

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Wednesday, August 14, 2019 — 12:30 to 3:30 p.m.

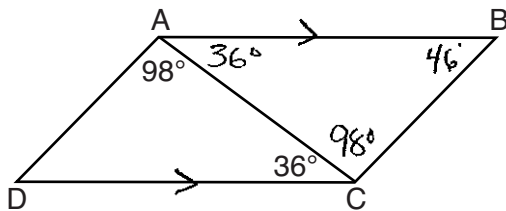
## MODEL RESPONSE SET

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Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



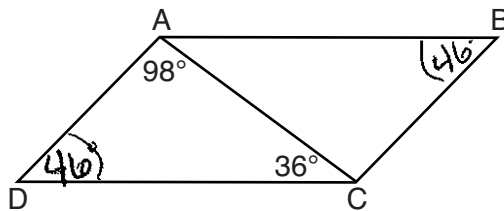
What is the measure of angle  $B$ ? Explain why.

$ABCD$  is a  $\parallel$ -gram, so  $\overline{AB} \parallel \overline{CD}$   
By alt. int.  $\angle$  theorem,  $\angle BCA = 98^\circ$ ,  
 $\angle BAC = 36^\circ$ .  $\angle$ 's in  $\triangle BAC$  must  
add up to  $180^\circ$ , so  $\angle B = 46^\circ$   
Since  $36 + 98 = 134$ , and  $180 - 134 = 46$ .

**Score 2:** The student gave a complete and correct response.

Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



$$\begin{array}{r} 180 \\ - 98 \\ - 36 \\ \hline 46^\circ \end{array}$$

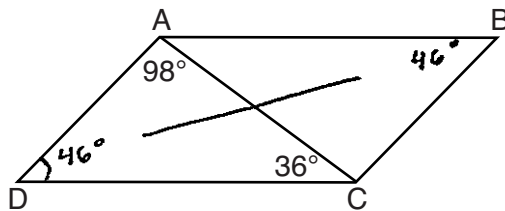
What is the measure of angle  $B$ ? Explain why.

$\angle D = 46^\circ$  because the sum of a  $\Delta$  in  $\Delta ACD$ .  
 $\angle D \cong \angle B$  because in a parallelogram,  
opposite  $\angle$ 's are  $\cong$ .  
So  $\boxed{\angle B = 46^\circ}$ .

**Score 2:** The student gave a complete and correct response.

Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

$$98 + 36 = 134$$

$$180 - 134 = 46$$

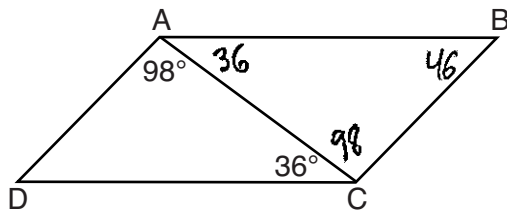
$$m\angle B = 46^\circ$$

First, I found  $m\angle D$ . A triangle has  $180^\circ$ , so I added  $m\angle A$  &  $m\angle C$  from  $\triangle DAC$  and got  $134^\circ$ . Then, I subtracted that from  $180$  in order to get  $m\angle D$ .  $m\angle D = 46^\circ$ . Finally, I know opp. angles in a parallelogram are congruent, so  $m\angle D = m\angle B$ . By chain rule,  $m\angle B$  would be  $46^\circ$ .

**Score 2:** The student gave a complete and correct response.

Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

$$98 + 36 = 134$$

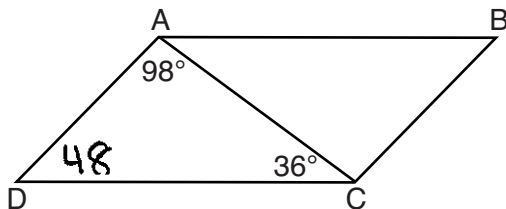
$$180 - 134 = 46$$

$m\angle B$  is  $46^\circ$  because of  $\cong$  alternate interior angles at  $\angle A + \angle C$  and  $\overline{AB} \parallel \overline{CD}$

**Score 2:** The student gave a complete and correct response.

Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

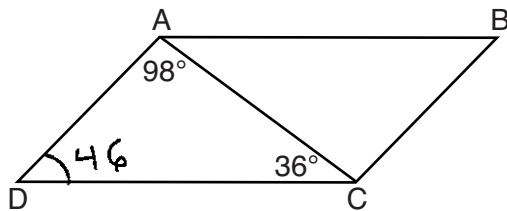
$98 + 34 = 132$   
48

Angle B is 48 because in a parallelogram opposite angles are  $\cong$ . If  $\angle D$  is 48, then  $\angle B$  is 48.

**Score 1:** The student made a transcription error. The student wrote an appropriate explanation.

Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

$$98 + 36 = 134$$

$$\begin{array}{r} 180 \\ -134 \\ \hline 46 \end{array}$$

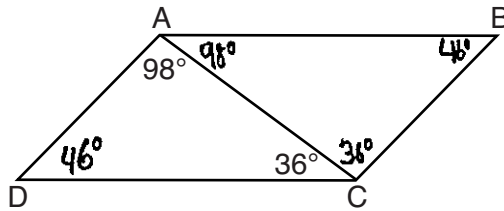
$$\angle D \cong \angle B$$

$$\angle D = 46$$

**Score 1:** The student found  $m\angle B = 46^\circ$ , but did not write an explanation.

**Question 25**

**25** In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

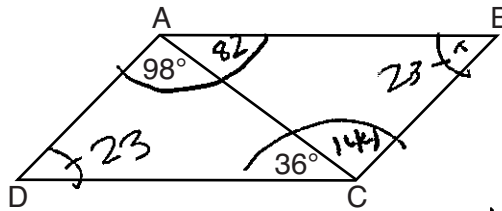
Angle B is  $46^\circ$  because the two triangles are the same so they have the same angles

**Score 1:** The student found  $m\angle B = 46^\circ$ , but the explanation is incomplete.



Question 25

25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



$$\begin{array}{r} 180 \\ - 36 \\ \hline 144 \end{array}$$

$a = bh$

What is the measure of angle  $B$ ? Explain why.

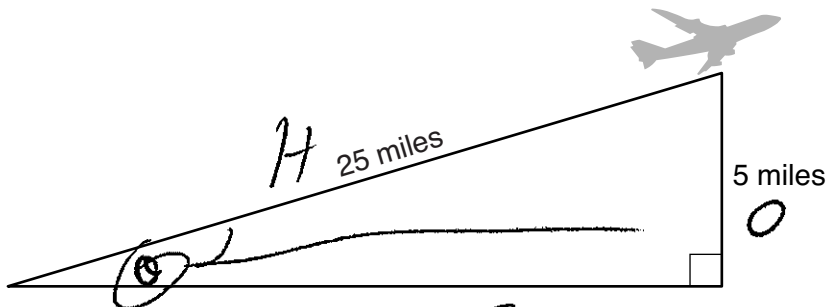
The measure of angle  $B$  is  $23^\circ$  because in a parallelogram, opposite sides are congruent and a triangle equals to  $180^\circ$ . If you add the two measures and subtract  $180$  you get  $144$ , and divided by  $2$  the measure is  $23^\circ$ .

$$\begin{array}{r} 180 \\ - 96 \\ \hline 84 \end{array}$$

**Score 0:** The student gave a completely incorrect response.

Question 26

26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.



$$\sin^{-1} = \frac{5}{25}$$

To the nearest tenth of a degree, what was the angle of elevation?

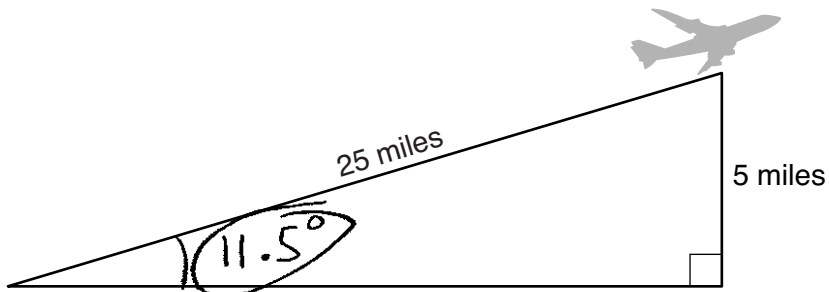
$$\sin^{-1}\left(\frac{5}{25}\right) = 11.53695903$$

11.5

**Score 2:** The student gave a complete and correct response.

**Question 26**

**26** An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.



To the *nearest tenth of a degree*, what was the angle of elevation?

$$\sin x = \frac{5}{25}$$

$$\sin^{-1}\left(\frac{5}{25}\right)$$

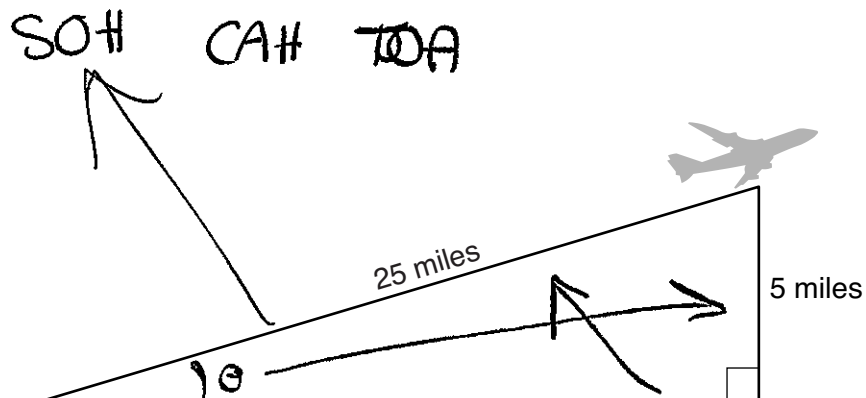
$$11.537$$

$$11.5^\circ$$

**Score 2:** The student gave a complete and correct response.

Question 26

- 26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.



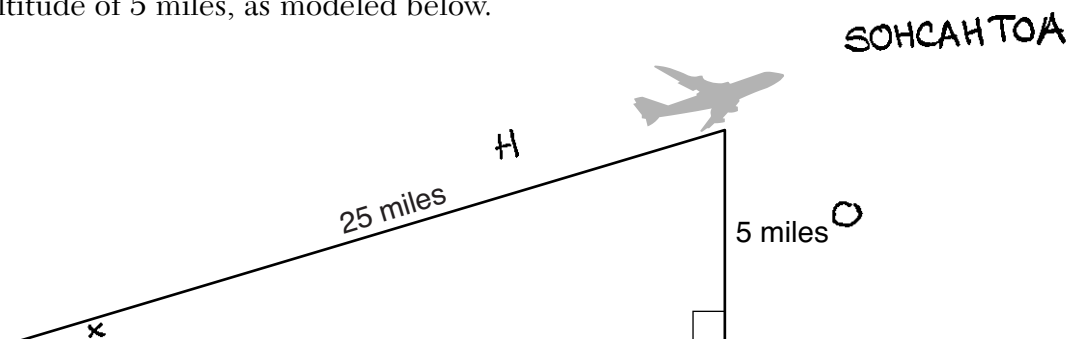
To the nearest tenth of a degree, what was the angle of elevation?

$$\sin \theta = \frac{5}{25}$$

**Score 1:** The student wrote a correct trigonometric equation, but no further correct work was shown.

Question 26

- 26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.



To the nearest tenth of a degree, what was the angle of elevation?

$$\sin x = \frac{5}{25}$$
$$x = \sin^{-1}\left(\frac{5}{25}\right)$$
$$x = 11.53695903$$
$$x = 12^\circ$$

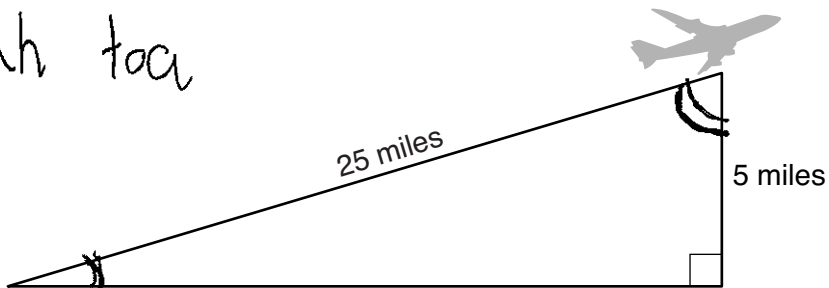
The  $\angle$  of elevation is  $12^\circ$

**Score 1:** The student did not round the answer to the nearest tenth of a degree.

Question 26

26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.

soh cah toa



To the nearest tenth of a degree, what was the angle of elevation?

$$\frac{\tan x}{1} = \frac{5}{25}$$

$$25x = 5$$

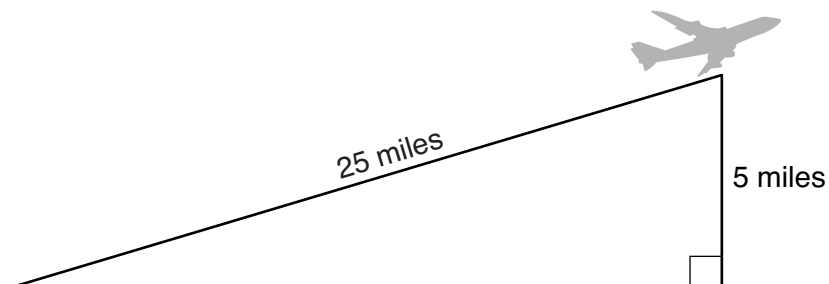
$$x = .2$$

$$x = 20^\circ$$

**Score 0:** The student gave a completely incorrect response.

Question 26

26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.



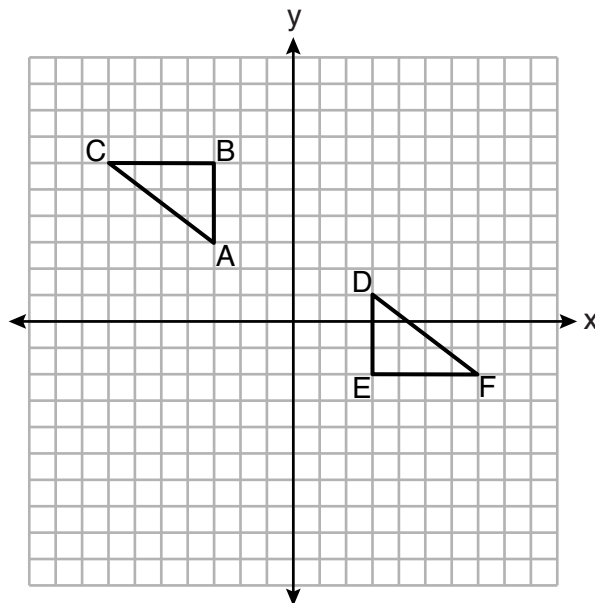
To the nearest tenth of a degree, what was the angle of elevation?

$$\begin{aligned} 5^2 + b^2 &= 25^2 \\ \cancel{25} + \cancel{b^2} &= 625 \\ -25 & \quad -25 \\ \hline \sqrt{b^2} &= \sqrt{600} \\ b &= 24.4 \end{aligned}$$

**Score 0:** The student gave a completely incorrect response.

**Question 27**

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

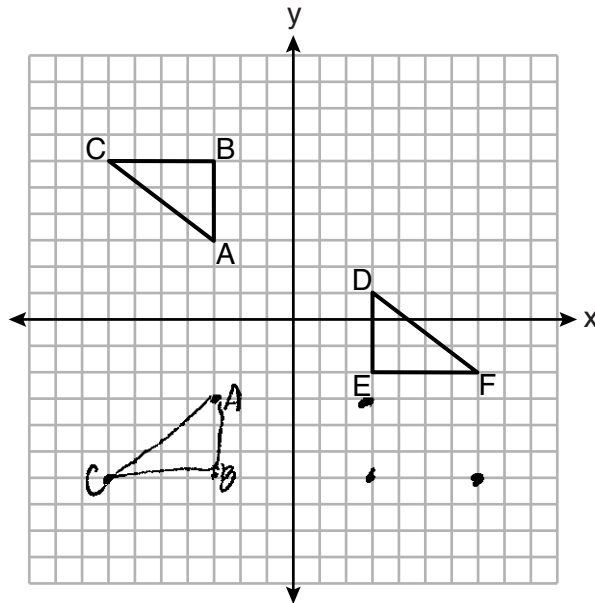
A reflection over the  $y$ -axis  
then a reflection over  $y=2$ .

