

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

**THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS**

**COURSE III**

Wednesday, August 16, 2000 — 8:30 to 11:30 a.m., only

**Notice . . .**

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**



Part I

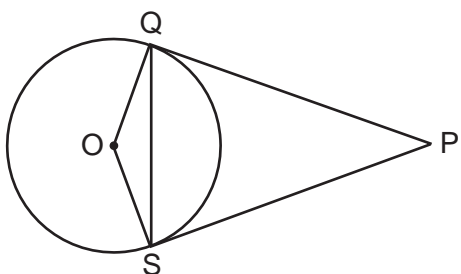
Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of  $\neq$  or in radical form. [60]

1 If  $f(x) = 3x - 4$  and  $g(x) = x^2$ , find the value of  $f(3) - g(2)$ .

2 What is the amplitude of the function  $y = 3 \sin 2x$ ?

3 What is the value of  $\sin (\text{Arc tan } \sqrt{3})$ ?

4 In the accompanying diagram,  $\overline{PQ}$  and  $\overline{PS}$  are tangents drawn to circle  $O$ , and chord  $\overline{QS}$  is drawn. If  $m\angle P = 40$ , what is  $m\angle PQS$ ?



5 An angle that measures  $\frac{5\pi}{3}$  radians is drawn in standard position. In which quadrant does the terminal side of the angle lie?

6 Express  $4\sqrt{-25} - 2\sqrt{-81}$  as a monomial in terms of  $i$ .

7 In  $\triangle ABC$ ,  $a = 6$ ,  $b = 10$ , and  $m\angle C = 30$ . Find the area of  $\triangle ABC$ .

8 Solve for the positive value of  $x$ :  $x^{\frac{4}{3}} + 2 = 18$

9 What is the solution set of the equation  $|2x - 1| = 5$ ?

10 If point  $A$  has coordinates  $(-3,4)$ , what are the coordinates of  $A'$ , the image of  $A$  under  $r_{y\text{-axis}} \circ D_2$ ?

11 Factor completely:  $3x^3 - 192x$

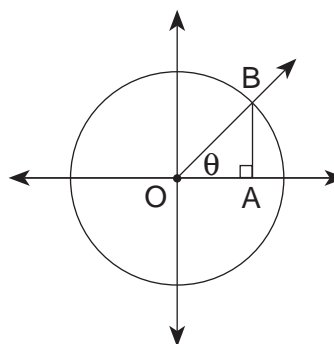
12 Evaluate:  $\sum_{k=0}^3 (2-k)^2$

13 Solve for the *smallest* non-negative value of  $\theta$ :

$$\sqrt{3 \cos \theta + 1} = 2$$

14 Express  $\frac{\frac{3}{x^2} + \frac{1}{x}}{1 - \frac{9}{x^2}}$  in simplest form.

15 Circle  $O$  has its center at the origin,  $OB = 1$ , and  $\overline{BA} \perp \overline{OA}$ . If  $m\angle BOA = \theta$ , which line segment shown has a length equal to  $\cos \theta$ ?



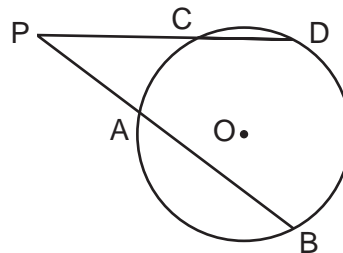
Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 In scientific notation, the number  $\frac{9}{1,000,000}$  is written as

- (1)  $9.0 \times 10^{-6}$                       (3)  $9.0 \times 10^6$   
 (2)  $9.0 \times 10^{-7}$                       (4)  $9.0 \times 10^7$

