641} = \sqrt{AC^2}$

AC = 37 inches

**Score 4:** The student gave a complete and correct response.

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.

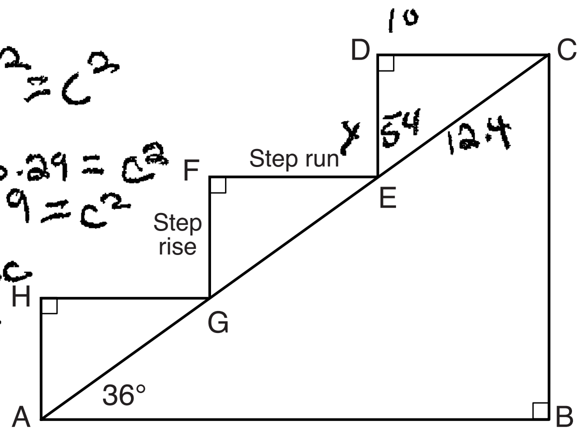
$10^2 + 7.3^2 = c^2$

$100 + 53.29 = c^2$

$153.29 = c^2$

$\sqrt{153.29} = c$

$12.4 = c$




$\text{Tan}(54) = \frac{10}{x}$

$\frac{10}{\text{tan}(54)} x$

$\cancel{90}$ 81

$-36$

$54$

7.3 in

If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

7.3 in

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

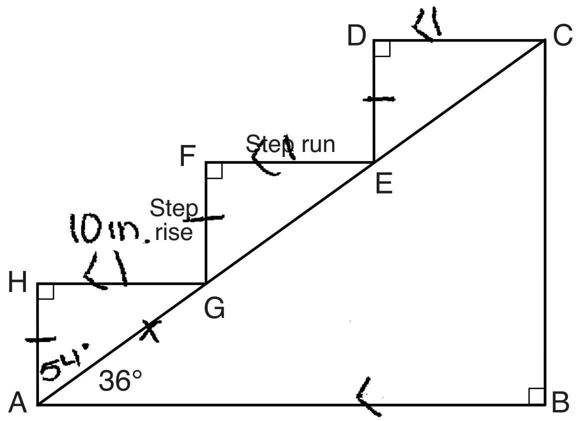
$(12.4)(3)$ $AC = 37.2$ in

$AC = 37$

**Score 4:** The student gave a complete and correct response.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

SOH CAH TOA

$\tan 54 = \frac{10}{x}$

$\frac{x \tan 54}{\tan 54} = \frac{10}{\tan 54}$

$x = 14.8$ in.

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$10^2 + 14.8^2 = x^2$

$100 + 219.04 = x^2$

$319.04 = x^2$

$x = \sqrt{319.04}$

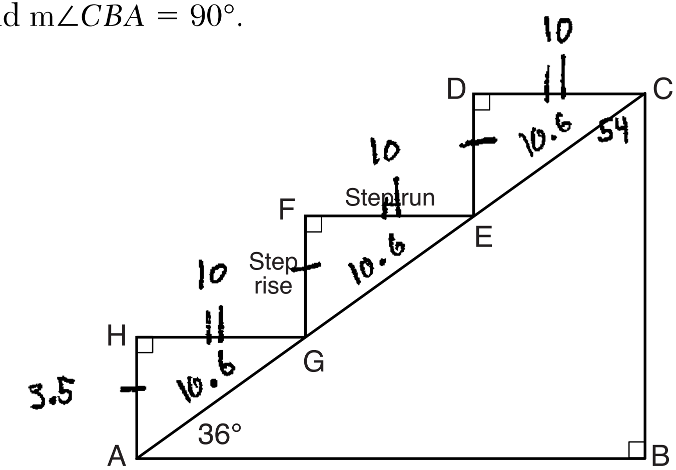
$x$ 3

$\overline{AC} = 53.6$ in.

$\overline{AC} = 54$

**Score 3:** The student made their calculations with the calculator in radian mode.

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

$2\sqrt{3} = 3.464101615$

$\boxed{\therefore \text{Each step rise is 3.5 inches}}$

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$a^2 + b^2 = c^2$

$10^2 + 3.5^2 = c^2$

$100 + 12.25 = \sqrt{112.25} = 10.6$ inches

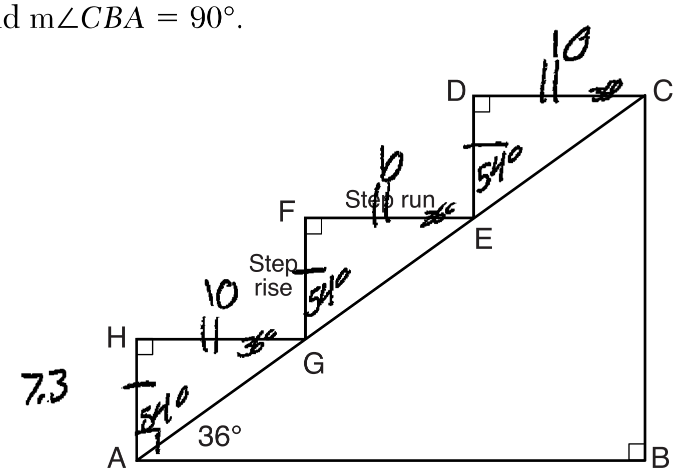
$10.6 \times 3 = 31.8$

$\boxed{\therefore \overline{AC} \text{ is 32 inches long}}$

**Score 2:** The student found an appropriate length of *AC* based on a completely incorrect length of each step rise.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

$\text{Tan}(36) = \frac{x}{10}$

$X = \text{Tan}(36)10$

$x \approx 7.26542528$

7.3 in.

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$10^2 + 7.3^2 = x^2$

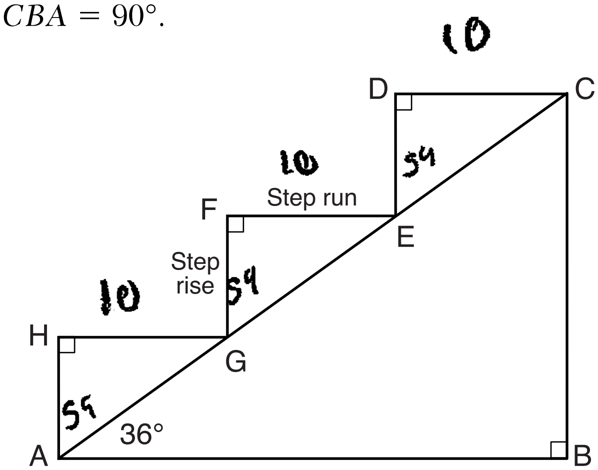
$100 + 57.78\text{ish} = 157.7864045$

$X \approx 12.56130584$

$AC \approx 13$ in.

**Score 2:** The student made a computational error when squaring 7.3. The student did not multiply by 3 to find the length of $\overline{AC}$.