**Score 2:** The student gave a complete and correct response.



**Question 27**

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



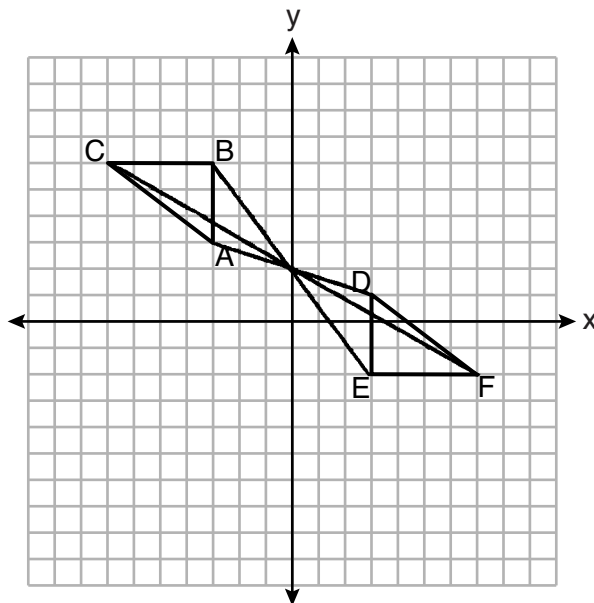
Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

1. reflection across the  $x$ -axis
2. reflection across  $y$ -axis
3. translation up 4 units

**Score 2:** The student gave a complete and correct response.

**Question 27**

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



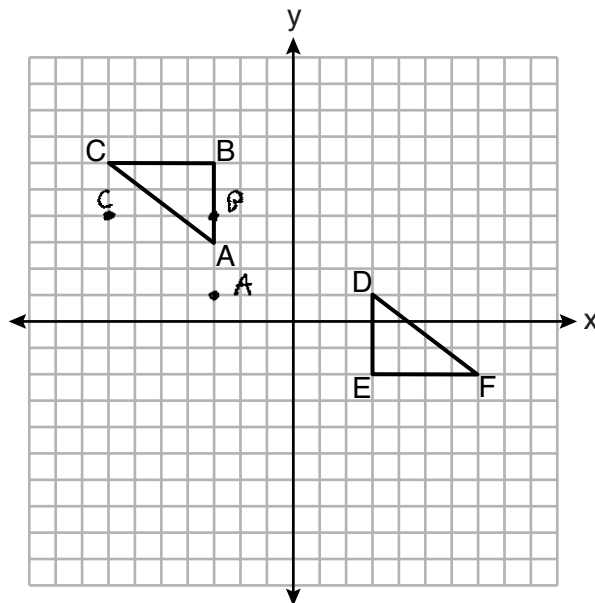
Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

A rotation of  $180^\circ$  around  
the point  $(0, 2)$

**Score 2:** The student gave a complete and correct response.

**Question 27**

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



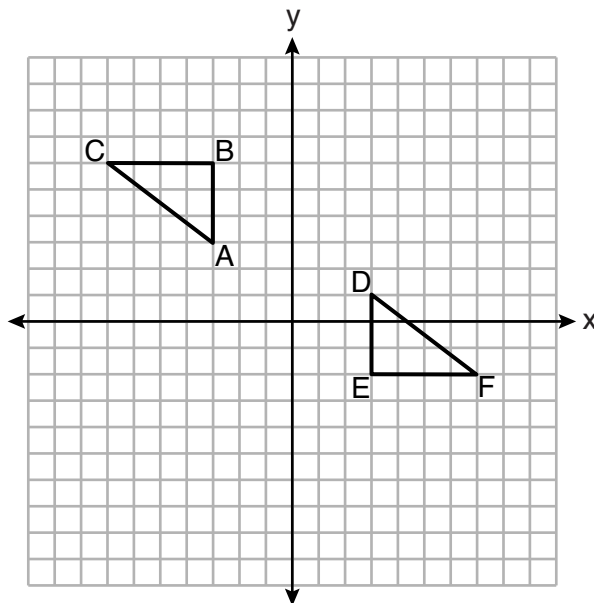
Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

Translate  $\triangle ABC$  down 2 and right  
6 then rotate it  $180^\circ$

**Score 1:** The student did not state point  $D$  as the center of rotation.

**Question 27**

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



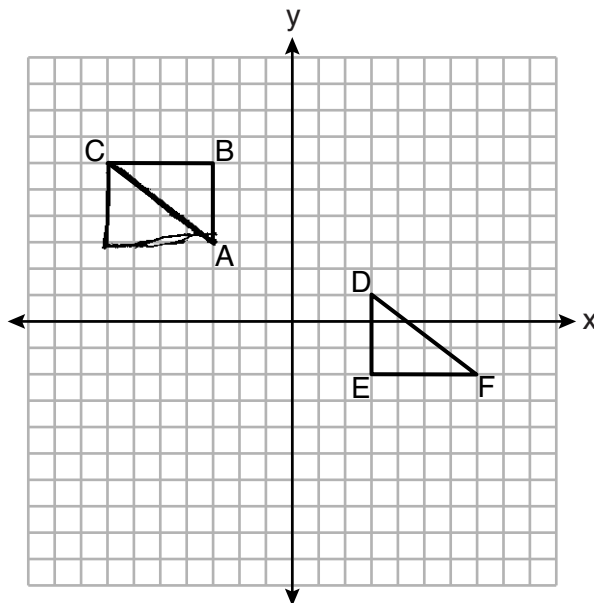
Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

Rotation of  $180^\circ$ , then a  
translation 4 units up

**Score 1:** The student did not state the origin as the center of rotation.

Question 27

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .



Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

$\triangle ABC$  would have to undergo a rotation of  $90^\circ$ , then translated 10 and 5.

**Score 0:** The student gave a completely incorrect response.

**Question 28**

**28** The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(4 - (-2))^2 + (-1 - (-1))^2}$$

$$0 + 25$$

$$d = \sqrt{25}$$

$$d = 5$$

Height

$$d = \sqrt{(10 - (-2))^2 + (-1 - (-1))^2}$$

$$d = \sqrt{12^2 + 0}$$

$$d = \sqrt{144}$$

$$d = 12$$

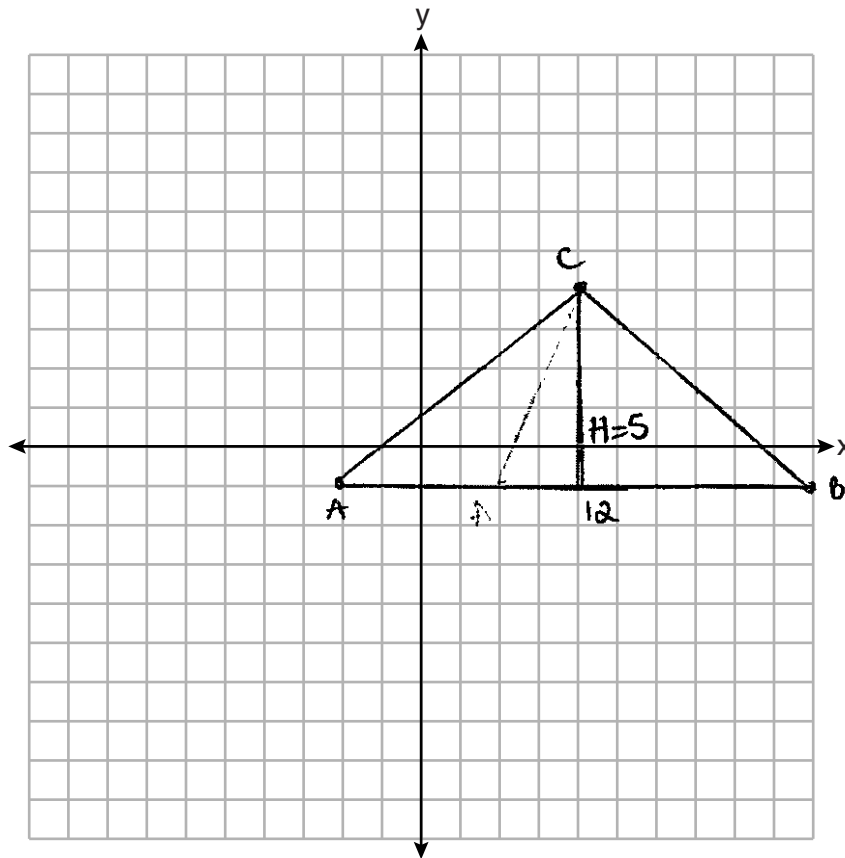
AB

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(12)(5)$$

$$A = \frac{1}{2}60$$

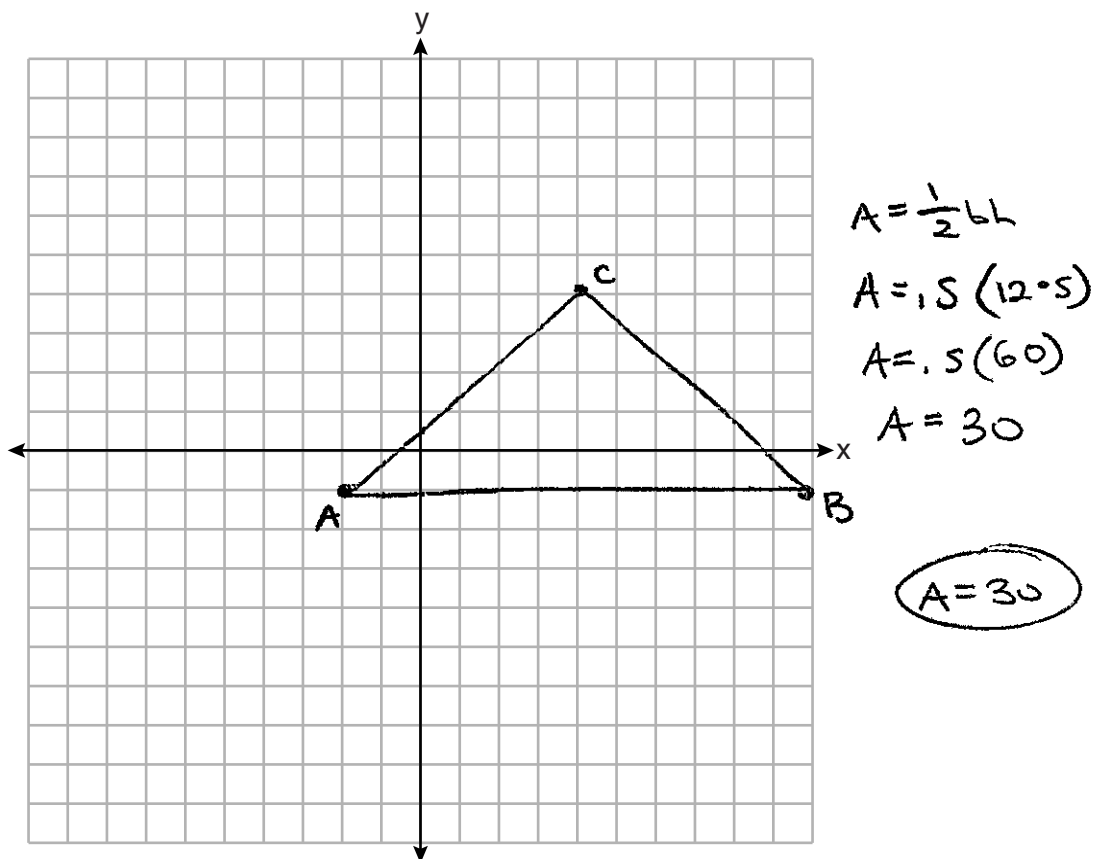
$$A = 30 \text{ units}^2$$



**Score 2:** The student gave a complete and correct response.

**Question 28**

**28** The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]



**Score 2:** The student gave a complete and correct response.

Question 28

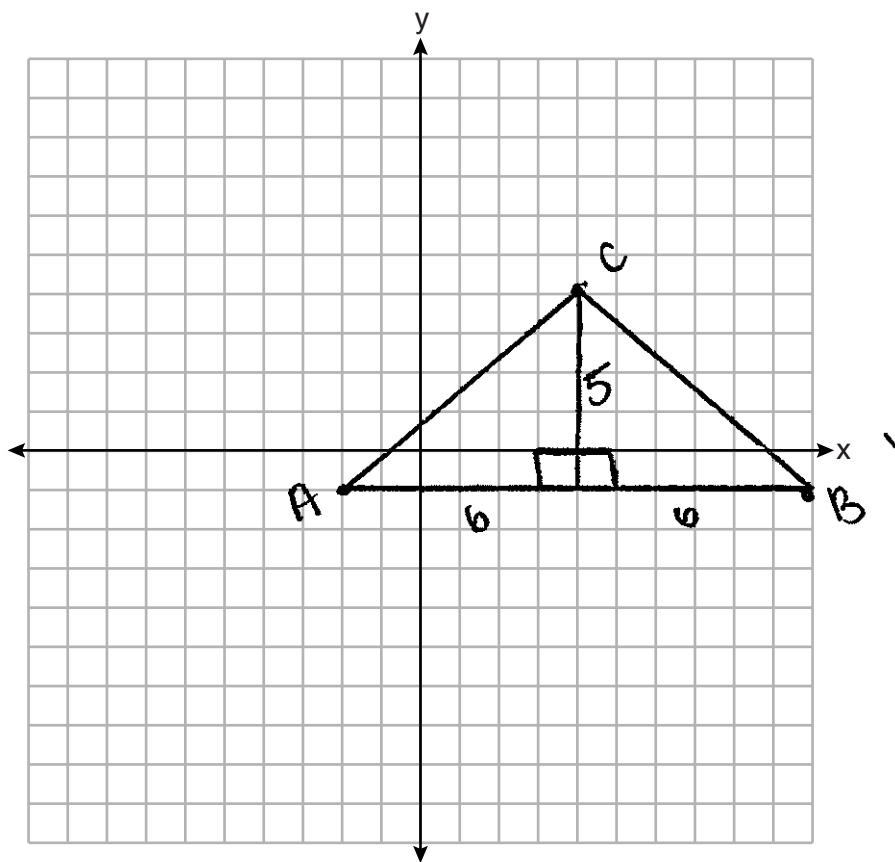
28 The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(6)$$

$$A = 15(2) = 30$$

$$\text{area of } \triangle ABC = 30 \text{ units}^2$$



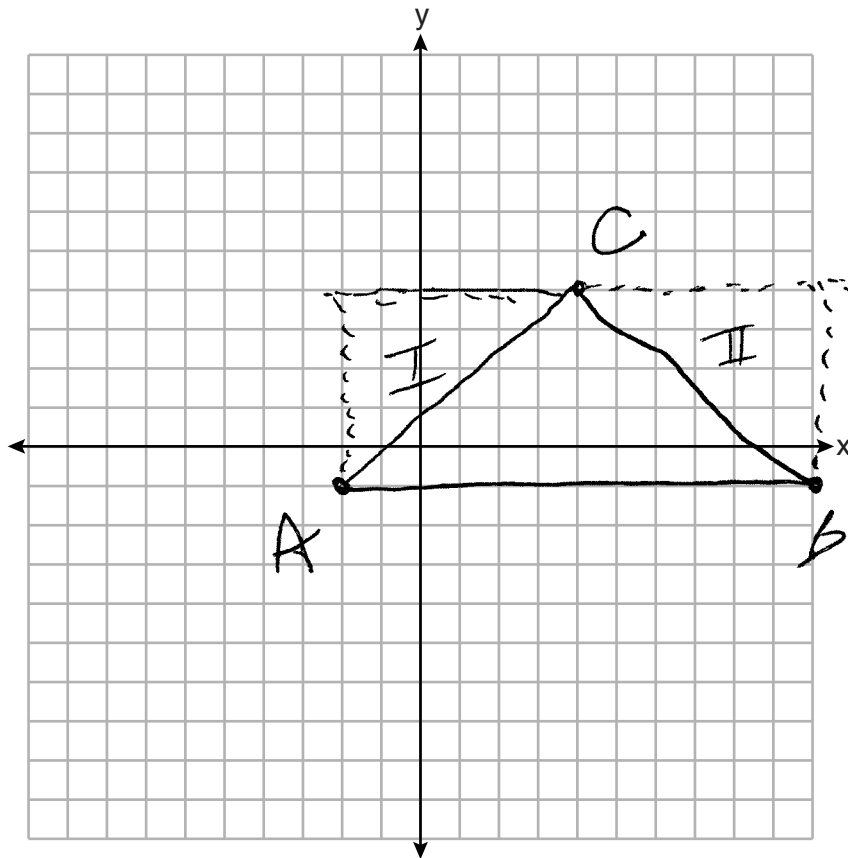
**Score 2:** The student gave a complete and correct response.



