The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, August 17, 2022 — 12:30 to 3:30 p.m.

MODEL RESPONSE SET

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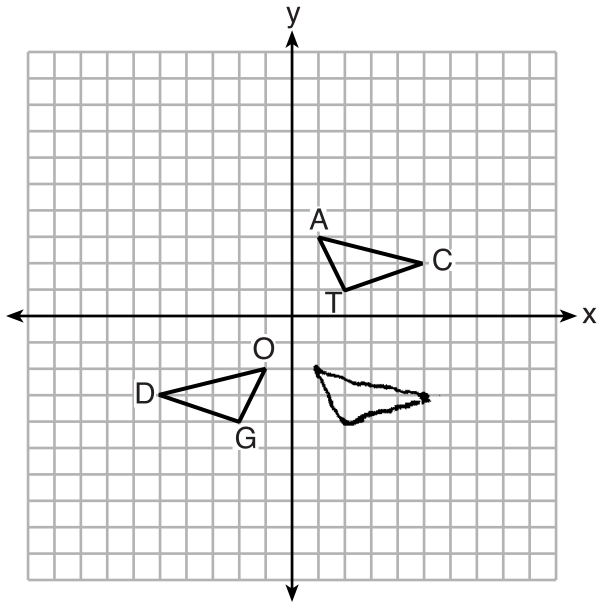
Updated 08/19/22 to correct the graphics
on pages 59 and 62.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

Reflection over the y-axis
and a
translation of 0,5.

$D(-5,-3) \xrightarrow{r_{y\text{axis}}} \xrightarrow{T_{0,5}} C(5,2)$

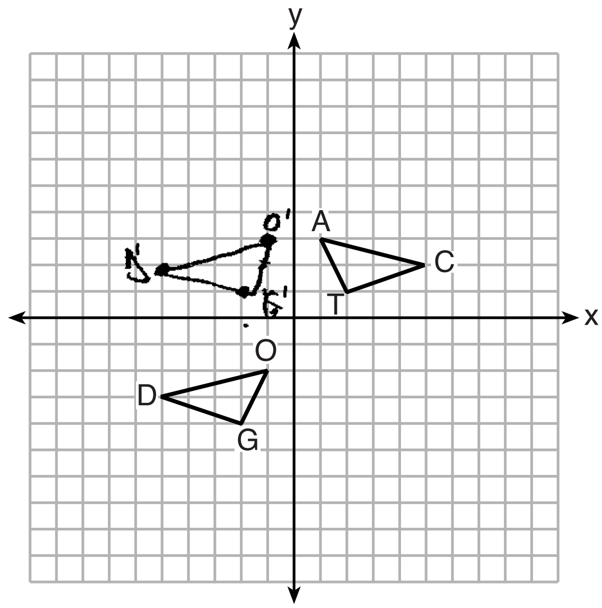
$O(-1,-2) \xrightarrow{r_{y\text{axis}}} \xrightarrow{T_{0,5}} A(1,3)$

$G(-2,-4) \xrightarrow{r_{y\text{axis}}} \xrightarrow{T_{0,5}} T(2,1)$

Score 2: The student gave a complete and correct response.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



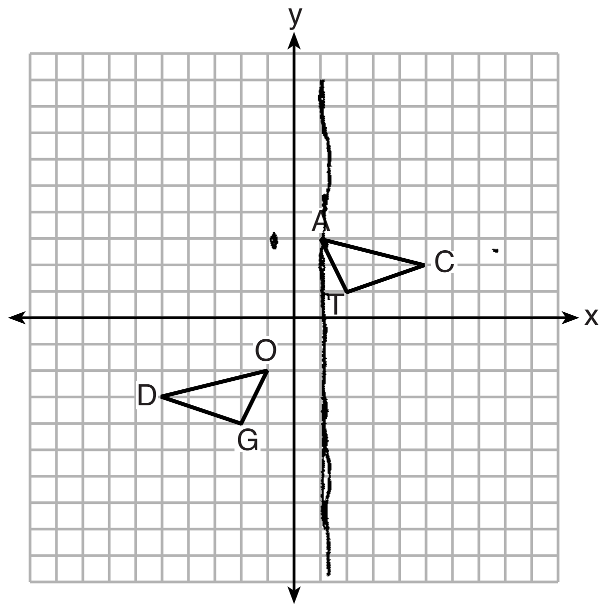
Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

1. Translate $\triangle$ DOG 5 units up
2. Reflection over y-axis

Score 2: The student gave a complete and correct response.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

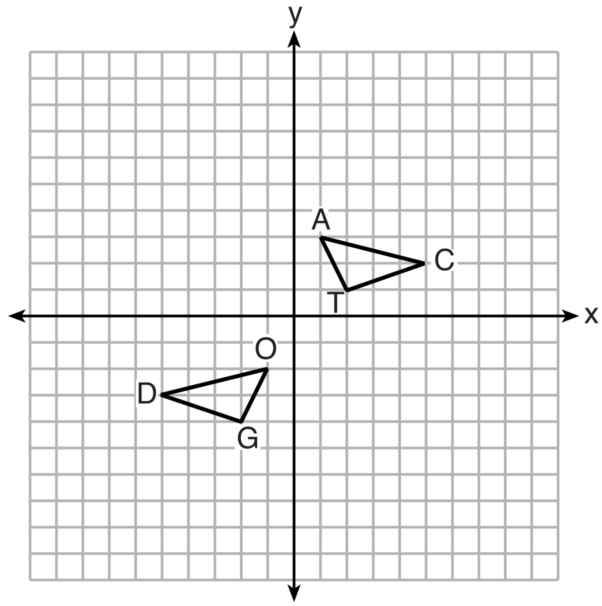
– Translate $\triangle DOG$ up 5 and right 1

– reflect $\triangle DOG$ over the line $x = 1$

Score 1: The student translated up 5 and right 1 instead of up 5 and right 2.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



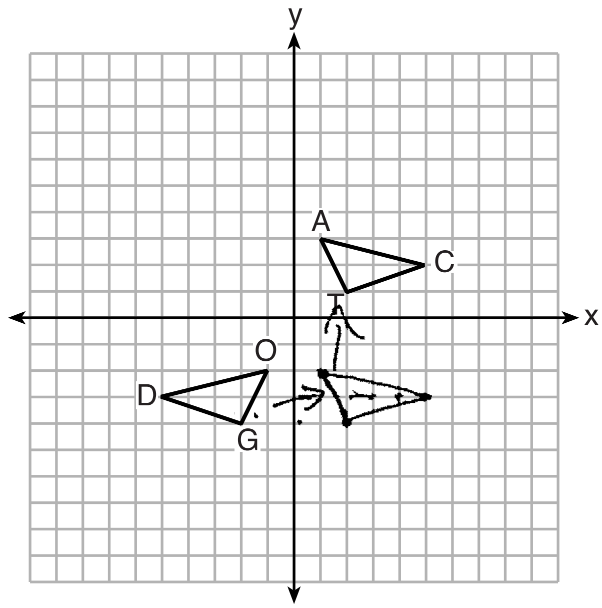
Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

reflection and translation

Score 1: The student identified an appropriate sequence of transformations, but did not describe the specific sequence of transformations.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



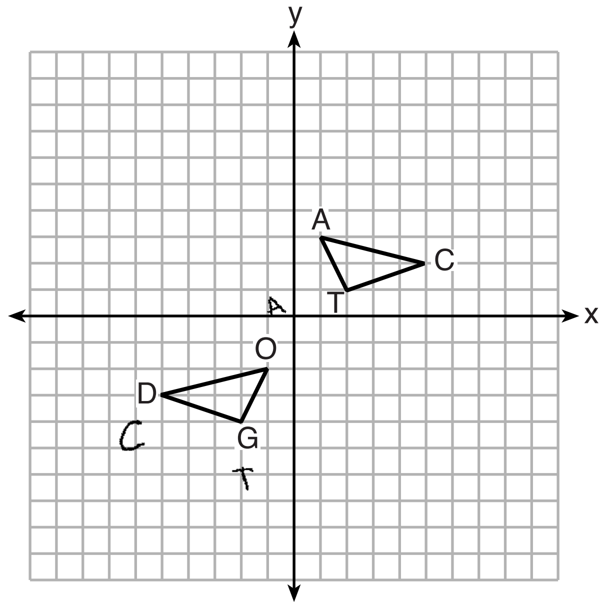
Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

a reflection of $\triangle$DOG over the y-axis, then a translation up 3 to map onto $\triangle$CAT.

Score 1: The student gave a partially correct response by stating a correct line of reflection, but the translation was not stated correctly.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

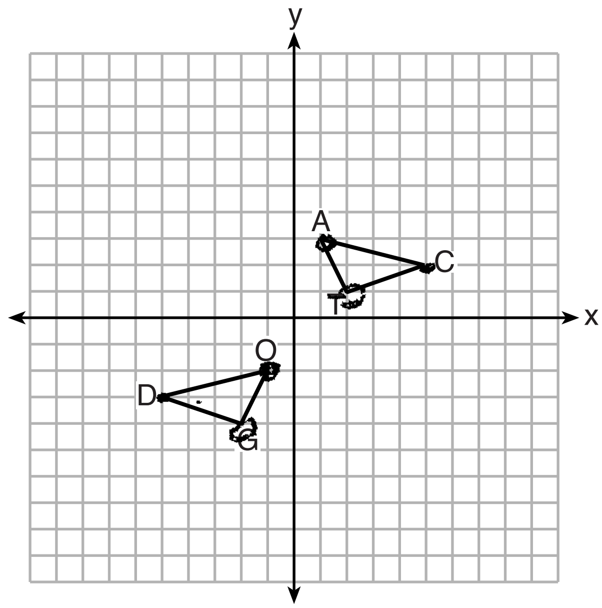
Step ① Reflection over y axis for $\triangle CAT$
Step ② Transformation over x axis for $\triangle CAT$

Now, C maps over D
A maps over O
T maps over G

Score 0: The student incorrectly mapped $\triangle CAT$ onto $\triangle DOG$, and incorrectly described the second transformation.

Question 25

25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.

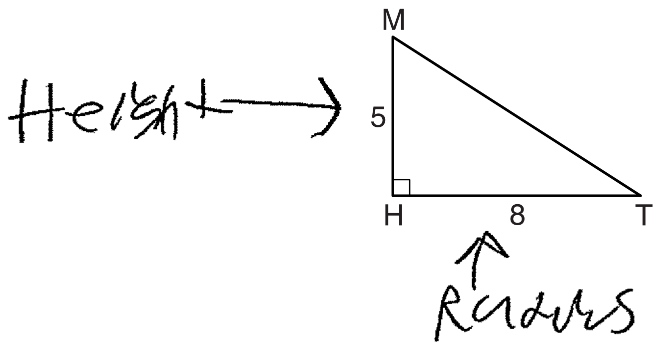


Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

$\triangle DOG$ is rotated 180° around the origin.

Score 0: The student gave a completely incorrect response.

26 In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$.



Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

Cone Volume formula $= V = \frac{1}{3}\pi R^2 H$

$V = \frac{1}{3}\pi 8^2 \cdot 5$

$V = \frac{1}{3} 64\pi \cdot 5$

$\frac{1}{3} 320\pi$

106.66π

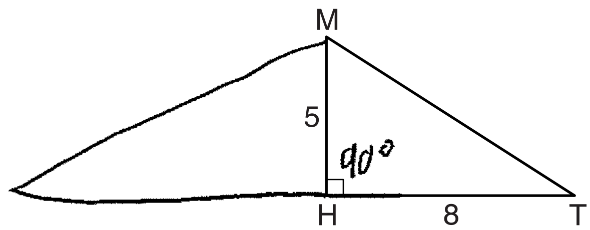
335.08

$\boxed{V = 335.1}$

Score 2: The student gave a complete and correct response.

Question 26

26 In right triangle *MTH* shown below, m$\angle H = 90°$, $HT = 8$, and $HM = 5$.



Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

$\cancel{A = \frac{bh}{2}}$

$\cancel{A = \frac{1}{2}(8 \cdot 5)}$

$\cancel{A = \frac{1}{2}C}$

$V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi(4)^2 \cdot 5$

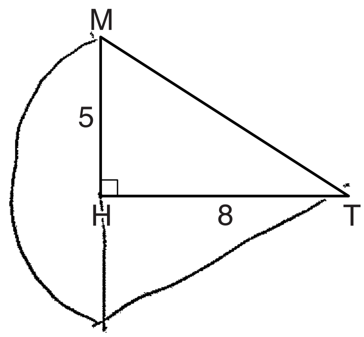
$V = \frac{1}{3}\pi(80)$

$\boxed{V = 83.8}$

Score 1: The student used the incorrect radius, $r = 4$, but found an appropriate volume.

Question 26

26 In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$.



$\frac{1}{3}(\pi r^2 h)$

Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

$\frac{\pi r^2 h}{3} = V$

$\frac{\pi (5)^2 8}{3}$

$\frac{\pi (25) 8}{3}$

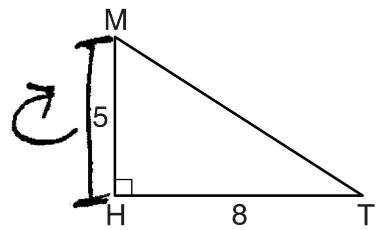
$\frac{200}{3} \pi$

$\boxed{209.4 \approx V}$

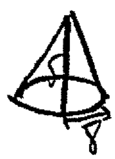
Score 1: The student rotated the triangle around the wrong leg, but found an appropriate volume.

Question 26

26 In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$.



Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi 8^2(5)$

$V = 21.\overline{33}(\pi)(5)$

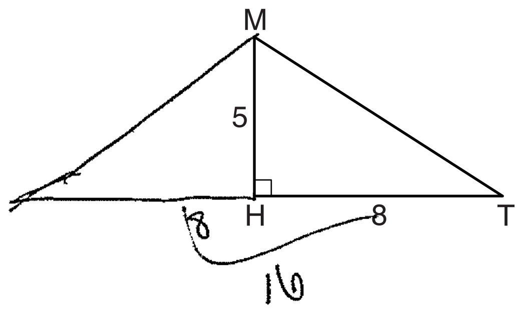
$V = 67.23(5)$

$\boxed{V = 336.2}$

Score 1: The student made a computational error when multiplying $21.\overline{33}(\pi)$.

Question 26

26 In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$.



Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

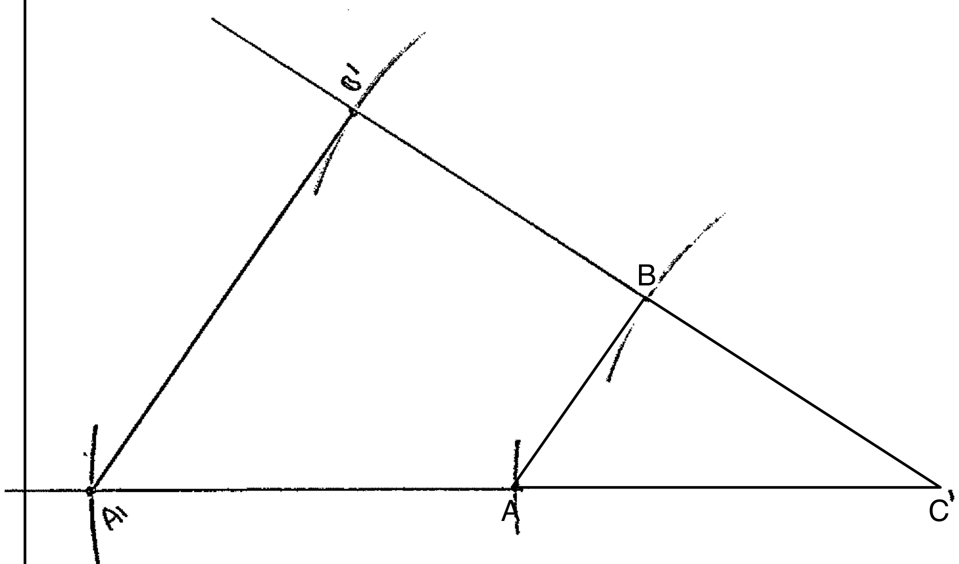
$V = 1/3(16)(5)$

$V = 26.67$

Score 0: The student gave a completely incorrect response.

