

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/ TRIGONOMETRY

Friday, June 19, 2015 — 9:15 a.m. – 12:15 p.m.

SAMPLE RESPONSE SET

Table of Contents

Question 28	2
Question 29	5
Question 30	9
Question 31	12
Question 32	17
Question 33	21
Question 34	26
Question 35	30
Question 36	34
Question 37	40
Question 38	50
Question 39	58

Question 28

28 Solve algebraically for x :

$$5^{4x} = 125^{x-1}$$

$$\frac{4x \log 5}{\log 5} = \frac{(x-1) \log 125}{\log 5}$$

$$4x = (x-1) \cdot 3$$

$$4x = 3x - 3$$

$$\begin{array}{r} -3x \\ \hline \end{array}$$

$$\boxed{x = -3}$$

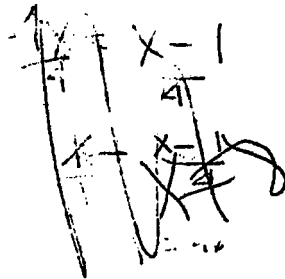
Score 2: The student has a complete and correct response.

Question 28

28 Solve algebraically for x :

$$5^{4x} = 125^{x-1}$$

$$5^{4x} = 125^{x-1}$$



$$\frac{4x \log 5}{\log 5} = (x-1) \frac{\log 125}{\log 5}$$

$$\frac{4x}{x-1} = 4$$

Score 1: The student wrote a correct logarithmic equation, but failed to complete the process to get the solution.

Question 28

28 Solve algebraically for x :

$$5^{4x} = 125^{x-1}$$

$$20x = 125x - 125$$

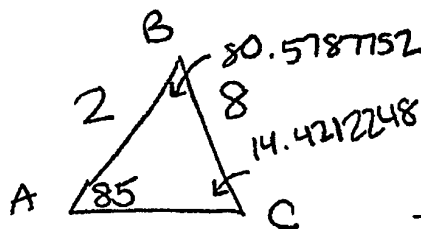
$$20x + 125 = 125x$$

$$x = \frac{125}{105}$$

Score 0: The student gave a completely incorrect answer.

Question 29

29 In triangle ABC , determine the number of distinct triangles that can be formed if $m\angle A = 85$, side $a = 8$, and side $c = 2$. Justify your answer.



$$\frac{8}{\sin 85} = \frac{2}{\sin X}$$

$$\frac{2 \sin 85}{8} = \frac{8 \sin X}{8}$$

$$\sin X = .2490486745$$

$$X = 14.4212248$$

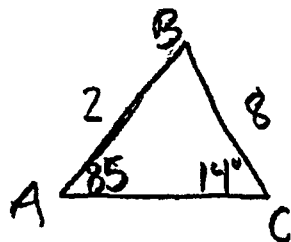
$$180 - 14.4212248 = 165.5787752$$

Only one triangle can be made because $\angle C$'s alternate \angle would result in the triangle's \angle 's adding up to more than 180° , which cannot happen.

Score 2: The student has a complete and correct response.

Question 29

29 In triangle ABC , determine the number of distinct triangles that can be formed if $m\angle A = 85^\circ$, side $a = 8$, and side $c = 2$. Justify your answer.



$$\frac{\sin 85}{8} = \frac{\sin x}{2}$$

$$\frac{8 \sin x}{8} = \frac{2 \sin 85}{8}$$

$$\sin x = .2490486745$$

$$\sin^{-1}(\text{Ans}) = 14^\circ$$

$$\begin{array}{r} 180 \\ -99 \\ \hline \end{array}$$

$$\begin{array}{r} 85 \\ +14 \\ \hline 99 \end{array}$$

$\Delta 1$	85	99	81	✓
$\Delta 2$	85	81	14	✓

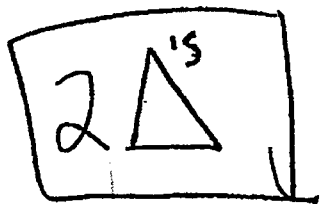
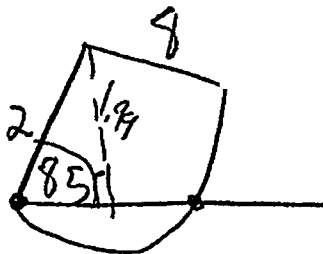
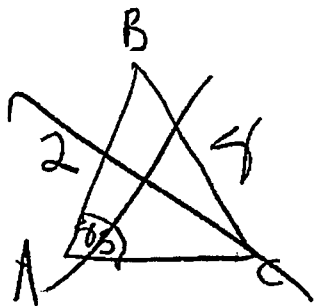
$$\boxed{2\Delta}$$

$$\begin{array}{r} 85 \\ +81 \\ \hline 166 \end{array} \quad \begin{array}{r} 180 \\ -166 \\ \hline 14 \end{array}$$

Score 1: The student found all three angles correctly ($\Delta 2$), but no further correct work was shown.