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

$\tan 54 = \frac{10}{x}$ = step rise

$\boxed{x = 7 \text{ in}}$

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$7^2 + 10^2 = c^2$

$49 + 100 = c^2$

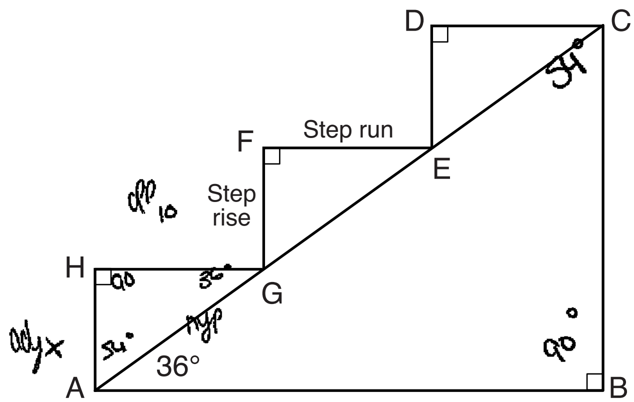
$149 = c^2$

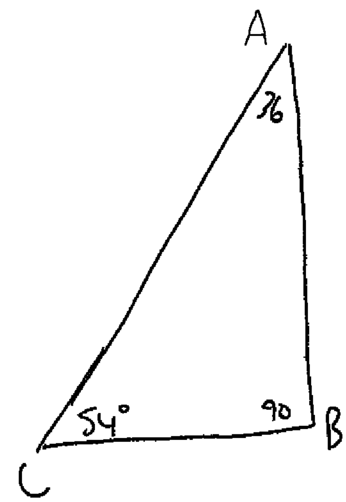
$c = \sqrt{149} = \boxed{12.2 \text{ in}}$

**Score 2:** The student made a rounding error in finding the length of each step rise. The student did not multiply by 3 to find the length of $\overline{AC}$.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*. sohcahtoa

$\tan(54) = \frac{10}{x}$ $\quad x = \frac{\tan(54)}{10}$

$x = .14$

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

sohcahtoa

$\sin(54)\frac{10}{x}$

$x = \frac{\sin(54)}{10}$

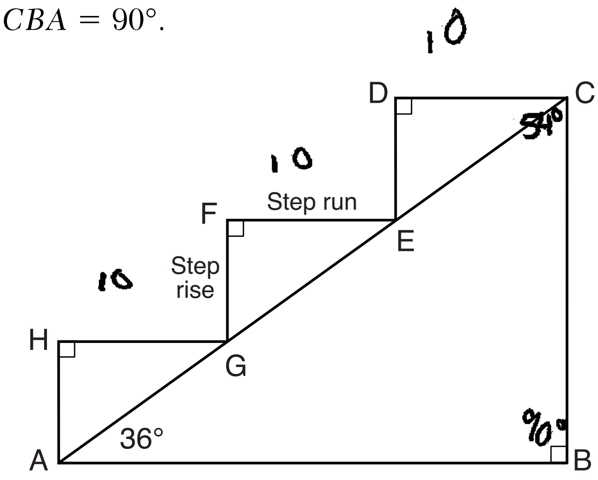
$.08$

$AC = .24$

**Score 1:** The student wrote a correct trigonometric equation, but no further correct work was shown.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




$$\begin{array}{r} 90 \\ +36 \\ \hline 126 \end{array} \qquad \begin{array}{r} 180 \\ -126 \\ \hline 54 \end{array}$$

If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

7.2

Determine and state the length of $\overline{AC}$, to the *nearest inch*.

$7.2^2 + 10^2 - x^2$

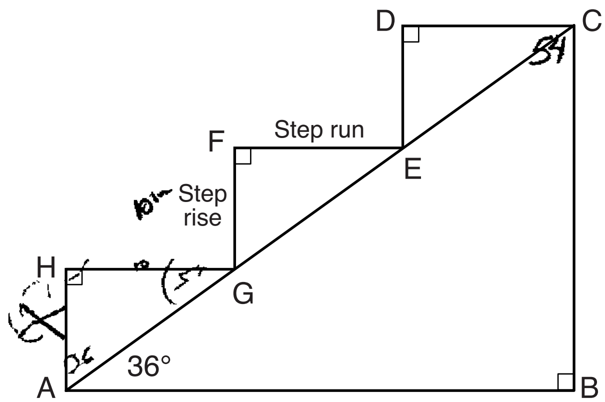
$51.84 + 100 = \sqrt{151.84^2}$

$\overline{AC} = 12.3$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

## Question 33

**33** A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $m\angle CBA = 90°$.




If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*.

$$\frac{\tan 54}{10} = \frac{x}{180} \qquad 10x = (\tan 54)(180)$$

$$10x = 247.7$$

$$\boxed{x = 24.8}$$

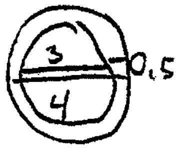
Determine and state the length of $\overline{AC}$, to the *nearest inch*.

30 inches.

**Score 0:** The student gave a completely incorrect response.

## Question 34

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.




$V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi(2)^3$
$V = \frac{4}{3}\pi(8)$
$V = 10.\overline{666}\pi$
$V = 33.51032164$

$V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi(1.5)^3$
$V = \frac{4}{3}\pi(3.375)$
$V = 14.13716694$

$33.51032164 - 14.13716694 = 19.3731547$

19.4 $cm^3$ of chocolate

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

$1.308 \cdot 19.4 = 25.3752$
$25.3752 \cdot 8 = 203.0016$

Total mass of chocolate in the box is 203 grams

**Score 4:** The student gave a complete and correct response.

## Question 34

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi 2^3$

$V = 33.5103216383$

$\boxed{V = 33.5 \text{ cm}^3}$

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

$(33.5)(1.308) = 43.818$

$(43.818)(8) = 350.544$

$\boxed{351 \text{ grams}}$

**Score 3:** The student found the volume and mass of 8 solid spheres, but no further correct work was shown.

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.

268.0825
− 113.0735

Chocolate is 154.98 ~ 155.0 $cm^3$

$V = \frac{4}{3}\pi r^3$

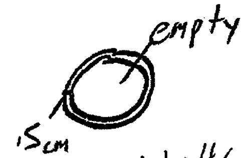
$V = \frac{4\pi 4^3}{3}$

$V = \frac{256\pi}{3}$

$V = 85.33\pi$

$V = 268.0825\ cm^3$

Outside




$V = \frac{4}{3}\pi r^3$

$V = \frac{4\pi 3^3}{3}$

$V = 113.0973\ cm$

Inside

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

155.0 × 8

8 chocolate volume is 1240 $cm^3$

$D = \frac{m}{V}$

$1.308 = \frac{m}{1240\ cm^3}$

1621.92 grams ~~mass~~

Total mass of chocolate is 1622 grams

**Score 3:** The student used the diameters instead of the radii when calculating the volumes.

## Question 34

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.

$4 - .5$

$d = 3.5$

$\frac{3.5}{2} = 1.75 = r$

$V = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2)^3 \qquad \frac{4}{3}\pi(1.75)^3$

$= 33.51032164 - 22.4492975$

$V = 11.06102414$

$\boxed{V = 11.1}\ cm^3$

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

$11.1 \times 8 = 88.8$

$\times 1.380$

$122.544 = \boxed{123 \text{ grams}}$

**Score 2:** The student used an incorrect radius of 1.75 in finding the volume and transcribed the density incorrectly.

## Question 34

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi 2^3$

$V = \frac{\cancel{4}^1}{3}\pi \cdot \frac{\cancel{8}^2}{1}$

$V = \frac{2}{3}\pi$

$V = 2.1$

Amount of chocolate in chocolate ball = 2.1 cm³

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

$d = \frac{m}{V}$

$2.1 \cdot 1.308 = \frac{m}{\cancel{2.1}}\ \cancel{2.1}$

$2.7468 \cdot 8$

$m = 2.7468$

mass = 22 g

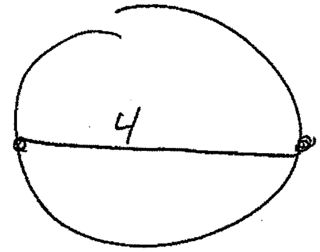
**Score 1:** The student made one computational error when finding the volume of one solid sphere. The student made a conceptual error by using the volume of a solid sphere to find the total mass.

