

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, January 24, 2018 — 1:15 to 4:15 p.m., only

MODEL RESPONSE SET

Table of Contents

Question 25	2
Question 26	8
Question 27	14
Question 28	17
Question 29	22
Question 30	26
Question 31	31
Question 32	36
Question 33	40
Question 34	47
Question 35	52
Question 36	59
Question 37	68

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

$$\begin{aligned}i^2 &= -1 \\ &\text{not } 1, \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4(-1) \\ &= 6 + 10i + 4 \\ &= 10 + 10i\end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

ELIZABETH SUBSTITUTED 1 RATHER THAN -1 FOR i^2

$$6 + 10i - 4(-1)$$

$$6 + 10i + 4$$

Score 2: The student gave a complete and correct response.

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4\textcircled{1} \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

$$10 + 10i$$

Score 2: The student gave a complete and correct response.

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

$$4i^2 = 4$$

$$6 + 10i + 4 = 10 + 10i$$

$$\boxed{1 + i}$$

Score 1: The student made an error by dividing the complex number by 10.

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

The error in this process is that i^2 does not equal 1, it equals 0. The correct product is $6 + 10i$.

$$\begin{aligned}&= 6 + 10i - 4(0) \\ &= 6 + 10i + 0 \\ &= 6 + 10i\end{aligned}$$

Score 1: The student used 0 for i^2 .

Question 25

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$\begin{aligned}(2 + 4i)(3 - i) & \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i\end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

$$\begin{aligned}(4i+2) (-i+3) \\ = 4i^2 + 12i - 2i + 6 \\ = 4i^2 + 10i + 6\end{aligned}$$

Score 0: The student did not identify the error or value of i^2 , and did not determine the correct product.

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$$D = (1.223)(2.652)^A$$

Score 2: The student gave a complete and correct response.

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

→ lists + spreadsheets
↓
data + statistics

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$$y = 1.223(2.652)^x$$

Score 2: The student gave a complete and correct response. The variables x and y are accepted as the regression equation was not restricted in terms of A and D .

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$$y = ab^x$$

$$a = 1.223034549$$

$$b = 2.652024589$$

$$y = 1.22 * 2.65^x$$

Score 1: The student did not round correctly.

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$$1.223 (2.657)^A$$

Score 1: The student wrote an expression, not an equation.

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$y =$

$$D = 2.991A^2 - 1.866A + 2.188$$

Score 1: The student used an incorrect regression model, but rounded correctly.

Question 26

26 A runner is using a nine-week training app to prepare for a “fun run”. The table below represents the amount of the program completed, A , and the distance covered in a session, D , in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

$$A = A_0 e^{k(t-t_0)} + B_0$$
$$A = \frac{4}{9} e^{(9)} + 2$$

Score 0: The student gave a completely incorrect response.

Question 27

27 A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk.

$$\boxed{\frac{1}{8} + \frac{1}{6} = \frac{1}{x}}$$

Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

~~(8)(10)(10)~~ ~~(6)(10)(10)~~ ~~(1)(10)(10)~~

$$\frac{1}{8} + \frac{1}{6} = \frac{1}{x}$$
$$6x + 8x = 48$$
$$14x = 48$$
$$\frac{14x}{14} = \frac{48}{14}$$
$$x = 3.4 \text{ hours}$$

Score 2: The student gave a complete and correct response.

Question 27

27 A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk.

$$\frac{1}{8} + \frac{1}{6} = \frac{1}{t_b}$$

Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

$$\frac{1}{8} + \frac{1}{6} = \frac{7}{24}$$

$$\frac{7}{24} = 0.3 \text{ hours}$$

Score 1: The student only stated a correct equation.

Question 27

27 A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk.

$$\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$$

Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

2.9 hours

Score 0: The student did not write a correct equation to solve for t_b and showed no further correct work.

Question 28

28 Completely factor the following expression:

$$x^2 + 3xy + 3x^3 + y$$

$$x^2 + 3xy + 3x^3 + y.$$

$$3x^3 + x^2 + 3xy + y \quad \checkmark$$

$$(3x^3 + x^2) + (3xy + y)$$

$$x^2(3x + 1) + y(3x + 1)$$

$$(x^2 + y)(3x + 1)$$

Score 2: The student gave a complete and correct response.

Question 28

28 Completely factor the following expression:

$$x^2 + 3xy + 3x^3 + y$$

$$\begin{aligned} & 3x^3 + x^2 + 3xy + y \\ & x^2(3x+1) + y(3x+1) \\ & \boxed{x^2 + y(3x+1)} \end{aligned}$$

Score 1: The student did not put parentheses around $x^2 + y$.

Question 28

28 Completely factor the following expression:

$$\begin{array}{l} x^2 + 3xy \quad \{ \quad 3x^3 + y \\ x^2 + 3x^3 \quad \{ \quad 3xy + y \end{array}$$

$x^2(1+3x) + y(3x+1)$

Score 1: The student did not completely factor the expression.

Question 28

28 Completely factor the following expression:

$$x^2 + 3xy + 3x^3 + y$$

$$\begin{aligned} & 3x^3 + x^2 + 3xy + y \\ & x^2(3x+1) + y(3x+1) \\ & (x^2 + y)(3x+1)(3x+1) \end{aligned}$$

Score 1: The student did not recognize $3x + 1$ as the GCF.

Question 28

28 Completely factor the following expression:

$$x^2 + 3xy + 3x^3 + y$$

$$\begin{array}{l} \boxed{x^2 + 3xy + 3x^3 + y} \\ \boxed{x^2 + 3xy^2 + 3x + y} \\ x(x+3)y(x+3) \\ \boxed{(x+y)(x+3)} \end{array} \quad \begin{array}{c} \cancel{3} \\ \cancel{3} \quad \cancel{1} \\ \cancel{1} \end{array}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 29

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month.

Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

$$\frac{20e^{.05t}}{30} = \frac{30e^{.03t}}{30}$$

$$\frac{2}{3} \frac{e^{.05t}}{e^{.05t}} = \frac{e^{.03t}}{e^{.05t}}$$

$$\frac{2}{3} = e^{-.02t}$$

$$\frac{\ln\left(\frac{2}{3}\right)}{-0.02} = \frac{-0.02t}{-0.02}$$

$$20.27325541 = t$$

20.3 months

Score 2: The student gave a complete and correct response.

Question 29

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month.

Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

$$20 e^{.05t} = 30 e^{.03t}$$

intersection: (20.273..., 55.113...)

20.3 months

Score 2: The student gave a complete and correct response.

Question 29

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month.

Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

$$R(m) = 20e^{.5m} \quad F(m) = 30e^{.3m}$$

$$\frac{20e^{.5m}}{30} = \frac{30e^{.3m}}{30}$$

$$\ln \frac{20}{30} = e^{-.2m} \quad | \ln$$

$$-.405465 = \frac{-.2m}{-2}$$

$$m = 2.0273$$

$$m = 2.0$$

Score 1: The student used incorrect rates, but rounded correctly.

Question 29

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month.

Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

$$P = 20$$
$$r = 0.05$$

$$P = 30$$
$$r = 0.03$$

$$P(1+r)^t = P(1+r)^t$$
$$20(1+0.05)^t = 30(1+0.03)^t$$
$$20(1.05)^t = 30(1.03)^t$$
$$21^t = 30.9^t$$
$$t \log 21 = \frac{t \log 30.9}{\log 21}$$
$$t = 1.126$$

1 month

Score 0: The student made several errors.

Question 30

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

min

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

min of $h(x)$ is -1

q has smaller min

Score 2: The student gave a complete and correct response.

Question 30

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

$$h(x) =$$

function $q(x)$ has a smaller minimum because the min of $h(x)$ is 0.4 while the min of $q(x)$ is -8

Score 1: The student incorrectly identified the minimum of h .

Question 30

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2,2]$. Justify your answer.

q has a smaller minimum value because
it goes down to -8.

Score 1: The student gave an incomplete justification.

Question 30

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

~~$q(x)$~~

$$h(x) = 2\sin 3(-2) + 1 = \overline{0.1743} \quad 0.79$$
$$h(x) = 2\sin 3(2) + 1 = 1.209$$

$q(x)$ because -8 is less than
 ~~0.79~~ 0.79

Score 1: The student made an error by using the wrong mode.

