The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA 2/ TRIGONOMETRY

**Tuesday,** January 28, 2014 — 1:15 – 4:15 p.m.

## SAMPLE RESPONSE SET

### Table of Contents


Question 28 . . . . . . . . . . . . . . . . . . . 2
Question 29 . . . . . . . . . . . . . . . . . . . 5
Question 30 . . . . . . . . . . . . . . . . . . . 9
Question 31 . . . . . . . . . . . . . . . . . . 12
Question 32 . . . . . . . . . . . . . . . . . . 16
Question 33 . . . . . . . . . . . . . . . . . . 21
Question 34 . . . . . . . . . . . . . . . . . . 25
Question 35 . . . . . . . . . . . . . . . . . . 28
Question 36 . . . . . . . . . . . . . . . . . . 31
Question 37 . . . . . . . . . . . . . . . . . . 37
Question 38 . . . . . . . . . . . . . . . . . . 44
Question 39 . . . . . . . . . . . . . . . . . . 49

**Question 28**

**28** Show that $\sec\theta \sin\theta \cot\theta = 1$ is an identity.

$$\frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\sin\theta}}{1} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}} = 1$$

$$\frac{1}{1} = 1$$

$$1 = 1$$

**Score 2:** The student has a complete and correct response.

**Question 28**

**28** Show that $\sec\theta \sin\theta \cot\theta = 1$ is an identity.

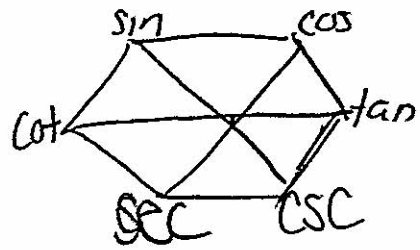
$$\frac{1}{\cos\theta} \quad \frac{\sin\theta}{1} \quad \frac{1}{\tan\theta}$$

$$\frac{1}{\cos\theta} \quad \frac{\sin\theta}{1} \quad \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos^2\theta}$$

**Score 1:** The student made a substitution error by replacing $\frac{1}{\tan\theta}$ with $\frac{\sin\theta}{\cos\theta}$.

**Question 28**

**28** Show that $\sec\theta \sin\theta \cot\theta = 1$ is an identity.



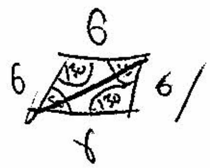

$$\frac{\cos\theta}{1} \cdot \frac{\csc}{1} \cdot \frac{\cos\theta}{\sin\theta} = 1$$

$$\sec\theta \cdot \sin\theta \cdot \cot\theta = 1$$

**Score 0:** The student made multiple errors when substituting for $\sec\theta$ and $\sin\theta$.

## Question 29

**29** Find, to the *nearest tenth of a square foot*, the area of a rhombus that has a side of 6 feet and an angle of 50°.



ab sin C

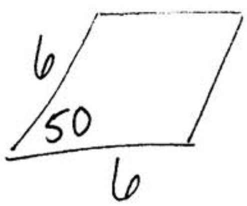
36 sin 130

27.6

**Score 2:** The student has a complete and correct response.

**Question 29**

**29** Find, to the *nearest tenth of a square foot*, the area of a rhombus that has a side of 6 feet and an angle of 50°.



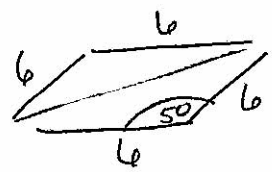

Area = $6 \cdot 6 \sin 50$

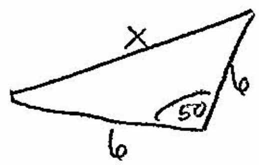
Area = 27.6 feet

**Score 1:** The student stated the wrong units.

## Question 29

**29** Find, to the *nearest tenth of a square foot*, the area of a rhombus that has a side of 6 feet and an angle of 50°.




$K = \frac{1}{2}(6)(6)(\sin 50)$

$K = 3(6)(\sin 50)$

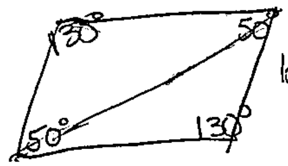
$K = 18 \cdot \sin 50$

$\boxed{K = 13.8 \text{ ft}^2}$

**Score 1:** The student made a conceptual error by not doubling the area of the triangle.

## Question 29

**29** Find, to the *nearest tenth of a square foot*, the area of a rhombus that has a side of 6 feet and an angle of 50°.