Question 29

29 In triangle ABC , determine the number of distinct triangles that can be formed if $m\angle A = 85^\circ$, side $a = 8$, and side $c = 2$. Justify your answer.



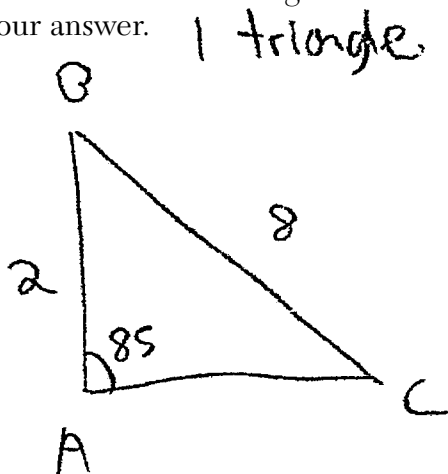
$$\frac{2}{\sin 90} = \frac{x}{\sin 85}$$

$$\frac{2 \sin 85}{\sin 90} = \frac{x \sin 90}{\sin 90}$$

Score 1: The student found the height of the triangle correctly, but determined the number of possible triangles incorrectly.

Question 29

29 In triangle ABC , determine the number of distinct triangles that can be formed if $m\angle A = 85$, side $a = 8$, and side $c = 2$. Justify your answer.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 2^2 + b^2 &= 8^2 \\ 4 + x^2 &= 64 \\ -4 &\quad -4 \\ \hline \sqrt{x^2} &= \sqrt{60} \\ x &= 2.7 \end{aligned}$$

Score 0: The student obtained a correct response by an obviously incorrect procedure.

Question 30

30 The probability that Kay and Joseph Dowling will have a redheaded child is 1 out of 4. If the Dowlings plan to have three children, what is the *exact* probability that only one child will have red hair?

$$3C_1 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^2$$

$$\frac{27}{64}$$

Score 2: The student has a complete and correct response.

Question 30

30 The probability that Kay and Joseph Dowling will have a redheaded child is 1 out of 4. If the Dowlings plan to have three children, what is the *exact* probability that only one child will have red hair?

$$N = 3$$

$$R = 1$$

$$F = 3/4$$

$$S = 1/4$$

$$3C_1 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^1$$

$$\frac{81}{256}$$

Score 1: The student made an error in the exponent when writing the probability of “not red.”

Question 30

30 The probability that Kay and Joseph Dowling will have a redheaded child is 1 out of 4. If the Dowlings plan to have three children, what is the *exact* probability that only one child will have red hair?

$$\frac{1}{4}$$

$${}_4C_1$$

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$${}_4C_1 \cdot \frac{1}{4} \wedge 1 \cdot \frac{3}{4} \wedge 3$$

$$\frac{27}{256}$$

Score 0: The student made one conceptual error by using ${}_4C_1$, and one computational error when solving.

Question 31

31 If $\log_{(x+1)} 64 = 3$, find the value of x .

$${}^3\sqrt{(x+1)^3} = {}^3\sqrt{64}$$

$$x+1 = 4$$

$$\begin{array}{r} -1 \quad -1 \\ \hline \end{array}$$

$$\boxed{x=3}$$

Score 2: The student has a complete and correct response.

Question 31

31 If $\log_{(x+1)} 64 = 3$, find the value of x .

$$\log_{(x+1)} 64 = 3$$

$$\frac{64}{3^{x+1}} = 3$$

$$(x+1)^3 = 64$$

$$2^3 = 8$$

$$3^3 = 27$$

$$5^3 = 125$$

$$4^3 = 64$$

$$(3+1)^3 = 64$$

$$x = 3$$

Score 2: The student has a complete and correct response by using trial and error after writing a correct exponential equation.

Question 31

31 If $\log_{(x+1)} 64 = 3$, find the value of x .

$$\log_{x+1} 64 = 3$$

$$(x+1)^3 = 64$$

$$(x+1)(x+1)(x+1)$$

$$(x^2 + 2x + 1)(x+1)$$

$$x^3 + x^2 + 2x^2 + 2x + 1x + 1$$

$$x^3 + 3x^2 + 3x + 1$$

$$x^3 + 3x^2 + 3x = -1$$

$$x(x^2 + 3x + 3) = -1$$

$$x(x+1)(x+3) = -1$$

$$x = -1$$

$$x+1 = -1$$

$$x = -2$$

$$x+3 = -1$$

$$x = -4$$

$$\{x \mid -1, -2, -4\}$$

Score 1: The student wrote a correct exponential equation.

Question 31

31 If $\log_{(x+1)} 64 = 3$, find the value of x .

$$\frac{\log 64}{\log x+1} = 3$$

$$\frac{\log 64}{3} = \log x+1$$

$$.602\dots = \log x+1$$

$$-.397\dots = \log x$$

$$x = .4$$

Score 1: The student did not put parentheses around $x + 1$ when rewriting the equation, but found an appropriate answer.

