

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA 2/ TRIGONOMETRY

Friday, January 29, 2016 — 9:15 a.m. – 12:15 p.m.

## SAMPLE RESPONSE SET

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**Question 28**

**28** Solve algebraically for  $x$ :

$$\sqrt{2x + 1} + 4 = 8$$

$$\sqrt{2x + 1} = 4$$

$$2x + 1 = 16$$

$$x = 7.5$$

**Score 2:** The student has a complete and correct response.

Question 28

28 Solve algebraically for x:

$$\begin{array}{r} \sqrt{2x+1} + 4 = 8 \\ -4 \quad -4 \\ \hline (\sqrt{2x+1})^2 = (4)^2 \\ 2x+1 = 16 \\ \begin{array}{r} \swarrow \\ 2x+1 = 16 \\ \swarrow \quad \searrow \\ -1 \quad -1 \\ \hline 2x = 15 \\ \swarrow \quad \searrow \\ 2 \quad 2 \\ \hline x = 7.5 \end{array} \quad \begin{array}{r} \swarrow \\ 2x+1 = -16 \\ \swarrow \quad \searrow \\ -1 \quad -1 \\ \hline 2x = -17 \\ \swarrow \quad \searrow \\ 2 \quad 2 \\ \hline x = -8.5 \text{ Reject} \end{array} \end{array}$$

**Score 1:** The student made an error by treating the square root as an absolute value.

**Question 28**

28 Solve algebraically for  $x$ :

$$\sqrt{2x + 1} + 4 = 8$$

$$\sqrt{2x + 1} + 4 = 8$$

$$2x + 1 + 4 = 64$$

$$2x = 59$$

$$x = 29.5$$

**Score 1:** The student made an error when squaring both sides of the equation.

**Question 28**

28 Solve algebraically for  $x$ :

$$\sqrt{2x + 1} + 4 = 8$$

$$\sqrt{2x + 1} = 4$$

$$2x + 1 = 4$$

$$2x = 3$$

$$x = \frac{3}{2}$$

**Score 1:** The student made a conceptual error by not squaring both sides of the equation.

Question 28

28 Solve algebraically for  $x$ :

$$\sqrt{2x+1} + 4 = 8$$

$$\sqrt{2x+1} + 4 = 8$$

~~-4~~  
-4

$$(\sqrt{2x+1})^2 = (4)^2$$

$$4x^2 + 1 = 16$$

~~-1~~  
-1

$$4x^2 = 15$$

~~-4~~  
-4

$$\sqrt{x^2} = \sqrt{3.75}$$

$$x = 1.936491673$$

**Score 0:** The student made one error when squaring the radical and a second error by not stating  $\pm\sqrt{3.75}$ .

**Question 29**

**29** Factor completely:

$$x^3 + 3x^2 + 2x + 6$$

$$x^3 + 3x^2 + 2x + 6$$

$$x^2(x+3) + 2(x+3)$$

$$(x^2 + 2)(x + 3)$$

**Score 2:** The student has a complete and correct response.

**Question 29**

**29** Factor completely:

$$x^3 + 3x^2 + 2x + 6$$

$$x^2(x+3) + 2(x+3)$$

$$(x^2+2)(x+3)$$

$$(x-2)(x-1)(x+3)$$

**Score 1:** The student incorrectly factored  $(x^2 + 2)$ .

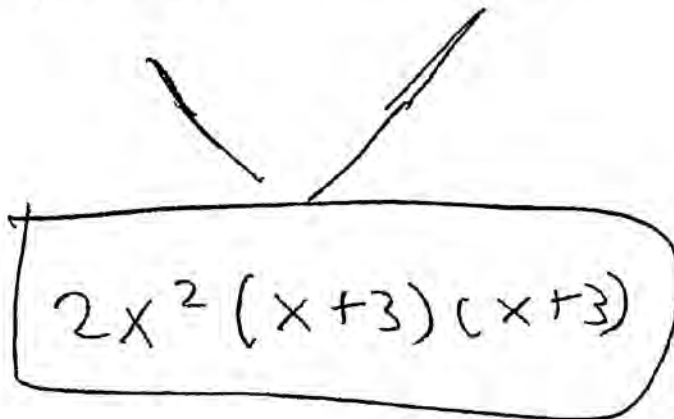


**Question 29**

**29** Factor completely:

$$x^3 + 3x^2 + 2x + 6$$

$$\begin{array}{ll} x^3 + 3x^2 & 2x + 6 \\ x^2(x + 3) & 2(x + 3) \end{array}$$


$$2x^2(x + 3)(x + 3)$$

**Score 1:** The student made an error in factoring by grouping.

Question 29

29 Factor completely:

$$x^3 + 3x^2 + 2x + 6$$

$$\begin{aligned} & x^3 + 3x^2 + 2x + 6 \\ & \underline{x^2}(x+3) + \underline{2}(x+3) \\ & (x^2+2)(x+3) \\ & \begin{array}{l} x^2+2=0 \\ -2 \quad -2 \\ \sqrt{x^2} = \sqrt{-2} \\ x = \sqrt{-2} \end{array} \quad \begin{array}{l} x+3=0 \\ -3 \quad -3 \\ x = -3 \end{array} \end{aligned}$$

**Score 1:** The student made an error by treating the expression as an equation.

**Question 29**

**29** Factor completely:

$$x^3 + 3x^2 + 2x + 6$$

$$X(x^2 + 3x + 2) + 6$$

$$(x+6)(x^2 + 3x + 2) = 0$$

$$(x+6)(x+2)(x+1) = 0$$

$$X+6=0 \quad X+2=0 \quad X+1=0$$

$$X=-6 \quad X=-2 \quad X=-1$$

**Score 0:** The student factored by grouping incorrectly and treated the expression as an equation.

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$8^{x+1} = 16$$

$$2^{3x+3} = 2^4$$

$$3x+3 = 4$$
$$-3 \quad -3$$

$$3x = 1$$
$$\frac{3x}{3} = \frac{1}{3}$$

$$x = \frac{1}{3}$$

**Score 2:** The student has a complete and correct response.

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$\frac{\log 16}{\log 8} = x + 1$$

$$1.\bar{3} = x + 1$$

$$0.\bar{3} = x$$

**Score 2:** The student has a complete and correct response.

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$\begin{aligned} \sqrt[8]{16} &= x + 1 \\ \sqrt[8]{8} &= x \end{aligned}$$

**Score 2:** The student has a complete and correct response.

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$\begin{aligned}8(x+1) &= 16 \\8x + 8 &= 16 \\8x &= 8 \\x &= 1\end{aligned}$$

**Score 1:** The student made an error by not raising 8 to the power of  $(x + 1)$ .

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$\log_8 16 = x + 1$$

$$1.204119983 = x + 1$$

-1                      -1

$$-0.204119983 = x$$

**Score 1:** The student made an error by calculating  $\log 16$ .



Question 30

30 Solve algebraically for the *exact* value of  $x$ :

$$\begin{aligned} \log_8 16 &= x + 1 \\ \log 16 &= \log 8^{(x+1)} \\ 2^4 &= (2^3)^{(x-1)} \\ 4 &= 3x - 3 \\ +3 & \quad +3 \\ \hline 7 &= 3x \\ \frac{7}{3} &= x \end{aligned}$$

**Score 1:** The student made a transcription error by writing  $(x - 1)$  instead of  $(x + 1)$ .

**Question 30**

**30** Solve algebraically for the *exact* value of  $x$ :

$$\log_8 16 = x + 1$$

$$16(x+1) = 8$$

$$x+1 = \frac{1}{2}$$

$$x = -\frac{1}{2}$$

**Score 0:** The student wrote a completely incorrect response.

Question 31

31 Determine how many eleven-letter arrangements can be formed from the word  
"CATARAUGUS."

~~CATARAUGUS.~~

3 A'S

2 T'S

2 U'S

11!