$b = 6.0000\,0046$

$6 = a$

2 sets of parallel lines

supplementary consecutive angles

diagonals are =

equal sides

$a^2+b^2=c^2$

$6^2+6^2=c^2$

$36+36$

$72=c^2$

$c \approx 8.4853$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\cancel{\frac{6}{\sin 50^\circ} = \frac{b}{\sin 130^\circ}}$$

$$\frac{4.596267}{\sin 50^\circ} = \frac{\sin 50^\circ\, b}{\sin 50^\circ}$$

$$\frac{4.596267}{.766044431} = b$$

$b = 6.0000046$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(6)$

$A = \frac{1}{2}(36)$

$A = 18$

$18(2) = 36$

$A = 36\text{ft}$

**Score 0:** The student made multiple conceptual errors, including the use of the Pythagorean Theorem and the incorrect use of the Law of Sines.

**Question 30**

**30** The following is a list of the individual points scored by all twelve members of the Webster High School basketball team at a recent game:

2 2 3 4 6 7 9 10 10 11 12 14

Find the interquartile range for this set of data.

$10.5 - 3.5 = \boxed{7}$

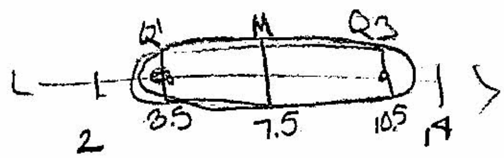
**Score 2:** The student has a complete and correct response.

**Question 30**

**30** The following is a list of the individual points scored by all twelve members of the Webster High School basketball team at a recent game:

2 2 3 4 6 7 9 10 10 11 12 14

Find the interquartile range for this set of data.




$3.5 \leq x \leq 10.5$

**Score 1:** The student made a conceptual error by expressing the interquartile range as an interval.

**Question 30**

**30** The following is a list of the individual points scored by all twelve members of the Webster High School basketball team at a recent game:

2 2 3 4 6 7 9 10 10 11 12 14

Find the interquartile range for this set of data.

2 2 3 4 6 7 | 9 10 10 11 12 14

$\boxed{\{3, 11\}}$

**Score 0:** The student made two conceptual errors. The quartiles were found incorrectly and the interquartile range was expressed as a set.

**Question 31**

**31** Determine algebraically the $x$-coordinate of all points where the graphs of $xy = 10$ and $y = x + 3$ intersect.

$\frac{xy}{x} = \frac{10}{x}$ $\quad y = \frac{10}{x}$ $\quad y = x + 3$

$x + 3 = \frac{10}{x}$

$x(x+3) = 10$

$x^2 + 3x = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x - 2 = 0 \quad x + 5 = 0$

$+2 \; +2 \quad -5 \; -5$

$\boxed{x = 2} \quad \boxed{x = -5}$

2, 5. $\quad$ -5, -2

**Score 2:** The student has a complete and correct response.

## Question 31

**31** Determine algebraically the $x$-coordinate of all points where the graphs of $xy = 10$ and $y = x + 3$ intersect.

$x(x+3)=10$

$x^2+3x=10$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5,2$

$y=-5+3=-2$

$y=3+2=5$

$(-5,-2),(2,5)$

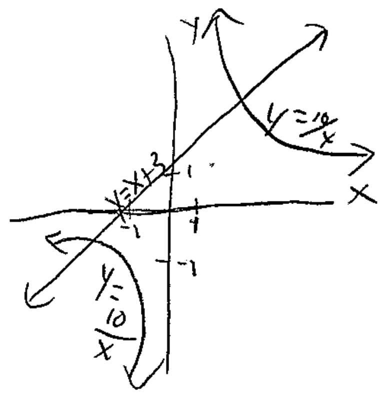
**Score 2:** The student has a complete and correct response, with correct work beyond the solutions.