Question 30

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

x	$h(x)$
-2	1.96
-1	.72
0	1
1	1.29
2	.45

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

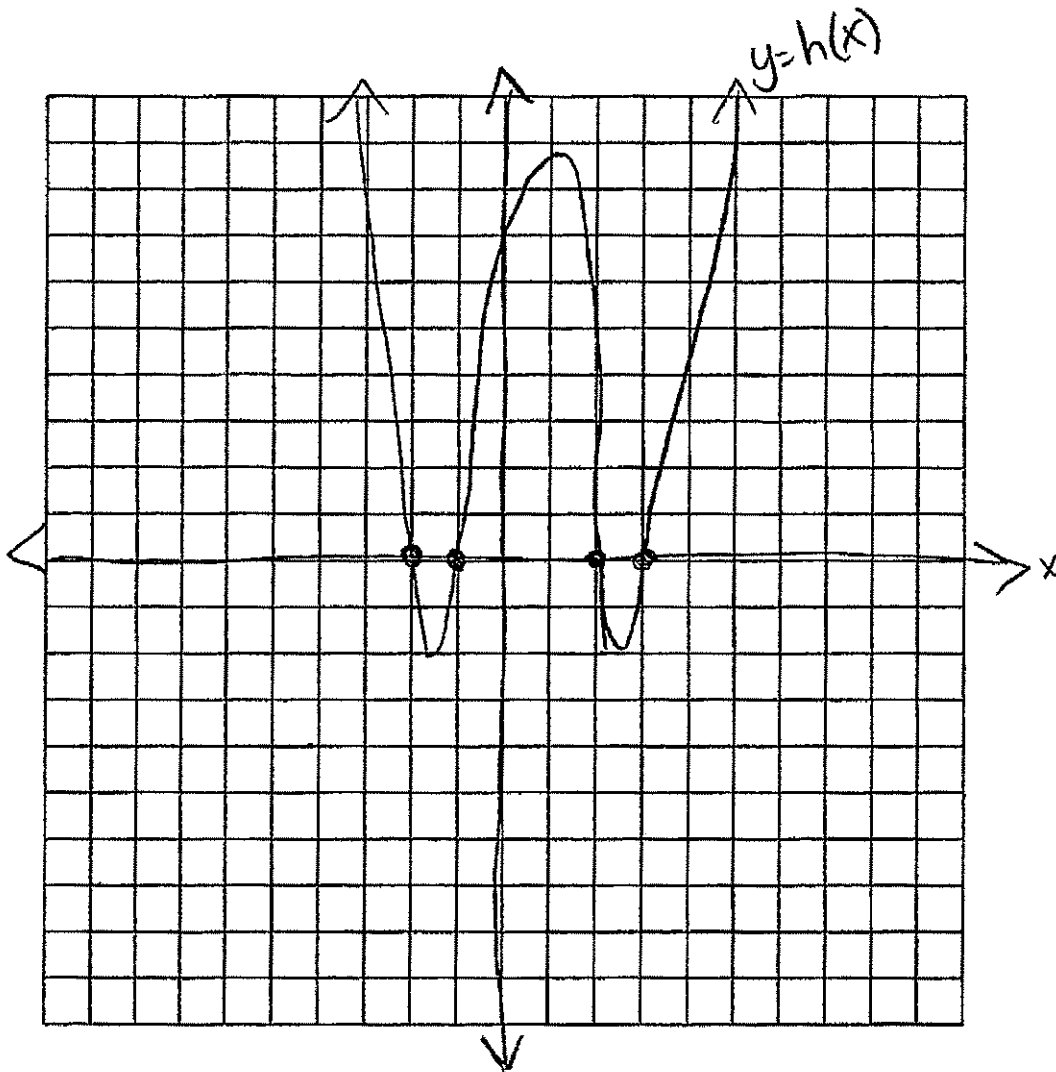
$h(x) = 2\sin 3x + 1$ has the smaller minimum because the amplitude is 2 & it is shifted 1 up so it can never be greater than -2 & $q(x)$ goes down to -8.

Score 0: The student did not show enough correct work to receive any credit.

Question 31

31 The zeros of a quartic polynomial function h are $-1, \pm 2,$ and 3 .

Sketch a graph of $y = h(x)$ on the grid below.

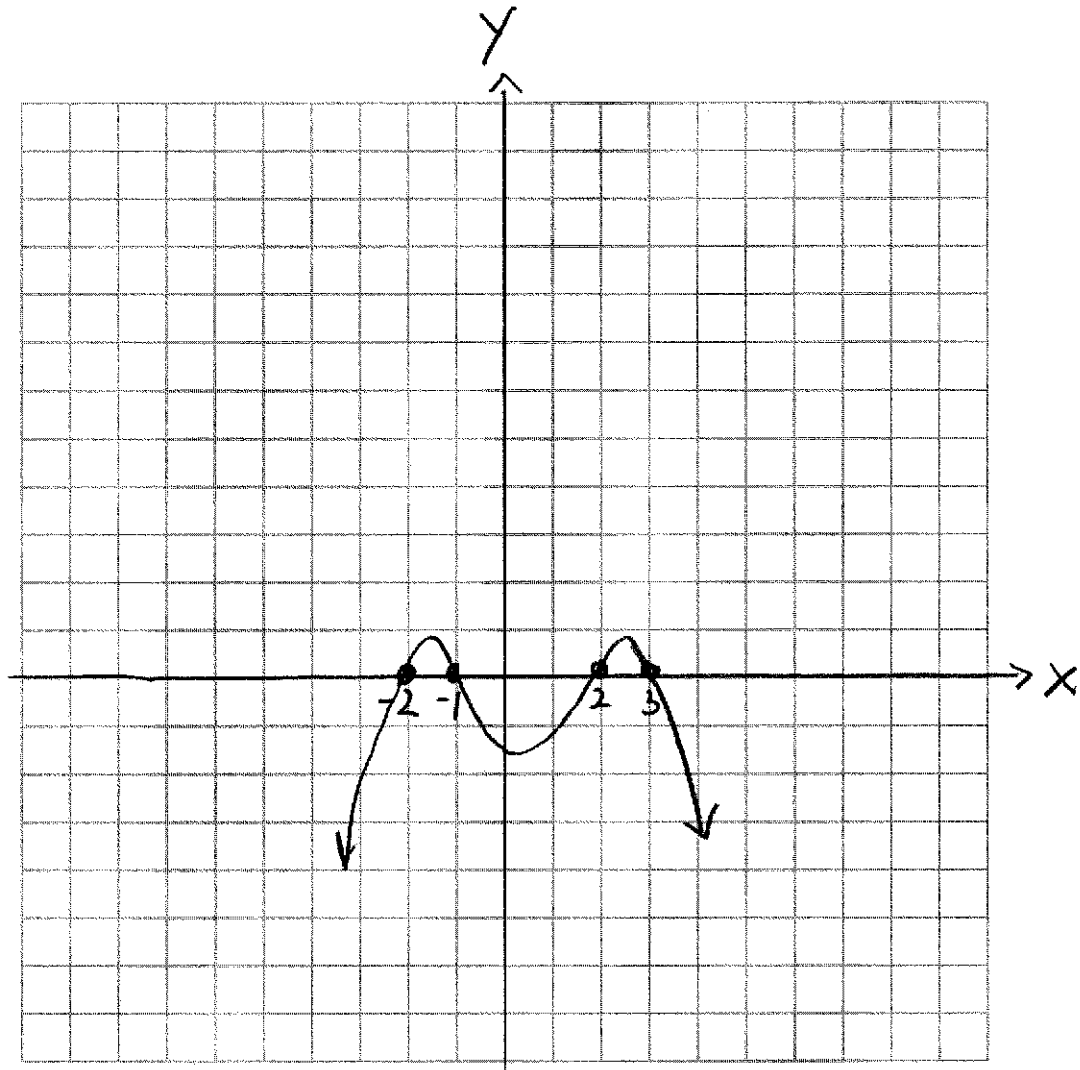


Score 2: The student gave a complete and correct response.

Question 31

31 The zeros of a quartic polynomial function h are $-1, \pm 2,$ and 3 .

Sketch a graph of $y = h(x)$ on the grid below.

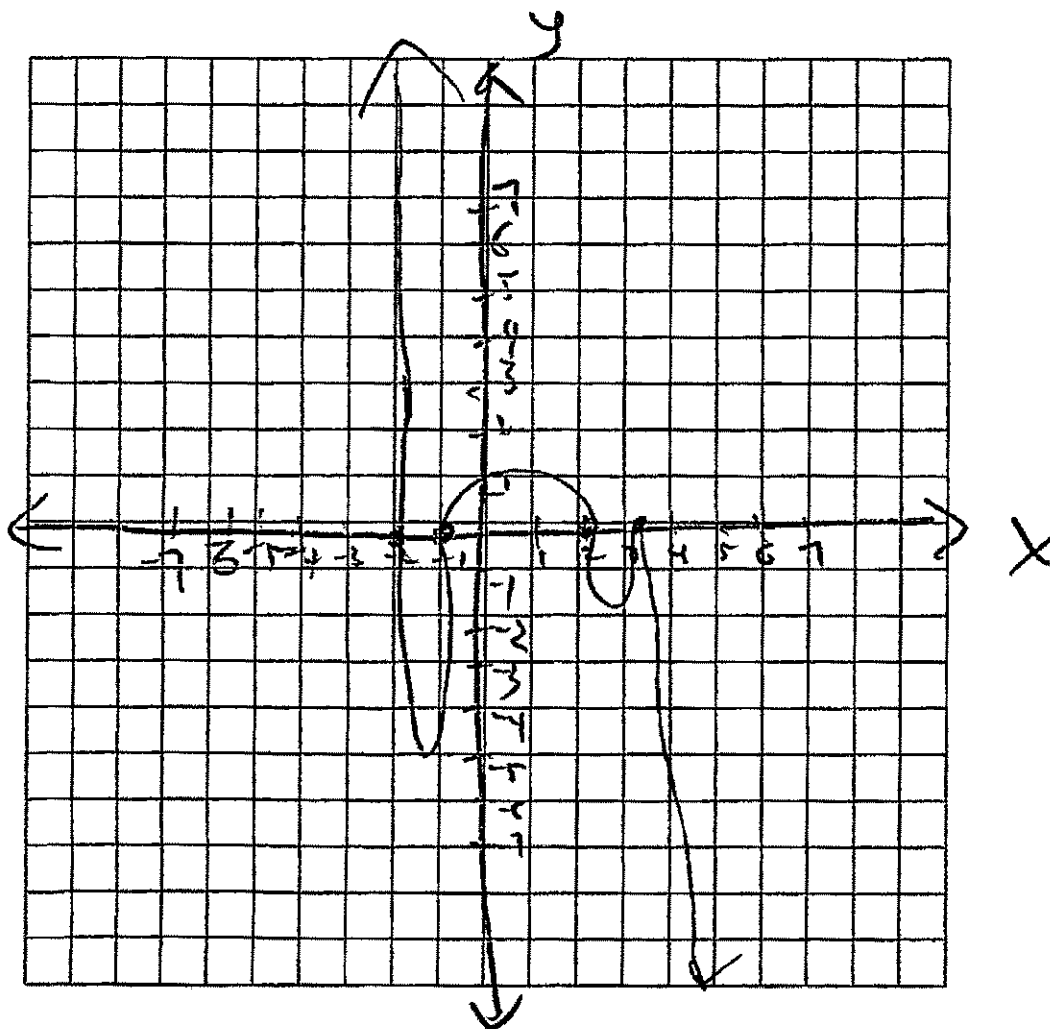


Score 2: The student gave a complete and correct response.

Question 31

31 The zeros of a quartic polynomial function h are $-1, \pm 2,$ and 3 .

Sketch a graph of $y = h(x)$ on the grid below.