$$\frac{11!}{3! 2! 2!}$$

1663200

**Score 2:** The student has a complete and correct response.

**Question 31**

**31** Determine how many eleven-letter arrangements can be formed from the word "CATTARAUGUS."

$$\frac{11!}{3 \cdot 2 \cdot 2 \cdot 2} = 1,663,200$$

**Score 2:** The student has a complete and correct response.

**Question 31**

**31** Determine how many eleven-letter arrangements can be formed from the word "CATTARAUGUS."

$$\frac{11!}{3 \cdot 2 \cdot 2} = 3,326,400$$

**Score 1:** The student divided by an incorrect denominator.

**Question 31**

**31** Determine how many eleven-letter arrangements can be formed from the word "CATTARAUGUS."

$$\frac{11!}{3! + 2! + 2!}$$

$$\frac{11!}{10}$$

3,991,680

**Score 1:** The student added in the denominator instead of multiplying.

**Question 31**

**31** Determine how many eleven-letter arrangements can be formed from the word "CATTARAUGUS."

$$11! = 39,916,800$$

**Score 0:** The student only evaluated  $11!$ .

Question 32

32 Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$$\frac{-130^\circ}{1} \cdot \frac{\pi}{180^\circ} = -\frac{13\pi}{18} = \boxed{-2.27}$$

**Score 2:** The student has a complete and correct response.



**Question 32**

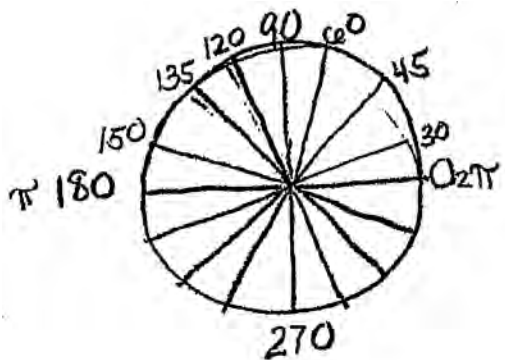
**32** Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$-130^\circ$  in calculator  
Switch to radian mode  
 $-2.27$

**Score 2:** The student has a complete and correct response.

Question 32

32 Express  $-130^\circ$  in radian measure, to the nearest hundredth.



$$\frac{-130}{180} \quad \frac{-13}{18}$$

$$\textcircled{-.72}$$

**Score 1:** The student did not include  $\pi$  in the formula.

**Question 32**

**32** Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$$\begin{aligned} \pi &= 180^\circ \\ x &= -130^\circ \end{aligned}$$

$$-130/x = 180/y$$

$$\boxed{-0.72 \pi}$$

**Score 1:** The student did not express the answer to the nearest hundredth.

**Question 32**

**32** Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$$130 \cdot \left(\frac{1}{180}\right) = \frac{13}{18}$$

$-13\pi$
$18$

**Score 1:** The student did not express the answer to the nearest hundredth.

**Question 32**

**32** Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$$\begin{aligned} -130 \cdot \frac{\pi}{180} &= \frac{-130\pi}{180} = \frac{-13\pi}{18} = \\ &= -2.27\pi \end{aligned}$$

**Score 1:** The student incorrectly included  $\pi$  in the final answer.

**Question 32**

**32** Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

$$-130^\circ \cdot \frac{\pi}{180^\circ} = \frac{-130\pi}{180} = \frac{-13}{18}\pi \approx .72\pi$$

**Score 0:** The student made an error when dividing  $-13$  by  $18$  and did not express the answer to the nearest hundredth.

Question 32

32 Express  $-130^\circ$  in radian measure, to the *nearest hundredth*.

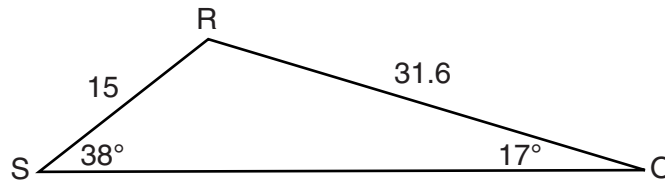
$$-130 \cdot \frac{180}{\pi}$$

$$\frac{-23400}{\pi} = -7442.451337$$

**Score 0:** The student used the wrong conversion and did not round to the nearest hundredth.

Question 33

33 Determine the area, to the nearest integer, of  $\triangle SRO$  shown below.



$$K = \frac{1}{2} ab \sin C$$

$$K = \frac{1}{2} (15)(31.6) \sin 125$$

$$K = 194.1390345$$

$$K = 194$$

$$\begin{array}{r} 38 \\ + 17 \\ \hline \end{array}$$

$$55$$

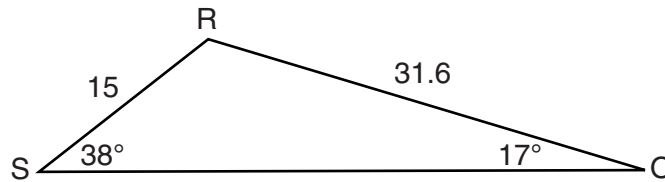
$$\begin{array}{r} 180 \\ - 55 \\ \hline 125 \end{array}$$