**Question 28**

**28** The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

Area of  $\triangle ABC = 30$



$$A_{\text{rect}} = 5 \cdot 12 = 60$$

$$A_{\text{I}} = \frac{1}{2}(5)(6) = 15$$

$$A_{\text{II}} = \frac{1}{2}(5)(6) = 15$$

$$A_{\triangle ABC} = 60 - 15 - 15 = 30$$

30

**Score 2:** The student gave a complete and correct response.

Question 28

28 The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$\boxed{60}$

$$\sqrt{(-2+10)^2 + (-1+4)^2}$$

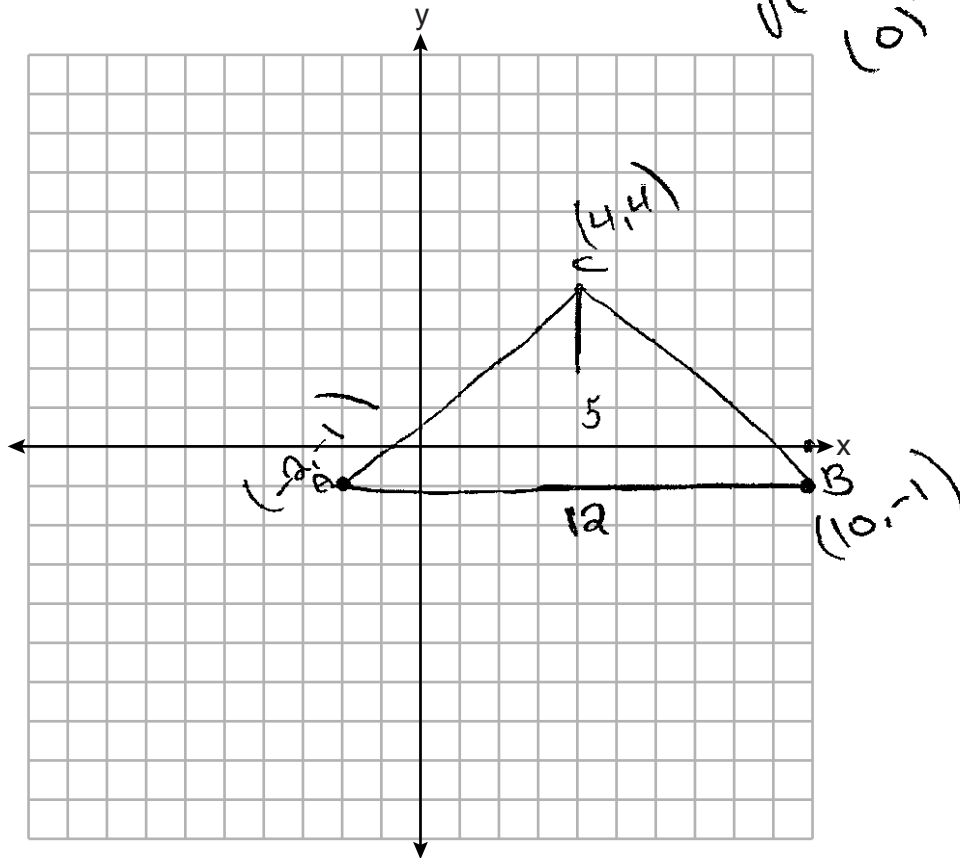
$$\sqrt{(-12)^2 + 3^2}$$

$$\sqrt{144}$$

$$\frac{12 \times 5}{2}$$

$$\sqrt{(4+4)^2 + (4+1)^2}$$

$$(0)^2 + (5)^2$$



**Score 1:** The student made an error by not multiplying the product of the base and height by  $\frac{1}{2}$ .

Question 28

28 The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(-1 - -1)^2 + (10 - -2)^2}$$

$$\sqrt{0 + 144}$$

$$(4, -1)$$

$$(4, -4)$$

~~$$AC = \sqrt{(4 - -1)^2 + (4 - -1)^2}$$

$$= \sqrt{25 + 9}$$

$$= \sqrt{34}$$~~

$$12$$

$$\sqrt{(4 - 4)^2 + (-4 - -1)^2}$$

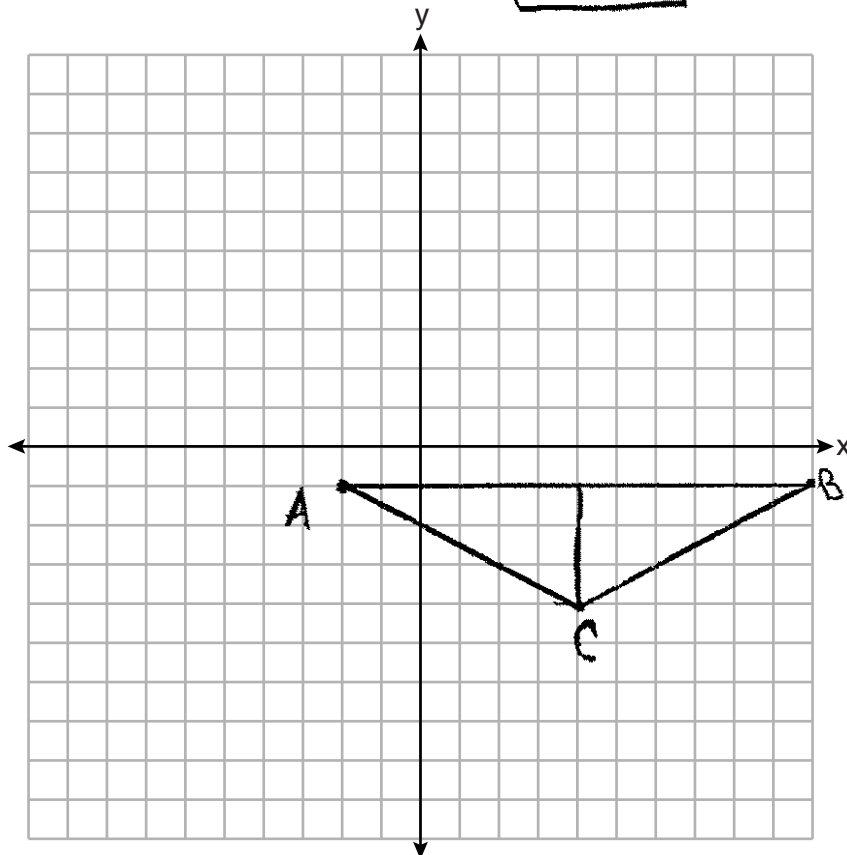
$$\sqrt{9}$$

$$3$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} 12 \cdot 3$$

$$A = 18$$



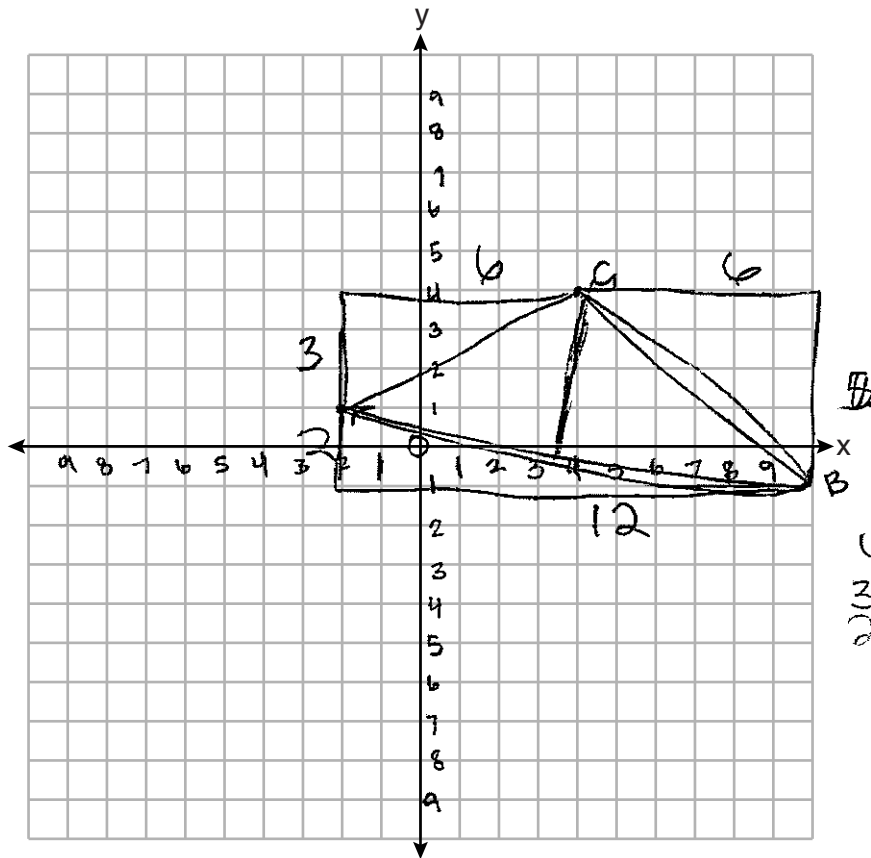
**Score 1:** The student graphed  $\triangle ABC$  incorrectly, but found the area of the triangle of equal difficulty.

**Question 28**

**28** The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(\sqrt{148})(3)$$



$$3^2 + 6^2 = c^2$$

$$9 + 12 = c^2$$

$$21 = c^2$$

$$\sqrt{21} = c$$

$$2^2 + 12^2 = c^2$$

$$4 + 144 = c^2$$

$$148 = c^2$$

$$\sqrt{148} = c$$

$$6^2 + 5^2 = c^2$$

$$36 + 25 = c^2$$

$$25 = c^2$$

$$6 = c^2$$

$$\sqrt{6} = c$$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

Question 28

28 The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$d = \sqrt{(10 - (-2))^2 + (-1 - (-1))^2}$$

$$d = \sqrt{144}$$

$$d = 12$$

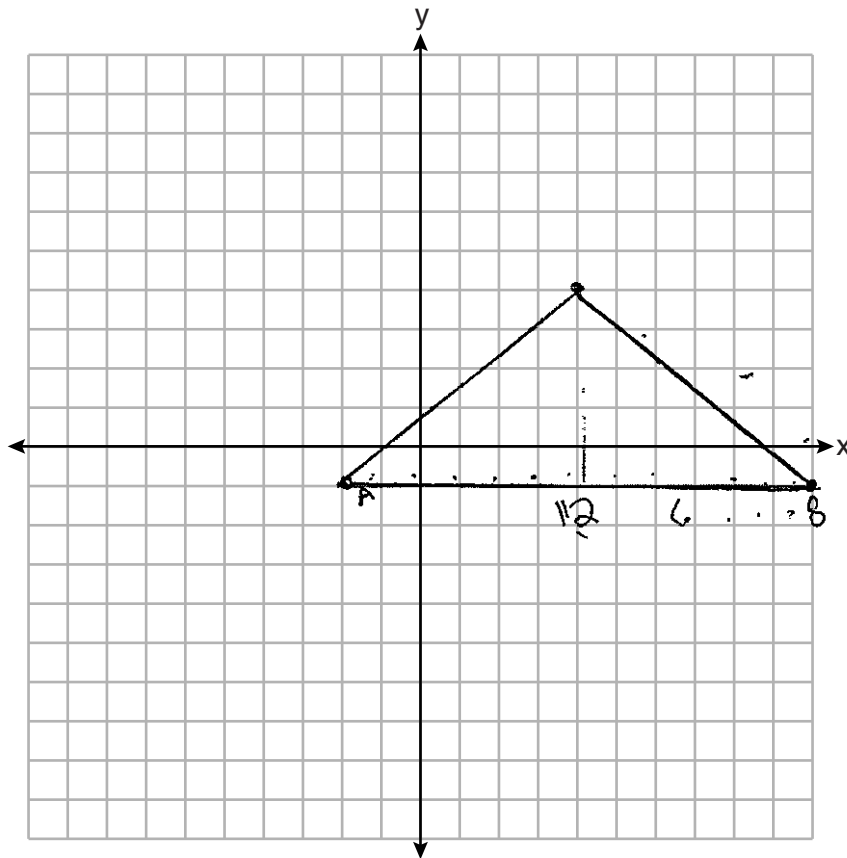
$$A_{\Delta} = \frac{1}{2}(b \cdot h)$$

$$d = \sqrt{(4 - 10)^2 + (4 - (-1))^2}$$

$$d = \sqrt{36 + 25}$$

$$d = \sqrt{61}$$

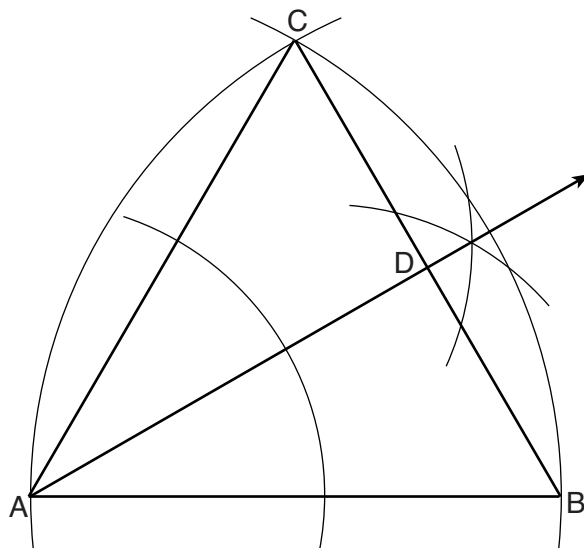
$A_{\Delta}$



**Score 0:** The student did not show enough correct relevant work to receive any credit.

Question 29

29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.

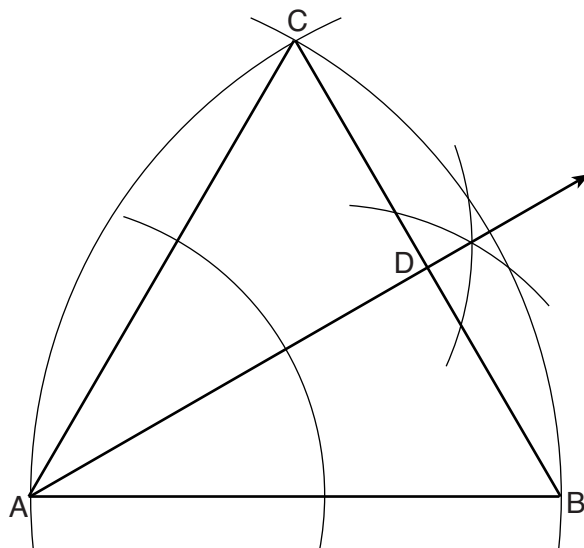


1.  $\triangle CAB$  is an equilateral triangle because all its sides are the radii of congruent circles.
2. All the angle measures in an equilateral triangle are  $60^\circ$ .
3.  $\angle CAD$  is  $30^\circ$  because an angle bisector was constructed, and angle bisectors cut an angle into 2 congruent pieces, so  $60^\circ \div 2 = 30^\circ$ .

**Score 2:** The student gave a complete and correct response.

Question 29

29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.

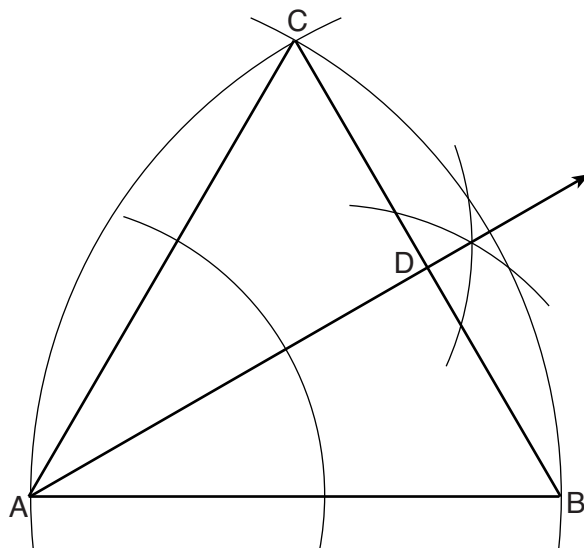


$m\angle CAD$  is  $30^\circ$  because  
 $\angle CAB$  is  $60^\circ$  because  $\triangle ABC$  is  
equilateral. Then  $\angle CAB$  was  
cut in half making  $\angle CAD$   
equal to  $30^\circ$

**Score 2:** The student gave a complete and correct response.

Question 29

29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.



