**Large-Type Edition**

**The University of the State of New York**
**REGENTS HIGH SCHOOL EXAMINATION**

# GEOMETRY

**Wednesday,** August 14, 2019 — 12:30 to 3:30 p.m., only

**Student Name:** _____

**School Name:** _____

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice…**
**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

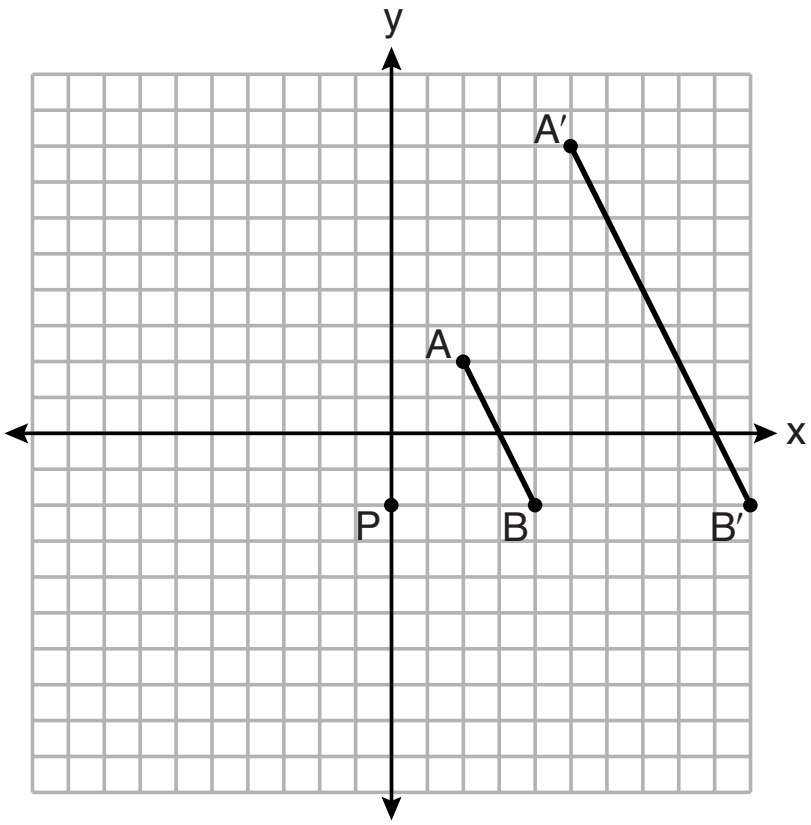
## DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

## Part I

**Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.** [48]

**Use this space for computations.**

**1** On the set of axes below, $\overline{AB}$ is dilated by a scale factor of $\frac{5}{2}$ centered at point $P$.




**Question 1 is continued on the next page.**

**Use this space for computations.**

Which statement is always true?

(1) $\overline{PA} \cong \overline{AA'}$

(2) $\overline{AB} \parallel \overline{A'B'}$

(3) $AB = A'B'$

(4) $\frac{5}{2}(A'B') = AB$

**2** The coordinates of the vertices of parallelogram $CDEH$ are $C(-5,5)$, $D(2,5)$, $E(-1,-1)$, and $H(-8,-1)$. What are the coordinates of $P$, the point of intersection of diagonals $\overline{CE}$ and $\overline{DH}$?

(1) $(-2,3)$

(2) $(-2,2)$

(3) $(-3,2)$

(4) $(-3,-2)$

**3** The coordinates of the endpoints of $\overline{QS}$ are $Q(-9,8)$ and $S(9,-4)$. Point $R$ is on $\overline{QS}$ such that $QR{:}RS$ is in the ratio of 1:2. What are the coordinates of point $R$?

(1) $(0,2)$

(2) $(3,0)$

(3) $(-3,4)$

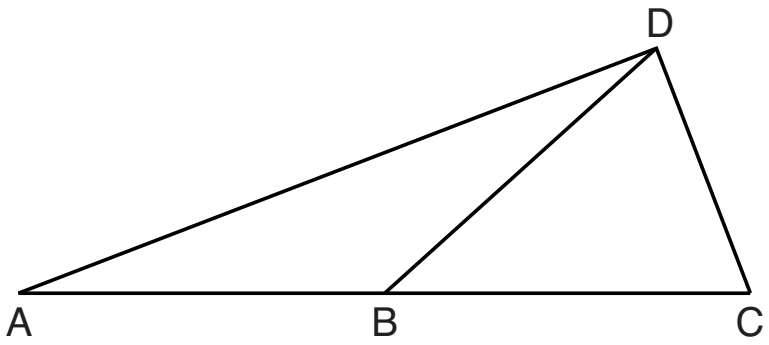
(4) $(-6,6)$

**Use this space for computations.**

**4** If the altitudes of a triangle meet at one of the triangle's vertices, then the triangle is

(1) a right triangle
(2) an acute triangle
(3) an obtuse triangle
(4) an equilateral triangle

**5** In the diagram below of $\triangle ACD$, $\overline{DB}$ is a median to $\overline{AC}$, and $\overline{AB} \cong \overline{DB}$.




If $m\angle DAB = 32°$, what is $m\angle BDC$?

(1) 32°
(2) 52°
(3) 58°
(4) 64°

**Use this space for computations.**

**6** What are the coordinates of the center and the length of the radius of the circle whose equation is $x^2 + y^2 = 8x - 6y + 39$?

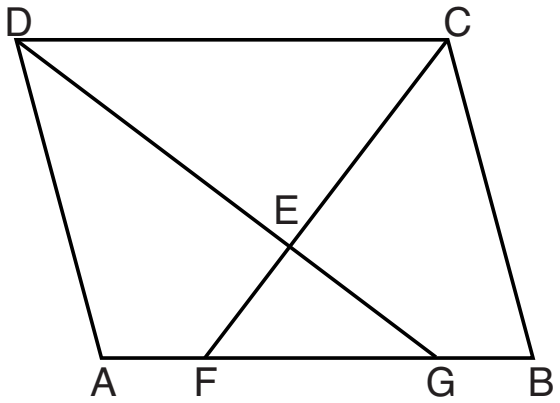
(1) center $(-4,3)$ and radius 64

(2) center $(4,-3)$ and radius 64

(3) center $(-4,3)$ and radius 8

(4) center $(4,-3)$ and radius 8

**7** In the diagram below of parallelogram $ABCD$, $\overline{AFGB}$, $\overline{CF}$ bisects $\angle DCB$, $\overline{DG}$ bisects $\angle ADC$, and $\overline{CF}$ and $\overline{DG}$ intersect at $E$.