## Question 34

**34** A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere.

A

$V = \frac{4}{3}\pi r^3$



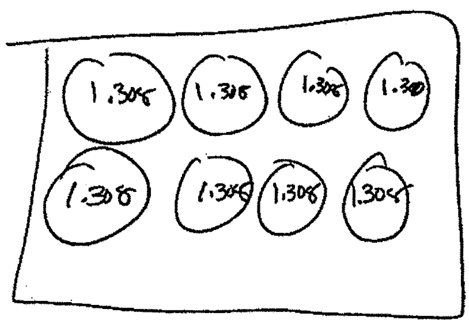

$V = \frac{4}{3}\pi 2^3$

$V = \frac{4}{3}\pi 8$

$\cancel{V = 10.7\pi}$

$\boxed{V = 33.6\text{ cm}^3}$

The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm$^3$, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.




33.

$0.5(8) = \boxed{4 \text{ grams}}$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

Parallelogram

$m$ of $\overline{MH} = \frac{rise}{run} = \frac{6}{10} = \frac{3}{5}$

$m$ of $\overline{AT} = \frac{rise}{run} = \frac{6}{10} = \frac{3}{5}$

Same Slope

$\therefore \overline{MH} \parallel \overline{AT}$

$m$ of $\overline{MA} = \frac{rise}{run} = \frac{-5}{3}$

$m$ of $\overline{HT} = \frac{rise}{run} = \frac{-5}{3}$

Same Slope

$\therefore \overline{MA} \parallel \overline{HT}$

∴ quad MATH is a parallelogram since both pairs of opposite sides are parallel

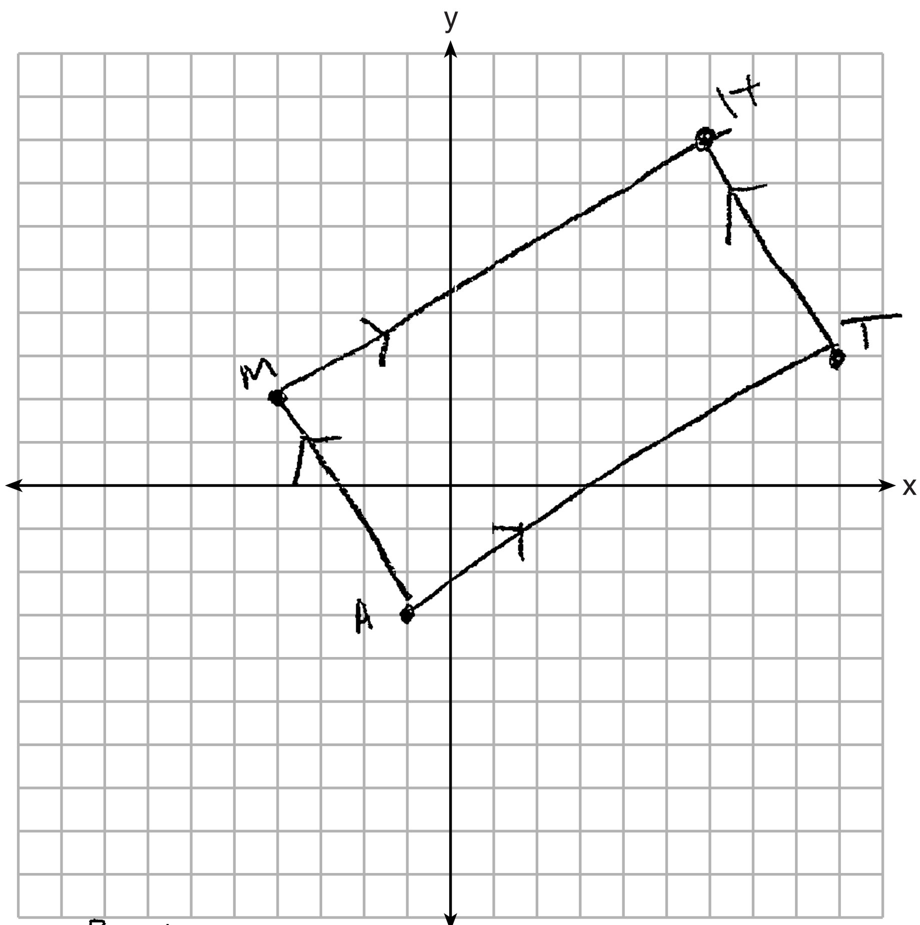
**Score 6:** The student gave a complete and correct response.

## Question 35

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




Rectangle

m of $\overline{MA} = \frac{rise}{run} = \frac{-5}{3}$

m of $\overline{AT} = \frac{rise}{run} = \frac{6}{10} = \frac{3}{5}$

Negative recipracals

$\therefore$ $\overline{MA} \perp \overline{AT} \rightarrow \angle A$ is a right angle

$\therefore$ Parallelogram MATH is a rectangle since it has right angles.

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

Slope $\overline{MH} = \frac{6}{10}$
Slope $\overline{AT} = \frac{6}{10}$
Same slope $\therefore \parallel$ lines

need at least one pair of $\parallel$ sides to be a parallelogram
MATH is a parallelogram

slope $\overline{MA} = -\frac{5}{3}$
Slope $\overline{HT} = -\frac{5}{3}$
Same Slope $\therefore \parallel$ lines

**Score 5:** The student made an incorrect conclusion of "at least one pair of parallel sides" to conclude *MATH* is a parallelogram.

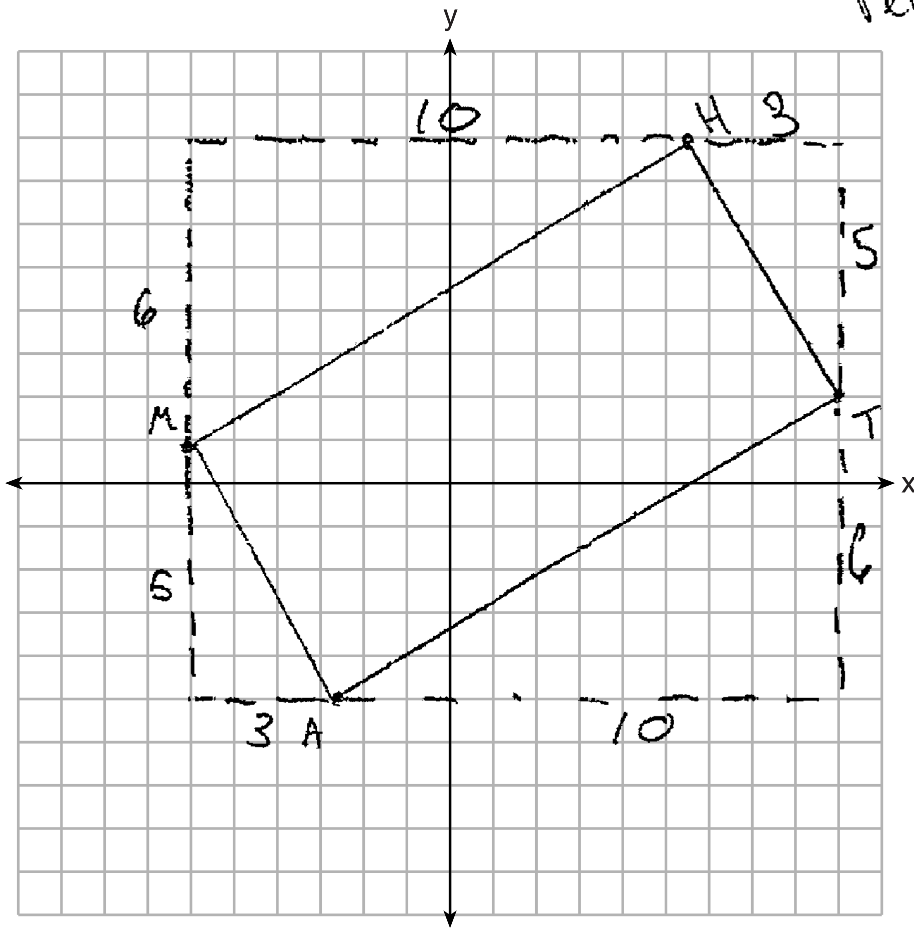
## Question 35

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]

