# THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS COURSE II 

Wednesday, June 19, 2002 - 1:15 to 4:15 p.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

## Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of $\pi$ or in radical form. [60]

1 What is the identity element for the system defined by the table shown below?

| $*$ | $L$ | $U$ | $C$ | $K$ | $Y$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $L$ | $U$ | $C$ | $K$ | $Y$ | $L$ |
| $U$ | $C$ | $K$ | $Y$ | $L$ | $U$ |
| $C$ | $K$ | $Y$ | $L$ | $U$ | $C$ |
| $K$ | $Y$ | $L$ | $U$ | $C$ | $K$ |
| $Y$ | $L$ | $U$ | $C$ | $K$ | $Y$ |

2 In the accompanying diagram, $\overrightarrow{Q T} \perp \overleftrightarrow{P Q R}$ at $Q$, $\overrightarrow{Q W} \perp \overrightarrow{Q S}$ at $Q$, and $m \angle S Q P=25$. Find $m \angle T Q W$.


3 In $\triangle A B C$, a line parallel to $\overline{A B}$ intersects $\overline{A C}$ at $D$ and $\overline{B C}$ at $E, C D=8, A C=24$, and $B C=30$. Find the length of $\overline{E C}$.

4 In $\triangle G E M, \mathrm{~m} \angle G=37$ and $\mathrm{m} \angle M=100$. Which side of $\triangle G E M$ is the longest?

5 If each exterior angle of a regular polygon measures $90^{\circ}$, how many sides does the polygon have?

6 If the binary operation $\odot$ is defined by $a \odot b=a^{b}-b^{a}$, what is the value of $4 \odot 3$ ?

7 Factor completely: $a x^{2}-16 a$

8 The coordinates of $C$ are $(x, 5)$, the coordinates of $D$ are $(6,9)$, and the coordinates of the midpoint, $M$, of $\overline{C D}$ are $(-1,7)$. Find $x$.

9 A dilation maps $(6,10)$ to $(3,5)$. What are the coordinates of the image of $(12,4)$ under the same dilation?

10 If $\overleftrightarrow{M A} \perp \overleftrightarrow{T H}$ and the slope of $\overleftrightarrow{M A}$ is $\frac{2}{7}$, what is the slope of $\overleftrightarrow{T H}$ ?

11 The diagonals of a rhombus are 20 centimeters and 48 centimeters. Find the number of centimeters in the length of one side of the rhombus.

12 As shown in the accompanying diagram, a kite is flying at the end of a 25 -meter string. If the string makes an angle of $65^{\circ}$ with the ground, how high, to the nearest meter, is the kite?


13 Solve for the positive value of $x: \quad \frac{4}{x+3}=\frac{x-5}{5}$

14 A bag contains two white marbles, four blue marbles, and six red marbles. What is the probability that three marbles chosen randomly from the bag without replacement are all white?

Directions (15-34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

15 Which property is true for all trapezoids?
(1) Only two opposite sides are parallel.
(2) Consecutive angles are supplementary.
(3) The base angles are congruent.
(4) All angles are equal.

16 Which argument illustrates the Law of Detachment?
(1) $\frac{a \vee a b}{\sim b}$
(3) $\frac{a}{a \rightarrow \sim b} \underset{a \sim b}{a n}$
(2) $\frac{\begin{array}{l}a \rightarrow b \\ b \rightarrow c\end{array}}{\therefore a \rightarrow c}$
(4) $\frac{a \wedge^{\sim b}}{\therefore a}$

17 How many different nine-letter permutations can be formed from the nine letters of the word "MILLENIUM"?
(1) 9 !
(3) $\frac{9!}{3!}$
(2) $\frac{9!}{6!}$
(4) $\frac{9!}{2!2!2!}$

18 What is the equation of the axis of symmetry of the parabola represented by the equation $y=2 x^{2}+16 x-11$ ?
(1) $x=-8$
(3) $x=-4$
(2) $x=8$
(4) $x=4$

19 What are the roots of the equation $x^{2}-5 x-2=0$ ?
(1) $\frac{5 \pm \sqrt{17}}{2}$
(3) $\frac{5 \pm \sqrt{33}}{2}$
(2) $\frac{-5 \pm \sqrt{17}}{2}$
(4) $\frac{-5 \pm \sqrt{33}}{2}$

20 In the accompanying diagram of $\triangle A B C$, $A B=4 x-3, B C=2 x+7, A C=5 x-1$, and the perimeter of $\triangle A B C$ is 58 .