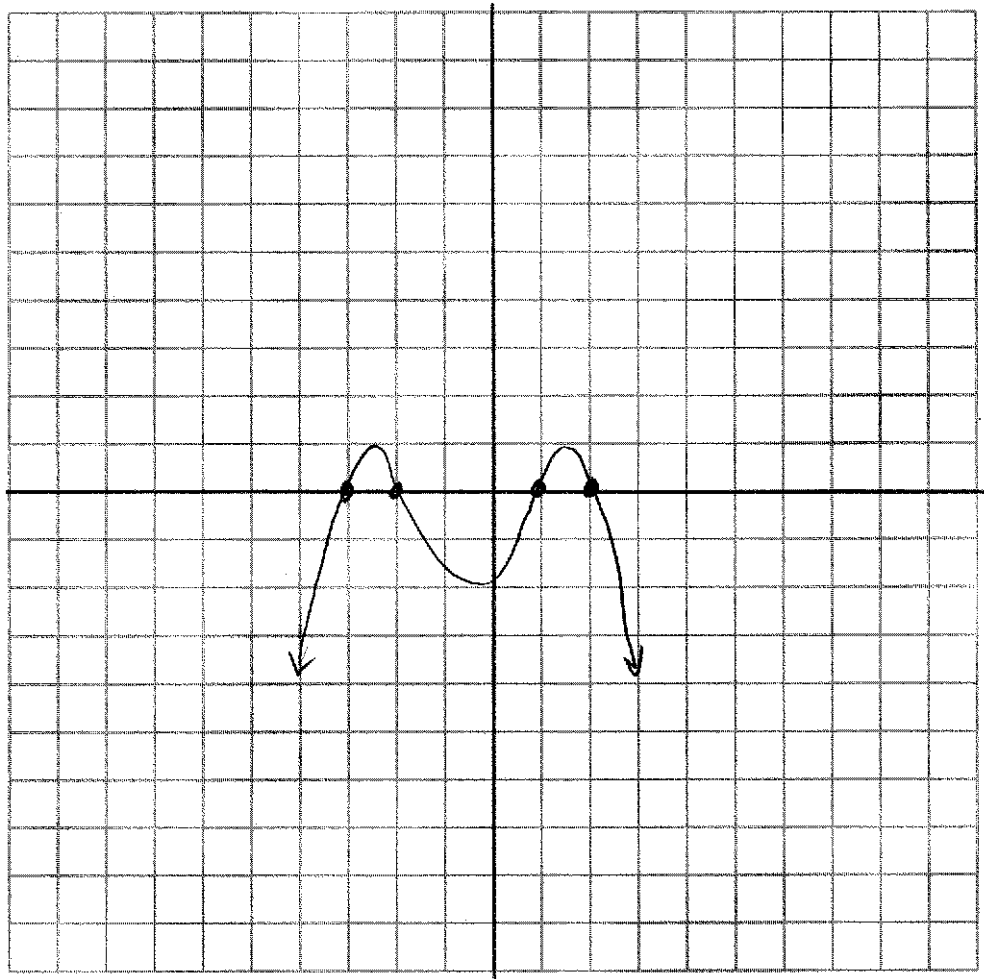


Score 1: The student made one graphing error.

Question 31

31 The zeros of a quartic polynomial function h are $-1, \pm 2,$ and 3 .

Sketch a graph of $y = h(x)$ on the grid below.

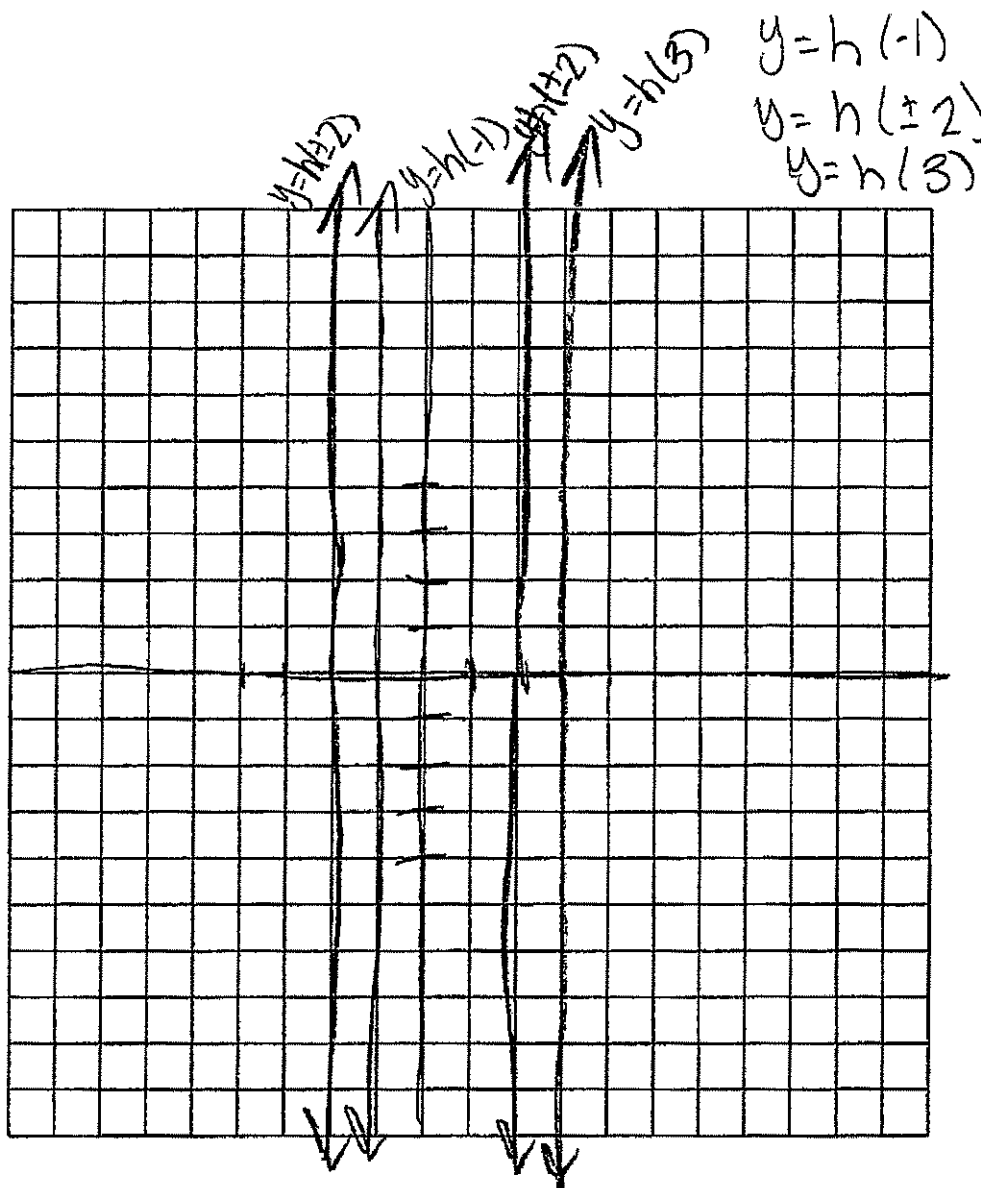


Score 1: The student incorrectly interpreted the zeros.

Question 31

31 The zeros of a quartic polynomial function h are -1 , ± 2 , and 3 .

Sketch a graph of $y = h(x)$ on the grid below.



Score 0: The student gave a completely incorrect response.

Question 32

32 Explain why $81^{\frac{3}{4}}$ equals 27.

$$\log_{81} 27 = \frac{3}{4}$$

$$\left(\sqrt[4]{81}\right)^3 = 27$$

$81^{\frac{3}{4}}$ equals 27 because 81 with four roots gives you 3 and 3 cubed is 27, therefore $81^{\frac{3}{4}}$ is 27.

Score 2: The student gave a complete and correct response.

Question 32

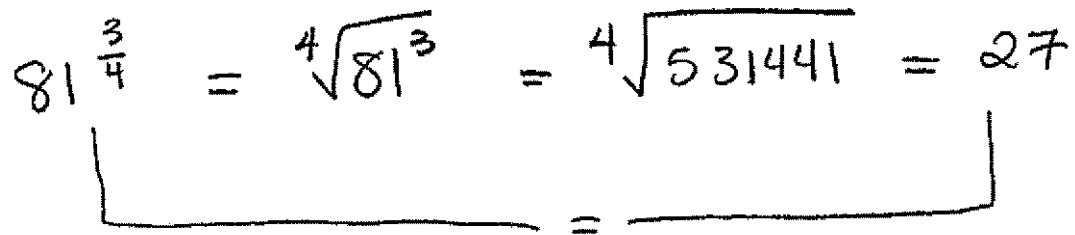
32 Explain why $81^{\frac{3}{4}}$ equals 27.

because when a number is raised by a fraction, the denominator becomes the index of the root and 81 is inside the root, raised to the power of 3, the numerator.

Score 1: The student gave an incomplete explanation because no reference was made to 27.

Question 32

32 Explain why $81^{\frac{3}{4}}$ equals 27.

$$81^{\frac{3}{4}} = \sqrt[4]{81^3} = \sqrt[4]{531441} = 27$$


Score 1: The student gave a justification and did not write an explanation.

Question 32

32 Explain why $81^{\frac{3}{4}}$ equals 27.

It doesn't, it equals ~~108~~ 108.

Score 0: The student gave completely incorrect response.