If $m\angle B = 75°$, then the measure of $\angle EFA$ is

(1) 142.5°

(2) 127.5°

(3) 52.5°

(4) 37.5°

**Use this space for computations.**

**8** What is an equation of a line that is perpendicular to the line whose equation is $2y + 3x = 1$?

(1) $y = \frac{2}{3}x + \frac{5}{2}$

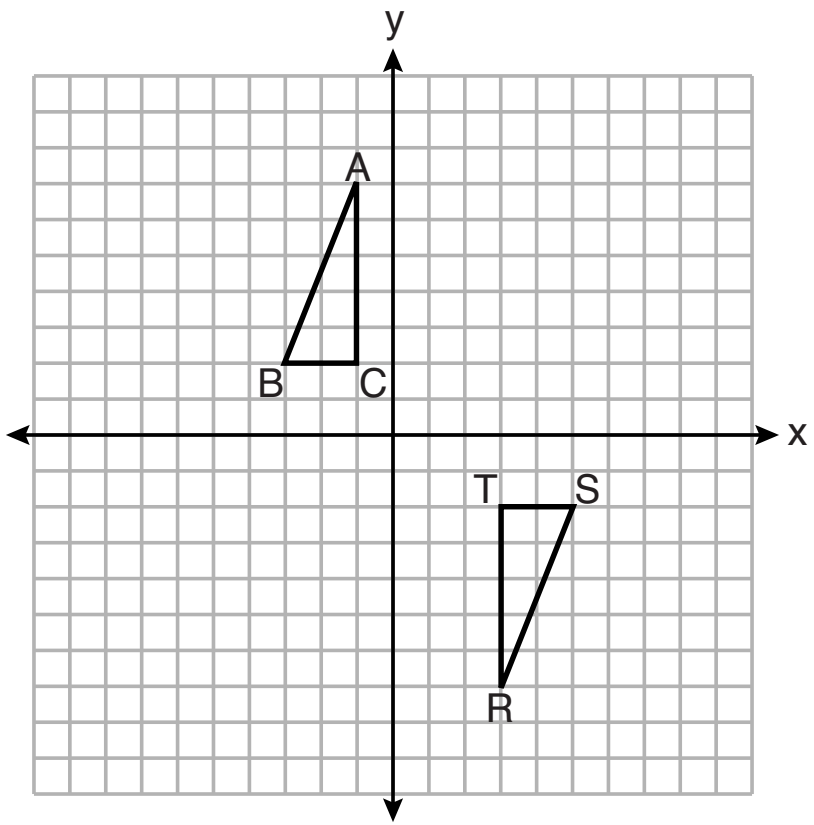
(2) $y = \frac{3}{2}x + 2$

(3) $y = -\frac{2}{3}x + 1$

(4) $y = -\frac{3}{2}x + \frac{1}{2}$

**Use this space for computations.**

**9** Triangles $ABC$ and $RST$ are graphed on the set of axes below.




Which sequence of rigid motions will prove $\triangle ABC \cong \triangle RST$?

(1) a line reflection over $y = x$

(2) a rotation of 180° centered at (1,0)

(3) a line reflection over the $x$-axis followed by a translation of 6 units right

(4) a line reflection over the $x$-axis followed by a line reflection over $y = 1$

**Use this space for computations.**

**10** If the line represented by $y = -\frac{1}{4}x - 2$ is dilated by a scale factor of 4 centered at the origin, which statement about the image is true?

(1) The slope is $-\frac{1}{4}$ and the $y$-intercept is $-8$.

(2) The slope is $-\frac{1}{4}$ and the $y$-intercept is $-2$.

(3) The slope is $-1$ and the $y$-intercept is $-8$.

(4) The slope is $-1$ and the $y$-intercept is $-2$.

**11** Square *MATH* has a side length of 7 inches. Which three-dimensional object will be formed by continuously rotating square *MATH* around side $\overline{AT}$?

(1) a right cone with a base diameter of 7 inches

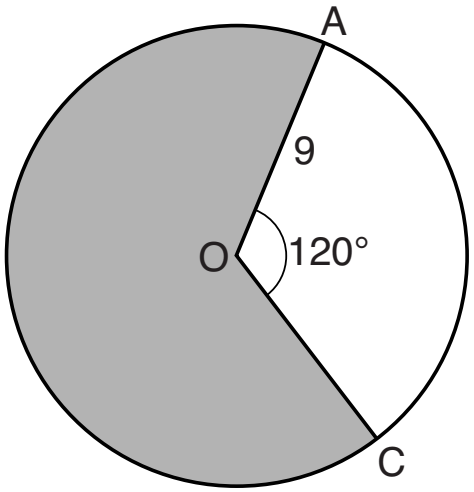
(2) a right cylinder with a diameter of 7 inches

(3) a right cone with a base radius of 7 inches

(4) a right cylinder with a radius of 7 inches

**Use this space for computations.**

**12** Circle $O$ with a radius of 9 is drawn below. The measure of central angle $AOC$ is 120°.




What is the area of the shaded sector of circle $O$?

(1) $6\pi$

(2) $12\pi$

(3) $27\pi$

(4) $54\pi$

**Use this space for computations.**

**13** In quadrilateral $QRST$, diagonals $\overline{QS}$ and $\overline{RT}$ intersect at $M$. Which statement would always prove quadrilateral $QRST$ is a parallelogram?

(1) $\angle TQR$ and $\angle QRS$ are supplementary.