## Question 31

**31** Determine algebraically the $x$-coordinate of all points where the graphs of $xy = 10$ and $y = x + 3$ intersect.

$\frac{xy}{x} = \frac{10}{x}$

$y = \frac{10}{x}$




| x | $Y_1$ ($y=\frac{10}{x}$) | $Y_2$ ($y=x+3$) |
|---|---|---|
| -5 | -2 | -2 |
| -4 | -2.5 | -1 |
| -3 | -3.33 | 0 |
| -2 | -5 | 1 |
| -1 | -10 | 2 |
| 0 | Error | 3 |
| 1 | 10 | 4 |
| 2 | 5 | 5 |

-5, -2

2, 5

**Score 1:** The student correctly solved the system of equations graphically.

**Question 31**

**31** Determine algebraically the $x$-coordinate of all points where the graphs of $xy = 10$ and $y = x + 3$ intersect.

$$\frac{xy}{x} = \frac{10}{x}$$

$$y = \frac{10}{x} \qquad y = x + 3$$

$$\frac{10}{2} = 5 \qquad 2 + 3 = 5$$

$$x = 5$$

**Score 0:** The student correctly solved for $y = \frac{10}{x}$, but no further correct work is shown. The $x$-coordinate that the student wrote is incorrect.

**Question 32**

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$|-4x+5| < 13$

$-4x+5 < 13$ $\quad$ $-4x+5 > -13$

$\phantom{-4x+}-5 \quad -5$ $\quad$ $\phantom{-4x+}-5 \quad -5$

$\frac{-4x}{-4} < \frac{8}{-4}$ $\quad$ $\frac{-4x}{-4} > \frac{-18}{-4}$

$x > -2$ $\quad$ $x < 4.5$

$\boxed{\{x \mid -2 < x < 4.5\}}$

**Score 2:** The student has a complete and correct response.

## Question 32

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$-4x + 5 < 13$

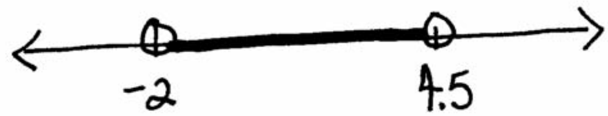
$-4x < 8$

$x > -2$

$-4x + 5 > -13$

$-4x > -18$

$x < 9/2$




**Score 2:** The student has a complete and correct response.

## Question 32

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$|-4x+5| = 13$

$|-4x+5| = 13$

$-4x + \cancel{5} = 13$
$\quad -\cancel{5} \quad -5$

$\frac{-\cancel{4}x}{-\cancel{4}} = \frac{8}{-4}$

$\boxed{x = -2}$

$4x - \cancel{5} = 13$
$\quad +\cancel{5} \quad +5$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{18}{4}$

$\boxed{x = \frac{9}{2}}$

**Score 1:** The student correctly solved the absolute value inequality as an absolute value equation. This is considered a conceptual error.

## Question 32

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$|-4x+5| < 13$

/ \

$-4x+5 < 13$ $\quad$ $-4x+5 > -13$

$-4x < 8$ $\quad$ $-4x > -18$

$\boxed{x > -2}$ $\quad$ $\boxed{x < 4.5}$

**Score 1:** The answer is not expressed as a conjunction.

**Question 32**

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$-4x+5<13$

$+5 \quad +5$

$\frac{-4x}{-4}<\frac{18}{-4}$

$x<-2$

or

$-4x+5<-13$

$-5 \quad -5$

$\frac{-4x}{-4}<\frac{18}{-4}$

$x<4.5$

**Score 0:** The student made more than one conceptual error.

## Question 33

**33** Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$4xi + 5yi^8 + 6xi^3 + 2yi^4$

$4xi - 6xi + 5y + 2y$

$-2xi + \cancel{5}y$

$\boxed{7y - 2xi}$

**Score 2:** The student has a complete and correct response.

**Question 33**

**33** Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$$4x(\sqrt{-1}) + 5y(1) + 6x(-i) + 2y(1)$$

$$4x\sqrt{-1} + 5y - 6xi + 2y$$

$$\boxed{7y + 4x\sqrt{-1} - 6xi}$$

**Score 1:** The student did not express the answer in simplest form. The $\sqrt{-1}$ should have been simplified to $i$.