Question 27

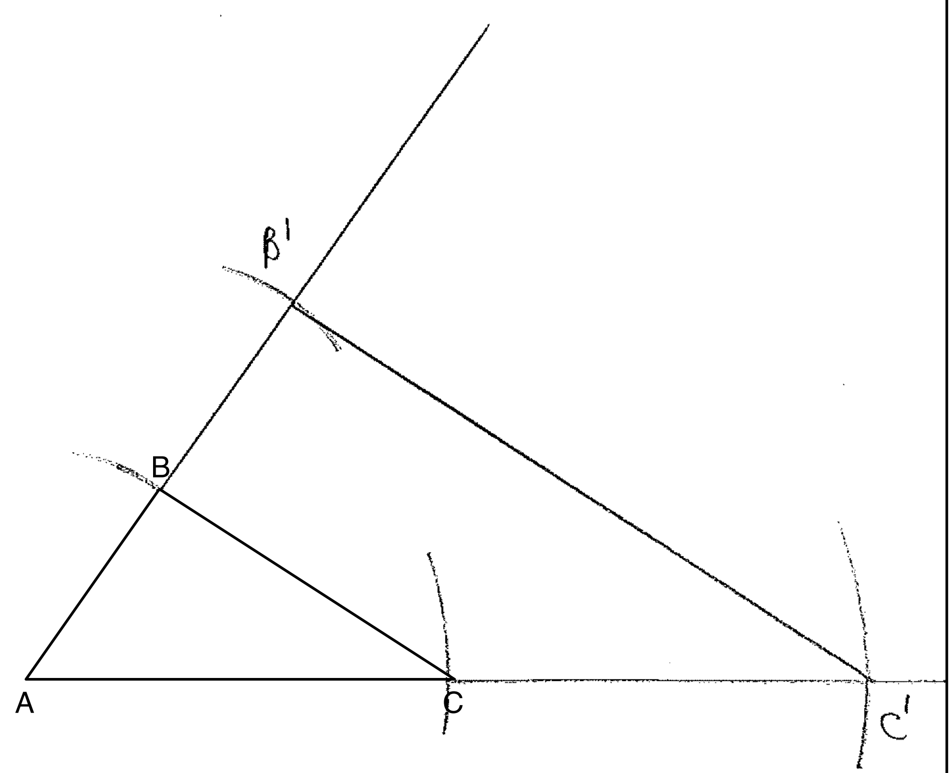
27 Using a compass and straightedge, dilate triangle ABC by a scale factor of 2 centered at C. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 27

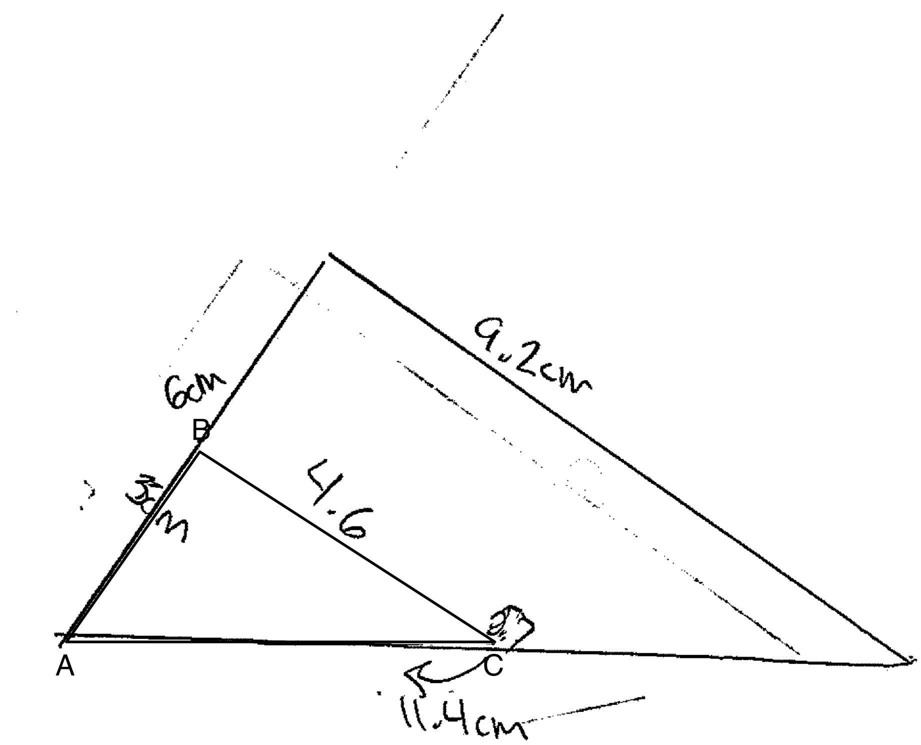
27 Using a compass and straightedge, dilate triangle *ABC* by a scale factor of 2 centered at *C*. [Leave all construction marks.]



Score 1: The student made an appropriate construction, but used vertex *A* as the center of dilation.

Question 27

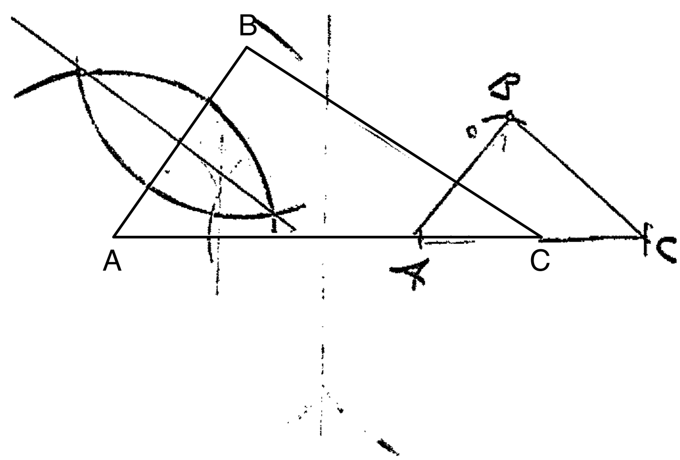
27 Using a compass and straightedge, dilate triangle ABC by a scale factor of 2 centered at C. [Leave all construction marks.]



Score 0: The student gave a completely incorrect response.

Question 27

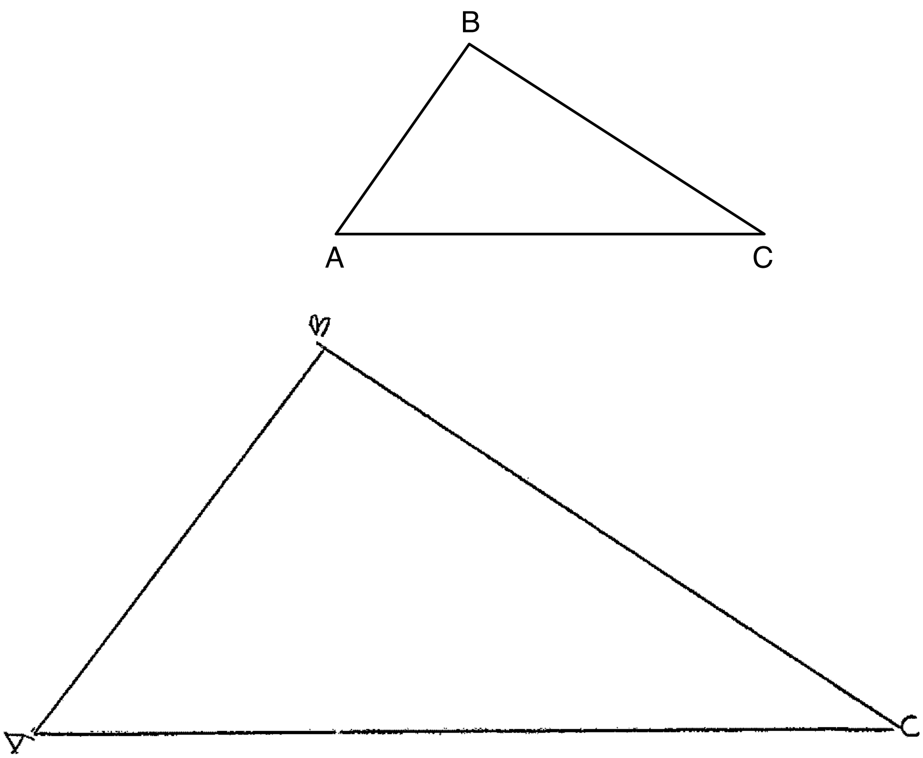
27 Using a compass and straightedge, dilate triangle *ABC* by a scale factor of 2 centered at *C*. [Leave all construction marks.]



Score 0: The student gave a completely incorrect response.

Question 27

27 Using a compass and straightedge, dilate triangle *ABC* by a scale factor of 2 centered at *C*. [Leave all construction marks.]

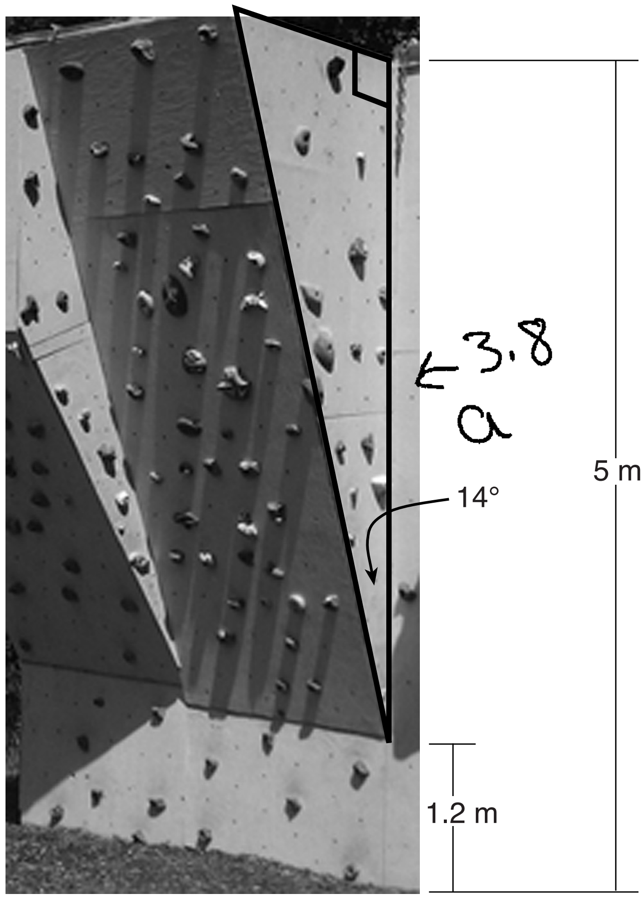


Score 0: The student gave a completely incorrect response.

Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

O   son cah toa



Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

$\frac{\cos 14}{1} = \frac{3.8}{x} = \boxed{x = 3.92 \text{ m.}}$

$x \cos 14 = 1(3.8)$

Score 2: The student gave a complete and correct response.

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

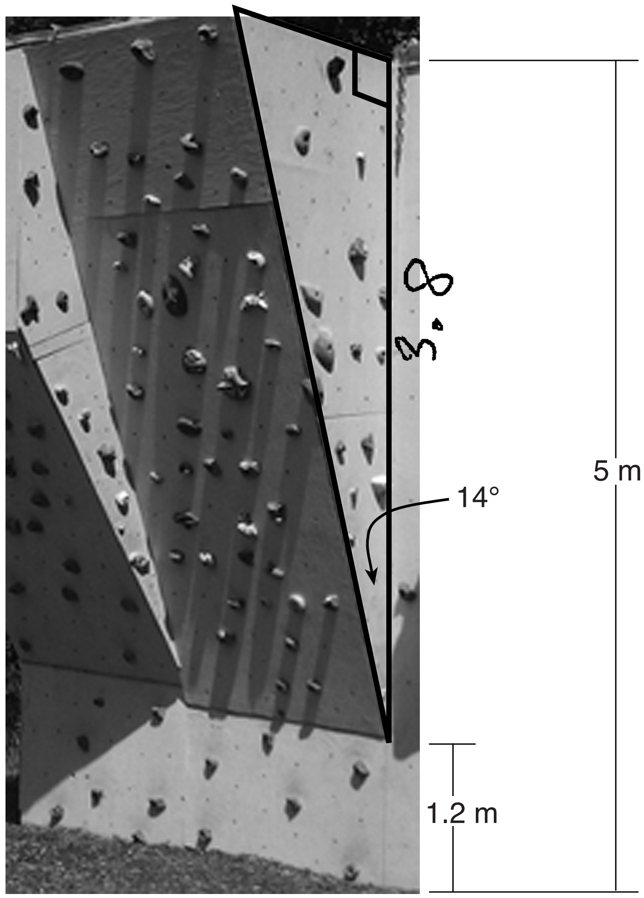
Let:
Hypotenuse = x

$\frac{\cos(14)}{1} = \frac{3.8}{x}$

$\frac{.9702957263x}{.9702957263} = \frac{3.8}{.9702957263}$

$x = 3.916331791$

$x \approx 3.92$



Length
$5 - 1.2 = 3.8$

Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

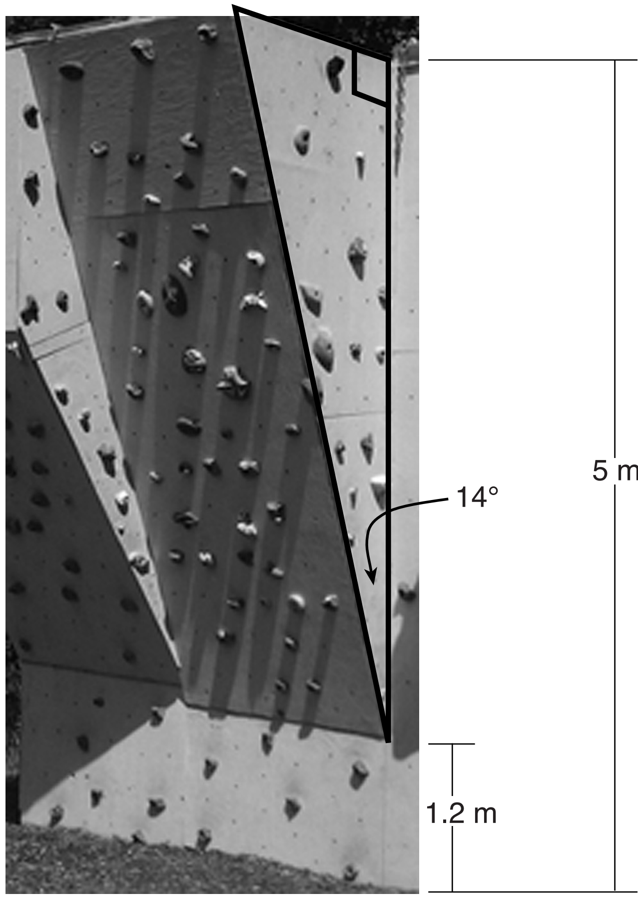
Length of section of slanted wall is 3.92 meters

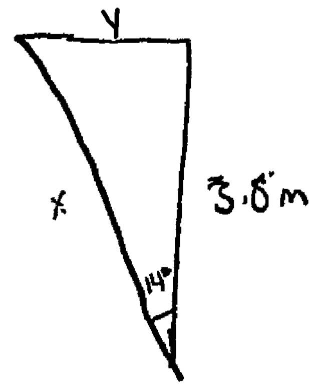
Score 2: The student gave a complete and correct response.

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

5
-1.2
3.8

SOHCAHTOA





$\frac{\tan 14}{1} = \frac{Y}{3.8}$

$Y = .95$

$3.8^2 + .95^2 = x^2$

$14.44 + .9025 = x^2$

$\sqrt{15.3425} = \sqrt{x^2}$

$x = 3.916$

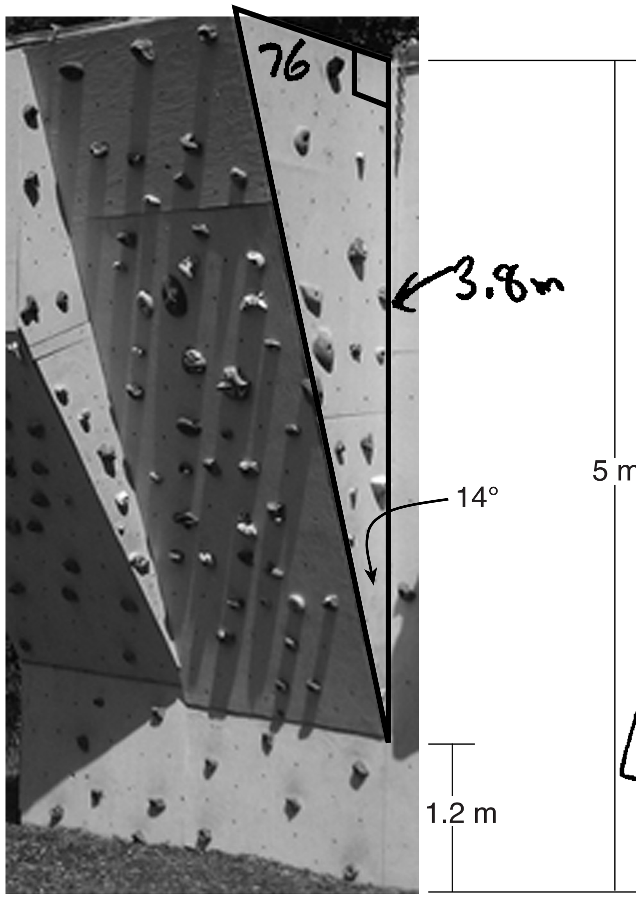
$x = 3.92$

Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

Score 2: The student gave a complete and correct response.

Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.



$\sin 76 (x) = \frac{3.8}{\cancel{x}(x)}$

$\frac{\sin 76(x)}{\sin 76} = 3.8$

$x = 3.9\text{ m}$

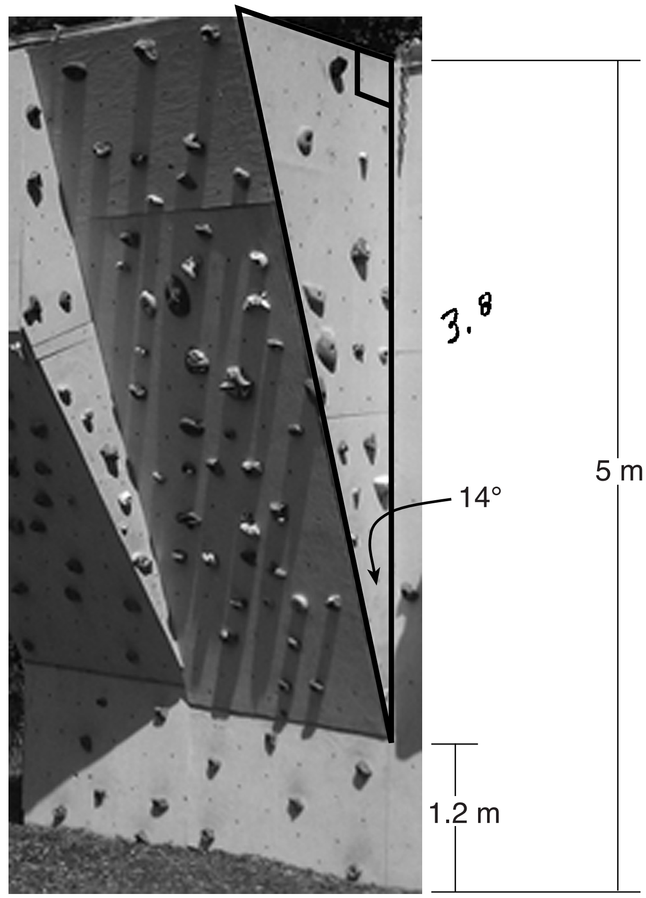
$\boxed{\text{hypotenuse} = 3.9\text{m}}$

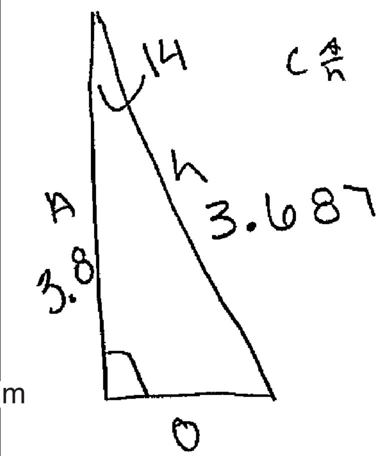
Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

Score 1: The student made a rounding error.

Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.





$$\cos 14 = \frac{3.8}{x}$$

.9703

Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

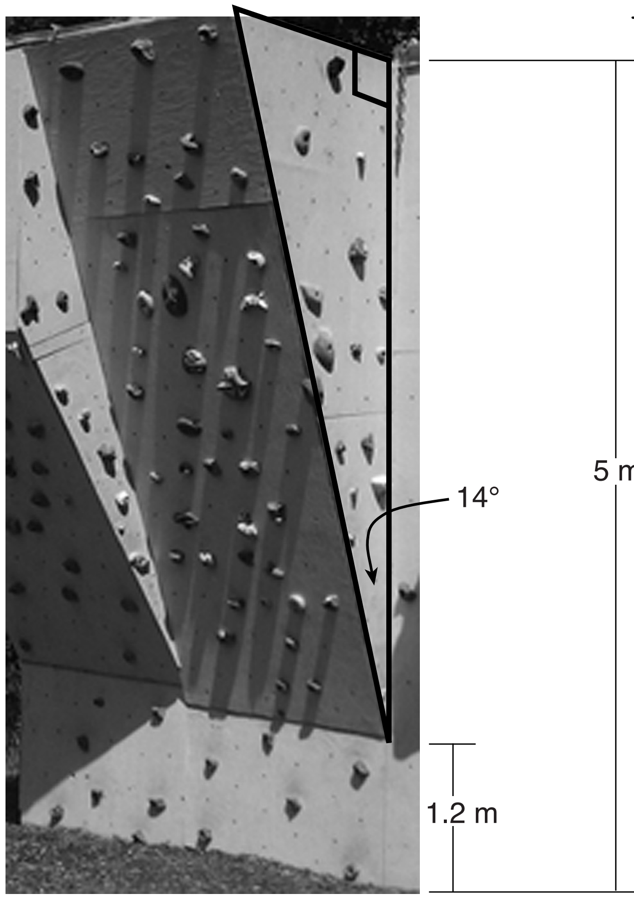
$\boxed{\text{3.687 meters}}$

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work was shown.

Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

soh cah toa



Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

$\cos 14 = \frac{5}{x}$ $\quad x = 5.15\text{ m}$

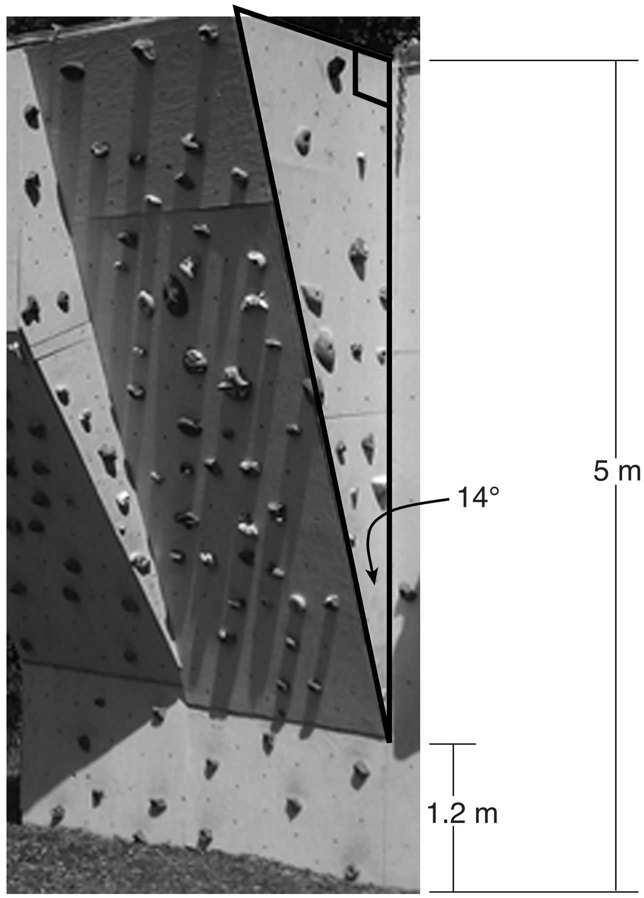
Score 1: The student used the incorrect height, but found an appropriate hypotenuse length.

Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$5-1.2$



Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

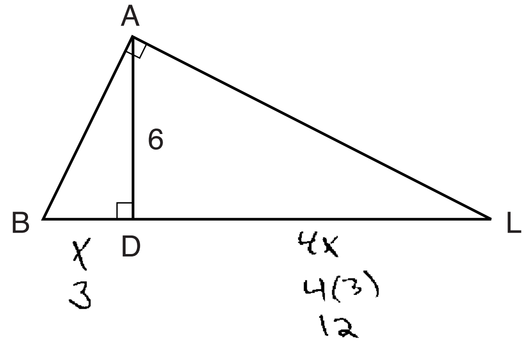
3.8m

$\frac{\sin(90)}{1} = \frac{3.8}{x}$

$3.8 = x(\sin(90)$ $x = 3.8$

Score 0: The student gave a completely incorrect response.

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

$$\frac{6}{x} = \frac{4x}{6}$$

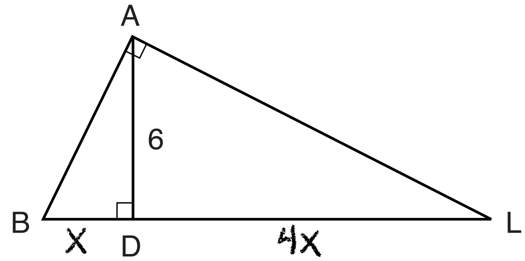
$$4x^2 = \cancel{36}$$
$$-36 \quad \cancel{-36}$$
$$4x^2 - 36 = 0$$
$$4(x^2 - 9) = 0$$
$$4(x+3)(x-3) = 0$$
$$\cancel{x = -3} \quad x = 3$$
reject

$BD = 3$

Score 2: The student gave a complete and correct response.

Question 29

29 In the diagram below of right triangle *BAL*, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

$(AB)^2 = x^2 + 36$

$(AL)^2 = 16x^2 + 36$

$(BL)^2 = 25x^2$

$25x^2 = 16x^2 + 36 + x^2 + 36$

$8x^2 - 72 = 0$

$8(x^2 - 9) = 0$

$\cancel{8 = 0} \quad x^2 - 9 = 0$

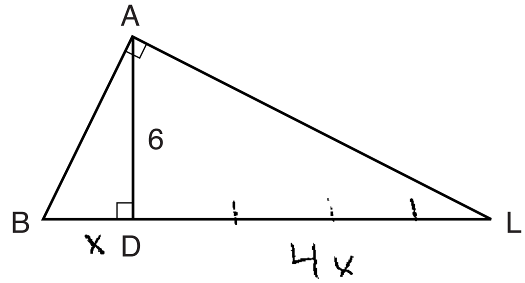
$(x+3)(x-3) = 0$

$\cancel{x = -3} \quad x = 3$

$BD = 3$

Score 2: The student gave a complete and correct response.

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

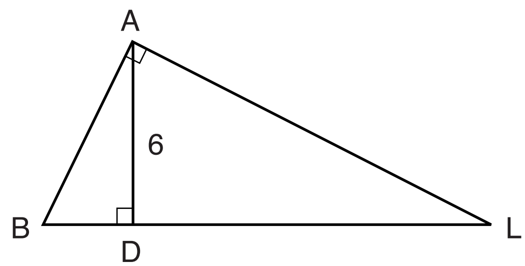
$$\frac{x}{6} = \frac{6}{4x}$$

$4x$

Score 1: The student wrote a correct equation to find the length of $\overline{BD}$, but no further correct work was shown.

Question 29

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

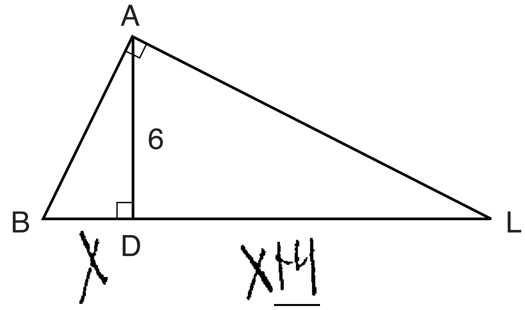
$\overline{BD} = 3$

$\overline{DL} = 12$

Score 1: The student found the length of $\overline{BD}$, but no work was shown.

Question 29

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

$\overline{BD} = 6$

$$\frac{x}{6} = \frac{6}{x+4}$$

S = 4

P = 6

$x^2 + 4x = 36$

$x^2 - 4x - 36 = 0$

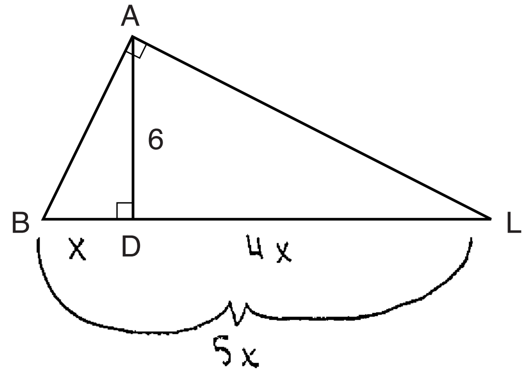
$(x+4)\ (x-6)$

$x - 6 = 0$
$+6$
$x = 6$ ✓

$x + 4 = 0$
-4
$x = -4$
✗

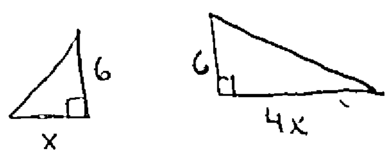
Score 0: The student did not show enough correct relevant work to receive any credit.

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

find $\overline{BD}$ $BD = x$

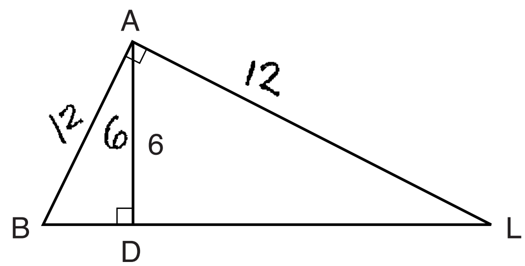


$$\frac{x}{6} = \frac{4x}{6}$$

$$6x = 24x$$

Score 0: The student did not show enough correct relevant work to receive any credit.

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

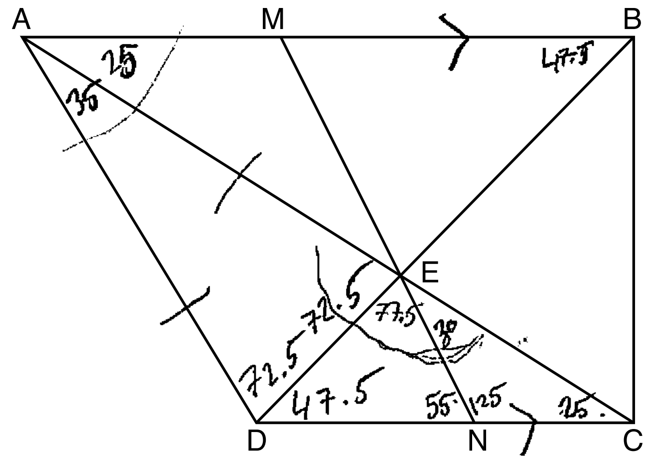
12 6 = 72

72 ÷ 3 = 24

$\boxed{BD \cong 24}$

Score 0: The student gave a completely incorrect response.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.

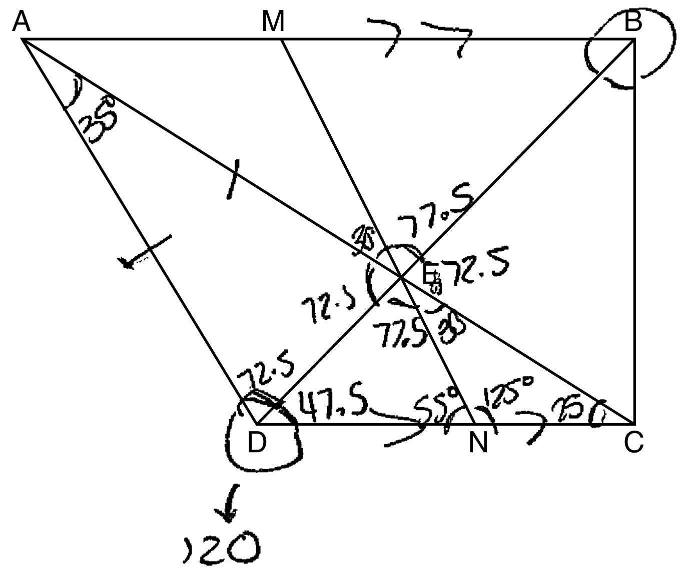


If m$\angle DAE = 35°$, m$\angle DCE = 25°$, and m$\angle NEC = 30°$, determine and state m$\angle ABD$.

m$\angle ABD = 47.5$

Score 2: The student gave a complete and correct response.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.



If m∠DAE = 35°, m∠DCE = 25°, and m∠NEC = 30°, determine and state m∠ABD.

180
-35
145
145/2
72.5

30
+25
55

180
-55
125

180
125°
55

77.5
+55
132.5

180
-132.5
47.5

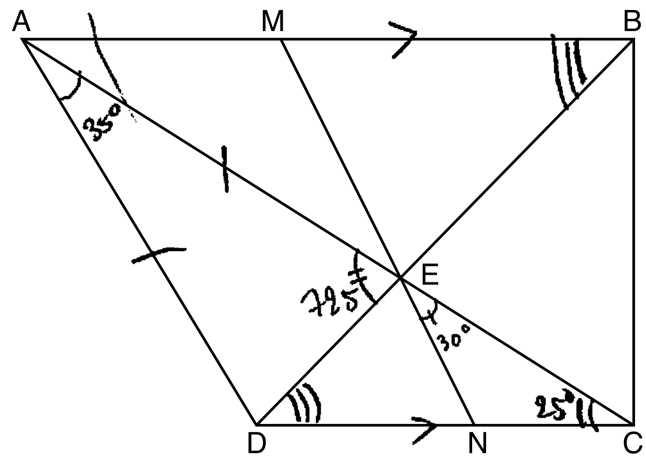
72.5
+30
102.5

180
-102.5
77.5

72.5
+47.5
120

Score 2: The student gave a complete and correct response.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.



If m$\angle DAE$ = 35°, m$\angle DCE$ = 25°, and m$\angle NEC$ = 30°, determine and state m$\angle ABD$.

$\overline{AD} \cong \overline{AE} \Rightarrow$ ADE is a isoseeless triangle

$m\angle DEA = \dfrac{180° - m\angle DAE}{2} = \dfrac{180° - 35°}{2} = 72.5°$

$m\angle AED = m\angle EDC + m\angle ECD$

$\Rightarrow 72.5° = m\angle EDC + 25°$

$\Rightarrow m\angle EDC = 47.5°$

$AB \parallel CD \Rightarrow m\angle ABD = m\angle EDC$ (alternate interior angles)

$\Rightarrow m\angle ABD = 47.5°$

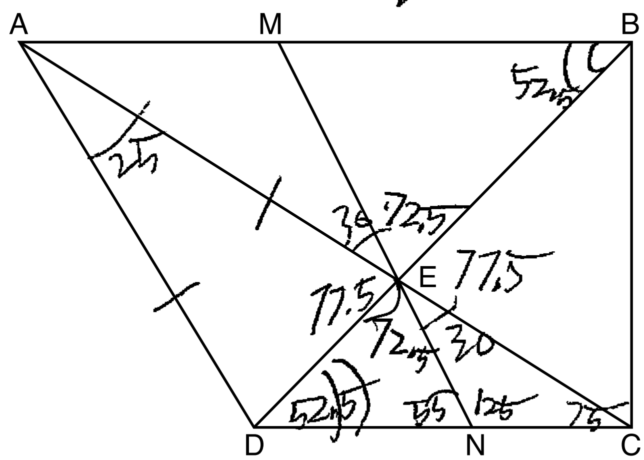
Score 2: The student gave a complete and correct response.

Question 30

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.

$72.5 + 55 = 127.5$

$180 - 127.5 = 52.5$



$180 - 125 = 55$

$180 - 25 - 30 = 125$

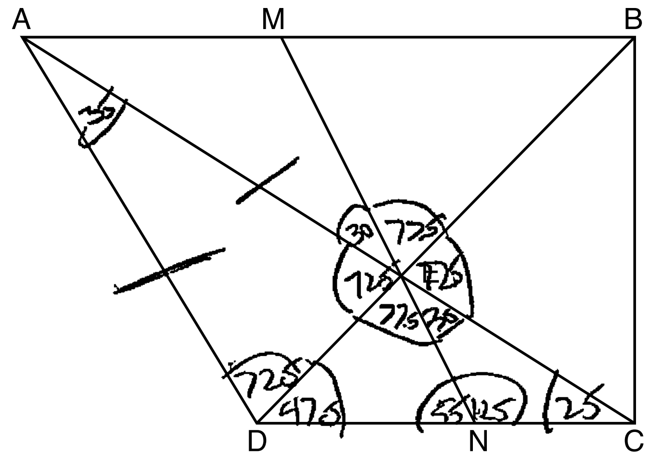
$30 + 77.5 = 107.5$ $180 - 107.5 = 72.5$

If m∠DAE = 35°, m∠DCE = 25°, and m∠NEC = 30°, determine and state m∠ABD.