**Score 2:** The student has a complete and correct response.



**Question 33**

**33** Determine the area, to the *nearest integer*, of  $\triangle SRO$  shown below.



$$180 - (38 + 17) = 125$$

$$A = 15(31.6) \sin 125$$

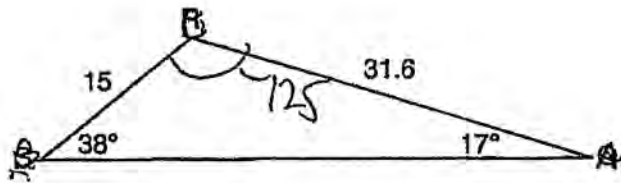
$$388.278069$$

$$A \approx 388$$

**Score 1:** The student did not divide by 2.

Question 33

33 Determine the area, to the nearest integer, of  $\triangle SRO$  shown below.



$$\frac{1}{2} ab \sin C$$

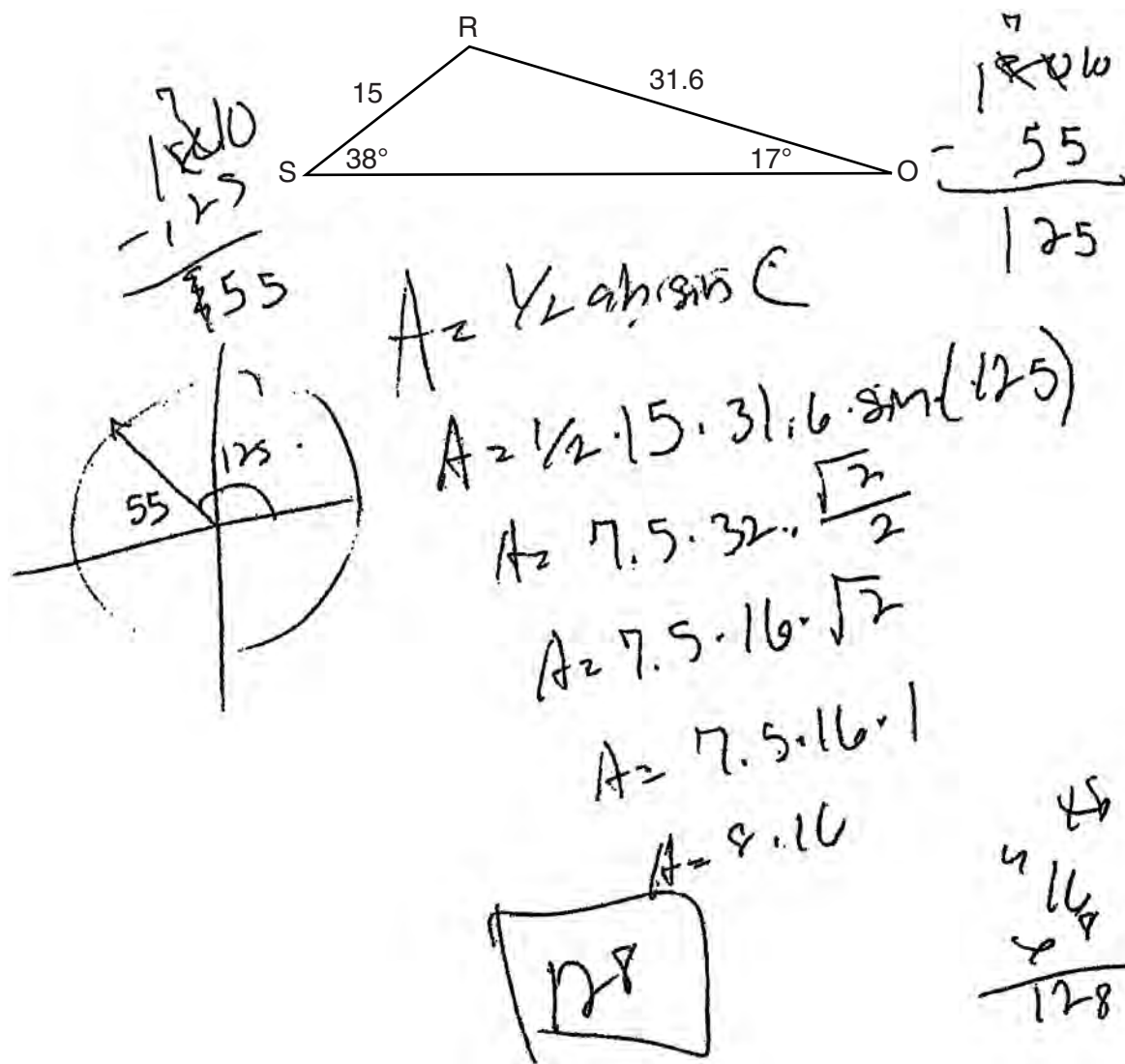
$$\frac{1}{2} (15)(31.6) \sin(125)$$

$$K = 194.13$$

**Score 1:** The student did not round to the nearest integer.

Question 33

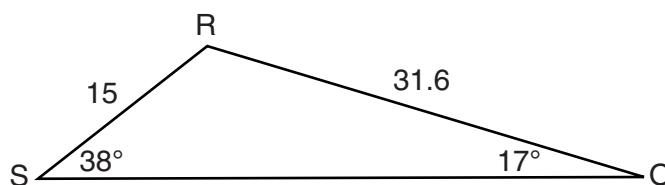
33 Determine the area, to the nearest integer, of  $\triangle SRO$  shown below.



**Score 1:** The student substituted correctly into the area formula.

**Question 33**

**33** Determine the area, to the *nearest integer*, of  $\triangle SRO$  shown below.

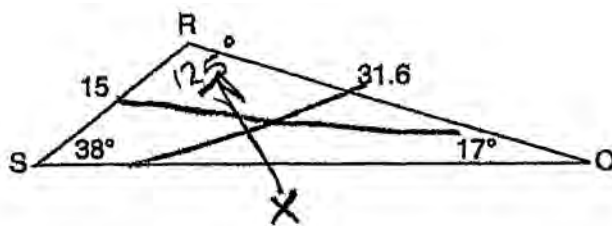


$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 15 \times 31.6 \\ &= 237 \end{aligned}$$

**Score 0:** The student used the incorrect formula.

Question 33

33 Determine the area, to the nearest integer, of  $\triangle SRO$  shown below.



$$\frac{x}{\sin 125^\circ} = \frac{31.6}{\sin 38^\circ}$$

$$25.885 =$$

$$42.0445$$

42

Score 0: The student wrote irrelevant work.

### Question 34

34 Prove that the equation shown below is an identity for all values for which the functions are defined:

$$\csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta$$

$$\frac{1}{\cancel{\sin \theta}} \cdot \cancel{\sin^2 \theta} \cdot \frac{\cos \theta}{\cancel{\sin \theta}} = \cos \theta$$

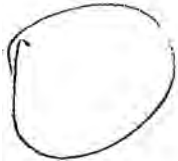
$$\cos \theta = \cos \theta$$

**Score 2:** The student has a complete and correct response.

### Question 34

34 Prove that the equation shown below is an identity for all values for which the functions are defined:

$$\csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta$$



$$\frac{1}{\sin 30} \sin^2 30 \frac{1}{\tan 30} = \cos 30$$

$$(2)(.25)(.132050808) = .8660254038$$

$$(.5)(.132) = .066$$

$$.066 = .066$$

**Score 1:** The student did not prove the equation works for all values of  $\theta$ .

### Question 34

34 Prove that the equation shown below is an identity for all values for which the functions are defined:

$$\csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta$$

$$\frac{1}{\sin \theta} \cdot (\cancel{1 - \cos^2 \theta}) \cdot \frac{\cos \theta}{\sin \theta}$$

**Score 1:** The student wrote all the trigonometric functions in terms of  $\sin \theta$  and  $\cos \theta$ , but showed no further correct work.



### Question 34

34 Prove that the equation shown below is an identity for all values for which the functions are defined:

$$\begin{aligned} \cancel{\csc \theta} \cdot \sin^2 \theta \cdot \cot \theta &= \cos \theta \\ \left(\frac{1}{\sin \theta}\right)(\sin^2 \theta)\left(\frac{\sin \theta}{\cos \theta}\right) &= \cos \theta \\ \sin^2 \theta + \cos^2 \theta &= 1 \end{aligned}$$

**Score 0:** The student did not substitute for  $\cot \theta$  correctly and showed no further correct work.

Question 35

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$\begin{array}{r} (2x^3 + \frac{3}{4}x^2 - \frac{2}{9}) \\ - (\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x) \\ \hline \frac{2}{3}x^3 + \frac{11}{8}x^2 - \frac{7}{9}x - \frac{2}{9} \end{array}$$

**Score 2:** The student has a complete and correct response.

**Question 35**

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$2x^3 + \frac{3}{4}x^2 - \frac{2}{9} - \left( \frac{4}{3}x^3 + \frac{5}{8}x^2 - \frac{7}{9}x \right)$$

$$\frac{2}{3}x^3 + \frac{11}{8}x^2 - \frac{7}{9}x - \frac{2}{9}$$

**Score 2:** The student has a complete and correct response.

Question 35

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$\left(2x^3 + \frac{3}{4}x^2 - \frac{2}{9}\right) - \left(\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}\right)$$

$$\cancel{2x^3} + \cancel{\frac{3}{4}x^2} - \cancel{\frac{2}{9}} - \cancel{\frac{4}{3}x^3} + \cancel{\frac{5}{8}x^2} - \cancel{\frac{7}{9}}$$

$$\frac{2}{3}x^3 + \frac{11}{8}x^2 - 1$$

**Score 1:** The student made a transcription error by not writing  $-\frac{7}{9}x$ .

Question 35

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$\left(\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x\right) - \left(2x^3 + \frac{3}{4}x^2 - \frac{2}{9}\right)$$
$$\left(\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x\right) - 2x^3 - \frac{3}{4}x^2 + \frac{2}{9}$$

$$-\frac{5}{8}x^2 - \frac{6}{8}x^2 = -\frac{11}{8}x^2$$

$$-\frac{2}{3}x^3 - \frac{11}{8}x^2 + \frac{7}{9}x + \frac{2}{9}$$

**Score 1:** The student subtracted in the wrong order.

**Question 35**

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$\begin{array}{r} 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \\ - \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \end{array}$$

$$\frac{2}{3}x^3 + \frac{1}{8}x^2 + \frac{7}{9}x - \frac{2}{9}$$

**Score 1:** The student did not distribute the negative.

Question 35

35 Find the difference when  $\frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x$  is subtracted from  $2x^3 + \frac{3}{4}x^2 - \frac{2}{9}$ .

$$\begin{array}{r} 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \\ - \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \\ \hline \frac{2}{3}x^3 - \frac{5}{8}x^2 + \frac{5}{9}x \end{array}$$

**Score 0:** The student did not distribute the negative and combined unlike terms.

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$(x^2 + 10x + 25) - 8 = 0$$

$$(x + 5)^2 - 33 = 0$$

$$(x + 5)^2 = 33$$

$$x + 5 = \pm \sqrt{33}$$

$$x = -5 \pm \sqrt{33}$$

**Score 4:** The student has a complete and correct response.



Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$x^2 + 10x = 8$$

$$x^2 + 10x + 25 = 8 + 25$$

$$(x + 5)^2 = 33$$

$$x + 5 = \pm\sqrt{33}$$

$$x = -5 \pm\sqrt{33}$$

$$x = -5 \pm 5.744562647$$

**Score 3:** The student did not give the exact values of  $x$  as the final answer.

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$\begin{aligned}x^2 + 10x - 8 &= 0 \\ &\quad +8 \quad +8 \\ \hline x^2 + 10x &= 8 \\ x^2 + 10x + 25 &= 8 + 25 \\ (x + 5)^2 &= 33 \\ \sqrt{(x + 5)^2} &= \sqrt{33} \\ x + 5 &= \sqrt{33} \\ x &= \sqrt{33} - 5\end{aligned}$$