MATH is a rectagle




m of $\overline{HM}$

negative recipricals make $\perp$ lines

$\frac{6}{10} \to \frac{3}{5}$

m of $\overline{MA}$

negative recipricals

$-\frac{5}{3}$ and $\frac{3}{5}$

$\perp$ lines form right angles

$\therefore \angle HMA$ is a right angle.

need a right angle and 2 pairs of $\parallel$ sides to have a rectangle

$x_2\ y_2 \qquad x_1\ y_1$

**35** The vertices of quadrilateral *MATH* have coordinates *M*(−4,2), *A*(−1,–3), *T*(9,3), and *H*(6,8).

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

parallel $\begin{cases} \overline{MH} = \dfrac{y_2-y_1}{x_2-x_1} \quad \dfrac{8-2}{6-(-4)} = \dfrac{\cancel{6}^{\,3}}{\cancel{10}_{\,5}} = \dfrac{3}{5} \text{ same slope} \\ \overline{AT} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-(-3)}{9-(-1)} = \dfrac{\cancel{6}^{\,3}}{\cancel{10}_{\,5}} = \dfrac{3}{5} \end{cases}$

parallel $\begin{cases} \overline{MA} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-3-2}{-1-(-4)} = \dfrac{-5}{3} \text{ same slope} \\ \overline{HT} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-8}{9-6} = \dfrac{5}{3} \end{cases}$

opposite sides
are parallel
therefore quadrilateral
MATH is a parallelogram

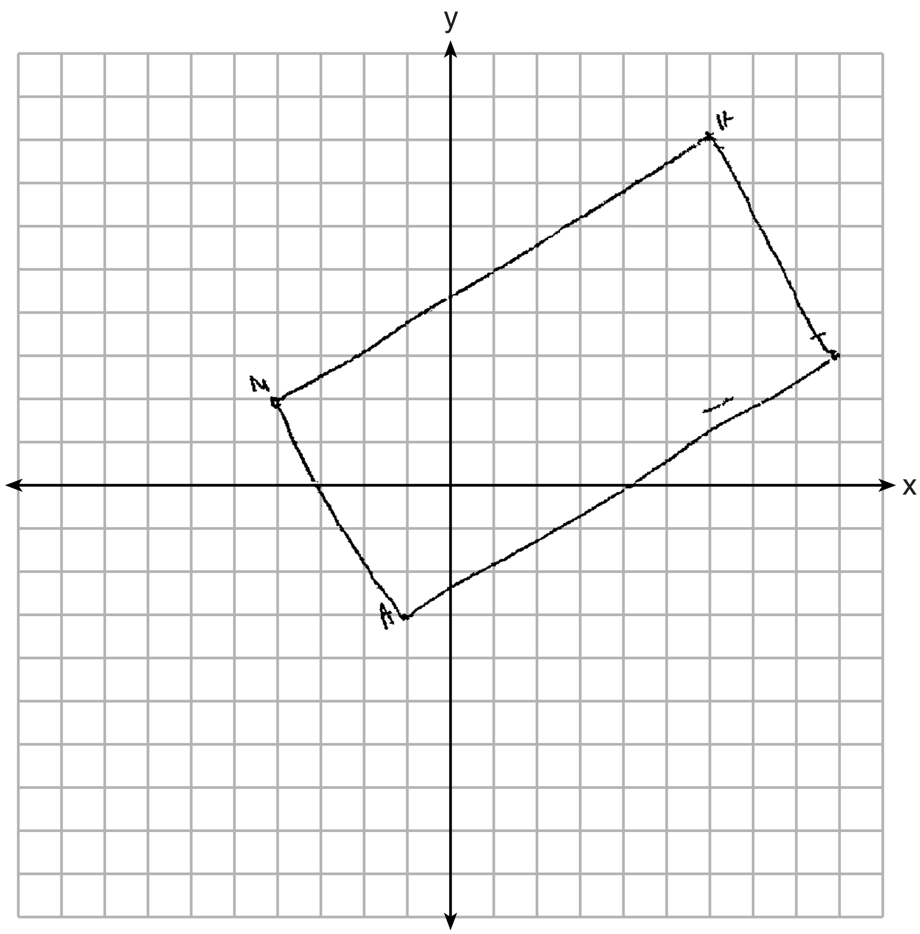
**Score 5:** The student had an incomplete reason when proving *MATH* is a rectangle.

## Question 35

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




$MA = -\frac{5}{3}$ neg. rec. perpendicular

$AT = \frac{3}{5}$

$MH = \frac{3}{5}$ neg. rec. perpendicular

$HT = -\frac{5}{3}$

$\overline{MA}$ is $\perp$ $\overline{AT}$ and $\overline{MH}$ is $\perp$ $\overline{HT}$ therefore quadrilateral MATH is a rectangle

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$D = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$D_{MA} = \sqrt{(-4 - -1)^2 + (+2 - -3)^2}$

$D_{MA} = \sqrt{(-3)^2 + (5)^2}$

$D_{MA} = \sqrt{9+25}$

$D_{MA} = \sqrt{36}$

$D_{MA} = 6$

$D_{TH} = \sqrt{(9-6)^2 + (3-8)^2}$

$D_{TH} = \sqrt{(3)^2 + (-5)^2}$

$D_{TH} = \sqrt{9+25}$

$D_{TH} = \sqrt{36}$

$D_{TH} = 6$

Same length ∴ congruent

$D_{MH} = \sqrt{(6 - -4)^2 + (8-2)^2}$

$D_{MH} = \sqrt{(10)^2 + (6)^2}$

$D_{MH} = \sqrt{100+36}$

$D_{MH} = \sqrt{136}$

$D_{AT} = \sqrt{(9 - -1)^2 + (3 - -3)^2}$

$D_{AT} = \sqrt{100+36}$

$D_{AT} = \sqrt{136}$

Same length ∴ congruent

Conclusion:

