

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 28, 2015 — 9:15 a.m.

SAMPLE RESPONSE SET

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Question 29

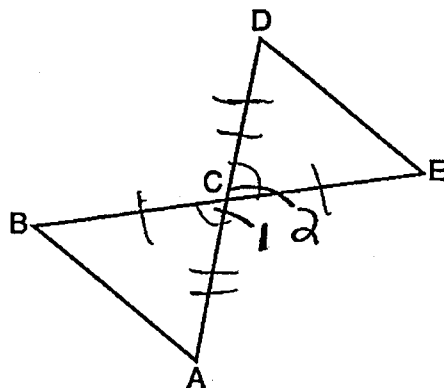
29 Given: \overline{BE} and \overline{AD} intersect at point C

$$\overline{BC} \cong \overline{EC}$$

$$\overline{AC} \cong \overline{DC}$$

\overline{AB} and \overline{DE} are drawn

Prove: $\triangle ABC \cong \triangle DEC$



S	R
$\overline{BC} \cong \overline{EC}$	Given
$\overline{AC} \cong \overline{DC}$	Given
$\angle 1 \cong \angle 2$	Intersecting lines form congruent vertical angles.
$\triangle ABC \cong \triangle DEC$	SAS postulate

Score 2: The student has a complete and correct proof.

Question 29

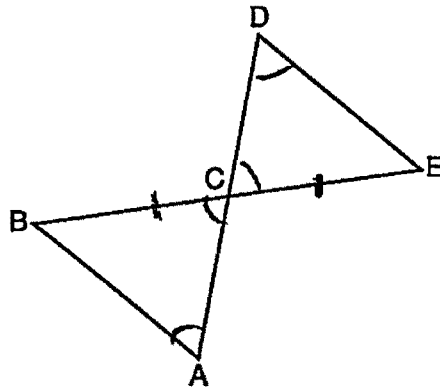
29 Given: \overline{BE} and \overline{AD} intersect at point C

$$\overline{BC} \cong \overline{EC}$$

$$\overline{AC} \cong \overline{DC}$$

\overline{AB} and \overline{DE} are drawn

Prove: $\triangle ABC \cong \triangle DEC$



statement	reason
1. $\overline{BC} \cong \overline{EC}$ $\overline{AC} \cong \overline{DC}$	1. given
2. $\angle BCA \cong \angle DCE$	2. vertical \angle s are \cong
3. $\angle D \cong \angle A$	3. alt. interior \angle s are \cong
4. $\triangle ABC \cong \triangle DEC$	4. AAS \cong AAS

Score 1: The student made a conceptual error in line 3, but had an appropriate concluding statement.

Question 29

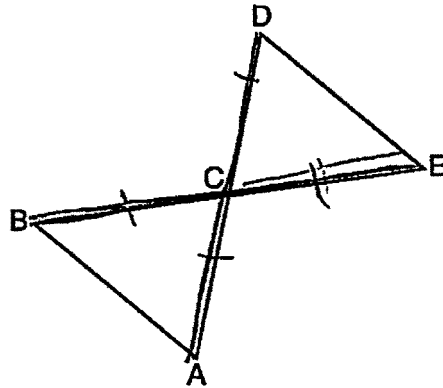
29 Given: \overline{BE} and \overline{AD} intersect at point C

$$\overline{BC} \cong \overline{EC}$$

$$\overline{AC} \cong \overline{DC}$$

\overline{AB} and \overline{DE} are drawn

Prove: $\triangle ABC \cong \triangle DEC$

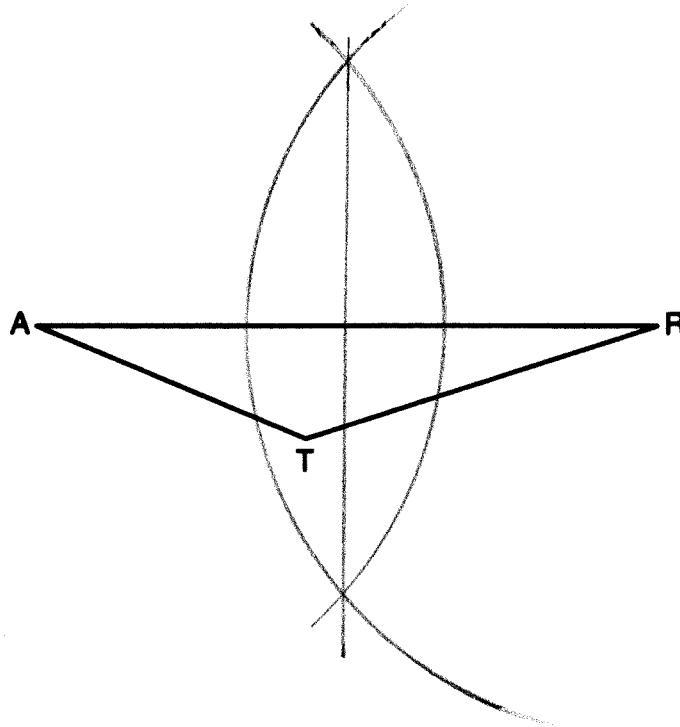


G	R
<p>① \overline{BE} + \overline{AD} intersect at point C $\overline{BC} \cong \overline{EC}$ $\overline{AC} \cong \overline{DC}$ \overline{AB} and \overline{DE} are drawn</p>	<p>① Given</p>
<p>② $\overline{BE} \cong \overline{AD}$</p>	<p>② Def of \perp bisect</p>
<p>③ $\overline{AD} \cong \overline{EB}$</p>	<p>③ CPCTC</p>
<p>④ $\overline{DA} \cong \overline{DA}$</p>	<p>④ reflexive prop</p>
<p>⑤ $\overline{BA} \cong \overline{DE}$</p>	<p>⑤</p>
<p>⑥ $\triangle ABC \cong \triangle DEC$</p>	<p>⑥ AAS Thm</p>

Score 0: The student has only the given correct.

Question 30

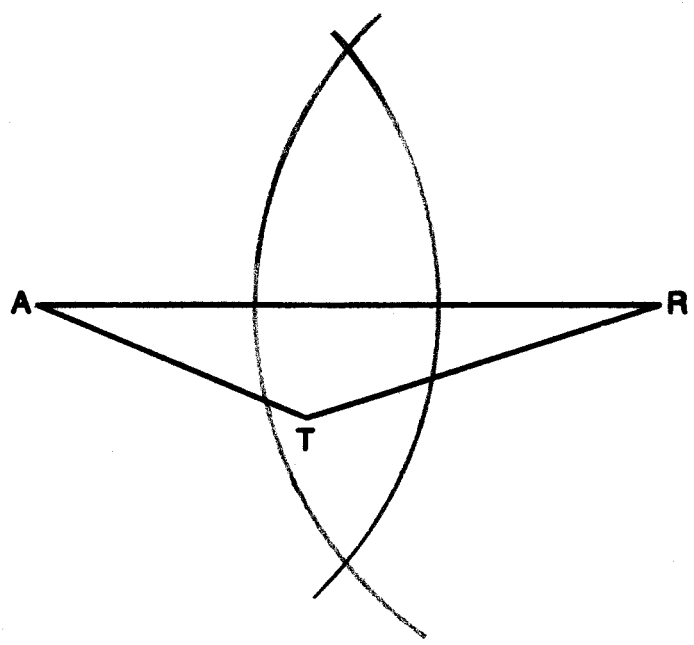
30 Using a compass and straightedge, construct the perpendicular bisector of side \overline{AR} in $\triangle ART$ shown below. [Leave all construction marks.]



Score 2: The student drew a correct construction showing all appropriate arcs, and the perpendicular bisector is drawn.

Question 30

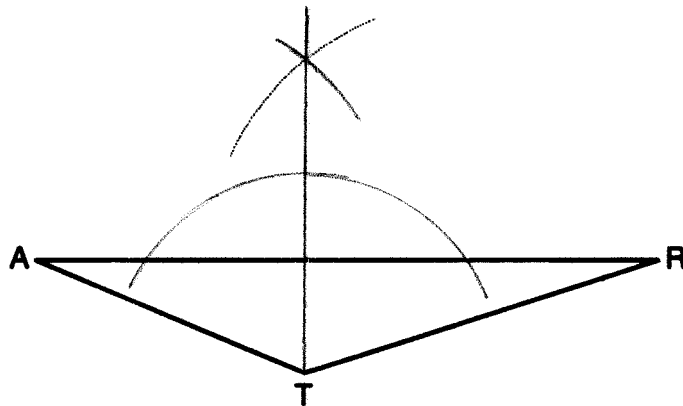
30 Using a compass and straightedge, construct the perpendicular bisector of side \overline{AR} in $\triangle ART$ shown below. [Leave all construction marks.]



Score 1: The student constructed all appropriate arcs, but did not draw the perpendicular bisector.

Question 30

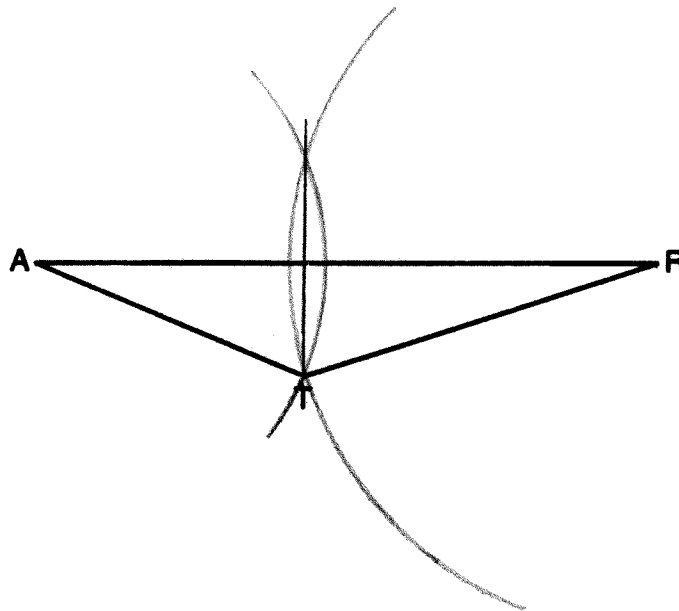
30 Using a compass and straightedge, construct the perpendicular bisector of side \overline{AR} in $\triangle ART$ shown below. [Leave all construction marks.]



Score 1: The student constructed a line perpendicular to \overline{AR} from vertex T . The line does not bisect \overline{AR} .