$\triangle ABC$  is equilateral

$$\angle CAD = \frac{1}{2} \angle CAB$$

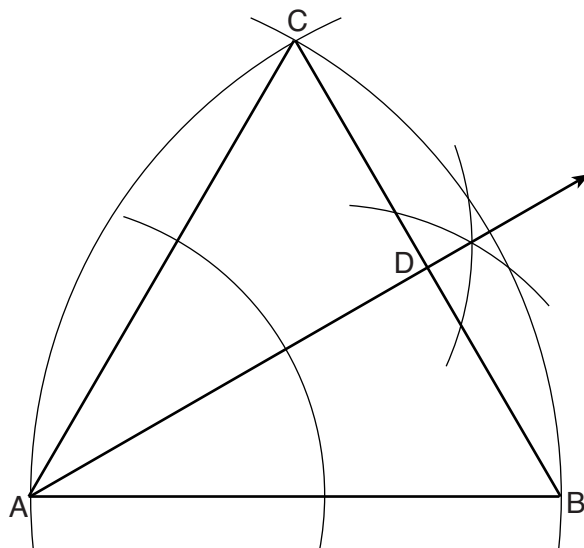
$\angle CAD$  is half of  $\angle CAB$   
because it was bisected

**Score 1:** The student wrote a correct explanation, but did not state that the angle measures  $30^\circ$ .



**Question 29**

**29** Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.

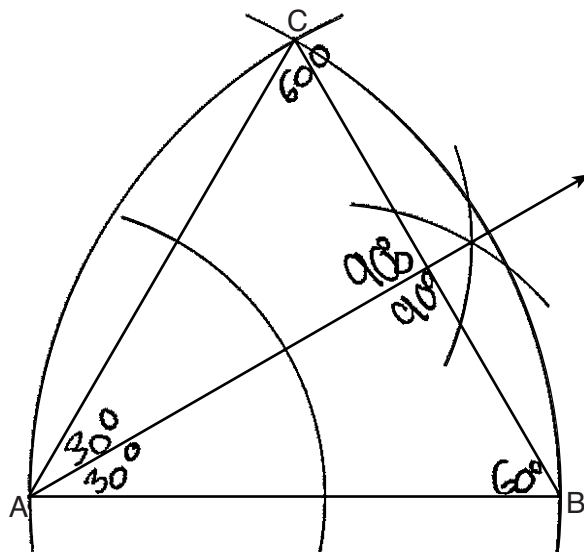


This is a  $30^\circ$  angle because the compass was open to a little more than half of  $\overline{AB}$ .

**Score 1:** The student stated that the angle measures  $30^\circ$ , but the explanation is incorrect.

Question 29

29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.



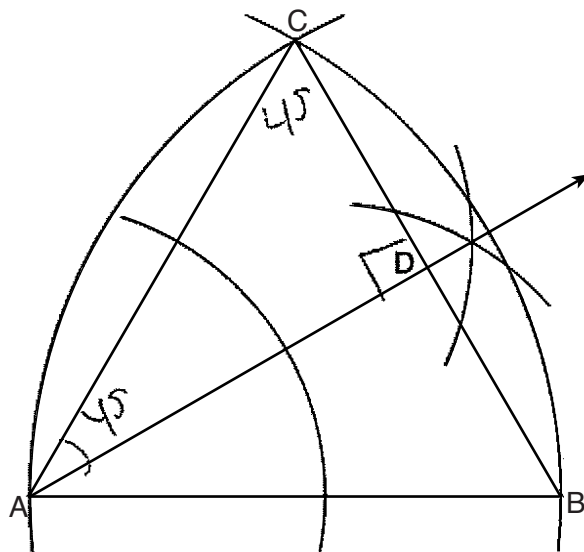
Its an equilateral triangle  
So  $\angle A$ ,  $\angle C$  and  $\angle B$  is  $60^\circ$ .

**Score 1:** The student stated that the angle measures  $30^\circ$ , but the explanation is incomplete.



Question 29

29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.

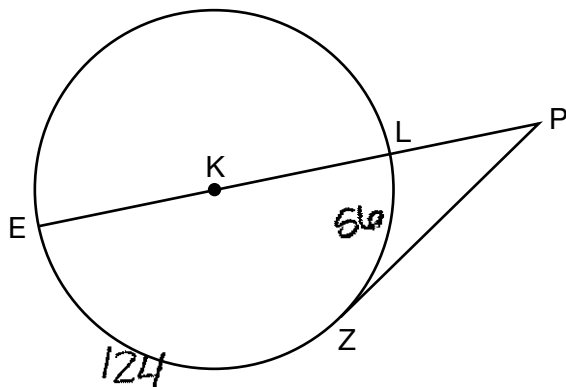


$\angle CAD$  is  $45^\circ$  b/c  $\triangle CAD$   
is a 45, 45, 90 triangle

**Score 0:** The student gave a completely incorrect response.

Question 30

30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



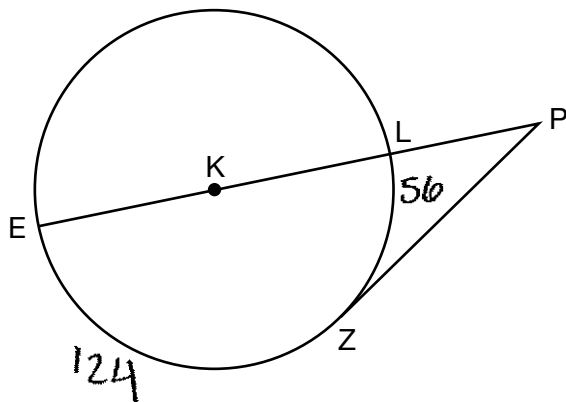
If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

$$m\angle P = \frac{1}{2} (\widehat{EZ} - \widehat{ZL})$$
$$m\angle P = \frac{1}{2} (124 - 56)$$
$$m\angle P = \frac{1}{2} (68)$$
$$m\angle P = 34$$

**Score 2:** The student gave a complete and correct response.

Question 30

30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

$$P = \frac{1}{2}(124 - 56)$$

$$P = \frac{1}{2}(68)$$

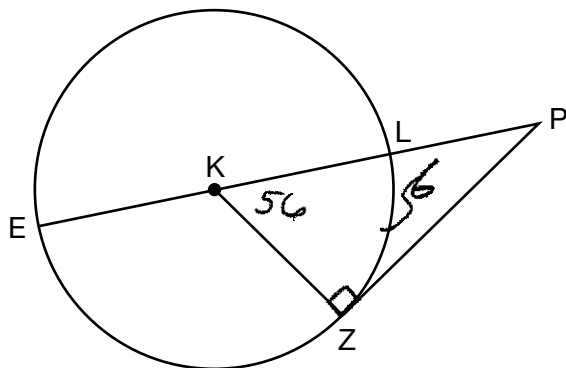
$$P = 34^\circ$$

$$m\angle P = 34^\circ$$

**Score 2:** The student gave a complete and correct response.

Question 30

30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

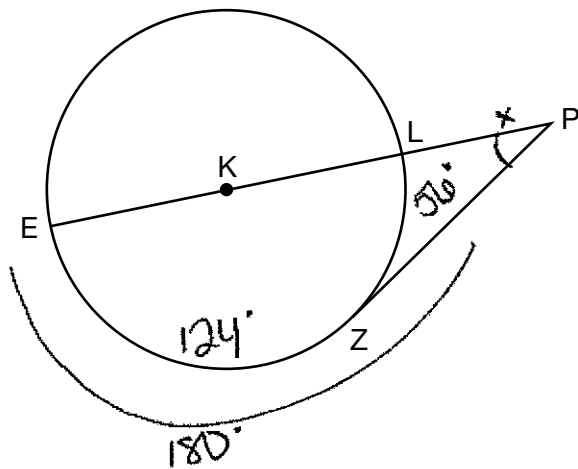
$$\begin{array}{r} 90 \\ - 56 \\ \hline 34 \end{array}$$

$$m\angle P = 34$$

**Score 2:** The student gave a complete and correct response.

**Question 30**

**30** In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



$$\begin{array}{r} 180 \\ - 56 \\ \hline 124 \end{array}$$

If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

$$2 \sqrt{124} \quad \begin{array}{r} 62 \\ \hline \end{array}$$

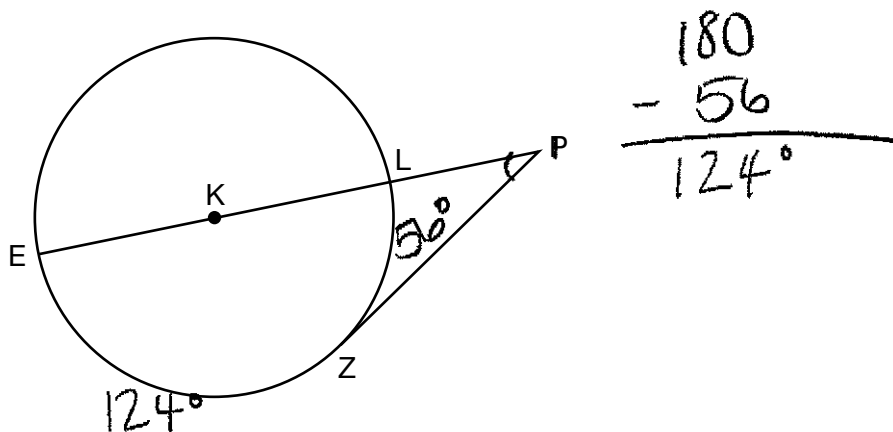
angle P has 62°

**Score 1:** The student made an error by thinking  $m\angle P = \frac{1}{2}(m\widehat{EZ})$ .



Question 30

30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

$$m\angle P = \text{major arc} - \text{minor arc}$$

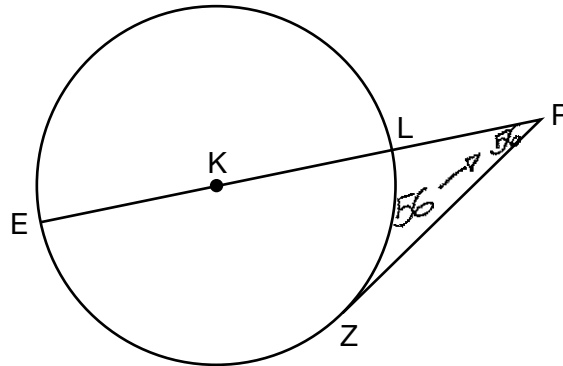
$$m\angle P = 124 - 56$$

$$m\angle P = 68^\circ$$

**Score 1:** The student made an error by thinking  $m\angle P = m\widehat{EZ} - m\widehat{LZ}$ .

Question 30

30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



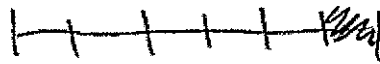
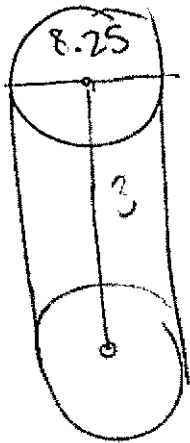
If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

measure of angle  $P \rightarrow 56^\circ$

**Score 0:** The student gave a completely incorrect response.

Question 31

31 A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.



↓  
 $\frac{1}{2}$  of basin

$$V = \pi r^2 h$$

$$V = \pi (4.125)^2 (2.5)$$

$$V = 42.5390625\pi$$

$$V = 134 \text{ ft}^3$$

**Score 2:** The student gave a complete and correct response.

**Question 31**

**31** A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.

Handwritten student work showing two diagrams of a cylinder and calculations for volume.

Diagram 1 (Left): A cylinder with a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. The formula  $V = \pi r^2 h$  is written above it. The calculation is  $V = \pi (4\frac{1}{8})^2 \cdot 3$ , resulting in  $V = 160.2368487989$ . The final answer is  $134 \text{ feet}^3$ .

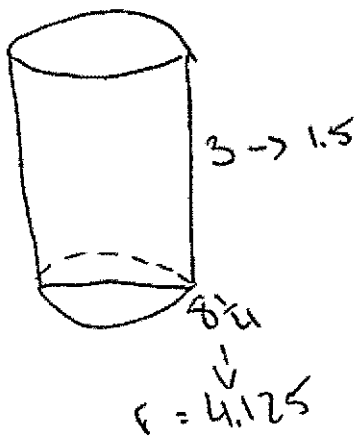
Diagram 2 (Right): A cylinder with a diameter of  $8\frac{1}{4}$  feet and a height of  $2\frac{1}{2}$  feet. The formula  $V = \pi r^2 h$  is written above it. The calculation is  $V = \pi (4\frac{1}{8})^2 \cdot 2.5$ , resulting in  $V = 26.72808124167$ .

The number 134 is written below the two diagrams, indicating the final answer.

**Score 2:** The student gave a complete and correct response.

Question 31

- 31 A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.

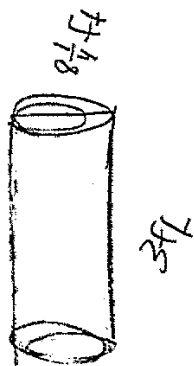


$$V_{\text{Cylinder}} = \pi r^2 h$$
$$V = \pi (4.125)^2 (1.5)$$
$$V \approx 80 \text{ ft}^3$$

**Score 1:** The student used a height of 1.5 feet to find the volume of the water in the basin.

Question 31

- 31 A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.



$$V = \pi r^2 h$$
$$\pi (4.125)^2 (3 \text{ ft})$$

$$V = 160.4 \text{ ft}^3$$

$$3 \div \frac{1}{2} = 6$$

$$6 \sqrt{160.4}$$

$$= 26.7 \text{ ft}^3$$

**Score 1:** The student found the volume of the whole cylinder and the volume of the empty space, but did not subtract to find the volume of water.

Question 31

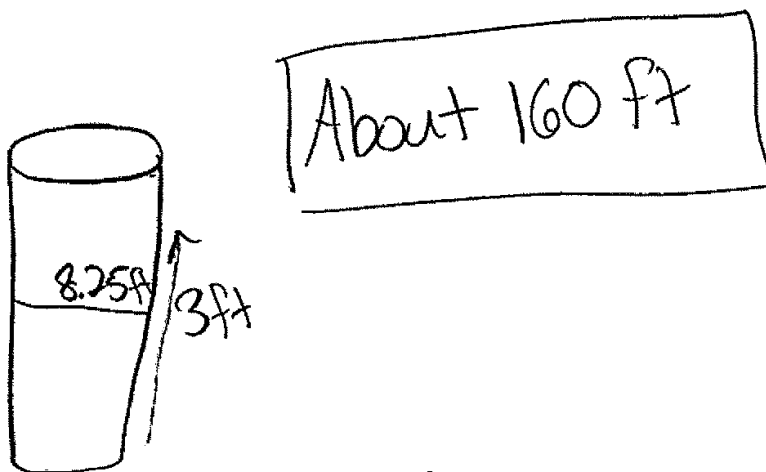
31 A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.

$$V = \pi r^3 h$$
$$V = \pi \left(\frac{33}{8}\right)^3 (2.5)$$
$$V = 551 \text{ ft}^3$$
$$d = 8\frac{1}{4}$$
$$r = \frac{33}{8}$$

**Score 1:** The student made an error by cubing the radius to find the volume of the water in the basin.

Question 31

- 31 A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.



Hand-drawn diagram of a cylinder. The diameter is labeled as 8.25 ft. The height is labeled as 3 ft. A box contains the answer "About 160 ft".