$\angle ABD = 52.5$

Score 1: The student mislabeled ∠*DAE* in the diagram, but found an appropriate measure of ∠*ABD*.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.

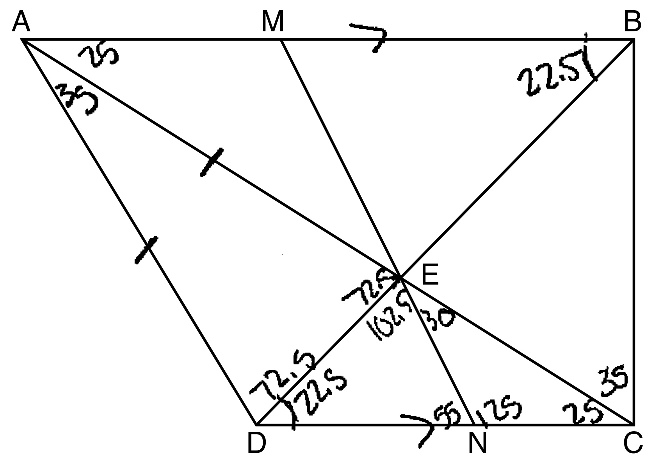


If $m\angle DAE = 35°$, $m\angle DCE = 25°$, and $m\angle NEC = 30°$, determine and state $m\angle ABD$.

Score 1: The student appropriately labeled the diagram, but did not state $m\angle ABD$.

Question 30

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.

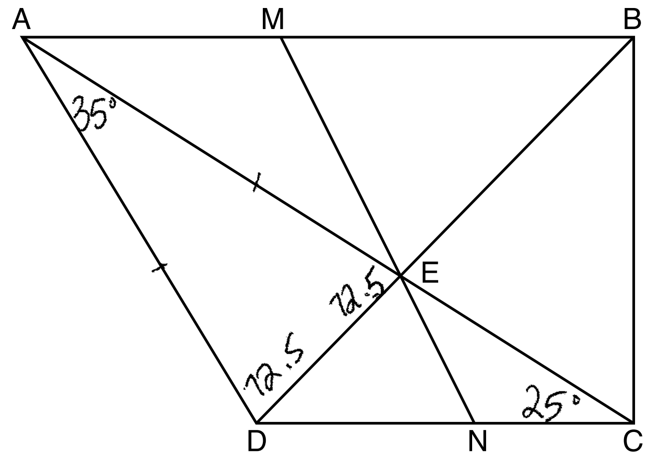


If m$\angle DAE = 35°$, m$\angle DCE = 25°$, and m$\angle NEC = 30°$, determine and state m$\angle ABD$.

m$\angle ABD = 22.5$

Score 1: The student made an error when finding m$\angle DEN$, but an appropriate measure was found for angle ABD. The measure of angle BCE is not necessary in finding m$\angle ABD$.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.



If m∠DAE = 35°, m∠DCE = 25°, and m∠NEC = 30°, determine and state m∠ABD.

$$180 - 35 = \frac{145}{2} = 72.5$$

Score 1: The student found m∠ADE and m∠AED, but m∠ABD was not stated.

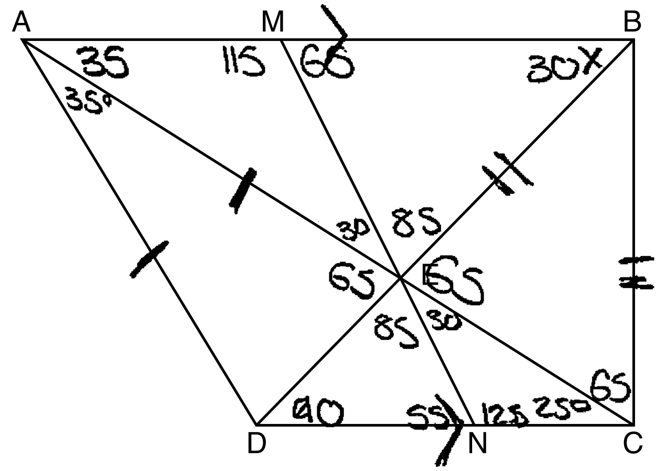
Question 30

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.

30+65=95

180-95=85

.85+55=140



30+25=55

180-55=

125

35+30=65

180-65=115

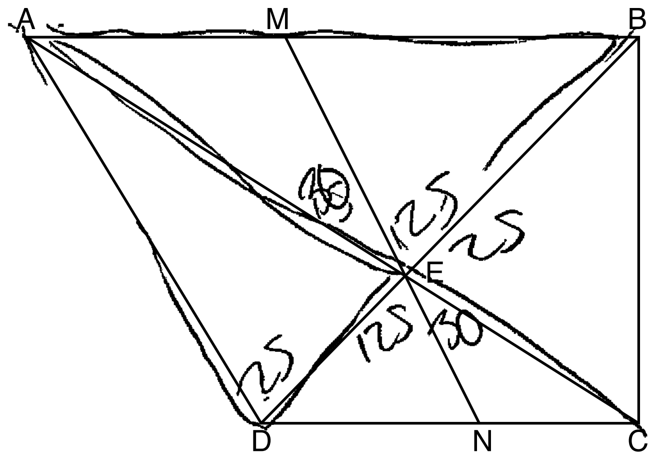
90-25=65

If m∠DAE = 35°, m∠DCE = 25°, and m∠NEC = 30°, determine and state m∠ABD.

m∠ABD=30°

Score 0: The student did not show enough correct relevant work to receive any credit.

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.



If m$\angle DAE = 35°$, m$\angle DCE = 25°$, and m$\angle NEC = 30°$, determine and state m$\angle ABD$.

$m\angle ABD = 180°$

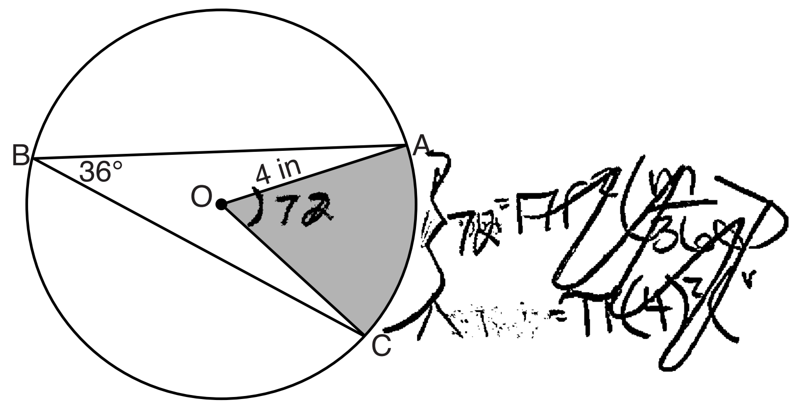
$25+30=55$

$180-55=125$

Score 0: The student gave a completely incorrect response.

Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

$A_{shade} = \pi r^2 \left(\frac{m}{360}\right)$

$A_{shad} = \pi(4)^2 \left(\frac{72}{360}\right)$

$A_{shade} = 16\pi(.2)$

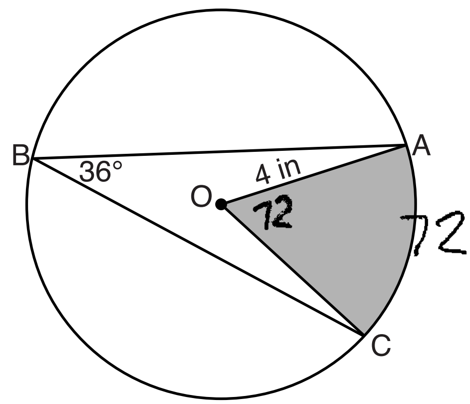
$A_{shade} = 3.2\pi$

$A_{shade} = 10.1 \text{ in}^2$

Score 2: The student gave a complete and correct response.

Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

$$A = \pi r^2 \cdot \frac{x}{360}$$

$$A = \pi 4^2 \cdot \frac{72}{360}$$

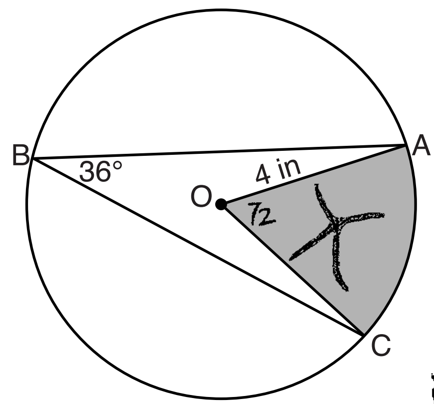
$$A = \pi 16 \cdot \frac{1}{5}$$

$$A = \pi \frac{16}{5}$$

$$A = 10.1 \text{ in}^2$$

Score 2: The student gave a complete and correct response.

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



x = area of shaded sector

Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

$$\frac{72}{360} = \frac{x}{16\pi}$$

$$\frac{1}{5} = \frac{x}{16\pi}$$

$$\frac{16\pi}{5}\,in^2 = x$$

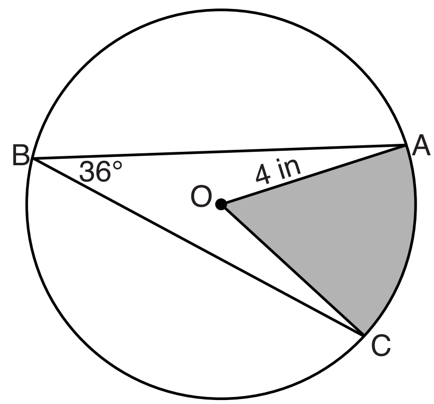
$$x = \frac{16\pi}{5}\,in^2$$

$$x = 10.1\,in^2$$

Score 2: The student gave a complete and correct response.

Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

Area of sector $= \left(\frac{m\text{Arc}}{360°}\right)\pi r^2$

Area of sector $= \left(\frac{36}{360°}\right)\pi \cdot 4^2$

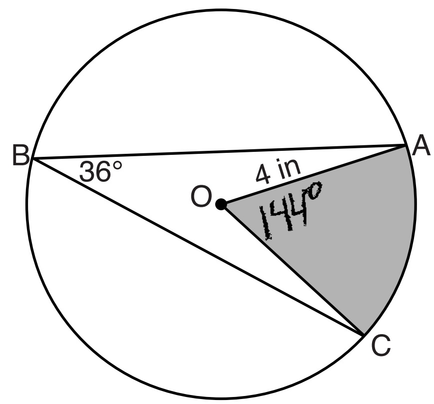
Area of sector $= \left(\frac{36°}{360°}\right)\pi \cdot 16$

$\boxed{\text{Area of sector} = 5.0}$

Score 1: The student used an incorrect measure for arc AC.

Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



$\frac{n}{360} \cdot \pi r^2$

$\frac{n}{360} \cdot \pi(4)^2$

$\frac{n}{360} \cdot 50.26$

Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

$\frac{144}{360} \cdot 50.26$

$.4 \cdot 50.26$

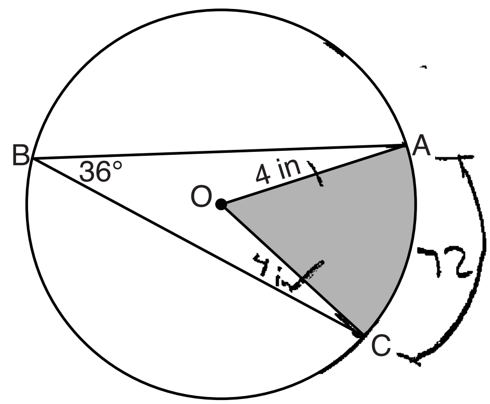
20.104

20.1 in^2

Score 1: The student used an incorrect measure for angle AOC.

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.

$\frac{36}{1/2} = 72$



$\overset{\frown}{AC} = 72$

$AO = 4$

$A = \frac{bh}{2}$

$A = \frac{1}{2}\, 72 \cdot 4$

$A = \frac{1}{2}\, 288$

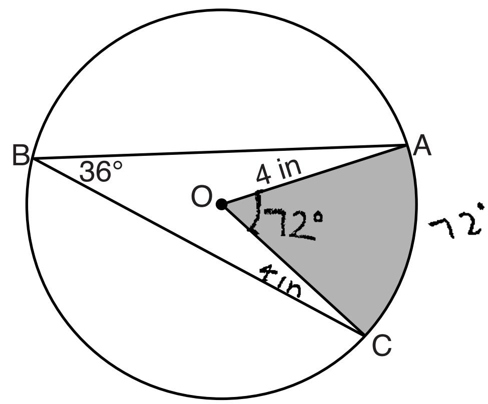
$\boxed{A = 144}$

Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



$A = \pi r^2$

$= \pi(4)^2$

$= 16\pi$

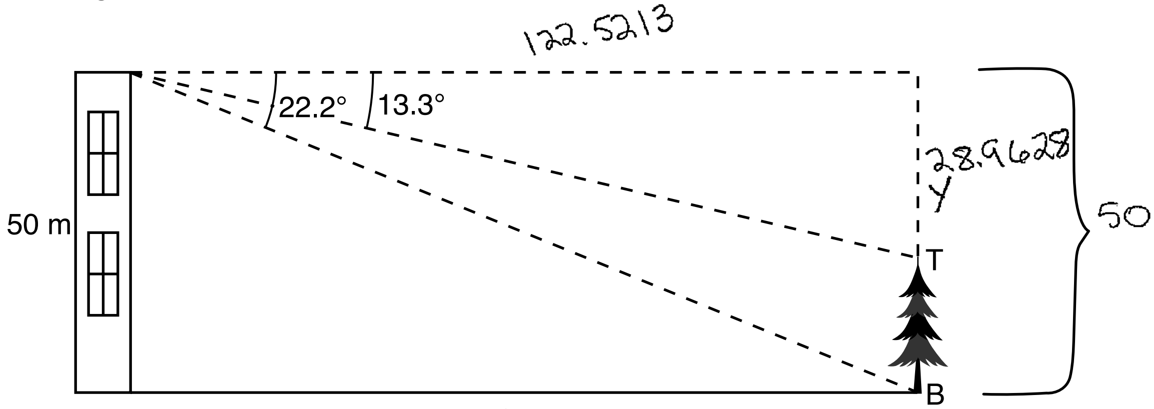
$A = 50.26548$

Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.



S O/H C A/H T O/A

Determine and state, to the *nearest meter*, the height of the tree.

$\tan 22.2 = \frac{50}{x}$

$\frac{\tan 22.2 x}{\tan 22.2} = \frac{50}{\tan 22.2}$

$x = 122.5213$

$\tan 13.3 = \frac{y}{122.5213}$

$28.9628 = y$

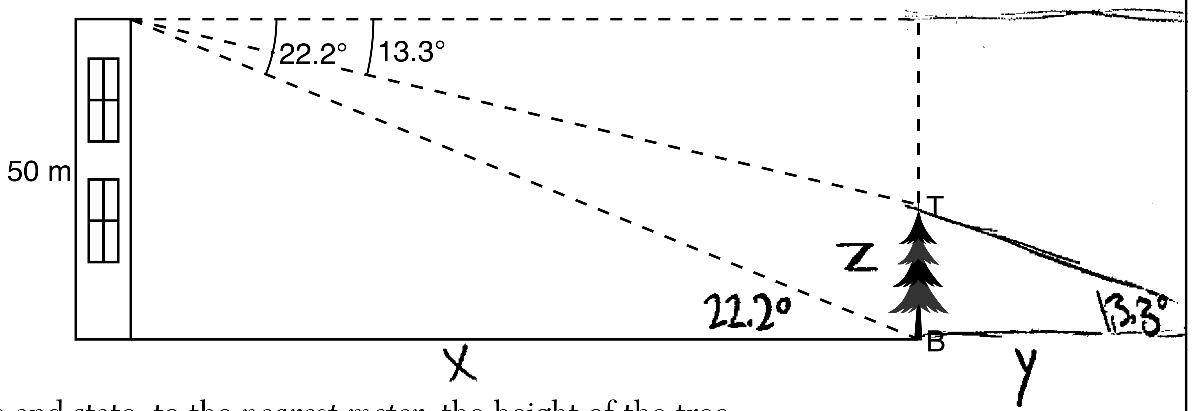
$50 - 28.9628 = 21.0372$

The tree is 21 meters tall.

Score 4: The student gave a complete and correct response.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.



Determine and state, to the *nearest meter*, the height of the tree.

$\tan 22.2 = \frac{50}{x}$

$x = \frac{50}{\tan 22.2}$

$x = 122.521$

$\tan 13.3 = \frac{50}{x+y}$

$x+y = \frac{50}{\tan 13.3}$

$x+y = 211.515$

$122.521 + y = 211.515$

$y = 88.994$

$\tan 13.3 = \frac{z}{88.994}$

$z = 88.994 \cdot \tan 13.3$

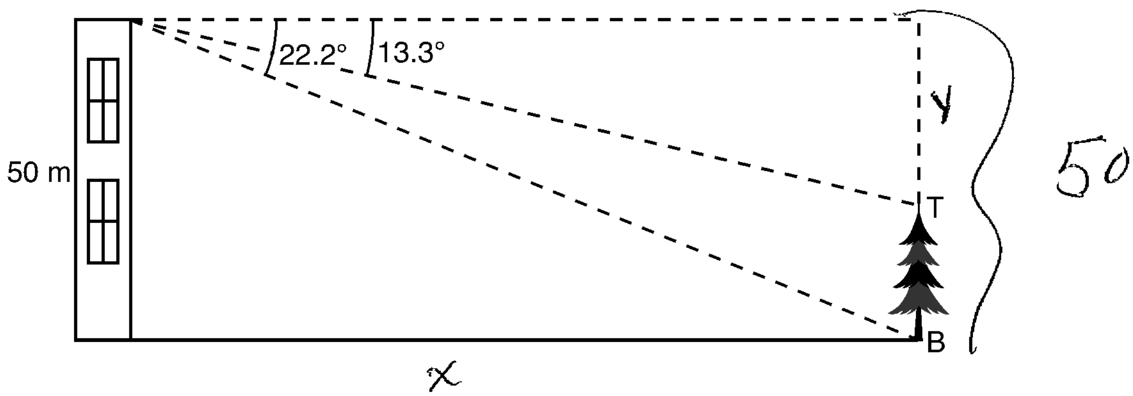
$z = 21.0373$

The tree is about 21 m tall.