- 17 For which value of  $\theta$  is the fraction  $\frac{1}{\cos \theta}$  undefined?
- (1)  $\neq$  (3)  $\frac{\pi}{4}$   
 (2)  $\frac{\pi}{2}$  (4) 0
- 18 What is the solution of the inequality  $x^2 + 2x - 15 < 0$ ?
- (1)  $x < -5$  or  $x > 3$  (3)  $x < -3$  or  $x > 5$   
 (2)  $-5 < x < 3$  (4)  $-3 < x < 5$
- 19 The roots of the equation  $x^2 + kx + 3 = 0$  are real if the value of  $k$  is
- (1) 0 (3) 3  
 (2) 2 (4) 4
- 20 The heights of the members of a high school class are normally distributed. If the mean height is 65 inches and a height of 72 inches represents the 84th percentile, what is the standard deviation for this distribution?
- (1) 7 (3) 12  
 (2) 11 (4) 137
- 21 What is the greatest possible integral value of  $x$  for which  $\sqrt{x - 5}$  is an imaginary number?
- (1) 5 (3) 3  
 (2) 6 (4) 4
- 22 The expression  $\log 4x$  is equivalent to
- (1)  $\log x^4$  (3)  $\log 4 + \log x$   
 (2)  $4 \log x$  (4)  $(\log 4)(\log x)$
- 23 Which value of  $\theta$  satisfies the equation  $2 \cos^2 \theta - \cos \theta = 0$ ?
- (1)  $\frac{\pi}{3}$  (3)  $\frac{\pi}{6}$   
 (2)  $\frac{\pi}{4}$  (4) 0
- 24 If the probability that Mike will successfully complete a foul shot is  $\frac{4}{5}$ , what is the probability that he will successfully complete exactly three of his next four foul shots?
- (1)  $\frac{64}{625}$  (3)  $\frac{256}{625}$   
 (2)  $\frac{192}{625}$  (4)  $\frac{64}{125}$

- 25 In  $\triangle ABC$ ,  $a = 1$ ,  $b = 1$ , and  $m\angle C = 120$ . The value of  $c$  is
- (1) 1 (3)  $\sqrt{2.5}$   
 (2)  $\sqrt{2}$  (4)  $\sqrt{3}$
- 26 In the accompanying diagram,  $\overline{PAB}$  and  $\overline{PCD}$  are secants drawn to circle  $O$ ,  $PA = 8$ ,  $PB = 20$ , and  $PD = 16$ .



What is  $PC$ ?

- (1) 6.4 (3) 12  
 (2) 10 (4) 40
- 27 When the graphs of the equations  $xy = -16$  and  $y = x$  are drawn on the same set of axes, what is the total number of common points?
- (1) 1 (3) 3  
 (2) 2 (4) 0
- 28 The inverse of the function  $y = 2x - 5$  is
- (1)  $y = \frac{1}{2}(x + 5)$  (3)  $y = 2x + 5$   
 (2)  $y = \frac{1}{2}(x - 5)$  (4)  $y = 5 - 2x$
- 29 As  $x$  increases from  $\neq$  to  $2\neq$ , the value of  $\sin x$
- (1) increases, only  
 (2) decreases, only  
 (3) increases, then decreases  
 (4) decreases, then increases
- 30 If  $3^x \cdot y = 3^{x+1}$ , what is the value of  $y$ ?
- (1) 1 (3) 3  
 (2)  $-1$  (4)  $\frac{1}{3}$
- 31 If  $m\angle A = 32$ ,  $a = 5$ , and  $b = 3$ , it is possible to construct
- (1) an obtuse triangle  
 (2) two distinct triangles  
 (3) no triangles  
 (4) a right triangle

32 The expression  $\frac{\sin x \cdot \cos x}{\tan x}$  is equivalent to

- (1) 1 (3)  $\cos x$   
 (2)  $\sin^2 x$  (4)  $\cos^2 x$

33 Which field property is *not* satisfied by the set of integers for addition and multiplication?

- (1) identity for multiplication  
 (2) inverses for multiplication  
 (3) identity for addition  
 (4) closure for addition

34 What is the fourth term of the expansion  $(2x - y)^7$ ?

- (1)  $16x^4y^3$  (3)  $-560x^4y^3$   
 (2)  $35x^3y^4$  (4)  $-560x^3y^4$

35 If  $\sec x < 0$  and  $\tan x < 0$ , then the terminal side of angle  $x$  is located in Quadrant

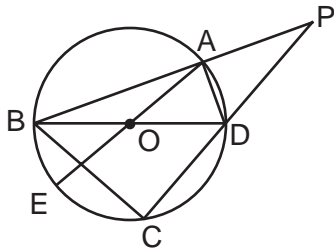
- (1) I (3) III  
 (2) II (4) IV

Answers to the following questions are to be written on paper provided by the school.