**Question 33**

**33** Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$$-4xi + 5y - 6xi + 2y$$
$$-2xi + 7y$$

**Score 1:** The student did not write the solution in $a + bi$ form.

## Question 33

**33** Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$4xi - 5y + 6xi - 2y$

$10xi - 7y$

**Score 0:** The student made one conceptual error in replacing $i$ and did not put the answer in $a + bi$ form.

## Question 34

**34** In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for $a_n$, the $n^{\text{th}}$ term of this sequence.

$d = 4$

$a_n = a_1 + (n-1)d$

$$\boxed{a_n = 7 + (n-1)4}$$

$$\begin{array}{r} 31 \\ -19 \\ \hline 12 \end{array}$$

7 11 15 19 23 27 31

**Score 2:** The student has a complete and correct response.

## Question 34

**34** In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for $a_n$, the $n^{\text{th}}$ term of this sequence.

7, 11, 15, 19, 23, 27, 31

$$an = \frac{n(a1 + an)}{2}$$

$$an = 7\frac{(7+4)}{2}$$

$$an = \frac{7(11)}{2} = \frac{77}{2} = \boxed{38.5}$$

**Score 1:** The student found the first term, 7, and the common difference of 4. No further correct work is shown.

## Question 34

**34** In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for $a_n$, the $n^{\text{th}}$ term of this sequence.

$a_4 = 19$

$a_7 = 31$

$a_n = \frac{n}{2}(19+1)$

$a_n = \frac{n}{2}(18)$

$2 \cdot 31 = 2 \cdot \frac{n}{2}(18)$

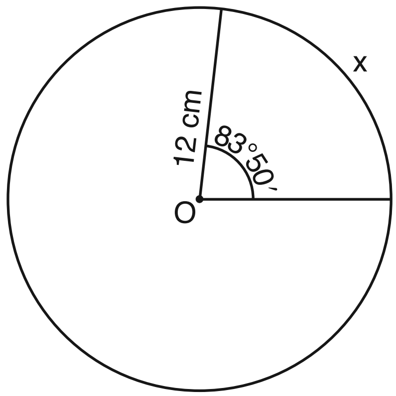
$\frac{62}{18} = \frac{n(18)}{18}$

$\boxed{3.44 = n}$

**Score 0:** The student response is completely incoherent.

## Question 35

**35** Circle $O$ shown below has a radius of 12 centimeters. To the *nearest tenth of a centimeter*, determine the length of the arc, $x$, subtended by an angle of 83°50′.




$S = r\theta$

$S = 12\left(\frac{83°50'\pi}{180}\right)$

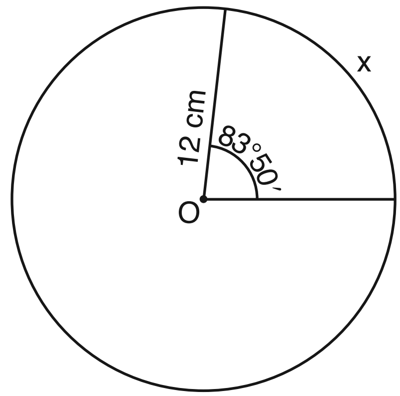
$S = 17.55801228$

$\boxed{S = 17.6 \text{ cm}}$

**Score 2:** The student has a complete and correct response.

## Question 35

**35** Circle $O$ shown below has a radius of 12 centimeters. To the *nearest tenth of a centimeter*, determine the length of the arc, $x$, subtended by an angle of $83°50'$.




$$\frac{83°50'}{360} = \frac{x}{144\pi}$$

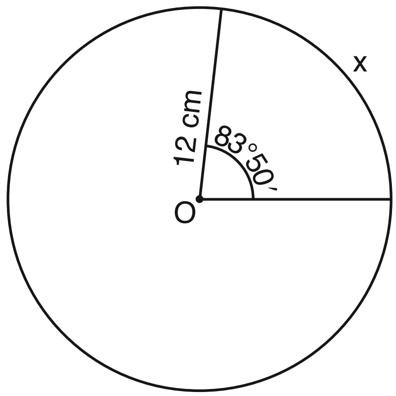
$$360x = 37925.30651$$

$$x = 105.3 \text{ cm}$$

**Score 1:** The student made a conceptual error by using the area of a circle rather than the circumference.

## Question 35

**35** Circle $O$ shown below has a radius of 12 centimeters. To the *nearest tenth of a centimeter*, determine the length of the arc, $x$, subtended by an angle of $83°50'$.




