

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Wednesday, June 19, 2013 — 9:15 a.m. to 12:15 p.m., only

## SAMPLE RESPONSE SET

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Practice Papers—Question 29

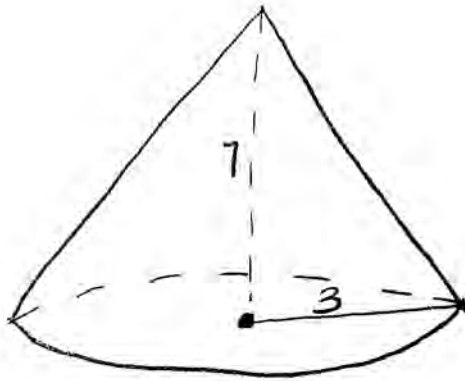
- 29 A right circular cylinder has a height of 7 inches and the base has a diameter of 6 inches. Determine the lateral area, in square inches, of the cylinder in terms of  $\pi$ .

$$2\pi rh \quad LA = 42\pi$$

Score 2: The student used the correct formula and found the correct lateral area.

Practice Papers—Question 29

- 29 A right circular cylinder has a height of 7 inches and the base has a diameter of 6 inches. Determine the lateral area, in square inches, of the cylinder in terms of  $\pi$ .



$$l = \sqrt{7^2 + 3^2}$$

$$l = \sqrt{58}$$

$$\begin{aligned} SA &= \pi r l \\ &= \pi(3)(\sqrt{58}) \\ &= 22.8\pi \end{aligned}$$

Score 1: The student draws a cone and finds the appropriate lateral surface area. (The number of decimal places in front of  $\pi$  is irrelevant.)

Practice Papers—Question 29

- 29 A right circular cylinder has a height of 7 inches and the base has a diameter of 6 inches. Determine the lateral area, in square inches, of the cylinder in terms of  $\pi$ .

$L = \pi r l$   
 $l = 9.2$

$L = \pi 3 (9.2)$   
 $27.6\pi$

$7^2 + 6^2 = x^2$   
 $49 + 36 = x^2$   
 $\sqrt{85} = x$   
 $9.21954457$   
 $\approx 9.2$

$LA = 27.6\pi \text{ in}^2$


Score 0: The student used the formula for the lateral area of a right circular cone. When attempting to find the slant height, the student used the diameter. These are two conceptual errors.

Practice Papers—Question 30

30 Determine, in degrees, the measure of each interior angle of a regular octagon.

int / ext.

$$\text{int} \angle = \boxed{135^\circ}$$


$$\frac{360}{8} = 45$$

$$\begin{array}{r} 180 \\ -45 \\ \hline 135 \end{array}$$

Score 2: The student divided the sum of the exterior angles by the number of sides to find an exterior angle. Then the student found the supplement of this angle to find the interior angle.

Practice Papers—Question 30

30 Determine, in degrees, the measure of each interior angle of a regular octagon.

Octagon . = 8 sides

$$\frac{360}{n} \rightarrow \frac{360}{8} = 45$$

Measure of each interior angle is  
 $45^\circ$

Score 1: The student made a conceptual error by using  $360^\circ$  as the sum of the interior angles of an octagon.

Practice Papers—Question 30

30 Determine, in degrees, the measure of each interior angle of a regular octagon.

$$180(n-2)$$

$$180(8-2)$$

$$1080^\circ$$

Score 1: The student showed work to find the sum of the interior angles of an octagon, but did not find the measure of each interior angle.

Practice Papers—Question 30

30 Determine, in degrees, the measure of each interior angle of a regular octagon.

$$\frac{(n-2)180}{n}$$

$$\frac{(5-2)(180)}{2}$$

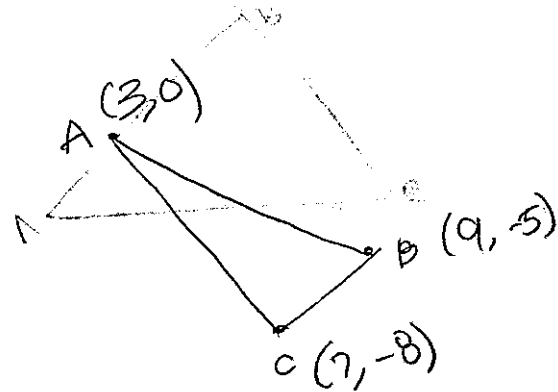
$$\frac{3(180)}{2} = 270^\circ$$

Score 0: The student wrote the correct formula for finding an interior angle of a regular polygon, but made a conceptual error by using 5 as the number of sides in an octagon. A second conceptual error was made by dividing by 2 instead of the number of sides.



Practice Papers—Question 31

31 Triangle  $ABC$  has vertices at  $A(3,0)$ ,  $B(9,-5)$ , and  $C(7,-8)$ . Find the length of  $\overline{AC}$  in simplest radical form.



$$d = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$$

$$\sqrt{(-8 - 0)^2 + (7 - 3)^2}$$

$$-8^2 + 4^2$$

$$\sqrt{64 + 16}$$

$$\sqrt{80}$$

$$\sqrt{20} \sqrt{4}$$

$$\sqrt{4} \sqrt{5} \quad 2$$

$$2\sqrt{5}$$

$$2 \times 2\sqrt{5}$$

$$\boxed{4\sqrt{5}}$$

Score 2: The student showed appropriate work and simplified the radical completely.

Practice Papers—Question 31

31 Triangle  $ABC$  has vertices at  $A(3,0)$ ,  $B(9,-5)$ , and  $C(7,-8)$ . Find the length of  $\overline{AC}$  in simplest radical form.

$$\text{distance: } \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$\overline{AC}: \sqrt{(-8 - 0)^2 + (7 - 3)^2}$$

$(3,0)(7,-8)$        $-8^2 + 4^2$

$$64 + 16$$

$$\sqrt{80}$$

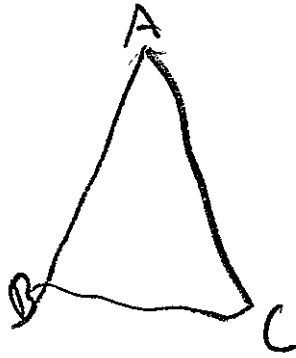
$$\sqrt{4} \times \sqrt{20}$$

$$2\sqrt{20}$$

Score 1: The student showed appropriate work, but did not simplify the radical completely.

Practice Papers—Question 31

31 Triangle  $ABC$  has vertices at  $A(3,0)$ ,  $B(9,-5)$ , and  $C(7,-8)$ . Find the length of  $\overline{AC}$  in simplest radical form.



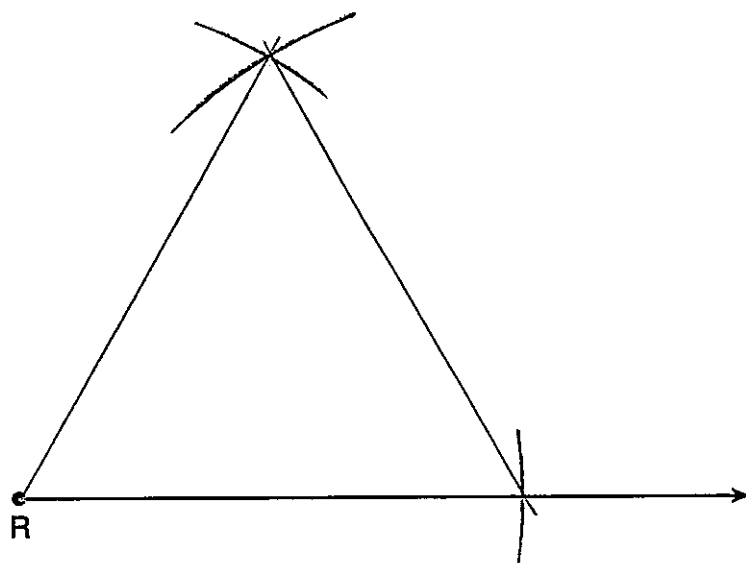
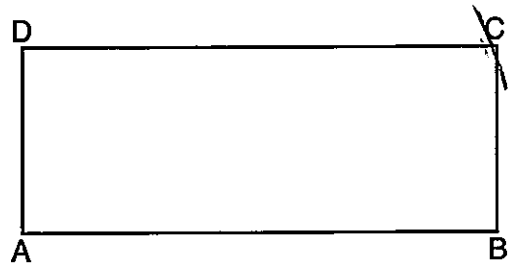
$$AC = \frac{8-0}{7-3} = \frac{-8}{4} = \boxed{-2}$$

$$\boxed{AC = -2}$$

Score 0: The student found the correct slope of  $\overline{AC}$ , which is irrelevant to this problem.