Question 31

31 If $\log_{(x+1)} 64 = 3$, find the value of x .

$$3^{x+1} = 64$$

$$3^{(x+1)} = 2^6$$

$$x+1 = 6$$
$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$x = 5$$

Score 0: The student made one conceptual error by confusing the base and exponent and a second conceptual error by equating the exponents of different bases.

Question 32

32 Factor completely: $x^3 - 6x^2 - 25x + 150$

$$x^3 - 6x^2 - 25x + 150$$

$$x^2(x-6) - 25(x-6)$$

$$(x^2 - 25)(x-6)$$

$$(x+5)(x-5)(x-6)$$

Score 2: The student has a complete and correct response.

Question 32

32 Factor completely: $x^3 - 6x^2 - 25x + 150$

$$x^3 - 6x^2 - 25x + 150$$

$$x^2(x-6) - 25(x-6)$$

$$(x^2 - 25)(x-6)$$

Score 1: The student did not completely factor the expression.

Question 32

32 Factor completely: $x^3 - 6x^2 - 25x + 150$

$$x^3 - 6x^2 - 25x + 150$$

$$x^2(x-6) - 25(x-6)$$

$$(x^2 - 25)(x-6)^2$$

$$(x+5)(x-5)(x-6)(x-6)$$

Score 1: The student made an error when factoring out the $(x - 6)$.

Question 32

32 Factor completely: $x^3 - 6x^2 - 25x + 150$

$$x^3 - 6x^2 - 25x + 150$$

$$x(x^2 - 6x - 25) + 150$$

$$x^2(x - 6 - 25) + 150$$

$$x^2(x - 31) + 150$$

Score 0: The student made multiple errors.

Question 33

33 Express $xi^8 - yi^6$ in simplest form.

A handwritten diagram consisting of a rectangular box containing the expression $X+Y$. Above the box, the letter X is written to the left and the letter Y is written to the right. Above the box, there is a circled minus sign $-$.

$$i^8 = 1$$
$$i^6 = -1$$

$$i^8 x - i^6 y$$
$$(x \cdot 1) - (y \cdot -1)$$
$$(x) + (-y)$$

Score 2: The student has a complete and correct response.

Question 33

33 Express $xi^8 - yi^6$ in simplest form.

The student has written the expression $xi^8 - yi^6$. Two arrows point from above to the exponents 8 and 6. Below the expression, there is a scribble that appears to be $x + y$ with a diagonal slash through it.

Score 2: The student has a complete and correct response.

Question 33

33 Express xi^8/yi^6 in simplest form.

$$\begin{aligned} &xi^2 - yi^6 \\ &x(-1) - y(-1) \\ &\underline{-x + y} \end{aligned}$$

Score 1: The student made an error in simplifying i^8 .

Question 33

33 Express $xi^8 - yi^6$ in simplest form.

$$xi^0 - yi^2$$
$$x(1) - y(-1)$$

$$x + (+y)$$

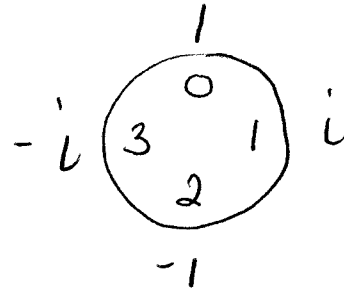
$$\boxed{\begin{array}{r} x + y \\ - \end{array}}$$

Score 1: The student made an error after the correct answer was written.

Question 33

33 Express $xi^8 - yi^6$ in simplest form.

$$xi^8 - yi^6$$



$$(x-y)i^2$$

$$(x-y)(-1)$$

$$x+y$$

Score 0: The student obtained a correct response by an obviously incorrect procedure.

Question 34

34 Given the equation $3x^2 + 2x + k = 0$, state the sum and product of the roots.

~~$$0 = x^2 - 5x + 6$$
$$(x-3)(x-2)$$
$$x=3 \quad x=2$$~~

$$\frac{-2}{3} = \text{sum}$$
$$\frac{k}{3} = \text{product}$$

$$\frac{-b}{a} = \text{sum}$$
$$\frac{c}{a} = \text{product}$$

Score 2: The student has a complete and correct response.

Question 34

34 Given the equation $3x^2 + 2x + k = 0$, state the sum and product of the roots.

$$3x^2 + 2x + k = 0$$

$$a = 3 \quad b = 2 \quad c = k$$

~~$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$~~

~~$$= \frac{-2 \pm \sqrt{(2)^2 - 4(3)(k)}}{2(3)}$$~~

~~$$= \frac{-2 \pm \sqrt{4 - 12k}}{6}$$~~

$$\text{sum} = -\frac{b}{a} = \boxed{-\frac{2}{3}}$$

$$\text{prod} = -\frac{c}{a} = \boxed{-\frac{k}{3}}$$

Score 1: The student correctly stated the sum.

Question 34

34 Given the equation $3x^2 + 2x + k = 0$, state the sum and product of the roots.

$$\text{sum} = \frac{-b}{2a} \quad \frac{-2}{2(3)} \quad \frac{-2}{6} = \text{sum} \left(-\frac{1}{3} \right)$$

Score 0: The student used the wrong formula for the sum and did not find the product.