Question 30

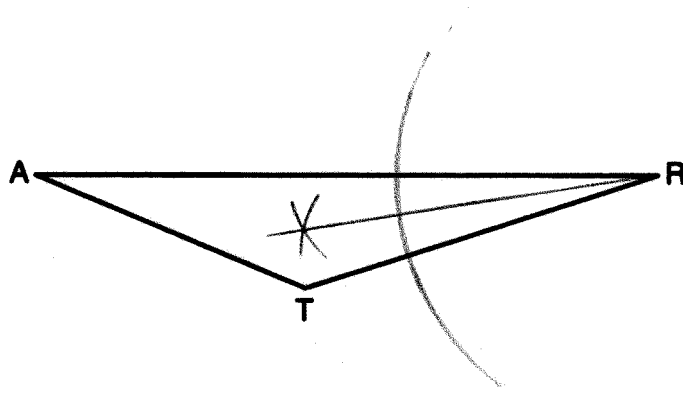
30 Using a compass and straightedge, construct the perpendicular bisector of side \overline{AR} in $\triangle ART$ shown below. [Leave all construction marks.]



Score 1: The student constructed a line perpendicular to \overline{AR} , but it does not bisect \overline{AR} .

Question 30

30 Using a compass and straightedge, construct the perpendicular bisector of side \overline{AR} in $\triangle ART$ shown below. [Leave all construction marks.]



Score 0: The student constructed the bisector of angle R . This construction is not relevant to the problem.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

10 side

$$\frac{180(n-2)}{n}$$

$$\frac{180(10-2)}{10}$$

$$\frac{180(8)}{10}$$

$$\frac{1440}{10}$$

$$1 \text{ Int } \angle = 144$$

Score 2: The student has a complete and correct response to find the measure of an interior angle.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

10



$$\text{exterior } \angle = \frac{360^\circ}{10} = 36^\circ$$

$$\begin{array}{r} 180 \\ - 36 \\ \hline 144 \end{array}$$

$$\text{An interior } \angle = 144^\circ$$

Score 2: The student has a complete and correct response. The student showed the correct work to find the measure of an exterior angle and used it to find the correct measure of an interior angle.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

180

180 3

180 4

180 5

180 6

180 7

180 8

180 9

180 10

1440

Score 1: The student did not show enough work to earn full credit, but the correct answer was stated.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

$$(n-2) \cdot 180$$

$$(10-2) \cdot 180$$

$$8 \cdot 180$$

$$1440$$

Score 1: The student showed appropriate work to find the sum of the interior angles of a decagon, but no further correct work is shown.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

$$\frac{360}{10} = 36$$

Score 1: The student showed appropriate work to find the measure of an exterior angle of a decagon, but no further correct work is shown.

Question 31

31 Determine and state the measure, in degrees, of an interior angle of a regular decagon.

$$10 \overline{) 180} \quad 18^\circ$$

Score 0: The student showed no relevant work.

Question 32

32 Write an equation of a line that is parallel to the line whose equation is $3y = x + 6$ and that passes through the point $(-3, 4)$.

$$\frac{3y}{3} = \frac{x+6}{3}$$

$$y = \frac{1}{3}x + 2$$

$$4 = \frac{1}{3}(-3) + b$$

$$4 = -1 + b$$

$$5 = b$$

$$y = \frac{1}{3}x + 5$$

Score 2: The student has a complete and correct response.

Question 32

32 Write an equation of a line that is parallel to the line whose equation is $3y = x + 6$ and that passes through the point $(-3, 4)$.

$$y = \frac{1}{3}x + 2$$

$$\frac{1}{3} = \frac{y - 4}{x + 3}$$

$$y - 4 = \frac{1}{3}(x + 3)$$

Score 2: The student has a complete and correct response.

Question 32

32 Write an equation of a line that is parallel to the line whose equation is $3y = x + 6$ and that passes through the point $(-3, 4)$.

$$\begin{aligned} & 3y = x + 6 \quad (-3, 4) \\ & \div 3 \quad \div 3 \\ \hline & y = \frac{1}{3}x + 2 \quad \leftarrow \text{substitute} \\ & \quad \quad \quad \downarrow \\ & 4 = \frac{1}{3}(-3) + b \\ & 4 = -1 + b \\ & +1 + 1 \\ & \boxed{5 = b} \end{aligned}$$

Score 1: The student found the correct slope and y -intercept, but did not write an equation.

Question 32

32 Write an equation of a line that is parallel to the line whose equation is $3y = x + 6$ and that passes through the point $(-3, 4)$.

$$\frac{3y = x + 6}{3 \quad 3}$$

$$y = 3x + 2$$

~~$$y = 3x + 2$$~~

$$4 = 3(-3) + b$$

$$4 = -9 + b$$

$$13 = b$$

$$y = 3x + 13$$

Score 1: The student made an error when solving for y . The student wrote an appropriate equation.

Question 32

32 Write an equation of a line that is parallel to the line whose equation is $3y = x + 6$ and that passes through the point $(-3, 4)$.

$$\frac{3Y}{3} = \frac{x + 6}{3}$$

$$Y = \frac{x}{3} + 2$$

$$(-3, 4)$$

$$m = -3$$

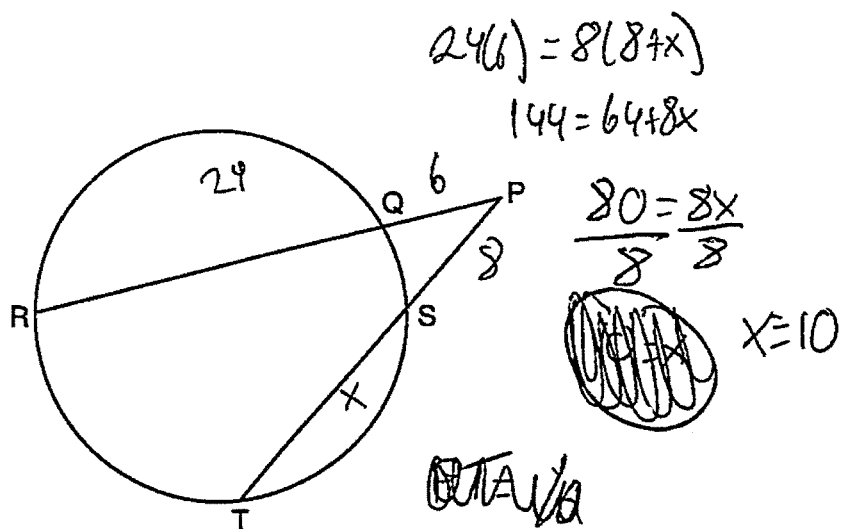
$$b = 4$$

$$Y = -3x + 4$$

Score 0: The student used the wrong slope and made a conceptual error when finding the value of b .

Question 33

33 In the diagram below, secants \overline{PQR} and \overline{PST} are drawn to a circle from point P .



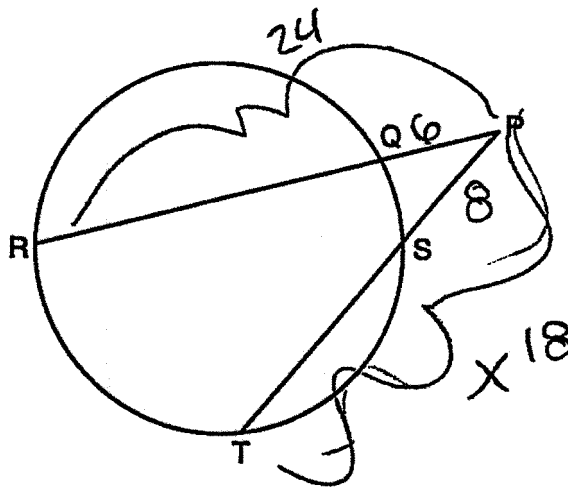
If $PR = 24$, $PQ = 6$, and $PS = 8$, determine and state the length of \overline{PT} .

$PT = 18$

Score 2: The student has a complete and correct response.

Question 33

33 In the diagram below, secants \overline{PQR} and \overline{PST} are drawn to a circle from point P .



If $PR = 24$, $PQ = 6$, and $PS = 8$, determine and state the length of \overline{PT} .

$$(PR)(PQ) = (PT)(PS)$$
$$(24)(6) = (x)(8)$$

$$\frac{144}{8} = \frac{8x}{8}$$

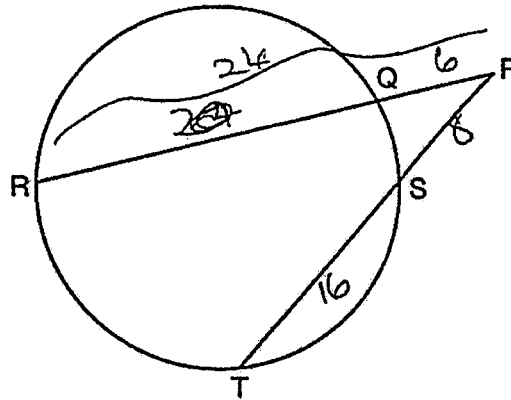
$$\boxed{x=18}$$

$$\overline{PT} = 18$$

Score 2: The student has a complete and correct response.

Question 33

33 In the diagram below, secants \overline{PQR} and \overline{PST} are drawn to a circle from point P .



If $PR = 24$, $PQ = 6$, and $PS = 8$, determine and state the length of \overline{PT} .

~~(PR)~~

~~$(PS)(QR) = (PS)(ST)$~~

~~$(6)(18) = (8)(x)$~~

~~$108 = 8x$~~

$(PQ)(PR) = (PS)(PT)$

$(6)(24) = (8)(x+8)$

$144 = 16 + 8x$

$-16 \quad -16$

$128 = 8x$

$\frac{128}{8} = \frac{8x}{8}$

$x = 16$

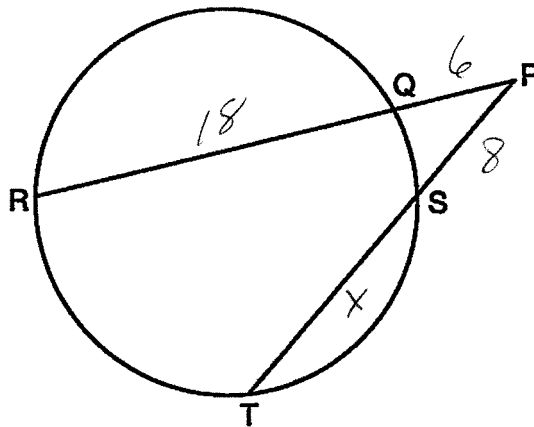
$16 + 8 = 24$

$PT = 24$

Score 1: The student made a computational error when distributing 8. The student showed appropriate work to find a length for \overline{PT} .