$C = 2\pi r$

$C = 2\pi(12)$

$C = 24\pi$

$$\frac{83}{360} = \frac{50}{x}$$

$$\frac{83x}{83} = \frac{18000}{83}$$

$x = 216.87$

$$\frac{83}{360} = \frac{\phantom{x}}{2\pi}$$

**Score 0:** The student response is completely incoherent.

**Question 36**

**36** Solve algebraically for all exact values of $x$ in the interval $0 \le x < 2\pi$:

$$2\sin^2 x + 5\sin x = 3$$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$ or $\sin x + 3 = 0$

$2\sin x = 1$ $\qquad$ $\sin x = -3$

$\sin x = \frac{1}{2}$ $\qquad$ (IMPOSSIBLE)

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = 30^\circ \quad x = 150^\circ$

$\frac{\pi}{6}$ and $\frac{5\pi}{6}$

**Score 4:** The student has a complete and correct response.

## Question 36

**36** Solve algebraically for all exact values of $x$ in the interval $0 \leq x < 2\pi$:

$$2\sin^2 x + 5\sin x = 3$$

$$-3 \quad -3$$

$$2x^2 + 5x - 3 = 0$$

$$(2x-1)(x+3) = 0$$

$$2x - 1 = 0 \qquad x + 3 = 0$$

$$+1 \; +1 \qquad -3 \; -3$$

$$\frac{2x}{2} = \frac{1}{2} \qquad x = -3$$

$$x = \frac{1}{2}$$

$$\sin x = \frac{1}{2} \qquad \cancel{\sin x = -3}$$

$$x = 30$$

$$\frac{\pi}{6}$$

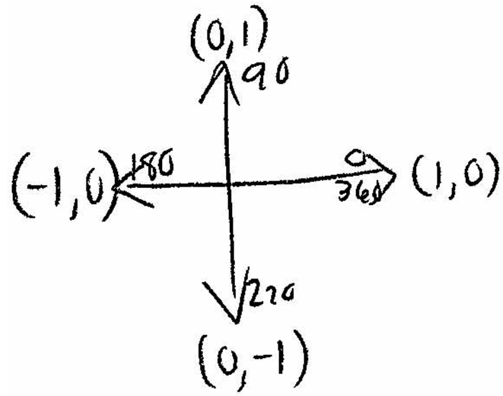
$$\frac{5\pi}{6}$$

**Score 4:** The student has a complete and correct response.

**Question 36**

**36** Solve algebraically for all exact values of $x$ in the interval $0 \leq x < 2\pi$:

$$2\sin^2 x + 5\sin x = 3$$




$2\sin^2 x + 5\sin x - 3 = 0$

$\sin^2 x + 5\sin x - 6 = 0$

$(\sin x + \frac{3}{2})(\sin x - \frac{2}{2}) = 0$

$\cancel{\sin x = \frac{-3}{2}}$ $\quad$ $\sin x = 1$

$90°$

$\left\{\frac{\pi}{2}\right\}$

**Score 3:** The student made a factoring error.

## Question 36

**36** Solve algebraically for all exact values of $x$ in the interval $0 \leq x < 2\pi$:

$$2\sin^2 x + 5\sin x = 3$$

$2x^2 + 5x = 3$

$-3 \quad -3$

$2x^2 + 5x - 3 = 0$

$(2x - 1)(2x + 6)$

$(2x - 1)(x + 3) = 0$

| | |
|---|---|
| $2x - 1 = 0$ | $x + 3 = 0$ |
| $+1 \quad +1$ | $-3 \quad -3$ |
| $\frac{2x}{2} = \frac{1}{2}$ | $x = -3$ |
| $x = .5$ | |