Question 33

33 Given $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$\begin{aligned} & (2x^2 + x - 3)(x - 1) - ((2x^2 + x - 3) + (x - 1)) \\ & (2x^3 + x^2 - 3x - 2x^2 - x + 3) - (2x^2 + 2x - 4) \\ & (2x^3 - x^2 - 4x + 3) - (2x^2 + 2x - 4) \\ & 2x^3 - 3x^2 - 6x + 7 \end{aligned}$$

Score 4: The student gave a complete and correct response.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$\begin{aligned} & (2x^2 + x - 3)(x - 1) \\ & 2x^3 - 2x^2 + x^2 - x - 3x + 3 \\ & (2x^3 - x^2 - 4x + 3) - (2x^2 + 2x - 4) \\ & 2x^3 - 3x^2 - 6x - 1 \end{aligned}$$

$$\begin{aligned} & 2x^2 + x - 3 + x - 1 \\ & 2x^2 + 2x - 4 \end{aligned}$$

Score 3: The student made a transcription error copying the sum.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$(2x^2 + x - 3)(x - 1) - (2x^2 + x - 3 + x - 1)$$

$$2x^3 + x^2 - 3x - 2x^2 - x - 3 - (2x^2 + 2x - 4)$$

$$2x^3 + x^2 - 6x - 1$$

Score 3: The student correctly found the product and sum, but subtracted incorrectly.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$\begin{array}{r} 2x^2 + x - 3 \\ + x - 1 \\ \hline 2x^2 + 2x - 4 \end{array}$$

Ans: $2x^2 + 2x - 4$

Score 2: The student found the correct sum only.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$g(x) = x - 1$$

$$\begin{aligned} f(x-1) &= 2(x-1)^2 + x - 1 - 3 \\ &= 2x^2 - 4x + 2 + x - 1 - 3 \end{aligned}$$

$$f(g(x)) = 2x^2 - 3x - 2$$

$$2x^2 + x - 3 + x - 1$$

$$f(x) + g(x) = 2x^2 + 2x - 4$$

$$(2x^2 - 3x - 2) - (2x^2 + 2x - 4)$$

$$\boxed{2x^2} - 3x - 2 - \boxed{2x^2} - 2x + 4$$

$$\boxed{-5x + 2}$$

$$\begin{aligned} (x-1)(x-1) \\ x^2 - x - x + 1 \\ 2(x^2 - 2x + 1) \\ 2x^2 - 4x + 2 \end{aligned}$$

Score 2: The student made a conceptual error by performing a composition.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$\begin{array}{r} 2x^2 + x - 3 \\ + \quad \quad x - 1 \\ \hline 2x^2 + 2x - 2 \end{array}$$

Score 1: The student made an error computing the sum.

Question 33

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - [f(x) + g(x)]$ as a polynomial in standard form.

$$\begin{array}{l} 2x^2 + x - 3 - 1 \\ 2x^2 - 2x - 7 - 2x^2 + x - 3 + x - 1 \\ -2x - 2 - 2x^2 \end{array}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

M = Math as
favorite
subject

$$P(M) = \frac{1}{4}$$

$$P(M \text{ OR } J) = \frac{47}{108}$$

J = Junior

$$P(J) = \frac{116}{459}$$

$$P(M \text{ AND } J) = P(M) + P(J) - P(M \text{ OR } J)$$

$$= \frac{1}{4} + \frac{116}{459} - \frac{47}{108} = \boxed{\frac{31}{459}}$$

Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

$$P(M) = \frac{1}{4} = .25$$

If Independent:

$$P(A \text{ AND } B) = P(A) P(B)$$

$$P(J) = \frac{116}{459} = .2527$$

$$P(M \text{ AND } J) = \frac{31}{459} = 0.0675$$

$$P(M) \cdot P(J) = \frac{1}{4} \cdot \frac{116}{459} = \frac{29}{459} = 0.0631$$

No, $P(M)$ and $P(J)$ are not independent of each other because $P(M \text{ AND } J) \neq P(M) \cdot P(J)$

Score 4: The student gave a complete and correct response.

Question 34

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

$$\frac{47}{108} = \frac{1}{4} + \frac{116}{459} - x$$

$$\frac{799}{1836} = \frac{459}{1836} + \frac{464}{1836} - x$$

$$x = \frac{124}{1836} = \frac{31}{459}$$

Are the events “the student is a junior” and “the student’s favorite subject is Math” independent of each other? Explain your answer.

$$P(j \text{ and } m) = P(j)P(m)$$

$$\frac{31}{459} = \frac{116}{459} \cdot \frac{1}{4}$$

$$\frac{31}{459} \neq \frac{116}{1836}$$

Score 3: The student did not provide an explanation.

Question 34

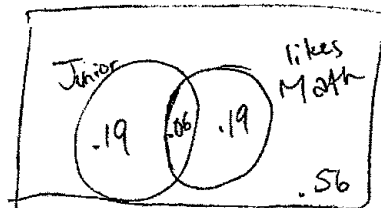
34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

$$P(M) = .25$$

$$P(J) = .25$$

$$P(J \text{ or } M) = .44$$

$$P(J \text{ and } M) = \boxed{.06}$$



Are the events “the student is a junior” and “the student’s favorite subject is Math” independent of each other? Explain your answer.

$$P(J) \stackrel{?}{=} P(J|M)$$

$$.25 \stackrel{?}{=} \frac{.06}{.25}$$

$$.25 \neq .27$$

NOT INDEPENDENT

Score 2: The student did not find the exact probability and did not provide an explanation.

Question 34

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

$$\frac{1}{4} + \frac{106}{459} - \frac{47}{108} = \frac{7}{153}$$

Are the events “the student is a junior” and “the student’s favorite subject is Math” independent of each other? Explain your answer.

Score 1: The student made one transcription error and did not write an explanation.

Question 34

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

$$\frac{1}{4} \times \frac{\overset{29}{\cancel{116}}}{459} \times \frac{47}{\cancel{108} 27} = \frac{1363}{99572}$$

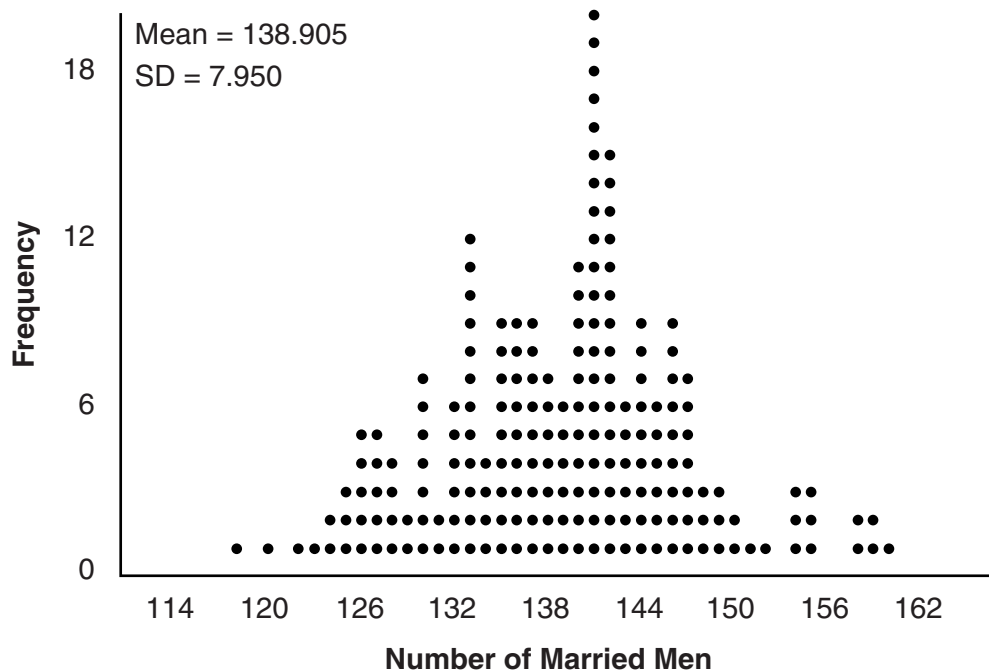
Are the events “the student is a junior” and “the student’s favorite subject is Math” independent of each other? Explain your answer.

They are independent of each other because you don't have to be a junior to like math or like math to be a junior.

Score 0: The student did not show enough correct work to receive any credit.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

$$\begin{aligned} \bar{x} &= 138.905 & \bar{x} - 2(\sigma) & & \bar{x} & & \bar{x} + 2(\sigma) \\ \sigma &= 7.950 & 138.905 - 2(7.950) & \text{---} & 138.905 + 2(7.950) & & \\ & & \boxed{123 \text{ --- } 155} & & & & \end{aligned}$$

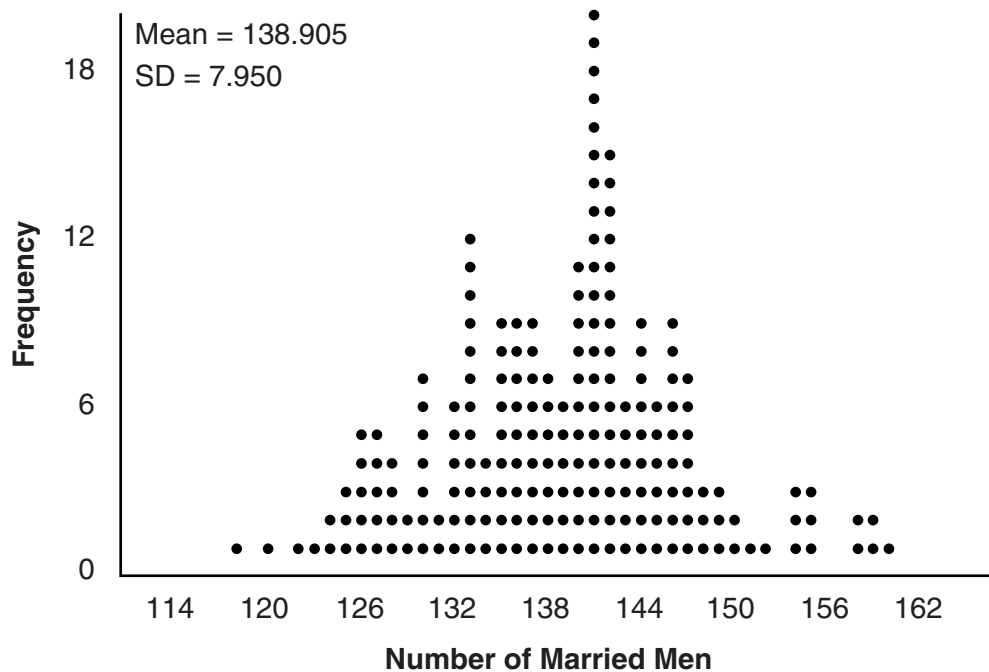
- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

My results from part a do not contradict this claim because half of 250 is 125. My result was between 123 and 155.