Question 33

33 In the diagram below, secants \overline{PQR} and \overline{PST} are drawn to a circle from point P .



If $PR = 24$, $PQ = 6$, and $PS = 8$, determine and state the length of \overline{PT} .

$$6 \cdot 18 = 8 \cdot x$$
$$108 = 8x$$
$$13\frac{1}{2} = x$$

Score 0: The student made a conceptual error in finding the length of \overline{ST} , and did not find the length of \overline{PT} .

Question 34

34 The slope of \overline{QR} is $\frac{x-1}{4}$ and the slope of \overline{ST} is $\frac{8}{3}$. If $\overline{QR} \perp \overline{ST}$, determine and state the value of x .

$$\begin{array}{l} \overline{ST} = \frac{8}{3} \\ \overline{QR} = \frac{x-1}{4} \end{array} \quad \frac{8}{3} \perp \frac{-3}{8}$$

$$\frac{x-1}{4} \times \frac{-3}{8}$$

$$-12 = 8(x-1)$$

$$\begin{array}{r} -12 = 8x - 8 \\ +8 \quad \quad +8 \end{array}$$

$$\frac{-4}{8} = \frac{8x}{8}$$

$$x = -0.5$$

Score 2: The student has a complete and correct response.

Question 34

34 The slope of \overline{QR} is $\frac{x-1}{4}$ and the slope of \overline{ST} is $\frac{8}{3}$. If $\overline{QR} \perp \overline{ST}$, determine and state the value of x .

$$\begin{array}{r} \frac{\cancel{8} - 1}{4} = \frac{\cancel{8}}{3} \\ 3x - 3 = 32 \\ \hline 3x = 35 \\ \frac{3x}{3} = \frac{35}{3} \\ x = 11.6667 \end{array}$$

Score 1: The student made a conceptual error by setting the slopes equal, but found an appropriate value of x .

Question 34

34 The slope of \overline{QR} is $\frac{x-1}{4}$ and the slope of \overline{ST} is $\frac{8}{3}$. If $\overline{QR} \perp \overline{ST}$, determine and state the value of x .

$$\frac{x-1}{4} = \frac{3}{8}$$

$$12 = 8(x-1)$$

$$12 = 8x - \cancel{8} \\ + 8 \quad + 8$$

$$\frac{20}{8} = \frac{8x}{8}$$

$$\boxed{x=2.5}$$

Score 1: The student made a conceptual error by using a slope of $\frac{3}{8}$, but found an appropriate value of x .

Question 34

34 The slope of \overline{QR} is $\frac{x-1}{4}$ and the slope of \overline{ST} is $\frac{8}{3}$. If $\overline{QR} \perp \overline{ST}$, determine and state the value of x .

The student's work is as follows:

$$\frac{x-1}{4} = \frac{3}{8}$$
$$6x - 8 = 12$$
$$\begin{array}{r} 6x - 8 = 12 \\ +8 \quad +8 \\ \hline 6x = 20 \\ \frac{6x}{6} = \frac{20}{6} \\ x = \frac{10}{3} \end{array}$$

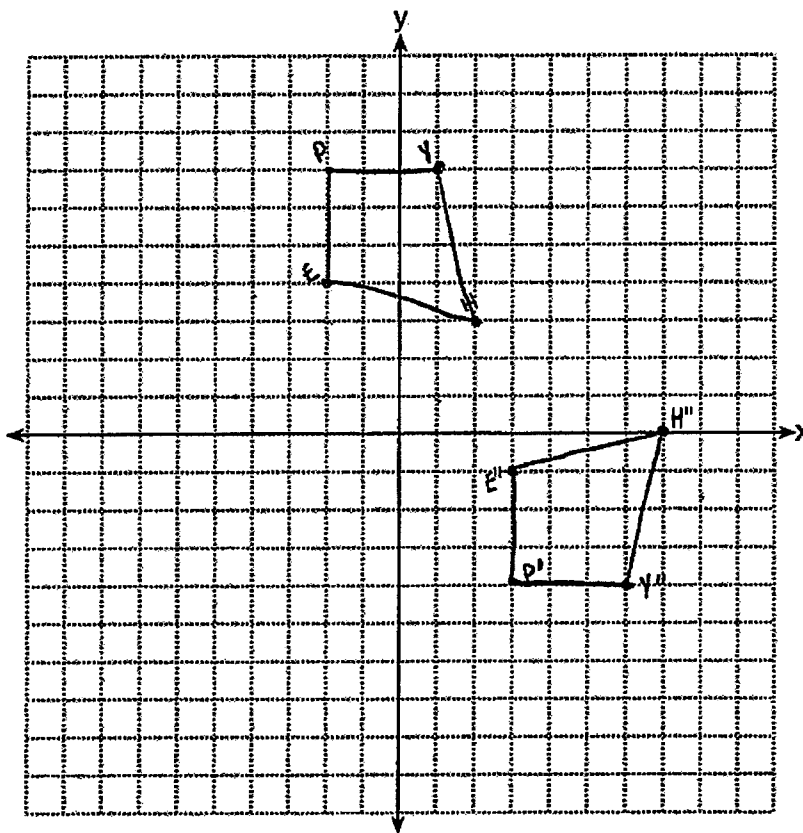
The final answer $x = \frac{10}{3}$ is circled.

Score 0: The student made a conceptual error by using a slope of $\frac{3}{8}$ and made two computational errors.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$\begin{array}{r} H: 2, 3 \\ + 5, -3 \\ \hline H' (7, 0) \\ H'' (7, 0) \end{array}$	$\begin{array}{r} Y: 1, 7 \\ + 5, -3 \\ \hline (6, 4) \\ (6, -4) \end{array}$	$\begin{array}{r} P: -2, 7 \\ + 5, -3 \\ \hline (3, 4) \\ (3, -4) \end{array}$	$\begin{array}{r} E: -2, 4 \\ + 5, -3 \\ \hline (3, 1) \\ (3, -1) \end{array}$
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Score 4: The student has a complete and correct response.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$$H' = (7, 0)$$

$$Y' = (6, 4)$$

$$P' = (3, 4)$$

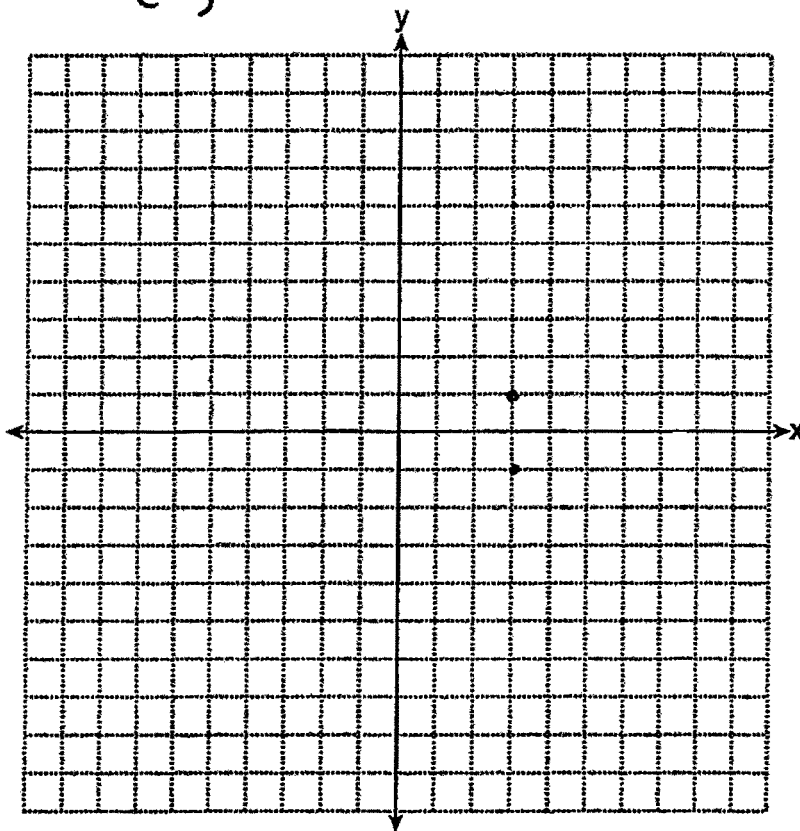
$$E' = (3, 1)$$

$$H'' = (7, 0)$$

$$Y'' = (6, -4)$$

$$P'' = (3, -4)$$

$$E'' = (3, -1)$$

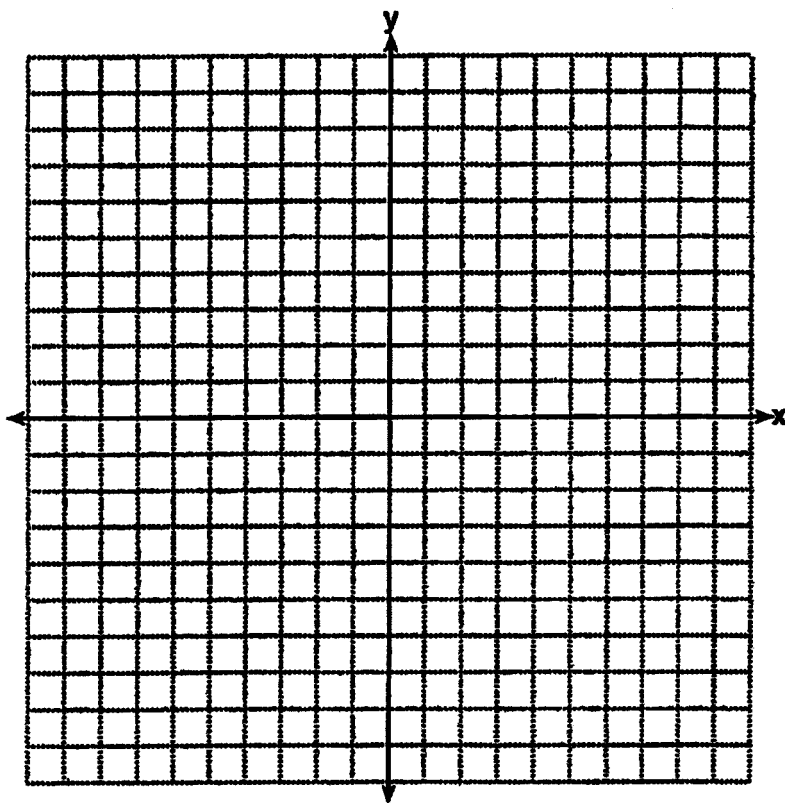


Score 4: The student has a complete and correct response. The student showed the middle step in mapping $HYPE$ onto $H''Y''P''E''$. The correct coordinates were stated.