$\sin^{-1}(\sin x) = (.5)\sin^{-1}$ | ~~$\sin^{-1}(\sin x) = (-3)\sin^{-1}$~~

$\boxed{30^\circ}$

$-6$

| | |
|---|---|
| 1 | −6 |
| −1 | 6 |
| 2 | −3 |
| −2 | 3 |

**Score 2:** The student correctly found $\sin x = 0.5$ and $\sin x = -3$.

## Question 36

**36** Solve algebraically for all exact values of $x$ in the interval $0 \le x < 2\pi$:

$$2\sin^2 x + 5\sin x = 3$$

$2\sin^2 + 5\sin x - 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$

$x = \dfrac{-5 \pm \sqrt{49}}{4}$

$x = \dfrac{-5 \pm 7}{4}$

$\sin x = 3$

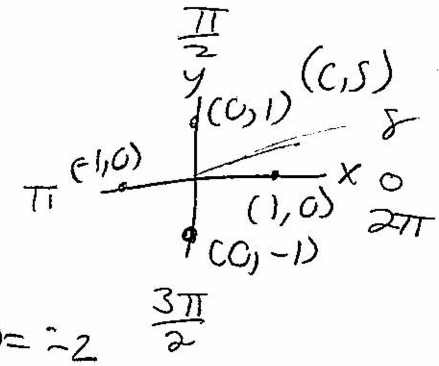
$\sin x = -.5$

**Score 1:** A correct substitution into the quadratic formula is made, but no further correct work is shown.

**Question 36**

**36** Solve algebraically for all exact values of $x$ in the interval $0 \leq x < 2\pi$:

$$2 \sin^2 x + 5 \sin x = 3$$




$2\sin^2 x + 5\sin x - 3 = 0$

$2\sin^2 x + 5\sin x = 3$

$\sin x(2\sin + 5) = 3$

Sin

$\sin x(2\sin) = -2$

$\cancel{2\pi}$

$\frac{8}{360}\pi$

$\boxed{\frac{\pi}{45}}$

$\cancel{\frac{-2}{360}}$

**Score 0:** The student made more than one conceptual error.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$n=5$ $p=.28$ $K=4$
geraniums

1- binomialcdf(5,(.28),(3)

.024

**Score 4:** The student has a complete and correct response.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$${}_5C_4(.28)^4(.72)^1 + {}_5C_5(.28)^5(.72)^0$$

$$0.024$$

**Score 4:** The student has a complete and correct response.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$$_5C_4\,(.28)^4(.72) + {}_5C_5\,(.28)^5(.72)^0 =$$

$$.0236486528$$

$$2.4\%$$

**Score 4:** The student has a complete and correct response. The answer of 2.4% is mathematically equivalent to 0.024.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$n = 5$

$r = 4, 5$

$s = .28$

$f = .72$

${}_5C_4 (.72)(.28)^4 = .022$

${}_5C_5 (.72)(.28)^5 = {}^{+} .001$

$.023$

**Score 3:** The student made one rounding error.

**Question 37**

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

4 + 5

$_5C_4 \; (.28)^4 \times (.72)^1 \quad + \quad _5C_5 \; (.28)^5 \times (.72)^0$

$0.022127616 \quad + \quad 0.0017210368$

$= .023\%$

**Score 2:** The student made one rounding error and expressed the answer as a percent.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$${}_5C_4\,(.28)^4\,(.72)^1$$

$$5 \cdot .00614656 \cdot .72$$

$$\boxed{.02}$$

**Score 1:** The student found a correct probability for exactly four out of five, and did not round to the *nearest thousandth*.

## Question 37

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$${}_nC_r \cdot p^r \cdot q^{n-1}$$

$${}_5C_4 \left(\tfrac{7}{25}\right)^4 \left(\tfrac{18}{25}\right)^4$$
$${}_5C_3 \left(\tfrac{7}{25}\right)^3 \left(\tfrac{18}{25}\right)^3$$
$${}_5C_2 \left(\tfrac{7}{25}\right)^2 \left(\tfrac{18}{25}\right)$$
$${}_5C_1 \left(\tfrac{7}{25}\right)^1 \left(\tfrac{18}{25}\right)^1$$
$${}_5C_0 \left(\tfrac{7}{25}\right)^0 \left(\tfrac{18}{25}\right)^0$$

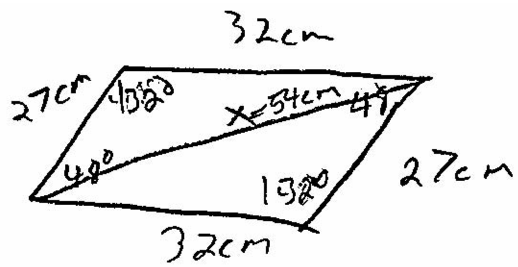
$${}_5C_4 \left(\tfrac{7}{25}\right)^4 \left(\tfrac{18}{25}\right)^3 = .01147$$
$${}_5C_5 \left(\tfrac{7}{25}\right)^5 \left(\tfrac{18}{25}\right)^4 =$$

$$\boxed{\frac{247}{250}}$$

**Score 0:** The student made two conceptual errors. Incorrect exponents were written, and then the student subtracted this answer from 1.

## Question 38

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.




$$x^2 = 32^2 + 27^2 - 2(32)(27)\cos 132^\circ$$

$$x^2 = 1024 + 729 - 1728\cos 132^\circ$$

$$x^2 = 1753 - 1728\cos 132^\circ$$

$$\sqrt{x^2} = \sqrt{2909.257688}$$

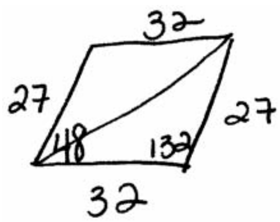
$$x = 53.9$$

$$\approx 54\text{ cm}$$

**Score 4:** The student has a complete and correct response.

## Question 38

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.