Practice Papers—Question 32

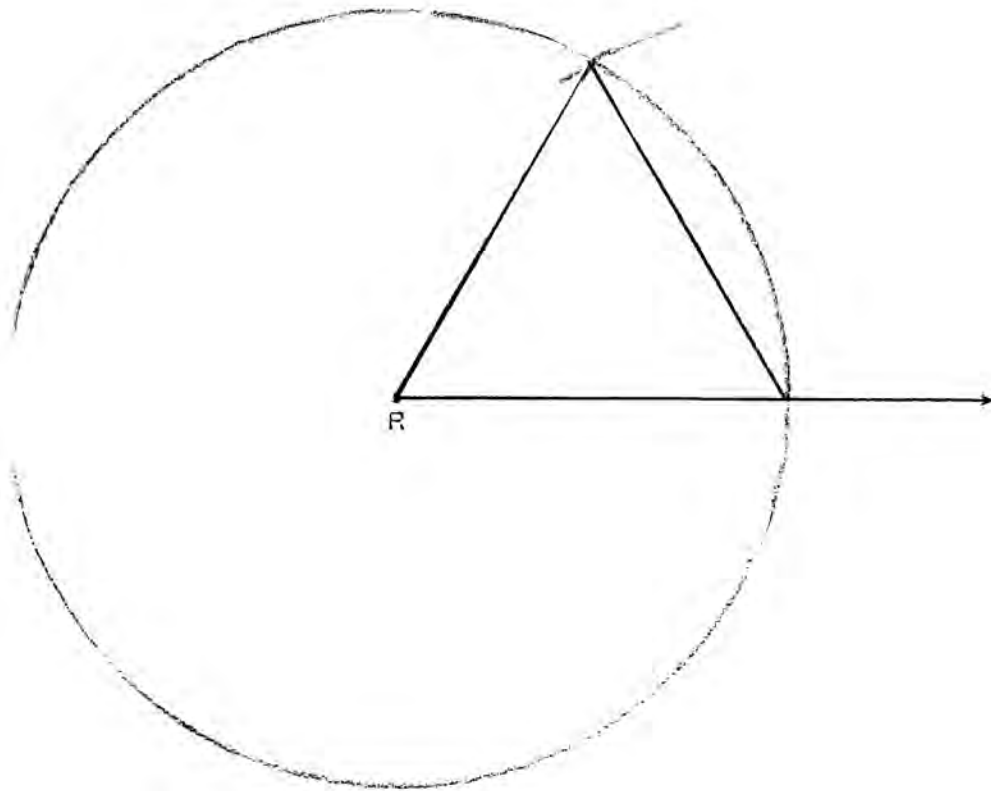
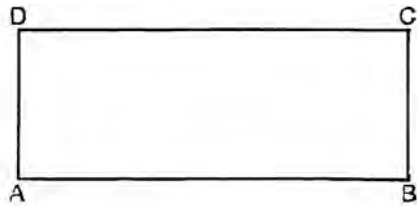
32 On the ray drawn below, using a compass and straightedge, construct an equilateral triangle with a vertex at  $R$ . The length of a side of the triangle must be equal to a length of the diagonal of rectangle  $ABCD$ .



Score 2: The student did a correct construction using the length of a diagonal as each side of the triangle.

**Practice Papers—Question 32**

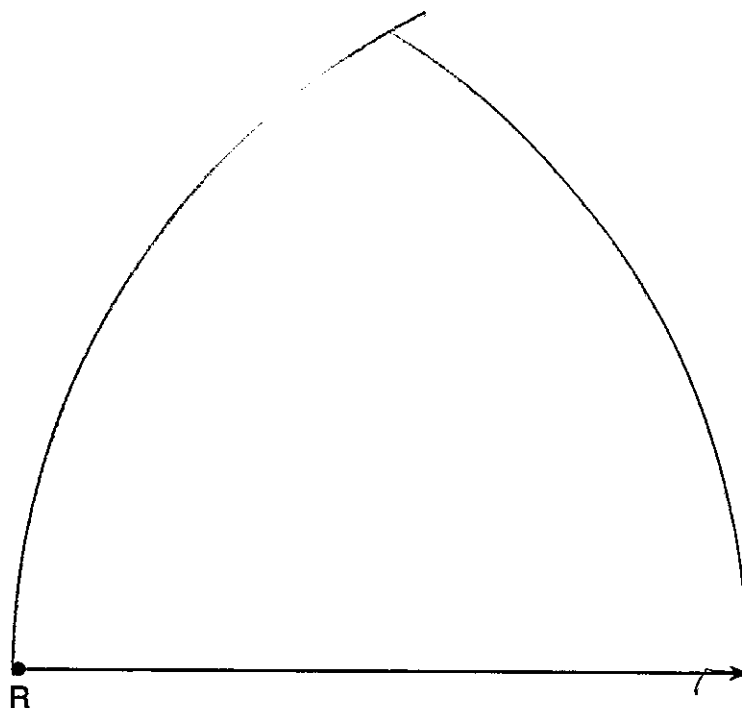
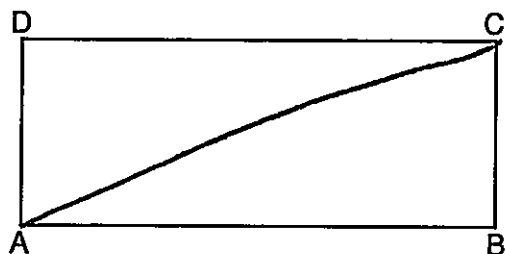
**32** On the ray drawn below, using a compass and straightedge, construct an equilateral triangle with a vertex at  $R$ . The length of a side of the triangle must be equal to a length of the diagonal of rectangle  $ABCD$ .



Score 1: The student showed a correct construction of an equilateral triangle, but used the length of  $\overline{AB}$  as the side of the triangle.

Practice Papers—Question 32

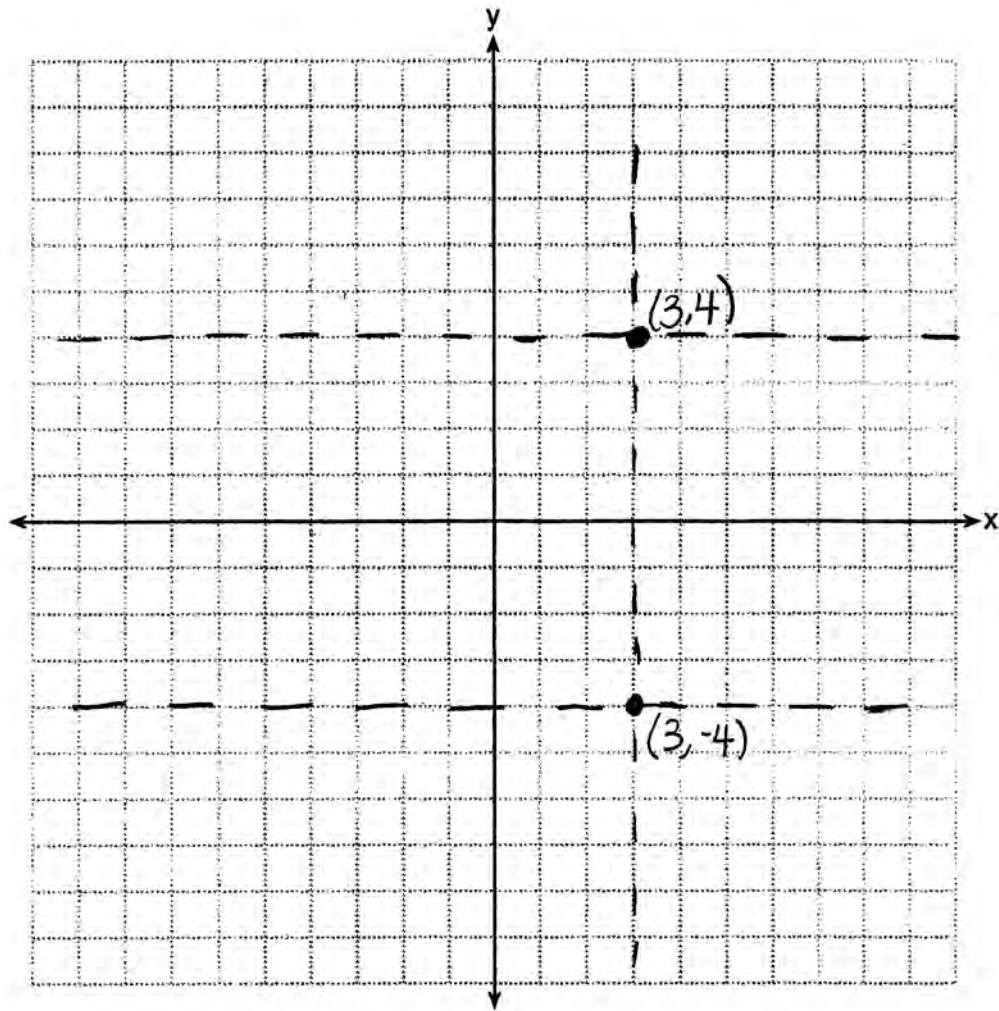
32 On the ray drawn below, using a compass and straightedge, construct an equilateral triangle with a vertex at  $R$ . The length of a side of the triangle must be equal to a length of the diagonal of rectangle  $ABCD$ .



Score 0: The student used the ray as one side of an equilateral triangle and made appropriate arcs. However, the student did not draw a triangle.