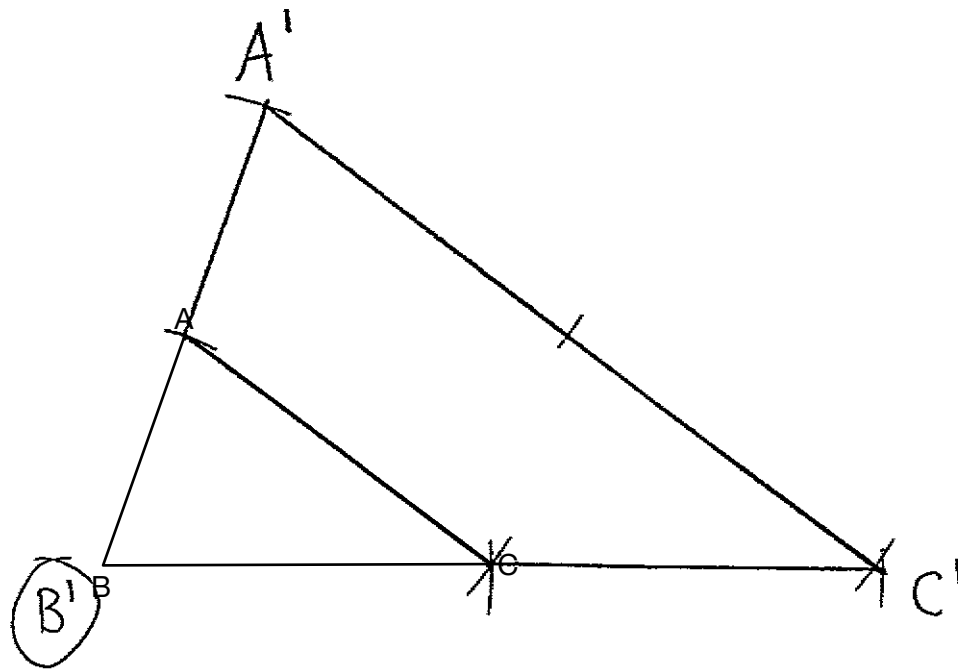
$$V = \pi r^2 h$$
$$V = \pi (4.125)^2 (3)$$
$$V = \pi (17.0156)(3)$$
$$V = 160 \text{ ft}$$

**Score 0:** The student did not show enough correct relevant work to receive any credit.



**Question 32**

**32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



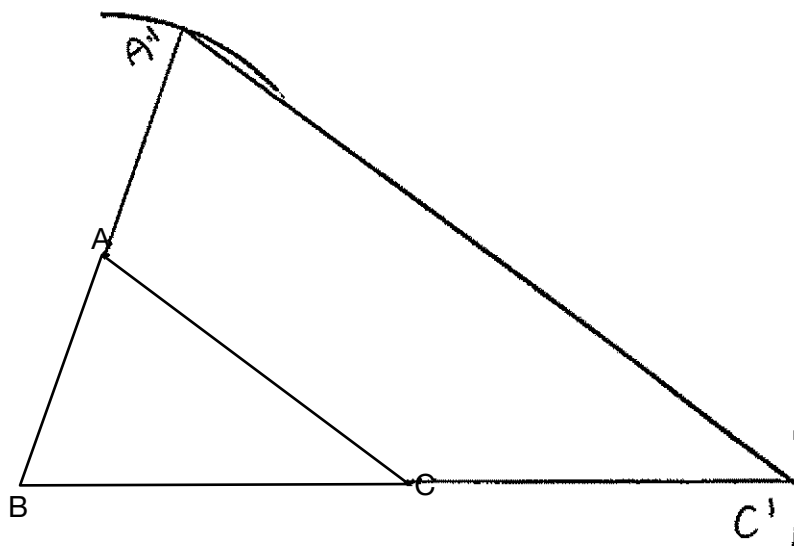
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

Yes,  $\triangle A'B'C'$  is similar to  $\triangle ABC$ !  
A dialation of 2 means that all sides  
are multiplied by 2!  
\*Similar  $\triangle$ 's have side lengths that  
are in proportion.\*

**Score 4:** The student gave a complete and correct response.

**Question 32**

- 32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



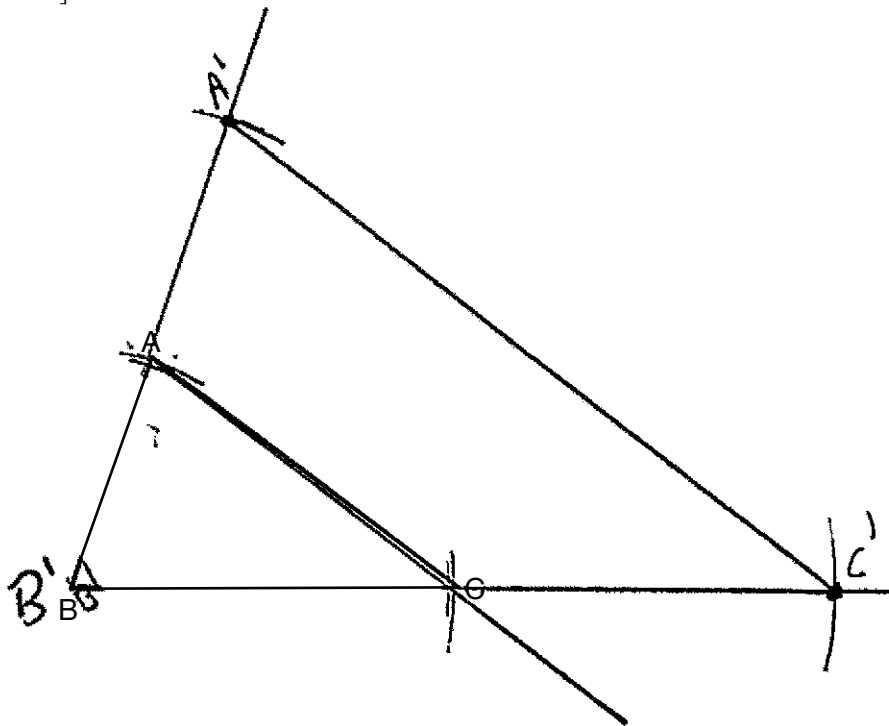
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

The image of  $\triangle ABC$  is similar to the triangle because since a dilation isn't a rigid motion, it preserves the angle measures but creates proportional side lengths. Similar triangles have congruent angles and proportional side lengths, meaning that a dilation creates an image similar to the original triangle.

**Score 4:** The student gave a complete and correct response.

Question 32

32 Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



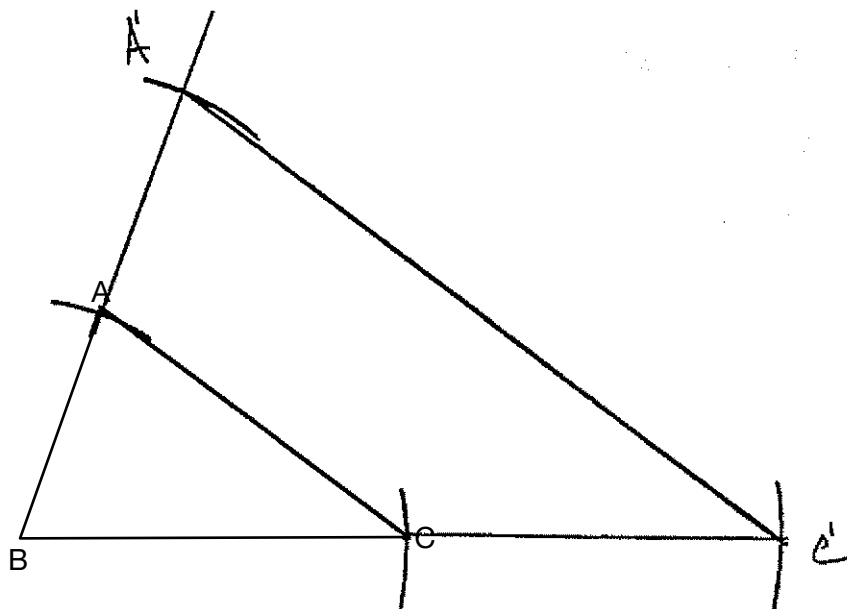
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

Yes because ~~this is~~ a dilation preserves  
angle measure, so this could be similar by the  
AA~thm

**Score 4:** The student gave a complete and correct response.

**Question 32**

**32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



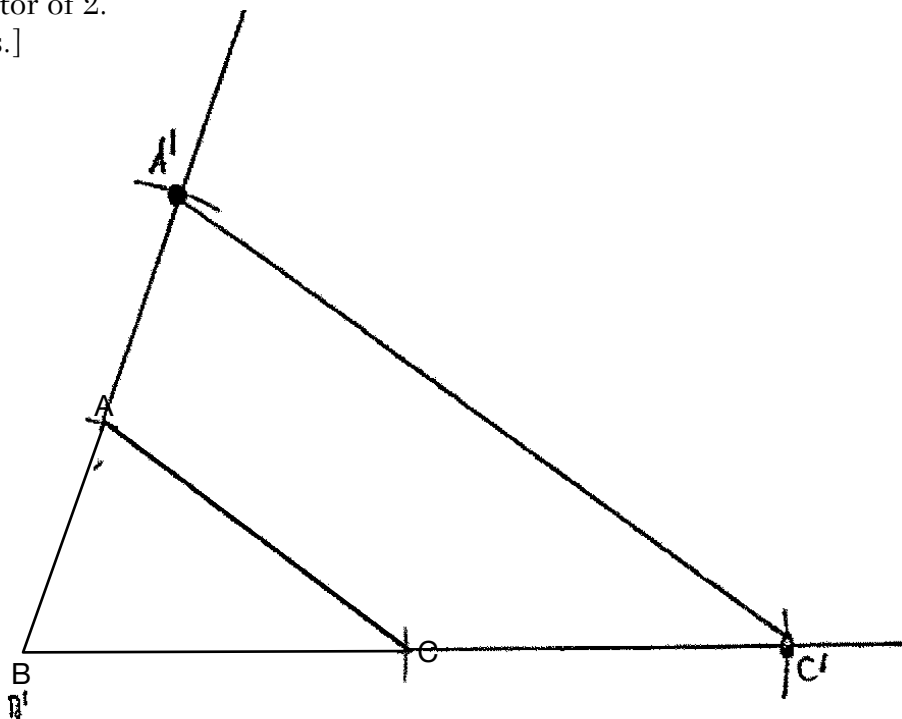
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

yes because a dialation is not isometric, it doesn't change sides. Also, the sides would be porportial.

**Score 3:** The student wrote a partially incorrect explanation by stating that the dilation doesn't change the sides.

**Question 32**

**32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]

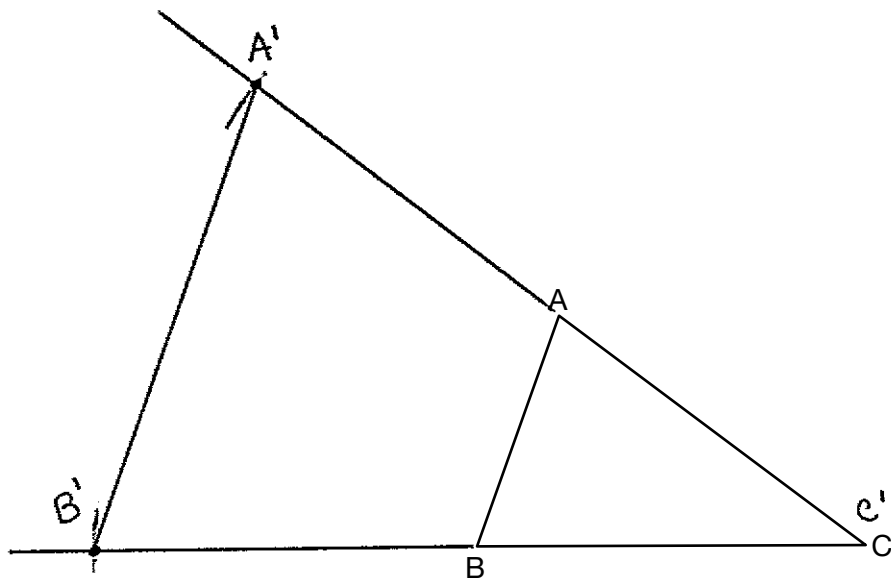


Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

**Score 2:** The student made a correct construction, but no further correct work was shown.

**Question 32**

**32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



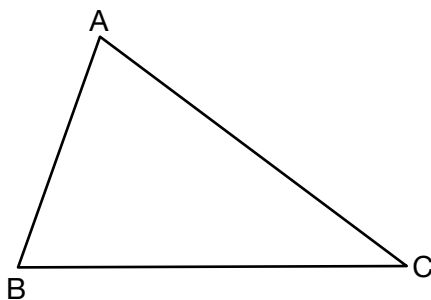
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

$\triangle ABC$  is similar to  $\triangle A'B'C'$  because dilations preserve shape.

**Score 1:** The student made an appropriate construction centered at a point other than  $B$ .  
The student wrote an incorrect explanation.

**Question 32**

- 32** Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



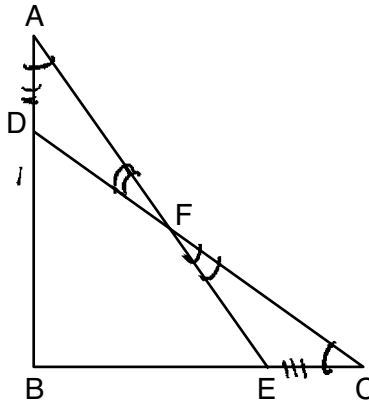
Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

No. Dilation is the only rigid motion that does not preserve size of a image. This transformation does not lead the triangle to be similar or remain the same as the original triangle.

**Score 0:** The student gave a completely incorrect response.

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



Prove:  $\triangle AFD \cong \triangle CFE$

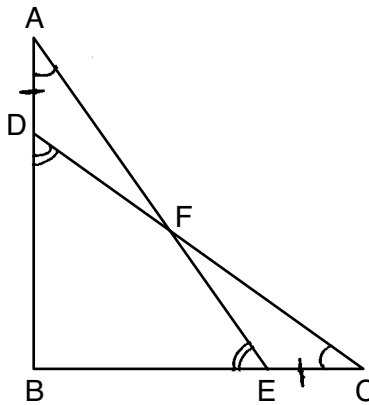
Statement	Reason
1. $\triangle ABE \cong \triangle CBD$	1. Given
2. $\angle A \cong \angle C$	2. CPCTC
3. $\angle AFD \cong \angle CFE$	3. Vertical $\angle$ s are $\cong$ ,
4. $\overline{AB} \cong \overline{CB}$ ; $\overline{DB} \cong \overline{EB}$	4. CPCTC
5. $\overline{AD} \cong \overline{CE}$	5. Subtraction (4)
6. $\triangle AFD \cong \triangle CFE$	6. AAS

**Score 4:** The student gave a complete and correct response.



Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



Prove:  $\triangle AFD \cong \triangle CFE$

①  $\triangle ABE \cong \triangle CBD$   
 Given

②  $\angle A \cong \angle C$   
 CPCTC

③  $\angle BDC \cong \angle BEA$   
 CPCTC

④  $\angle ADF; \angle BDC$   
 $\angle CEF; \angle BEA$  } Both pairs are linear pairs and supplement angles

⑤  $\angle ADF \cong \angle CEF$   
 supplements of  $\cong$  angles are  $\cong$

⑥  $\overline{AB} \cong \overline{CB}$   
 CPCTC

⑦  $\overline{BD} \cong \overline{BE}$   
 CPCTC

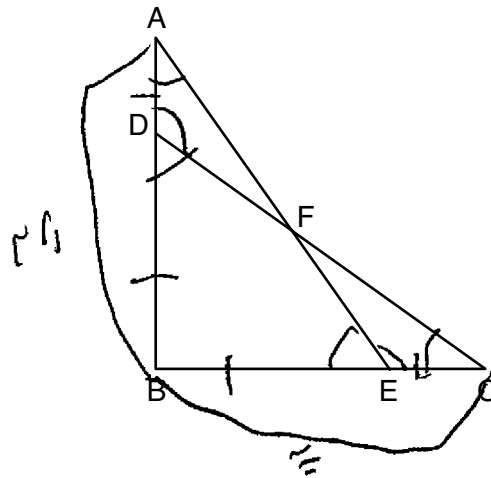
⑧  $\overline{AD} \cong \overline{CE}$   
 Subtraction

$\triangle AFD \cong \triangle CFE$   
 ASA

Score 4: The student gave a complete and correct response.

