The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 25, 2024 — 1:15 to 4:15 p.m., only

MODEL RESPONSE SET

Table of Contents

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$x^3+4x^2-9(x+4)$

$x(x^2+4x)-9(x+4)$

$(x+4)(x+3)(x-3)$

Score 2: The student gave a complete and correct response.

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$x^3+4x^2-9x-36$

$x=-4,-3,3$

$(x+4)(x+3)(x-3)$

Score 2: The student gave a complete and correct response.

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$x^2(x+4) - 9(x+4)$

$(x+4)(x^2-9)$

$(x+4)(x+3)^2$

Score 1: The student made one factoring error.

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$x^2(x+4) - 9(x+4)$

$(x^2-9)(x+4)$

$(x+3)(x-3)(x+4)$

$x=-3 \quad x=3 \quad x=-4$

Score 1: The student made a conceptual error by solving for x.

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$(x^3 + 4x^2) - (9x + 36)$

$x^2(x + 4) - 9(x - 6)$

$(x^2 - 9)(x + 4)(x - 6)$

$(x + 3)(x - 3)(x + 4)(x - 6)$

Score 0: The student made multiple factoring errors.

Question 25

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$x(x^2 + 4x - 9) - 36$

$x^2(x + 4) - 9(x + 4)$

$(x^2 - 9) = 0 \quad (x + 4) = 0$

$x^2 - 9 = 0$
$+9 \quad +9$
$x^2 = \sqrt{9}$
$x = 3$

$x + 4 = 0$
$-4 \quad -4$
$x = -4$

Score 0: The student did not write the expression in factored form and made a conceptual error by solving for x.

Question 26

26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.

$x+4=0$
$-4 \quad -4$
$x=-4$

$f(-4)=2(-4)^3+10(-4)^2+4(-4)-16$
$=2(-64)+10(16)+4(-4)-16$
$=-128+160-16-16$
$=32-32$
$f(-4)=0 \rightarrow$ it is a factor

By using the remainder Theorm, I can conclude that if I plugged in -4 for x, then if the answer is 0 then x+4 is a factor of $2x^3+10x^2+4x-16$.

Score 2: The student gave a complete and correct response.

Question 26

26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.

$$\begin{array}{r|cccc} -4 & 2 & 10 & 4 & -16 \\ & \downarrow & -8 & -8 & 16 \\ \hline & 2 & 2 & -4 & 0 \end{array}$$

Yes because there is a 0 remainder.

Score 2: The student gave a complete and correct response.

Question 26

26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.

$2(x^3+5x^2+2x-8)$

$2(x^3-x^2+6x^2+2x-8)$

$2(x-1)x(x^2+6x+8)$

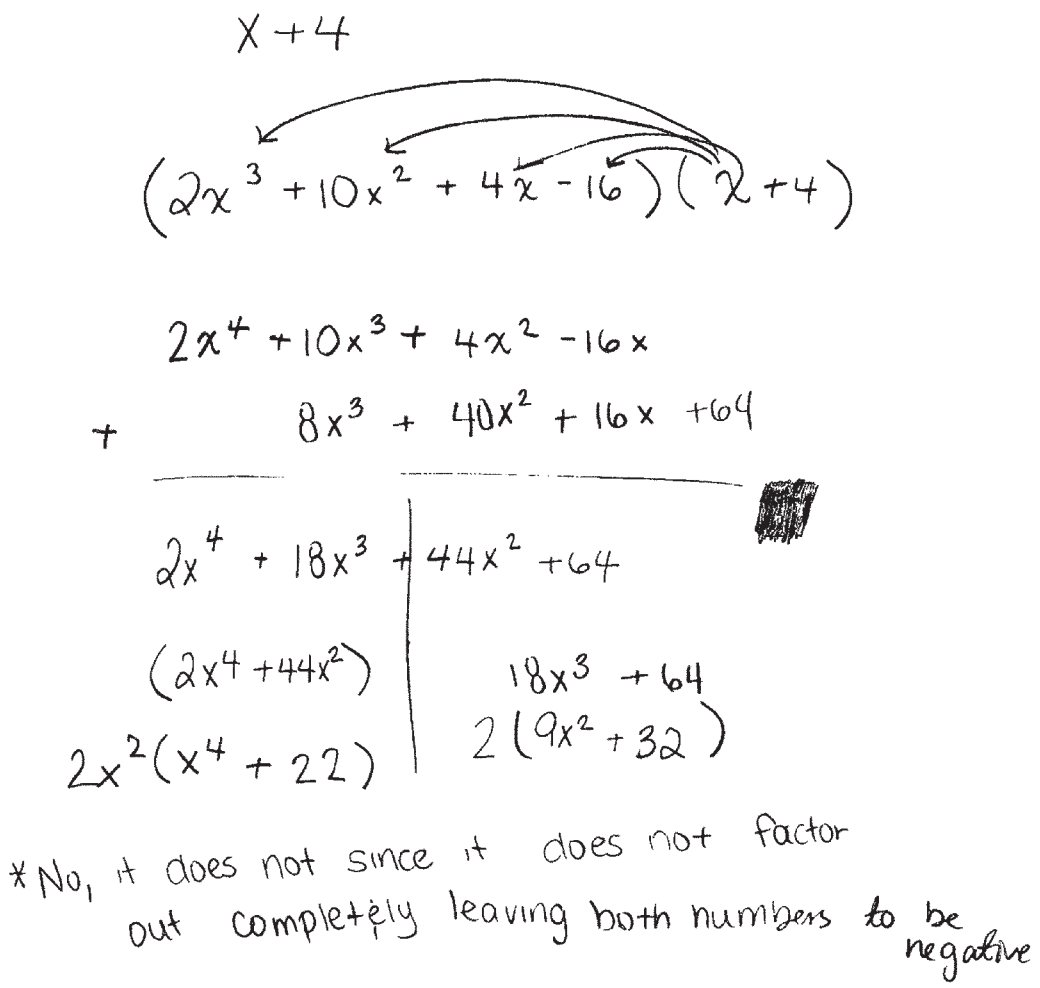
$(4x+2x)$

$2(x-1)x(x+4)x(x+2)$

Score 1: The student did not provide an explanation.

Question 26

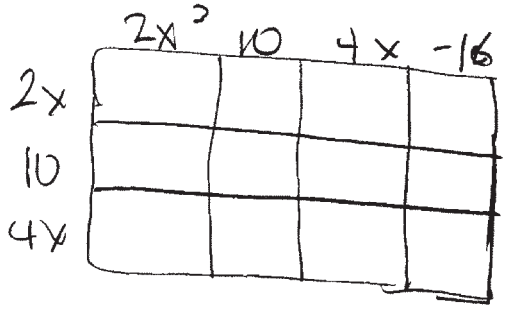
26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.



Score 0: The student made multiple errors.

Question 26

26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.



Yes because using the factor graph you can multiply it

Score 0: The student did not show enough correct work to receive any credit.

Question 27

27 An initial investment of \$1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

$V(t) = 1000(1.01)^{4t}$

t	$V(t)$
2	1083
3	1127
4	1173
5	1220
6	1270
7	1321

$\frac{y^2 - y^1}{x^2 - x^1} - \frac{1321 - 1083}{7 - 2} = \frac{238}{5}$

$= 47.6$

$\approx \$48$

Score 2: The student gave a complete and correct response.

Question 27

27 An initial investment of \$1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

$$\frac{1082.8567 - 1321.29046}{2 - 7}$$

\$48. per year

Score 2: The student gave a complete and correct response.

Question 27

27 An initial investment of \$1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

$V(2) = 1000(1.01)^{4(2)}$

$= 1082.856706$

$V(7) = 1000(1.01)^{4(7)}$

$= 1321.920967$

1082.856706

Aroc: $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1321.920967 - 1082.856706}{7 - 2}$

$= 47.8128522$

$\approx \$48$

Score 1: The student made an error evaluating $V(7)$.

Question 27

27 An initial investment of \$1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

2 years = 1082.4

7 years = 1321.3

$$\frac{\begin{array}{r} 1321.3 \\ -\ 1082.4 \\ \hline 238.9 \end{array}}{5}$$

"AROC = \$47.68 per year"

Score 1: The student made a rounding error.

Question 27

27 An initial investment of $1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

$V(2) = 1000(1.01)^{4(2)} \rightarrow 108.29$

$V(7) = 114.95$

$114.95 - 108.29 = 6.66$

6.66

Score 0: The student made multiple errors.

Question 28

28 When $\left(\frac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.

$\left(\frac{1}{y^{2/3}}\right) y^4$

$\frac{y^4}{y^{2/3}}$

$y^{10/3} = y^n$

$n = 10/3$

$n = \frac{10}{3}$ because when solving for n, fractions can be multiplied to values, but then exponents in a fraction are subtracted from one another $(4 - 2/3)$. which resulted in the answer of $10/3$.

Score 2: The student gave a complete and correct response.

Question 28

28 When $\left(\frac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.

$y^{-2/3} \cdot y^4$

$y^{3.\overline{3}}$

$n = 3.\overline{3}$

Score 2: The student gave a complete and correct response.