(2) $\overline{QM} \cong \overline{SM}$ and $\overline{QT} \cong \overline{RS}$

(3) $\overline{QR} \cong \overline{TS}$ and $\overline{QT} \cong \overline{RS}$

(4) $\overline{QR} \cong \overline{TS}$ and $\overline{QT} \parallel \overline{RS}$

**14** A standard-size golf ball has a diameter of 1.680 inches. The material used to make the golf ball weighs 0.6523 ounce per cubic inch. What is the weight, to the *nearest hundredth of an ounce*, of one golf ball?

(1) 1.10

(2) 1.62

(3) 2.48

(4) 3.81

**Use this space for computations.**

**15** Chelsea is sitting 8 feet from the foot of a tree. From where she is sitting, the angle of elevation of her line of sight to the top of the tree is 36°. If her line of sight starts 1.5 feet above ground, how tall is the tree, to the *nearest foot*?

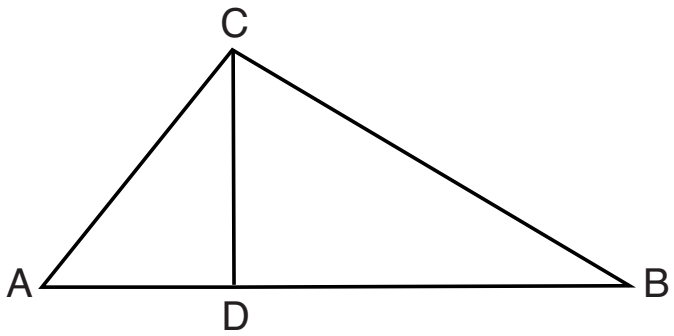
(1) 8

(2) 7

(3) 6

(4) 4

**16** In the diagram below of right triangle $ABC$, altitude $\overline{CD}$ intersects hypotenuse $\overline{AB}$ at $D$.




Which equation is always true?

(1) $\frac{AD}{AC} = \frac{CD}{BC}$

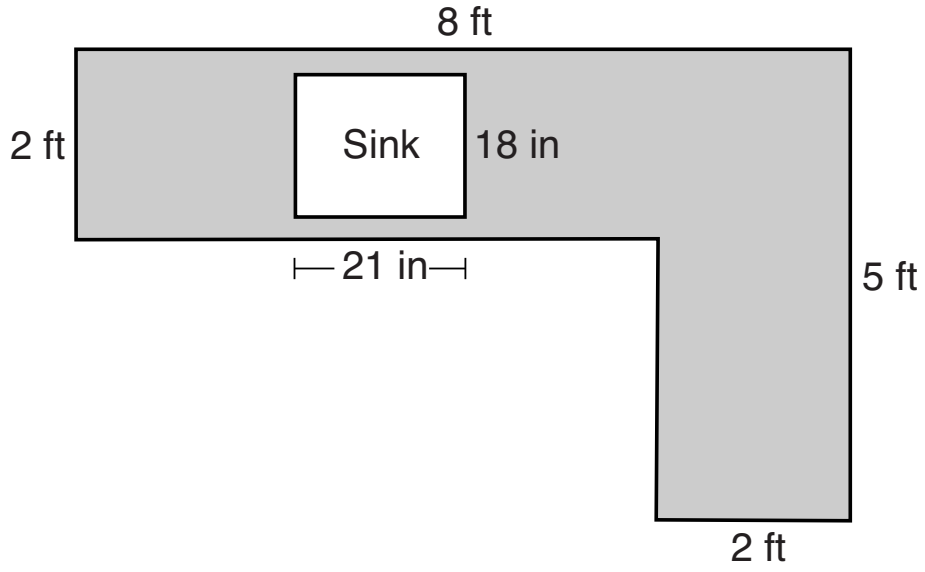
(2) $\frac{AD}{CD} = \frac{BD}{CD}$

(3) $\frac{AC}{CD} = \frac{BC}{CD}$

(4) $\frac{AD}{AC} = \frac{AC}{BD}$

**Use this space for computations.**

**17** A countertop for a kitchen is modeled with the dimensions shown below. An 18-inch by 21-inch rectangle will be removed for the installation of the sink.




What is the area of the top of the installed countertop, to the *nearest square foot*?

(1) 26

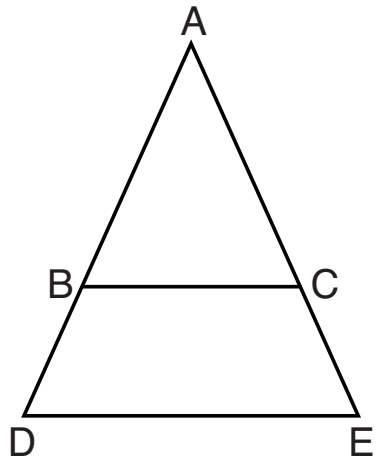
(2) 23

(3) 22

(4) 19

**Use this space for computations.**

**18** In the diagram below, $\overline{BC}$ connects points $B$ and $C$ on the congruent sides of isosceles triangle $ADE$, such that $\triangle ABC$ is isosceles with vertex angle $A$.




If $AB = 10$, $BD = 5$, and $DE = 12$, what is the length of $\overline{BC}$?

(1) 6

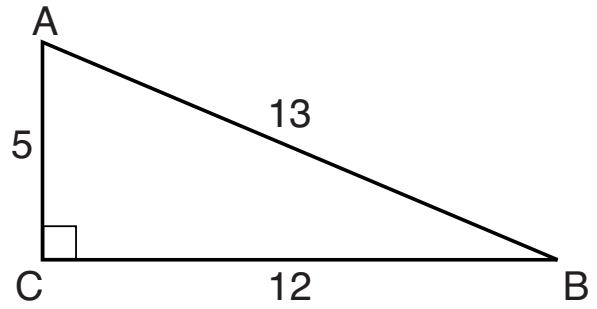
(2) 7

(3) 8

(4) 9

**Use this space for computations.**

**19** In $\triangle ABC$ below, angle $C$ is a right angle.




Which statement must be true?

(1) $\sin A = \cos B$

(2) $\sin A = \tan B$

(3) $\sin B = \tan A$

(4) $\sin B = \cos B$

**20** In right triangle $RST$, altitude $\overline{TV}$ is drawn to hypotenuse $\overline{RS}$. If $RV = 12$ and $RT = 18$, what is the length of $\overline{SV}$?