Question 35

35 Quadrilateral *HYPE* has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$H(2,3)$		$(7,0)$		$(7,0)$
$Y(1,7)$	$\xrightarrow{T_{5,-3}}$	$(6,4)$	$\xrightarrow{r_x}$	$(6,-4)$
$P(-2,7)$		$(3,4)$		$(3,-4)$
$E(-2,4)$		$(3,1)$		$(3,-1)$

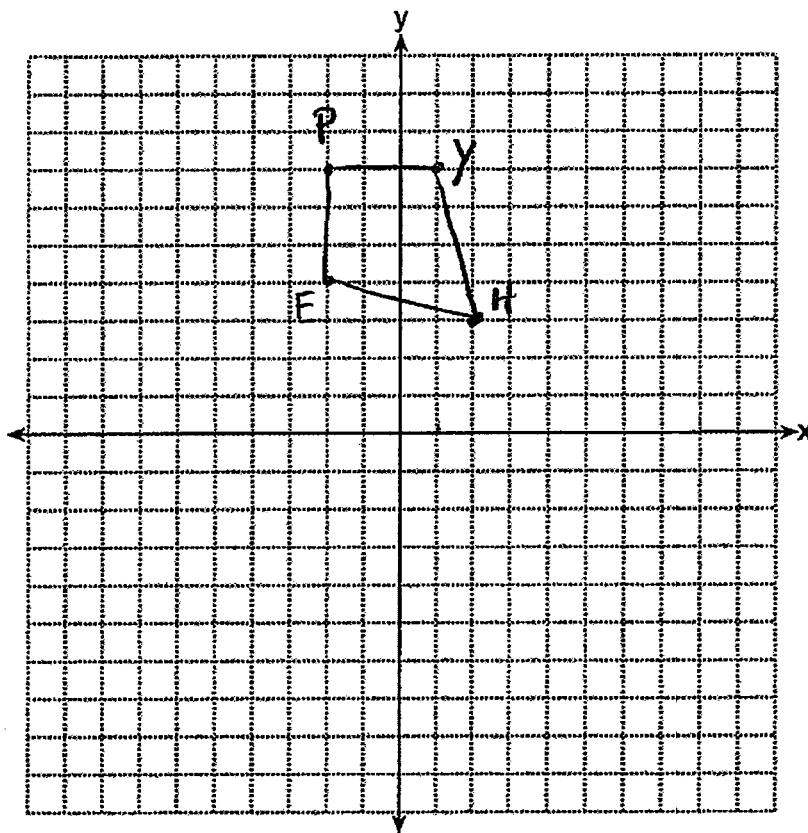


Score 4: The student showed the minimum amount of work to receive full credit.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$T_{5,-3}$
 $H'(7,0)$ $Y'(6,4)$ $P'(3,4)$ $E'(3,-1)$
 $r_{x\text{-axis}}$
 $H''(7,0)$ $Y''(6,-4)$ $P''(3,-4)$ $E''(3,1)$

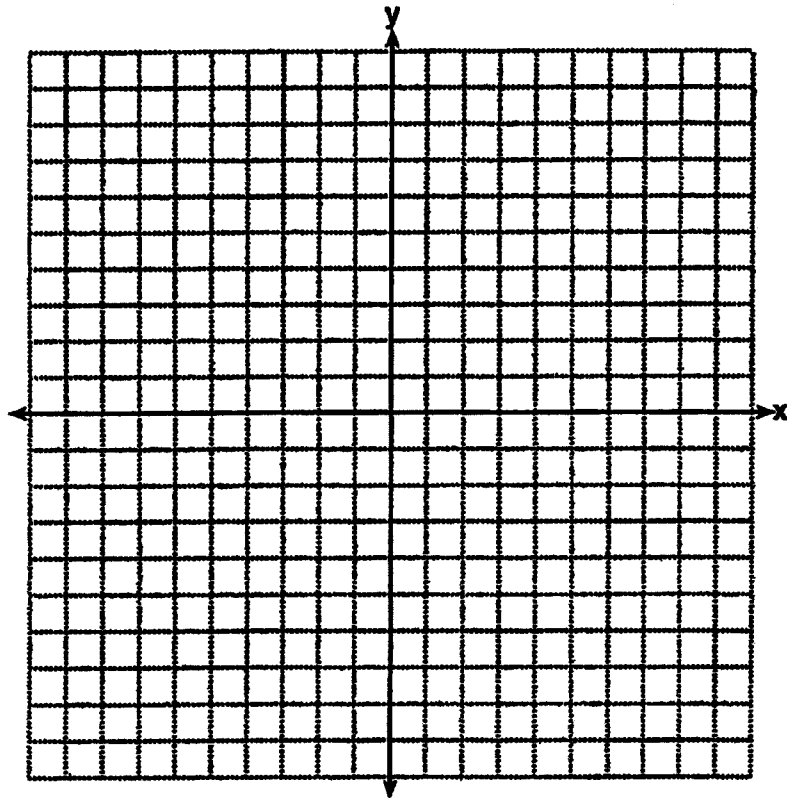


Score 3: The student made a computational error in finding the coordinates for E' . Appropriate coordinates for $H''Y''P''E''$ were stated.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$H(2,3) H'(7,0) H''(0,7)$
 $Y(1,7) Y'(6,4) Y''(4,6)$
 $P(-2,7) P'(3,4) P''(4,3)$
 $E(-2,4) E'(3,1) E''(1,3)$



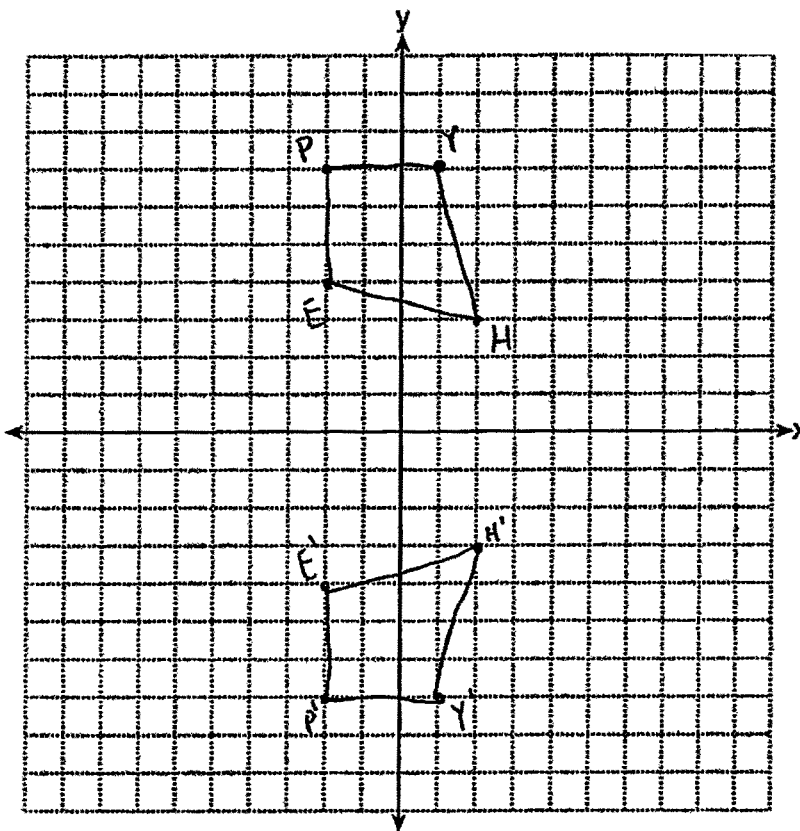
Score 2: The student stated the correct coordinates of $H'Y'P'E'$, but made a conceptual error in finding $H''Y''P''E''$.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$$H' (2, -3) \quad Y' (1, -7) \quad P' (-2, -2) \quad E' (-2, -4)$$

$$H'' (7, -6) \quad Y'' (6, -10) \quad P'' (3, -10) \quad E'' (3, -7)$$



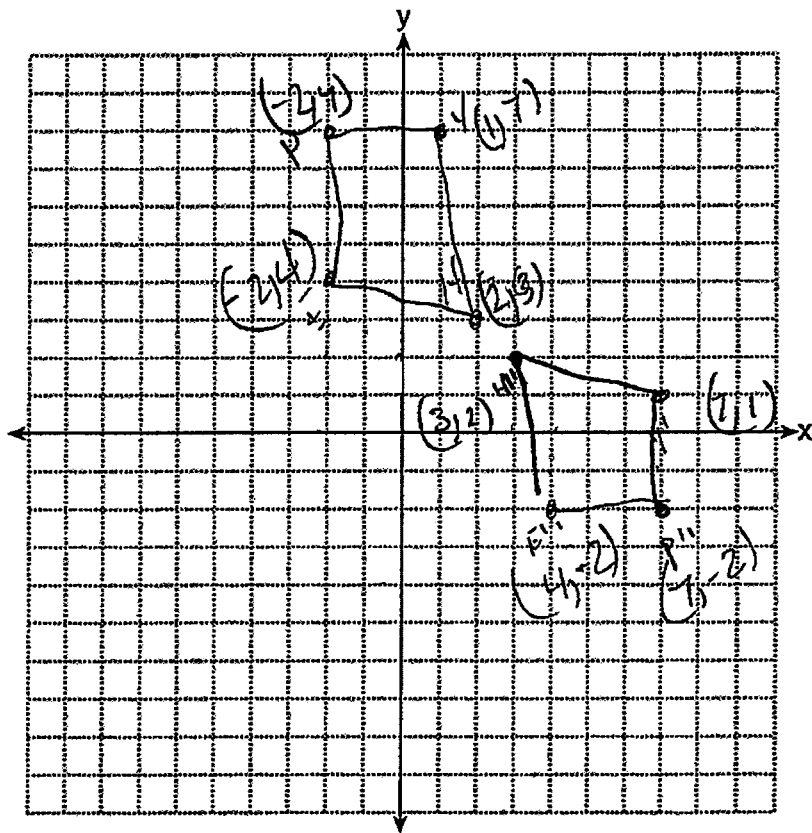
Score 2: The student made a conceptual error by performing the reflection before the translation.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$\begin{matrix} 2, 3 \\ +5, -3 \\ H' 7, 0 \end{matrix}$
 $\begin{matrix} Y 1, 7 \\ +5, -3 \\ Y' 6, 4 \end{matrix}$
 $\begin{matrix} P -2, 7 \\ 5, -3 \\ P' 3, 4 \end{matrix}$
 $\begin{matrix} E -2, 4 \\ +5, -3 \\ E' 3, 1 \end{matrix}$