Question 28

28 When $\left(\frac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.

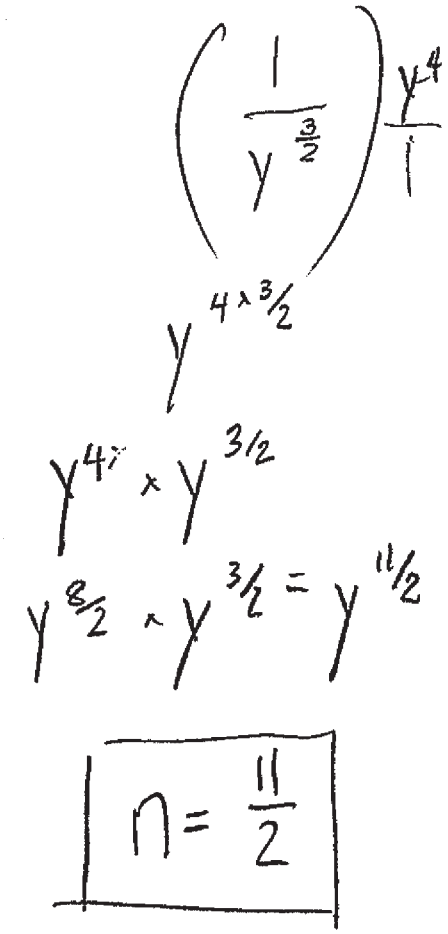
$(x^{\frac{-2}{3}}) y^4$

$y^{\frac{10}{3}}$

Score 1: The student did not state the value of n.

Question 28

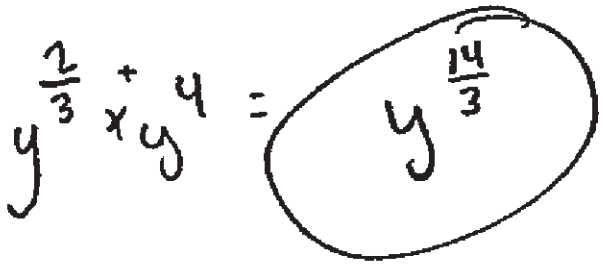
28 When $\left(\frac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.



Score 0: The student made multiple errors.

Question 28

28 When $\left(\frac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.



Score 0: The student made a conceptual error and did not state the value of n.

Question 29

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

normal cdf(67, 72, 64.7, 4.3) = 25%

Score 2: The student gave a complete and correct response.

Question 29

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

0.25158052412885

25%

Score 2: The student gave a complete and correct response.

Question 29

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

25%

using graphing calculator

Score 1: The student did not show work.

Question 29

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

64.7 mm

$z = \frac{67 - 64.7}{4.3}$

$z = .628$

$z = \frac{72 - 64.7}{4.3}$

$z = 1.698$

$.628 < z < 1.698$

.220

22%

Score 1: The student made a computational error finding the first z-score.

Question 29

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

Mean = 64.7

Standard deviation = 4.3

lower bound = 67

upper bound = 72

normalcdf(67, 72, 4.3, 64.7)

= 0.185549731

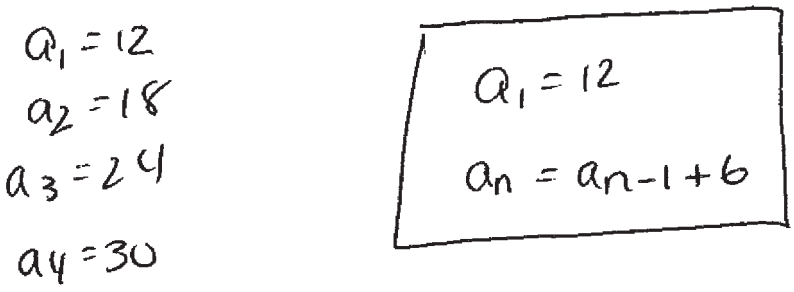
18.5549731

18%

Score 0: The student made multiple errors.

Question 30

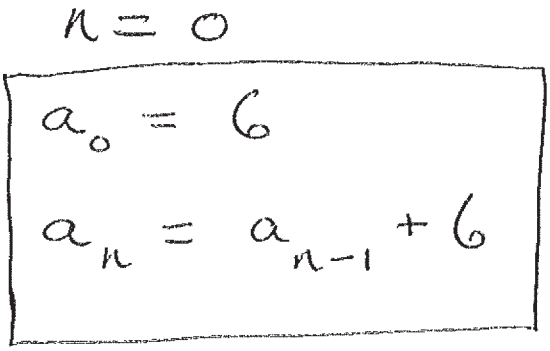
30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.



Score 2: The student gave a complete and correct response.

Question 30

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.



Score 2: The student gave a complete and correct response.

Question 30

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$a_1 = 12$

$a_2 = 18$

$a_3 = 24$

$a_4 = 30$

$a_n = a_{n-1} + 6$

Score 1: The student did not state a_1, in the answer.

Question 30

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$a_1 = 6 + 6(1)$
$a_1 = 12 + 6 = 12$

$a_1 = 12$

$a_n = a_1$

Score 1: The student only stated a_1, correctly.

Question 30

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$a_n = 6 + 6n$

$a_n = a_1 + 6n$

Score 0: The student did not show enough correct work to receive any credit.

Question 30

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$a_n = 6 + 6n$

$a_n = 6 + (a_{n-1})\,6$

Answer

Score 0: The student did not show enough correct work to receive any credit.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.

$(2xi^3 - 3y)(2xi^3 - 3y)$

$4x^2i^6 - 6xyi^3 - 6xyi^3 + 9y^2$

$4x^2i^6 - 12xyi^3 + 9y^2$

$9y^2 + 4x^2(-1) - 12xyi^3$

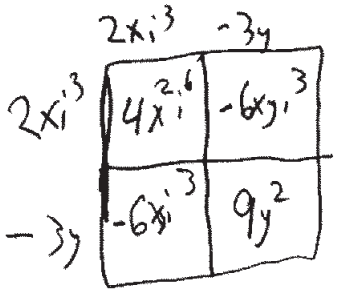
$9y^2 - 4x^2 - 12xyi^3$

$9y^2 - 4x^2 + 12xyi$

Score 2: The student gave a complete and correct response.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.



$4x^2i^6 - 12xyi^3 + 9y^2$

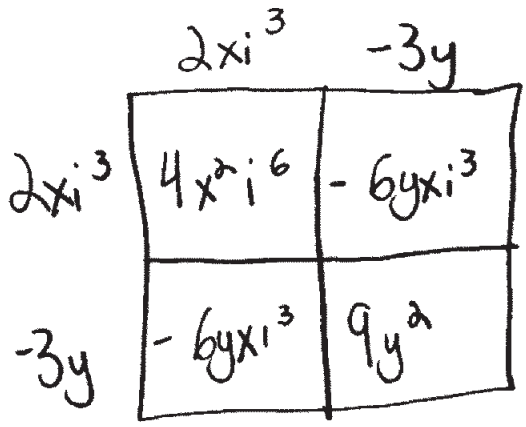
$-4x^2 + 12xyi^3 + 9y^2$

$9y^2 - 4x^2 + 12xyi^3$

Score 2: The student gave a complete and correct response.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.



$-12yxi^3 + 4x^2i^6 + 9y^2$

Score 1: The student did not write the answer in simplest form.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.

0 1
1 i
2 -1
3 i

	$2xi^3$	$-3y$
$2xi^3$	$4x^2i^6$	$-6xyi^3$
$-3y$	$-6xyi^3$	$9y^2$

$4x^2i^6 - 12xyi^3 + 9y^2$

$4x^2(-1) - 12xy(-i) + 9y^2$

$-4x^2 + 12xyi + 9y^2$

$$\frac{-12 \pm \sqrt{12^2 - 4(-4)(9)}}{2(-4)} = \frac{-12 \pm \sqrt{288}}{-8}$$

$$= \frac{-12 \pm \sqrt{36}\sqrt{8}}{-8} \cdot \frac{-12 \pm \sqrt{36}\sqrt{4}\sqrt{2}}{-8}$$

$$= \frac{-12 \pm 6 \cdot 2\sqrt{2}}{-8} = \frac{-12 \pm 12\sqrt{2}}{-8}$$

$$= \frac{-3 \pm 3\sqrt{2}}{-2}$$

$$\frac{3}{2} \pm \frac{3}{2}\sqrt{2}$$

Score 1: The student made a conceptual error by solving the expression as an equation.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.

$(2xi^3-3y)(2xi^3-3y)$

$4xi^6 - 6xi^3y - 6xi^3y + 9y^2$

$4xi^6 \quad -12xi^3y + 9y^2$

$-4x \cdot -12x \quad 9y$

$-16x + 9y$

Score 0: The student made multiple errors.

Question 31

31 Express $(2xi^3 - 3y)^2$ in simplest form.

$(2xi^3 - 3y)(2xi^3 - 3y)$

$4x^2i^9 - 6xi^3y - 6xi^3y + 9y^2$

$4x^2i^9 - 12xiy + 9y^2$

Score 0: The student made multiple errors.

Question 32

32 A survey was given to 1250 randomly selected high school students at the end of their junior year. The survey offered four post-graduation options: two-year college, four-year college, military, or work. Of the 1250 responses, 475 chose a four-year college. State *one* possible conclusion that can be made about the population of high school juniors, based on this survey.