Practice Papers—Question 33

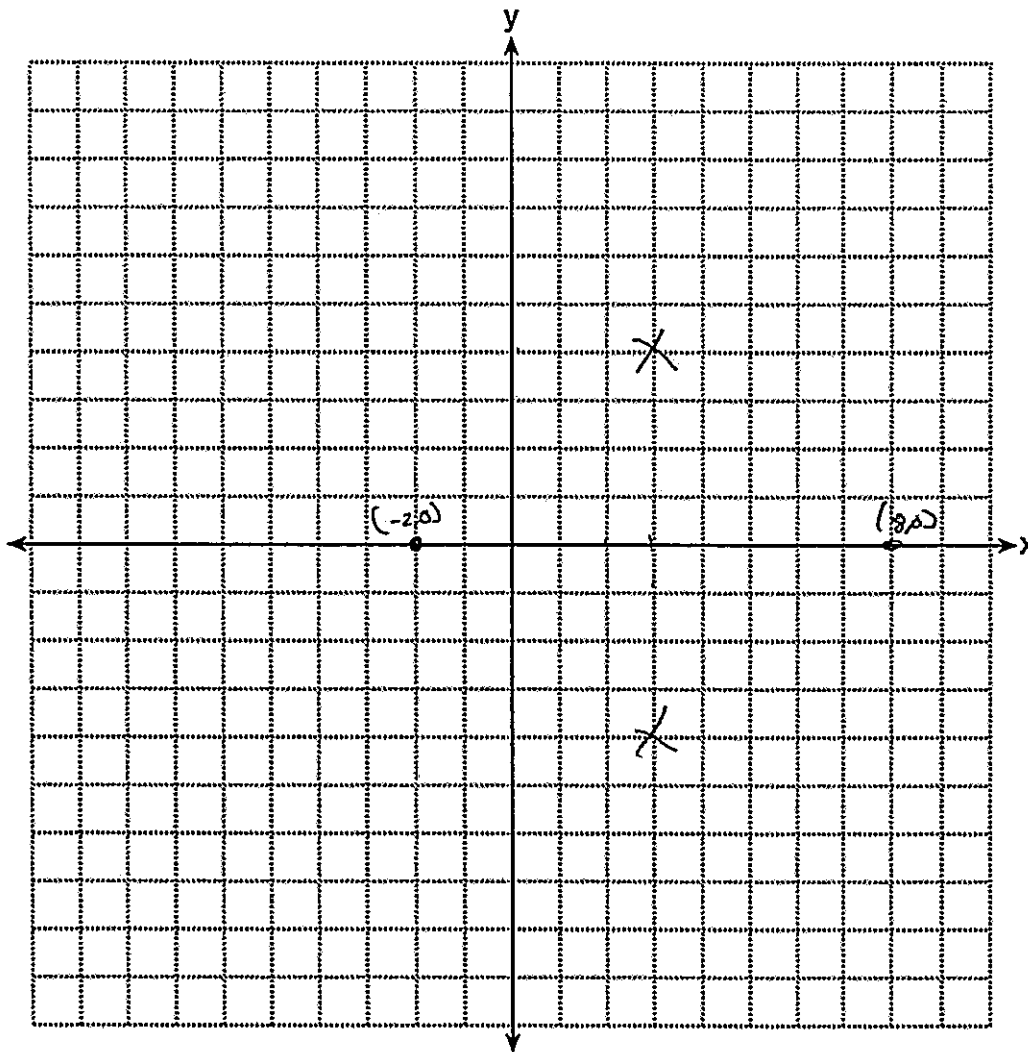
- 33 On the set of axes below, graph the locus of points 4 units from the  $x$ -axis and equidistant from the points whose coordinates are  $(-2,0)$  and  $(8,0)$ .  
Mark with an **X** all points that satisfy *both* conditions.



Score 2: The student graphed both loci correctly and labeled the points that satisfy both conditions with coordinates instead of an **X**.

Practice Papers—Question 33

33 On the set of axes below, graph the locus of points 4 units from the  $x$ -axis and equidistant from the points whose coordinates are  $(-2,0)$  and  $(8,0)$ . Mark with an **X** all points that satisfy *both* conditions.

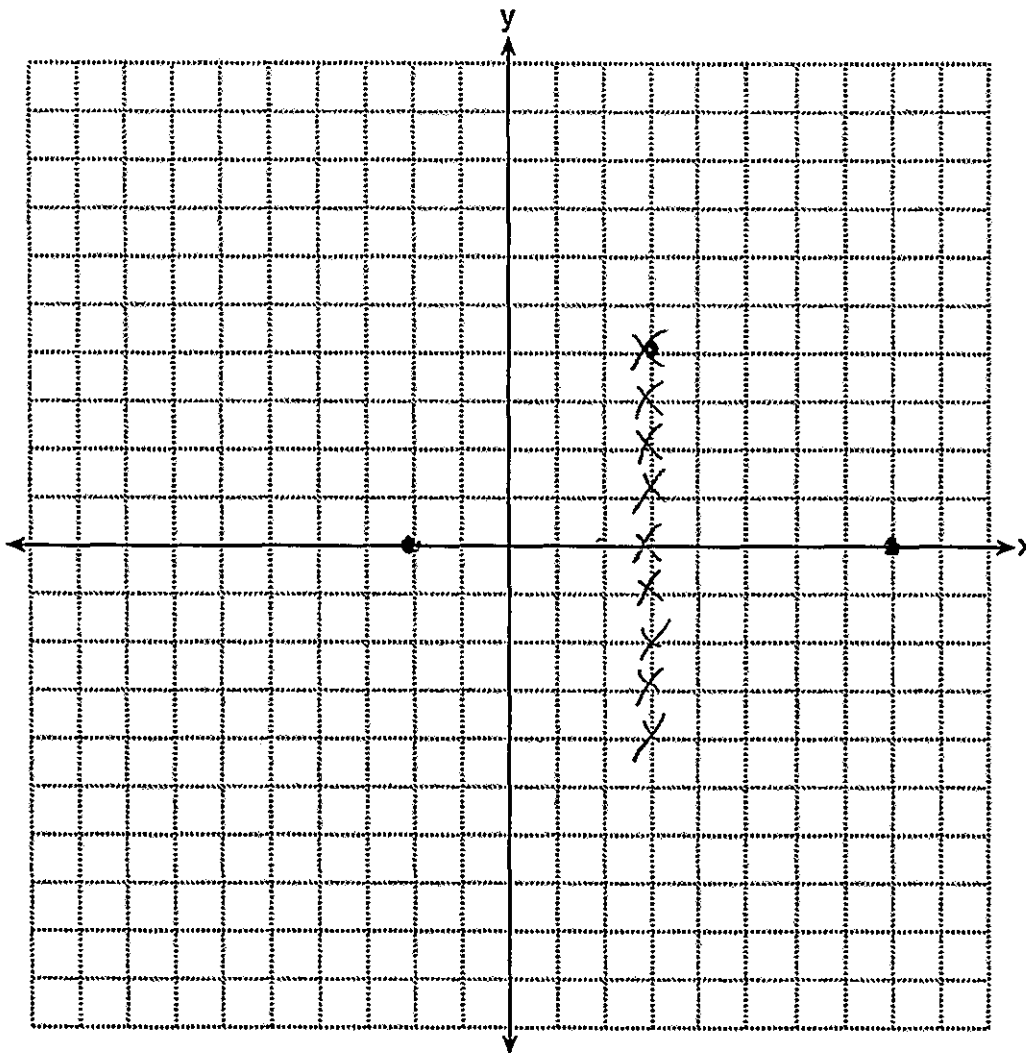


Score 1: The student located the two points that satisfy both conditions and marked them with an **X**, but did not graph either locus.



Practice Papers—Question 33

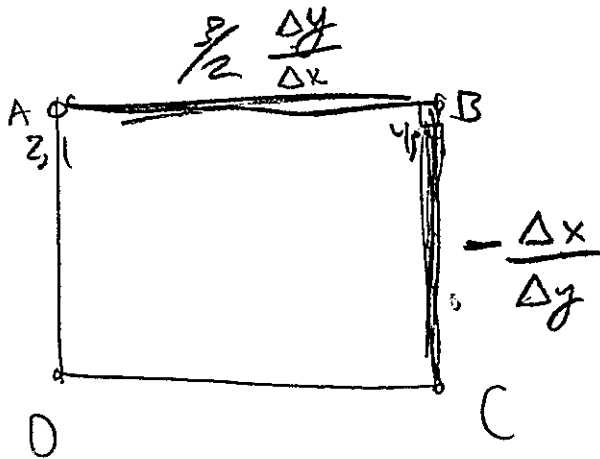
33 On the set of axes below, graph the locus of points 4 units from the  $x$ -axis and equidistant from the points whose coordinates are  $(-2,0)$  and  $(8,0)$ . Mark with an **X** all points that satisfy *both* conditions.



Score 0: The student did not graph either locus correctly and marked multiple points with an **X**.

Practice Papers—Question 34

34 The coordinates of two vertices of square  $ABCD$  are  $A(2,1)$  and  $B(4,4)$ .  
Determine the slope of side  $\overline{BC}$ .



Handwritten calculation for the distance  $AB$ :