Score 4: The student gave a complete and correct response.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

$$138.905 - 2(7.95) = 123.005$$

$$138.905 + 2(7.95) = 154.805$$

123 to 155

- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

$\frac{123}{250} = .492$ No, since .5 falls within the 95% interval of .492 to .50 it is not a rare event

$\frac{155}{250} = .62$ therefore this proportion could happen

Score 4: The student gave a complete and correct response.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

$$2(7.950) = 15.9$$

$$138.905 \pm 15.9$$

$$(123.005, 154.805)$$

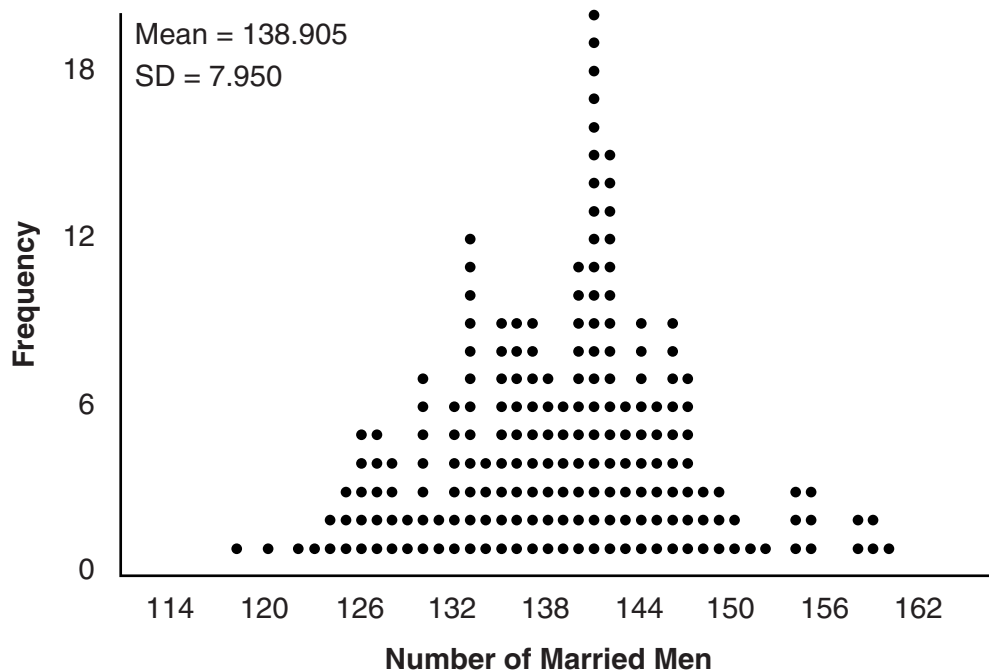
- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

No because $250(.5) = 125$. Even though it is not likely, 125 is in the interval which makes it plausible

Score 3: The student made a rounding error creating the interval.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

139 - 143

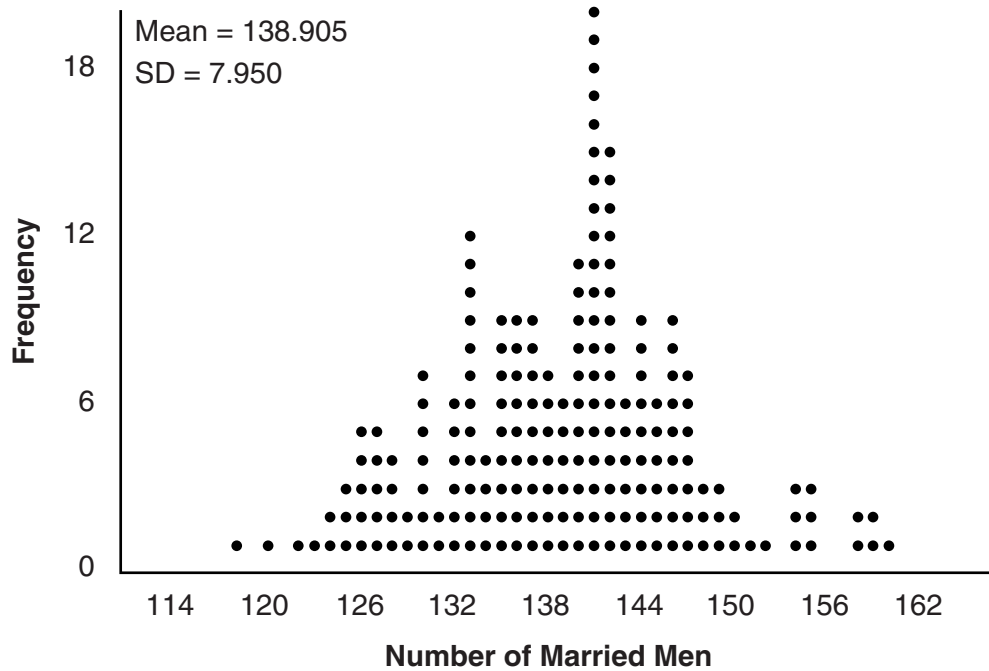
- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

Yes because 50% of 250 is 125, which doesn't fit into the interval.

Score 2: The student stated an incorrect interval, but gave a complete explanation based on the interval.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

$$138.905 + 2(7.95) = 154.805$$

$$138.905 - 2(7.95) = 123.005$$

$$123 < x < 155$$

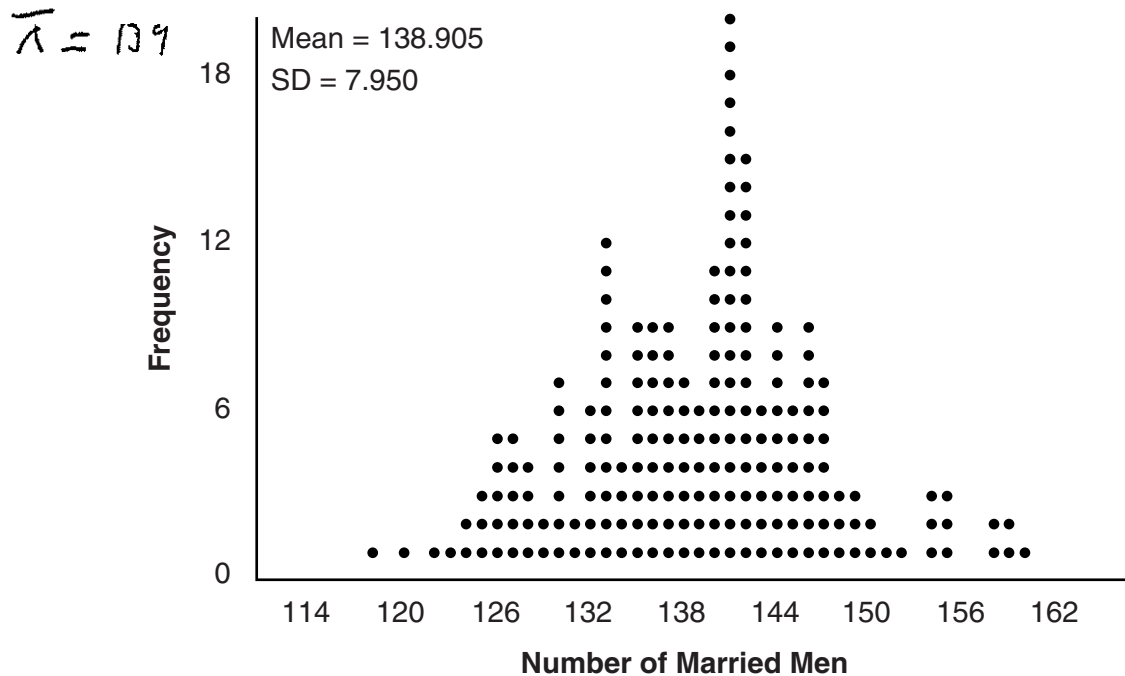
- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

It is possible, but not highly likely, that 125 men out of 250 are married.

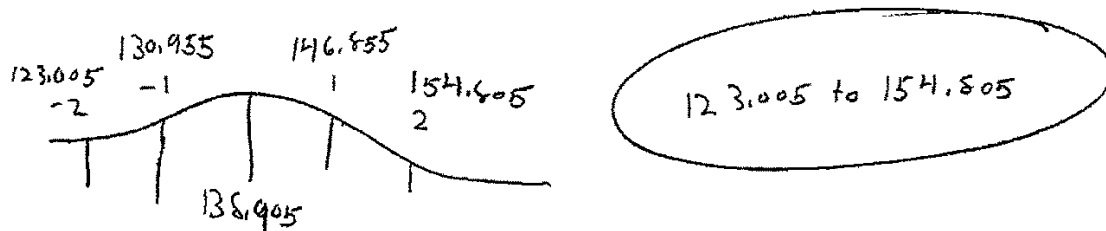
Score 2: The student only received credit for the correct interval.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

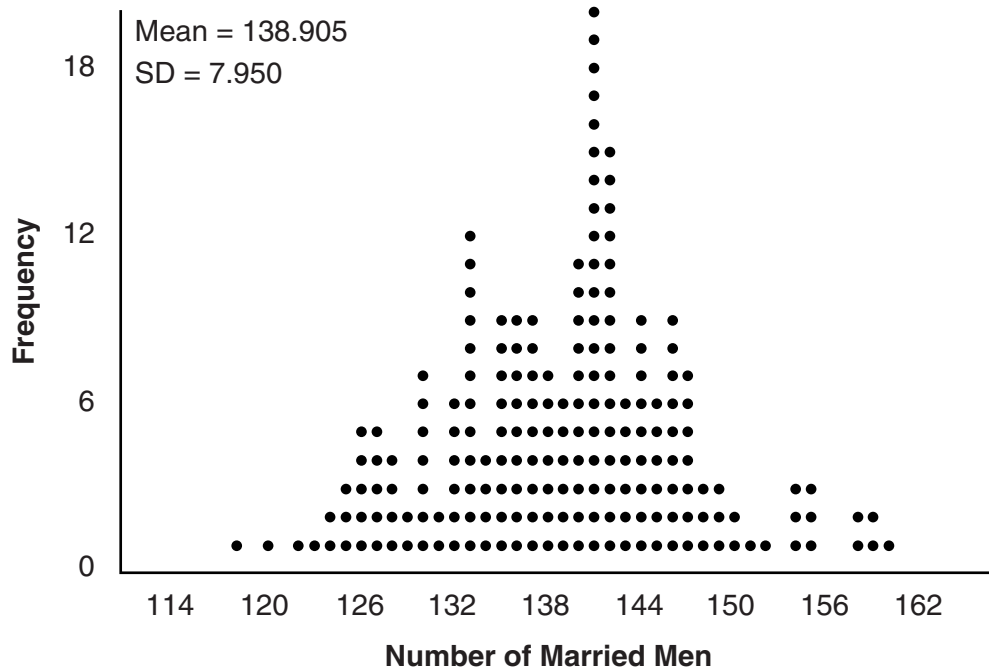


- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

Score 1: The student made a rounding error creating the interval and did not provide an explanation.