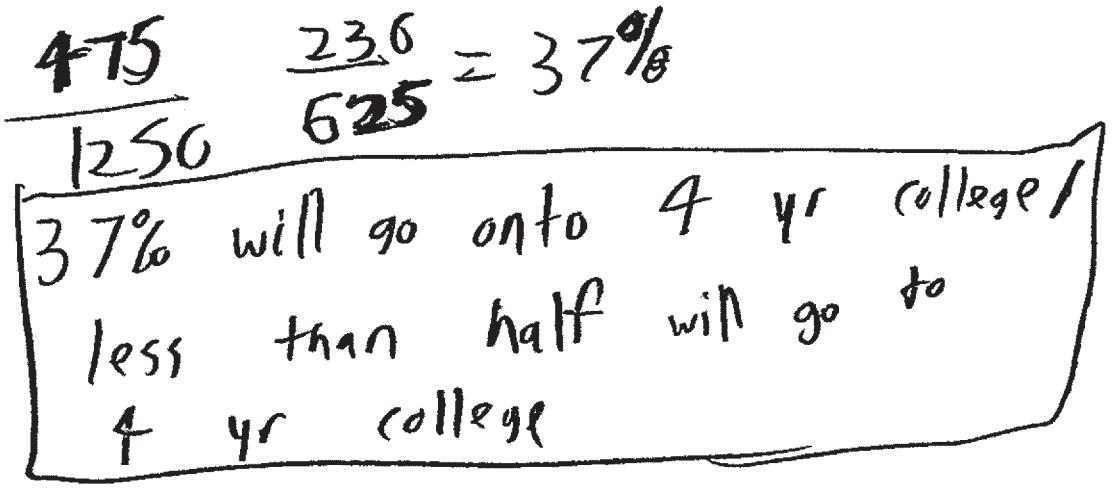
$\frac{475}{1250} = 38\%$

The population of high school juniors that would chose a four-year college would probably be about 38% who would chose a 4-year college whereas 62% would choose a different option based on the survey

Score 2: The student gave a complete and correct response.

Question 32

32 A survey was given to 1250 randomly selected high school students at the end of their junior year. The survey offered four post-graduation options: two-year college, four-year college, military, or work. Of the 1250 responses, 475 chose a four-year college. State *one* possible conclusion that can be made about the population of high school juniors, based on this survey.



Score 1: The student gave a correct conclusion based on incorrect work.

Question 32

32 A survey was given to 1250 randomly selected high school students at the end of their junior year. The survey offered four post-graduation options: two-year college, four-year college, military, or work. Of the 1250 responses, 475 chose a four-year college. State *one* possible conclusion that can be made about the population of high school juniors, based on this survey.

One possible conclusion is that the juniors are looking for more education to get better jobs.

Score 0: The student did not show enough relevant work to receive any credit.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15		1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500}$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{15}$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

$P(a) = P(a|b)$

$\frac{45}{1500} = \frac{3}{15}$

$0.03 \neq 0.2$

Not independent

Score 4: The student gave a complete and correct response.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15	1485	1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500}$ or $\frac{3}{100}$ or 3%

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{15}$ or $\frac{1}{5}$ or 20%

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

No b/c someone is more likely to be allergic to milk if they are allergic to nuts

Score 4: The student gave a complete and correct response.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	total 45
Not Allergic to Milk	12	1443	1455
	total 15	1485	1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500}$ 0.03

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

A given B → denom

15-peanuts $\frac{3}{15}$ 0.2

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

equal eachother

NO

Score 3: The student did not justify the answer.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15	1485	1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500} = .03$ 3%

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{15} = .002$ .2%

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

$P(N \cap M) = P(N) \cdot P(M)$ They are dependent

$\frac{3}{1500} = \frac{15}{1500} \cdot \frac{45}{1500}$

$.002 \neq .01 \cdot .03$

$.002 \neq$

Score 3: The student made a computational error.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15	1485	1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500}$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{15}$ or $\frac{1}{5}$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

Nut Allergy $\frac{15}{1500}$ milk allergy $\frac{45}{1500}$ Both allergy $\frac{3}{1500}$ no allergy $\frac{1443}{1500}$

An allergy: $\frac{60}{1500}$

They are independent since there is such a difference in the # of people allergic to nuts & milk. 45 is many more than 15.

Score 2: The student incorrectly determined independence and gave an incorrect justification.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts
Allergic to Milk	3	42
Not Allergic to Milk	12	1443

Determine the probability that a randomly selected survey respondent is allergic to milk.

$P = \frac{45}{1500} = \frac{9}{300} = \frac{3}{100}$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$P = \frac{3}{1500}$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

nut allergies and milk allergies are not independent events.

Score 1: The student received one point for the first part.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15	1485	1500

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{45}{1500}$

$= .03$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{1500}$

$= .002$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

The are dependent b/c they are the same.

Score 1: The student received one point for the first part.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts
Allergic to Milk	3	42
Not Allergic to Milk	12	1443

Determine the probability that a randomly selected survey respondent is allergic to milk.

$\frac{42}{1500} = \frac{21}{750} = \frac{7}{250}$

$\frac{7}{250}$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$\frac{3}{1500} = \frac{1}{500}$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

it's not because most people are not allergic to milk and nut

Score 0: The student did not show enough correct work to receive any credit.

Question 33

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts
Allergic to Milk	3	42
Not Allergic to Milk	12	1443

Determine the probability that a randomly selected survey respondent is allergic to milk.

12 out of 1443 1 out of

$\frac{12}{1443} = .08$

percent chance

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

3 out of 1500 $\frac{3}{1500}$ 1 out of 500

$= .02$ percent chance

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer.

No they are not because there is a outside event so it dependents.

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x-1}$

$2x = 6 + 2\sqrt{x-1}$
$-6 \quad -6$

$\frac{2x-6}{2} = \frac{2\sqrt{x-1}}{2}$

$(x-3)^2 = (\sqrt{x-1})^2$

$(x-3)(x-3) = x-1$

$x^2 - 3x - 3x + 9 = x - 1$
$-x \quad +1 \quad -x+1$

$x^2 - 7x + 10 = 0$

$a=1 \quad b=-7 \quad c=10$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{7 \pm \sqrt{49-4(1)(10)}}{2}$

$x = \frac{7 \pm \sqrt{9}}{2}$

$x = \frac{7 \pm 3}{2}$

$x = \frac{7+3}{2} = \frac{10}{2} = 5$

$x = \frac{7-3}{2} = \frac{4}{2} = 2$ extraneous

$x = 5$
$x = 2$ extraneous

check

$2(5) = 6 + 2\sqrt{5-1}$
$10 = 10$ ✓

$2(2) = 6 + 2\sqrt{2-1}$
$4 \neq 8$

Score 4: The student gave a complete and correct response.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$-2x = 6 + 2\sqrt{x-1}$
$-6 \quad -6$

$\frac{2x-6}{2} = \frac{2\sqrt{x-1}}{2}$

$(x-3)^2 = \sqrt{x-1}^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5)$

$x \neq 2$ reject $\quad x = 5$ ✓

Score 4: The student gave a complete and correct response.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x-1}$

$-1 \quad -6$

$\frac{2x-6}{2} = \frac{2\sqrt{x-1}}{2}$

$(x-3)^2 = (\sqrt{x-1})^2$

$x^2-6x+9 = x-1$

$-x+1 \quad -x+1$

$x^2-7x+10 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{7 \pm \sqrt{49-40}}{2}$

$x = \frac{7 \pm 3}{2}$

$\frac{7+3}{2} = 5$

$\frac{7-3}{2} = 2$

Score 3: The student did not reject $x = 2$.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$2x = 6 + 2\sqrt{x-1}$

$\frac{2x-6}{2} = \frac{2\sqrt{x-1}}{2}$

$(x-3)^2 = (\sqrt{x-1})^2$

$(x-3)(x-3) = x-1$

$x^2 - 3x - 3x + 9 = x - 1$

$x^2 - 6x + 9 = x - 1$

$x^2 - 6x + 10 = x$

$x^2 - 7x + 10$

$(x-5)(x+2) = 0$

$x = 5 \quad x = -2$

$x = -2, 5$

Score 2: The student made a factoring error and did not reject $x = -2$.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$2x-6=2\sqrt{x-1}$

$x-3=\sqrt{x-1}$

$x^2-6x+9=x-1$

$x^2-7x+10=0$

$x=\frac{7\pm\sqrt{(-7)^2-4(10)}}{2}$

$x=\frac{7\pm3}{2}$

$x=4 \quad x=5$

Score 2: The student made a computational error and did not reject $x = 4$.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$2x - 6 = 2(\sqrt{x} - 1)$
$2x - 6 = 2\sqrt{x} - 2$
$(2x - 4)^2 = (2\sqrt{x})^2$
$4x^2 - 16x + 16 = 4x$
$4x^2 - 20x + 16 = 0$
$x^2 - 5x + 4 = 0$
$(x - 4)(x + 1) = 0$
$x = 4 \quad x = -1$

Score 1: The student made multiple errors and did not reject their solutions.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$$\frac{2x}{2} = \frac{6}{2} + \frac{2\sqrt{x-1}}{2}$$

$$x^2 = 3^2 + \sqrt{x-1}^2$$

$$x^2 = 3 + x - 1$$

$$x^2 = 2 + x$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1)$$

$$x - 2 = 0 \quad | \quad x + 1 = 0$$

$$x = 2 \quad | \quad x = -1$$

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x-1}$

$(2x-6)^2 = (2(\sqrt{x-1}))^2$

$= 2(x-1)$

$(2x-6)(2x-6) = 2x-4$

$4x^2 - 12x - 12x + 36 = 2x-4$

$-2x$

$4x^2 - 24x + 36 =$

$4x^2 - 26x + 40 = 0$

$40 \times 4 = 160$

-26

Score 0: The student did not show enough correct work to receive any credit.