$$D = \sqrt{(2-4)^2 + (1-4)^2}$$

$$D^2 = \sqrt{-2^2 + -3^2}$$

$$D^2 = 4 + 9$$

$$D = \sqrt{15}$$

$$\frac{4-1}{4-2} = \frac{3}{2}$$

$$\frac{3}{2} = \frac{\Delta y}{\Delta x}$$

$$\frac{3}{2}$$

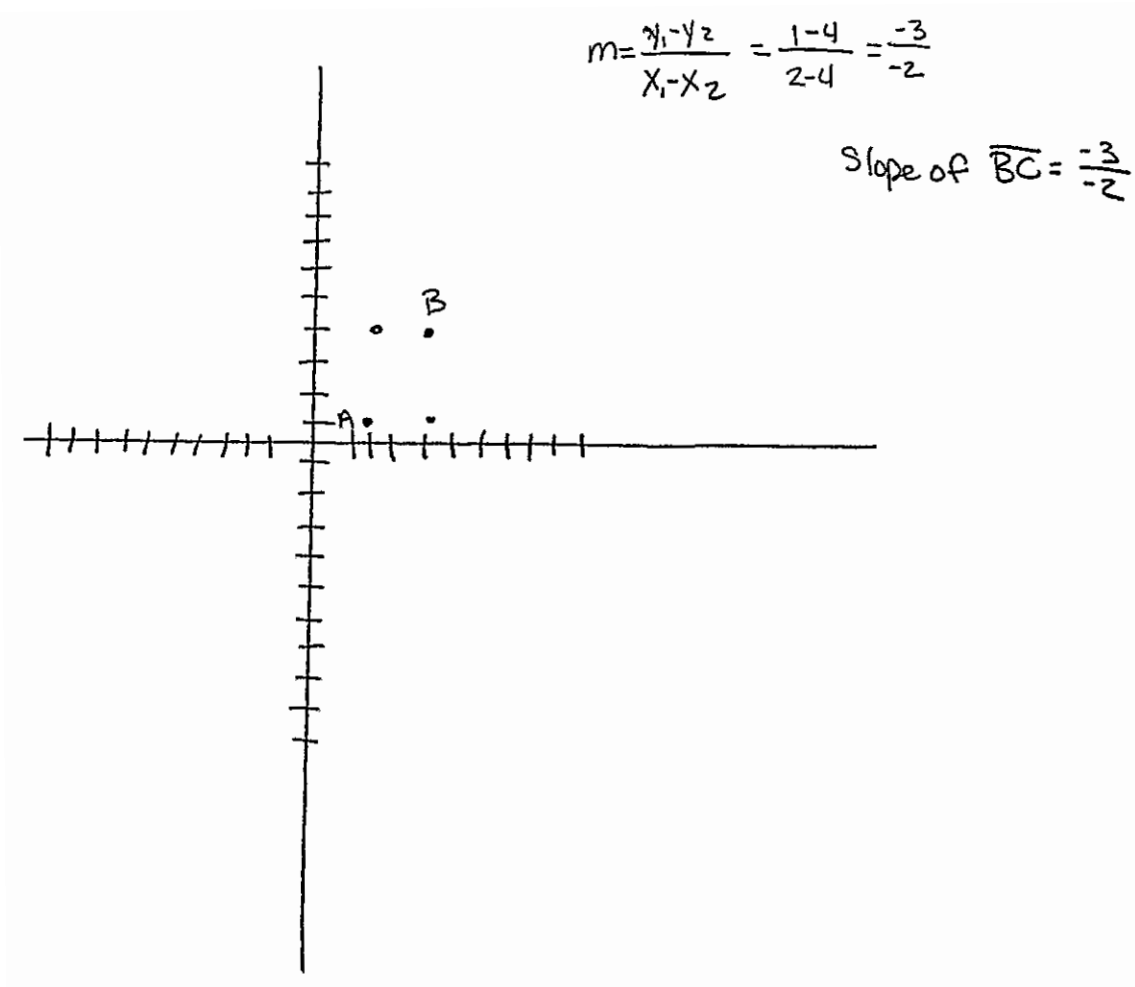
$$\frac{-2}{3}$$

$$-\frac{2}{3} = -\frac{\Delta x}{\Delta y}$$

Score 2: The student found the slope of  $\overline{AB}$  correctly and used the negative reciprocal to find the slope of  $\overline{BC}$ .

Practice Papers—Question 34

- 34 The coordinates of two vertices of square  $ABCD$  are  $A(2,1)$  and  $B(4,4)$ .  
Determine the slope of side  $\overline{BC}$ .



Score 1: The student found the slope of  $\overline{AB}$  correctly, but made a conceptual error in stating that the slope of  $\overline{BC}$  was the same.

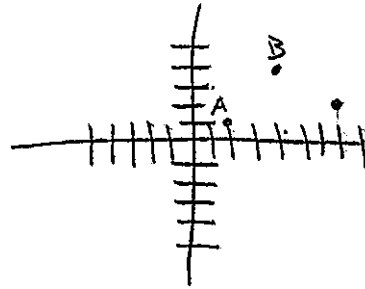
Practice Papers—Question 34

34 The coordinates of two vertices of square  $ABCD$  are  $A(2,1)$  and  $B(4,4)$ .  
Determine the slope of side  $\overline{BC}$ .

$$m = \left( \frac{y_2 - y_1}{x_2 - x_1} \right) = \left( \frac{4 - 1}{4 - 2} \right) = \left( \frac{3}{2} \right)$$

$$\perp \frac{3}{2} = -\frac{2}{3}$$

$$m_{\overline{BC}} = -\frac{2}{3}x$$



Score 1: The student included  $x$  when writing the slope as  $\frac{2}{-3}$  in the circled solution, which is a conceptual error.

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**Practice Papers—Question 34**

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- 34 The coordinates of two vertices of square  $ABCD$  are  $A(2,1)$  and  $B(4,4)$ .  
Determine the slope of side  $\overline{BC}$ .

$$BC = \frac{4-0}{4-1} = \left| \frac{4}{3} \right|$$

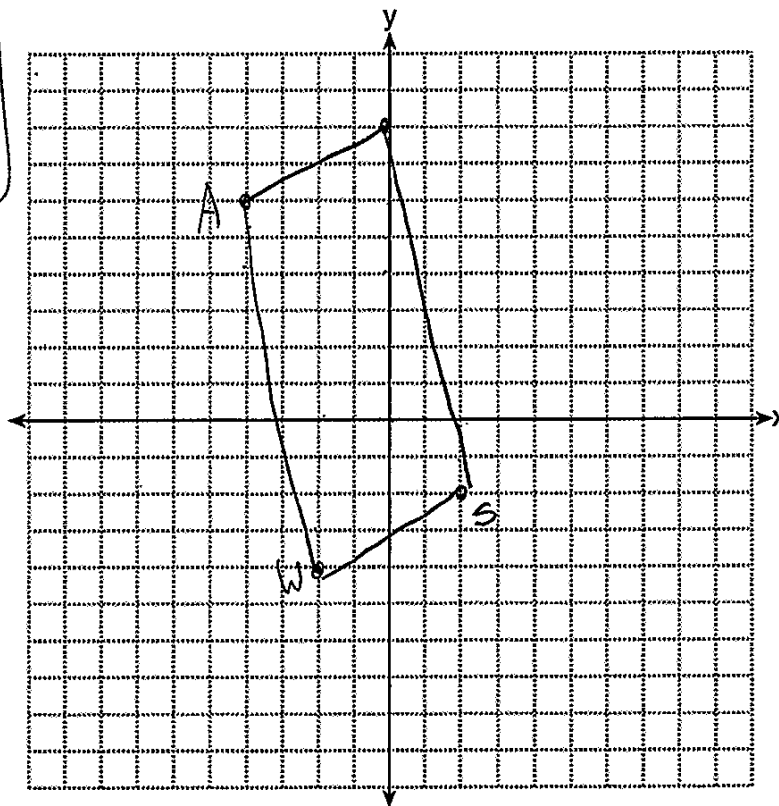
Score 0: The student does not show appropriate work and the solution is incorrect.

Practice Papers—Question 35

35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2, -2)$ ,  $W(-2, -4)$ ,  $A(-4, 6)$ , and  $N(0, 8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4, -2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]

$$\begin{aligned} S(2, -2) &\rightarrow (1, -1) \rightarrow (5, -3) \\ W(-2, -4) &\rightarrow (-1, -2) \rightarrow (3, -4) \\ A(-4, 6) &\rightarrow (-2, 3) \rightarrow (2, 1) \\ N(0, 8) &\rightarrow (0, 4) \rightarrow (4, 2) \end{aligned}$$

$$\begin{aligned} S'' &(5, -3) \\ W'' &(3, -4) \\ A'' &(2, 1) \\ N'' &(4, 2) \end{aligned}$$



Score 4: The student applied the correct transformation rules to find the coordinates of  $S''W''A''N''$ .

Practice Papers—Question 35

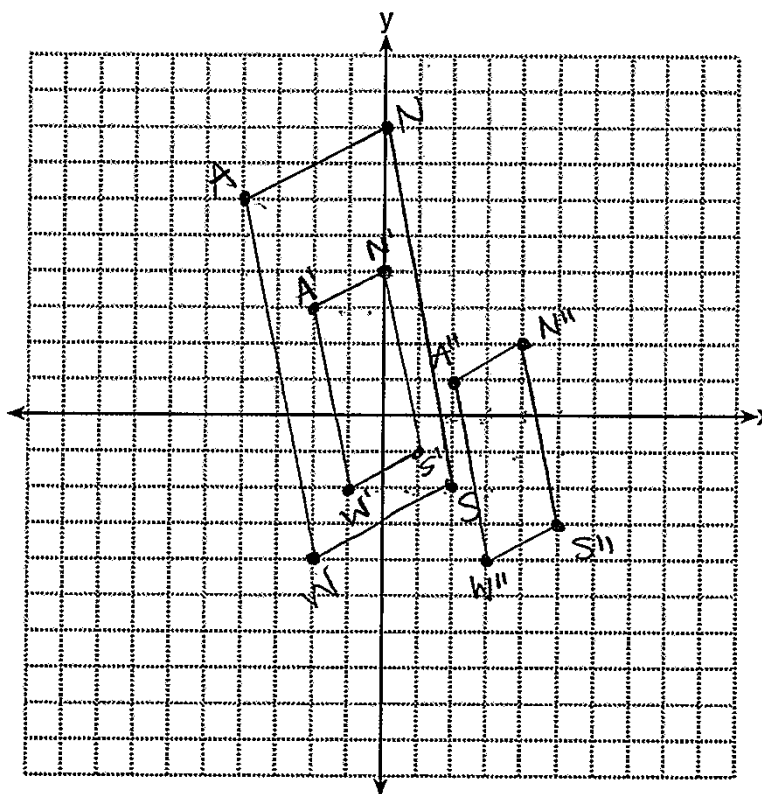
35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2,-2)$ ,  $W(-2,-4)$ ,  $A(-4,6)$ , and  $N(0,8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4,-2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]

$$S''(5,-3)$$

$$W''(3,-4)$$

$$A''(2,1)$$

$$N''(4,2)$$

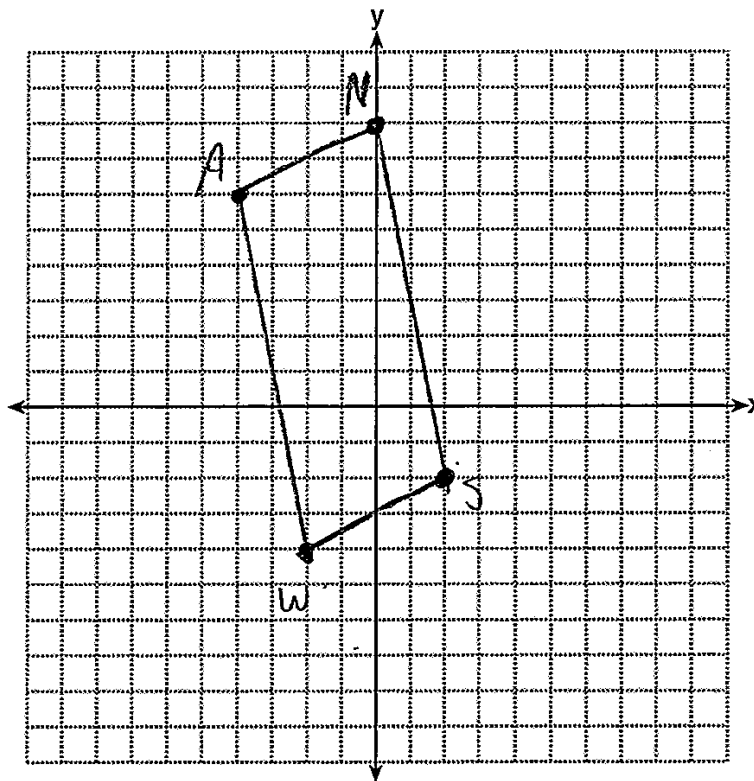


Score 4: The student used a graphic solution to find the coordinates of  $S''W''A''N''$ .