(1) $6\sqrt{5}$

(2) 15

(3) $6\sqrt{6}$

(4) 27

**Use this space for computations.**

**21** What is the volume, in cubic centimeters, of a right square pyramid with base edges that are 64 cm long and a slant height of 40 cm?

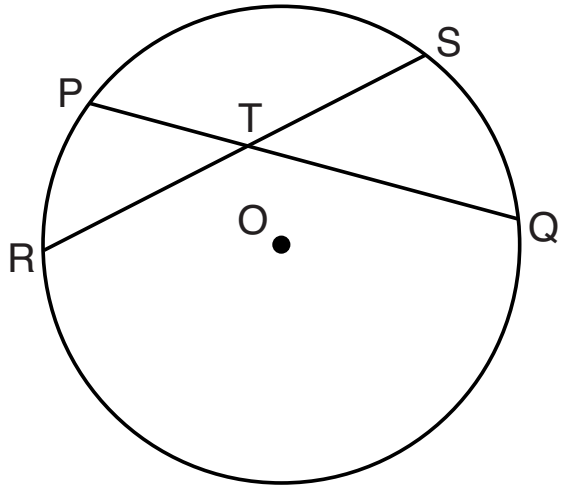
(1) 8192.0

(2) $13{,}653.\overline{3}$

(3) 32,768.0

(4) $54{,}613.\overline{3}$

**22** In the diagram below, chords $\overline{PQ}$ and $\overline{RS}$ of circle $O$ intersect at $T$.




Which relationship must always be true?

(1) $RT = TQ$

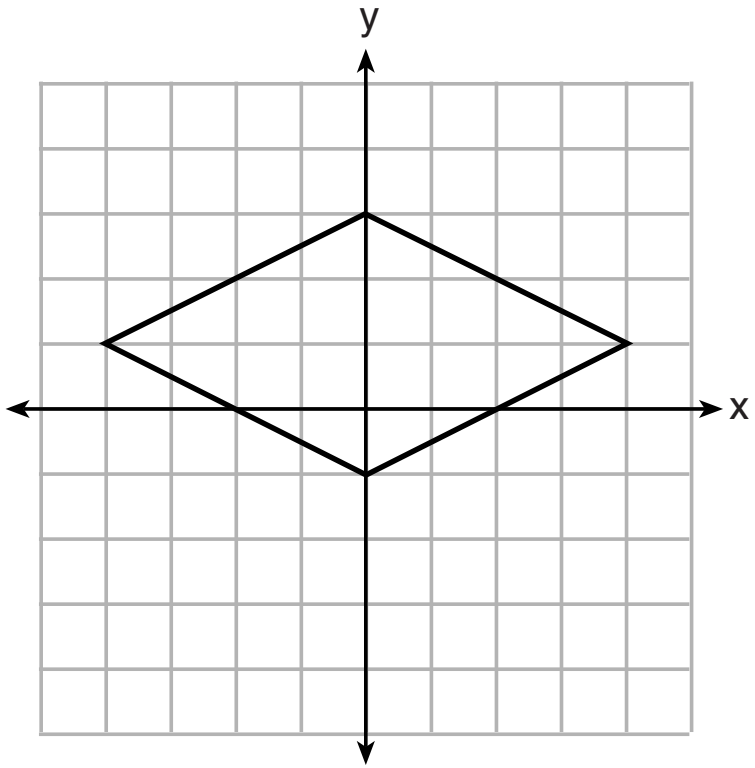
(2) $RT = TS$

(3) $RT + TS = PT + TQ$

(4) $RT \times TS = PT \times TQ$

**Use this space for computations.**

**23** A rhombus is graphed on the set of axes below.




Which transformation would carry the rhombus onto itself?

(1) 180° rotation counterclockwise about the origin

(2) reflection over the line $y = \frac{1}{2}x + 1$

(3) reflection over the line $y = 0$

(4) reflection over the line $x = 0$

**Use this space for computations.**

**24** A 15-foot ladder leans against a wall and makes an angle of 65° with the ground. What is the horizontal distance from the wall to the base of the ladder, to the *nearest tenth of a foot*?

(1) 6.3

(2) 7.0

(3) 12.9

(4) 13.6

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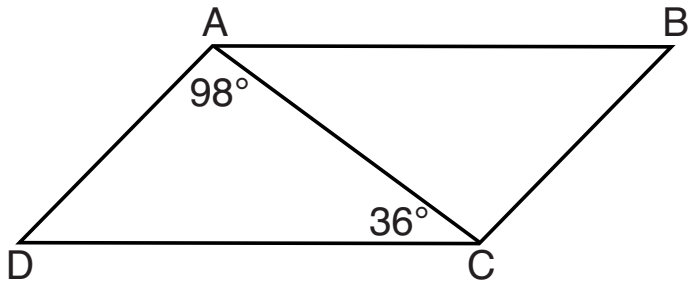
**GO RIGHT ON TO THE NEXT PAGE ➡**

## Part II

**Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [14]

**25** In parallelogram $ABCD$ shown below, m$\angle DAC = 98°$ and m$\angle ACD = 36°$.

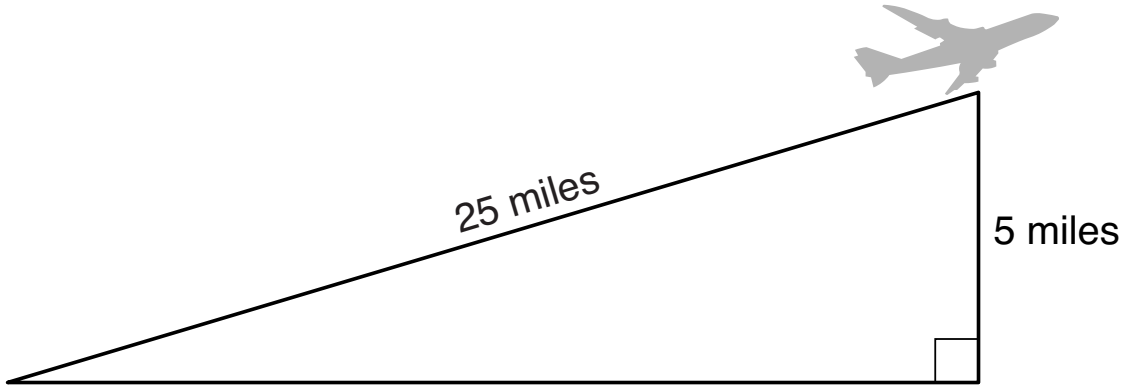



What is the measure of angle $B$? Explain why.