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



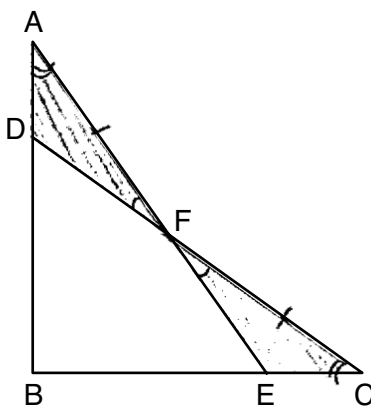
Prove:  $\triangle AFD \cong \triangle CFE$

Statement	Reason
1. $\triangle ABE \cong \triangle CBD$	1. Given
2. $\angle A \cong \angle C$	2. CPCTC
3. $\angle BDC \cong \angle BEA$	3. CPCTC
4. $\angle ADF \cong \angle CEF$	4. They are supplements of two $\cong$ angles so they must be $\cong$
5. $\overline{AB} \cong \overline{CB}$	5. CPCTC
6. $\overline{BD} \cong \overline{BE}$	6. CPCTC
7. $\overline{AD} \cong \overline{CE}$	7. subtraction
8. $\triangle AFD \cong \triangle CFE$	8. ASA postulate

**Score 3:** The student is missing a statement and reason to prove  $\angle ADF \cong \angle CEF$ .

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



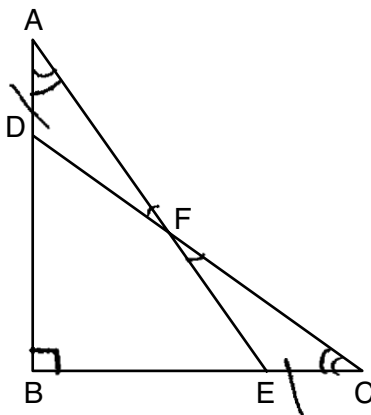
Prove:  $\triangle AFD \cong \triangle CFE$

- |  |                                    |
|--|------------------------------------|
| 1. $\triangle ABE \cong \triangle CBD$             | 1. Given                           |
| 2. $\angle A \cong \angle C$                       | 2. CPCTC                           |
| 3. $\angle AFD$ & $\angle CFE$ vertical $\angle$ s | 3. vertical $\angle$ definition    |
| 4. $\angle AFD \cong \angle CFE$                   | 4. vertical $\angle$ s are $\cong$ |
| 5. $\overline{AE} \cong \overline{CD}$             | 5. CPCTC                           |
| 6. $\overline{AF} \cong \overline{FC}$             | 6. segment subtraction             |
| 7. $\triangle AFD \cong \triangle CFE$             | 7. ASA (2, 4, 6)                   |

**Score 2:** The student wrote two correct statements and reasons, but two statements and/or reasons are missing or incorrect.

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



Prove:  $\triangle AFD \cong \triangle CFE$

Statements:

1.  $\triangle ABE \cong \triangle CBD$
2.  $\angle B \cong \angle B$
3.  $\angle AFD \cong \angle CFE$
4.  $\angle A \cong \angle C$
5.  $BA - DB \cong BC - BE$
6.  $\triangle AFD \cong \triangle CFE$

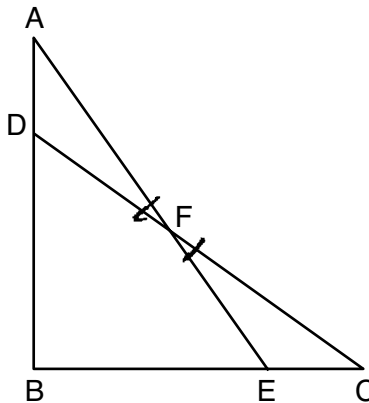
Reasons:

1. Given
2. Reflexive Property.
3. Vertical angles are congruent
4. CPCTC
5. Subtraction postulate
6. AAS

**Score 2:** The student wrote two correct statements and reasons, but is missing two statements and reasons to prove  $\overline{AD} \cong \overline{CE}$ .

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



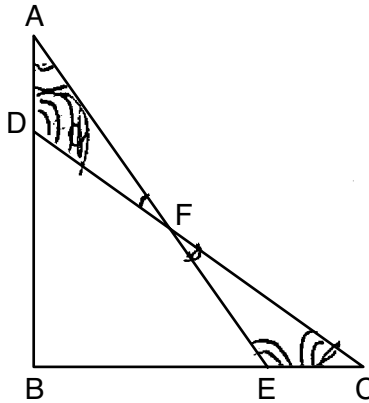
Prove:  $\triangle AFD \cong \triangle CFE$

Statement	Reason
1.) $\triangle ABE \cong \triangle CBD$	1.) Given.
2.) $\angle AFD \cong \angle CFE$	2.) Vertical $\angle$ s are $\cong$ .
3.) $\overline{DE} \cong \overline{EA}$	3.) OPCTC
4.) $\overline{DC} - \overline{FC} = \overline{DF}$ $\overline{AE} - \overline{FE} = \overline{AF}$ $\overline{DC} = \overline{DE} = \overline{FC}$ $\overline{AE} - \overline{AF} = \overline{FE}$	4.) Subtraction property
5.) $\overline{AF} \cong \overline{FC}$ $\overline{EF} \cong \overline{DF}$	5.) Substitution
6.) $\triangle AFD \cong \triangle CFE$	6.) SAS.

**Score 1:** The student had one correct relevant statement and reason in step 2.

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



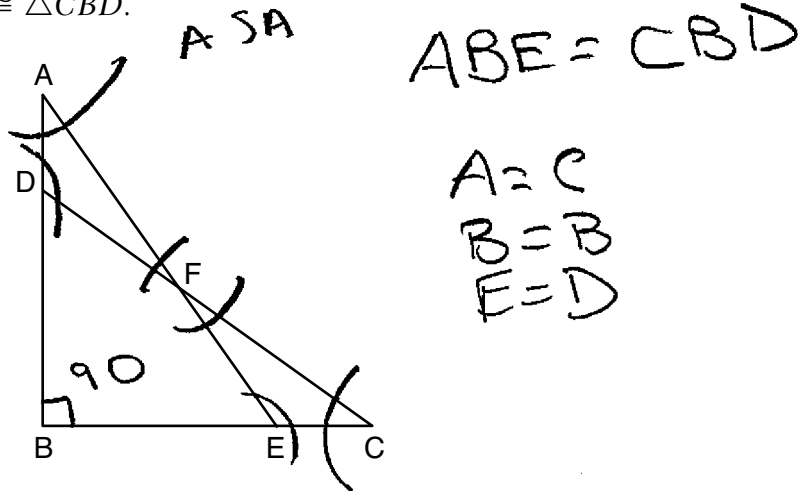
Prove:  $\triangle AFD \cong \triangle CFE$

$\angle AFD \cong \angle CFE$  / vertical  $\angle$ 's  
 $\angle FAD \cong \angle FEC$  / opposite interior  $\angle$ 's  
 $\angle ADF \cong \angle FCE$  / opposite interior  $\angle$ 's  
 $\triangle AFD \cong \triangle CFE$  / SSS

**Score 0:** The student did not show enough correct relevant work to receive any credit.

Question 33

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



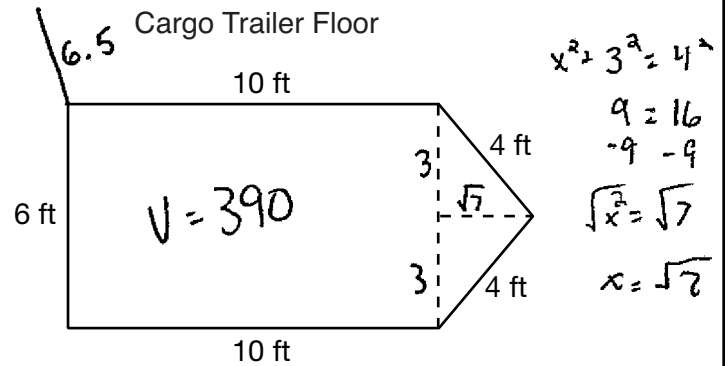
Prove:  $\triangle AFD \cong \triangle CFE$

Statement	reason
$\triangle ABE \cong \triangle CBD$	Given
$B \cong B$	reflexive property. its its own $\Delta = 90^\circ$ right triangle
$A \cong C$ $B \cong B$ $E \cong D$	supplementary $\angle$ 's $\triangle ABE \cong \triangle CBD$
$F \cong F$	reflexive property
$\overline{AF} \cong \overline{CF}$ $\overline{DF} \cong \overline{EF}$	F is intersection point for all line segments
$\triangle AFD \cong \triangle CFE$	ASA

**Score 0:** The student did not show enough correct relevant work to receive any credit.

**Question 34**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the *nearest cubic foot*?

441.5921

442 ft<sup>3</sup>

$10 \times 6 \times 6.5 = 390$

$2 \left( \frac{1}{2} (3)(\sqrt{7})(6.5) \right)$

51.59215057

**Score 4:** The student gave a complete and correct response.



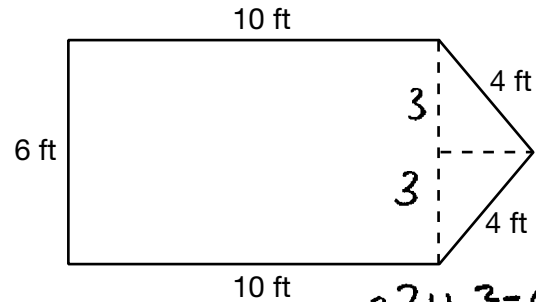
**Question 34**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



Cargo Trailer Floor



$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 x^2 + 3^2 &= 4^2 \\
 \sqrt{x^2} &= \sqrt{7} \\
 x &= 2.645751311
 \end{aligned}$$

If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$\begin{aligned}
 V_{\text{rect}} &= lwh \\
 11 &= 10 \cdot 6 \cdot 6.5 \\
 &= 390
 \end{aligned}$$

$$\begin{aligned}
 V_{\text{tri}} &= Bh \\
 &= \left(\frac{1}{2}bh\right)(h) \\
 &= \left(\frac{1}{2}(6 \cdot 2.645751311)\right)(6.5) \\
 &= 51.59215056
 \end{aligned}$$

$$\begin{aligned}
 V + V &= 441.5921506 \\
 &= 442 \text{ ft}^3
 \end{aligned}$$

**Score 4:** The student gave a complete and correct response.

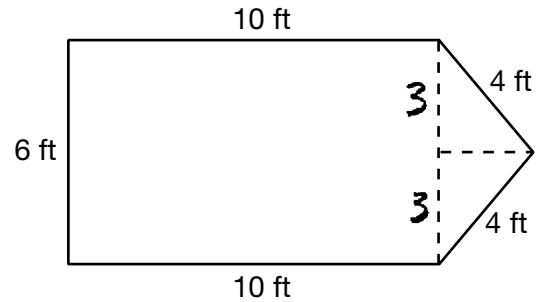
**Question 34**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$A = b \cdot h$$

$$A = 6 \cdot 10$$

$$A = 60$$

$$\begin{array}{r} 67.95 \\ \times 6.50 \\ \hline \end{array}$$

$$A = \frac{1}{2} b \cdot h$$

$$A = \frac{1}{2} 6 \cdot 2.65$$

$$A = 7.95$$

$$a^2 + 3^2 = 4^2$$

$$\begin{array}{r} a^2 + 9 = 16 \\ -9 \quad -9 \end{array}$$

$$\sqrt{a^2}$$

$$a = 2.65$$

$$V = 442 \text{ ft}^3$$

**Score 4:** The student gave a complete and correct response.

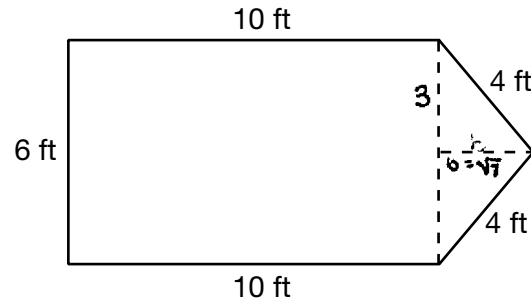
**Question 34**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

rectangular prism  $V = Bh$   $V = 6 \times 6.5 \times 10 = 390 \text{ ft}^3$

triangular prism  $V = Bh$   $V = \frac{1}{2} \cdot 6 \times \sqrt{7} \times 6.5 = 8.598691761 + 390 = 398.5986918$

$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 4^2$$

$$9 + b^2 = 16$$

$$-9 \quad -9$$

$$b^2 = 7$$

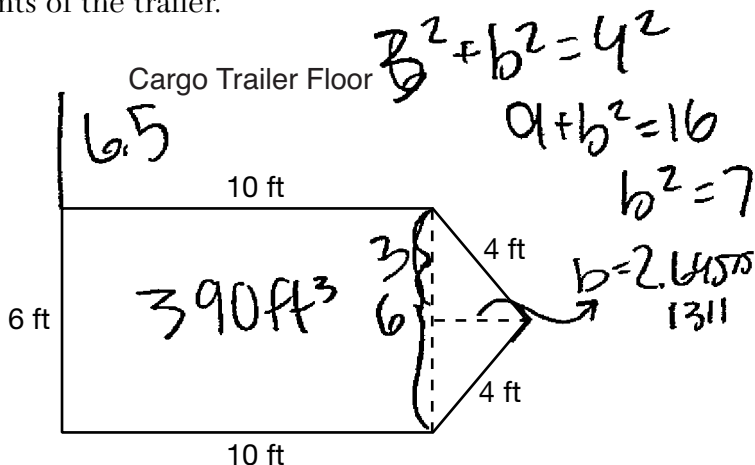
$$b = \sqrt{7}$$

Volume of the trailer:  
399

**Score 3:** The student made an error when determining the volume of the triangular prism.

**Question 34**

**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

Triangle =  $V = \frac{1}{2} \cdot b \cdot h$   
 $V = \frac{1}{2} \cdot 3 \cdot 2.645751311$   
 $V = 3.968626967 \times 2 = 7.937253933$

Rectangle =  $V = Bh$   
 $V = 60 \cdot 6.5$   
 $V = 390$

Volume of trailer =  
398 ft<sup>3</sup>

**Score 3:** The student did not multiply by the height of the trailer to determine the volume of the triangular prism.

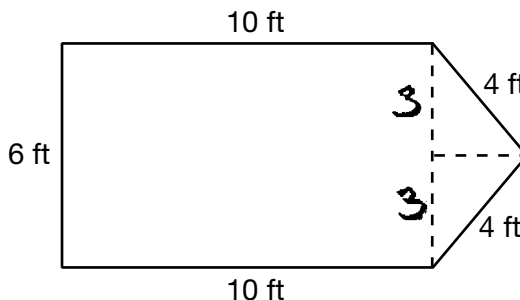
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**34** A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



Cargo Trailer Floor



$$\begin{aligned}
 3^2 + x^2 &= 4^2 \\
 9 + x^2 &= 16 \\
 -9 & \quad -9 \\
 \hline
 x^2 &= 7 \\
 \sqrt{x^2} &= \sqrt{7} \\
 x &\approx 2.6458
 \end{aligned}$$

If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$V_{\Delta} = \frac{1}{3} (6)(10)(2.6458)$$

$$V_{\Delta} = 52.916$$

$$V_{\square} = 6(10)(6.5)$$

$$V_{\square} = 390$$

$$\begin{array}{r}
 390 \\
 + 52.916 \\
 \hline
 442.916
 \end{array}$$

443

**Score 2:** The student found the height of the triangular base and the volume of the rectangular prism, but no further correct work was shown.

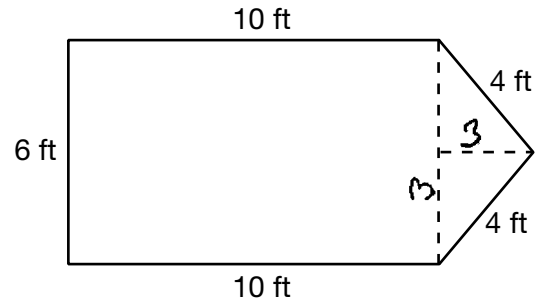
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Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$10 \times 6 \times 6.5 = 390$$

$$\frac{3 \times 3 \times 6.5}{2} = 29.25$$

$$419 \text{ ft}^3$$

$$390 + 29.25$$

$$\checkmark$$

$$419.25$$

$$419$$

**Score 2:** The student used an incorrect height of the base of the triangular prism. The student also incorrectly calculated by finding the volume of only half of the triangular prism.

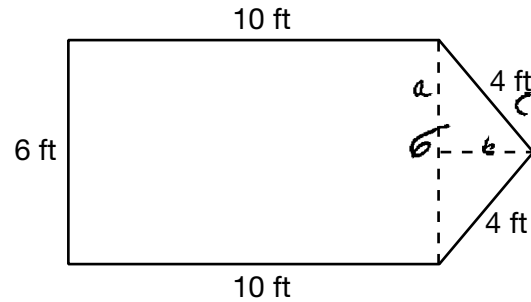
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Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$V_{RP} = Ah$$

$$V_{RP} = (10)(10)(6)$$

$$V_{RP} = 600 \text{ cu ft}$$

$$V_{RP} = (10)(10)(6.5)$$

$$V_{RP} = 650$$

$$V_{TP} = \frac{1}{2}bh$$

$$V_{TP} = \frac{1}{2}(6)(5.7)$$

$$V_{TP} = 7.937$$

$$650 + 7.937 =$$

$$\boxed{657.937 \text{ cu ft}}$$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 - 9 = 16$$

$$\sqrt{16} = 4$$

$$b = 5.7$$

**Score 1:** The student found the height of the triangular base, but no further correct work was shown.

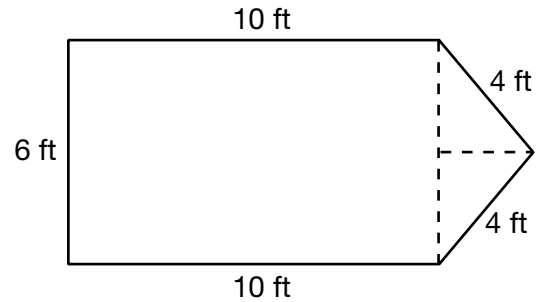
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Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$V = Bh$$

$$V = 6 \cdot 10 \cdot 6.5$$

$$V = 390$$

$$V = \frac{1}{2} Bh$$

$$V = 8 \cdot 6.5$$

$$V = 52$$

Volume is 442 ft.