Score 4: The student gave a complete and correct response.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, *T*, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, *B*, is 22.2°.



Determine and state, to the *nearest meter*, the height of the tree.

$\tan 22.2 = \frac{50}{x}$

$x = 122.52125$

$\tan 13.3 = \frac{y}{122.52125}$

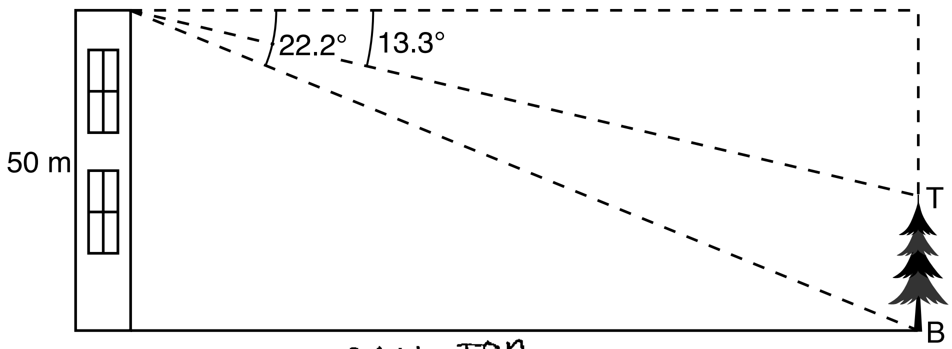
$y = 28$

$50 - 28 = 22\text{ m}$

Score 3: The student made a rounding error.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.



SOH-CAH-TOA

Determine and state, to the *nearest meter*, the height of the tree.

$\text{Tan}(22.2) = \frac{50}{\cancel{x}} \cancel{\cdot x}$

$\frac{50}{\tan(22.2)} = 122.5212599$

$\tan(22.2) = \frac{13.3}{x} \cdot x$

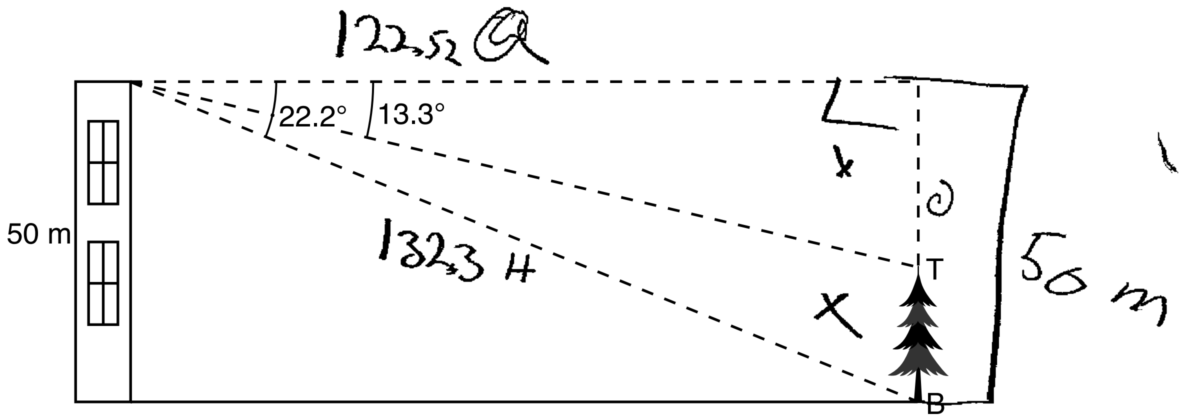
$\frac{13.3}{\tan(22.2)} = 32.59065513$

$$\begin{array}{r} 122.5212599 \\ -\ 32.59065513 \\ \hline 89.93060477 \end{array}$$

$\boxed{90\text{m}}$

Score 2: The student correctly found the horizontal distance between the building and the tree, but no further correct work was shown.

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, *T*, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, *B*, is 22.2°.



Determine and state, to the *nearest meter*, the height of the tree.

$S = \frac{O}{H} \quad C \frac{a}{H} \quad T \frac{O}{a}$

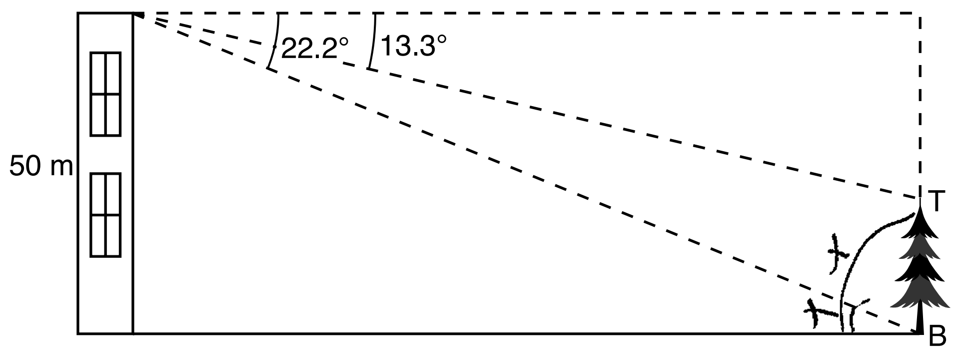
$- 50$

28.96

21 M

Score 1: The student found the correct height of the tree, but did not show enough work to receive additional credit.

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, *T*, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, *B*, is 22.2°.



SOH CAH TOA

Determine and state, to the *nearest meter*, the height of the tree.

$\sin x = \frac{22.2}{50}$

$x = 26.3593\ldots$

$x = 26$ ft

$\sin x = \frac{13.3}{50}$

$x = 15.4263\ldots$

$x = 15$ ft

$26 - 15 = 11$

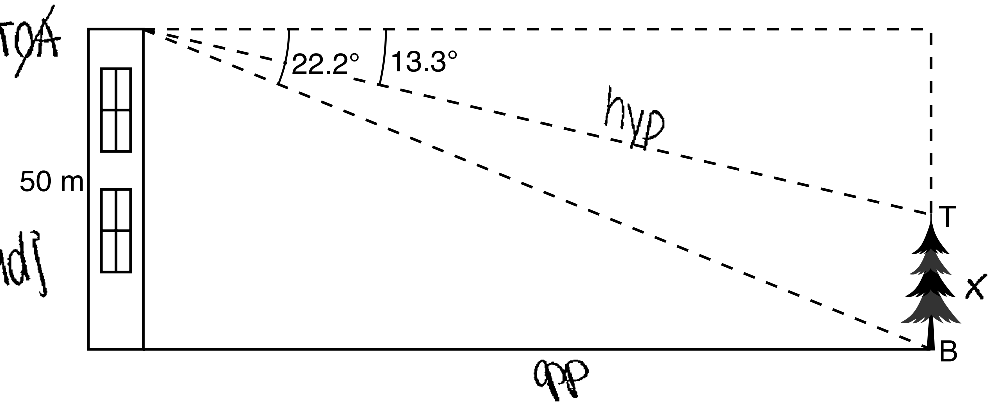
The tree is about 11 feet tall

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.

~~SOH~~ CAH ~~TOA~~



Determine and state, to the *nearest meter*, the height of the tree.

$\cos 22.2\left(\frac{50}{x}\right)$

74.26988 4935

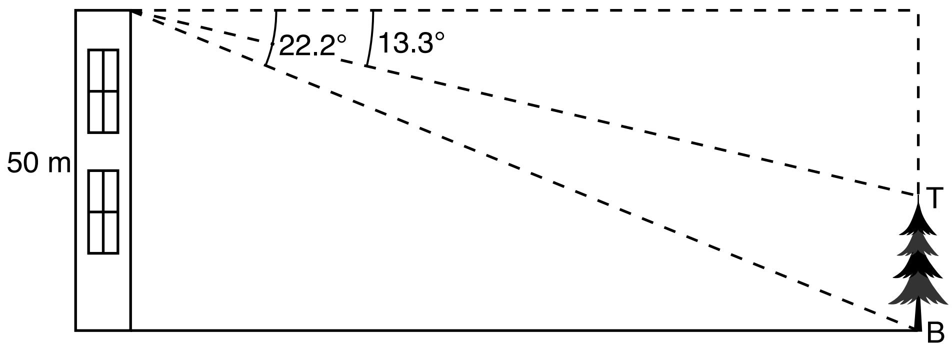
$\cos 13.3\left(\frac{50}{x}\right)$ ~~8.54907520883~~

5.7 meters

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.



Determine and state, to the *nearest meter*, the height of the tree.

$\frac{\tan 22.2}{1} = \frac{x}{50}$

$x = 20.40462204$

$\frac{\tan 13.3}{1} = \frac{x}{50}$

$x = 11.81949975$

$$\begin{array}{r} 20.40462204 \\ -\ 11.8194995 \\ \hline 8.58512284 \end{array} \approx 9$$

$\boxed{9 \text{ meters.}}$

Score 0: The student gave a completely incorrect response.

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

$m_{\overline{HY}} = \frac{\Delta y}{\Delta x} = \frac{9-6}{2-(-3)} = \frac{3}{5}$

$m_{\overline{EP}} = \frac{\Delta y}{\Delta x} = \frac{-4+(+1)}{3-8} = \frac{-3}{-5} = \frac{3}{5}$

$m_{\overline{HE}} = \frac{\Delta y}{\Delta x} = \frac{6+(+4)}{-3-3} = \frac{10}{-6} = -\frac{5}{3}$

$m_{\overline{YP}} = \frac{\Delta y}{\Delta x} = \frac{9+(+1)}{2-8} = \frac{10}{-6} = -\frac{5}{3}$

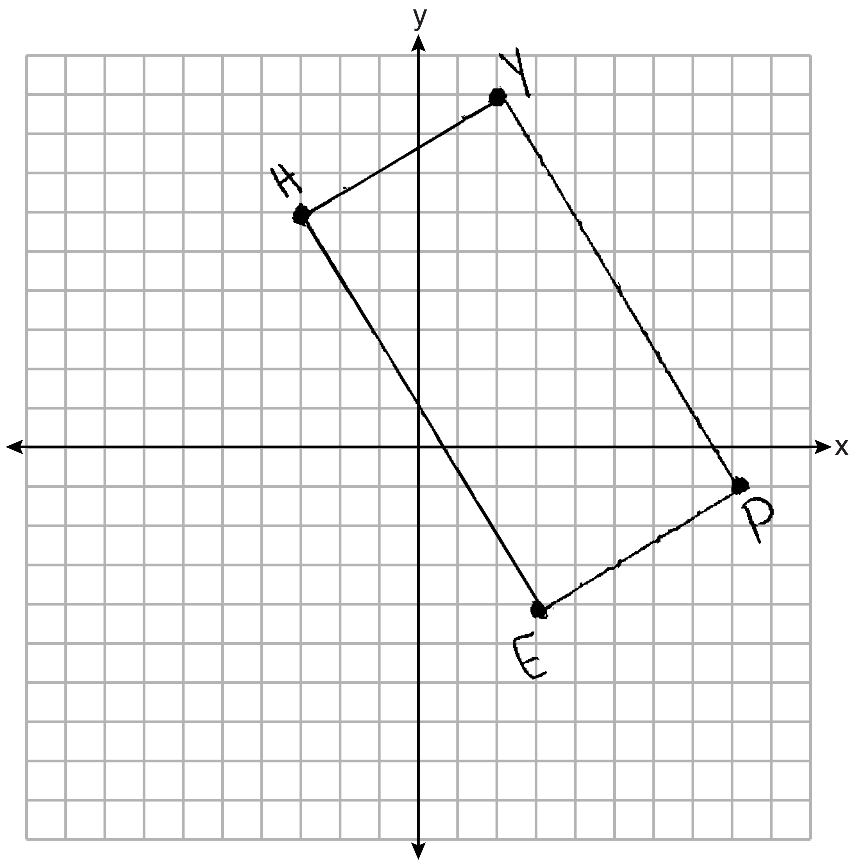
$\overline{HY} \parallel \overline{EP}$, $\overline{HE} \parallel \overline{YP}$ } since they have the same slope.

Quadrilateral HYPE is a parallelogram since both pairs of opposite sides are parallel

$\overline{HY} \perp \overline{YP}$ since their slopes are opposite reciprocals

∡Y is a rt. ∡ since ⊥ lines form rt ∡'s

Quadrilateral HYPE is a rectangle since it is a parallelogram w/ a rt. ∡



Score 4: The student gave a complete and correct response.

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

$HY = \sqrt{(2-{-3})^2+(9-6)^2}$ $YP = \sqrt{(8-2)^2+(-1-9)^2}$ $EP = \sqrt{(8-3)^2+(-1-{-4})^2}$

$HY = \sqrt{25+9}$ $YP = \sqrt{36+100}$ $EP = \sqrt{25+9}$

$HY = \sqrt{34}$ $YP = \sqrt{136}$ $EP = \sqrt{34}$

$HE = \sqrt{(3-{-3})^2+(-4-6)^2}$

$HE = \sqrt{36+100}$

$HE = \sqrt{136}$

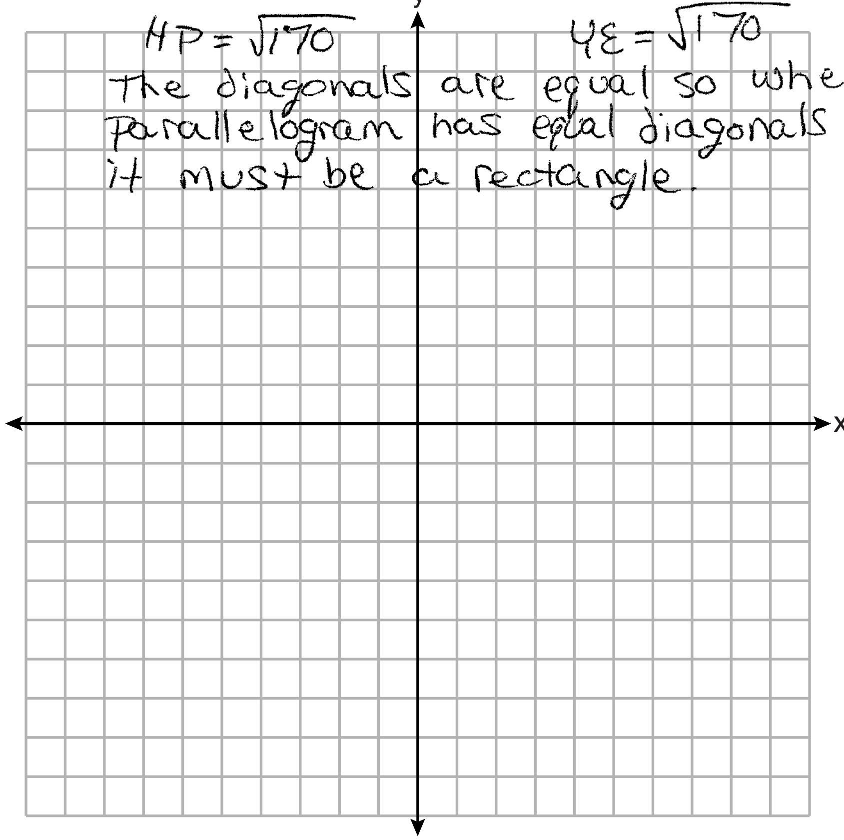
Both pairs of opposite sides are equal so HYPE is a parallelogram.

$HP = \sqrt{(8-{-3})^2+(-1-6)^2}$ $YE = \sqrt{(3-\cancel{2})^2+(-4-9)^2}$

$HP = \sqrt{121+49}$ $YE = \sqrt{1+169}$

$HP = \sqrt{170}$ $YE = \sqrt{170}$

The diagonals are equal so when a parallelogram has equal diagonals then it must be a rectangle.