Question 34

34 Given the equation $3x^2 + 2x + k = 0$, state the sum and product of the roots.

$$\frac{2a}{a} = \frac{2(3)}{3} = \frac{6}{3} = 2$$
$$\frac{-b}{a} = \frac{-2}{3}$$

Score 0: The student did not label the answers and used an incorrect formula.

Question 35

35 Determine which set of data given below has the stronger linear relationship between x and y . Justify your choice.

Set A

x	1	2	3	4	5	6
y	24	30	36	51	70	86

$$r = 0.9763726986$$

Set B

x	1	2	3	4	5	6
y	81	64	49	36	25	16

$$r = -0.9937474961$$

Set B

Score 2: The student has a complete and correct response.

Question 35

35 Determine which set of data given below has the stronger linear relationship between x and y . Justify your choice.

Set A

x	1	2	3	4	5	6
y	24	30	36	51	70	86

Set B

x	1	2	3	4	5	6
y	81	64	49	36	25	16

Set A, Lin Reg.

$$y = ax + b$$

$$a = 12.71928571$$

$$b = 5$$

$$r^2 = 0.9533036866$$

$$r = 0.9763726288$$

Set B, Lin Reg

$$y = ax + b$$

$$a = -13$$

$$b = 90.66666667$$

$$r^2 = 0.9875390863$$

$$r = -0.9937777963$$

Set A has a stronger linear relationship between the x and y variables because its value of r is closer to one.

Score 1: The student made one conceptual error by not realizing $|r|$ should be closest to 1.

Question 35

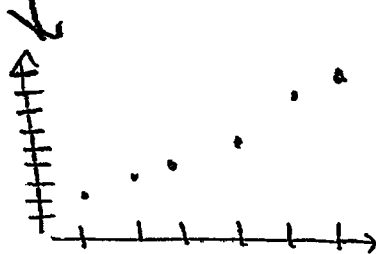
35 Determine which set of data given below has the stronger linear relationship between x and y . Justify your choice.

Set A

x	1	2	3	4	5	6
y	24	30	36	51	70	86

Set B

x	1	2	3	4	5	6
y	81	64	49	36	25	16



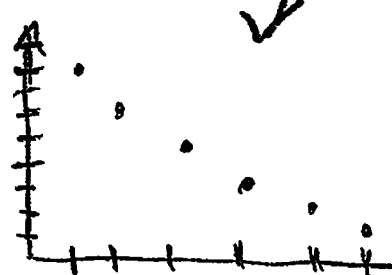
↑

$$y = ax + b$$

$$a = 13.63488415$$

$$b = 1.554802032$$

$$r = 0.9818226836$$



↑

$$y = ax + b$$

$$a = -13$$

$$b = 90.\bar{6}$$

$$r = -0.9937474963$$

Set B

Score 1: The student made one computational error, but made an appropriate choice.

Question 35

35 Determine which set of data given below has the stronger linear relationship between x and y . Justify your choice.

Set A

x	1	2	3	4	5	6
y	24	30	36	51	70	86

$y = 12.7x + 5$

Set B

x	1	2	3	4	5	6
y	81	64	49	36	25	16

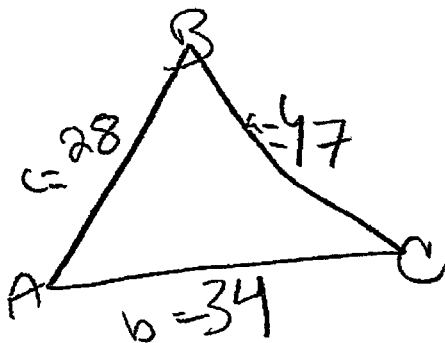
$y = -13x + 90.7$

Set A, because Set B includes negative numbers

Score 0: The student had a response that was incorrect.

Question 36

36 Find the measure of the smallest angle, to the nearest degree, of a triangle whose sides measure 28, 47, and 34.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$28^2 = 47^2 + 34^2 - 2(47)(34) \cos C$$

$$784 = 2209 + 1156 - 3196 \cos C$$

$$784 = 3365 - 3196 \cos C$$

$$\frac{-2581}{-3196} = \cos C$$

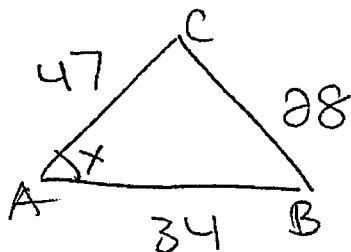
$$0.807571965 = \cos C$$

$$C = 36^\circ$$

Score 4: The student has a complete and correct response.