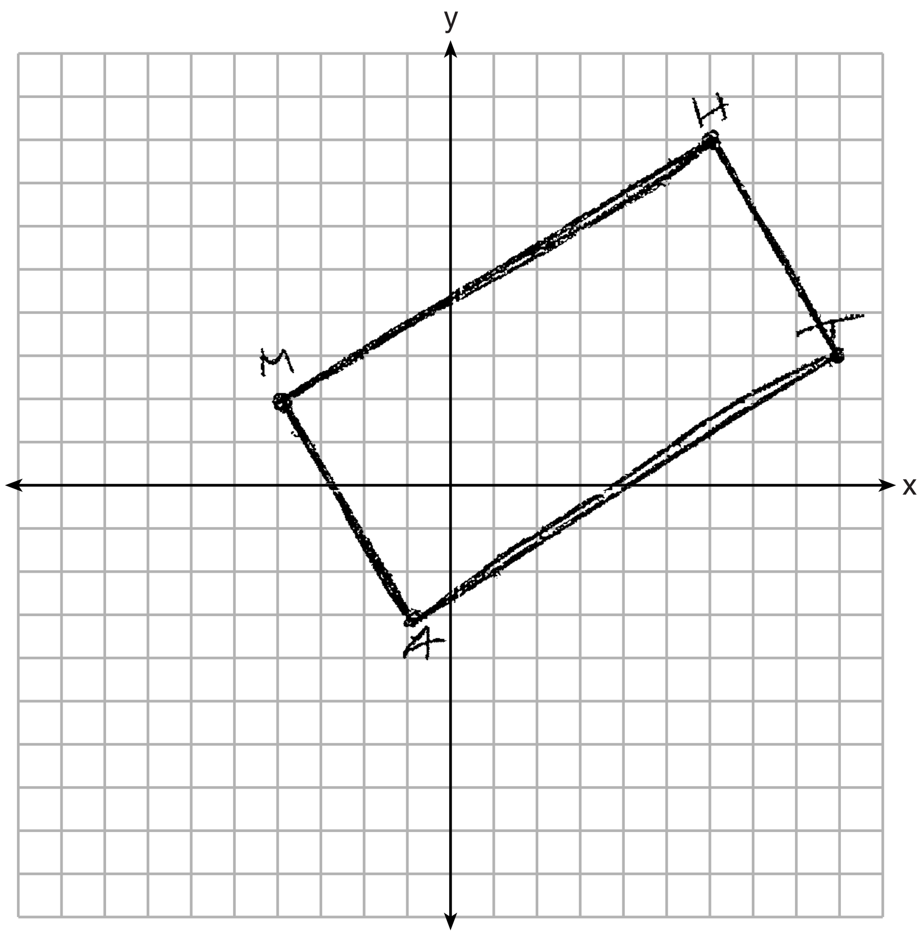
It is a parallelogram because it has 2 pairs of congruent sides.

**Score 5:** The student made a computational error in finding the lengths of $\overline{MA}$ and $\overline{TH}$.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




$m = \frac{\Delta y}{\Delta x}$

$M_{MA} = \frac{2 - -3}{-4 - -1}$

$M_{MA} = \frac{5}{-3}$

$M_{AT} = \frac{\cancel{8}3 - -3}{9 - -1}$

$M_{AT} = \frac{6}{10} = 3/5$

Negative reciprocal slopes

$\therefore \perp$

$\therefore$ rt. angle.

Conclusion: This quadrilateral is a rectangle because it has 2 pairs of congruent sides and one rt. angle.

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$m_{MH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{6+4} = \frac{6}{10} = \frac{3}{5}$

$m_{th} = \frac{8-3}{6-9} = \frac{5}{-3}$

$m_{At} = \frac{3+3}{9+1} = \frac{6}{10} = \frac{3}{5}$

$m_{AM} = \frac{2+3}{-4+1}$

$= \frac{5}{-3}$

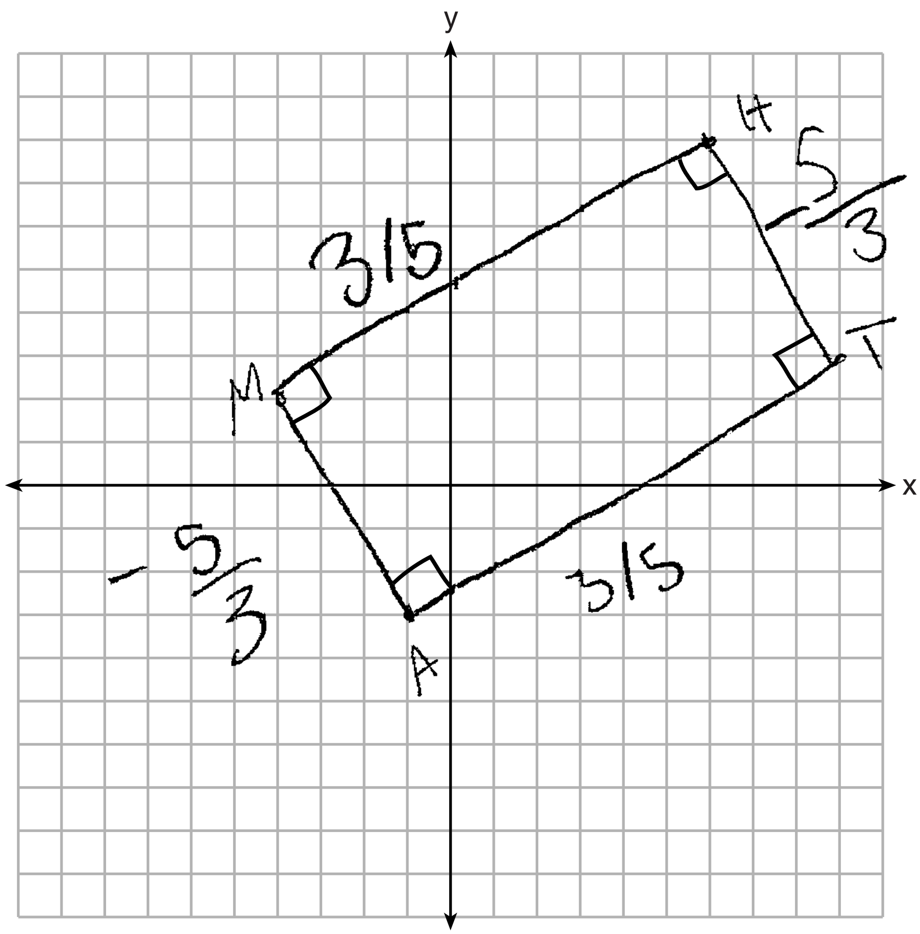
$\therefore$ It a $\boxed{P}$ because opposite sides are $\parallel$.

**Score 4:** The student made two incomplete concluding statements.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




$\boxed{Q}$ MATH is a rectangle because all the angles are right angles

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$\overline{MH} \cong \overline{AT}$ $\quad$ $\overline{MA} \cong \overline{HT}$

$(-4,2)\ (6,8)$ $\quad$ $(-1,-3)\ (9,3)$ $\quad$ $(-4,2)\ (-1,-3)$ $\quad$ $(6,8)\ (9,3)$

$\sqrt{(6-(-4))^2+(8-2)^2}$ $\quad$ $\sqrt{(9-(-1))^2+(3-(-3))^2}$ $\quad$ $\sqrt{(-1-(-4))^2+(-3-2)^2}$ $\quad$ $\sqrt{(9-6)^2+(3-8)^2}$

$\sqrt{10^2+6^2}$ $\quad$ $\sqrt{10^2+6^2}$ $\quad$ $\sqrt{3^2+-5^2}$ $\quad$ $\sqrt{3^2+-5^2}$

$\sqrt{100+36}$ $\quad$ $\sqrt{100+36}$ $\quad$ $\sqrt{9+25}$ $\quad$ $\sqrt{9+25}$