**Score 1:** The student found the volume of the rectangular prism, but no further correct work was shown.



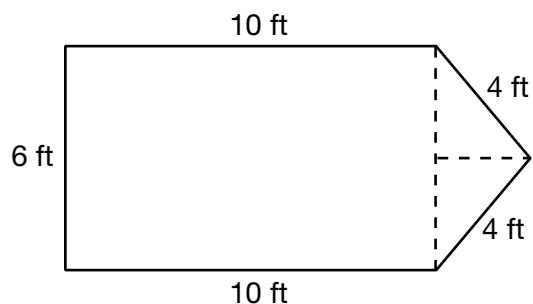
### Question 34

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Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$V = bh$$

$$V = 10(6.5)$$
$$V = 65$$

$$V = 4(6.5)$$
$$V = 26$$

$$65 + 26 = 91$$

The total volume is  $91 \text{ ft}^3$ .

**Score 0:** The student gave a completely incorrect response.

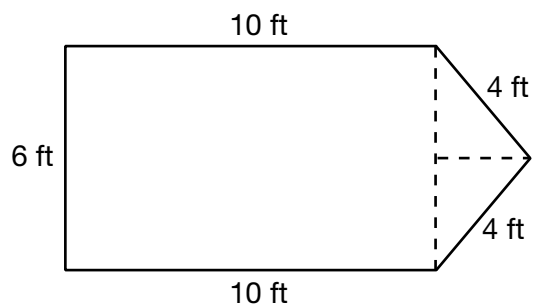
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Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the nearest cubic foot?

$$B = 6 + 10 + 10 + 4 + 4$$
$$B = 34$$

$$V = Bh$$

$$V = 34(6.5)$$

$$V = 221$$

The volume of the trailer is  
221 ft<sup>3</sup>

**Score 0:** The student gave a completely incorrect response.

### Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$AB = \sqrt{(-5-1)^2 + (3-2)^2} = \sqrt{37}$$

$$BC = \sqrt{(-5--6)^2 + (3--3)^2} = \sqrt{37}$$

$\triangle ABC$  is isosceles b/c  $AB=BC$

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$(0, -4)$

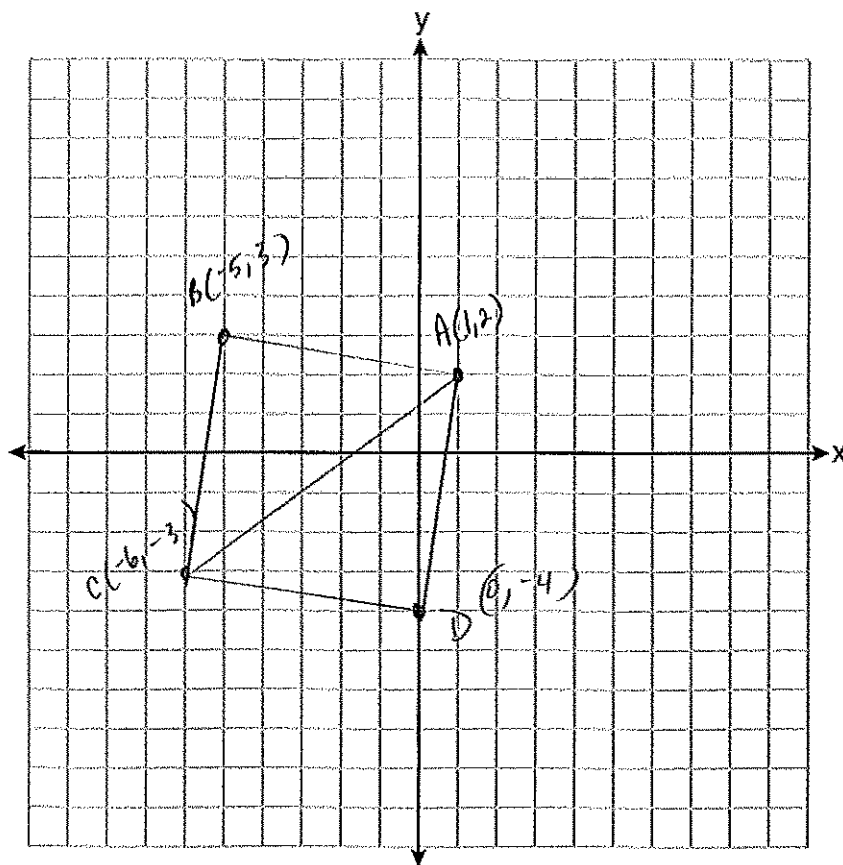
**Score 6:** The student gave a complete and correct response.

### Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

$$AD = \sqrt{1^2 + (2 - -4)^2} = \sqrt{37}$$
$$CD = \sqrt{(-6 - 0)^2 + (-3 - -4)^2} = \sqrt{37}$$
$$\text{slope of } AB = \frac{3 - 2}{-5 - 1} = \frac{-1}{6}$$
$$CB = \frac{3 - -3}{-5 - -6} = 6$$

$ABCD$  is a square b/c all 4 sides are  $\cong$   
and consecutive sides are  $\perp$  since their slopes  
are opp. signed reciprocals, so  $\angle B$  is a right  $\angle$ .



### Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} \text{distance from } A \text{ to } B &= \sqrt{(-5-1)^2 + (3-2)^2} = \sqrt{36+1} = \sqrt{37} \\ \text{distance from } B \text{ to } C &= \sqrt{(-6-5)^2 + (-3-3)^2} = \sqrt{1+36} = \sqrt{37} \end{aligned}$$

Since there are two congruent sides,  $\triangle ABC$  is isosceles.

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$$D = (0, -4)$$

**Score 5:** The student did not write the concluding statement when proving the square.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
 [The use of the set of axes below is optional.]

$$\overline{AB} = \sqrt{37}$$

$$\overline{BC} = \sqrt{37}$$

$$\overline{CD} = \sqrt{37}$$

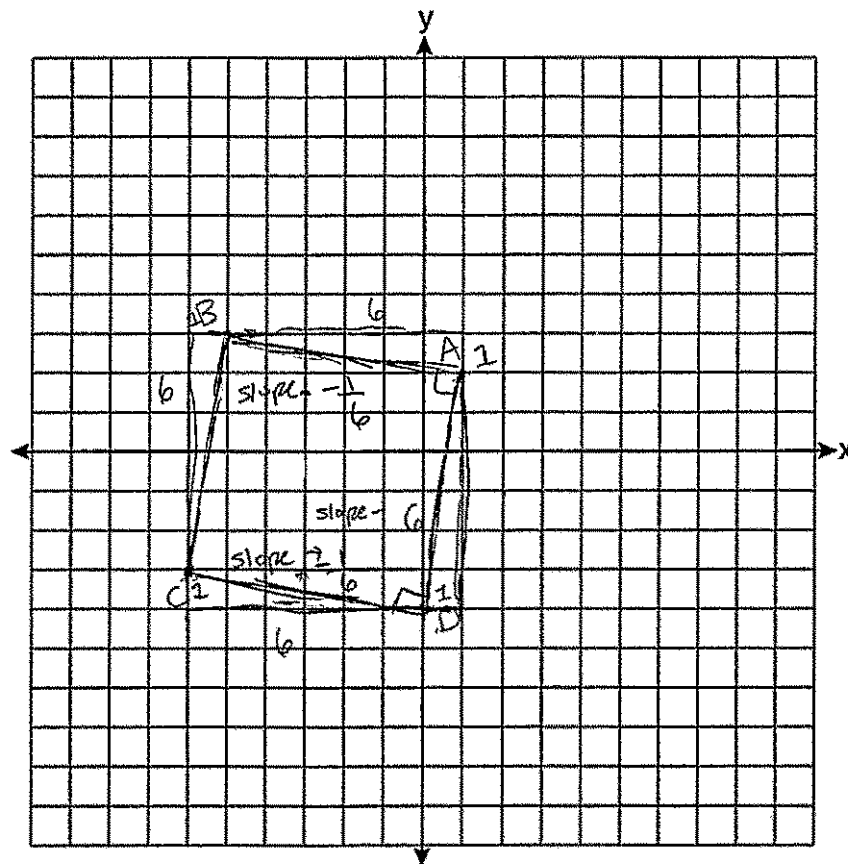
$$\overline{DA} = \sqrt{37}$$

$$\angle BAD = 90^\circ$$

$$\angle ADC = 90^\circ$$

(slopes are negative reciprocals, making them perpendicular)

$-\frac{1}{6}, 6$   
 $6, -\frac{1}{6}$  } slopes are negative reciprocals



### Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} AB - d &= \sqrt{(-5-1)^2 + (3-2)^2} \\ d &= \sqrt{36+1} \\ d &= \sqrt{37} \end{aligned}$$

$\overline{AB}$  and  $\overline{BC}$  are  
congruent so  $\triangle ABC$   
is isosceles.

$$\begin{aligned} BC - d &= \sqrt{(-6+5)^2 + (-3-3)^2} \\ d &= \sqrt{1+36} \\ d &= \sqrt{37} \end{aligned}$$

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$$D(0, -4)$$

**Score 4:** The student made a conceptual error by stating that all sides with the same length is a square.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

$$AB \sim d = \sqrt{37}$$

$$BC \sim d = \sqrt{37}$$

$$CD \sim d = \sqrt{(-6-0)^2 + (-3+4)^2}$$

$$d = \sqrt{36+1}$$

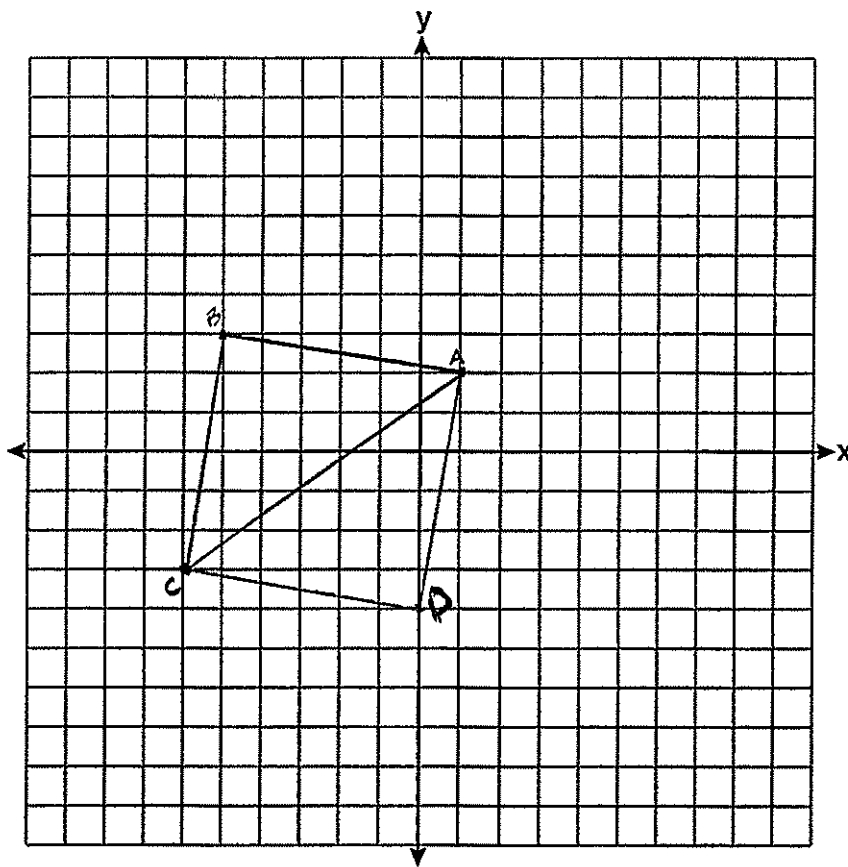
$$d = \sqrt{37}$$

$$DA \sim d = \sqrt{(0-1)^2 + (-4-2)^2}$$

$$d = \sqrt{1+36}$$

$$d = \sqrt{37}$$

All sides have the same length therefore it is a square.





**Question 35**

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$AB = \sqrt{(1 - (-5))^2 + (2 - 3)^2} = \sqrt{(6)^2 + (-1)^2} = \sqrt{36 + 1} = \sqrt{37}$$

$$BC = \sqrt{(-5 - (-6))^2 + (3 - (-3))^2} = \sqrt{(1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$CA = \sqrt{(-6 - 1)^2 + (-3 - 2)^2} = \sqrt{(-7)^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$$

$\therefore \triangle ABC$  is isosceles b/c  
two sides are congruent

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$(-1, -4)$

**Score 3:** The student proved  $\triangle ABC$  is isosceles. The student graphed points  $C$  and  $D$  incorrectly, which resulted in a quadrilateral that is not a square. The student made an appropriate concluding statement based on their work, but the level of difficulty was significantly reduced.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

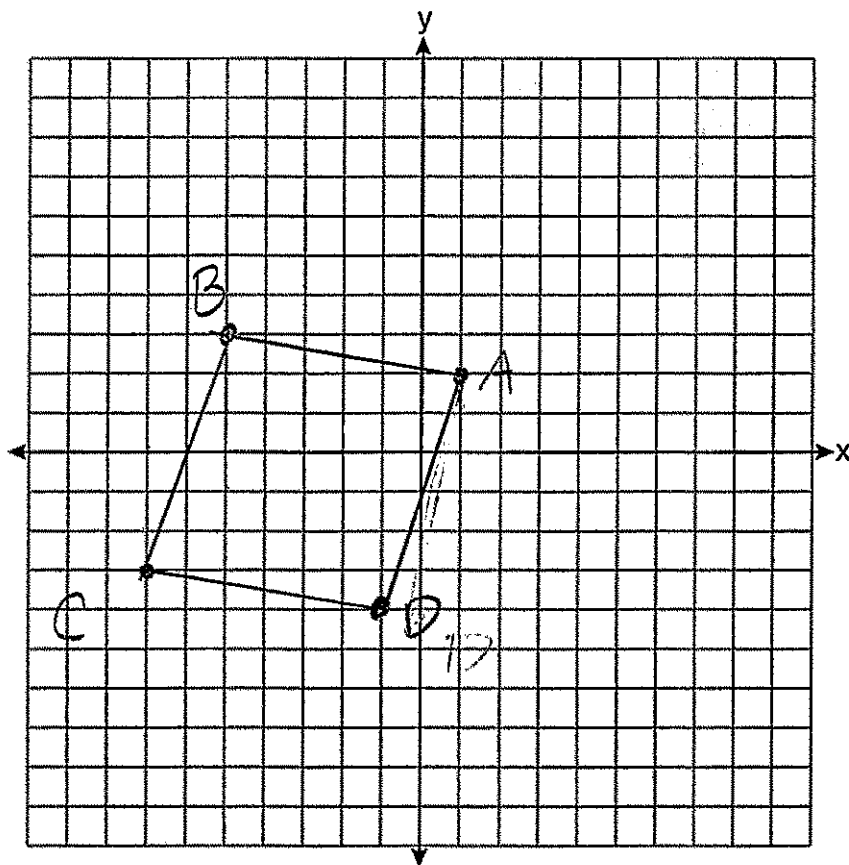
$$\overline{AB} =$$

$$\overline{BC} =$$

$$\overline{CD} = \sqrt{(-6 - (-1))^2 + (-3 - (-4))^2} = \sqrt{(-5)^2 + (1)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$\overline{DA} = \sqrt{(-1 - 1)^2 + (-4 - 2)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4 + 36} = \sqrt{40}$$

$\therefore ABCD$  is not a square  
b/c all sides are not congruent



Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$d = \sqrt{\Delta X^2 + \Delta Y^2}$$

$$\overline{AB} \quad d = \sqrt{(1+5)^2 + (2-3)^2} = \sqrt{36+1} = \sqrt{37} \approx$$

$$\overline{BC} \quad d = \sqrt{(-5+6)^2 + (3+3)^2} = \sqrt{1+36} = \sqrt{37} \approx$$

$$\overline{CA} \quad d = \sqrt{(-6-1)^2 + (-3-2)^2} = \sqrt{49+25} = \sqrt{74}$$

$\triangle ABC$  is isosceles because it has 2  $\cong$  sides and 1 non- $\cong$  side

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

**Score 3:** The student proved  $\triangle ABC$  is isosceles, but one computational error was made. The coordinates of point  $D$  were not stated. In proving  $ABCD$  is a square, the student was missing the length of  $\overline{AD}$ .