Question 36

36 Find the measure of the smallest angle, to the *nearest degree*, of a triangle whose sides measure 28, 47, and 34.



SAS

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$28^2 = 47^2 + 34^2 - 2(47)(34) \cos X$$

$$784 = 2209 + 1156 - 3196 \cos X$$

$$784 = 3365 - 3196 \cos X$$

$$\begin{array}{r} -3365 \\ \hline -2581 \end{array} = \begin{array}{r} -3196 \cos X \\ \hline -3196 \end{array}$$

$$\cos X = \frac{-2581}{-3196}$$

$$\cos X = .807571965$$

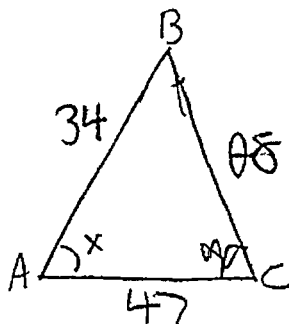
$$\cos X = 36.14062034$$

$$\boxed{\cos X \approx 36^\circ}$$

Score 3: The student should have indicated that $x = 36^\circ$.

Question 36

36 Find the measure of the smallest angle, to the *nearest degree*, of a triangle whose sides measure 28, 47, and 34.



$$K = \frac{1}{2} ab \sin C$$

$$K = \frac{1}{2} (34)(47) \sin C$$

$$K = 799 \sin C$$

$$28^2 = 47^2 + 34^2 - 2(47)(34) \cos A$$

$$3784 = 2209 + 1156 - 3196 \cos A$$

$$784 = 3365 - 3196 \cos A$$

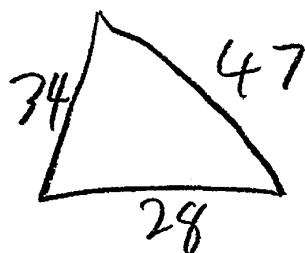
$$\frac{784}{-169} = \frac{169 \cos A}{-169}$$

$$615 = \cos A$$

Score 2: The student made a correct substitution into the Law of Cosines, but no further correct work was shown.

Question 36

36 Find the measure of the smallest angle, to the *nearest degree*, of a triangle whose sides measure 28, 47, and 34.



$$28^2 = 34^2 + 47^2 - 2(34)(47)\cos x$$

$$28^2 = 1885 - 3196 \cos x$$

$$-1885$$

$$\frac{-1101}{-3196} = \frac{-3196 \cos x}{-3196}$$

$$.34449 = \cos x$$

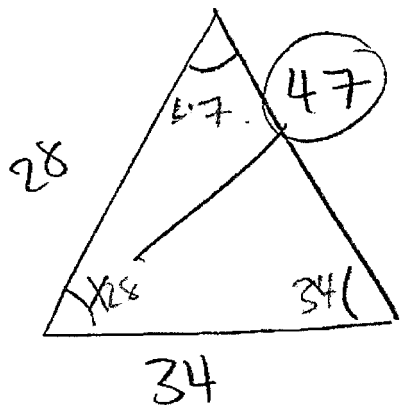
$$\cos^{-1} \quad \cos^{-1}$$

$$\boxed{69.85^\circ}$$

Score 2: The student made one computational error in obtaining 1885, and one rounding error by stating 69.85°.

Question 36

36 Find the measure of the smallest angle, to the nearest degree, of a triangle whose sides measure 28, 47, and 34.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$47^2 = 34^2 + 28^2 - 2(34)(28) \cos X$$

$$2209 = 1156 + 784 - (1904 \cos X)$$

$$2209 = 1940 - (1904 \cos X)$$

$$-1940 \quad -1940$$

$$\frac{269}{1904} = \frac{-1904 \cos X}{1904}$$

$$\frac{269}{1904} = \cos X$$

$$.141 \dots = \cos X$$

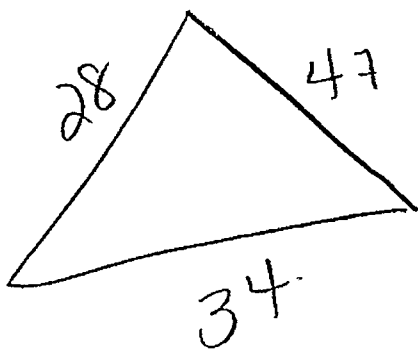
82°

~~82°~~

Score 1: The student made one conceptual error by not finding the measure of the smallest angle. One computational error was made by ignoring the negative sign.

Question 36

36 Find the measure of the smallest angle, to the *nearest degree*, of a triangle whose sides measure 28, 47, and 34.



$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$34 = (47)^2 + (28)^2 - 2(47)(28) \cos X$$
$$34 = 2993 - 2632 \cos X$$
$$-2959 = -2632 \cos X$$
$$1.12 = \cos X$$

Score 0: The student made one conceptual error by not finding the measure of the smallest angle. The student made a second conceptual error by not recognizing that a value of $\cos x$ cannot be greater than 1. The student also did not square the 34.