Score 4: The student gave a complete and correct response.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

$\cancel{HP = \sqrt{(2--3)^2+(9-6)^2}}$

$\cancel{= \sqrt{5^2+3^2}}$

$\cancel{= \sqrt{25+9}}$

$\cancel{= \sqrt{34}}$

$HP = \sqrt{(8--3)^2+(6--1)^2}$

$= \sqrt{11^2+7^2}$

$= \sqrt{121+49}$

$= \sqrt{170}$

$\overline{HP} \cong \overline{YE}$

Diagonals are $\cong$

$YE = \sqrt{(3-2)^2+(9--4)^2}$

$YE = \sqrt{1^2+13^2}$

$= \sqrt{1+169}$

$= \sqrt{170}$

$\overline{HP}$ midpoint

$\left(\frac{8-3}{2}, \frac{6-1}{2}\right)$

$\left(\frac{5}{2}, \frac{5}{2}\right)$

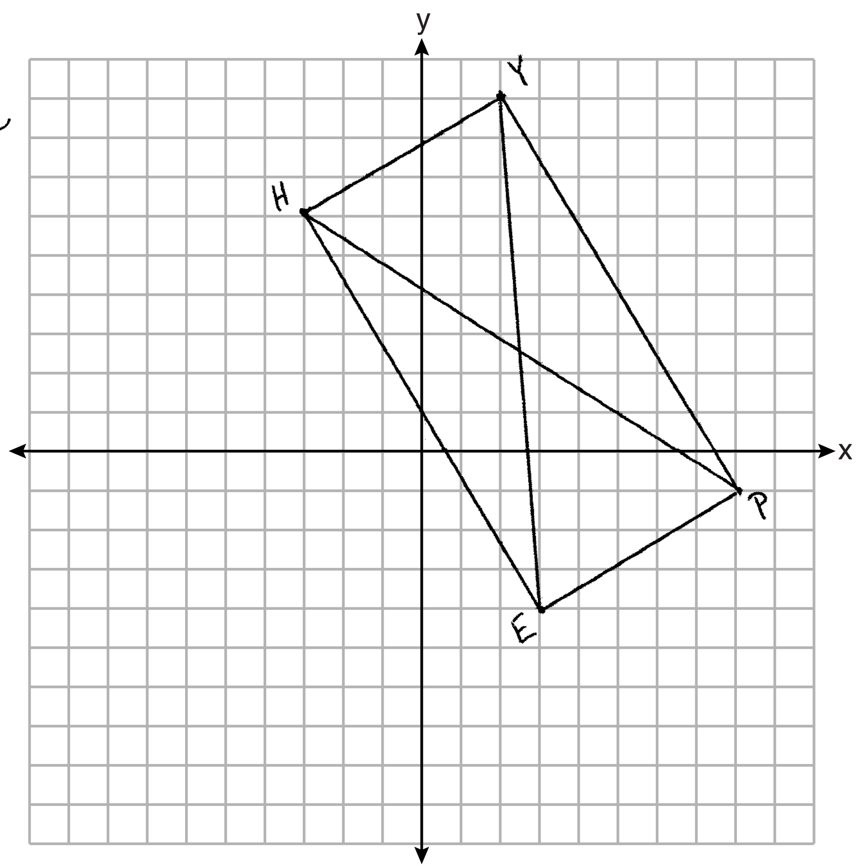
$\overline{YE}$ midpoint

$\left(\frac{3+2}{2}, \frac{9-4}{2}\right)$

$\left(\frac{5}{2}, \frac{5}{2}\right)$

same midpoint so diagonals bisect each other so HYPE is a parallelogram

parallelogram with $\cong$ diagonals is a rectangle



Score 4: The student gave a complete and correct response.

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

$\overline{HY}$ $\frac{3}{5}$ $\overline{EP}$ $\frac{-3}{-5}$ — Same slope parallel

$\overline{YP}$ $\frac{10}{-6}$ $\left(-\frac{5}{3}\right)$ $\overline{HE}$ $\frac{-10}{6}$ — Same slope parallel

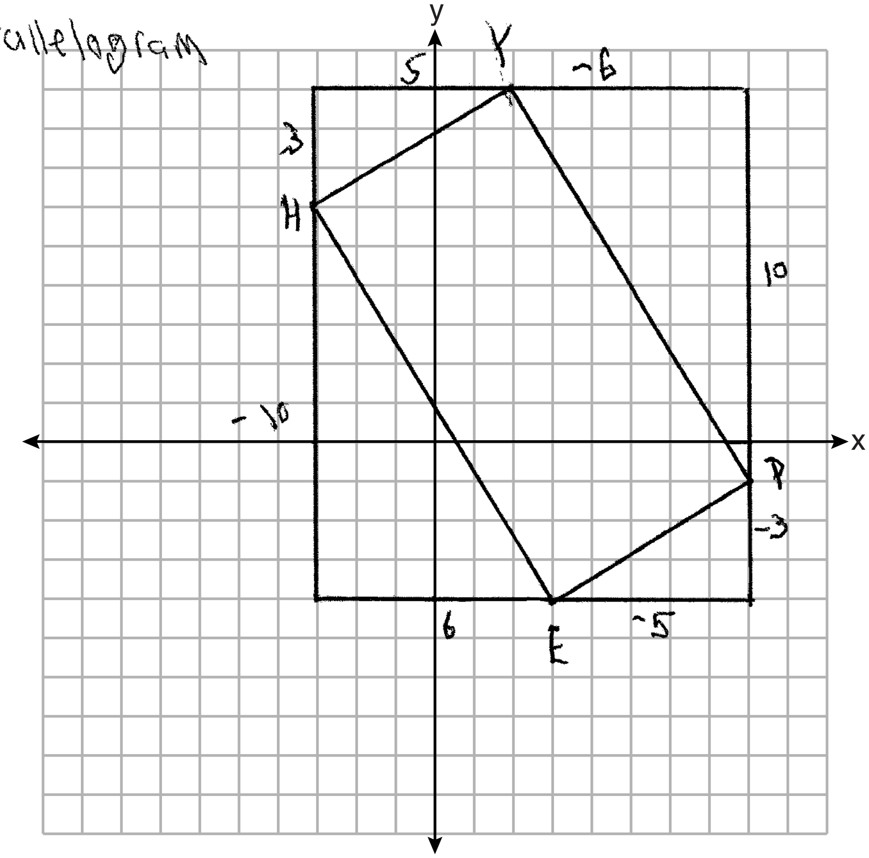
opp sides ∥ parallelogram

$\overline{HY} \perp \overline{YP}$

$\frac{3}{5} \perp -\frac{5}{3}$

neg rec slopes

rt∠



Score 3: The student did not write a concluding statement in proving a rectangle.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

$H(\overset{x_1,y_1}{-3,6})$
$Y(\overset{x_2,y_2}{2,9})$

$d=\sqrt{(2+3)^2+(9-6)^2}$
$=\sqrt{5^2+3^2}$
$=\sqrt{25+9}$
$=\boxed{\sqrt{34}}$

$E(\overset{x_1,y_1}{3,-4})$
$P(\overset{x_2,y_2}{8,-1})$

$d=\sqrt{(8-3)^2+(-1+4)^2}$
$=\sqrt{(5)^2+(3)^2}$
$=\sqrt{25+9}$
$=\boxed{\sqrt{34}}$

$H(\overset{x_1,y_1}{-3,6})$
$E(\overset{x_2,y_2}{3,-4})$

$d=\sqrt{(3+3)^2+(-4-6)^2}$
$=\sqrt{6^2+(-10)^2}$
$=\sqrt{36+100}$
$=\sqrt{136}$

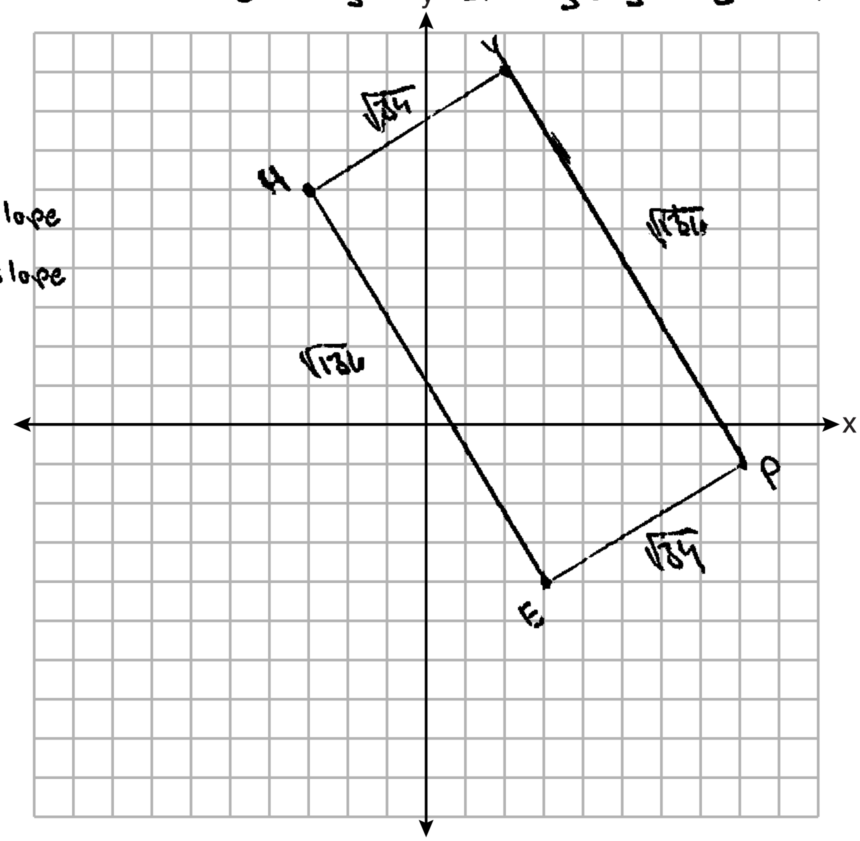
$P(\overset{x_1,y_1}{8,-1})$
$Y(\overset{x_2,y_2}{2,9})$

$d=\sqrt{(2-8)^2+(9+1)^2}$
$=\sqrt{-6^2+10^2}$
$=\sqrt{36+100}$
$=\sqrt{136}$

$\frac{\Delta y}{\Delta x}=\frac{9-6}{2--3}=\frac{3}{5}$ $\quad\frac{\Delta y}{\Delta x}=\frac{-1+4}{8-3}=\frac{3}{5}$ $\quad\frac{\Delta y}{\Delta x}=\frac{-4-6}{3--3}=\frac{-10}{6}$ $\quad\frac{\Delta y}{\Delta x}=\frac{9--1}{2-8}=\frac{10}{-6}$

$\overline{HY}$ slope = $\overline{EP}$ slope
$\overline{HE}$ slope = $\overline{YP}$ slope

$\overline{HY} \parallel \overline{EP}$
$\overline{HE} \parallel \overline{YP}$
$\overline{HY} \cong \overline{EP}$
$\overline{HE} \cong \overline{YP}$
$\boxed{P}$



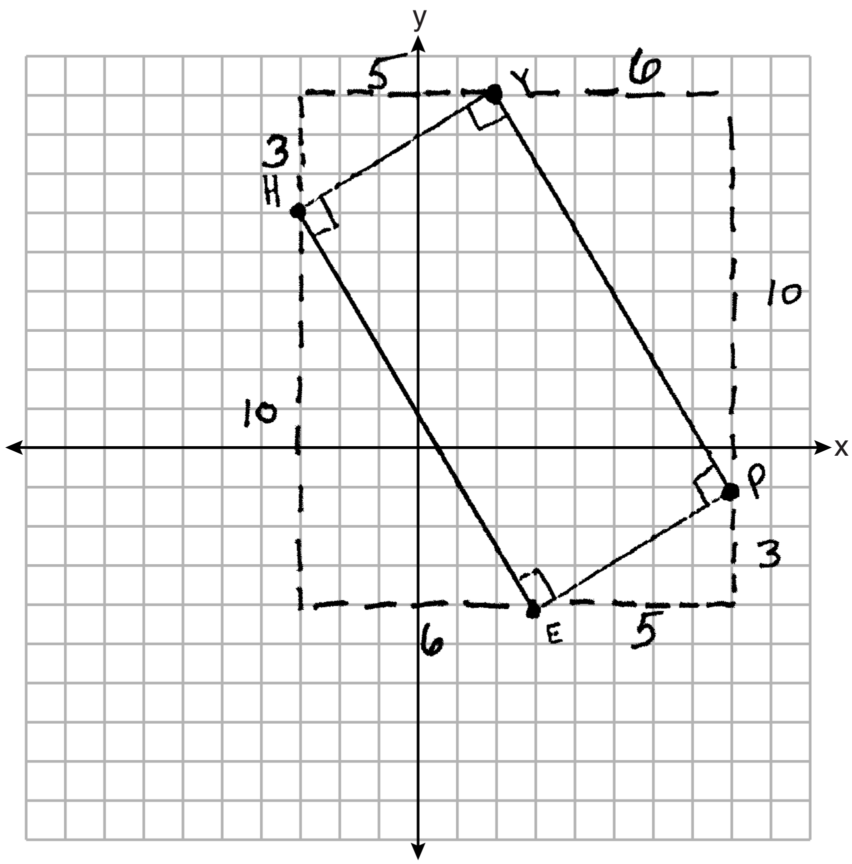
Score 2: The student proved *HYPE* is a parallelogram, but did not prove *HYPE* is a rectangle.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

Slope of $\overline{HY} = \frac{3}{5}$
Slope of $\overline{EP} = \frac{3}{5}$ ∥
Slope of $\overline{HE} = \frac{10}{6}$
Slope of $\overline{YP} = \frac{10}{6}$ ∥

quadrilateral HYPE is a rectangle because opposite sides are parallel, and it has four right angles

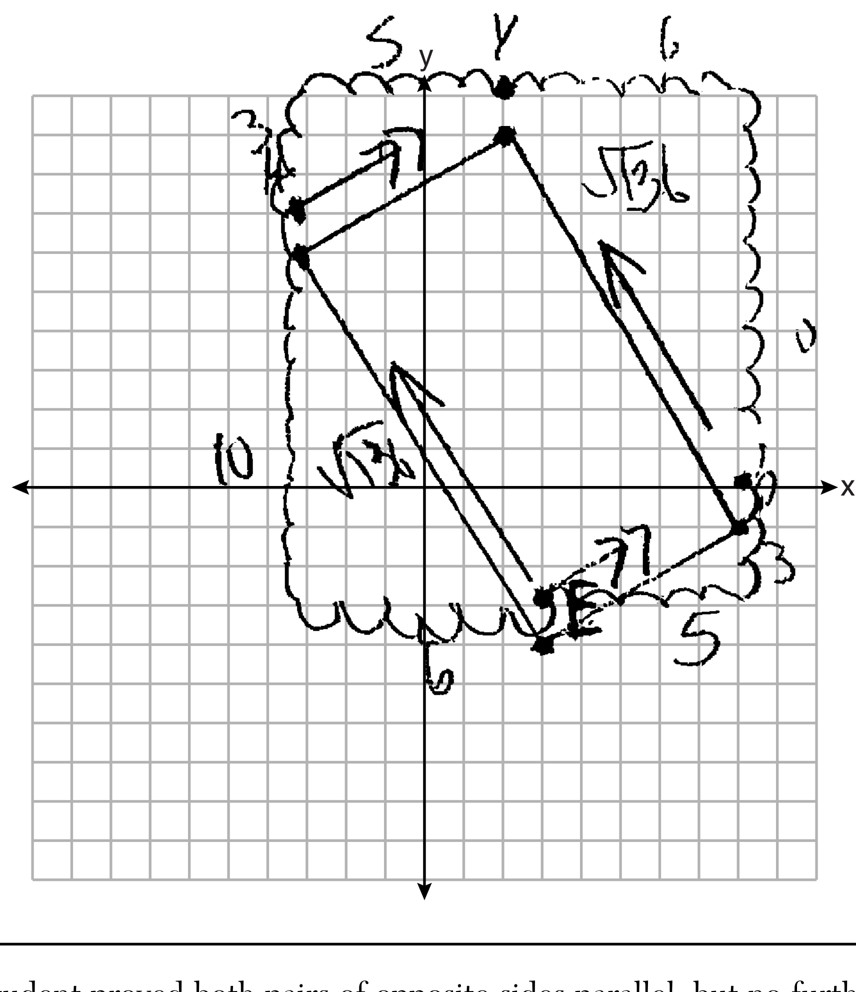


Score 1: The student made a conceptual error in proving a rectangle and a computational error in finding the slopes of $\overline{HE}$ and $\overline{YP}$.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

HYPE has opposite Sides parallel.
$\overline{HY}$ and $\overline{EP}$ have the Same Slope of $\frac{3}{5}$,
While $\overline{YP}$ and $\overline{HE}$ have a
Slope of $\frac{-5}{3}$ or $\frac{-10}{6}$ making them Parallel.



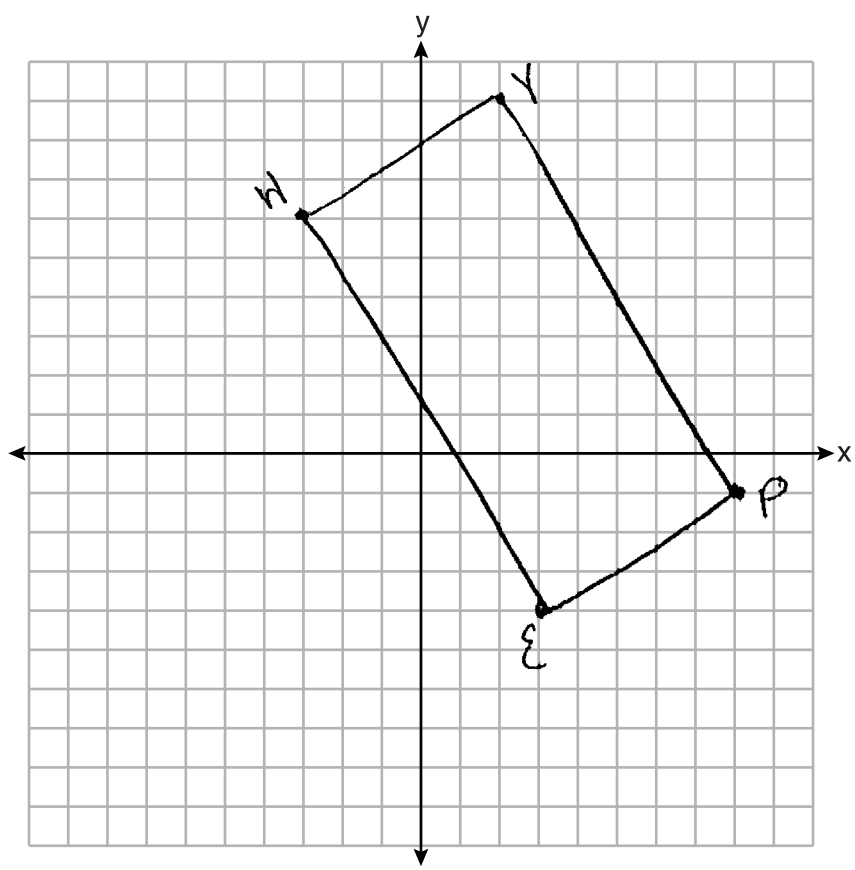
slopes
$\frac{3}{5}$ $\frac{-10}{6}$

Score 1: The student proved both pairs of opposite sides parallel, but no further correct work was shown.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

Hype is a rectangle because it has 2 pairs of parallel lines.