Question 35

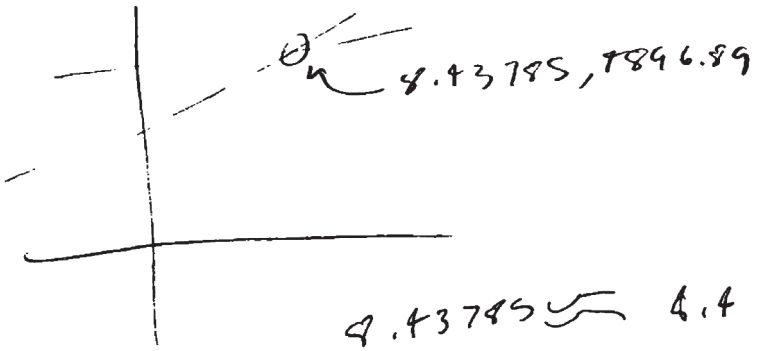
35 During the summer, Adam saved $4000 and Betty saved $3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

$A(t) = 4000(1+0.002)^{12t}$

$B(t) = 3500(1+0.01)^{4t}$

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

Graphed on calc roughly



It will take 8.4 years for the two accounts to have the same amount of money in them

Score 4: The student gave a complete and correct response.

Question 35

35 During the summer, Adam saved $4000 and Betty saved $3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

$A(t) = 4000\left(1 + \frac{0.024}{12}\right)^{12t}$

$B(t) = 3500\left(1 + \frac{0.04}{4}\right)^{4t}$

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

It will take about 8.4 years for the two accounts to have the same amount of money

Score 3: The student gave an incomplete justification.

Question 35

35 During the summer, Adam saved $4000 and Betty saved $3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

Adam

$A(t) = 4000(1 + \frac{0.024}{12})^{12t}$

0.002

Betty

$B(t) = 3500(1 + \frac{0.04}{4})^{4t}$

0.01

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

8 years for both accounts to have the same amount of money

Score 2: The student made a rounding error and gave an incomplete justification.

Question 35

35 During the summer, Adam saved $4000 and Betty saved $3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

$A(t) = 4000(1.024)^{\frac{1}{12}t}$ $B(t) = 3500(1.04)^{\frac{1}{4}t}$

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

17.1 years

$4000(1.024)^{\frac{1}{12}t} = 3500(1.04)^{\frac{1}{4}t}$

Score 2: The student gave the correct number of years based on their incorrect equations.

Question 35

35 During the summer, Adam saved \$4000 and Betty saved \$3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

$A(t) = 4000\left(1 + \frac{0.024}{12}\right)^{12t}$

$B(t) = 3500\left(1 + \frac{0.04}{12}\right)^{12t}$

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

$4008^{12t} = 3511.666667^{12t}$

Score 1: The student stated $A(t)$ correctly.

Question 35

35 During the summer, Adam saved \$4000 and Betty saved \$3500. Adam deposited his money in Bank *A* at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank *B* at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after *t* years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

A

$A = 4000\left(\frac{.024}{12}\right)^{12(t)}$

$B = 3500\left(\frac{.04}{4}\right)^{4(t)}$

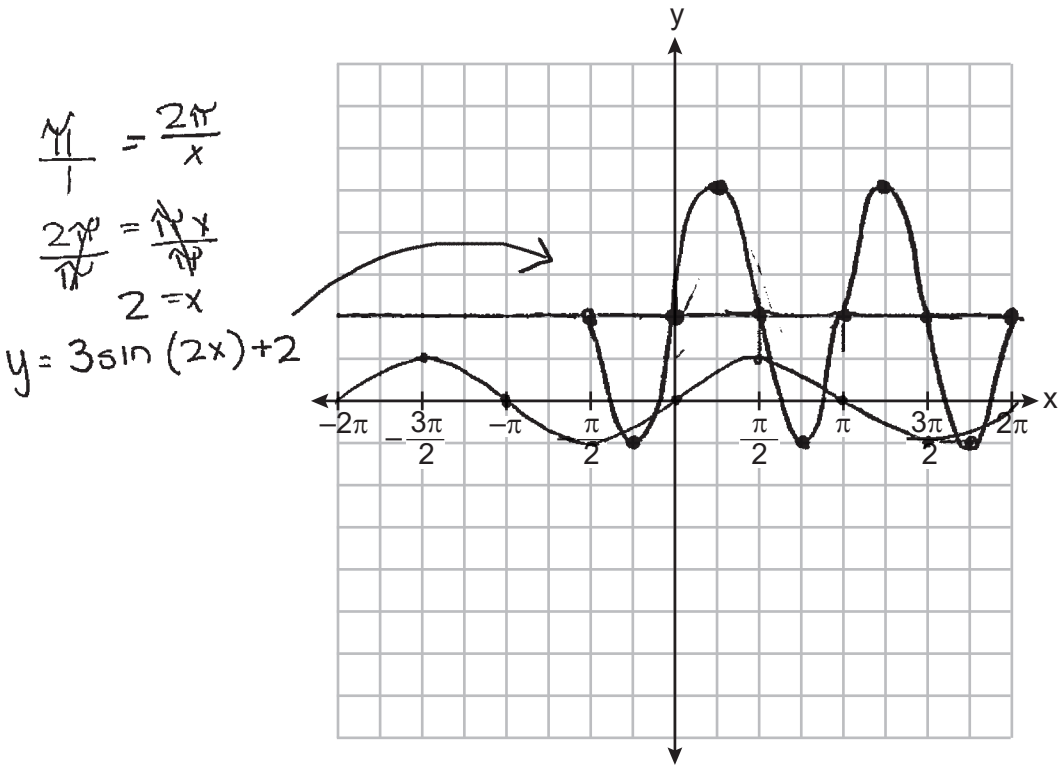
Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

5 years

Score 0: The student did not show enough correct work to receive any credit.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.



$\frac{\pi}{1} = \frac{2\pi}{x}$

$\frac{2\pi}{\pi} = \frac{\pi x}{\pi}$

$2 = x$

$y = 3\sin(2x) + 2$

Based on your graph, state an interval in which the graph is increasing.

$-\pi/4$ to $\pi/4$

Score 4: The student gave a complete and correct response.

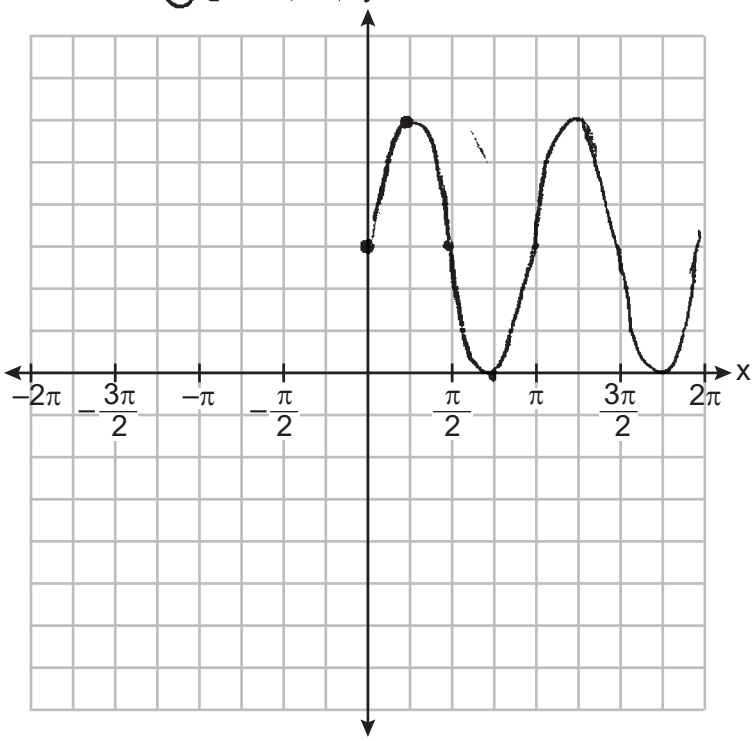
Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.

$\frac{\pi}{\frac{\pi}{2}}$

$\frac{\pi}{1} \cdot \frac{2}{\pi}$

$3\sin(2) + 2$



Based on your graph, state an interval in which the graph is increasing.