Question 35

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.



- a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

$$ME = 2 \left(\frac{7.95}{\sqrt{250}} \right) = 1.0056 = 1$$

$$137.905 \leq \mu \leq 139.905$$

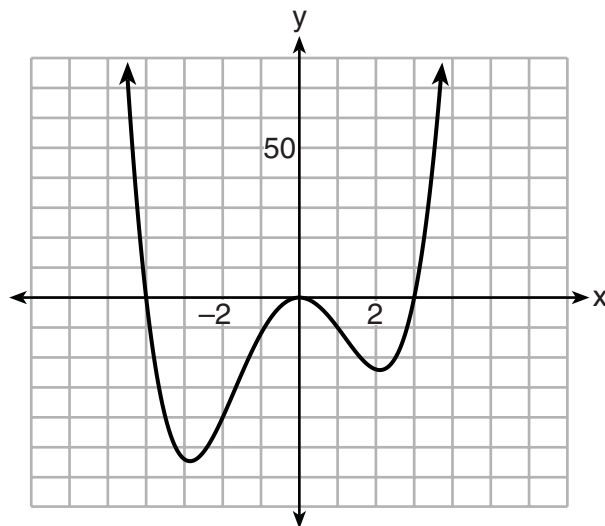
- b) A study claims “50 percent of men 21 and older in the United States are married.” Do your results from part *a* contradict this claim? Explain.

No they don't. We don't know the age of the men.

Score 0: The student did not show any correct work.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$f(x) = (x-3)(x^2)(x+4)$$

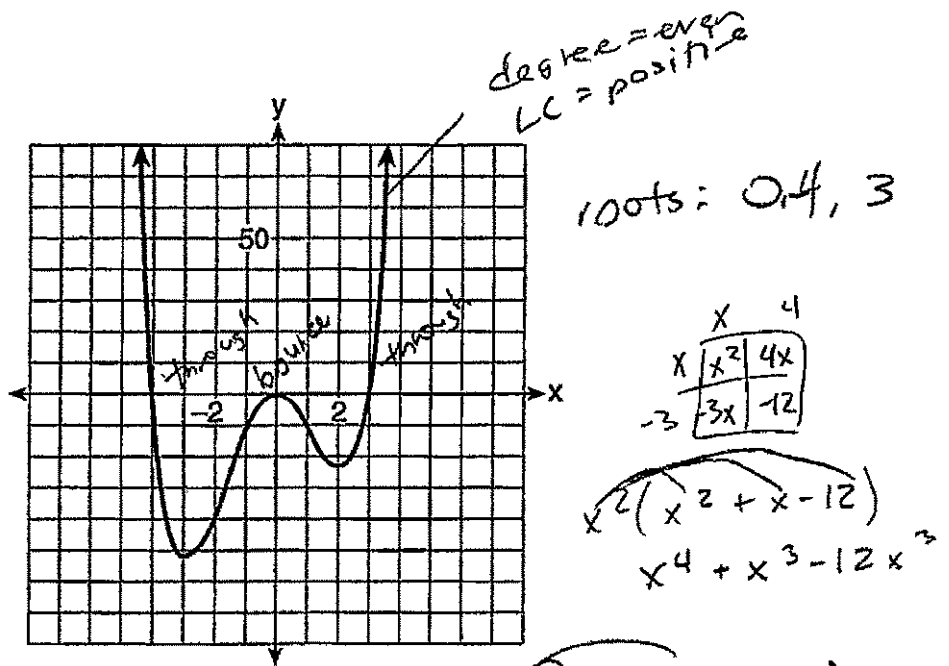
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x-1)(x+2)^2(x+6)$$

Score 4: The student gave a complete and correct response.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$y = x^4 + x^3 - 12x^2$$

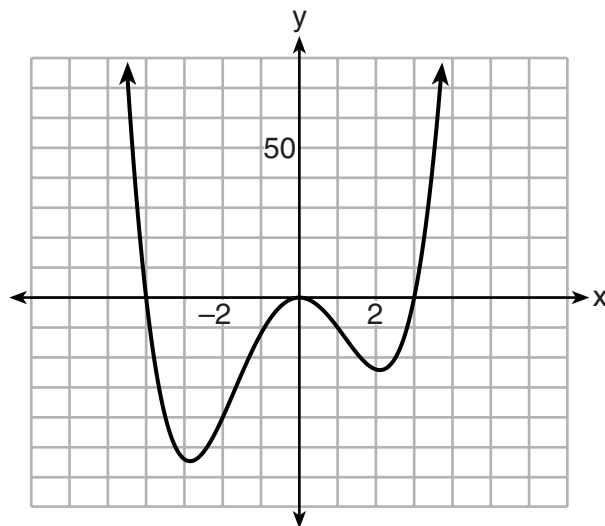
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x+2)^4 + (x+2)^3 - 12(x+2)^2$$

Score 4: The student gave a complete and correct response.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$y = (x + 4)(x)^2(x - 3)$$

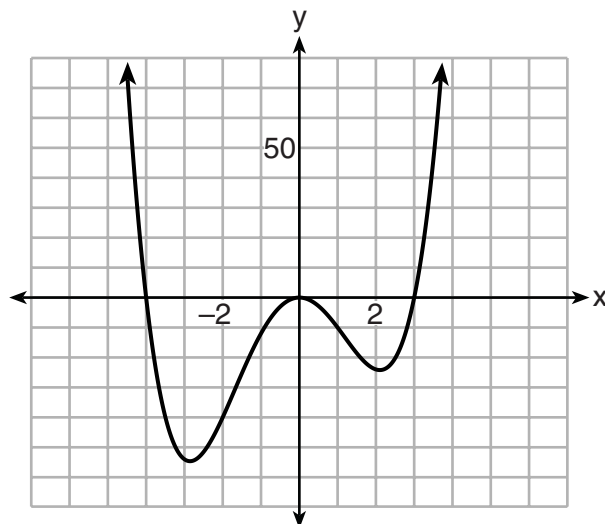
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$y = (x + 6)(x + 2)^2(x - 1)$$

Score 3: The student made a notation error by mislabeling $g(x)$.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$\begin{aligned} & (x^4 - 4x^3 + 4x^2 - 16x^2) \\ & (x^2 + 4x)(x^2 - 4x) \\ & (x + 4)(x + 0)(x + 0)(x - 4) \\ & f(x) = x^4 - 16x^2 \end{aligned}$$

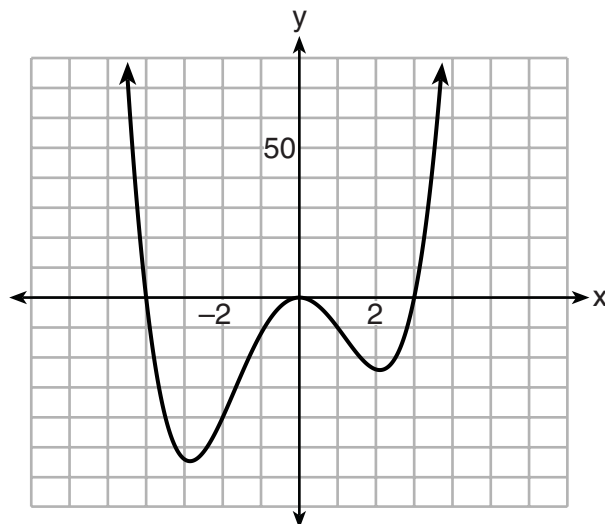
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x + 2)^4 - 16(x + 2)^2$$

Score 3: The student used $x - 4$ instead of $x - 3$.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$f(x) = (x-4)^3 (x-2)^2 (x-3)$$

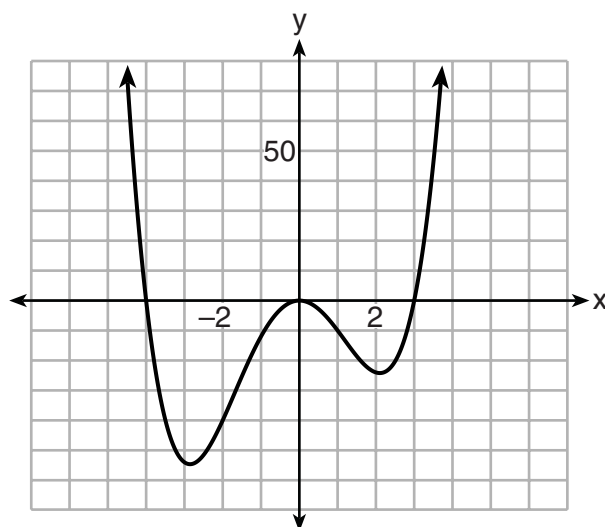
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x-2)^3 (x+2)^2 (x-1)$$