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
 [The use of the set of axes below is optional.]

$$\overline{CD} \quad d = \sqrt{(-6+1)^2 + (-5+4)^2} = \sqrt{25+2} = \sqrt{27}$$

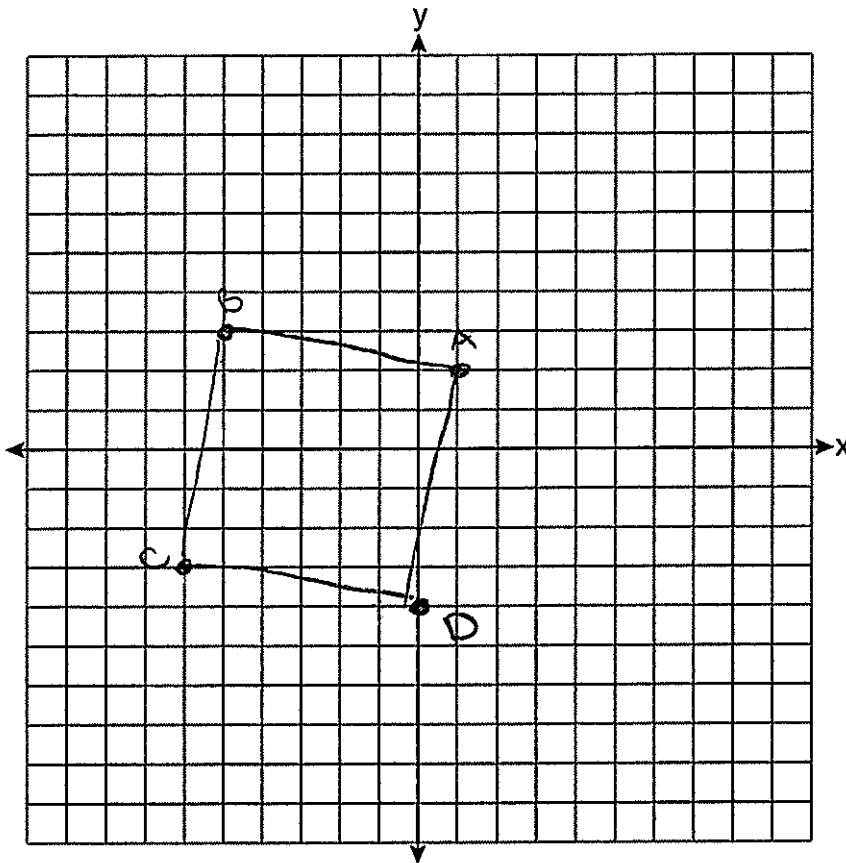
$$d = \sqrt{(-6-0)^2 + (-3+4)^2} = \sqrt{36+2} = \sqrt{38} \quad \checkmark$$

$$\sqrt{38} = \sqrt{38} = \sqrt{38}$$

$$\overline{CD} \cong \overline{CB} \cong \overline{BA} \cong \overline{AD}$$

$\overline{CB} \quad \frac{\Delta y}{\Delta x} = \frac{-3+4}{-6-0} = \frac{1}{-6}$   
 $\overline{AD} \quad \frac{\Delta y}{\Delta x} = \frac{2+4}{1-0} = \frac{6}{1}$

all sides are equal  
 and have rt  $\angle$ 's  
 $\perp$  slopes  
 $\perp$  lines form rt  $\angle$ 's



**Question 35**

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$d_{AB} = \sqrt{(-5-1)^2 + (3-2)^2} = \sqrt{(-6)^2 + (1)^2} = \sqrt{36+1} = \sqrt{37}$$

$$d_{BC} = \sqrt{(-6-5)^2 + (-3-3)^2} = \sqrt{(-11)^2 + (-6)^2} = \sqrt{121+36} = \sqrt{157}$$

$\overline{AB} \neq \overline{BC}$  therefore  $\triangle ABC$   
is ~~isosc.~~

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

?

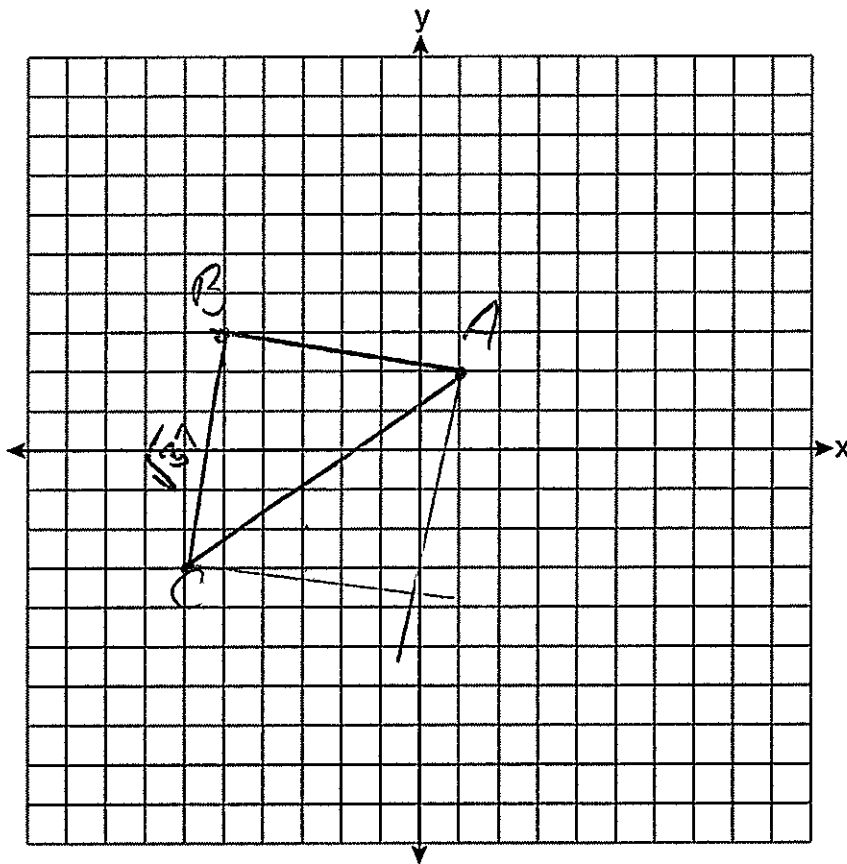
**Score 2:** The student proved  $\triangle ABC$  is isosceles, but no further correct work was shown.

**Question 35 continued**

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

?

$$\sqrt{37} = \sqrt{(x-1)^2 + (y-2)^2}$$



Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$\begin{array}{l} A(1,2) \quad \frac{2-3}{1-(-5)} = \frac{-1}{6} \\ B(-5,3) \end{array} \quad -1^2 + 6^2 = \sqrt{37}$$

$$\begin{array}{l} B(-5,3) \quad \frac{3-(-3)}{-5-(-6)} = \frac{6}{1} \\ C(-6,-3) \end{array} \quad 6^2 + 1^2 = \sqrt{37}$$

$$\begin{array}{l} C(-6,-3) \quad \frac{-3-2}{-6-1} = \frac{-5}{-7} \\ A(1,2) \end{array} \quad -5^2 + -7^2 = \sqrt{74}$$

$\therefore \triangle ABC$  is isosceles  
because 2 sides are  
equal, they have  
similar slopes

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$$D(0, -4)$$

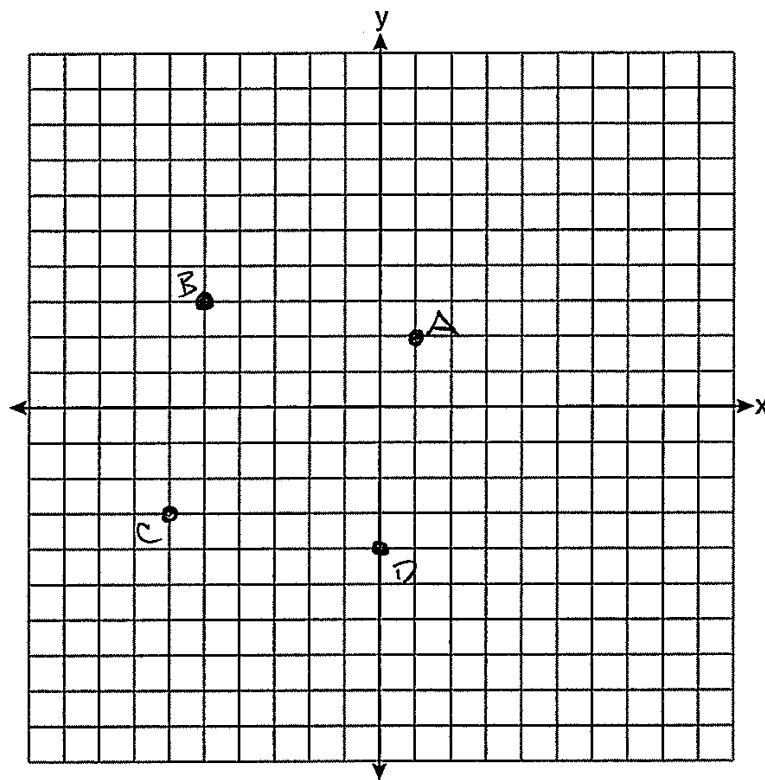
**Score 2:** The student showed the appropriate work to prove  $\triangle ABC$  is isosceles, but the concluding statement included an incorrect phrase. The student stated the coordinates of point  $D$ . The student did not show enough additional correct work to earn more credit.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

$$\begin{array}{l} C(-6,-3) \quad \frac{-3+4}{-6-0} \quad \frac{1}{-6} \\ D(0,4) \\ CD(-D(0,4) \quad \frac{4-2}{0-1} \quad \frac{2}{-1} \\ DA \quad A(1,2) \end{array}$$

$\therefore$  Quad  $ABCD$  is  
a square because  
all the slopes are equal.





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**Question 35**

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**35** The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

Triangle  $ABC$  is isosceles because the length of  $\overline{BA}$  is equal to the length of  $\overline{BC}$

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$(0, -4)$

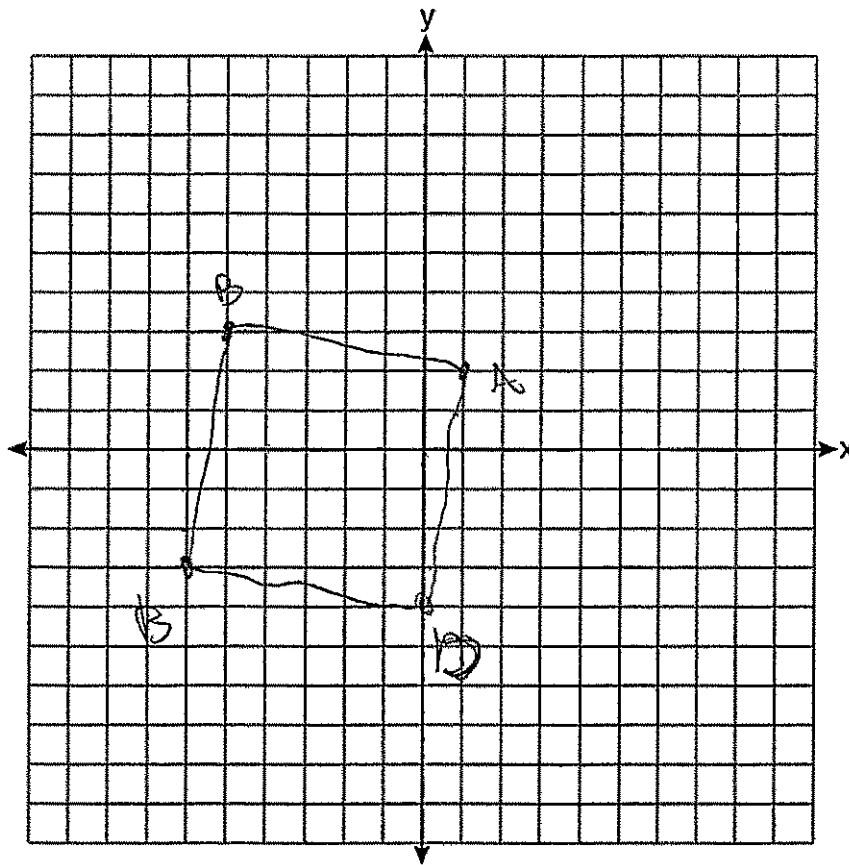
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**Score 1:** The student stated the coordinates of point  $D$ , but no further correct work was shown.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

quadrilateral  $ABCD$  is a square because the  
distance from  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ ,  $\overline{DA}$  are the same  
and all angle meet to form  $90^\circ$  angles



### Question 35

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

isosceles triangles  
Slope of  $\overline{AB} = \frac{6}{-1} = -\frac{1}{6}$  have to have a  $90^\circ$   
Slope of  $\overline{BC} = \frac{6}{1} = \frac{1}{6}$  angle which is what  
 $\angle B$  is  
↑  
Negative reciprocals  
Same slope  
Slope is  $\frac{3}{4}$

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

point  $D = (0, -4)$

**Score 1:** The student stated the coordinates of point  $D$ , but did not show enough correct work to earn additional credit.

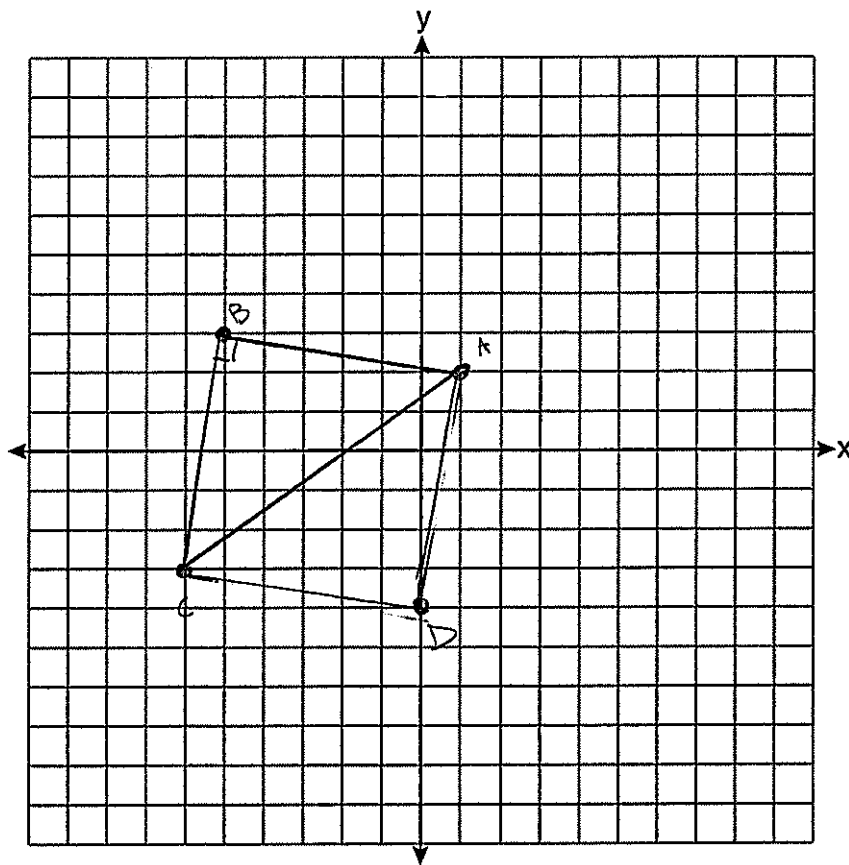
Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

4  $90^\circ$  angles

same slope  
 $\frac{6}{1} + \frac{-1}{6}$

↑  
negative reciprocals



**Question 35**

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$D = \sqrt{(y_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$A(1,2) \quad B(-5,3) \quad C(-6,-3)$$
$$\begin{matrix} x_1 & y_1 & x_2 & y_2 \end{matrix}$$
$$BC = \sqrt{(-6 - (-5))^2 + (-3 - 3)^2}$$

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$$D(-2, -)$$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

**Question 35 continued**

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