**Score 3:** The student did not write  $\pm\sqrt{33}$ .

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$x^2 + 10x = 8$$

$$x^2 + 10x + 25 = 8 - 25$$

$$(x + 5)^2 = -17$$

$$x + 5 = \pm \sqrt{-17}$$

$$x = -5 \pm i\sqrt{17}$$

**Score 2:** The student made a conceptual error by adding 25 to the left and subtracting 25 from the right.

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$x^2 + 10x - 8 = 0$$

$$a = 1 \quad b = 10 \quad c = -8$$

$$x = \frac{-10 \pm \sqrt{100 - (-32)}}{2}$$

$$x = \frac{-10 \pm \sqrt{132}}{2}$$

$$x = \frac{-10 \pm 2\sqrt{33}}{2}$$

$$x = -5 \pm \sqrt{33}$$

**Score 2:** The student used the quadratic formula to solve for  $x$ .

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$x^2 + 10x - 8 = 0$$

$$x^2 + 10x + 100 = 8 + 100$$

$$(x + 10)^2 = 108$$

$$x + 10 = \sqrt{108}$$

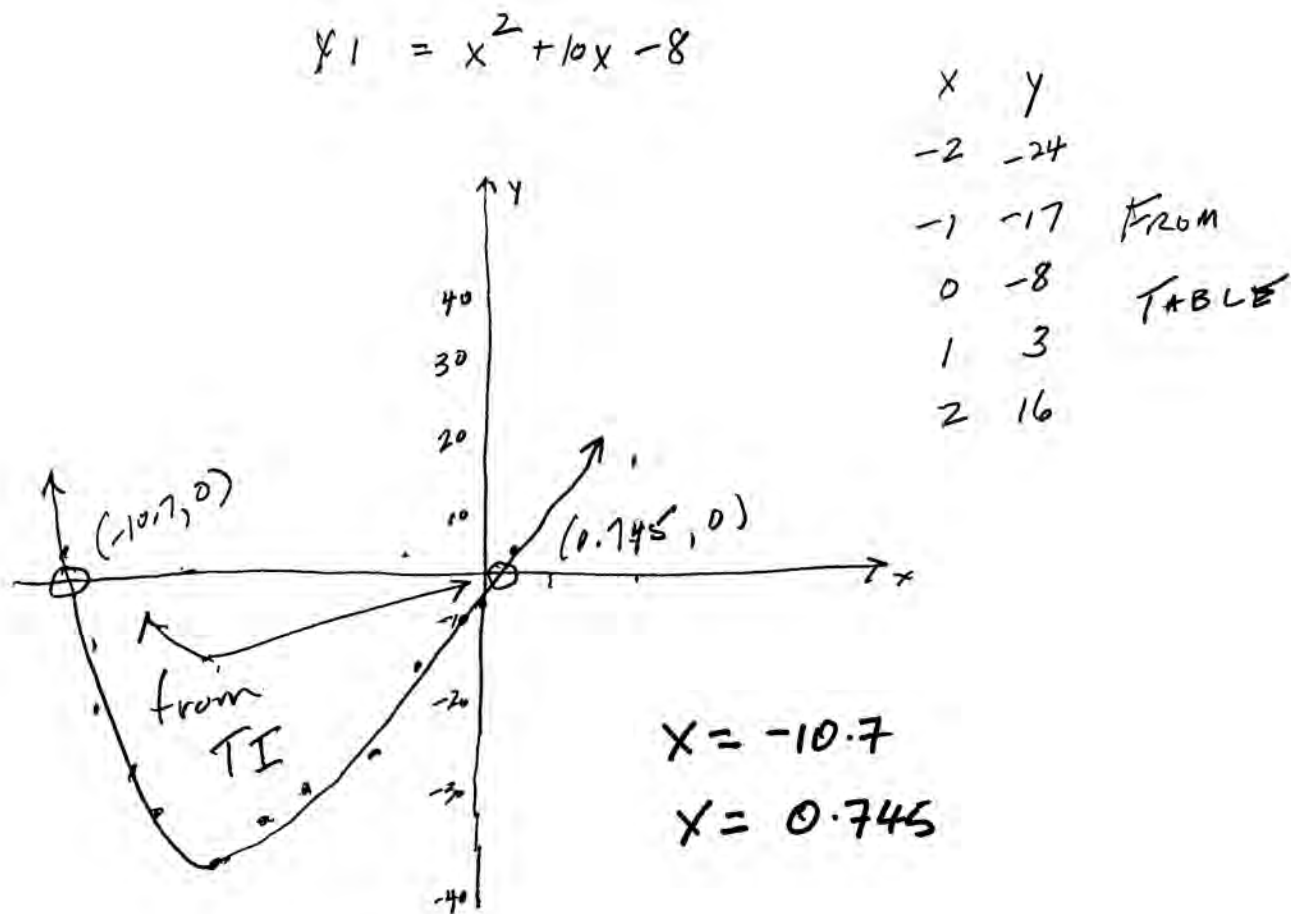
$$x = -10 + \sqrt{108}$$

$$x = -10 + 6\sqrt{3}$$

**Score 1:** The student made a conceptual error in completing the square by adding 100 to both sides and not writing  $\pm\sqrt{108}$ .

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.



**Score 1:** The student used a method other than completing the square and did not give exact values of  $x$ .

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$a = 1 \quad b = 10 \quad c = -8$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{132}}{2}$$

$$x = .7445626465$$

$$x = -10.74456265$$

**Score 1:** The student used the quadratic formula and did not give the exact value of  $x$ .

Question 36

36 Find the exact roots of  $x^2 + 10x - 8 = 0$  by completing the square.

$$\begin{aligned}x^2 + 10x &= 8 \\x^2 + 10x + 25 &= 8 \\(x+5)(x+5) &= 8 \\(x+5)^2 &= 8^2 \\(x+5)^2 &= 64 \\(x+5)^2 - 64 &= 0\end{aligned}$$

**Score 0:** The student made a conceptual error by not adding 25 to both sides of the equation and another conceptual error by squaring the 8. The student also did not solve for  $x$ .



**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

$$y = 2.19 \cdot (3.23)^x$$

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth.  
[Only an algebraic solution can receive full credit.]

$$426.21 = 2.19 \cdot (3.23)^x$$

$$194.62 = 3.23^x$$

$$\log_{3.23} 194.62 = x$$

$$x = 4.5$$

**Score 4:** The student has a complete and correct response.

Question 37

37 The table below gives the relationship between  $x$  and  $y$ .

$x$	1	2	3	4	5
$y$	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth.  
[Only an algebraic solution can receive full credit.]

$$y = a(B^{bx})$$
$$a = 2.19$$
$$B = 3.23$$
$$426.21 = 2.19(3.23^{bx})$$
$$194.6164384 = 3.23^{bx}$$
$$\log 194.6164384 = bx \log 3.23$$
$$x = 4.495617009$$
$$x = 4.5$$

**Score 4:** The student has a complete and correct response.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth.  
[Only an algebraic solution can receive full credit.]

$$y = ab^x$$
$$a = 8.26$$
$$b = 2.32$$

$$\frac{426.21}{8.26} = \frac{8.26}{8.26} \cdot 2.32^x$$

$$51.599 = 2.32^x$$

$$\frac{\log 51.599}{\log 2.32} = \frac{x \cdot \log 2.32}{\log 2.32}$$

$$x = 4.6859$$

$$\boxed{x = 4.7}$$

**Score 3:** The student solved an incorrect exponential regression equation appropriately.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

$$y = 2.19 (3.23)^x$$

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

$$426.21 = 2.19 (3.23)^x$$

**Score 2:** The student did not solve for  $x$ .

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

$$y = ab^x \quad a \approx 2.19 \quad b \approx 3.23 \quad y = 2.19 \times 3.23^x$$

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth.  
[Only an algebraic solution can receive full credit.]

$$426.21 = 2.19(3.23)^x$$

$$\log 426.21 = \log 2.19(3.23)^x$$

$$\log 426.21 = x \log 2.19(3.23)$$

$$\frac{\log 426.21}{\log 2.19(3.23)} = x$$

$$3.1 \approx x$$

**Score 2:** The student wrote a correct exponential regression equation, but made a conceptual error by not applying the product rule.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

nearest hundredth

$$y = ab^x$$

$$y = (2.19)(3.23)^x$$

$$\frac{426.21}{-2.19} = \frac{(2.19)(3.23)^x}{-2.19}$$

$$424.02 = 3.23^x$$

$$\frac{\log 424.02}{\log 3.23} = \frac{x \log 3.23}{\log 3.23}$$

$$5.2 = x$$

**Score 2:** The student wrote a correct exponential regression equation, but made a conceptual error by subtracting 2.19 instead of dividing.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

$$y = a \cdot b^x \quad y = .71(4.05)^x$$

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

$$\frac{426.21}{.71} = \frac{.71(4.05)^x}{.71}$$

$$606.2957746 = (4.05)^x$$

$$\frac{\ln 606.2957746}{\ln 4.05} = \frac{x \ln 4.05}{\ln 4.05}$$

$$x = 4.573779424$$

$$x = 4.57$$

**Score 2:** The student solved an incorrect exponential regression equation appropriately, but did not round 4.57 to the nearest tenth.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the *nearest hundredth*.

$$y = 8.26 (2.32)^x$$

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the *nearest tenth*.  
[Only an algebraic solution can receive full credit.]