Score 2: The student made two errors writing the equation for $f(x)$.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$f(x) = (x-3)(x-0)(x+4)$$

$$f(x) = (x^2 - 3x)(x+4) \quad x^3 + 4x^2 - 3x^2 - 12x = \boxed{x^3 + x^2 - 12x}$$

The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x-1)(x+2)(x+6)$$

$$(x-1)(x+2)$$

$$x^2 + 2x - x - 2$$

$$(x^2 + x - 2)(x+6)$$

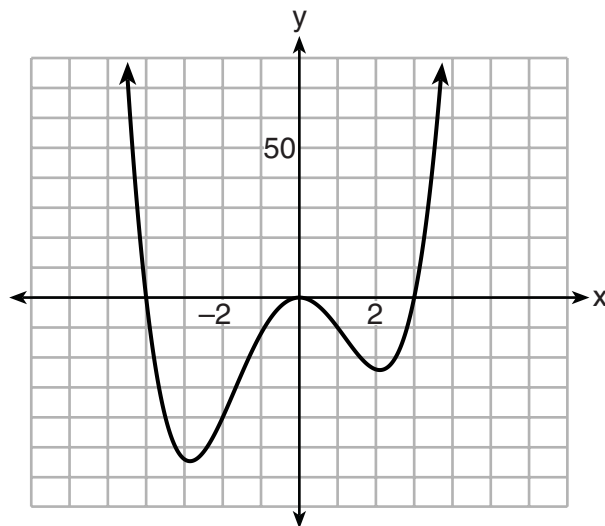
$$x^3 + 6x^2 + x^2 + 6x - 2x - 12$$

$$\boxed{x^3 + 7x^2 + 4x - 12}$$

Score 2: The student made a conceptual error not realizing $x = 0$ was a double root.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$f(x) = x^2$$

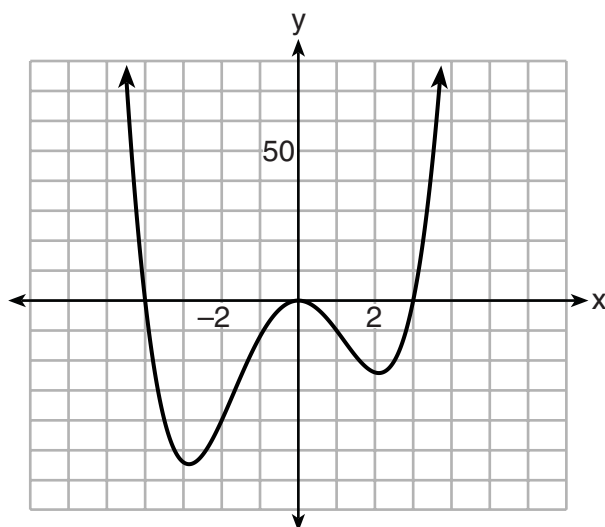
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = (x+2)^2$$

Score 1: The student only received credit for $g(x)$.

Question 36

36 The graph of $y = f(x)$ is shown below.



$$\begin{aligned}
 & (x^2 + 4x) (x - 3) \\
 & + 3 - 3x^2 + 4x^2 - 12x \\
 & x^3 + x^2 - 12x (x + 2) \\
 & x^4 + x^3 - 12x^2 \\
 & 2x^3 + 2x^2 - 14x
 \end{aligned}$$

Write an equation for $f(x)$.

$$\begin{aligned}
 f(x) &= (x + 4)(x - 3)(x) \\
 f(x) &= x^3 + x^2 - 12x
 \end{aligned}$$

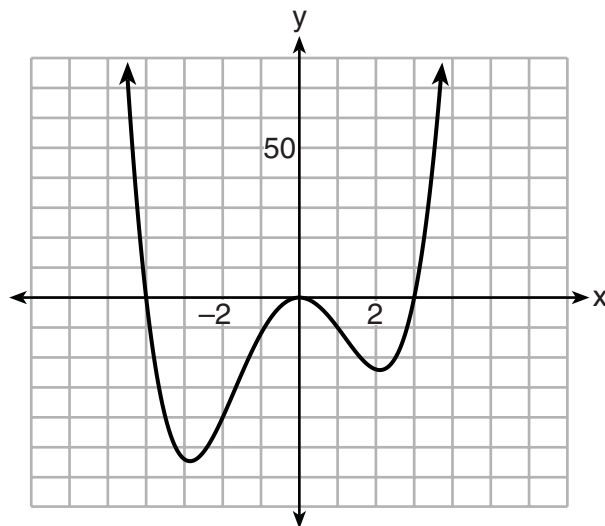
The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

$$g(x) = x^4 + 3x^3 - 10x^2 - 14x$$

Score 1: The student made a conceptual error not realizing $x = 0$ was a double root and incorrectly determined $g(x)$.

Question 36

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$(x+4)(x+0)(x-3) \quad x^3 + 4x^2 - 3x^2 + 4x - 3x - 12 = x^3 + x^2 + x - 12$$

The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

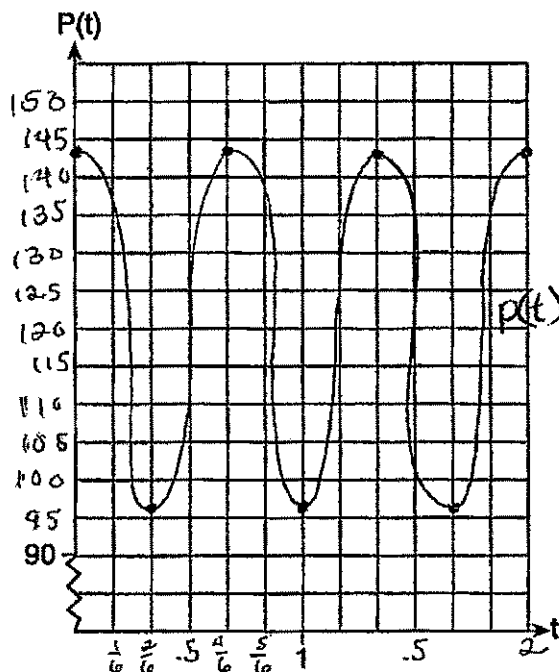
Score 0: The student did not show enough correct work to receive any credit.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



$$120 - 24 = 96 \text{ min}$$

$$120 + 24 = 144 \text{ max}$$

$$\frac{2\pi}{3\pi} = \frac{2}{3} = P$$

$$f = \frac{2\pi}{P}$$

$$f = \frac{2\pi}{\frac{2}{3}} = 3$$

$$f = 3$$

Score 6: The student gave a complete and correct response.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

$$p = \frac{2\pi}{3\pi} \rightarrow \boxed{\frac{2}{3}} \text{ every 2 seconds, the blood pressure oscillates 3 times}$$

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

$$\frac{144}{96} \quad \text{The given patient has high blood pressure. The values 144 and 96 are above 140 and over 90.}$$

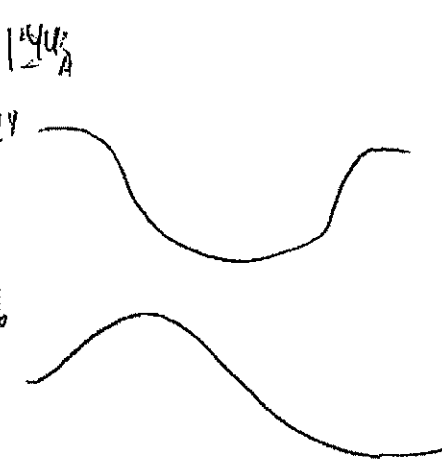
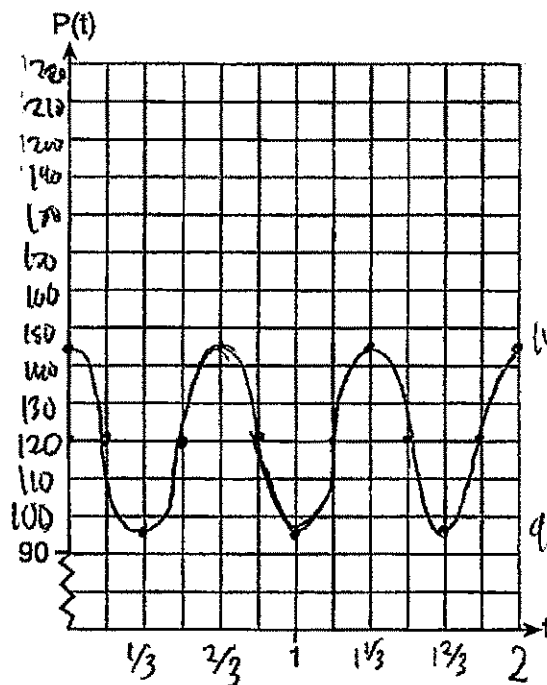
Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.

~~Ampl = 24~~
~~freq = 3\pi~~
~~per = 2/3~~
 $\frac{2\pi}{3\pi}$



Score 5: The student provided an incomplete explanation of period in the given contest.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

$2/3$: Entire length of each oscillation

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

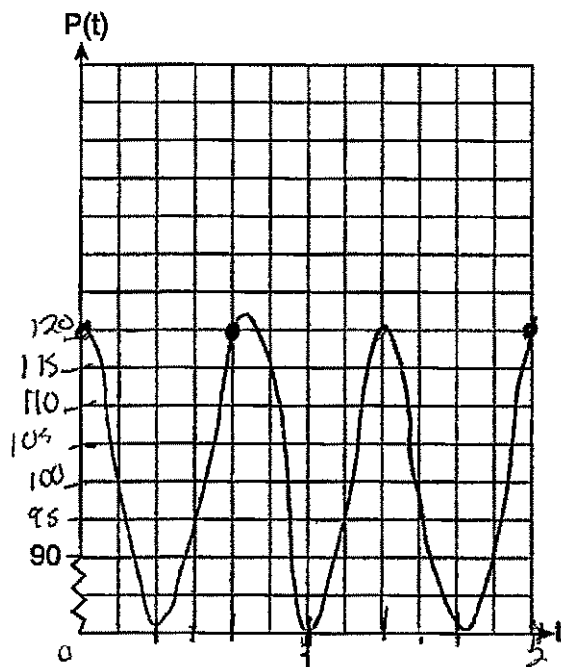
high, because it went as high as 144 and as low as 96 — mostly in the "high" range.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



$$24\cos(3\pi x) + 120$$

$$A = 24$$

$$F = 3\pi$$

$$P = \frac{24}{3\pi} = \frac{2}{3}$$

$$C.P = \frac{2}{3} \cdot 4 = \frac{8}{3}$$

$$P(t) = 24\cos(3\pi t) + 120$$

Score 4: The student made an error graphing the range and incorrectly interpreted the graph.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

Period = $\frac{2}{3}$ This is how long it takes for blood pressure to oscillate to its maximum.