Question 37

37 Solve algebraically for x:

$$\frac{(x+2)^3}{(x+2)x} + \frac{x(x)}{x+2(x)} - \frac{2}{x+2}$$

$$\frac{3}{-3} + \frac{3}{3+2} = -\frac{2}{3+2} \frac{3x+6}{x(x+2)} + \frac{x^2}{x(x+2)} = -\frac{2}{x+2}$$

$$\frac{1}{5} + \frac{3}{5} = -\frac{2}{5} \frac{x^2+3x+6}{x(x+2)} + \frac{-2}{x+2}$$

$$-\frac{2}{5} = -\frac{2}{5} \left[\frac{x^2+3x+6}{x(x+2)} + \frac{2}{x+2} \right] = 0$$

$$\frac{x^2+3x+6}{x(x+2)} + \frac{2x}{x(x+2)} = 0$$

$$\frac{x^2+5x+6}{x(x+2)} = 0$$

$$\frac{(x+2)(x+3)}{x(x+2)} = 0$$

$$\frac{x+3}{x} = 0$$

$$x+3 = 0$$

$$x = -3$$

Score 4: The student has a complete and correct response.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$$x+2 \left(\frac{3}{x} + \frac{x}{x+2} \right) = -\left(\frac{2}{x+2} \right) x+2$$

$$\cancel{x} \left(\frac{3x+6}{x} \right) + \frac{x}{\cancel{1}} = (-2)\cancel{1}$$

$$3x+6 + x^{\cancel{0}} = -2x$$

$$x^{\cancel{0}} + 3x + 6 = -2x$$

$\quad \quad \quad +2x \quad \quad +2x$

$$x^{\cancel{0}} + 5x + 6$$

$$(x+2)(x+3)$$

$$\cancel{x=-2} \quad x=-3$$

$$(x=-3)$$

Score 4: The student has a complete and correct response.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$$\frac{3}{x} + \frac{x}{x+2} + \frac{2}{x+2} = 0$$

$$\frac{3}{x} + \frac{x+2}{x+2} = 0$$

$$\frac{3}{x} + 1 = 0$$

$$3 + x = 0$$

$$x = -3$$

Score 4: The student has a complete and correct response.

Question 37

37 Solve algebraically for x:

$$\frac{(x+2)3}{(x+2)x} + \frac{x(x)}{x+2} = \frac{2(x)}{x+2(x)}$$

$$3x+6 + x^2 = -2x$$

$$-6 = x^2 - 2x$$

$$-3x - x^2 - 5x - 6 = 0$$

$$(-x+1)(x-6) = 0$$

$$-x+1=0$$

$$=1-1$$

$$\frac{-x}{-1} = \frac{-1}{-1}$$

$$\frac{-x}{-1} = \frac{-1}{-1}$$

$$x=1$$

$$x-6=0$$

$$+6 \quad +6$$

$$x=6$$

Score 3: The student made one factoring error, but stated an appropriate solution.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = \frac{2}{x+2}$$

$$\begin{array}{r} 3x+6 + x^2 = 2x \\ -2x \end{array}$$

$$x^2 + x + 6 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(6)(1)}}{2}$$

$$x = \frac{-1 \pm \sqrt{-23}}{2}$$

$$x = \frac{-1 \pm i\sqrt{23}}{2}$$

Score 3: The student made one computational error by not using the negative sign.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$$\frac{\overset{3x+6}{\cancel{3(x+2)}}}{x(x+2)} + \frac{x^2}{x(x+2)} = \frac{-2x}{x(x+2)}$$

$$\frac{x^2+3x+6}{\cancel{x(x+2)}} = \frac{-2x}{\cancel{x(x+2)}}$$

$$x^2+3x+6 = \frac{-2x}{+2x}$$

$$x^2+5x+6=0$$

$$(x+3)(x+2)=0$$

$$\cancel{x=-3} \quad x=-2$$

$$x = \emptyset$$

Score 3: The student made one error by also rejecting -3 .

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$$\frac{(x+2) 3}{(x+2) x} + \frac{x(x)}{x+2(x)} = -\frac{2(x)}{x+2(x)}$$

$$\frac{3x+6+x^2}{x^2+2x} = -2x$$

$$\frac{x^2+5x+6}{x^2+2x}$$

$$\frac{(x+3)(x+2)}{(x+2)(x)}$$

$$\frac{x+3}{x}$$

Score 2: The student made one conceptual error by not solving an equation.

Question 37

37 Solve algebraically for x:

$$\frac{\cancel{(x)}(x+2) \cdot 3}{1} + \frac{\cancel{(x)}(x+2)}{x+2} = \frac{\cancel{(x)}(x+2)}{x+2}$$

$$\begin{array}{r} 3x + 6 + x^2 = 2x \\ -2x \qquad -2x \\ \hline x^2 + x + 6 = 0 \end{array}$$

Score 1: The student made one computational error by not using the negative sign. The student did not solve for x.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$$\frac{3}{x} \left(\frac{x+2}{x+2} \right) + \frac{x}{x+2} = \frac{-2}{x+2}$$

$$\frac{5}{x+2} + \frac{x}{x+2} = \frac{-2}{x+2}$$

$$\frac{5+x}{x+2} = \frac{-2}{x+2}$$

$$5+x = -2$$

$$x = -3$$

Score 1: The student made one conceptual error by adding 2 to the numerator and denominator of the first fraction. The student made one computational error when solving the equation.