$H'' 3, 2$
 $Y'' 7, 1$
 $P'' 7, -2$
 $E'' 4, -2$

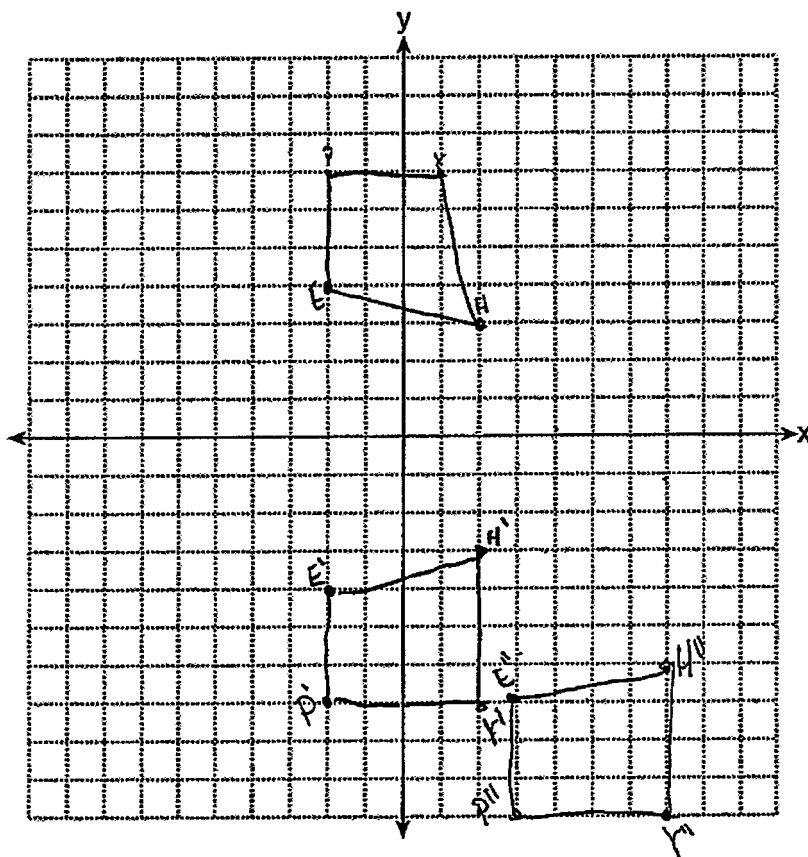


Score 1: The student stated appropriate coordinates for $H'Y'P'E'$, but the parentheses are missing. No further correct work is shown.

Question 35

35 Quadrilateral $HYPE$ has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$$H''(7,-6) \quad Y''(7,-10) \quad P''(3,-10) \quad E''(3,-7)$$

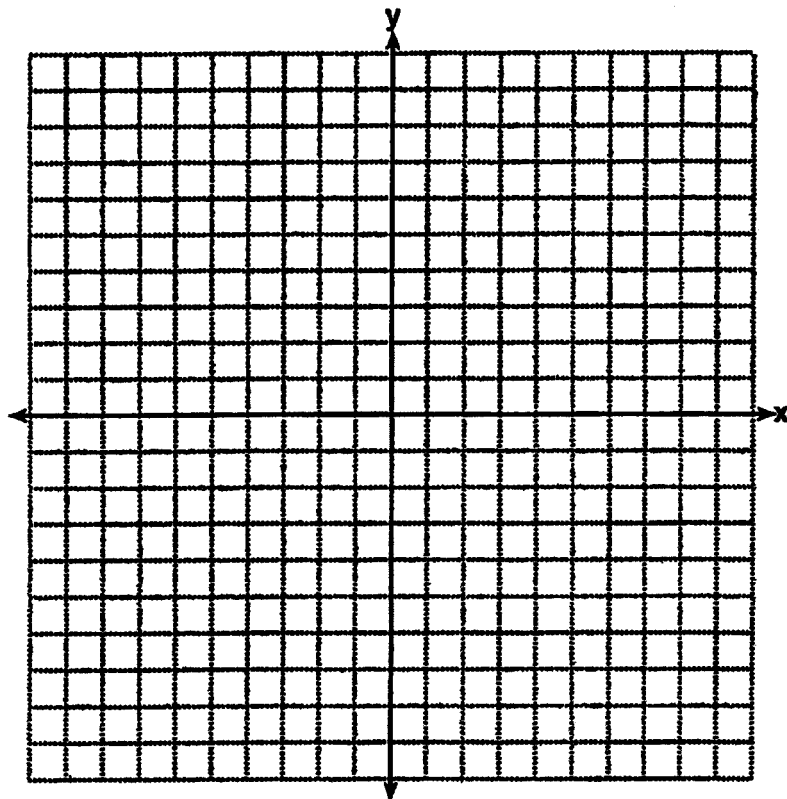


Score 1: The student made a conceptual error by performing the reflection before the translation and graphed Y'' incorrectly. Appropriate coordinates were stated.

Question 35

35 Quadrilateral *HYPE* has vertices $H(2,3)$, $Y(1,7)$, $P(-2,7)$, and $E(-2,4)$. State and label the coordinates of the vertices of $H''Y''P''E''$ after the composition of transformations $r_{x\text{-axis}} \circ T_{5,-3}$. [The use of the set of axes below is optional.]

$H(2,3)$ $Y(1,7)$ $P(-2,7)$ $E(-2,4)$
 $H'(10,-9)$ $Y'(5,-2)$ $P'(-10,-2)$ $E'(-10,12)$



Score 0: The student had a completely incorrect response.

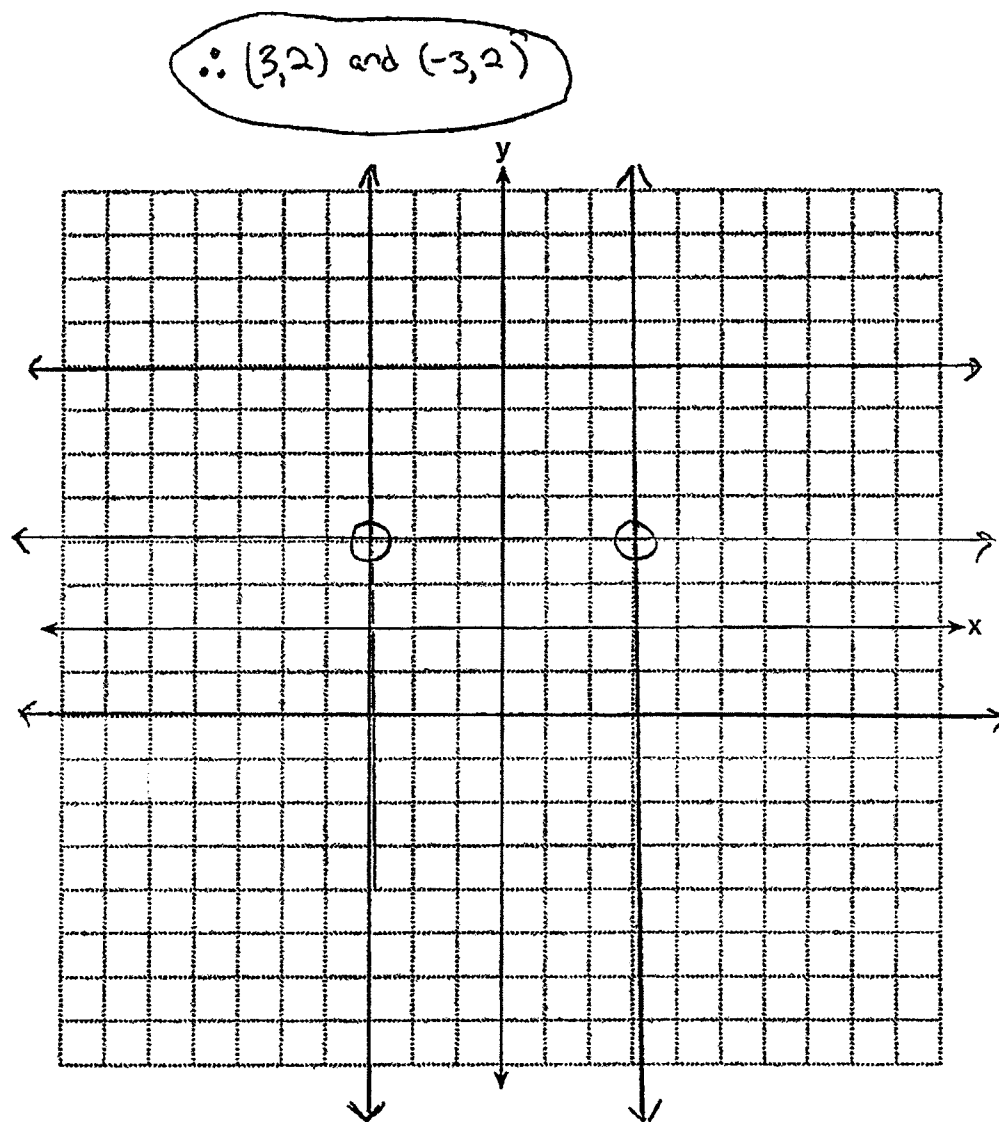
Question 36

36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

State the coordinates of the points that satisfy both loci.



Score 4: The student has a complete and correct response.