**Work space for question 25 is continued on the next page.**

**Question 25 continued**

**26** An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.

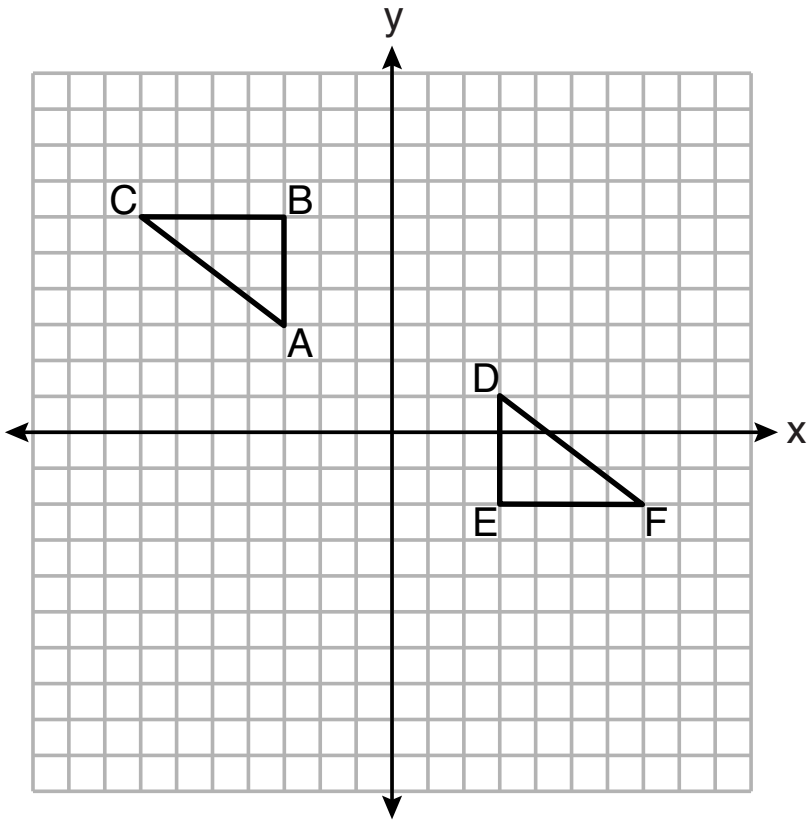



To the *nearest tenth of a degree*, what was the angle of elevation?

**Work space for question 26 is continued on the next page.**

**Question 26 continued**

**27** On the set of axes below, $\triangle ABC \cong \triangle DEF$.




Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.

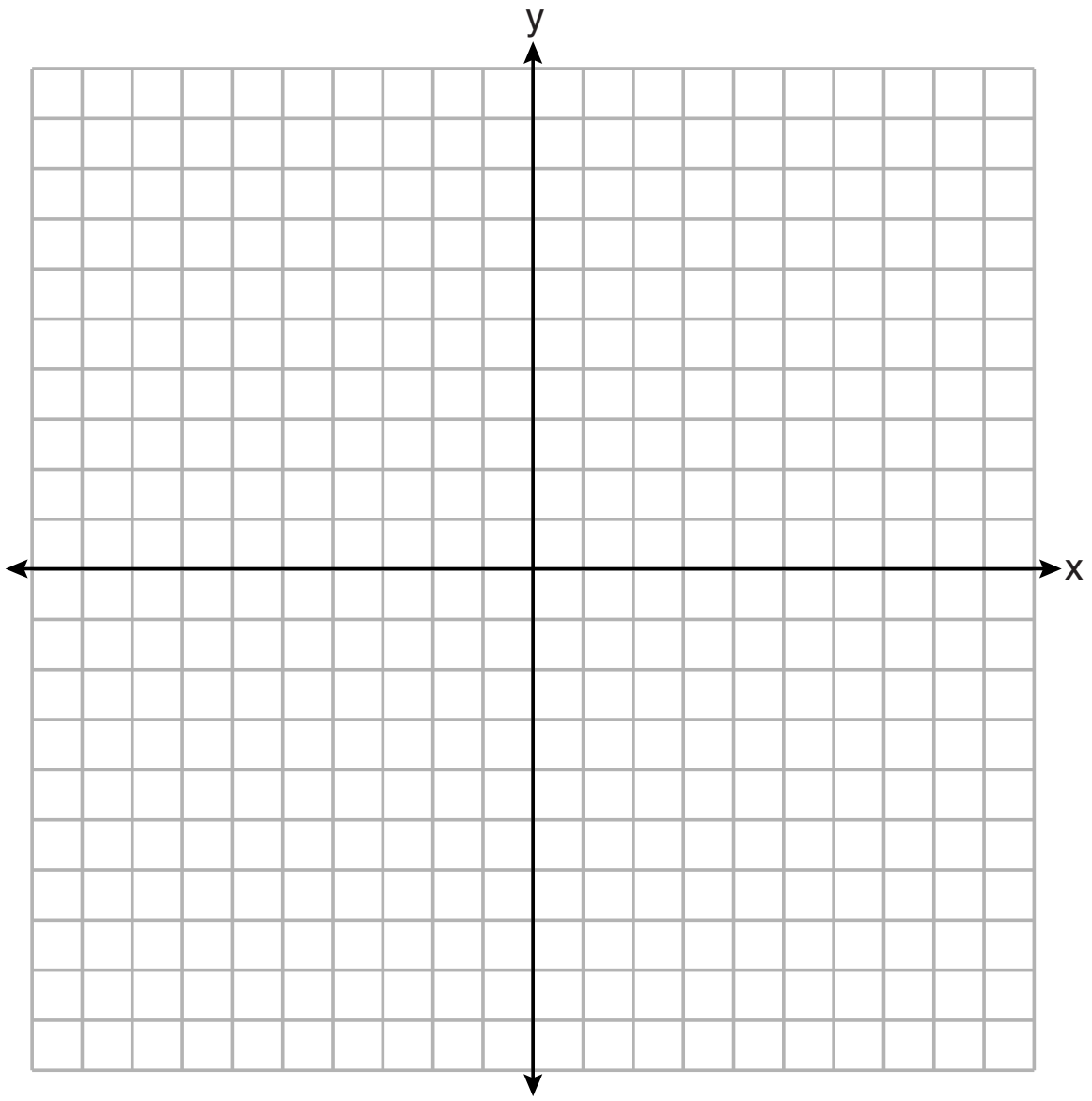
**Work space for question 27 is continued on the next page.**