**Score 1:** The student wrote an incorrect exponential regression equation.



**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth.  
[Only an algebraic solution can receive full credit.]

$$y = a \cdot b^x$$
$$a = 8.26$$
$$b = 2.32$$
$$\frac{426.21}{8.26} = \frac{8.26 \cdot 2.32^x}{8.26}$$
$$\frac{51.6}{2.32} = \frac{2.32^x}{2.32}$$
$$22.2$$

**Score 1:** The student solved an incorrect exponential equation, but made a conceptual error by dividing by 2.32.

**Question 37**

37 The table below gives the relationship between  $x$  and  $y$ .

<b>x</b>	1	2	3	4	5
<b>y</b>	4.2	33.5	113.1	268.1	523.6

Use exponential regression to find an equation for  $y$  as a function of  $x$ , rounding all values to the nearest hundredth.

Using this equation, predict the value of  $x$  if  $y$  is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

4.2

$$y = ax^2 + bx + c$$

$$426.21 = 37.7x^2 + -98.86x + 70.38$$

$$\bar{x} = 3$$

$$355.83 = 37.7x^2 + -98.86$$

$$Sx = 1.58113883$$

$$\theta x = 1.414213562$$

$$Sy = 213.485842x$$

$$\theta y = 190.9475425$$

**Score 0:** The student wrote completely incorrect work.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\int \cos^2 x - 1 = \cos x$$

$$\int \cos^2 x - \cos x - 1 = 0$$

$$a = 2 \quad b = -1 \quad c = -1$$

$$\cos x = \frac{1 \pm \sqrt{1 - (-8)}}{4} = \frac{1 \pm \sqrt{9}}{4}$$

$$\cos x = \frac{1 \pm 3}{4}$$

$$\cos x = \frac{1+3}{4} = 1$$

$$x = \arccos(1)$$

$$x = 0$$

$$\cos x = \frac{1-3}{4} = -\frac{1}{2}$$

$$\text{Q II, III; } 60$$

$$x = \arccos\left(-\frac{1}{2}\right)$$

$$x = 120, 240$$

**Score 4:** The student has a complete and correct response.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\cos 2x = \cos x$$

$$2\cos^2 x - 1 = \cos x$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x = 120 \\ 240$$

$$0$$

**Score 4:** The student has a complete and correct response.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\begin{aligned}\cos 2x &= \cos x \\ 2\cos^2 x - 1 &= \cos x \\ 2\cos^2 x - \cos x - 1 &= 0 \\ (2\cos x - 1)(\cos x + 1) &= 0 \\ \left. \begin{aligned} 2\cos x - 1 &= 0 \\ \cos x &= \frac{1}{2} \end{aligned} \right\} \begin{aligned} \cos x + 1 &= 0 \\ \cos x &= -1 \end{aligned} \\ x &= 60^\circ \text{ and } 300^\circ & x &= 180^\circ \\ \boxed{x = 60^\circ, 180^\circ, 300^\circ} \end{aligned}$$

**Score 3:** The student made one factoring error.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\cos 2x = \cos x$$

$$2 \cos^2 x - 1 = \cos x$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$\begin{array}{l|l} 2 \cos x + 1 = 0 & \cos x - 1 = 0 \\ \cos x = -\frac{1}{2} & \cos x = 1 \end{array}$$

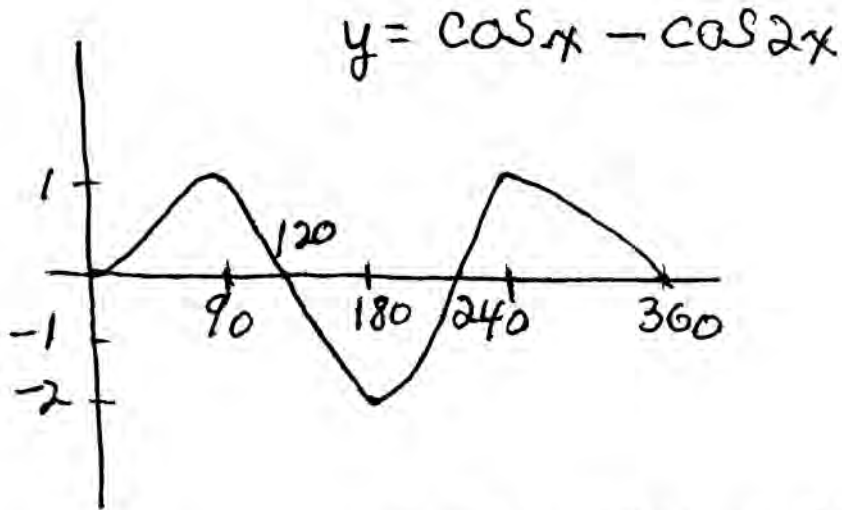
$$x = 120^\circ \text{ ; } 240^\circ$$

$$x = 0^\circ \text{ ; } 360^\circ$$

**Score 3:** The student stated a value that is not included in the domain.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .



$x = 0, 120, 240$

**Score 2:** The student used a method other than algebraic.

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\begin{array}{r} \cos 2x = \cos x \\ - \cos x \quad - \cos x \end{array}$$

$$\cos x = 0$$

$$x = \cos^{-1}(0)$$

$$= \underline{90^\circ, 270^\circ}$$

**Score 2:** The student made a conceptual error by subtracting  $\cos x$ .



Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0$$

$$\sin x = \frac{1}{2}$$

$$x = 30 \quad \text{I, II}$$

$$30, 150, 270$$

**Score 1:** The student made a conceptual error by using the formula for  $\sin 2x$  and did not find all values of  $x$ .

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$1 - 2\sin^2 x = \cos x$$

$$\cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\begin{array}{l|l} \cos x = 0 & \cos x - 1 = 0 \\ x = 90^\circ & \cos x = 1 \\ & x = 0^\circ \end{array}$$

$$x = 0^\circ, 90^\circ$$

**Score 1:** The student made a conceptual error when replacing  $1 - 2\sin^2 x$  with  $\cos^2 x$  and did not find all the values of  $x$ .

Question 38

38 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .

$$\frac{2 \sin x \cos x}{\cos x} = \frac{\cos x}{\cos x}$$

$$2 \sin x = 1$$

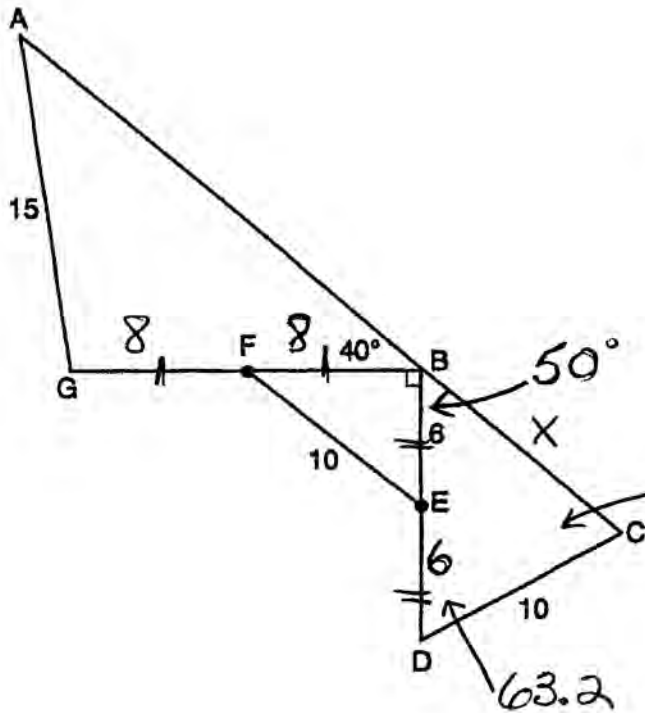
$$\sin x = \frac{1}{2}$$

$$x = 30$$

**Score 0:** The student made conceptual errors by using an incorrect substitution for  $\cos 2x$  and then dividing both sides by  $\cos x$ . The student did not find all values of  $x$ .