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

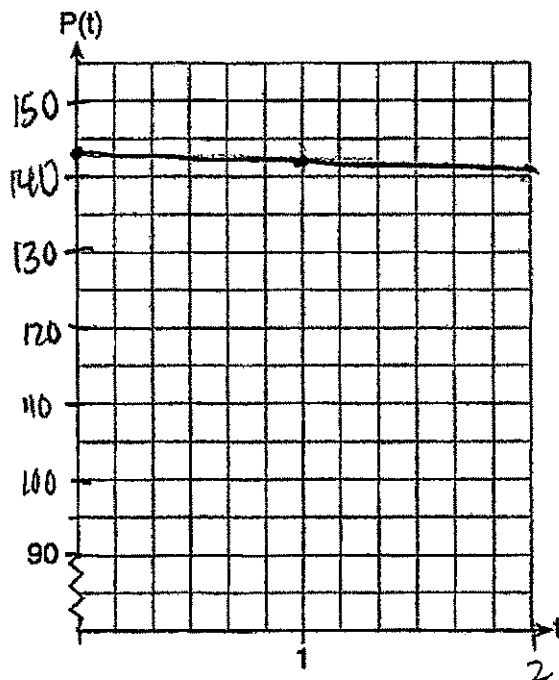
The patient is normal because the maximum is 120 and the minimum is 90.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



$$P(0) = 24 \cos(3\pi \cdot 0) + 120$$

$$(0, 144)$$

$$(1, 143.18)$$

$$(2, 142.78)$$

Score 3: The student correctly identified the period and, based on the graph, correctly classified and explained the patient's blood pressure.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

$\frac{1}{3}$, the normal curve for blood pressure.
(Shows min and max)

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

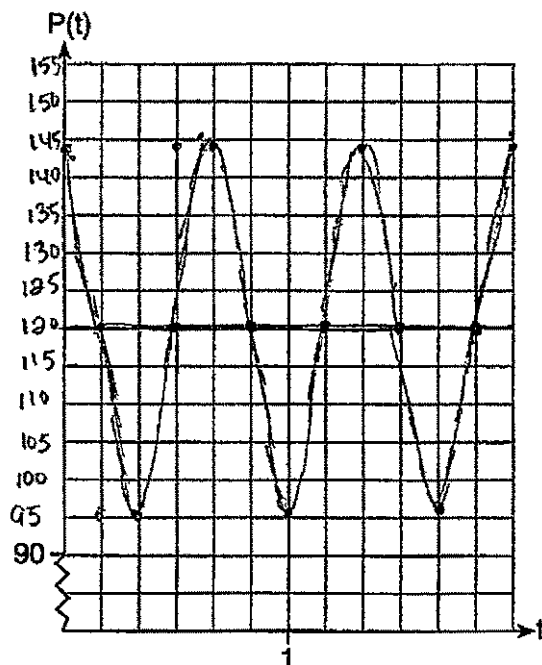
high b/c above 140

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



Score 2: The student received credit for a correct graph only.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

$$P = 2\pi / B \quad \textcircled{\pi/6}$$

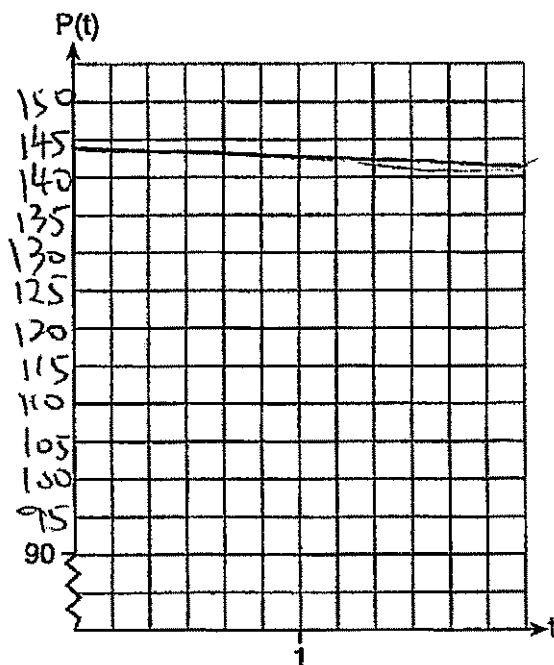
Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



Score 2: The student correctly classified and explained the patient's blood pressure based on the graph.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

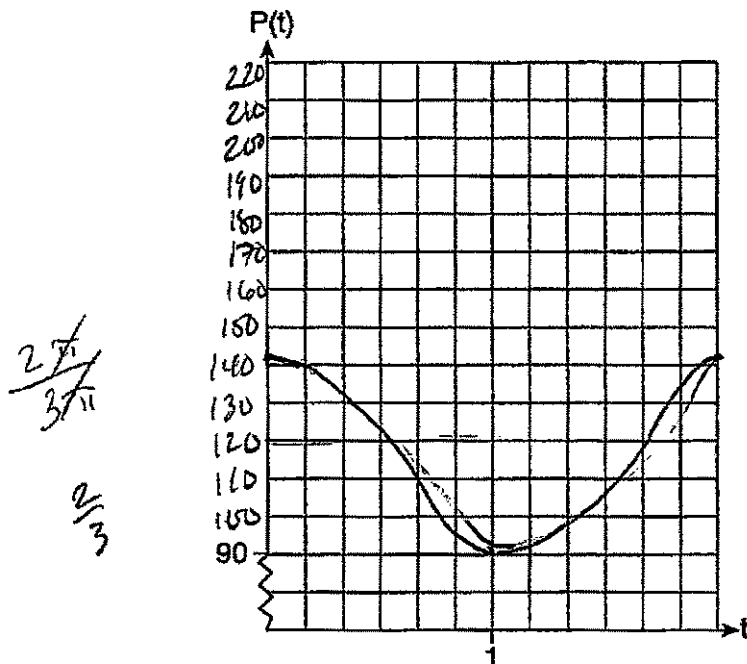
This patient's pressure is high because his average blood pressure is above 140.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



Score 1: The student correctly stated the period.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

$$\frac{2\pi}{\text{freq}} = \frac{2\pi}{3\pi} = \left(\frac{2}{3}\right) \text{ THIS VALUE REPRESENTS THE}$$

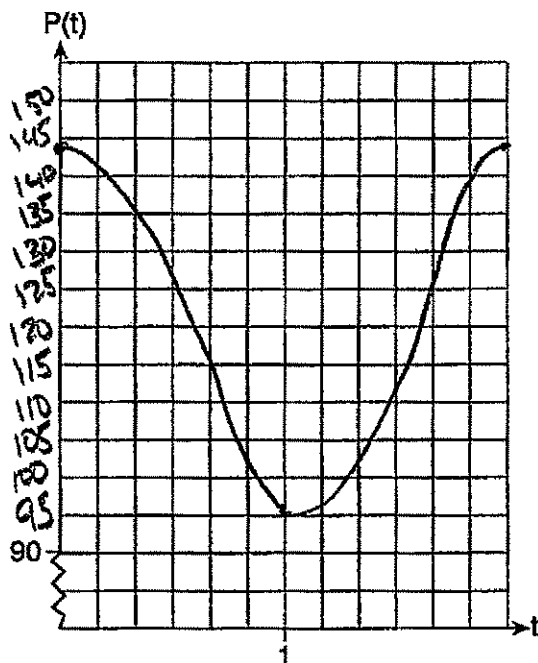
Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



Score 1: The student made one graphing error relating to the period.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

2, because one full wavelength occurs in the domain of $0 \leq t \leq 2$.

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

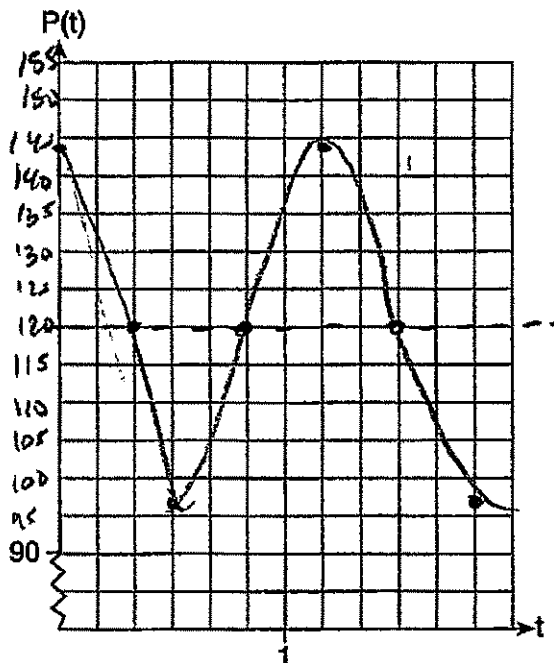
High blood pressure means that the blood pressure oscillates very high and low blood pressure means that it oscillates too low. Both are unhealthy and patients are at risk if not treated to be in the healthy range, 120 over 80.

Question 37

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \leq t \leq 2$.



Score 0: The student made multiple graphing errors and showed no further work.

Question 37 continued.

Determine the period of P . Explain what this value represents in the given context.

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.