$a^2 = b^2 + c^2 - 2bc\ \cos A$

$a^2 = (27)^2 + (32)^2 - 2(27)(32)\cos(132)$

$a^2 = 27.44$

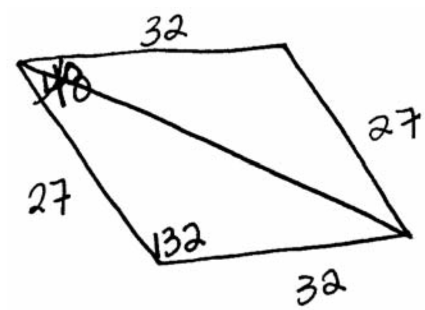
$\sqrt{27.44} = 5.24$

5cm

**Score 3:** The student made one computational error by using radian mode.

## Question 38

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.




$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 27 + 32^2 - 2(27)(32)\cos \cancel{\angle} 132$

$a^2 = 729 + 1024 - 2(27)(32)\cos \cancel{\angle} 132$

$a^2 = 1753 - 2(27)(32)\cos 132$

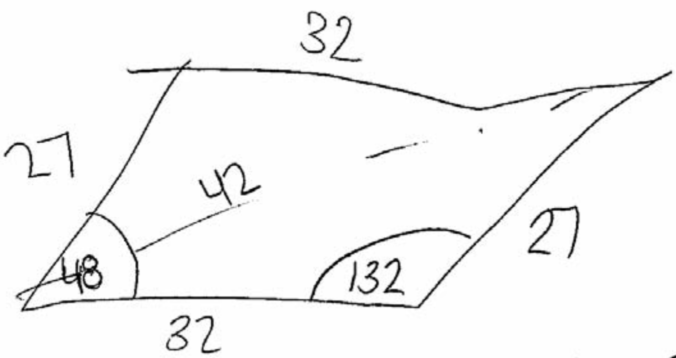
$a^2 = 1753 - 1728\cos 132$

$a^2 = \cancel{27.436}\ 55482$

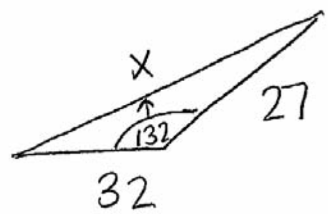
**Score 2:** The student made a correct substitution into the Law of Cosines.

## Question 38

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.




$$\begin{array}{r} 180 \\ -\ 48 \\ \hline 132 \end{array}$$




$32^2 + 27^2 = x^2$

$1024 + 729 = x^2$

$\sqrt{1753} = \sqrt{x^2}$

$x - 41.86884283$

$x \approx 42$ cm.

**Score 1:** The student drew a correctly labeled diagram. The remainder of the work shown is incorrect.

**Question 38**

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.


$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = (32)^2 + (27)^2 - 2(32)(27)\cos 48$$

$$a^2 = 1024 + 729 - 1728(\cos 48)$$

$$a^2 = 25\cos 48$$

$$\sqrt{a^2} = \sqrt{16.72826516}$$

$$a = 4.090020191$$

$$\boxed{a = 4.1\text{ cm}}$$

**Score 0:** The student made two conceptual errors by finding the shorter diagonal and combining terms incorrectly. There was also one rounding error.

## Question 39

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$$\log_{(x+3)}(2x+3)(x+5) = 2$$

$$(x+3)^2 = (2x+3)(x+5)$$

$$x^2+6x+9 = 2x^2+13x+15$$
$$-x^2-6x-9 \quad -x^2-6x-9$$

$$0 = x^2+7x+6$$

$$0 = (x+6)(x+1)$$

$\cancel{x=-6}$ reject | $x=-1$

**Score 6:** The student has a complete and correct response.

## Question 39

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$\log_{10} 2 \overset{?}{=} x$