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\frac{10}{\sin 50} = \frac{12}{\sin C}$$

$$\sin C = .91925$$

$$C = 66.8$$

$$\frac{X}{\sin 63.2} = \frac{12}{\sin 66.8}$$

$$X = 11.65$$

$$X = 11.7$$

Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

$$\frac{16}{\sin A} = \frac{15}{\sin 40}$$

$$\sin A = .6856$$

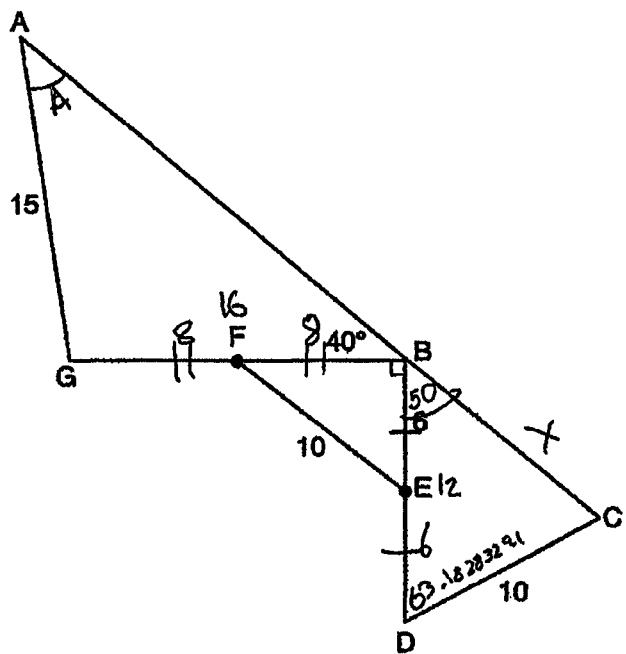
$$A = 43.28$$

$$A = 43.3$$

**Score 6:** The student has a complete and correct response.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$10^2 = 6^2 + x^2$$

$$100 = 36 + x^2$$

$$x = 8$$

$$\frac{15}{\sin 40} = \frac{16}{\sin A}$$

$m\angle A = 43.3^\circ$

$$\frac{10}{\sin 50} = \frac{12}{\sin C}$$

$$\frac{10}{\sin 50} = \frac{x}{\sin 63.18283291}$$

$\overline{BC} = 11.7$

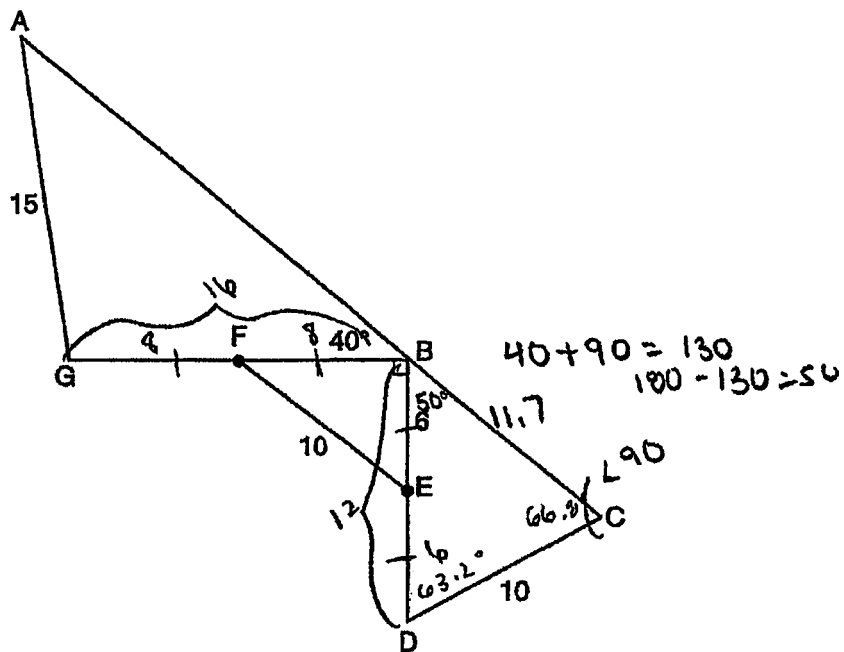
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

Score 6: The student has a complete and correct response.

Question 39

- 39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

$$\frac{15}{\sin 40^\circ} = \frac{16}{\sin x}$$

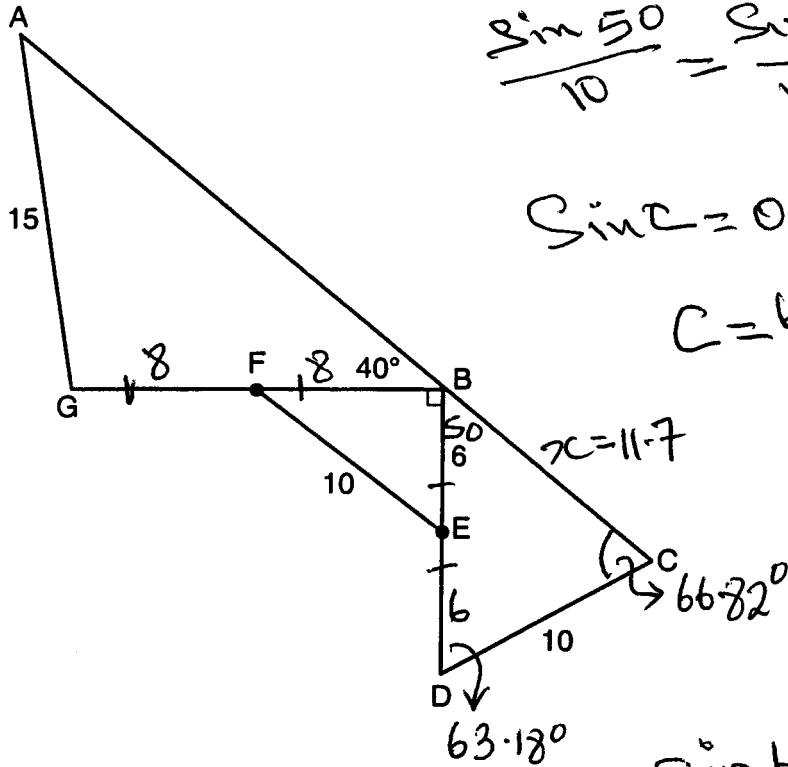
$$x = 43.2$$

$$m\angle A = 43.2^\circ$$

Score 5: The student made one rounding error in  $m\angle A$ .

Question 39

- 39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\frac{\sin 50}{10} = \frac{\sin C}{12}$$

$$\sin C = 0.919253333$$

$$C = 66.817^\circ$$

$$x = 11.7$$

$$C = 66.82^\circ$$

$$63.18^\circ$$

$$\frac{\sin 63.18}{x} = \frac{\sin 50}{10}$$

$$x = 11.65$$

$$11.7$$

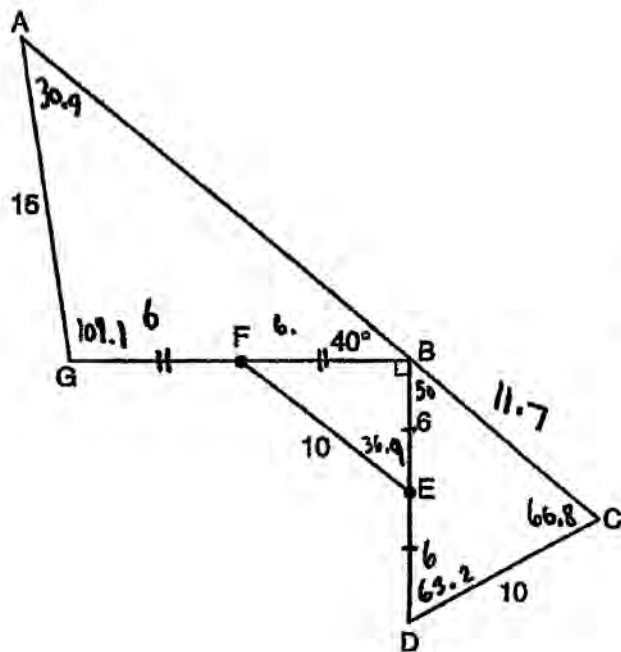
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 5:** The student showed appropriate work to find 11.7 and found  $BG$  to be 16.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\frac{15}{\sin 40} = \frac{12}{\sin x}$$

$$\frac{x}{\sin 101.1} = \frac{15}{\sin 40}$$

Find  $m\angle A$  to the nearest tenth.