Which type of triangle is $\triangle A B C$ ?
(1) equilateral
(3) right
(2) isosceles
(4) scalene

21 In the accompanying diagram, $\overline{A B}$ and $\overline{C D}$ intersect at $E, E$ is the midpoint of $\overline{A B}$, and $\angle A \cong \angle B$.


Which statement can be used to prove
$\triangle A D E \cong \triangle B C E$
(1) ASA $\cong$ ASA
(3) $\mathrm{SSS} \cong \mathrm{SSS}$
(2) $\mathrm{HL} \cong \mathrm{HL}$
(4) $\mathrm{SAS} \cong \mathrm{SAS}$

22 A translation maps $(x, y)$ to $(x-5, y+3)$. In which quadrant does the point $(-3,-2)$ lie under the same translation?
(1) I
(3) III
(2) II
(4) IV

23 In the accompanying diagram, $\overleftrightarrow{A B} \| \overleftrightarrow{C D}$, and $\overleftrightarrow{E F}$ is a transversal that intersects $\overleftrightarrow{A B}$ at $G$ and $\overleftrightarrow{C D}$ at $H$.


Which two angles are not always congruent?
(1) $\angle B G H$ and $\angle D H G$
(3) $\angle B G H$ and $\angle G H C$
(2) $\angle A G E$ and $\angle D H F$
(4) $\angle E G B$ and $\angle A G H$

24 Which ordered pair is a solution of the system of equations $y=x^{2}+6$ and $y=x+6$ ?
(1) $(1,6)$
(3) $(0,6)$
(2) $(2,8)$
(4) $(2,10)$

25 Which statement is logically equivalent to "If Andrea gets a job, she buys a new car"?
(1) Andrea gets a job and she buys a new car.
(2) If Andrea does not buy a new car, she does not get a job.
(3) If Andrea does not get a job, she does not buy a new car.
(4) If Andrea buys a new car, she gets a job.

26 When $\frac{x+4}{2}$ is divided by $\frac{x^{2}-16}{8}$, the quotient is
(1) $\frac{x-4}{4}$
(3) $\frac{1}{x-1}$
(2) $\frac{1}{x}$
(4) $\frac{4}{x-4}$

27 Which set of numbers could represent the lengths of the sides of a triangle?
(1) $\{1,1,3\}$
(3) $\{1,2,3\}$
(2) $\{1.7,3.6,5.3\}$
(4) $\{2.2,3.3,4\}$

28 Which equation represents all points in a plane 5 units from point (3,2)?
(1) $(x-3)^{2}+(y-2)^{2}=5$
(2) $(x-3)^{2}+(y-2)^{2}=25$
(3) $\left(x^{2}+9\right)+\left(y^{2}+4\right)=5$
(4) $(x+3)^{2}+(y+2)^{2}=25$

29 Two isosceles triangles with equal vertex angles are always
(1) equilateral
(3) right
(2) congruent
(4) similar

30 Side $\overline{A B}$ of parallelogram $A B C D$ is represented by the equation $y-7=\frac{3}{2} x$. Which equation could represent side $\overline{C D}$ ?
(1) $2 y+14=3 x$
(3) $3 y-3=-2 x$
(2) $3 y+9=2 x$
(4) $y-5=\frac{2}{3} x$

31 In the accompanying diagram, $A B C D$ is a square, diagonal $\overline{B D}$ is extended through $D$ to $E$, $\overline{A D} \cong \overline{D E}$, and $\overline{A E}$ is drawn.


What is $\mathrm{m} \angle B A E$ ?
(1) 22.5
(3) 112.5
(2) 45.0
(4) 135.0

32 If the statements $\sim r \rightarrow \sim g$ and $a \rightarrow g$ are true, which statement must also be true?
(1) $\sim a \rightarrow \sim r$
(3) $g \rightarrow a$
(2) $a \rightarrow r$
(4) $r \rightarrow a$

33 What is the area of an equilateral triangle whose side is 4 ?
(1) $4 \sqrt{3}$
(3) $8 \sqrt{3}$
(2) 8
(4) $16 \sqrt{3}$

34 What is the total number of points in a plane that are 3 units from the $y$-axis and also 4 units from the origin?
(1) 0
(3) 3
(2) 2
(4) 4

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct the locus of points equidistant from the sides of $\angle A B C$.