Question 37

37 Solve algebraically for x:

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

$x(x+2)$ $\frac{3}{\cancel{x(x+2)}} + \frac{x}{\cancel{x(x+2)}} = \frac{2}{\cancel{x(x+2)}}$

$$\begin{array}{r} 3 + x = 2 \\ -3 \quad - \\ \hline x = -5 \end{array}$$

Score 0: The student had a response that was completely incorrect.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.

a)

$$\sigma = 5.17$$

Determine the number of students whose scores are within one population standard deviation of the mean.

b) $\bar{x} = 84.46$

$$\begin{array}{ccc} -1SD & \bar{x} & 1SD \\ 79.29 & 84.46 & 89.63 \end{array}$$

17 students

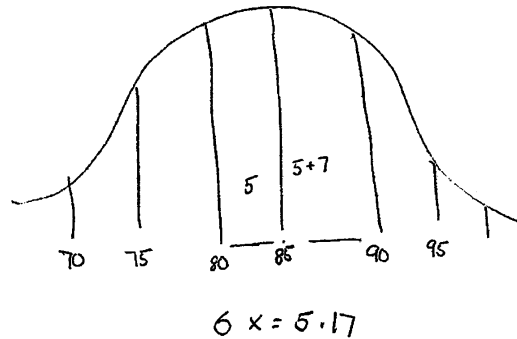
Score 4: The student has a complete and correct response.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.



Determine the number of students whose scores are within one population standard deviation of the mean.

17 students.

Score 3: The student had an appropriate answer based upon an incorrect mean of 85.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.

stat, Clac, 1-Var Stats L1, L2

$$\sigma_x = 5.17$$

$$\bar{x} = 84.6$$

Determine the number of students whose scores are within one population standard deviation of the mean.

21 students
 89.63
 79.29

Score 3: The student made one computational error by including students with a test score of 79.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*. 5.17

$$\begin{array}{l} 84 \div \text{mean} \\ - 5.17 \div \text{psd} \\ \hline \end{array}$$

Determine the number of students whose scores are within one population standard deviation of the mean.

$$78.83 - 84 - 89.17 \quad 21 \text{ students}$$

Score 3: The student had an appropriate answer based upon a prematurely rounded mean.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.

$$\sigma_x = 6.90$$

Determine the number of students whose scores are within one population standard deviation of the mean.

$$\bar{x} = 82.43$$

$$75.53 - 89.33$$

$$22$$

Score 2: The student made one conceptual error by not using the frequencies when finding the standard deviation, but found an appropriate number of students.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.

$$\sigma_x = 5.27$$

$$\bar{x} = 84.46$$

Determine the number of students whose scores are within one population standard deviation of the mean.

$$\begin{array}{r} 84.46 \\ + 5.27 \\ \hline 89.73 \end{array}$$

$$\begin{array}{r} 84.46 \\ - 5.27 \\ \hline 79.19 \end{array}$$

$$\begin{array}{r} 4 \\ 3 \\ 7 \\ + 5 \\ \hline \end{array}$$

21 students

Score 1: The student made one conceptual error by using the sample standard deviation. The student made one computational error by including the test score of 79.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76	1
79	4
83	5
85	7
88	5
94	3

Find the population standard deviation based on these data, to the *nearest hundredth*.

5.17

Determine the number of students whose scores are within one population standard deviation of the mean.

26

Calc { Stat → Edit
Enter list 1 and 2
Stat → 1-Var Stats
L1, L2
Enter

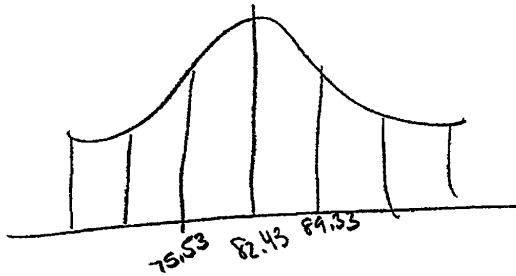
Score 1: The student found 5.17, but no further correct work was shown.

Question 38

38 The table below shows the final examination scores for Mr. Spear's class last year.

Test Score	Frequency
72	1
76 ✓	1
79 ✓	4
83 ✓	5
85 ✓	7
88 ✓	5
94	3

a) Find the population standard deviation based on these data, to the *nearest hundredth*.



a) $\sigma_x = 6.90$

b) Determine the number of students whose scores are within one population standard deviation of the mean.