$BC = 11.7$

Find  $BC$  to the nearest tenth.

$$\frac{10}{\sin 50} = \frac{12}{\sin x} \quad \frac{x}{\sin 63.2} = \frac{10}{\sin 50}$$

$m\angle A = 30.9$

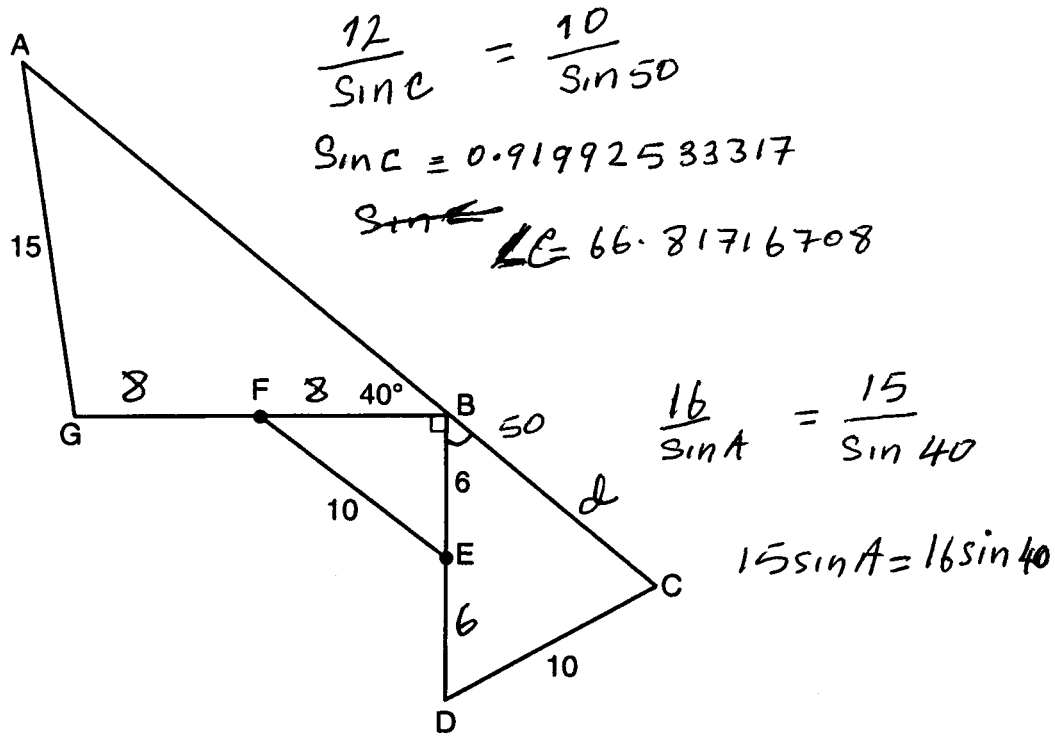
$$\frac{10}{\sin 90} = \frac{6}{\sin x}$$

Score 4: The student made one conceptual error by assuming  $\overline{GF}$  and  $\overline{FB}$  are congruent to  $\overline{BE}$ .



Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

$$\sin A = \frac{16 \sin 40}{15}$$

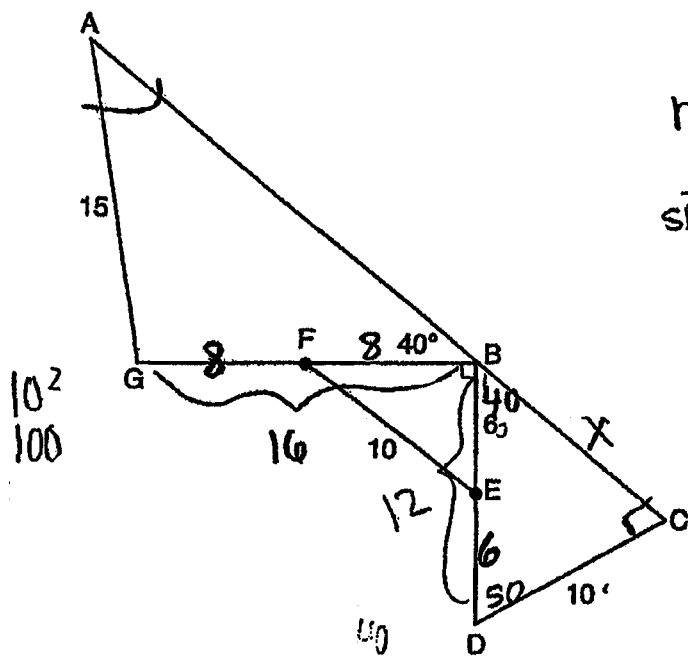
$$\sin A = 0.6026133841$$

$$A \approx 37.1$$

**Score 4:** The student found  $\angle C$  correctly, and  $BG = 16$  but made a computational error when evaluating  $\sin A$ .

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$m\angle A =$$

$$\frac{16}{\sin A} = \frac{15}{\sin 40}$$

$$\frac{15 \sin A}{15} = \frac{16 \sin 40}{15}$$



Q: I, ~~B~~  
 R: 43.28596  
θ: 43.3

$$180 - (40 + 90)$$

$$= 50$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$d = 10^2 + 12^2 - 2(10)(12) \cos 50$$

$$d = 89.730976$$

$$d = 89.7$$

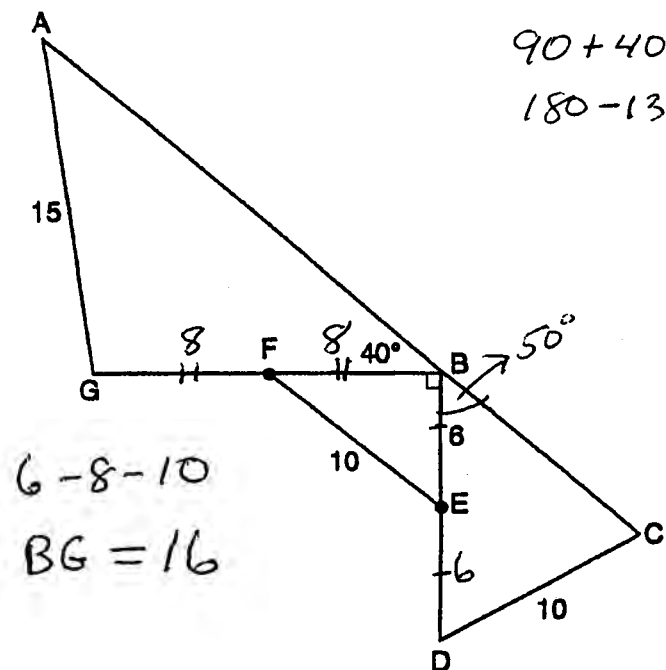
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 3:** The student correctly found  $\angle A$  and the student found  $BC$ .

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



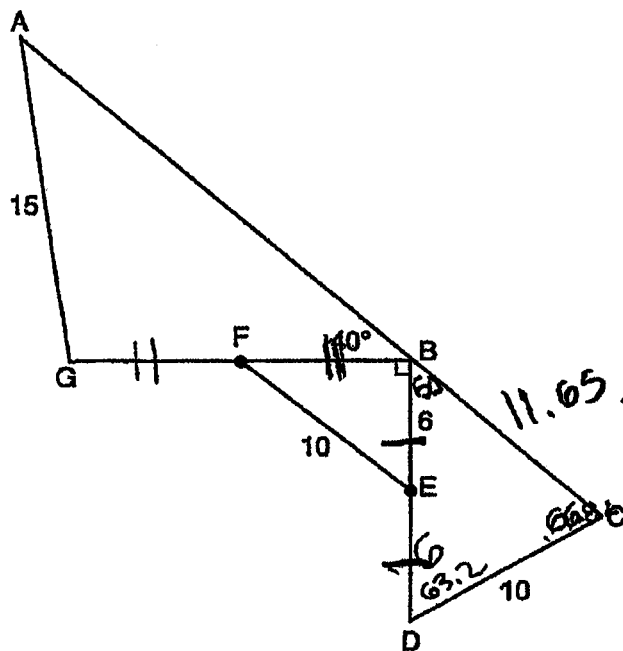
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 3:** The student found  $m\angle C$  and  $BG$ , but showed no further work.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\frac{x}{\sin 63.2} = \frac{10}{\sin 50}$$

$$\frac{\sin 50}{\sin 63.2} = \frac{10 \sin 63.2}{\sin 50}$$

$$11.65 \frac{10}{\sin 50} = \frac{12}{\sin \theta}$$

$$\frac{10 \sin \theta}{10} = \frac{12 \sin 50}{10}$$

$$\theta = 66.82$$

Find  $m\angle A$  to the nearest tenth.