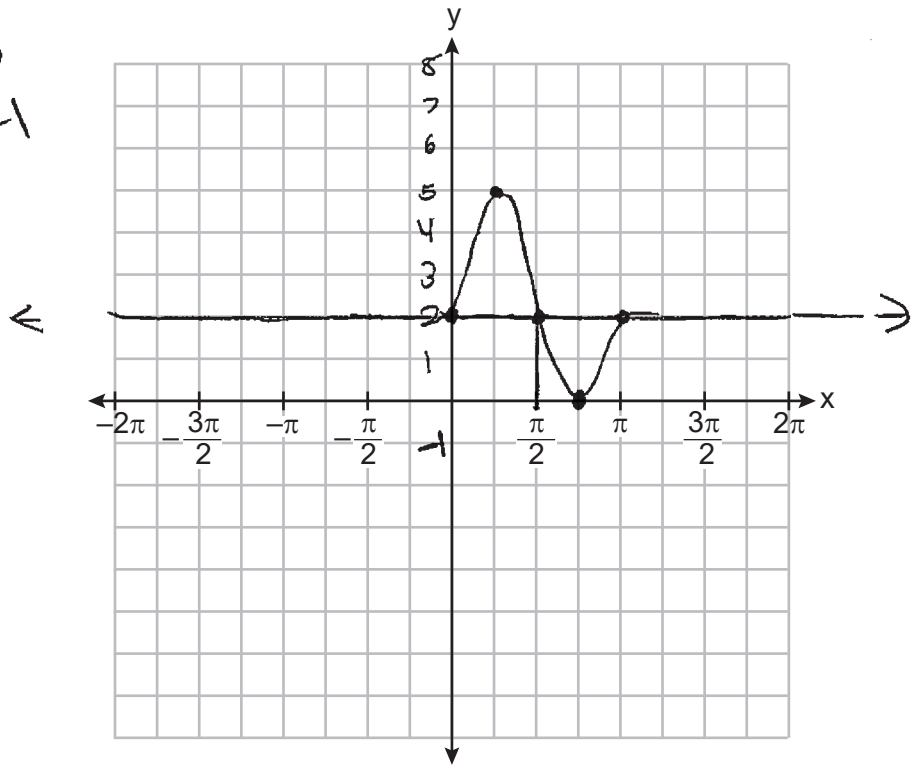
$0 - \frac{\pi}{4}$

Score 3: The student made one graphing error.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.

max = 5
min = −1



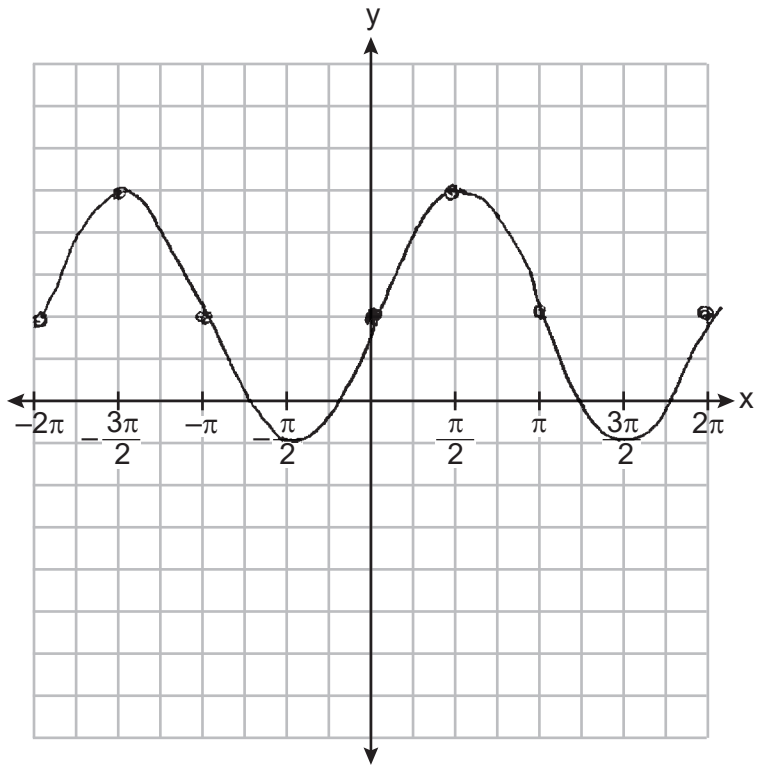
Based on your graph, state an interval in which the graph is increasing.

Interval = $0 \rightarrow \frac{\pi}{4}$

Score 3: The student made one graphing error.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.



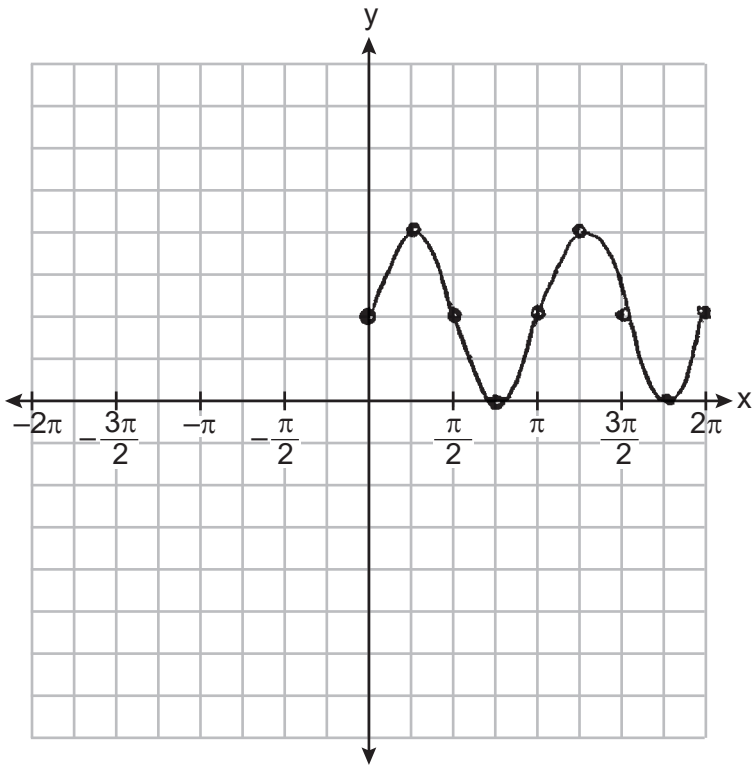
Based on your graph, state an interval in which the graph is increasing.

The graph is increasing from -2π to $-\frac{3\pi}{2}$, $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, and $\frac{3\pi}{2}$ to 2π.

Score 3: The student made one graphing error.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.



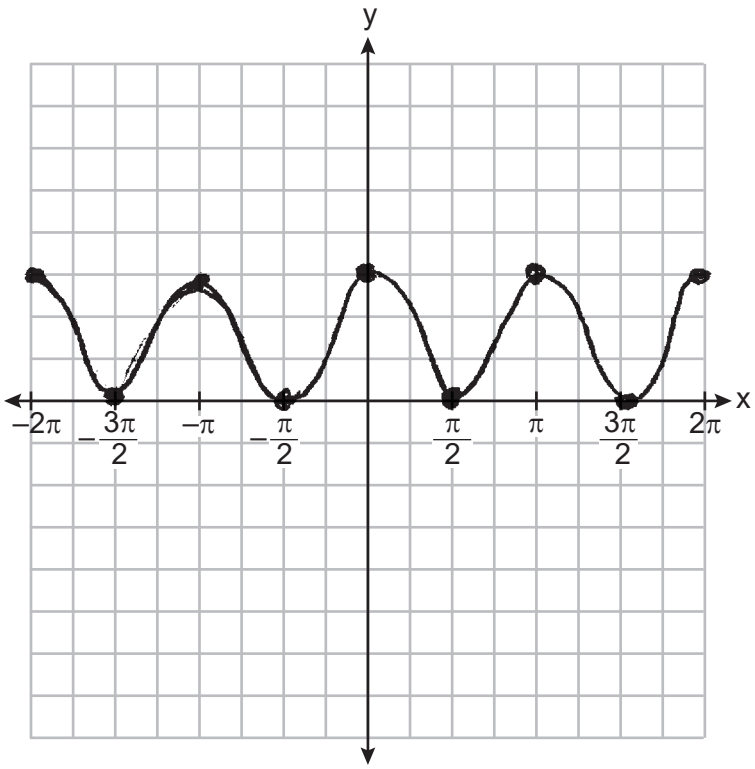
Based on your graph, state an interval in which the graph is increasing.

{x | 0 ≤ x ≤ 1}

Score 2: The student made one graphing error and stated an incorrect interval.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.



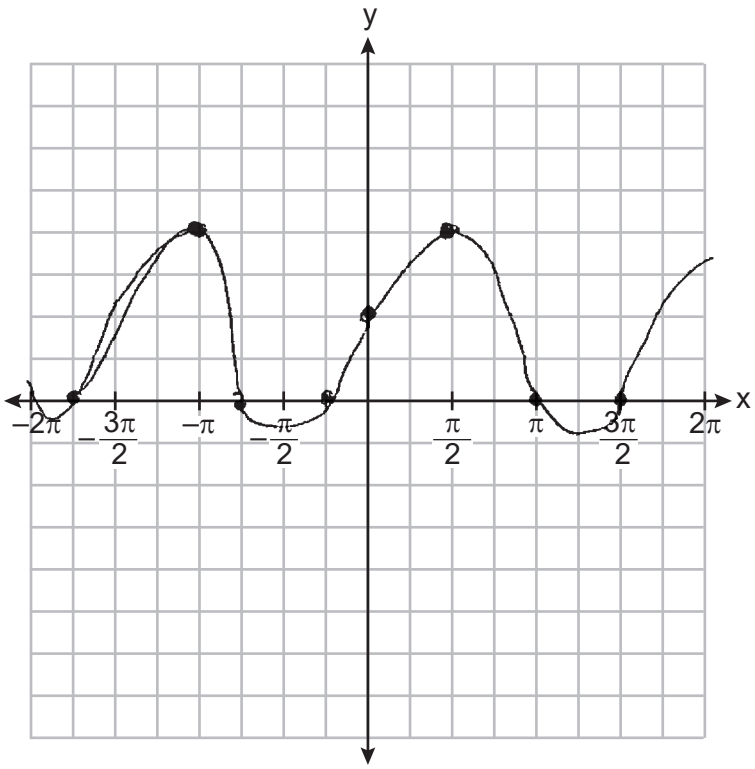
Based on your graph, state an interval in which the graph is increasing.