$\sqrt{136}$ $\quad$ $\sqrt{136}$ $\quad$ $\sqrt{34}$ $\quad$ $\sqrt{34}$

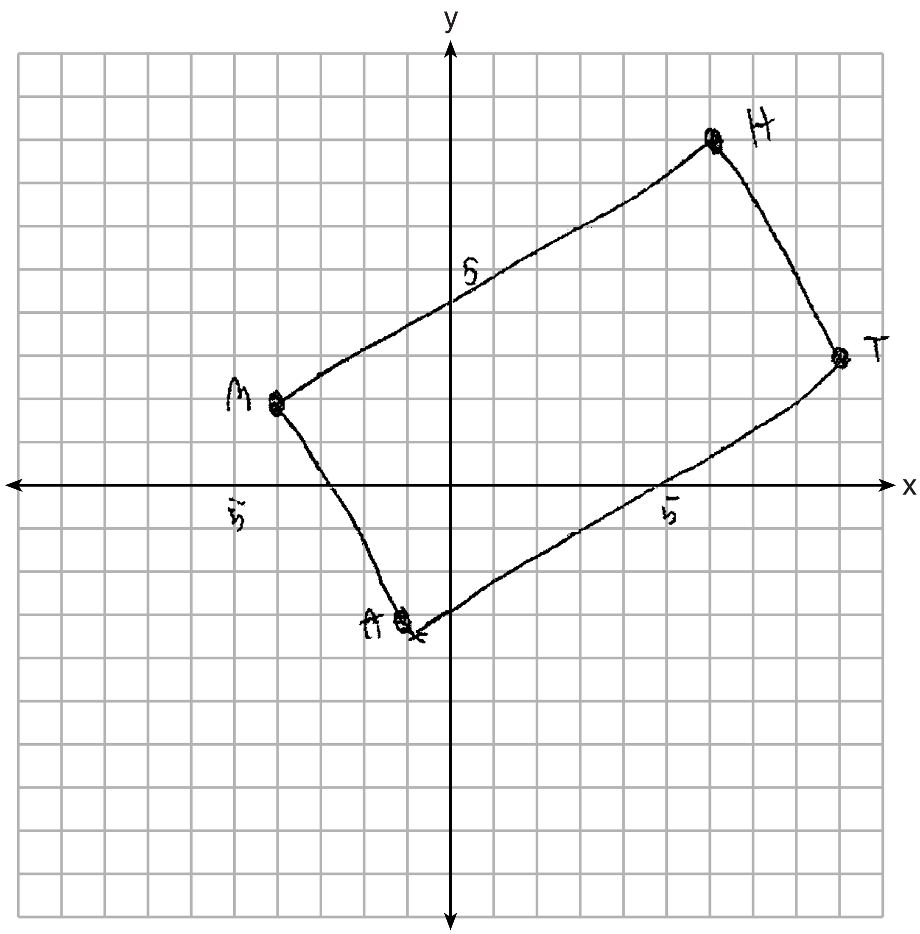
MATH is
a parallelogram
because opposite
sides are $\cong$

**Score 3:** The student proved *MATH* is a parallelogram, but no further correct work was shown.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]



## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$MH\ \frac{\Delta y}{\Delta x} = \frac{6}{10}$

$m\ AT = \frac{6}{10}$

$HT = \frac{5}{3}$

$MA = \frac{5}{3}$ same slope

same slope

$MH \parallel AT$ ∴ this quadrilateral is a parallelogram since all opposite sides are parallel.

$HT \parallel MA$

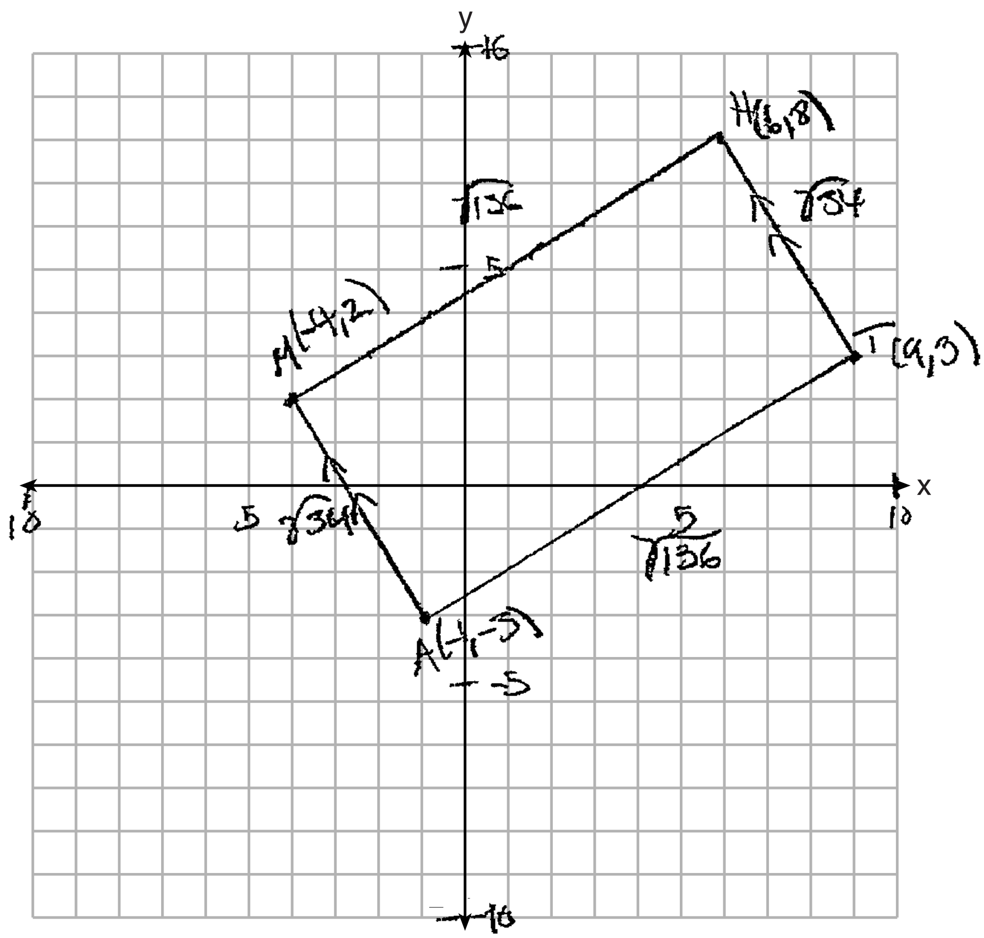
**Score 3:** The student found the slopes of $\overline{MA}$ and $\overline{HT}$ to be positive. The student had a conceptual error in proving *MATH* is a rectangle.

## Question 35

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

MH $\sqrt{(6+4)^2+(8-2)^2}$
$\sqrt{100+36}$
$\sqrt{136}$

AT $\sqrt{(9+1)^2+(3+3)^2}$
$\sqrt{136}$

HT $\sqrt{(9-6)^2+(3-8)^2}$
$\sqrt{3^2+(-5)^2}$
$\sqrt{34}$

MA $\sqrt{(-4+1)^2+(-3-2)^2}$
$\sqrt{(-3)^2+(-5)^2}$
$\sqrt{34}$

quadrilateral MATH is a rectangle because it has two pairs of parallel sides and two pairs of congruent sides

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$\overline{MH}$

$\underset{x_1\ y_1}{(-4,2)}\underset{x_2\ y_2}{(6,8)} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-2}{6--4} = \frac{6}{10} \div 2 = \frac{3}{5}$

$\overline{AT}$

$\underset{x_1\ y_1}{(-1,-3)}\underset{x_2\ y_2}{(9,3)} = \frac{y_2-y_1}{x_2-x_1} = \frac{3--3}{9--1} = \frac{6}{10} \div 2 = \frac{3}{5}$

$\overline{MA}$

$\underset{x_1\ y_1}{(-4,2)}\underset{x_2\ y_2}{(-1,-3)} = \frac{y_2-y_1}{x_2-x_1} = \frac{-3-2}{-1--4} = \frac{-5}{3}$

$\overline{HT}$

$\underset{x_1\ y_1}{(6,8)}\underset{x_2\ y_2}{(9,3)} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-8}{9-6} = \frac{-5}{3}$