$\log_{(x+3)}(2x^2+10x+3x+15) = 2$

$(x+3)^2 = 2x^2+10x+3x+15$

$x^2+6x+9 = 2x^2+13x+15$

$-x^2-6x-9 \quad -x^2-6x-9$

$0 = x^2+7x+6$

$0 = (x+6)(x+1)$

$x+6=0 \quad | \quad x+1=0$

$x=-6 \quad | \quad x=-1$

$\{-6,-1\}$

**Score 5:** The student did not reject $-6$.

**Question 39**

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$$\log_{(x+3)}(2x+3)(x+5)=2$$

$$\log_{(x+3)}(2x^2+10x+3x+15)=2$$

$$\log_{(x+3)}(2x^2+13x+15)=2$$

$$(x+3)^2=2x^2+13x+15$$

$$\cancel{x^2}+\cancel{9x}+\cancel{9}=2x^2+13x+15$$

$$-\cancel{x^2}-\cancel{9x}-\cancel{9} \quad -x^2-9x-9$$

$$x^2+4x+6=0$$

$$a=1 \quad b=4 \quad c=6$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2(a)}=\frac{-4\pm\sqrt{4^2-4(1)(6)}}{2(1)}=\frac{-4\pm\sqrt{16-24}}{2}=$$

$$\frac{-4\pm\sqrt{-8}}{2}=\frac{-4\pm 2i\sqrt{2}}{2}=\boxed{-2\pm i\sqrt{2}}$$

**Score 4:** The student made one computational error when squaring $x + 3$. The student also made an error in not discarding the imaginary solutions.

## Question 39

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$$\frac{\cancel{\log}((2x+3)(x+5))}{\cancel{\log}(x+3)} = \frac{2}{1}$$

$\boxed{\xi : 13}$

$(2x+3)(x+5) = 2x+6$

$2x^2 \cancel{+3x+10x} + 1\cancel{5} = 2x + \cancel{6}$ ($+13x$; $-6$, $-6$)

$2x^2 + 11x + 9 = 0$

$(2x+9)(x+1) = 0$

$\boxed{x \neq -\frac{9}{2}}$ $\boxed{x = -1}$

Chk.

$\log_{(-1+3)}(2(-1)+3) + \log_{(-1+3)}(-1+5) = 2$

$\log_{(2)}(1) + \log_{(2)}(4) = 2$

$2 = 2 \checkmark$

$\boxed{\therefore x = -1}$

$\cancel{\log_{(\frac{-9}{2}+3)}(2(\frac{-9}{2})+3) + \log_{(\frac{-9}{2}+3)}(\frac{-9}{2}+5) = 2}$

no solution

**Score 3:** The student made a conceptual error by canceling the logs.

**Question 39**

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$(x+3)^2 = 2x+3 + x+5$

$(x+3)\ (x+3) \quad = 3x+8$

$x^2 + 3x + 3x + 9$

$x^2 + 6x + 9 = 3x+8$

$\quad -3x - 8 \quad -3x \quad -8$

$x^2 + 3x + 1 = 0$

$a=1 \quad b=3 \quad c=1$

$\dfrac{-(b) \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-3 \pm \sqrt{9-4}}{2}$

$\dfrac{-3+-\sqrt{5}}{2}$

**Score 2:** The student made a conceptual error by adding the binomials. The student did not discard the solution outside the domain.

## Question 39

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$$\log_{(x+3)}(2x+3)(x+5) = 2$$

$$\log_{(x+3)}(2x^2+10x+3x+15) = 2$$

$$\log_{(x+3)}(2x^2+7x+15) = 2$$

$$(2x^2+7x+15) = (x+3)^2$$

$$(2x^2+7x+15) = \cancel{x^2}+9$$

$$-x^2 \qquad -x^2$$

$$2+7x+15 = 9$$

$$7x+17 = 9$$

$$-17 \quad -17$$

$$\frac{7x}{7} = \frac{-8}{7} = -\frac{8}{7}$$

$$\boxed{x = -\frac{8}{7}}$$

**Score 1:** The student correctly wrote $\log_{(x + 3)}(2x + 3)(x + 5) = 2$. The remainder of the work was incorrect.

**Question 39**

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$10x$

$2x$

$(2x+3)(x+5) = 2$

$2x^2 + 13x + 15 = 2$
$\phantom{2x^2 + 13x + 1}-2 \quad -2$

$2x^2 + 13x + 13 = 0$

$0 = (2x + 3) \quad (x+5) = 2$
$-3 \qquad\qquad\qquad\qquad -5$

$\frac{-3}{2} = \frac{2x}{2}$

$x = \frac{-3}{2}, \quad x = -3$

**Score 0:** The student made multiple errors in attempting to solve the log equation.