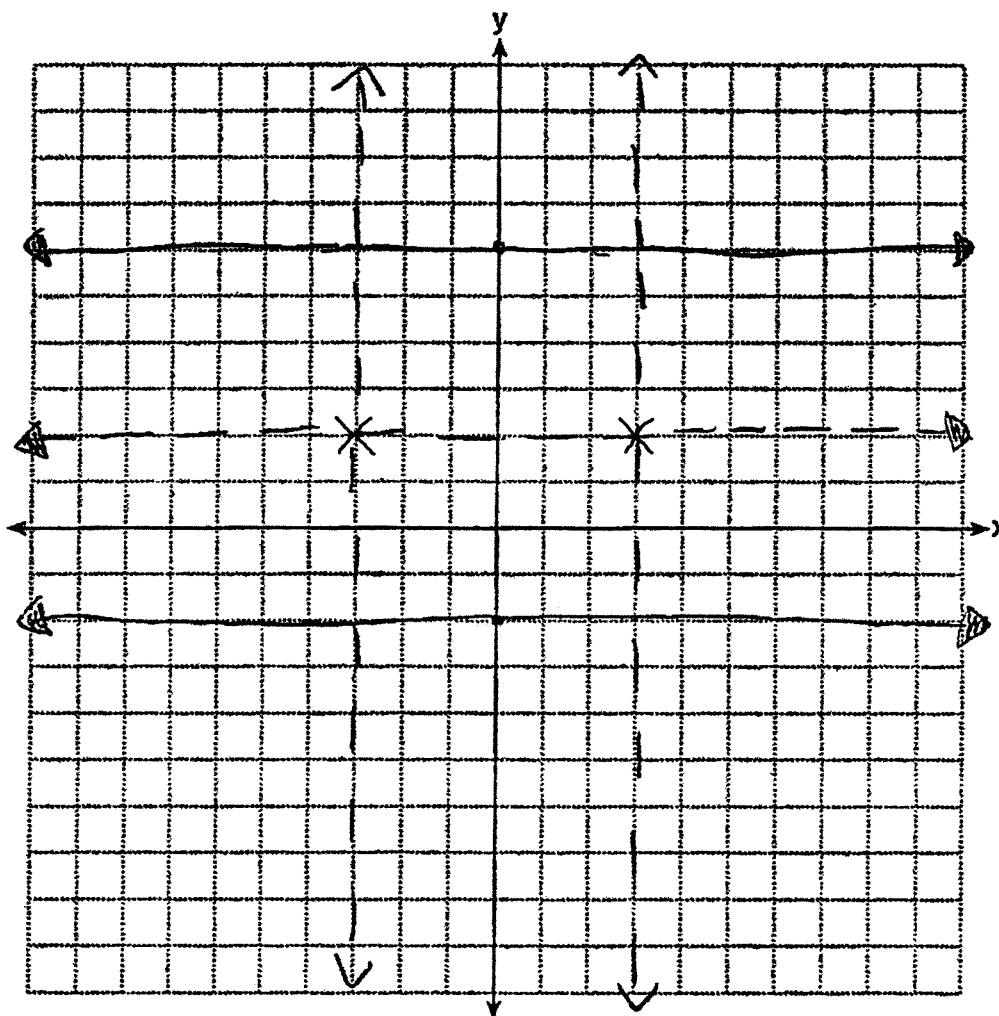
Question 36

36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

State the coordinates of the points that satisfy both loci.



Score 3: The student graphed both loci correctly and labeled points that satisfied both with an **X**, but the coordinates were not stated.

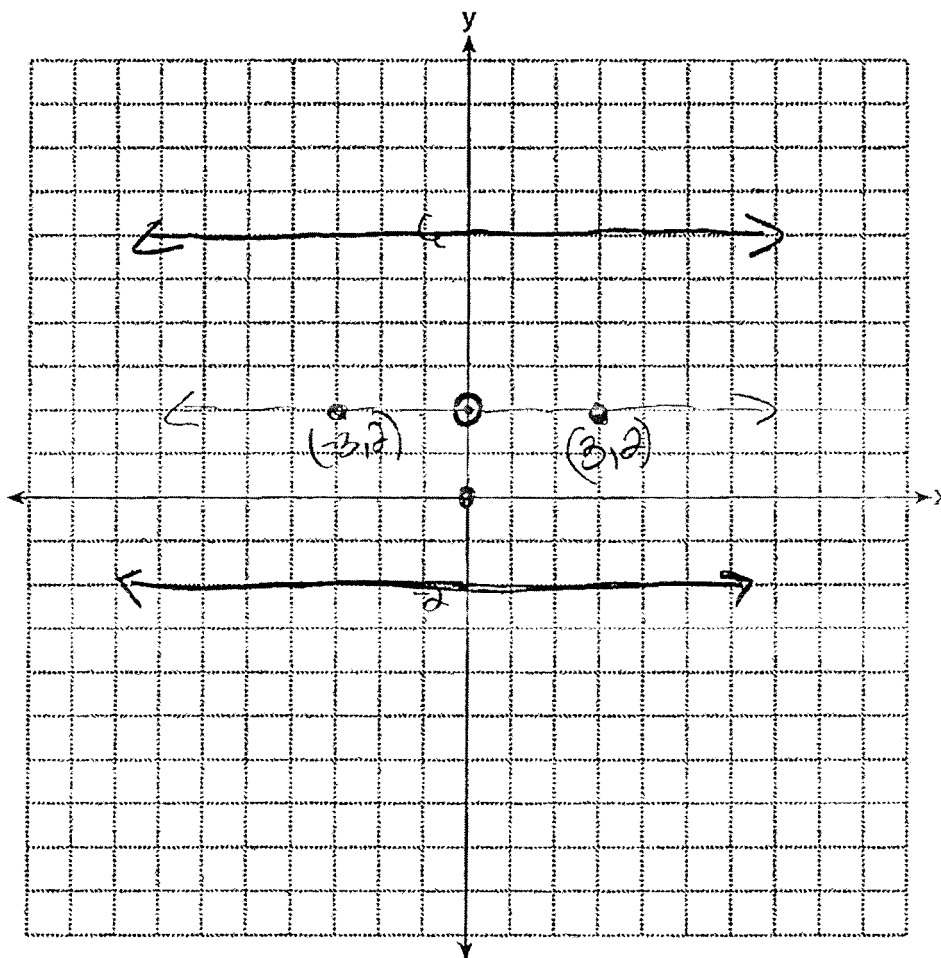
Question 36

36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

State the coordinates of the points that satisfy both loci.



Score 2: The student graphed one locus correctly and appropriate points were stated.

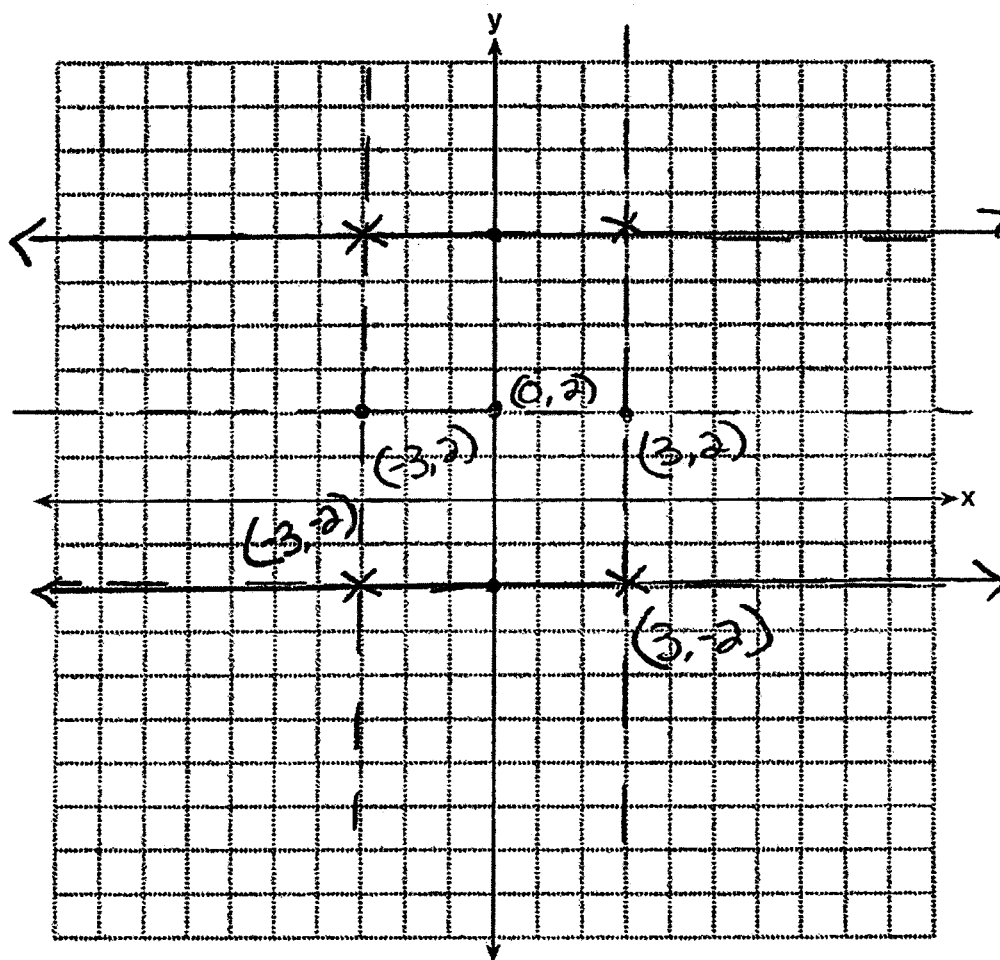
Question 36

36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

State the coordinates of the points that satisfy both loci.



Score 2: The student graphed both loci correctly, but the coordinates of more than two points were stated.

Question 36

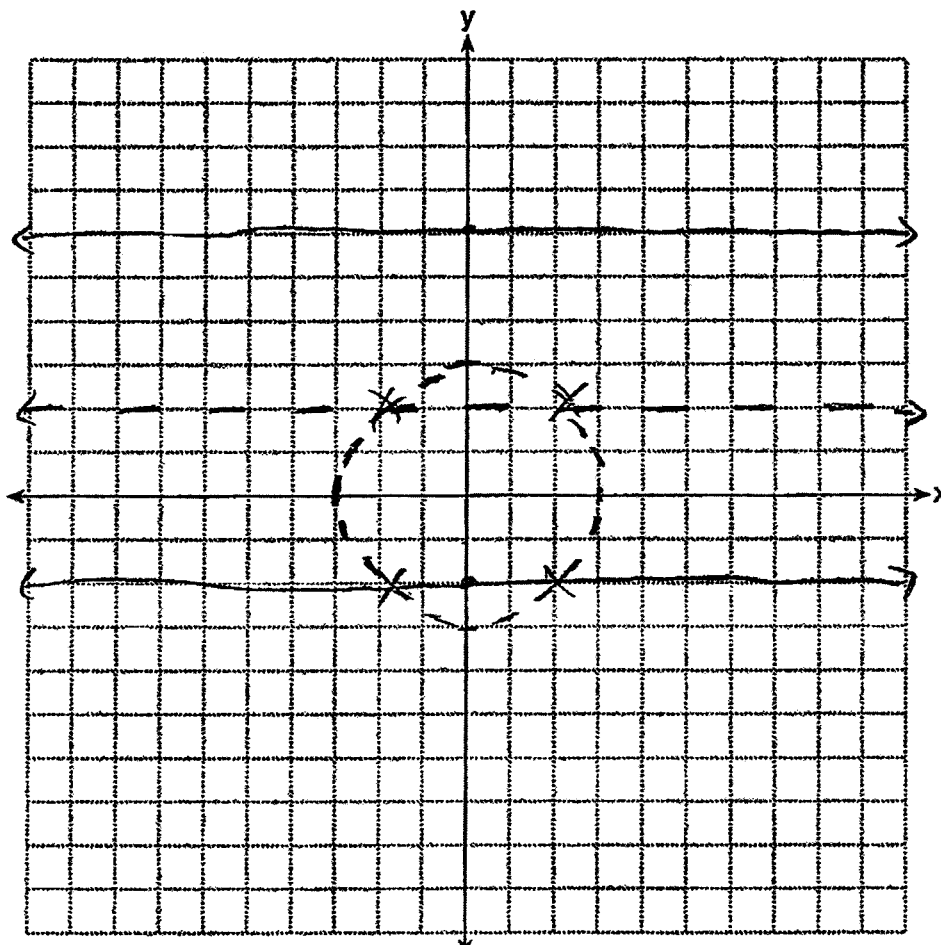
36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

State the coordinates of the points that satisfy both loci.

$$(-2, 2) (2, 2) (-2, -2) (2, -2)$$



Score 1: The student graphed only one locus correctly.