Practice Papers—Question 35

35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2, -2)$ ,  $W(-2, -4)$ ,  $A(-4, 6)$ , and  $N(0, 8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4, -2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]

$$\begin{aligned}
 S(2, -2) : D_{\frac{1}{2}} &\rightarrow S'(1, -1) : T_{4, -2} \rightarrow S''(5, -3) \\
 W(-2, -4) : D_{\frac{1}{2}} &\rightarrow W'(-1, -2) : T_{4, -2} \rightarrow W''(3, -4) \\
 A(-4, 6) : D_{\frac{1}{2}} &\rightarrow A'(-2, 3) : T_{4, -2} \rightarrow A''(2, 1) \\
 N(0, 8) : D_{\frac{1}{2}} &\rightarrow N'(0, 4) : T_{4, -2} \rightarrow N''(4, 2)
 \end{aligned}$$



Score 3: The student made one computational error in finding  $N''$ .

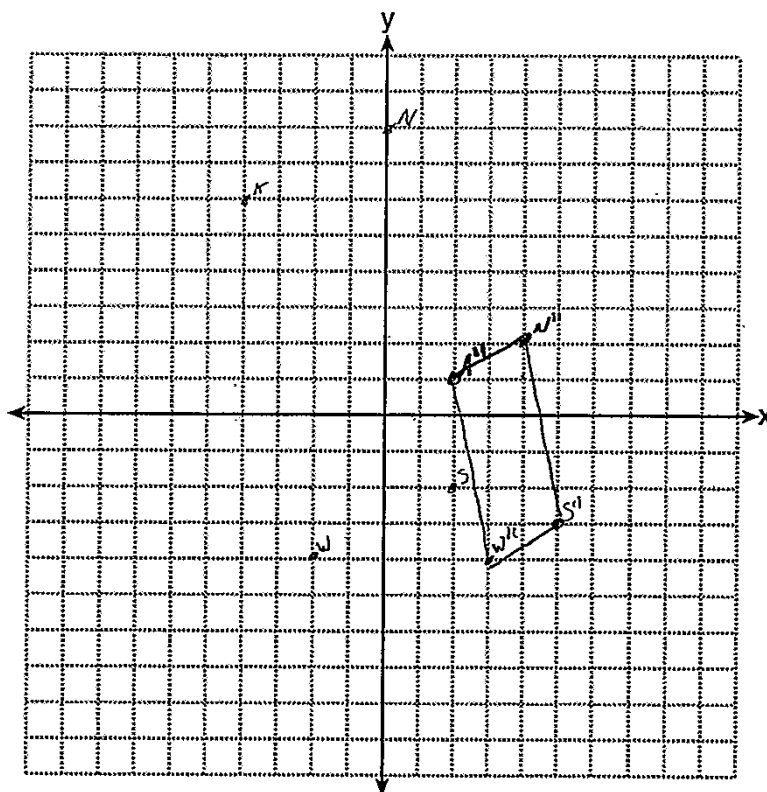


Practice Papers—Question 35

35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2,-2)$ ,  $W(-2,-4)$ ,  $A(-4,6)$ , and  $N(0,8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4,-2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]

$S' 1, -1$   
 $W' -1, -2$   
 $A' -2, 3$   
 $N' 0, 4$

$S'' 5, -3$   
 $W'' 3, -4$   
 $A'' 2, 1$   
 $N'' 4, 2$

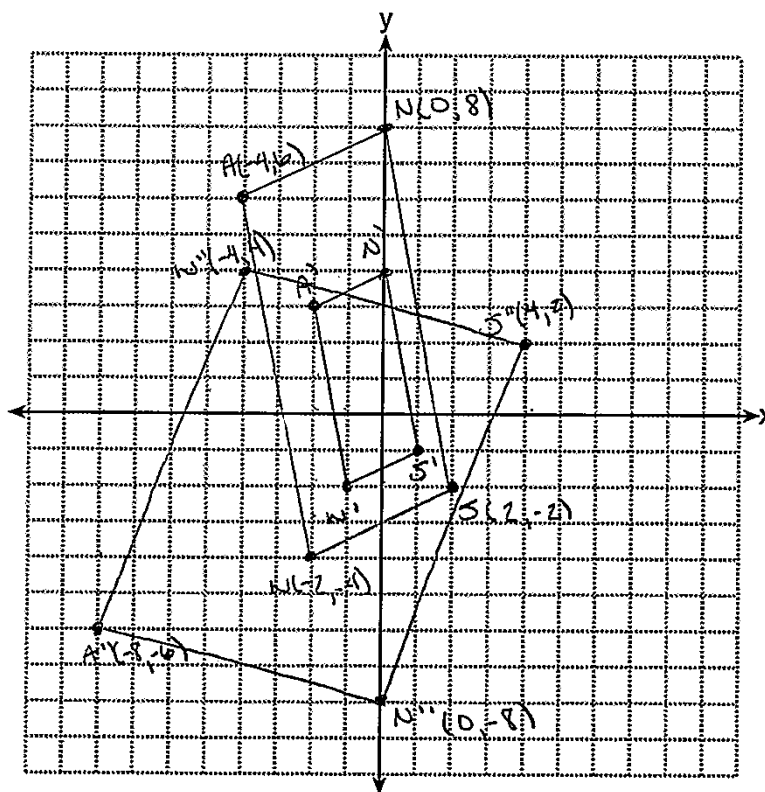


Score 3: The student applied the rules for composite transformations correctly, but did not put parentheses around the coordinates.

Practice Papers—Question 35

35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2,-2)$ ,  $W(-2,-4)$ ,  $A(-4,6)$ , and  $N(0,8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4,-2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]

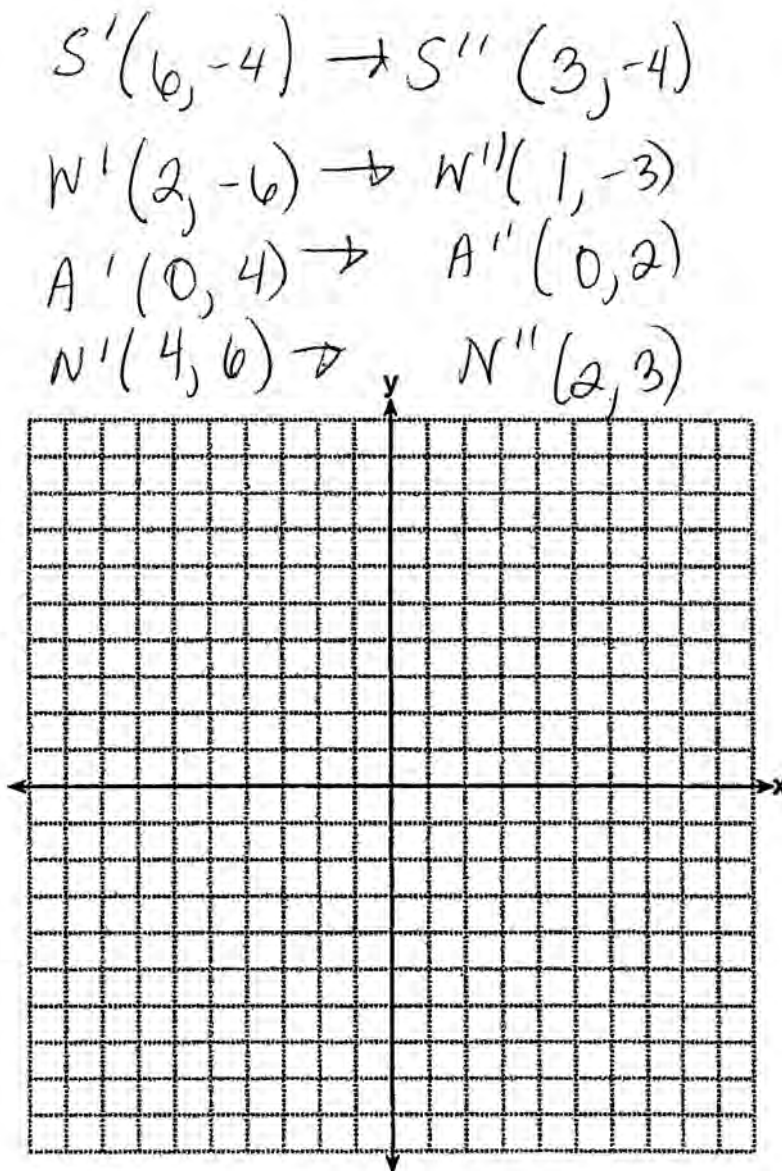
$D_{1/2}$	$S(2,-2) \rightarrow S'(1,-1)$	$T_{4,-2}$	$S'(1,-1) \rightarrow S''(4,2)$
	$W(-2,-4) \rightarrow W'(-1,-2)$		$W'(-1,-2) \rightarrow W''(-4,4)$
	$A(-4,6) \rightarrow A'(-2,3)$		$A'(-2,3) \rightarrow A''(-8,-6)$
	$N(0,8) \rightarrow N'(0,4)$		$N'(0,4) \rightarrow N''(0,-8)$



Score 2: The student made a conceptual error in doing the translation (multiplied by 4 and  $-2$  instead of adding 4 and  $-2$ ).