(π/2 to π)

Score 1: The student stated a correct interval only.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.



Based on your graph, state an interval in which the graph is increasing.

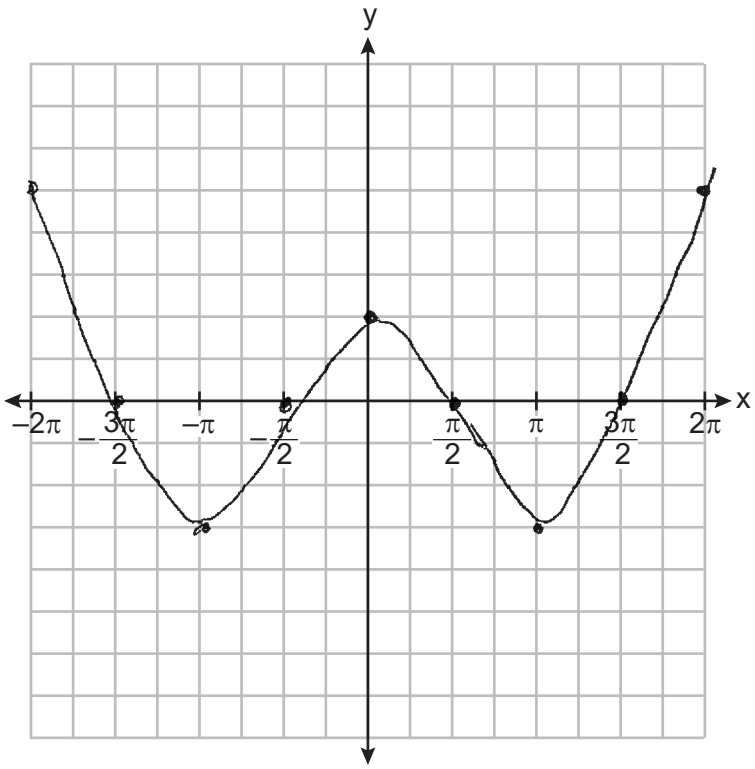
The graph is increasing on interval 1.

Score 0: The student did not show enough correct work to receive any credit.

Question 36

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.

A=3
D=2



Based on your graph, state an interval in which the graph is increasing.

(−π, ∞)

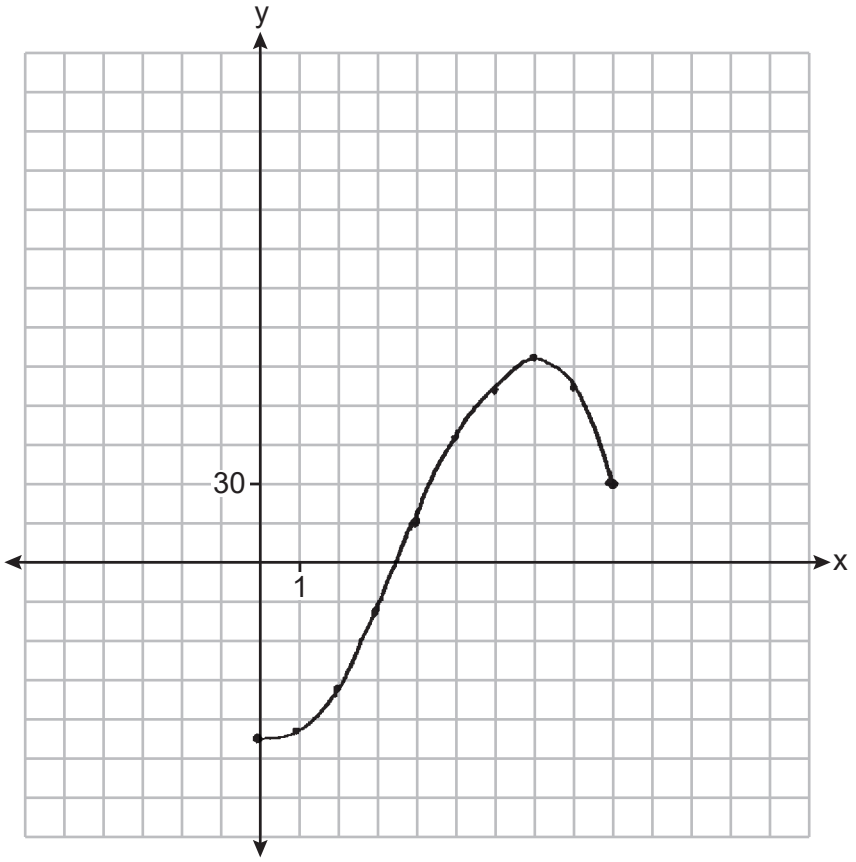
Score 0: The student did not show enough correct work to receive any credit.

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(7, 78) if 7,000 sweatshirts are sold, then profit is maximized at $78,000.

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

3,549 sweatshirts.

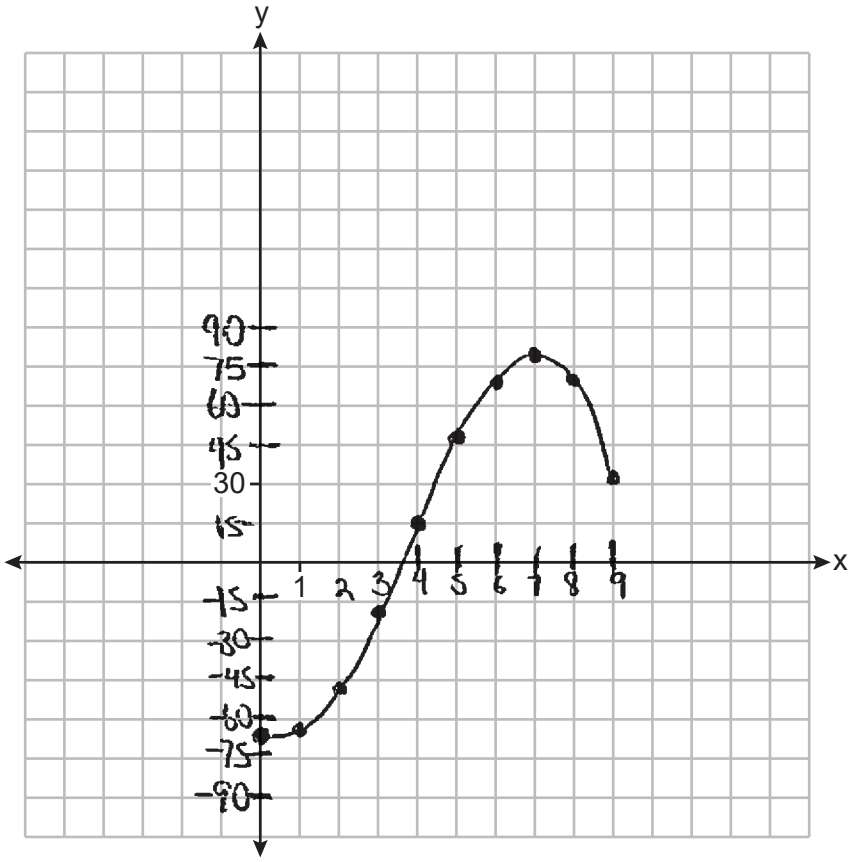
when $x = 3.548$, $p(x)$ is negative, meaning that a positive profit is not made when 3,548 sweatshirts are produced. 3,549 is the first x value where $p(x)$ is positive, making the minimum amount produced for a profit needs to be 3,549 whole sweatshirts

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



x	y
0	-69
1	-66
2	-47
3	-18
4	15
5	46
6	69
7	78
8	67
9	30

Question 37 is continued on the next page.

Score 5: The student made an error in units in the last part.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

2nd calc max
(6.969697, 77.990845)

(7, 78) is the local maximum of p.
If 7000 hooded sweatshirts are sold, the manufactur will profit with 78,000 dollars.