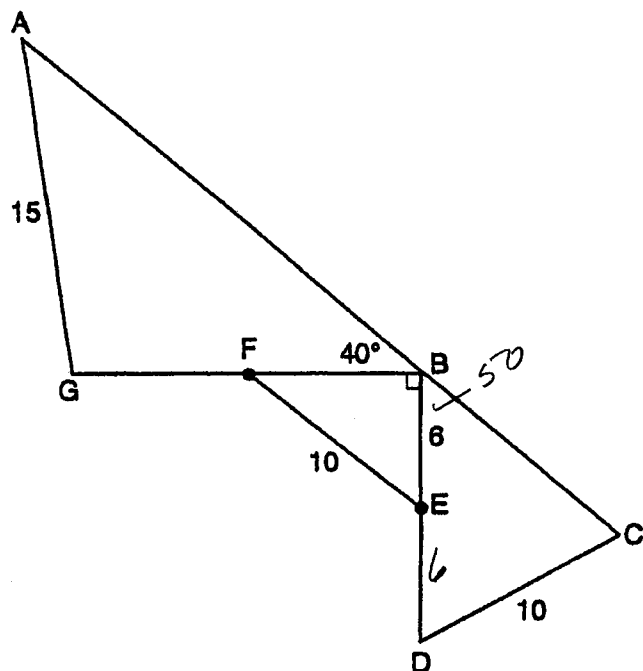
$$BC = 11.65$$

Find  $BC$  to the nearest tenth.

**Score 3:** The student showed appropriate work to find  $BC$ , but did not round properly.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\frac{12}{\sin C} = \frac{10}{\sin 50}$$

$$10 \sin C = 12 \sin 50$$

$$C = \sin^{-1} \left( \frac{12 \sin 50}{10} \right)$$

$$C \approx 66.81716709$$

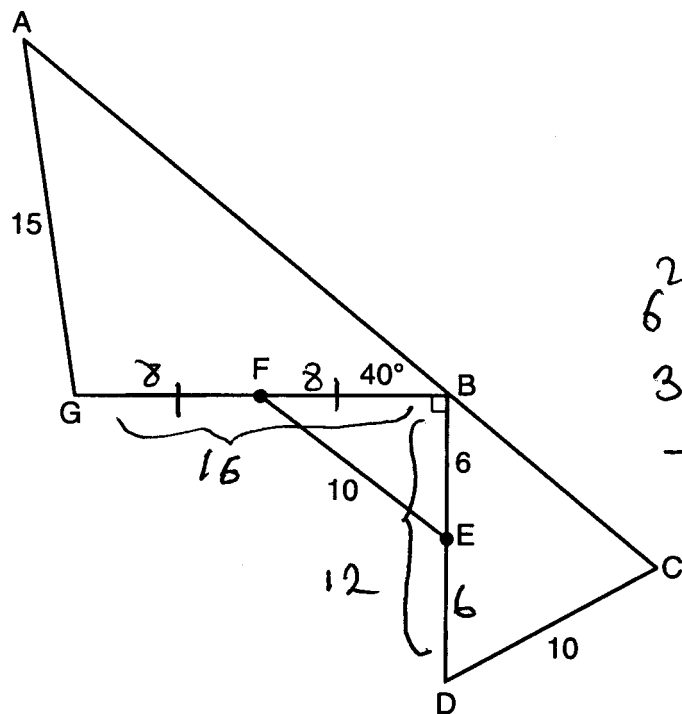
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 2:** The student found  $m\angle C$ , but showed no further work.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$\begin{aligned} 6^2 + b^2 &= 10^2 \\ 36 + b^2 &= 100 \\ -36 \quad -36 & \\ \hline b^2 &= 64 \\ b &= 8 \end{aligned}$$

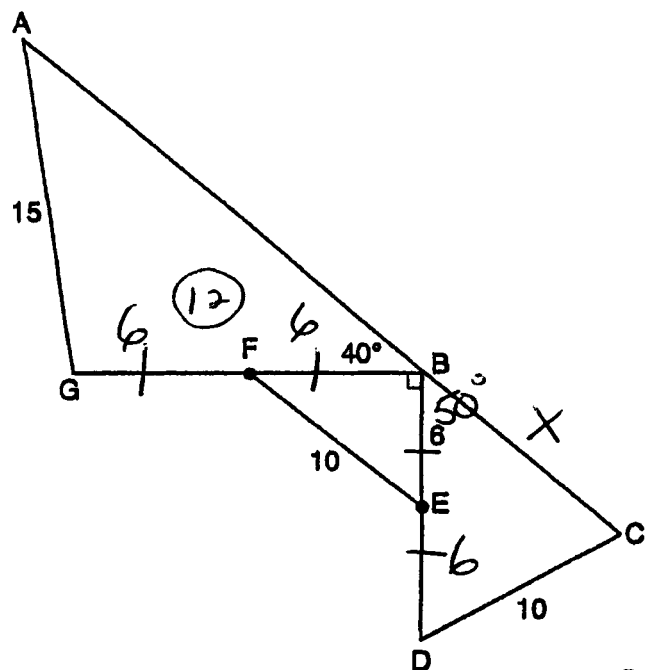
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 1:** The student found  $\overline{BG}$ , but showed no further work.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$X^2 = 12^2 + 10^2 - 2(12)(10)\cos 50$$

$$X^2 = 144 + 100 - 154.269$$

$$X^2 = 89.731$$

$$X = 9.47$$

$$X = 9.5$$

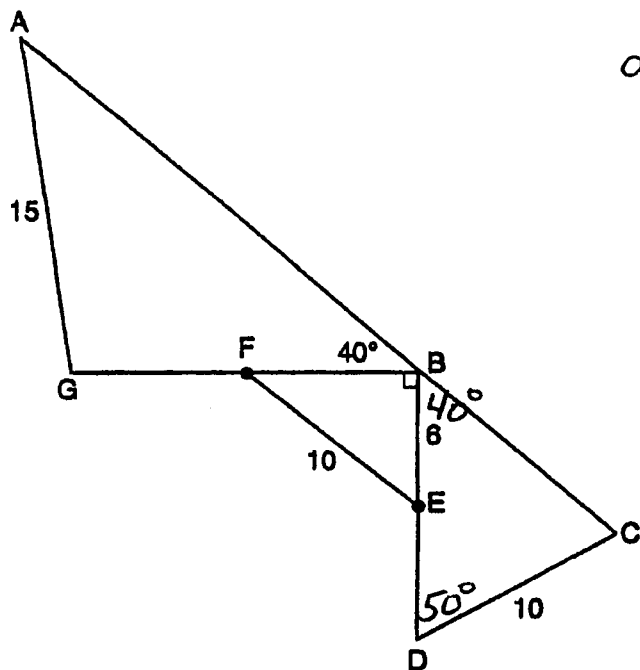
Find  $m\angle A$  to the nearest tenth.

Find  $BC$  to the nearest tenth.

**Score 0:** The student wrote a completely incorrect response.

Question 39

39 Given:  $DC = 10$ ,  $AG = 15$ ,  $BE = 6$ ,  $FE = 10$ ,  
 $m\angle ABG = 40$ ,  $m\angle GBD = 90$ ,  $m\angle C < 90$ ,  
 $\overline{BE} \cong \overline{ED}$ , and  $\overline{GF} \cong \overline{FB}$



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = 10^2 + 12^2 - 2(10)(12) \cos 50$$

$$a = 89.730976$$

$$a = 89.7$$

Find  $m\angle A$  to the nearest tenth. = 89.7

Find  $BC$  to the nearest tenth.

**Score 0:** The student wrote a completely incorrect response.