**Question 27 continued**

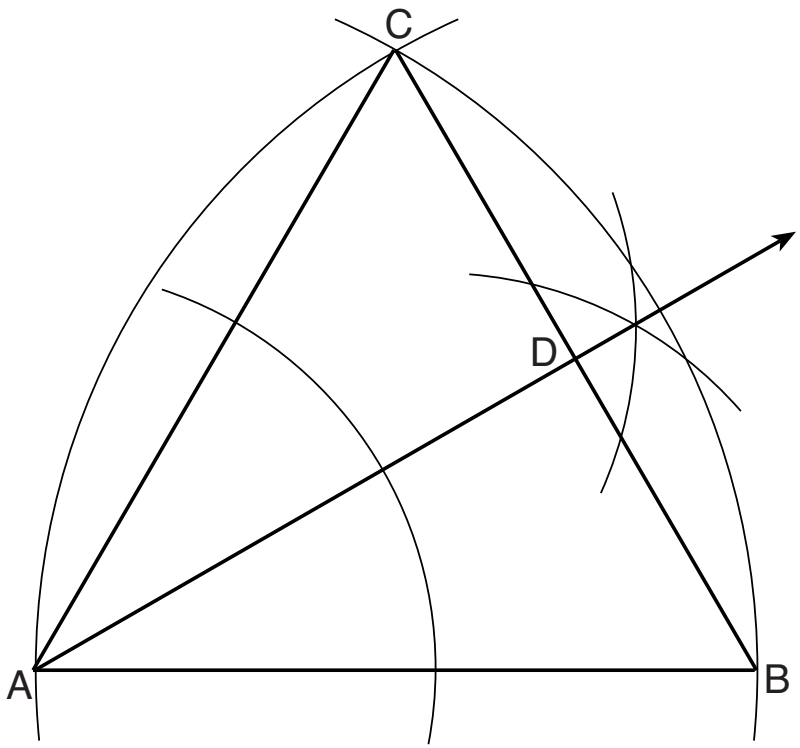
**28** The vertices of $\triangle ABC$ have coordinates $A(-2,-1)$, $B(10,-1)$, and $C(4,4)$. Determine and state the area of $\triangle ABC$. [The use of the set of axes on page 27 is optional.]

**The set of axes for question 28 is on the next page.**

**Question 28 continued**



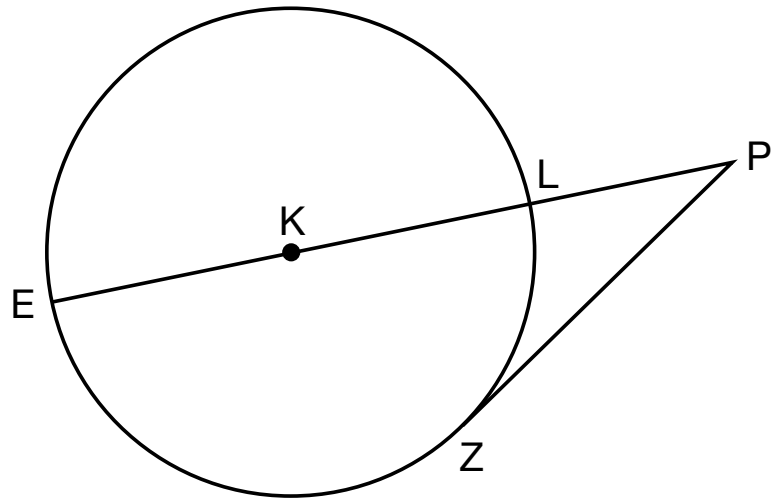
**29** Using the construction below, state the degree measure of $\angle CAD$. Explain why.




**Work space for question 29 is continued on the next page.**

**Question 29 continued**

**30** In the diagram below of circle $K$, secant $\overline{PLKE}$ and tangent $\overline{PZ}$ are drawn from external point $P$.




If $m\widehat{LZ} = 56°$, determine and state the degree measure of angle $P$.

**Work space for question 30 is continued on the next page.**

**Question 30 continued**

**31** A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of $8\frac{1}{4}$ feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of $\frac{1}{2}$ foot from the top.

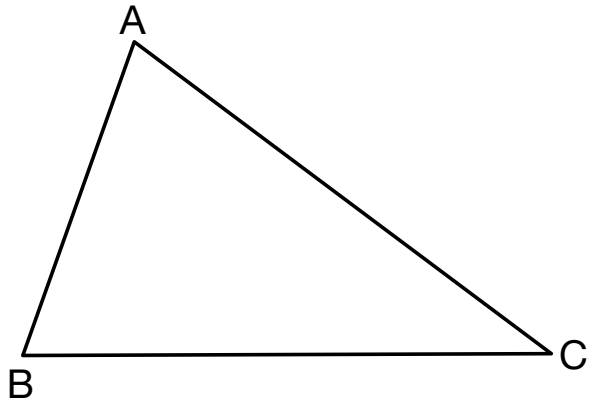
**Work space for question 31 is continued on the next page.**

**Question 31 continued**

## Part III

**Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

**32** Triangle $ABC$ is shown below. Using a compass and straightedge, construct the dilation of $\triangle ABC$ centered at $B$ with a scale factor of 2.
[Leave all construction marks.]