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

y=0 p(x)
2nd calc intersect (3.5488398, 0)

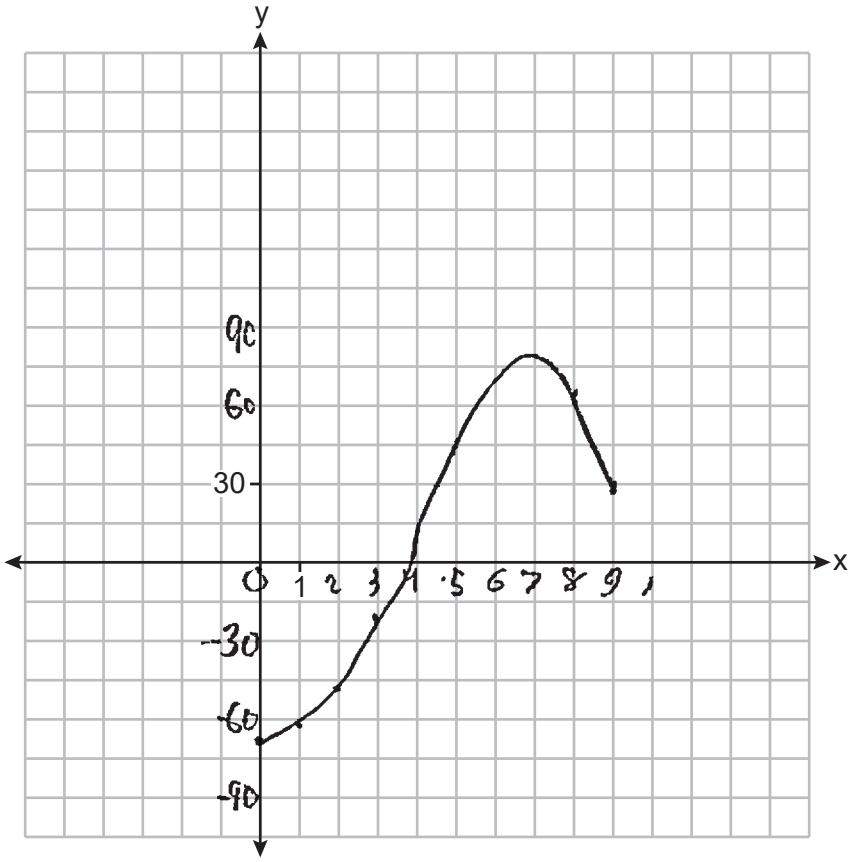
4 sweatshirts

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 5: The student made an error in units throughout the problem.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(7, 78) for every 7 sweatshirts sold, the profit is $78

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

4 sweatshirts. $P(x) = -x^3 + 11x^2 - 7x - 69$

$P(3) = -(3)^3 + 11(3)^2 - 7(3) - 69$

$P(3) = -27 + 99 - 21 - 69$

$P(3) = -18$

$P(4) = -64 + 176 - 28 - 69$

$P(4) = 15$

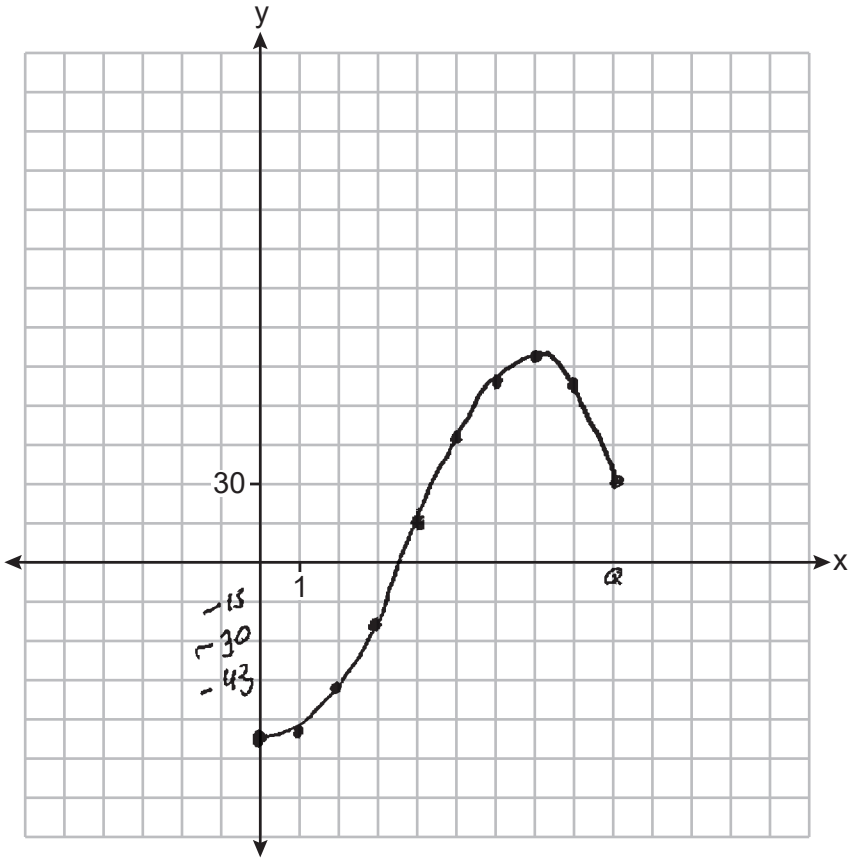
4 is the lowest whole integer that makes profit since 3 is negative, but 4 is positive.

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 4: The student stated the wrong coordinates and did not justify their answer to the last question.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(78, 7) If 7,000 sweatshirts are made and sold, there will be $78,000 made at most.

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

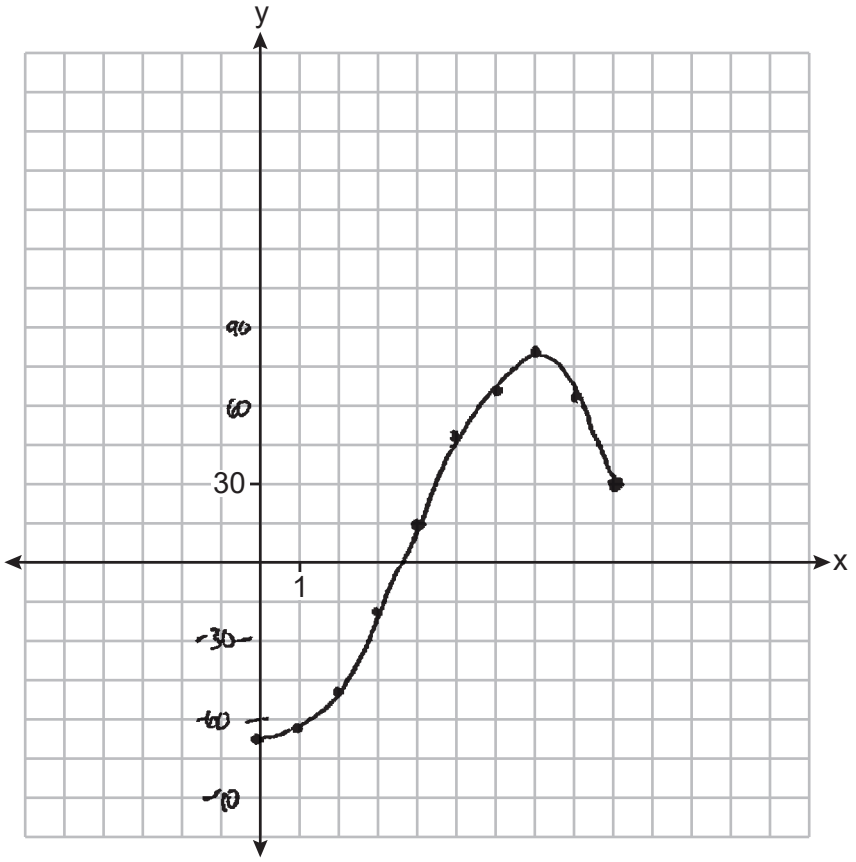
3,549 sweatshirts

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 3: The student received credit for the graph and stating the coordinates of the point.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(7, 78), this point represents that producing 7,000 sweatshirts is most profitable for the manufacturer.

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

$-x^3+11x^2-7x-69 > 0$

$x^3-11x^2+7x+69 < 0$

$x^3-11x^2+7x < -69$

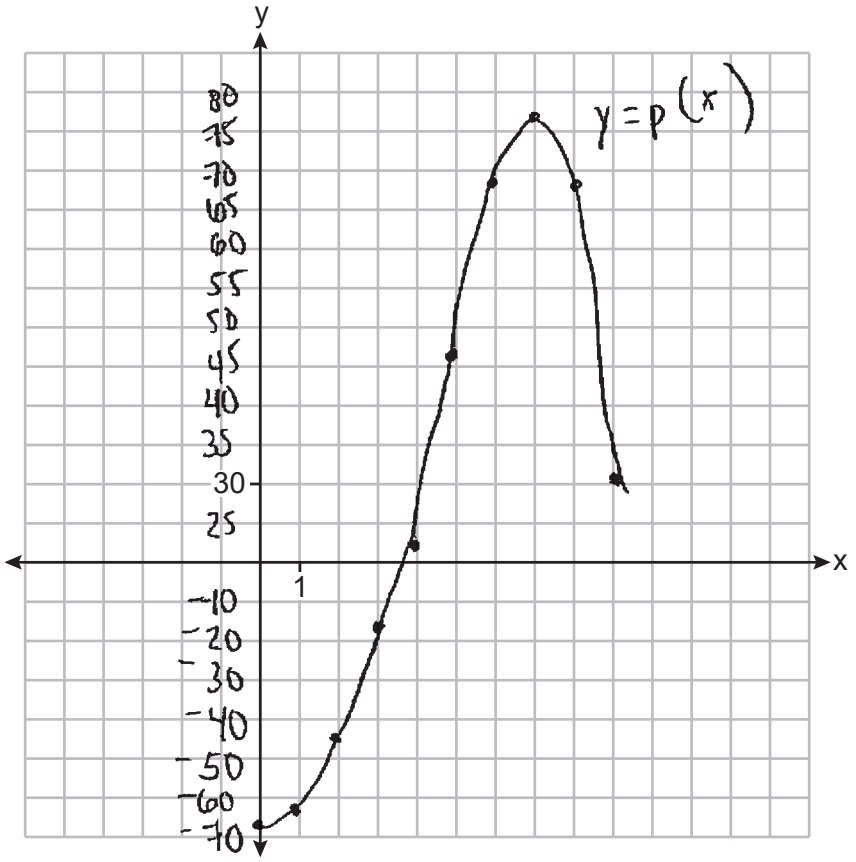
$x(x^2-11x+7) < -69$

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 3: The student made one graphing error and received no credit for the last part.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(7,78)