b) 5 students

Score 0: The student's response was completely incorrect since the student disregarded the frequency.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta = 2 \sec \theta - 3$$

$$\cos \theta \left(5 \cos \theta = \frac{2}{\cos \theta} - 3 \right)$$

$$5 \cos^2 \theta = 2 - 3 \cos \theta$$

$$5 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$(5 \cos \theta - 2)(\cos \theta + 1)$$

$$5 \cos \theta = 2$$

$$\cos \theta = \frac{2}{5}$$

$$\cos \theta = -1$$

$$\{66.4, 180, 293.6\}$$

Score 6: The student has a complete and correct response.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \left(\frac{1}{\sec \theta} \right) = 2 \sec \theta - 3$$

$$5 = 2 \sec^2 \theta - 3 \sec \theta$$

$$0 = 2 \sec^2 \theta - 3 \sec \theta - 5$$

$$0 = (2 \sec \theta - 5)(\sec \theta + 1)$$

$$2 \sec \theta - 5 = 0$$

$$\sec \theta + 1 = 0$$

$$2 \sec \theta = 5$$

$$\sec \theta = -1$$

$$\sec \theta = \frac{5}{2}$$

$$\cos \theta = -1$$

$$\cos \theta = \frac{2}{5}$$

$$\theta = 180$$

$$\theta = 66.4218215218$$

$$360 - \theta = 293.578178478$$

$$\boxed{66.4, 180, 293.6}$$

Score 6: The student has a complete and correct response.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta = 2 \left(\frac{1}{\cos \theta} \right) - 3$$

$$5 \cos^2 \theta = 2 - 3 \cos \theta$$

$$5 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$(5 \cos \theta - 2)(\cos \theta + 1) = 0$$

$$5 \cos \theta = 2$$

$$\cos \theta = \frac{2}{5}$$

$$\theta = 66.4$$

$$\cos \theta = -1$$

$$\theta = 180.0$$

Score 5: The student only found two correct values of θ .

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta = 2 \sec \theta - 3$$

$$\cos \theta \left(5 \cos \theta = \frac{2}{\cos \theta} - 3 \right)$$

$$5 \cos^2 \theta = 2 - 3 \cos \theta$$

$$5 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$(5 \cos \theta + 2)(\cos \theta - 1) = 0$$

$$\cos \theta = -\frac{2}{5} \quad \cos \theta = 1$$

$$113.6, 246.4, 0.0$$

$$\{114, 246, 0\}$$

Score 4: The student made one error in factoring the trinomial, and then made one rounding error when stating the final answer.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta - 2 \left(\frac{1}{\cos \theta} \right) + 3 = 0$$

$$5 \cos^2 \theta - 2 + 3 \cos \theta = 0$$

$$\cos \theta = \frac{-3 \pm \sqrt{9 - (4 \cdot 5 \cdot -2)}}{10}$$

$$\cos \theta = \frac{-3 \pm \sqrt{149}}{10}$$

$$\cos \theta = \frac{-3 \pm 7}{10}$$

$$\cos \theta = \frac{4}{10} \quad \text{or} \quad \cos \theta = -1$$

Score 4: The student found correct values for $\cos \theta$, but no further correct work was shown.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta = \frac{2}{\cos \theta} - 3$$

$$5 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

$$(5 \cos \theta + 2)(\cos \theta - 1) = 0$$

$$\cos \theta = -\frac{2}{5} \quad \cos \theta = 1$$

$$\cos \theta = -0.6 \quad \theta = 0$$

$$\theta = \cos^{-1}(-0.6)$$

$$\theta = 126.869897646$$

$$\theta = 127$$

$$\theta = 233$$

$$\boxed{0, 127, 233}$$

Score 3: The student made an error in factoring the trinomial, one computational error in writing the decimal, and rounded incorrectly.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$\begin{aligned} \cos \theta [5 \cos \theta = 2 \sec \theta - 3] \\ 5 \cos^2 \theta = 2 \cos \theta \frac{1}{\cos \theta} - 3 \cos \theta \\ 5 \cos^2 \theta = 2 - 3 \cos \theta \\ 5 \cos^2 \theta + 3 \cos \theta - 2 = 0 \\ \cancel{\cos \theta} (5 \cos \theta + 3) = \frac{2}{\cos \theta} \\ 5 \cos \theta + 3 = 0 \quad \frac{2}{\cos \theta} = 0 \\ \cos \theta = -\frac{3}{5} \end{aligned}$$

Score 2: The student wrote a correct quadratic equation in standard form, but no further correct work was shown.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the *nearest tenth of a degree*.

$$5 \cos \theta = 2 \left(\frac{1}{\cos \theta} \right) - 3$$

$$5 \cos^2 \theta = 2 - 3 \cos \theta$$

Score 1: The student wrote an equation in terms of cosine, but did not solve for $\cos \theta$.

Question 39

39 In the interval $0^\circ \leq \theta < 360^\circ$, solve the equation $5 \cos \theta = 2 \sec \theta - 3$ algebraically for all values of θ , to the nearest tenth of a degree.

$$5 \cos \theta = 0$$

$$\cos \theta = 0$$

$$\theta = 90.0$$

$$2 \sec \theta - 3 = 0$$

$$\sec \theta = \frac{3}{2}$$

$$\theta = 48.1897$$

$$\theta = 48.2$$

Score 0: The student had a response that was completely incorrect.