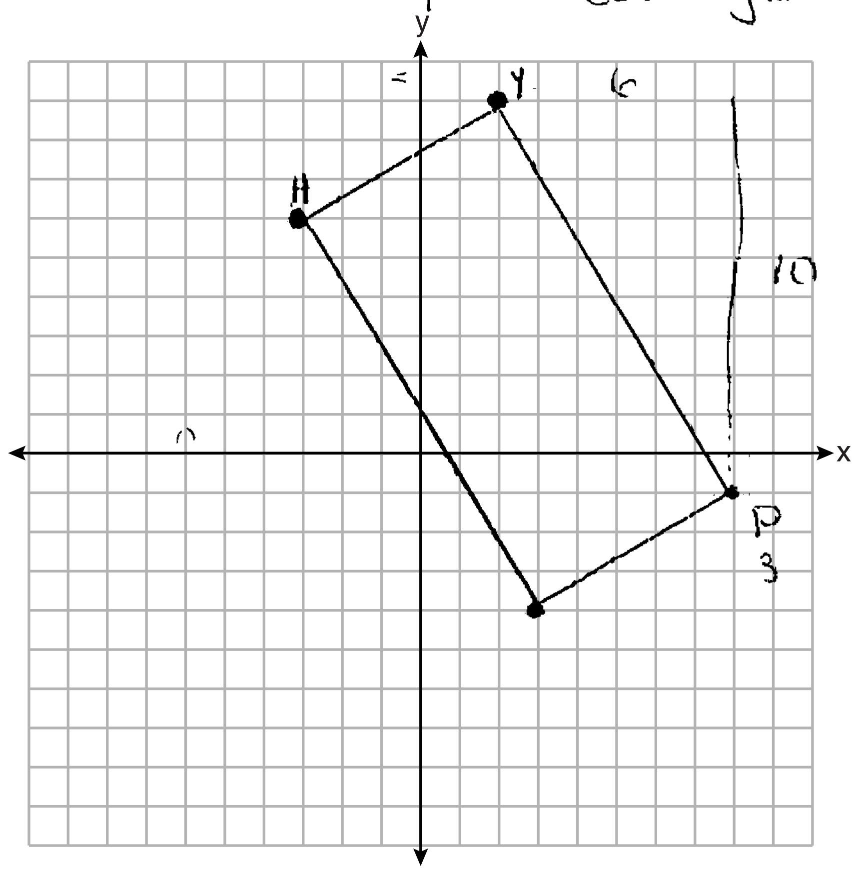


Score 0: The student did not show enough correct relevant work to receive any credit.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are *H*(−3,6), *Y*(2,9), *P*(8,−1), and *E*(3,−4). Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

Statement	Reason
① HYPE is a ▭	① Given
② $\overline{HY} \parallel \overline{EP}$	Same slope
③ $\overline{HE} \parallel \overline{YP}$	Same slope
④ $\overline{YP} \perp \overline{PE}$ & $\overline{HE} \perp \overline{HY}$	Defination of $\perp$
⑤ HYPE is a ▭	⑤ Defintion of Rectangle



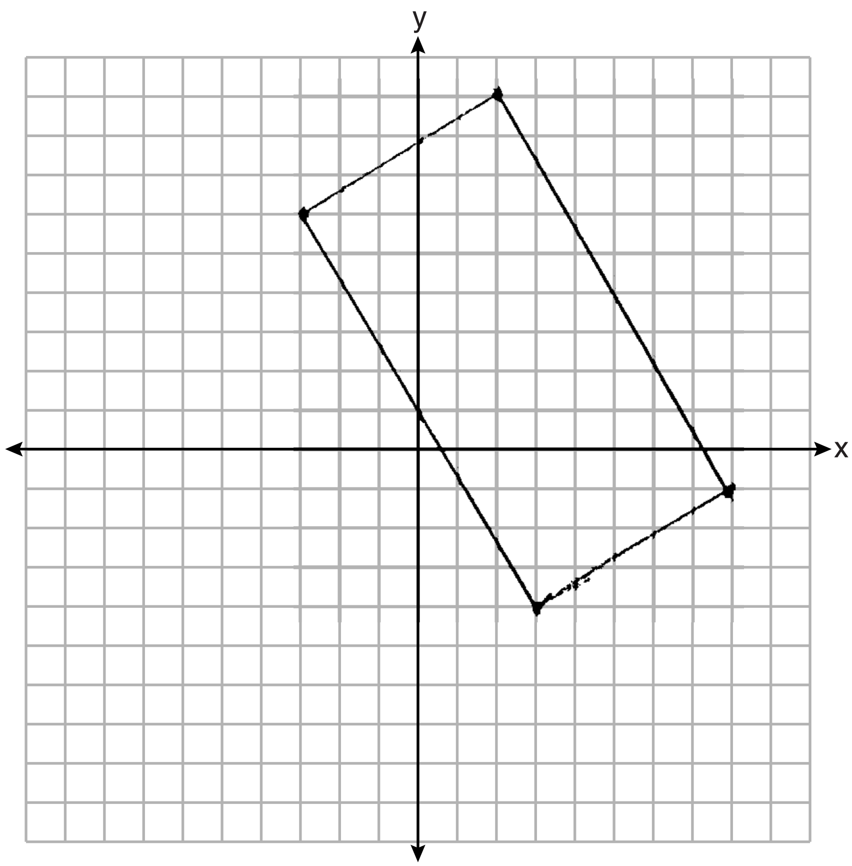
Score 0: The student did not show enough correct relevant work to receive any credit.

Question 33

33 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

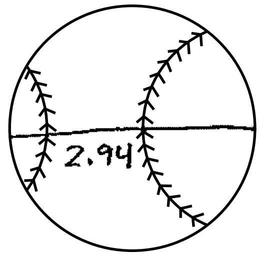
$\overline{HY}$ and $\overline{PE}$ both have the same slope while $\overline{YP}$ and $\overline{HE}$ have the same slope If two lines have the same slope then they are parallel Therefore HYPE has to pairs of parallel sides. If all sides of a quadrilateral are congruent, then opposite side are congruent HYPE has 2 pairs of congruent and parallel sides. Therefore HYPE is a rectangle



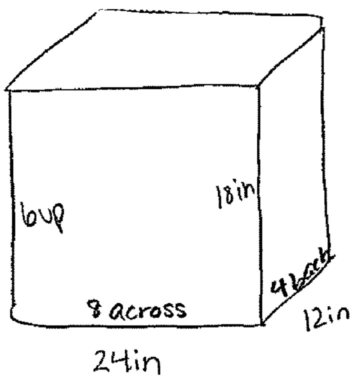
Score 0: The student did not show enough correct relevant work to receive any credit.

Question 34

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.



$8 \cdot 4 \cdot 6 = 192$ $\boxed{\text{192 baseballs}}$

$\frac{24}{2.94} = 8.16$

$\frac{18}{2.94} = 6.1$

$\frac{12}{2.94} = 4.08$

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

$V_{circle} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (1.47)^3$

$= 13.30578843 \text{ in}^3$

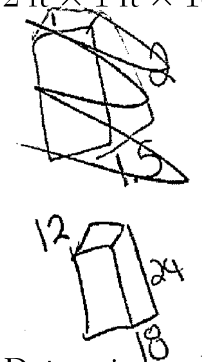
$13.30578843 \text{ in}^3 \times 192 = 2{,}554.711379 \text{ in}^3$

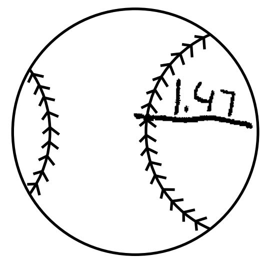
$2{,}554.711379 \text{ in}^3 \times 0.025 = 63.86778446 \text{ lbs}$

$\boxed{\text{64 pounds}}$

Score 4: The student gave a complete and correct response.

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.





$V = \dfrac{4\pi r^3}{3}$

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

$\cancel{1 \cdot 2 \cdot 1.5 = 3\,ft^3}$

$12 \cdot 18 \cdot 24 = 5184\ in^3$

$5184 \div 13.3058$

$V = \dfrac{4\pi(1.47)^3}{3}$

$V = 13.30578842$

$\boxed{\text{389 baseballs}}$

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

$0.025 \cdot 13.30578842 = 0.3326447108$

$0.3326447108 \cdot 389 = 129.3987925$

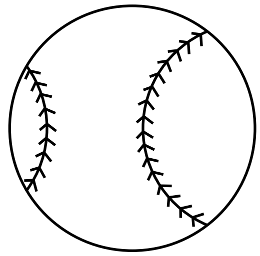
$\boxed{\text{129 lbs}}$

Score 3: The student made an error in finding the number of baseballs.

Question 34

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.

24in 12in 18in



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

24/2.94 12/2.94 18/2.94

8.2 4.1 6.1

8.2 × 4.1 × 6.1 = 205.1

205 baseballs can fit in the box

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi\, 1.47^3$

$V = 13.3$

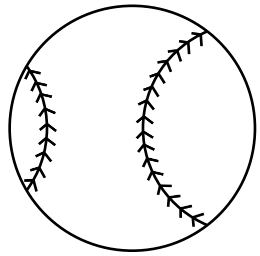
13.3 × .025 = .3325

.3325 × 205

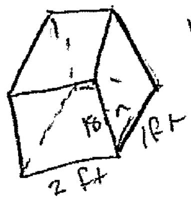
$\boxed{\text{68 Pounds}}$

Score 3: The student made an error in finding the number of baseballs.

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.



V prism $= Bh$
$= 2ft^2 \cdot 18 in$
$= 24 in^2 \cdot 18 in$
$= 432 in^3$

V ball $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\, 1.47^3$
$= 13.30578843$

number $= \frac{V\text{prism}}{V\text{ball}} = 32.46707268 =$ $\boxed{32 \text{ baseballs per box}}$

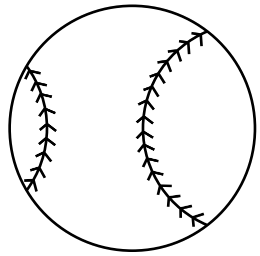
The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

Weight $= .025 \cdot 13.30578843$
$= .3326447108$ pound/ball
total weight $= .3326447108 \cdot 32$
$= 10.64463074$
$\boxed{\approx 11 \text{ pounds per box}}$

Score 2: The student found an appropriate weight of baseballs in a box, but no further correct work was shown.

Question 34

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

$V=(2)(1)(1.5)$
$V=3\text{ ft}$
$=36 \text{ in}^3$

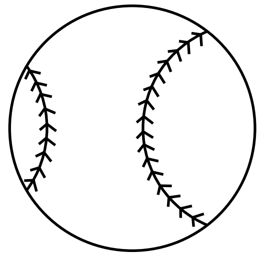
$V=\frac{4}{3}\pi r^3$
$V=\frac{4}{3}\pi(1.47)^3$
$V=13.3058 \text{ in}^3$

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

Score 1: The student found the volume of one baseball, but no further correct relevant work was shown.

Question 34

34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

$V = L \cdot W \cdot H$

$V = (24 \text{ in})(12 \text{ in})(18 \text{ in})$

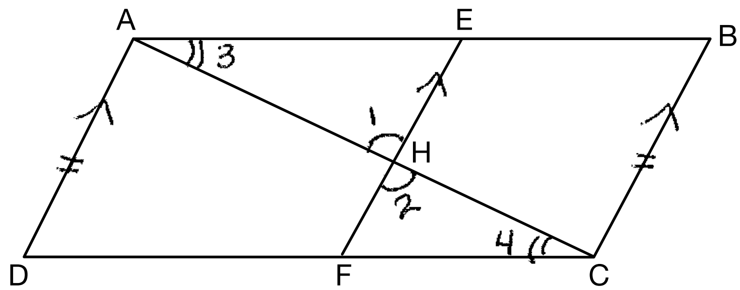
$V = 5184 \text{ in}^3$

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



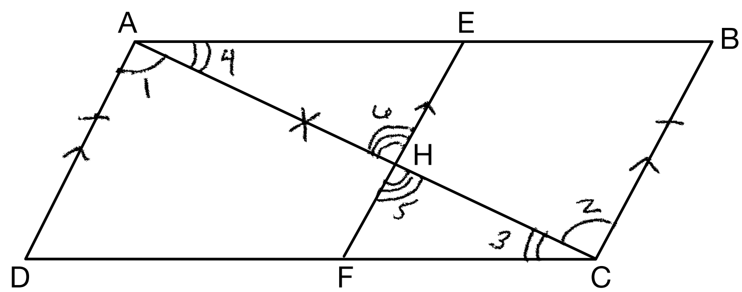
Prove: $(EH)(CH) = (FH)(AH)$

Statement	Reason
1. Quad ABCD, $\overline{AC}$ & $\overline{EF}$ intersect at H. $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. Transitive Postulate of parallel lines.
3. ABCD is a parallelogram	3. If 1 pair of opposite sides are ≅ and ∥, then Quad ABCD is a parallelogram.
4. ∡1 and ∡2 are vertical ∡s.	4. De inition of vertical ∡s.
5. ∡1 ≅ ∡2	5. vertical ∡'s are ≅
6. $\overline{AB} \parallel \overline{CD}$	6. In a parallelogram, opposite sides are ∥.
7. ∡3 ≅ ∡4	7. If 2 ∥ lines are cut by a transversal, then the alternate interior ∡'s are ≅.
8. △AHE ~ △CHF	8. AA ≅ AA
9. $\frac{EH}{FH} = \frac{AH}{CH}$	9. If 2 △'s are similar, their corresponding sides are in proportion.
10. (EH)(CH) = (FH)(AH)	10. In a proportion, the product of the means equals the product of the extremes.

Score 6: The student gave a complete and correct response.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



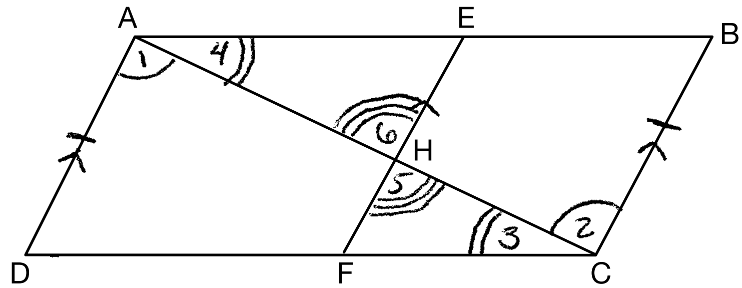
Prove: $(EH)(CH) = (FH)(AH)$

1. Quad $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. Transitive property
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive property
4. $\angle 1 \cong \angle 2$	4. When ∥ lines are cut by a transversal, alternate interior angles are ≅
5. $\triangle ADC \cong \triangle CBA$	5. SAS
6. $\angle 3 \cong \angle 4$	6. CPCTC
7. $\angle 5 \cong \angle 6$	7. Vertical angles are ≅
8. $\triangle HFC \sim \triangle HEA$	8. AA
9. $\dfrac{EH}{FH} = \dfrac{AH}{CH}$	9. Corresponding sides of similar triangles are in proportion
10. $(EH)(CH) = (FH)(AH)$	10. The product of the means equals the product of the extremes.

Score 6: The student gave a complete and correct response.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



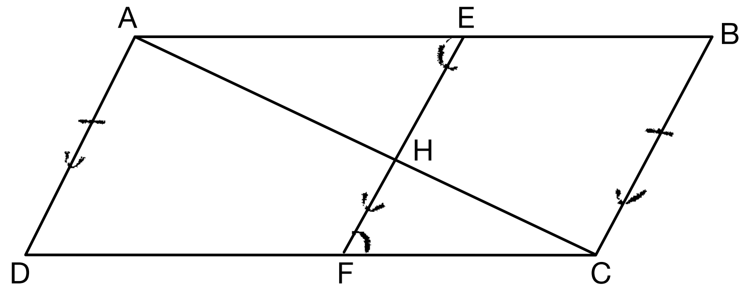
Prove: $(EH)(CH) = (FH)(AH)$

- Given quadrilateral ABCD, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$.
- Since $\overline{EF} \parallel \overline{BC}$ and $\overline{EF} \parallel \overline{AD}$ then $\overline{AD} \parallel \overline{BC}$ by the transitive property.
- So $\angle 1 \cong \angle 2$ because when two $\parallel$ lines are cut by a transversal, the alternate interior angles are congruent.
- Diagonal $\overline{AC} \cong \overline{AC}$ by reflexive $\therefore \triangle ADC \cong \triangle CBA$ by SAS.
- $\angle 3 \cong \angle 4$ because corresponding angles of $\cong$ triangles are $\cong$.
- $\angle 5 \cong \angle 6$ because vertical angles are $\cong$
- So $\triangle FHC \sim \triangle EHA$ by AA and then $\frac{EH}{FH} = \frac{AH}{CH}$ because corresponding sides of similar triangles are proportional.
- Therefore $(EH)(CH) = (FH)(AH)$ because the product of the means equals the product of the extremes.