Practice Papers—Question 35

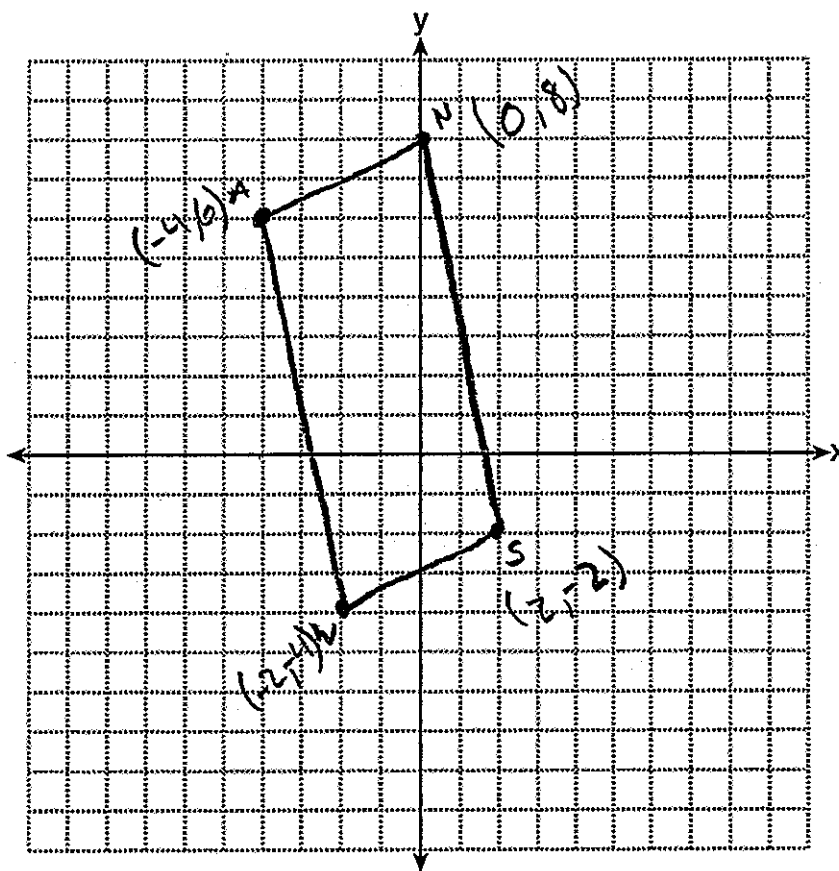
35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2,-2)$ ,  $W(-2,-4)$ ,  $A(-4,6)$ , and  $N(0,8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4,-2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]



Score 1: The student did the transformation first, which is a conceptual error, and then made an error in calculating the  $y$ -coordinate for  $S''$ .

Practice Papers—Question 35

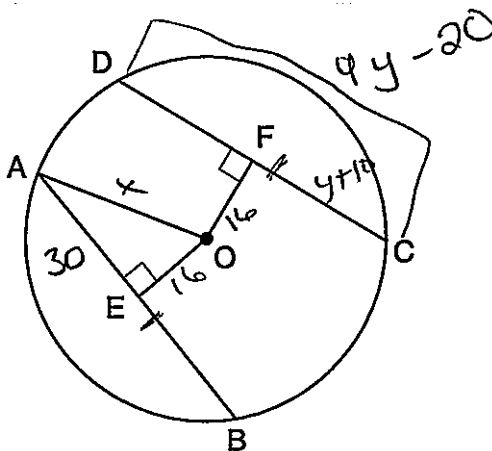
35 The coordinates of the vertices of parallelogram  $SWAN$  are  $S(2, -2)$ ,  $W(-2, -4)$ ,  $A(-4, 6)$ , and  $N(0, 8)$ . State and label the coordinates of parallelogram  $S''W''A''N''$ , the image of  $SWAN$  after the transformation  $T_{4, -2} \circ D_{\frac{1}{2}}$ . [The use of the set of axes below is optional.]



Score 0: The student graphed the parallelogram  $SWAN$  correctly, but did no other work.

Practice Papers—Question 36

36 In circle  $O$  shown below, chords  $\overline{AB}$  and  $\overline{CD}$  and radius  $\overline{OA}$  are drawn, such that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{OE} \perp \overline{AB}$ ,  $\overline{OF} \perp \overline{CD}$ ,  $OF = 16$ ,  $CF = y + 10$ , and  $CD = 4y - 20$ .



Determine the length of  $\overline{DF}$ .

$$\begin{aligned} y + 10 + y + 10 &= 4y - 20 \\ 2y + 20 &= 4y - 20 \\ \underline{y + 40} &= 2y \\ 20 &= y \end{aligned}$$

$$\begin{aligned} y + 10 \\ 20 + 10 \\ 30 \end{aligned}$$

$DF = 30$

Determine the length of  $\overline{OA}$ .

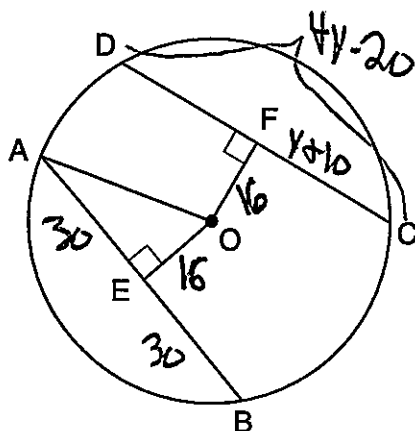
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 30^2 + 16^2 &= c^2 \\ 900 + 256 &= c^2 \\ \underline{\sqrt{1156}} &= \sqrt{c^2} \\ c &= 34 \end{aligned}$$

$OA = 34$

Score 4: The student showed appropriate work to find both lengths.

Practice Papers—Question 36

- 36 In circle  $O$  shown below, chords  $\overline{AB}$  and  $\overline{CD}$  and radius  $\overline{OA}$  are drawn, such that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{OE} \perp \overline{AB}$ ,  $\overline{OF} \perp \overline{CD}$ ,  $OF = 16$ ,  $CF = y + 10$ , and  $CD = 4y - 20$ .



Determine the length of  $\overline{DF}$ .

$$4y - 20 = 2y + 20$$

$$2y = 40 \quad y = 20$$

$$DF = 60$$

Determine the length of  $\overline{OA}$ .

$$16^2 + 30^2 = c^2$$

$$256 + 900 = c^2$$

$$\sqrt{1156} = c$$

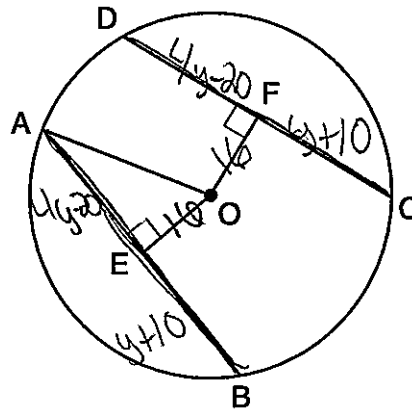
$$34 = c$$

$$AO = 34$$

Score 3: The student showed appropriate work to find  $y$ , but the length of  $\overline{DF}$  is incorrect. The student showed appropriate work to find the length of  $\overline{OA}$ .

Practice Papers—Question 36

36 In circle  $O$  shown below, chords  $\overline{AB}$  and  $\overline{CD}$  and radius  $\overline{OA}$  are drawn, such that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{OE} \perp \overline{AB}$ ,  $\overline{OF} \perp \overline{CD}$ ,  $OF = 16$ ,  $CF = y + 10$ , and  $CD = 4y - 20$ .



Determine the length of  $\overline{DF}$ .

$$4y - 20 = 2(y + 10)$$

$$4y - 20 = 2y + 20$$

$$-2y - 20 = 20$$

$$-2y = 40$$

$$y = -20$$

$$y = 10$$

$$DF = 20$$

Determine the length of  $\overline{OA}$ .

$$a^2 + b^2 = c^2$$

$$16^2 + 20^2 = c^2$$

$$256 + 400 = c^2$$

$$\sqrt{656} = c$$

$$\sqrt{656} = c$$

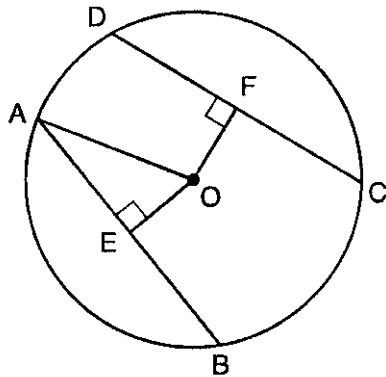
$$4041 = OA$$

- 4
- 9
- 16
- 25
- 36
- 49
- 64
- 81
- 100
- 121
- 144
- 169
- 196
- 225
- 256
- 289

Score 2: The student made a conceptual error in writing the equation to find  $y$ , but used this answer to do the rest of the problem correctly and found appropriate lengths.

Practice Papers—Question 36

36 In circle  $O$  shown below, chords  $\overline{AB}$  and  $\overline{CD}$  and radius  $\overline{OA}$  are drawn, such that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{OE} \perp \overline{AB}$ ,  $\overline{OF} \perp \overline{CD}$ ,  $OF = 16$ ,  $CF = y + 10$ , and  $CD = 4y - 20$ .



Determine the length of  $\overline{DF}$ .

$$4y - 20 = 2(y + 10)$$

$$4y - 20 = 2y + 20$$

$$2y = 40$$

$$y = 20$$

$$4y - 20 =$$

$$4(20) - 20$$

$$80 - 20$$

$$(60)$$

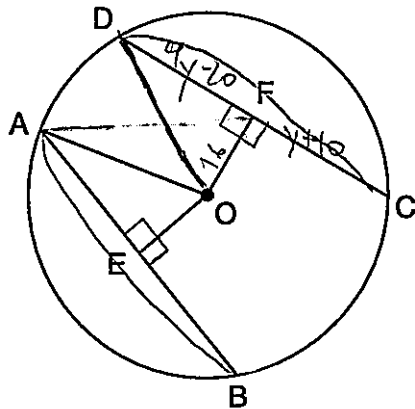
Determine the length of  $\overline{OA}$ .