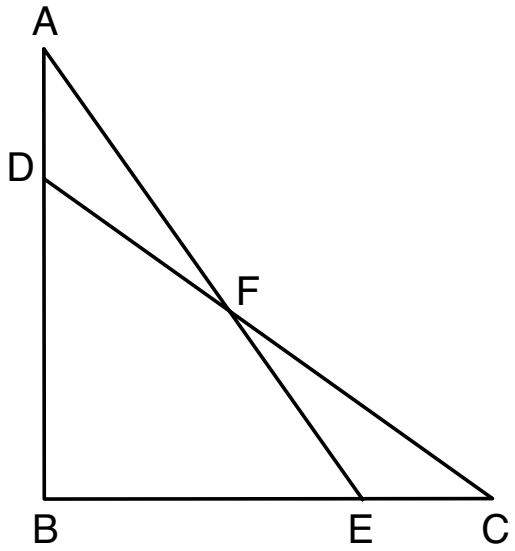



**Question 32 is continued on the next page.**

**Question 32 continued**

Is the image of $\triangle ABC$ similar to the original triangle? Explain why.

**33** In the diagram below, $\triangle ABE \cong \triangle CBD$.




Prove: $\triangle AFD \cong \triangle CFE$

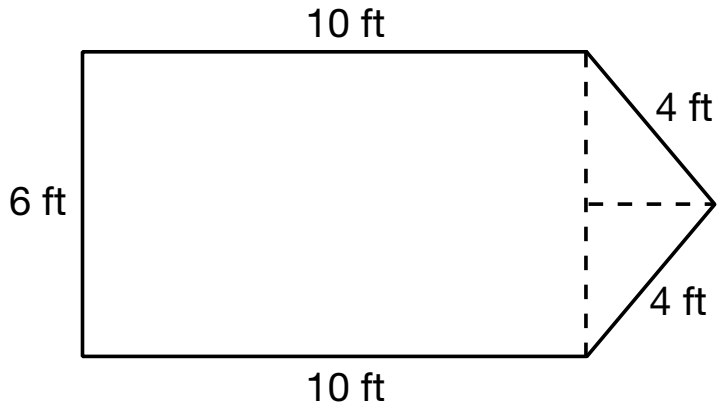
**Work space for question 33 is continued on the next page.**

**Question 33 continued**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer

Cargo Trailer Floor




If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the *nearest cubic foot*?

**Work space for question 34 is continued on the next page.**

**Question 34 continued**

## Part IV

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [6]

**35** The coordinates of the vertices of $\triangle ABC$ are $A(1,2)$, $B(-5,3)$, and $C(-6,-3)$.

Prove that $\triangle ABC$ is isosceles.

[The use of the set of axes on page 43 is optional.]

**Question 35 is continued on the next page.**

**Question 35 continued**

State the coordinates of point $D$ such that quadrilateral $ABCD$ is a square.

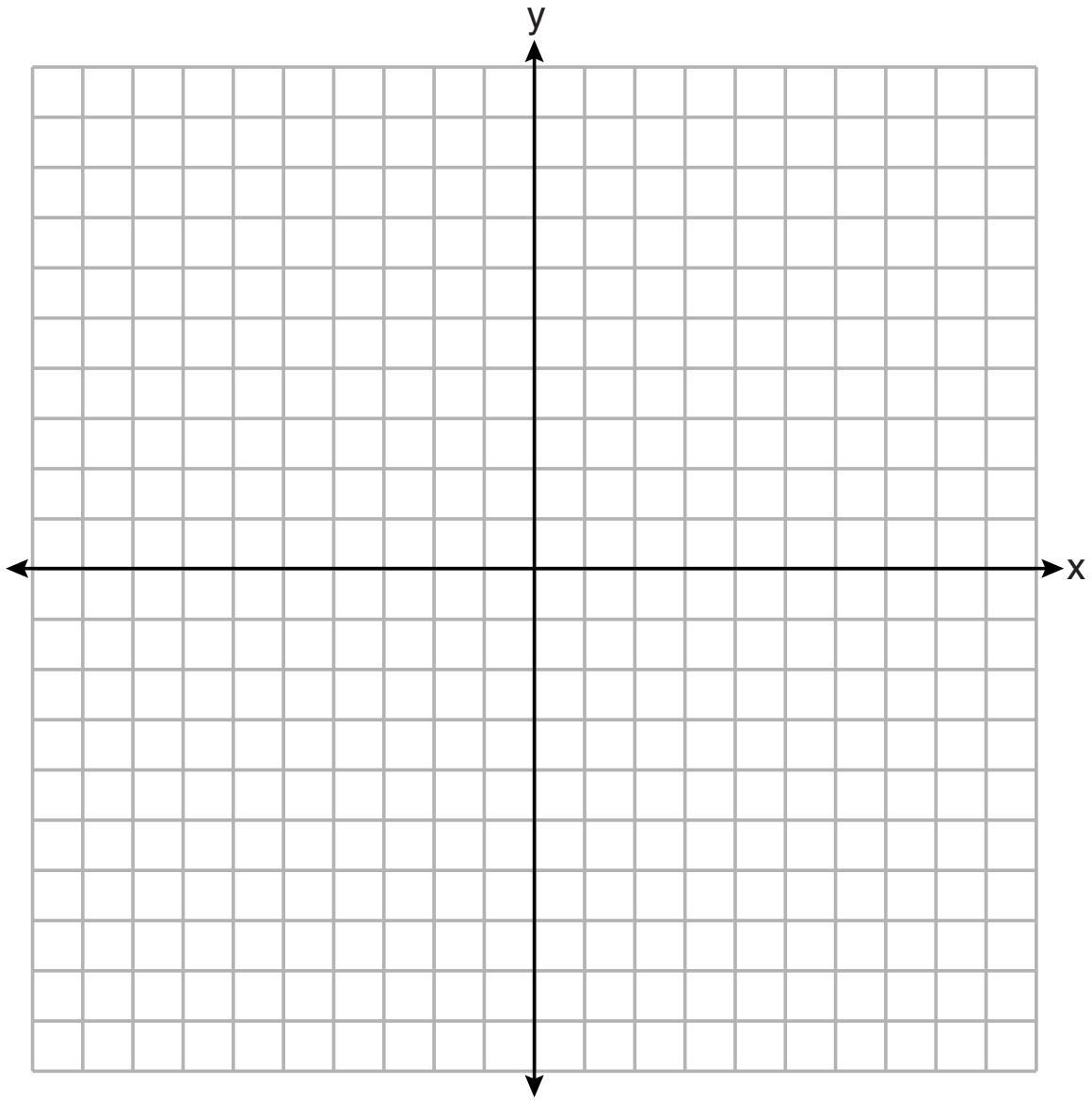
**Question 35 is continued on the next page.**

**Question 35 continued**

Prove that your quadrilateral *ABCD* is a square.
[The use of the set of axes on page 43 is optional.]