Score 6: The student gave a complete and correct response.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



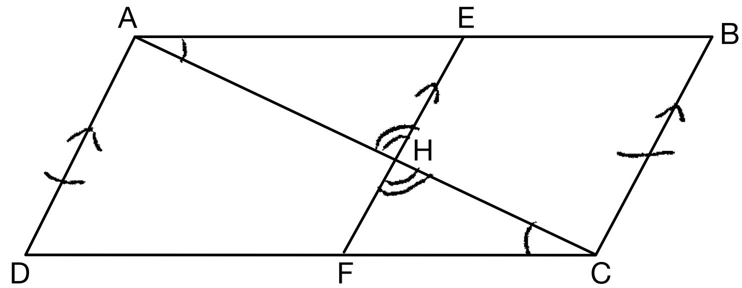
Prove: $(EH)(CH) = (FH)(AH)$

Statements	Reasons
1. Quad ABCD. $\overline{AC}$ intersects $\overline{EF}$ at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. Transitive Property
3. ABCD is a ▱	3. A quadrilateral with 2 opposite sides ≅ and ∥ is a ▱
4. $\overline{AB} \parallel \overline{DC}$	4. Def. of ▱
5 ∠AEH ≅ ∠CFH	5. If ∥ lines are cut by a transversal, alternate interior ∠s are ≅
6. ∠AHE ≅ ∠CHF	6. Vertical ∠s are ≅
7. △AHE ~ △CHF	7. AA~
8. $\frac{EH}{AH} = \frac{FH}{CH}$	8. Corresponding sides of similar △s are proportional
9. $(EH)(CH) = (FH)(AH)$	9. Substitution

Score 5: The student wrote an incorrect reason in step 9.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$

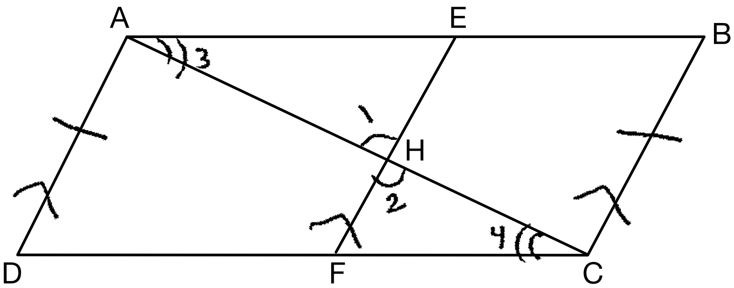


Prove: $(EH)(CH) = (FH)(AH)$ $\quad \frac{EH}{FH} = \frac{AH}{CH} \quad \triangle AHE \sim \triangle CHF$

S	R
① Quadrilateral ABCD, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	① Given
② Quad ABCD is a parallelogram	② If a quad has an opposite pair of sides ≅ and parallel, it is a parallelogram
③ ∠AHE and ∠CHF are vertical ∠s	③ Intersecting lines form vertical angles
④ ∠AHE ≅ ∠CHF	④ Vertical angles are ≅
⑤ $\overline{BA} \parallel \overline{DC}$	⑤ In a parallelogram, opposite sides are ∥
⑥ ∠EAH ≅ ∠HCF	⑥ If 2 ∥ lines are cut by a transversal, alternate interior angles are ≅
⑦ △AHE ~ △CHF	⑦ AA Similarity
⑧ $\frac{EH}{FH} = \frac{AH}{CH}$	⑧ Corresponding sides of similar △s are in proportion
⑨ (EH)(CH) = (FH)(AH)	⑨ In a proportion, the product of the means is equal to the product of the extremes

Score 5: The student did not state $\overline{AD} \parallel \overline{BC}$ to prove $ABCD$ is a parallelogram.

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



$\frac{EH}{FH} = \frac{AH}{CH}$

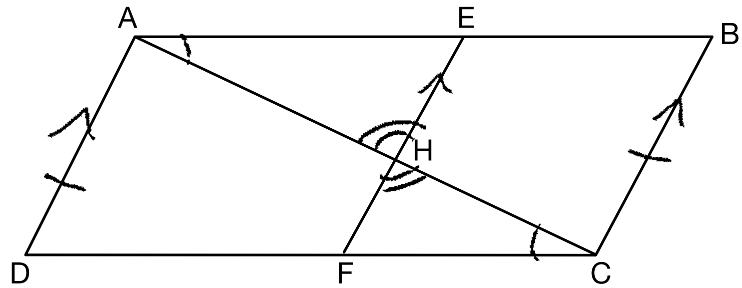
Prove: $(EH)(CH) = (FH)(AH)$

statements	reasons
① quad ABCD, $\overline{AC}$ + $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	① given
② $\overline{AB} \parallel \overline{CD}$	② opp sides of para are //
A ③ $\angle 3 \cong \angle 4$	③ if 2 lines // alt int ∠'s ≅
A ④ $\angle 1 \cong \angle 2$	④ vertical ∠'s ≅
⑤ $\triangle AHE \sim \triangle CHF$	⑤ AA
⑥ $\frac{EH}{FH} = \frac{AH}{CH}$	⑥ corr sides of ~Δ's are in proportion
⑦ $(EH)(CH) = (FH)(AH)$	⑦ prod of means = prod of extremes

Score 4: The student made a conceptual error by not proving $ABCD$ is a parallelogram.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



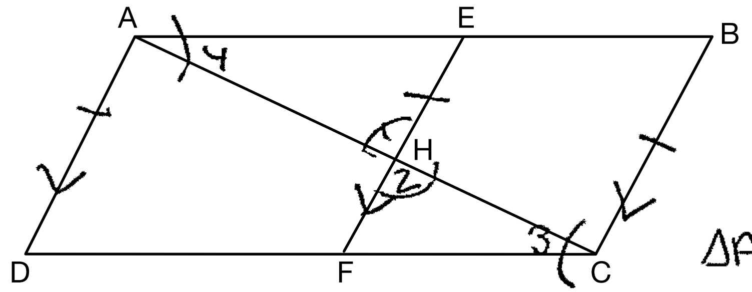
Prove: $(EH)(CH) = (FH)(AH)$

Statements	Reasons
1. Quad ABCD, $\overline{AC}$ and $\overline{EF}$ intersect at H	1. Given
2. $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$	2. Given
3. $\overline{AD} \parallel \overline{BC}$	3. If two lines are ∥ to the same line, then they are parallel
4. $\overline{AD} \cong \overline{BC}$	4. Given
5. Quad ABCD is a ▱	5. A quad with one pair of opposite sides that are ≅ and ∥, then it is a ▱
6. $\overline{AB} \parallel \overline{DC}$	6. def of parallelogram
7. ∡HAE ≅ ∡HCF	7. Alternate interior ∡s
8. ∡EHA and ∡FHC are vertical ∡s	8. def of vertical ∡s
9. ∡EHA ≅ ∡FHA	9. vertical ∡s are ≅
10. △AHE ~ △CHF	10. AA~
11. $\frac{EH}{FH} = \frac{AH}{CH}$	11. If two △s are similar, corresponding sides are in proportion
12. (EH)(CH) = (FH)(AH)	12. Cross products are equal

Score 4: The student gave an incorrect reason in step 7, and stated an incorrect angle in step 9.

35 Given: Quadrilateral *ABCD*, $\overline{AC}$ and $\overline{EF}$ intersect at *H*, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$

$\frac{EH}{FH} = \frac{AH}{CH}$



$\triangle AEH$
$\triangle FCH$

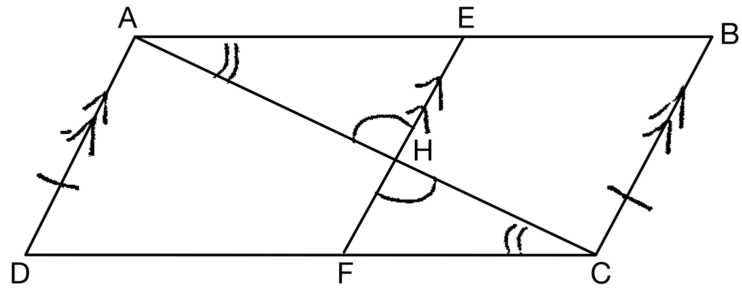
Prove: $(EH)(CH) = (FH)(AH)$

Statement	reason
1. Quad ABCD, $\overline{AC}$ and $\overline{EF}$ intersect at H $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$ $\overline{AD} \cong \overline{BC}$	1. Given
2. Quad ABCD is a parallelogram	2. A parallelogram has one pair of opposite sides congruent and parallel, then ABCD is a parallelogram.
3. $\angle 1 \cong \angle 2$	3. vertical angles are congruent
4. $\angle 3 \cong \angle 4$	4. If ∥ lines are cut by a transversal, the alternate int. angles are congruent
5. $\triangle AEH \sim \triangle CFH$	5. AA~Thm
6. $\frac{EH}{FH} = \frac{AH}{CH}$	6. corresponding sides of a congruent $\triangle$ are inporpotion
7. $EH \cdot CH = FH \cdot AH$	7. the product of the means is equal to the product of the extremes

Score 3: The student did not state $\overline{AD} \parallel \overline{BC}$ to prove *ABCD* is a parallelogram, did not state $\overline{AB} \parallel \overline{CD}$ to prove $\angle 3 \cong \angle 4$, and incorrectly stated congruent triangles in reason 6.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



Prove: $(EH)(CH) = (FH)(AH)$

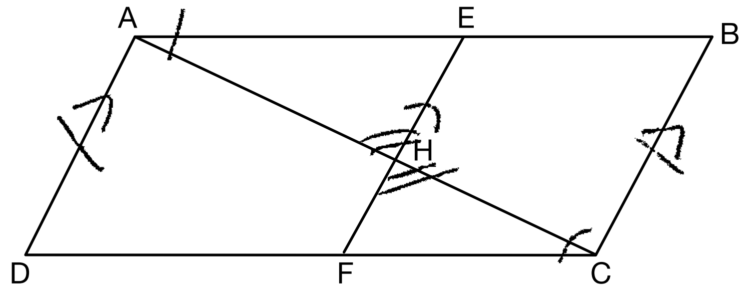
$\frac{EH}{CH} = \frac{FH}{AH}$ $EH \cong AH$ $FH \cong CH$

Statements	Reasons
1.) $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$	1.) Given
2.) $\overline{AD} \cong \overline{BC}$	2.) Given
3.) ABCD is PARA	3.) if opp sides $\cong$ & $\parallel$, PARA
4.) $\angle AHE \cong \angle FHC$	4.) vert $\angle$'s are $\cong$
5.) $\overline{AB} \parallel \overline{CD}$	5.) if PARA, opp sides $\parallel$
6.) $\angle BAC \cong \angle HCF$	6.) if lines $\parallel$, alt int $\angle$'s $\cong$
7.) $\triangle AEH \sim \triangle CFH$	7.) AA
8.) $(EH)(CH) = (FH)(AH)$	8.) if $\triangle$'s $\sim$, a proportion with sides of $\sim$ $\triangle$'s is correct

Score 3: The student did not state $\overline{AD} \parallel \overline{BC}$ to prove $ABCD$ is a parallelogram and gave no correct statements and reasons after step 7.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



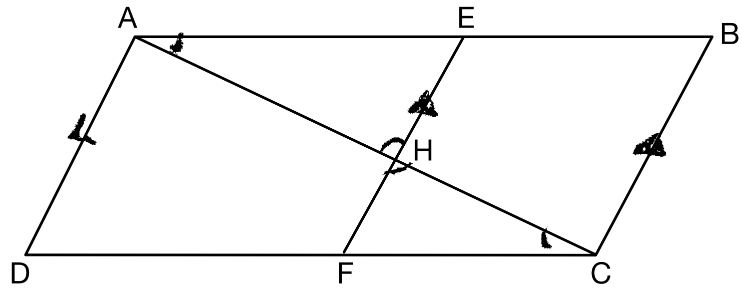
Prove: $(EH)(CH) = (FH)(AH)$

S	R
1) $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	1) given
2) $\angle EAH \cong \angle HCF$	2) ∥ lines form ≅ alt int. ∠'s
3) $\angle AHE \cong \angle FHC$	3) Vertical ∠'s are ≅
4) $\triangle AHE \sim \triangle CHF$	4) AA thm for similarity
5) $\frac{EH}{FH} = \frac{AH}{CH}$	5) Corresponding sides in ∼ △'s are proportional
6) $(EH)(CH) = (FH)(AH)$	6) Cross multiplying

Score 2: The student made a conceptual error by not proving $ABCD$ is a parallelogram, did not state $\overline{AB} \parallel \overline{CD}$ to prove $\angle EAH \cong \angle FCH$, and wrote an incorrect reason in step 6.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$

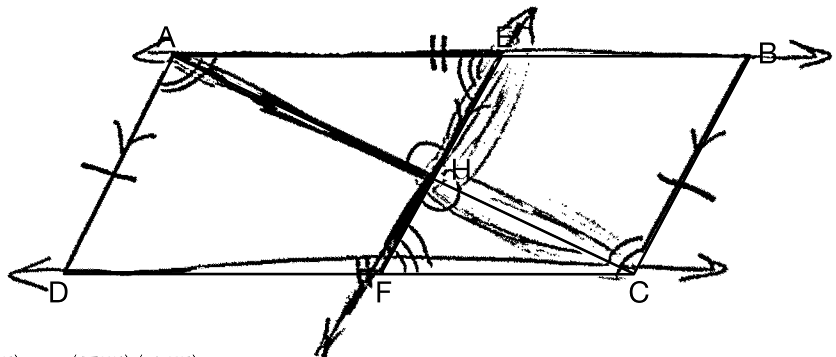


Prove: $(EH)(CH) = (FH)(AH)$

1. $\overline{AD} \parallel \overline{EF}$, $\overline{EF} \parallel \overline{BC}$	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. If two lines are parallel to the same line, then they are parallel to each other.
3. $\angle BAC \cong \angle ACD$	3. Alternate interior angles are congruent to each other.
4. $\angle EHA \cong \angle CHF$	4. Vertical angles are congruent to each other.
5. $\triangle AHE$ is similar to $\triangle CHF$	5. AA Similarity
6. $(EH)(CH) = (FH)(AH)$	6. Similar triangles are in proportion.

Score 2: The student wrote some correct relevant statements and reasons.

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



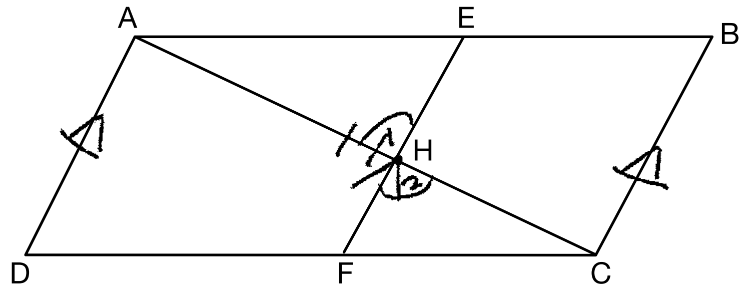
Prove: $(EH)(CH) = (FH)(AH)$

statement	reason
① quadrilateral ABCD, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$	① given
A ② $\angle AHE \cong \angle CHF$	② vertical angles are $\cong$
③ $\overline{AB} \parallel \overline{DC}$	③ opposite sides of a quadrilateral are parallel
A ④ $\angle AEH \cong \angle CFH$	④ two parallel lines cut by a transversal create congruent alternate interior angles
⑤ $\Delta AEH \sim \Delta CFH$	⑤ AA
⑥ $(EH)(CH) = (FH)(AH)$	⑥ corresponding parts of similiar triangles are similiar

Score 2: The student made a conceptual error in step 3 and gave no correct statements and reasons after step 5.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



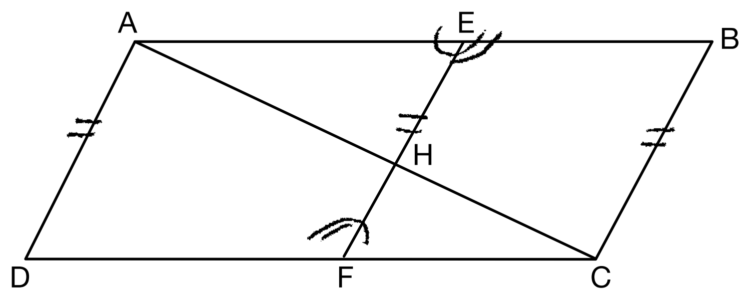
Prove: $(EH)(CH) = (FH)(AH)$

Statements	Reasons
1) Quadrilateral ABCD, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$	1) Given
2) $\angle 1, \angle 2$ are vert $\angle$'s	2) $\angle$'s that form a intersection are vert
3) $\angle 1 \cong \angle 2$	3) vert $\angle$'s are $\cong$
4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Prop
5) $\frac{EH}{FH} = \frac{CH}{AH}$	5) CSSTP
6) $(EH)(CH) = (FH)(AH)$	6) cross products

Score 1: The student only proved $\angle 1 \cong \angle 2$ correctly, and no further correct relevant work was shown.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



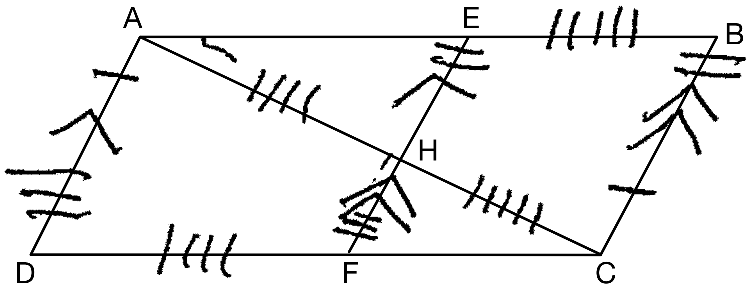
Prove: $(EH)(CH) = (FH)(AH)$

Statement	Reasoning
1. Quadrilateral ABCD, $\overline{AC}$ & $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$	1. Given
2. $\angle HEA \cong \angle HFC$	2. ∥ lines create ≅ alternate exterior angles
3. $\angle HEB \cong \angle HFD$	3. ∥ lines create ≅ alternate exterior angles.
4. $\overline{EH} \cong \overline{HF}$, $\overline{AH} \cong \overline{HC}$	4. They are proportional
5. $\frac{EH}{FH} = \frac{AH}{CH}$	5. proportional
6. (EH)(CH) = (FH)(AH)	6. cross multiplication

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 35

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



Prove: $(EH)(CH) = (FH)(AH)$

Statements	Reasons
1. Quadrilateral ABCD, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	1. given
2. AHFD and EHCB are parallelograms	2. They have opposite parallel sides
3. ▱ AHFD and ▱ EHCB have opposite congruent sides.	3. Parallelograms have opposite parallel sides.
4. (EH)(CH)=(FH)(AH)	4. Corresponding parts or Corresponding figures and equal

Score 0: The student did not show enough correct relevant work to receive any credit.