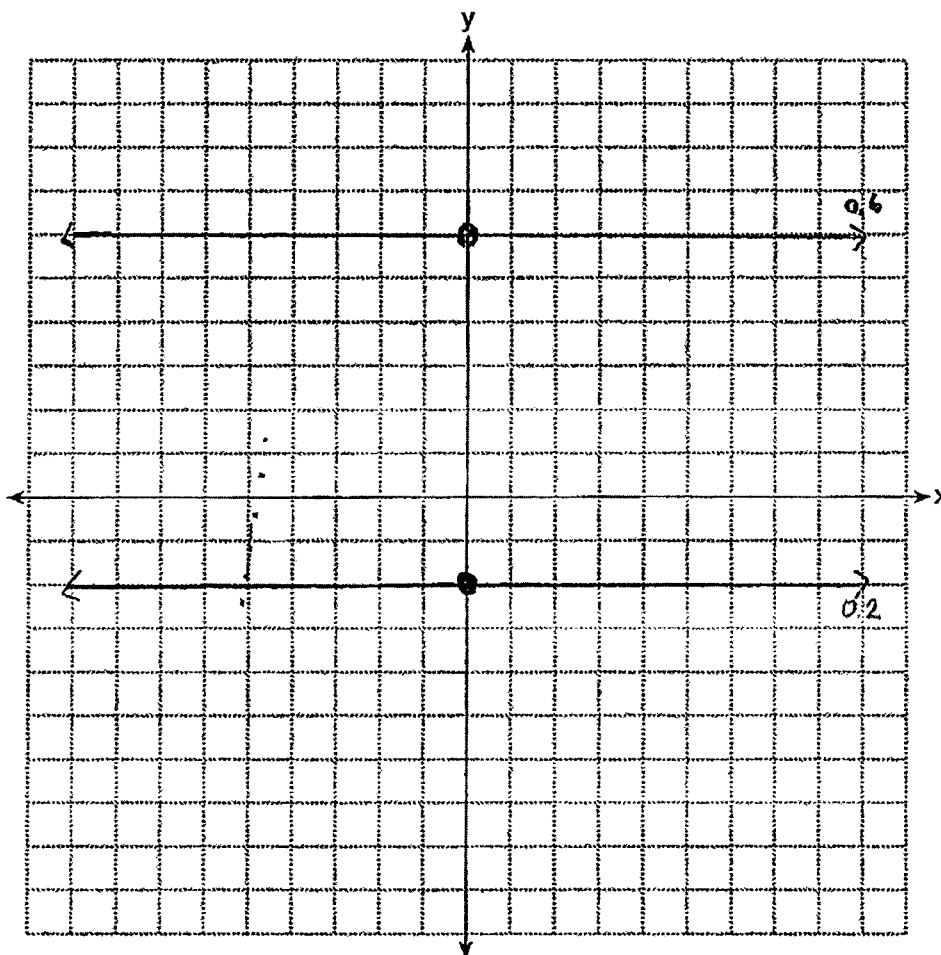
Question 36

36 On the set of axes below, graph two horizontal lines whose y -intercepts are $(0, -2)$ and $(0, 6)$, respectively.

Graph the locus of points equidistant from these horizontal lines.

Graph the locus of points 3 units from the y -axis.

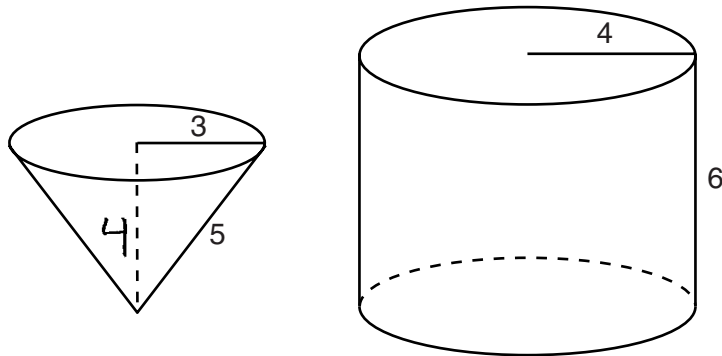
State the coordinates of the points that satisfy both loci.



Score 0: The student only graphed the two horizontal lines correctly.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (\pi r^2) h$$

$$V = \frac{1}{3} (\pi 3^2) 4$$

$$V = \frac{1}{3} (36\pi)$$

$$V = 12\pi$$

$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$b^2 = 16$$

$$b = 4$$

$$V = Bh$$

$$V = \pi r^2 (h)$$

$$V = \pi 4^2 (6)$$

$$V = 96\pi$$

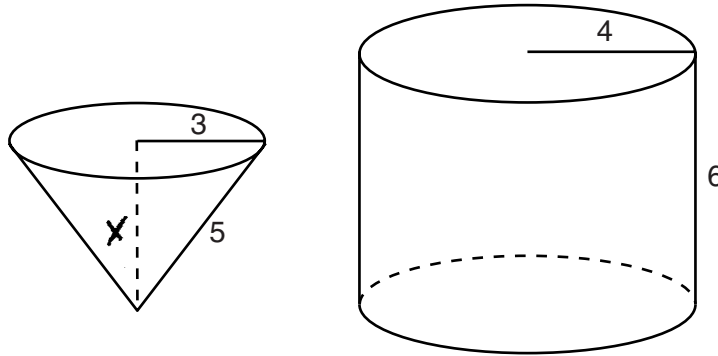
$$12\pi \overline{) 96\pi}$$

8 Cones

Score 4: The student has a complete and correct response.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

$$\begin{aligned}3^2 + x^2 &= 5^2 \\9 + x^2 &= 25 \\x^2 &= 16 \\x &= 4\end{aligned}$$

$$\begin{aligned}V &= \frac{1}{3} Bh \\V &= \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 4 \\V &= 37.6\end{aligned}$$

$$\begin{aligned}V &= Bh \\V &= \pi \cdot 4^2 \cdot 6 \\V &= 301.6\end{aligned}$$

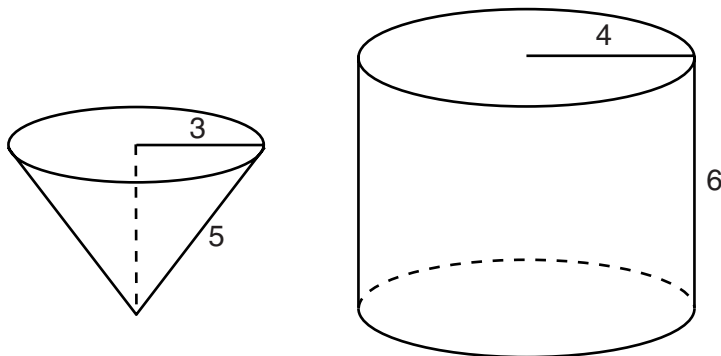
$$\frac{301.6}{37.6} = 8.02$$

9 cups

Score 3: The student made a rounding error when finding the volume of the cone. An appropriate answer was stated.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

$$V = \pi r^2 \cdot h$$

$$V = \pi 4^2 \cdot 6$$

$$V = 96\pi$$

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} (\pi r^2) \cdot 5$$

$$V = \frac{1}{3} (\pi 3^2) \cdot 5$$

$$V = 15\pi$$

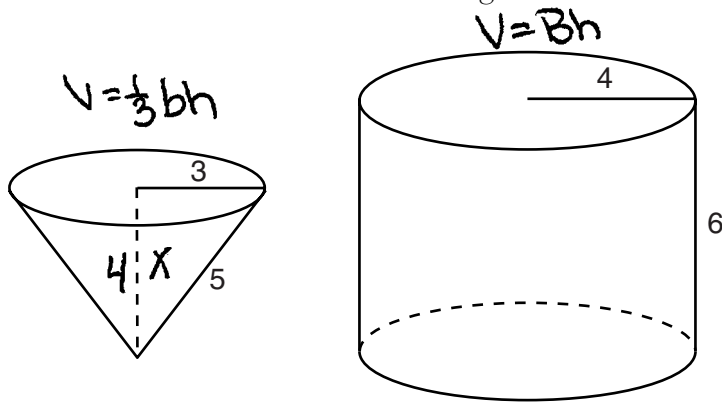
$$\frac{96}{15} = 6.4$$

7 cones of water

Score 2: The student made a conceptual error using 5 for the height when finding the volume of the cone. An appropriate solution was stated.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 3^2 + x^2 &= 5^2 \\
 9 + x^2 &= 25 \\
 -9 & \quad -9 \\
 \hline
 \sqrt{x^2} &= \sqrt{16} \\
 x &= 4
 \end{aligned}$$

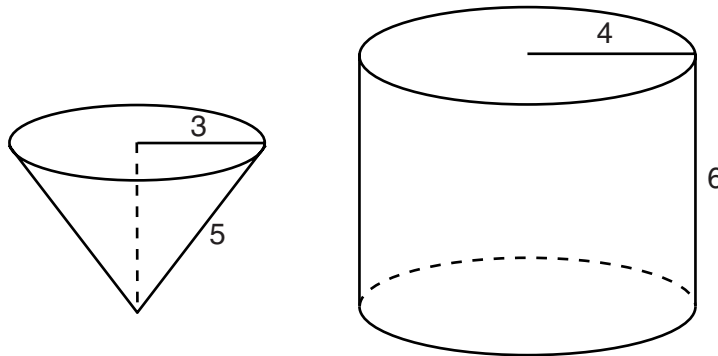
$$\begin{aligned}
 V &= \frac{1}{3}bh \\
 V &= \frac{1}{3}(4)(4) \\
 V &= 8
 \end{aligned}$$

$$\begin{aligned}
 V &= Bh \\
 V &= (8)(6) \\
 V &= 48 \text{ in}
 \end{aligned}$$