**The set of axes for question 35 is on the next page.**

**Question 35 continued**



# High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

| | |
|---|---|
| Triangle | $A = \frac{1}{2}bh$ |
| Parallelogram | $A = bh$ |
| Circle | $A = \pi r^2$ |
| Circle | $C = \pi d$ or $C = 2\pi r$ |
| General Prisms | $V = Bh$ |

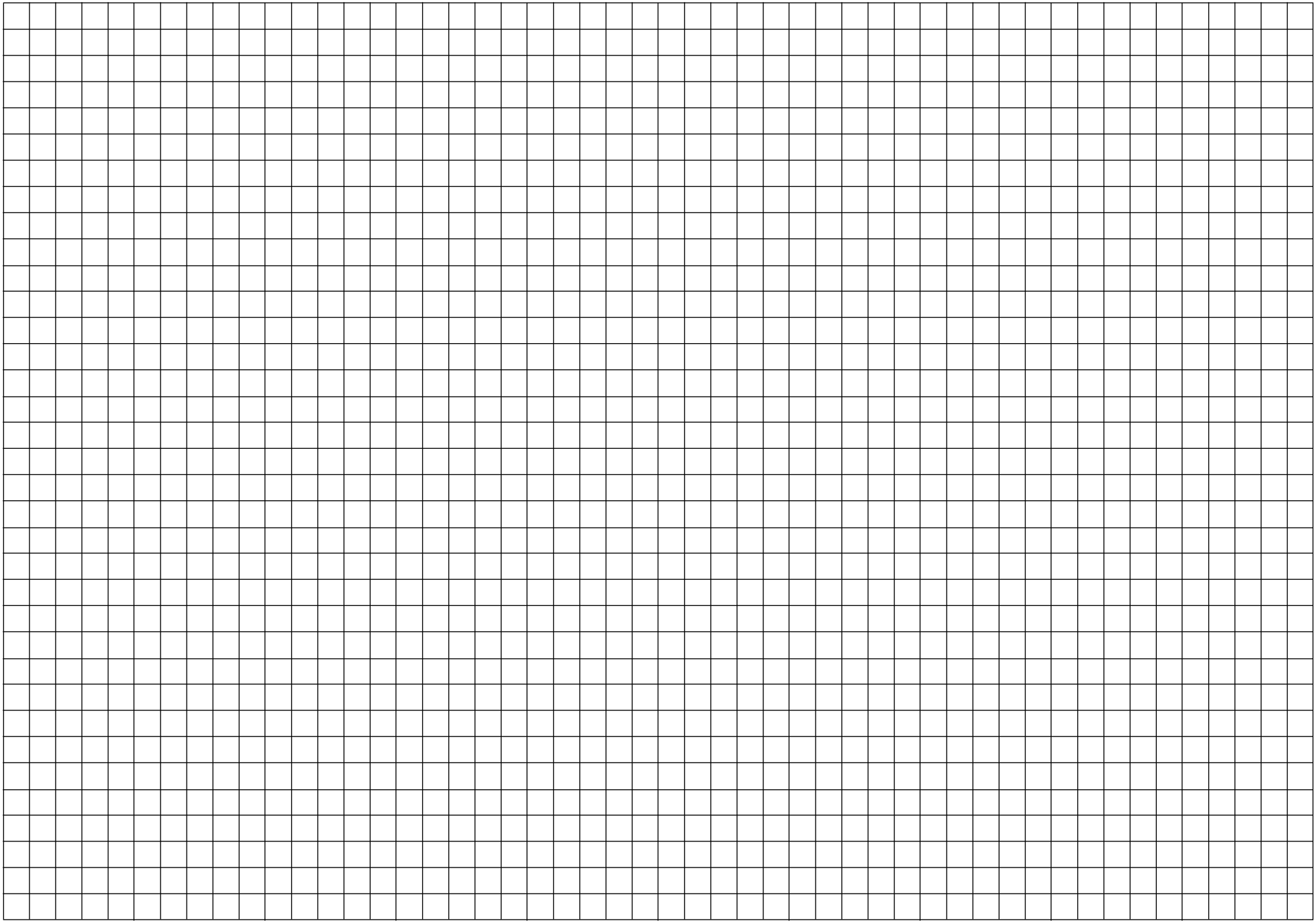
| | |
|---|---|
| Pythagorean Theorem | $a^2 + b^2 = c^2$ |
| Quadratic Formula | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Arithmetic Sequence | $a_n = a_1 + (n - 1)d$ |
| Geometric Sequence | $a_n = a_1 r^{n-1}$ |
| Geometric Series | $S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$ |

**The Reference Sheet is continued on the next page.**

| | |
|---|---|
| Cylinder | $V = \pi r^2 h$ |
| Sphere | $V = \frac{4}{3}\pi r^3$ |
| Cone | $V = \frac{1}{3}\pi r^2 h$ |
| Pyramid | $V = \frac{1}{3}Bh$ |

| | |
|---|---|
| Radians | 1 radian $= \frac{180}{\pi}$ degrees |
| Degrees | 1 degree $= \frac{\pi}{180}$ radians |
| Exponential Growth/Decay | $A = A_0 e^{k(t - t_0)} + B_0$ |

**Scrap Graph Paper — This sheet will *not* be scored.**



**Scrap Graph Paper — This sheet will *not* be scored.**