Score 1: The student wrote the correct equation and found  $y = 20$ , but no further correct work was shown.



Practice Papers—Question 36

36 In circle  $O$  shown below, chords  $\overline{AB}$  and  $\overline{CD}$  and radius  $\overline{OA}$  are drawn, such that  $\overline{AB} \cong \overline{CD}$ ,  $\overline{OE} \perp \overline{AB}$ ,  $\overline{OF} \perp \overline{CD}$ ,  $OF = 16$ ,  $CF = y + 10$ , and  $CD = 4y - 20$ .



Determine the length of  $\overline{DF}$ .

$$4y - 20 = y + 10$$

$$-y + 20 - y + 20$$

$$3y = 30$$

$$y = 30$$

$$4y - 20 = DF$$

$$4(30) - 20 = DF$$

$$120 - 20 = DF$$

$$100 = DF$$

Determine the length of  $\overline{OA}$ .

$$30^2 + 16^2 = c^2$$

$$900 + 256 = c^2$$

$$1156 = c^2$$

$$34 = c$$

$c = \text{radius}$

$$34 = \text{radius}$$

$\overline{OD}$  and  $\overline{OA}$  are radii,  
so they are of equal lengths

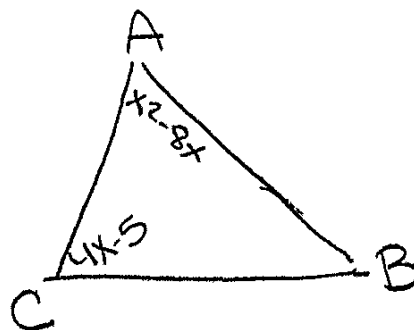
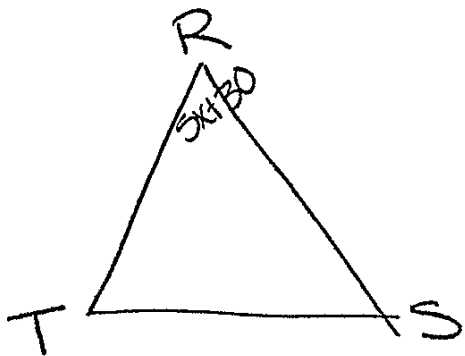
$$\boxed{OA = 34}$$

Score 0: The student made a conceptual error in writing the equation to solve for  $y$ , and then made a computational error when solving this equation. When solving for the length of  $\overline{OA}$ , the student made a second conceptual error by using their value for  $y$  instead of the length of  $\overline{DF}$  in the Pythagorean Theorem. The student got a correct answer for the second part by an incorrect method.

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]



$$\angle C = 4x - 5$$

$$\angle C = 4(15) - 5$$

$$\angle C = 60 - 5$$

$$\boxed{m\angle C = 55^\circ}$$

$$\begin{array}{r} x^2 - 8x = 5x + 30 \\ -5x \quad -5x \\ \hline \end{array}$$

$$x^2 - 13x - 30 = 0$$

$$\begin{array}{r} 1 \cdot 30 \\ 2 \cdot 15 \end{array}$$

$$\begin{array}{r} \cancel{1 \cdot 30} \\ -15 \quad \cancel{2} \\ \cancel{-13} \end{array}$$

$$\begin{array}{l} (x-15)(x+2) = 0 \\ \hline +15 \quad -2 \\ \boxed{x=15} \quad x=-2 \\ \text{reject} \end{array}$$

Score 4: The student wrote a correct equation and showed appropriate work to find  $m\angle C$ .

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]

$m\angle C = 105^\circ$   
 $5(15) + 30$   
 $75 + 30$   
 $105$

$5x + 30 = x^2 - 8x$   
 $-15x \quad -5x$

---

$x^2 - 13x - 30$   
 $-30$

---

$x^2 - 13x - 30$   
 $(x - 15)(x + 2)$

$x - 15 = 0$   
 $x + 15 = 25$   
 $x = 15$  (A)

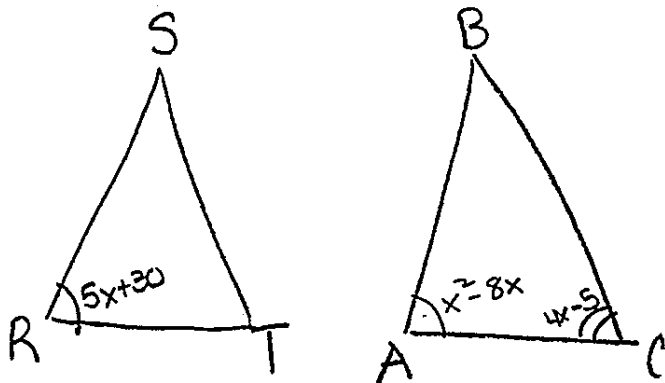
$x + 2 = 0$   
 $-2$   
 $x = -2$  (Reject)

Score 3: The student showed appropriate work to find  $x = 15$ , but no further correct work is shown.

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]



$$\begin{array}{r} 5x+30 = x^2-8x \\ \mp 5x \qquad \qquad -5x \\ \hline 30 = x^2-13x \\ -30 \qquad \qquad -30 \end{array}$$

$$\begin{array}{l} 1 \cdot 30 \\ 2 \cdot 15 \\ 3 \cdot 10 \\ 5 \cdot 6 \end{array}$$

$$0 = x^2 - 13x - 30$$

$$0 = (x-10)(x-3)$$

$$\begin{array}{l} x-10=0 \qquad x-3=0 \\ +10 \ +10 \qquad +3 \ +3 \\ \hline x=10 \qquad x=3 \end{array}$$

$$\begin{array}{l} 4x-5 \qquad 4x-5 \\ 4(10)-5 \qquad 4(3)-5 \\ 40-5 \qquad 12-5 \\ \hline 35 \qquad 7 \end{array}$$

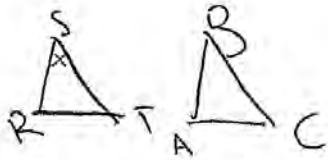
$$m\angle C = \{35, 7\}$$

Score 3: The student made a factoring error, but found appropriate measures for  $\angle C$ .

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]



$$m\angle A = x^2 - 8x$$

$$m\angle C = 4x - 5$$

~~$$x^2 - 8x + 4x - 5 + 5x + 30 = 180$$

$$x^2 - 9x + 25 = 180$$

$$\phantom{x^2 - 9x} - 25 \quad - 25$$


---


$$x^2 - 9x = 155$$

$$\phantom{x^2 - 9x} - 9 \quad - 9$$~~

$$5(15.5) + 30 = 107.5$$

$$m\angle C = 4x - 5$$

$$m\angle R = 5x + 30$$

$$4(15.5) - 5 = 57$$

$$m\angle C = 57$$

$$5x + 30 + 4x - 5 + x = 180$$

$$10x + 25 = 180$$

$$\phantom{10x + 25} - 25 \quad - 25$$


---


$$\frac{10x}{10} \quad \frac{155}{10}$$

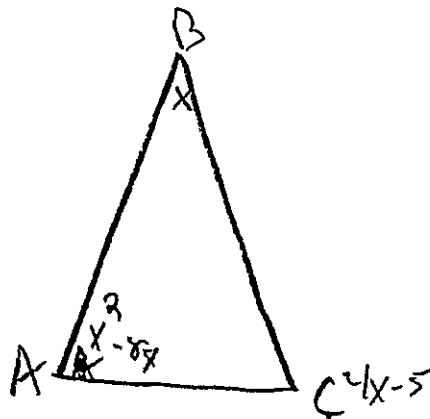
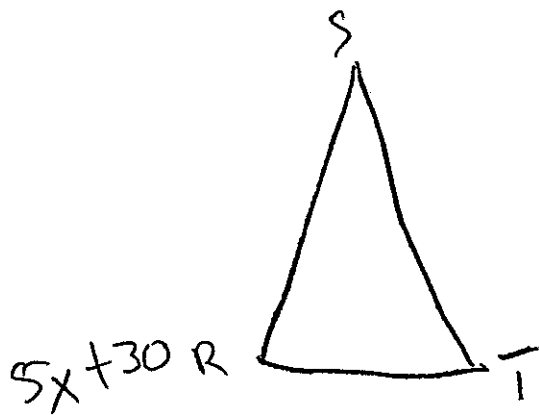
$$x = 15.5$$

Score 2: The student crossed out the first attempt. Then the student made a conceptual error in assuming that the measure of  $\angle S$  is  $x$ . The student used this value for  $x$  to find an appropriate measure for  $\angle C$ .

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]



$$\begin{array}{r}
 5x + 30 = x^2 - 8x \\
 -5x \qquad \qquad -5x \\
 \hline
 30 = x^2 - 13x
 \end{array}$$

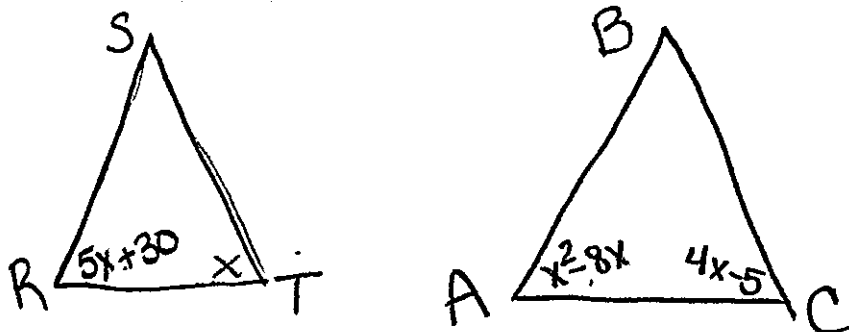
~~$$\begin{array}{r}
 x^2 - 8x + x + 4x - 5 = 180 \\
 x^2 - 3x - 5 = 180 \\
 \qquad \qquad +5 \qquad +5 \\
 \hline
 x^2 - 3x \qquad = 185 \\
 \qquad \qquad \underline{3x} \qquad \underline{3x} \\
 x^2 \qquad + = \cancel{x}
 \end{array}$$~~

Score 1: The student wrote a correct equation, but no further correct work was shown.