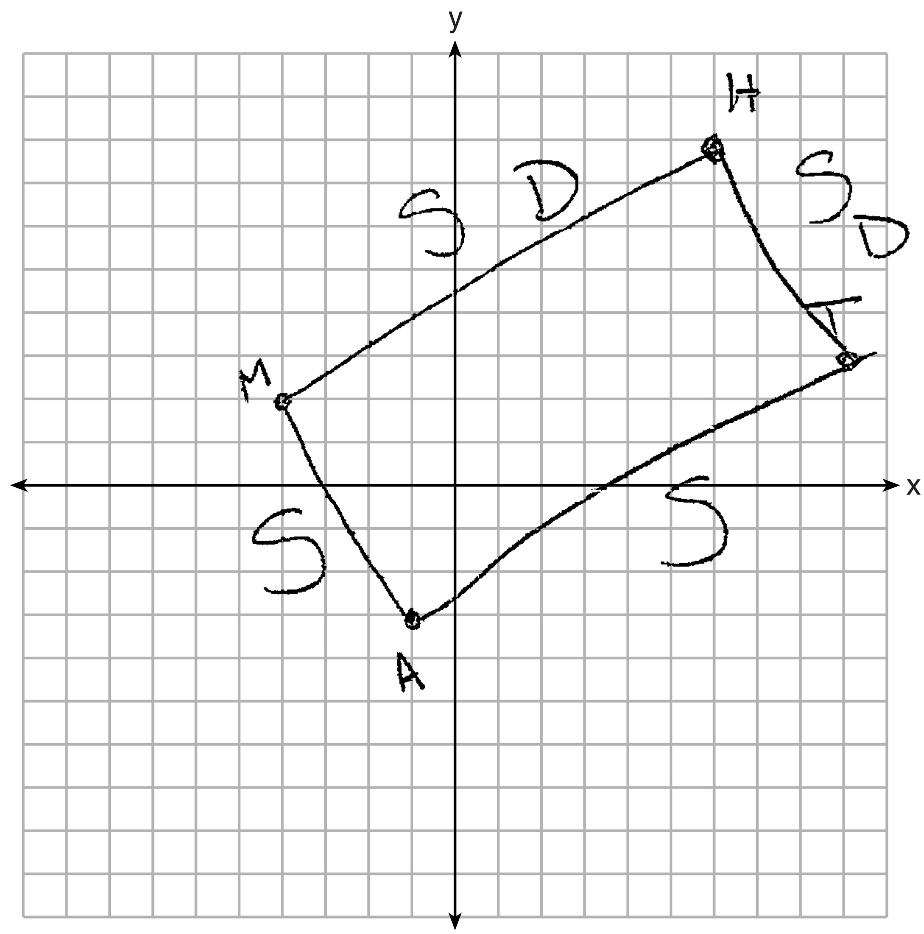
MATH is a ▱ because all opp. sides have the same slopes.

**Score 2:** The student did not connect the equal slopes to parallelism in proving *MATH* is a parallelogram. The student did not show enough relevant work to prove *MATH* is a rectangle.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




$\overline{MH}$

$(-4,2)(6,8)$ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$x_1 y_1$ $x_2 y_2$ $d=\sqrt{(6--4)^2+(8-2)^2}$

$d=\sqrt{10^2+6^2}$

$d=\sqrt{100+36}$

$d=\sqrt{136}$

$\overline{HT}$

$(6,8)(9,3)$

$x_1 y_1$ $x_2 y_2$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(9-6)^2+(3-8)^2}$

$d=\sqrt{(3)^2+(-5)^2}$

$d=\sqrt{9+25}$

$d=\sqrt{34}$

MATH is a rectangle because the slopes form neg. reciprocals and the distances of joining sides are different.

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

Plan:
·Slope ~~##~~ to show = slopes (parallel lines)

WORK:

$\underline{MH}$ $\frac{8-2}{6+4}$ $\frac{8}{10} = \frac{3}{5}$

$\underline{AT}$ $\frac{3+3}{9+1}$ $\frac{6}{10} = \frac{3}{5}$

$\underline{MA}$ $\frac{-3-2}{-1+4}$ $\frac{-5}{-3}$ $\frac{5}{3}$

$\underline{HT}$ $\frac{3-8}{9-6} = \frac{-5}{3}$

Conclusion

MATH is a parallelogram bc all 4 sides are parallel.

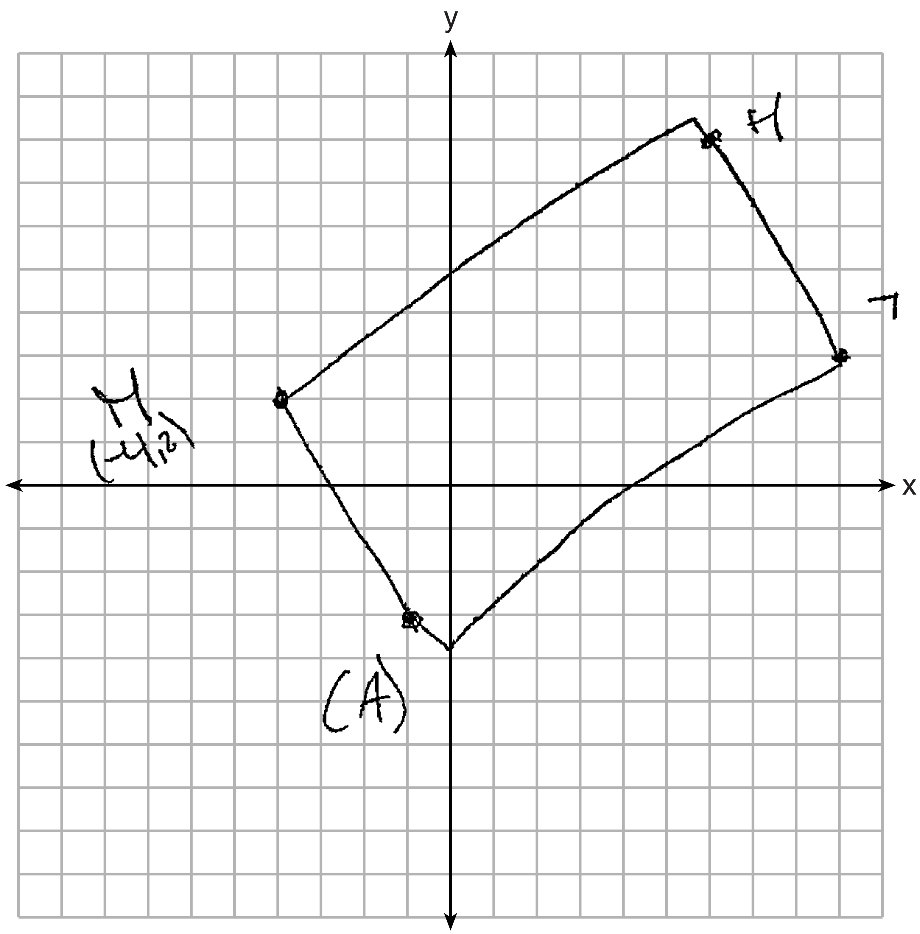
**Score 2:** The student made a computational error and wrote an incorrect conclusion in proving *MATH* is a parallelgram. The student made a conceptual error in proving *MATH* is a rectangle.

## Question 35

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]




Plan:
· midpoint 2x to show diagonals bisect.

mdpt

AH $\left(\frac{-1+6}{2}, \frac{-3+8}{2}\right)$

$(2.5, 2.5)$

MT $\left(\frac{-4+9}{2}, \frac{2+3}{2}\right)$

$(2.5, 2.5)$

Conclusion

MATH is a rectangle b/c their diagonals bisect eachother.

## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$\overline{MA} = \frac{2--3}{-4--1} = \frac{1}{-3} = \frac{-5}{3}$

$\overline{AT} = \frac{-3-3}{-1-9} = \frac{-6}{-10} = \frac{3}{5}$

$\overline{TH} = \frac{3-8}{9-6} = -\frac{5}{3}$

$\overline{MH} = \frac{2-8}{-4-6} = \frac{-6}{-10} = \frac{3}{5}$

$\overline{MA} \parallel MH$

$\overline{AT} \parallel \overline{TH}$

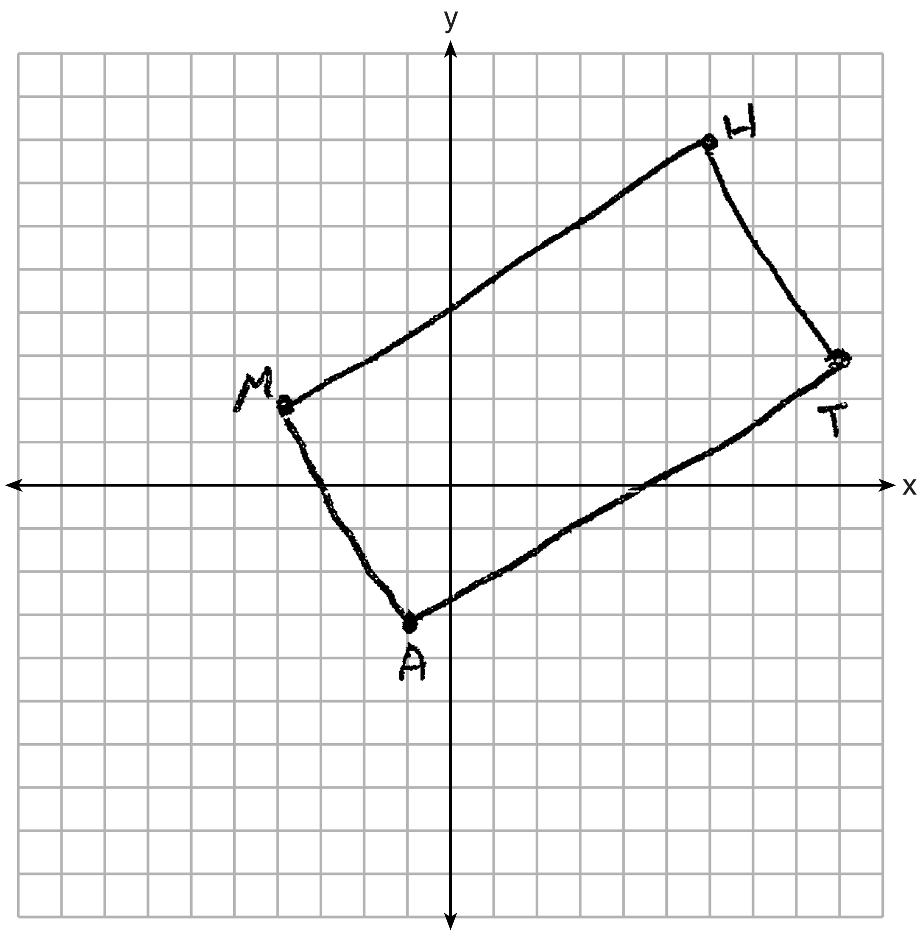
Opposite sides are parallel so quadrilateral MATH is a parallelogram and rectangle

**Score 1:** The student found the slopes of the four sides. No further correct work was shown.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]



## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

Slope of MH $= \frac{8-2}{6-4} = \frac{6}{2}$

Slope of AT $= \frac{3+3}{9+1} = \frac{6}{10}$

$MH = \sqrt{(6-4)^2+(8-2)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$AT = \sqrt{(-1-9)^2+(3--3)^2}$

$\sqrt{100+36}$

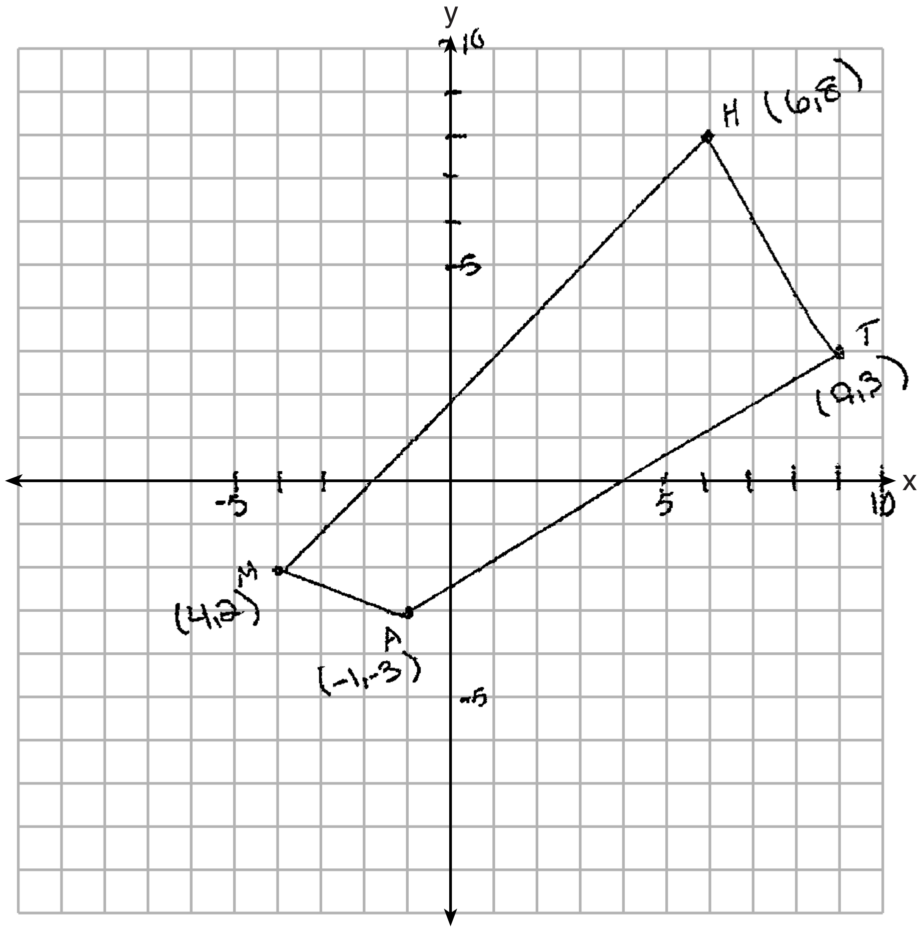
$\sqrt{136}$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]



## Question 35

**35** The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$.

Prove that quadrilateral *MATH* is a parallelogram.

[The use of the set of axes on the next page is optional.]

$\sqrt{(6-9)^2+(8-3)^2}$

$(-3)^2+(5^2)$

distance: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{9+25}\;\sqrt{34}$

$\sqrt{(-1+-4)^2+(-3,2)^2}$

$(3)^2+(-1)^2$

$9+1\;\sqrt{10}$

MH $\sqrt{(6(+-4)+(8,2)}$

$10+10=\sqrt{100}$

$10$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

**Question 35 continued**

Prove that quadrilateral *MATH* is a rectangle.

[The use of the set of axes below is optional.]

No
~~pa~~ this
a parallelogram
but ~~not~~ is
a rectangle