**Part II**

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 In the accompanying diagram of circle  $O$ , diameters  $\overline{BD}$  and  $\overline{AE}$ , secants  $\overline{PAB}$  and  $\overline{PDC}$ , and chords  $\overline{BC}$  and  $\overline{AD}$  are drawn;  $m\widehat{AD} = 40$ ; and  $m\widehat{DC} = 80$ .



Find:

- a  $m\widehat{AB}$  [2]  
 b  $m\angle BCD$  [2]  
 c  $m\angle BOE$  [2]  
 d  $m\angle P$  [2]  
 e  $m\angle PAD$  [2]

37 a On the same set of axes, sketch and label the graphs of the equations  $y = 4 \sin 2x$  and  $y = -2 \cos \frac{1}{2}x$  in the interval  $0 \leq x \leq 2\pi$ . [8]

b Based on the graph drawn in part a, how many values in the interval  $0 \leq x \leq 2\pi$  satisfy the equation  $4 \sin 2x = -2 \cos \frac{1}{2}x$ ? [2]

38 Given:  $f(x) = \log_3 x$

a On graph paper, sketch and label the graph of  $f(x) = \log_3 x$ . [4]

b On the same set of axes, rotate the graph drawn in part a  $90^\circ$  counterclockwise about the origin. Sketch this rotation and label it b. [4]

c Write an equation of the function graphed in part b. [2]

39 a Five marbles are in a jar. Two are red and three are white. Four marbles are selected at random with replacement.

(1) Find the probability that *at most* two red marbles are selected. [4]

(2) Find the probability that *at least* three red marbles are selected. [2]

b Find, to the *nearest tenth*, the standard deviation of this set of data. [4]

$x_i$	$f_i$
87	3
89	4
91	3
93	6
95	2

40 a In the interval  $0^\circ \leq \theta < 360^\circ$ , find all values of  $\theta$  that satisfy the equation  $1 + 2 \sin \theta = \csc \theta$ . [5]

b Prove the following identity:

$$\frac{\tan \theta}{\cot \theta} + 1 = \sec^2 \theta \quad [5]$$

41 a The roots of a quadratic equation are  $r_1 = 3 + 2i$  and  $r_2 = 3 - 2i$ .

(1) Find the sum of the roots  $r_1$  and  $r_2$ . [1]

(2) Find the product of the roots  $r_1$  and  $r_2$ . [2]

(3) Write a quadratic equation that has roots  $r_1$  and  $r_2$ . [2]

b Solve for  $x$ :  $\frac{4x}{x+2} - \frac{12}{x} = 1$  [5]

42 a Two forces of 130 and 150 pounds yield a resultant force of 170 pounds. Find, to the *nearest ten minutes* or *nearest tenth of a degree*, the angle between the original two forces. [7]

b Given:  $z_1 = 1 + 3i$  and  $z_2 = 5 + 2i$ . Plot  $z_1$ ,  $z_2$ , and  $z_1 + z_2$  on graph paper. [3]

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SEQUENTIAL MATH – COURSE III

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Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Pupil ..... Sex:  Male  Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- |          |          |          |          |
|----------|----------|----------|----------|
| 1 .....  | 11 ..... | 21 ..... | 31 ..... |
| 2 .....  | 12 ..... | 22 ..... | 32 ..... |
| 3 .....  | 13 ..... | 23 ..... | 33 ..... |
| 4 .....  | 14 ..... | 24 ..... | 34 ..... |
| 5 .....  | 15 ..... | 25 ..... | 35 ..... |
| 6 .....  | 16 ..... | 26 ..... |          |
| 7 .....  | 17 ..... | 27 ..... |          |
| 8 .....  | 18 ..... | 28 ..... |          |
| 9 .....  | 19 ..... | 29 ..... |          |
| 10 ..... | 20 ..... | 30 ..... |          |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

\_\_\_\_\_  
Signature

Tear Here

Tear Here

Tear Here

Tear Here