## Answers to the following questions are to be written on paper provided by the school.

## Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]
$36 a$ Sketch the graph of the equation $y=x^{2}-6 x+4$ for all values of $x$ in the interval $0 \leq x \leq 6$. [6]
$b$ Find the roots of the equation $x^{2}-6 x+4=0$ to the nearest hundredth. [4]
$37 a$ Perform the indicated operations and simplify:
(1) $\frac{x^{2}-x-6}{3 x^{2}-10 x+3} \cdot \frac{4 x^{2}-7 x-2}{x^{2}-4}$
(2) $\frac{x-6}{x^{2}-9}+\frac{5}{x-3} \quad[3]$
$b$ Solve for $x, x \neq \pm 3: \quad \frac{x}{x-3}=\frac{x-3}{x+3} \quad[3]$

38 Kayla, Alyssa, Juanita, Dominic, and Troy want to go to a concert, but they only have three tickets. They decide to select randomly who will go.
a How many different three-person groups can be selected? [2]
$b$ What is the probability that Dominic will be selected? [3]
$c$ What is the probability that Alyssa will not be selected? [2]
$d$ How many different three-person groups can be selected if Kayla must be a member? [3]

39 The center of circle $O$ is $(1,2)$, and $A$ and $B$ are endpoints of a diameter of circle $O$. The coordinates of $A$ are $(6,2)$.
$a$ On graph paper, sketch circle $O$. [2]
$b$ Write the equation of circle $O$. [2]
c State the coordinates of B. [2]
$d$ Find the length of diameter $\overline{A B}$. [2]
$e$ Prove algebraically or by coordinate geometry that point $C(-2,6)$ lies on circle $O$. [2]

40 Triangle $S A M$ has coordinates $S(3,4), A(3,-5)$, and $M(-4,-2)$.
$a$ On graph paper, graph and label $\triangle S A M$. [1]
$b$ Graph and label $\triangle S^{\prime} A^{\prime} M^{\prime}$, the image of $\triangle S A M$ after a reflection in the line $y=x$. [3]
$c$ Graph and label $\triangle S^{\prime \prime} A^{\prime \prime} M^{\prime \prime}$, the image of $\triangle S A M$ after a dilation of 2. [3]
$d$ Express in simplest form the ratio of the area of $\triangle S A M$ to the area of $\triangle S^{\prime \prime} A^{\prime \prime} M^{\prime \prime}$. [3]

Answers to the following questions are to be written on paper provided by the school.

## Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: Either South High won the championship or North High won the championship.
If Larissa scored 50 points and Tamara was in the game, Kate was not the point guard.
If Larissa did not score 50 points, North High did not win the championship.
Kate was the point guard.
Tamara was in the game.
Let S represent: "South High won the championship."
Let $N$ represent: "North High won the championship."
Let $L$ represent: "Larissa scored 50 points."
Let $T$ represent: "Tamara was in the game."
Let $K$ represent: "Kate was the point guard."
Prove: South High won the championship. [10]

42 Given: $\angle H \cong \angle J, K$ is the midpoint of $\overline{H J}$, and $\overline{I F} \cong \overline{I G}$.


Prove: $\quad \overline{F K} \cong \overline{G K} \quad[10]$

## The University of the State of New York

Regents High School Examination

## SEQUENTIAL MATH - COURSE II

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$$
\begin{aligned}
& \text { Part I Score } \\
& \text { Part II Score } \\
& \text { Part III Score } \\
& \text { Total Score } \\
& \text { Rater's Initials: }
\end{aligned}
$$

ANSWER SHEET
Student Sex:MaleFemale Grade

Teacher School $\qquad$
Your answers to Part I should be recorded on this answer sheet.

## Part I

Answer 30 questions from this part.

| 1 | 11 | 21 | $31 . . . . . . . . . . . . . . . . .$. |
| :---: | :---: | :---: | :---: |
| 2 | 12 | 22 | $32 . . . . . . . . . . . . . . . . .$. |
| 3 | 13 | 23 |  |
| 4 | 14 | 24 | 34 |
| 5 6 | 15 16 | 25 26 | 35 Answer question 35 on the other side of this sheet. |
| 7 | 17 | 27 |  |
| 8 | 18 | 28 |  |
| 9 | 19 | 29 |  |
| 10 | 20 | 30 |  |

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Your answers for Part II and Part III should be placed on paper provided by the school.
The declaration below should be signed when you have completed the examination.
I do hereby affirm, at the close of this examination, that $I$ had no unlawful knowledge of the questions or answers prior to the examination and that $I$ have neither given nor received assistance in answering any of the questions during the examination.