Score 1: The student showed appropriate work to find 4. No further correct work is shown.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

$$\begin{aligned} \text{Cone: } V &= \frac{1}{3} B h \\ &= \frac{1}{3} \pi (4)^2 \cdot 6 \\ &= \frac{1}{3} \pi (16) \cdot 6 \\ &= \frac{1}{3} \pi 22 \end{aligned}$$

$$\text{one cone} \approx 7.3 \pi \text{ in}^3$$

$$\text{one cone} \approx 23 \text{ in}^3$$

$$\begin{aligned} \text{cylinder: } V &= B h \\ &= \pi r^2 h \\ &= \pi (3)^2 (5) \\ &= \pi 45 \\ \text{one cylinder} &\approx 141 \text{ in}^3 \end{aligned}$$

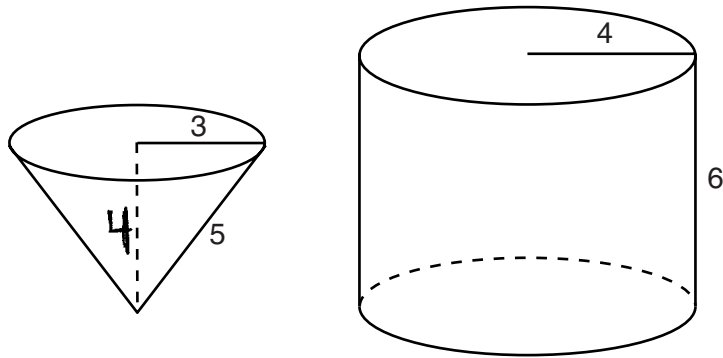
$$141 \div 23 \approx 6.1$$

\therefore 7 full cones of water are needed to fill the cylinder with water completely

Score 0: The student made more than one conceptual error.

Question 37

37 In the diagram below, a right circular cone with a radius of 3 inches has a slant height of 5 inches, and a right cylinder with a radius of 4 inches has a height of 6 inches.



Determine and state the number of full cones of water needed to completely fill the cylinder with water.

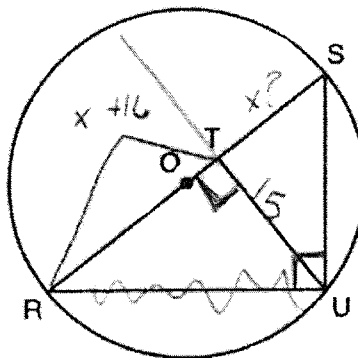
Score 0: The student stated 4, the height of the cone, but no work is shown.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



15^2

~~$x + 16 = 30$
 $225 = 30 \cdot 16$
 $x = 14$~~

~~$\frac{x+16}{15} = \frac{15}{x}$~~

~~$x^2 + 16x = 225$~~

~~$x^2 + 16x - 225 = 0$~~

~~$(x + 25)(x - 9)$~~

~~$x = 25$~~ $x = 9$

$TS = 9$

$RU = 5\sqrt{34}$

~~$a^2 + b^2 = c^2$
 $15^2 + 23^2 = c^2$
 $225 + 529 = c^2$
 $\sqrt{754} = c$~~

$25^2 + 15^2 = c^2$
 $\sqrt{850} = c$
 $\sqrt{34 \cdot 25} = c$

$5\sqrt{34}$

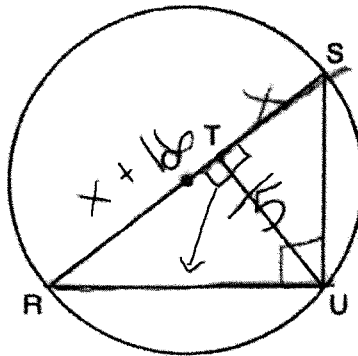
Score 6: The student has a complete and correct response.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{15}{x+16} = \frac{x}{15}$$

$$x^2 + 16x = 225$$

$$x^2 + 16x - 225 = 0$$

$$(x+25)(x-9) = 0$$

$$x = \cancel{25} \quad \boxed{x = 9}$$

$$\overline{RT} = x + 16$$

$$\overline{RT} = 9 + 16$$

$$\overline{RT} = 25$$

$$25^2 + 15^2 = \overline{RU}^2$$

$$850 = \overline{RU}^2$$

$$\boxed{\sqrt{850} = \overline{RU}}$$

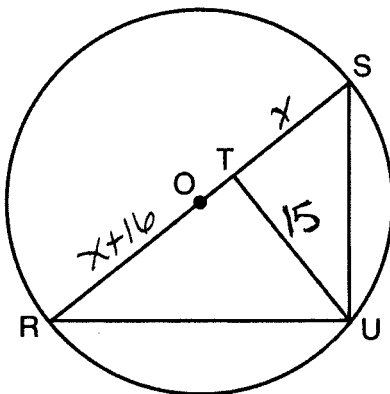
Score 5: The student showed appropriate work to find the lengths of \overline{TS} and \overline{RU} , but did not simplify the radical.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{x+16}{15} = \frac{15}{x}$$

$$x^2 + 16x = 225$$

$$x^2 + 16x - 225 = 0$$

$$(x-9)(x+25) = 0$$

$$x-9 = 0$$

$$x+25 = 0$$

$$x = 9, TS$$

$$x = -25$$

$$x + 16 = 25, RT$$

$$x + x + 16 = 36, RS$$

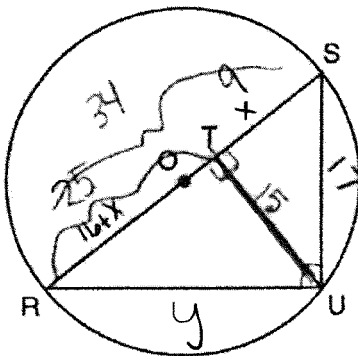
Score 4: The student showed appropriate work to find $TS = 9$ and $RT = 25$. No further correct work is shown.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{LL}{SL} = \frac{15}{x} = \frac{16+x}{15}$$

$$225 = x^2 + 16x$$

$$x^2 + 16x - 225 = 0$$

$$(x+25)(x-9) = 0$$

$$\boxed{x=9} \text{ ; } 25 \text{ reject}$$

$$225 = x^2 + 16$$

$$\frac{x}{15} = \frac{16+x}{15}$$

$$\frac{LL}{h} = \frac{25}{y} = \frac{4}{15}$$

$$\sqrt{y^2} = \sqrt{375}$$

$$RU = \sqrt{375}$$

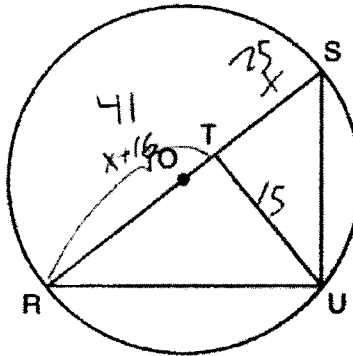
Score 3: The student made a conceptual error in finding the length of \overline{RU} , and the radical was not written in simplest radical form.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{proj_1}{alt} = \frac{alt}{proj_2}$$

$$TS = 25$$

$$\frac{x}{15} = \frac{15}{x+16}$$

$$\begin{aligned} 225 &= x^2 + 16x \\ -225 &\quad -225 \\ \hline 0 &= x^2 + 16x - 225 \end{aligned}$$

$$\begin{aligned} (x + 9)(x - 25) \\ \cancel{x = -9} \quad x = 25 \end{aligned}$$

$$15^2 + 41^2 = x^2$$

$$225 + 1681 = x^2$$

$$1906 = x^2$$

$$x = 43.65$$

$$x = \sqrt{43}$$

~~$$x = 43$$~~

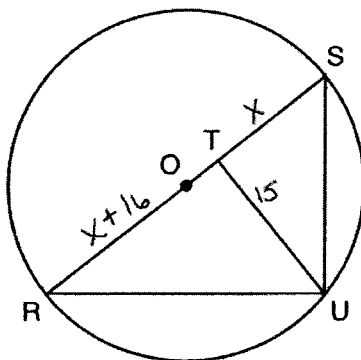
Score 3: The student made a factoring error in finding the length of \overline{TS} . The student made a conceptual error in finding the length of \overline{RU} .

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{x}{15} = \frac{15}{x+16}$$

$$x(x+16) = 225$$

$$x^2 + 16x - 225 = 0$$

$$(x \quad)(\quad) \quad ??$$

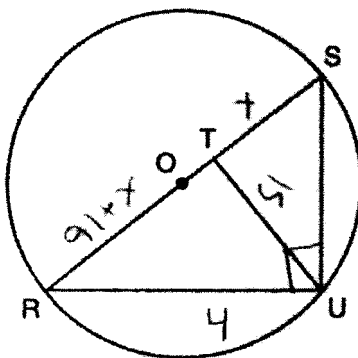
Score 2: The student wrote a correct quadratic equation. No further correct work is shown.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .



$$\frac{51}{9+x} = \frac{x}{15}$$

$$(9+x)(x)$$

$$\frac{15}{x+16} = \frac{x}{15}$$

Score 1: The student wrote a correct proportion. No further correct work is shown.

Question 38

38 In the diagram below, right triangle RSU is inscribed in circle O , and \overline{UT} is the altitude drawn to hypotenuse \overline{RS} . The length of \overline{RT} is 16 more than the length of \overline{TS} and $TU = 15$.

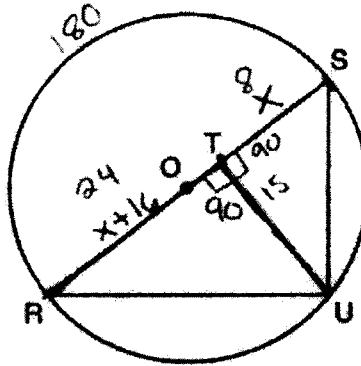
Find the length of \overline{TS} .

Find, in simplest radical form, the length of \overline{RU} .

$$\begin{aligned} x+16 &= x \\ -x-16 & \quad -x \end{aligned}$$

$$\frac{2x+16}{2} \quad \frac{2}{2}$$

$$x=8$$



$$\overline{TS} = 24$$

$$\overline{RU} = 18.73$$

$$\begin{aligned} 24^2 + 15^2 &= \\ 576 + 225 &= 351 \end{aligned}$$

Score 0: The student showed no relevant work.