When there are 7,100 sweatshirts sold, $78,000 are made in profit

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

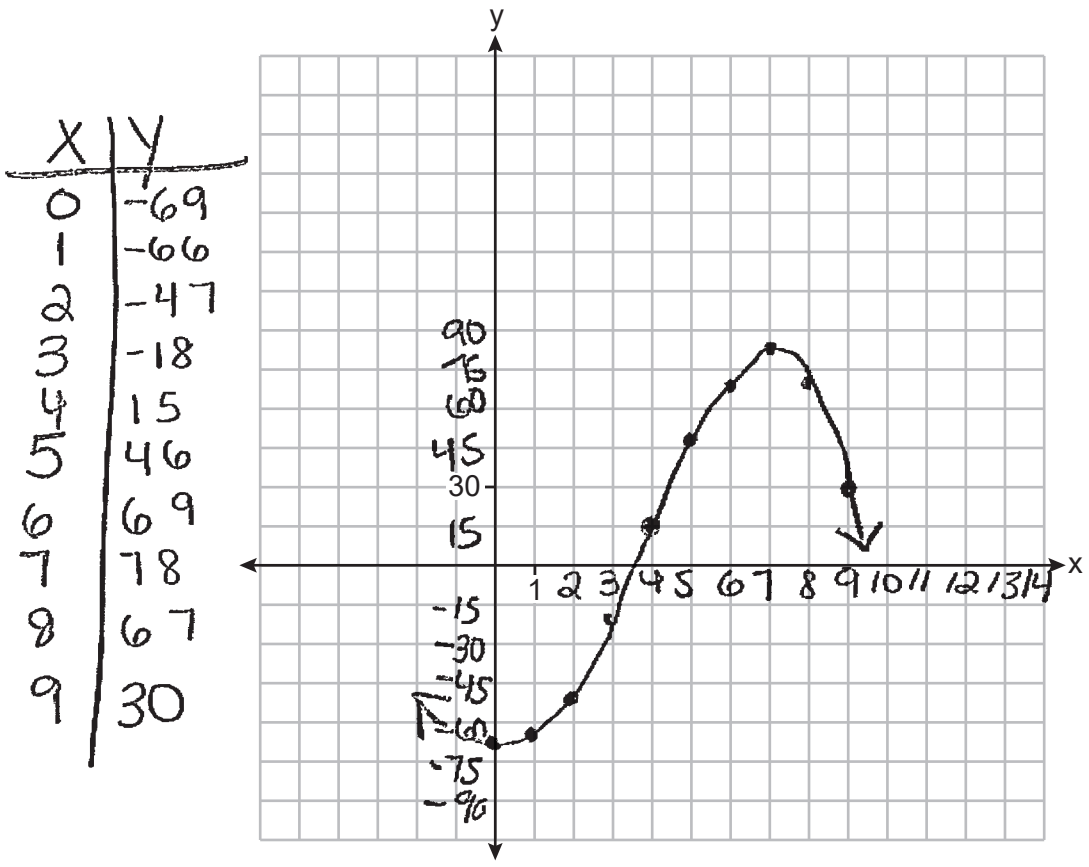
4000 sweatshirts

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 2: The student received one point for the graph and one for stating the coordinates.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer.* Explain what this point represents in terms of the number of sweatshirts sold and profit.

MAX=(7, 78) every 7 sweatshirts has a 78% profit

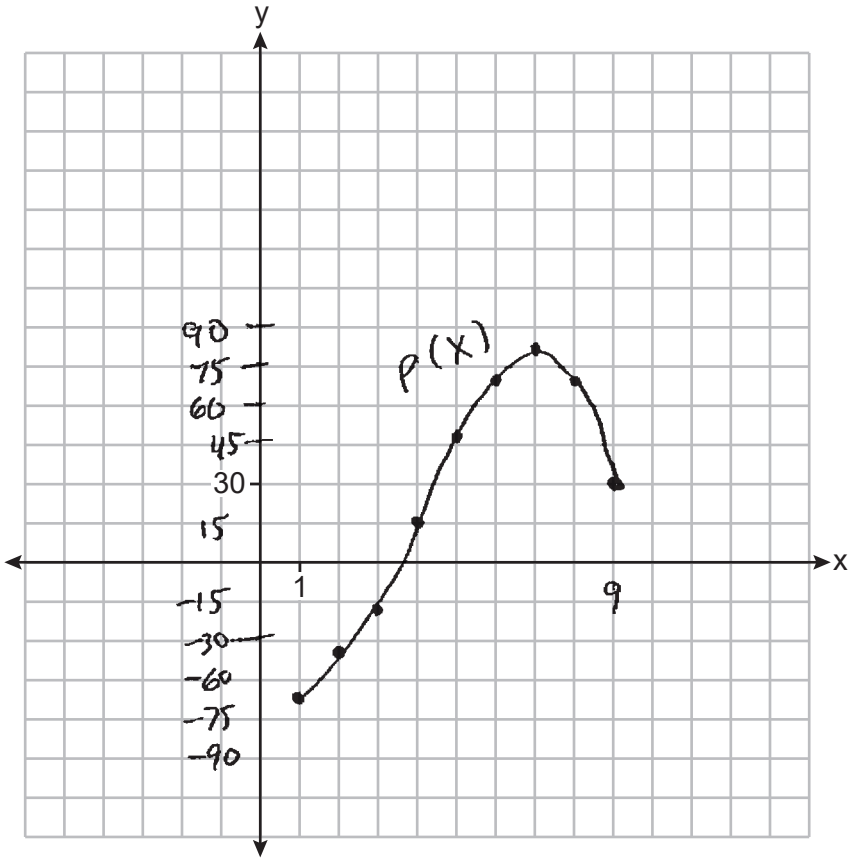
Determine how many sweatshirts, to the *nearest whole sweatshirt,* the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 1: The student received one point for stating the coordinates.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(7, 78)

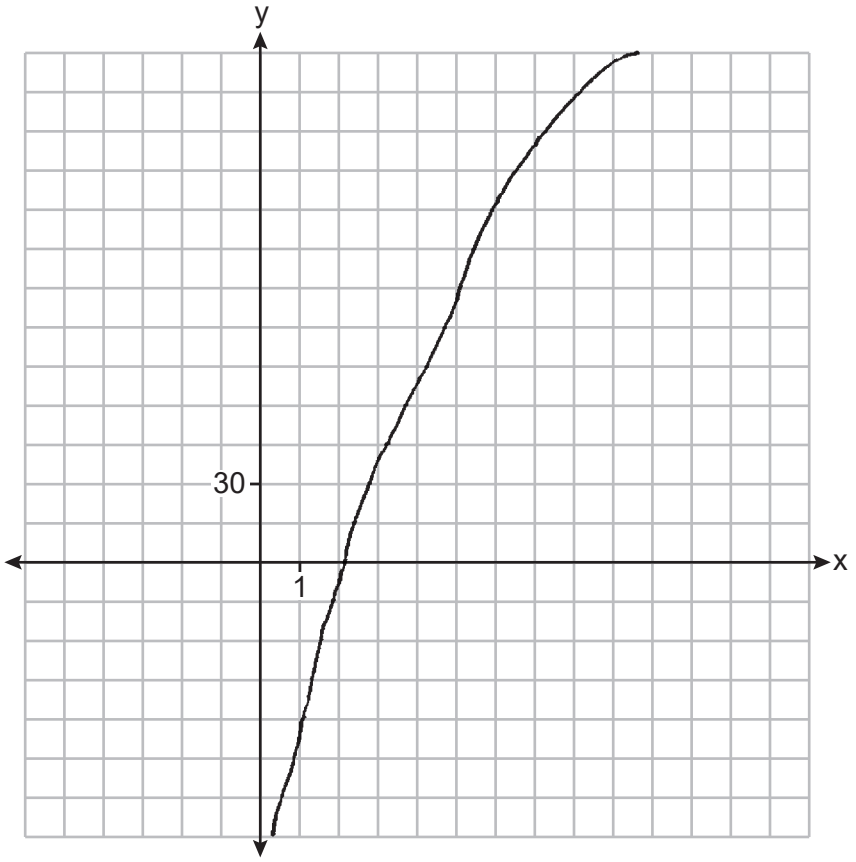
Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

Question 37

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \leq x \leq 9$, on the set of axes below.



Question 37 is continued on the next page.

Score 0: The student showed no correct work.

Question 37

Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

(9, 1488)

All values: 4155

The number of sweatshirts sold made tremendous profit.

-69 1091
-64 1488
-31
36
143
296
501
764

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

36 sweatshirts