Practice Papers—Question 37

37 If  $\triangle RST \sim \triangle ABC$ ,  $m\angle A = x^2 - 8x$ ,  $m\angle C = 4x - 5$ , and  $m\angle R = 5x + 30$ , find  $m\angle C$ .

[Only an algebraic solution can receive full credit.]



$$\frac{x^2 - 8x}{4x - 5} = \frac{5x + 30}{x}$$

$$20x + 25 = 2x^2 - 8x$$

$$28x + 25 = 2x^2$$

$$\frac{2x^2}{2} - \frac{28x}{2} - \frac{25}{2} = 0$$

$$x^2 - 14x - 12.5 = 0$$

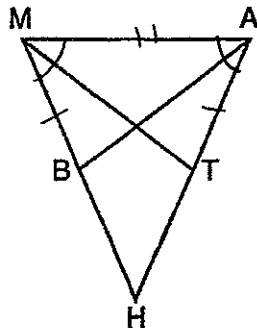
$$(x + 4)(x - 3.5) = 0$$

Score 0: The student's work is completely incorrect, irrelevant, and incoherent.

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



S	R
1. $\triangle MAH$ , $\overline{MH} \cong \overline{AH}$ medians $\overline{AB}$ and $\overline{MT}$	1. given
2. B is midpt of $\overline{MH}$ T is midpt of $\overline{AH}$	2. def of median
3. $\overline{MB} = \frac{1}{2} \overline{MH}$ $\overline{AT} = \frac{1}{2} \overline{AH}$	3. def of midpt
4. $\overline{MB} \cong \overline{AT}$	4. Halves of $\cong$ s are $\cong$
5. $\angle AMH \cong \angle MAH$	5. If 2 sides of a $\triangle$ are $\cong$ , the $\angle$ 's opposite these sides are $\cong$
6. $\overline{MA} \cong \overline{MA}$	6. Reflexive Property
7. $\triangle AMB \cong \triangle MAT$	7. SAS
8. $\angle MBA \cong \angle ATM$	8. CPCTC

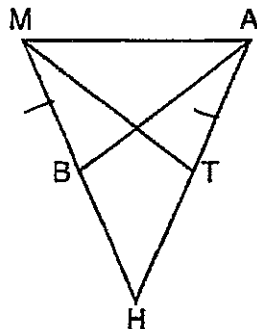
Score 6: The student showed a complete and correct proof.



Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



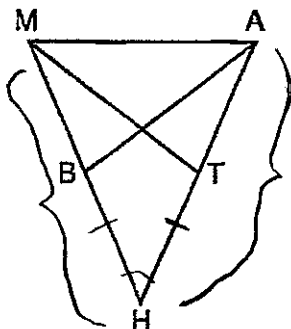
STATEMENTS	REASONS
1) $\triangle MAH$ , $\overline{MH} = \overline{AH}$ $\overline{AB}$ AND $\overline{MT}$ ARE MEDIANS	1) GIVEN
2) B IS MIDPOINT $\overline{MH}$ T IS MIDPOINT $\overline{AH}$	2) DEFINITION OF MEDIAN
3) $\overline{MB} = \frac{1}{2} \overline{MH}$ $\overline{AT} = \frac{1}{2} \overline{AH}$	3) DEFINITION OF MIDPOINT
4) $\overline{MB} \cong \overline{AT}$	4) $\frac{1}{2}$ 'S OF '='S ARE =
5) $\overline{MA} \cong \overline{MA}$	5) REFLEXIVE
6) $\overline{BA} = \overline{TM}$	6) IF 2 SIDES OF $\triangle \cong$ MEDIANS DRAWN TO THESE SIDES $\cong$
7) $\triangle MBA \cong \triangle ATM$	7) SSS

Score 5: The student proved  $\triangle MBA \cong \triangle ATM$ , but did not do the last step to prove  $\angle MBA \cong \angle ATM$ .

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



1  $\triangle MAH$ ,  $\overline{MH} \cong \overline{AH}$ , medians  $\overline{AB}$  &  $\overline{MT}$

2 B is midpoint of  $\overline{MH}$ , T is midpoint of  $\overline{AH}$

3  $\overline{BH} = \frac{1}{2}MH$ ,  $\overline{TH} = \frac{1}{2}AH$

4  $\overline{BH} \cong \overline{TH}$

5  $\angle H \cong \angle H$

6  $\triangle MHT \cong \triangle AHB$

7  $\angle HBA \cong \angle HTM$

1 given

2 medians connects vertex to midpoint of opposite side

3 midpoint divides line segment in half

4 halves of equals are equal

5 reflexive property

6. SAS

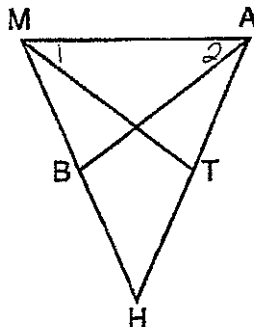
7. CPCTC

Score 4: The student proved  $\triangle MHT \cong \triangle AHB$ , but is missing two steps to prove  $\angle MBA \cong \angle ATM$ .

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



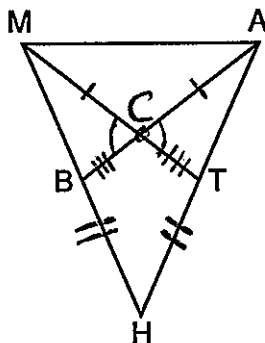
Statement	Reason
① $\overline{MH} \cong \overline{AH}$ median $\overline{AB}$ + $\overline{MT}$ $\triangle MAH$	① given
② $\angle 1 \cong \angle 2$	② angle bisector divides $\angle$ in $\frac{1}{2}$
③ $\angle MAT \cong \angle AMB$	③ 2 sides $\triangle \cong$ opposite $\angle$ s $\cong$
④ $\overline{MA} \cong \overline{MA}$	④ reflexive property
⑤ $\triangle MAB \cong \triangle AMT$	⑤ ASA
⑥ $\angle MBA \cong \angle ATM$	⑥ CPCTC

Score 3: The student made a conceptual error in line 2, but from there the student proved  $\angle MBA \cong \angle ATM$ .

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



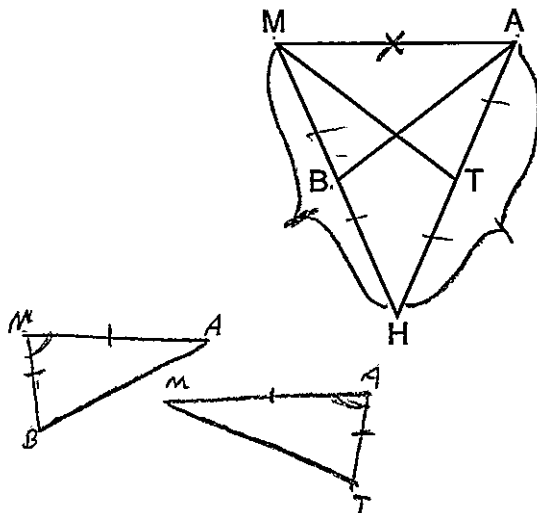
Statements	Reasons
① $\overline{MH} \cong \overline{AH}$ , $\overline{AB}$ is a median to $\overline{MH}$ . $\overline{MT}$ is a median to $\overline{AH}$ .	① Given
② $\overline{AB} \cong \overline{MT}$ $\overline{BC} \cong \overline{CT}$ $\overline{MC} \cong \overline{AC}$	② When medians bisect two congruent sides then they must be congruent too. When the 2 medians intersect they form congruent lines.
③ $\angle MCB \cong \angle ACT$	③ Vertical Angles theorem
④ $\triangle MBC \cong \triangle ATC$	④ SAS
⑤ $\angle B \cong \angle T$	⑤ CPCTC

Score 2: The student made some relevant statements, but four statements or reasons are missing or are incorrect.

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



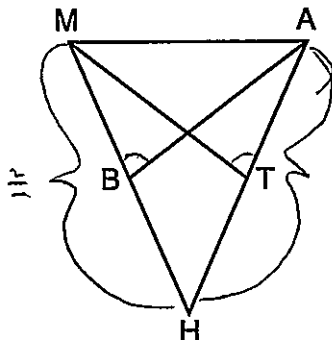
state ment	reason
1) $\overline{MH} \cong \overline{AH}$ , medians $\overline{AB} \text{ \& } \overline{MT}$	give
2) $\overline{MA} \cong \overline{MA}$	3) reflexive prop.
3) $\angle M \cong \angle A$	3) in a $\triangle$ if 2 sides are $\cong$ the $\angle$ 's opp those sides are $\cong$
4) $\angle$ $\cong$ $\angle$	4)

Score 1: The student has only one or two correct statements and reasons, but no further correct work is shown.

Practice Papers—Question 38

38 In the diagram of  $\triangle MAH$  below,  $\overline{MH} \cong \overline{AH}$  and medians  $\overline{AB}$  and  $\overline{MT}$  are drawn.

Prove:  $\angle MBA \cong \angle ATM$



statement	Reason
$\overline{MH} \cong \overline{AH}$	Given
$\overline{MB} \cong \overline{AT}$	Medians are congruent
$\overline{BH} \cong \overline{HT}$	Medians are congruent
$\angle MBA$ & $\angle HBA = 180^\circ$	Supplementary angles
$\angle ATM$ & $\angle HTM = 180^\circ$	Supplementary angles
$\angle MBA \cong \angle MTA$	Medians are congruent

Score 0: The student wrote no relevant statements.