The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m.

MODEL RESPONSE SET

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Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

LCD: $3x$

$$3x\left(\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}\right)$$

$$3 - x = -1$$
$$-3 \qquad\qquad -3$$

$$\frac{-x}{-1} = \frac{-4}{-1}$$

$$x = 4$$

Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{3}{3}\left(\frac{1}{x}\right)\left(-\frac{1}{3}\right)\frac{x}{x} = \frac{-1}{3x}$$

$$\frac{3}{3x} - \frac{1x}{3x} = \frac{-1}{3x}$$

$$3 - 1x = -1$$
$$-3 \qquad\quad -3$$

$$\frac{-1x}{-1} = \frac{-4}{-1}$$

$$x = 4$$

Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{(3)1}{(3)x} - \frac{(x)1}{(x)3} = \frac{-1}{3x}$$

$$\frac{3}{3x} - \frac{x}{3x} = \frac{-1}{3x}$$

$$\frac{3-x}{3x} = \frac{-1}{3x}$$

Score 1: The student only found a common denominator and combined like terms.

Question 25

25 Solve for x: $\frac{1}{\cancel{x}} - \frac{1}{\cancel{3}} = -\frac{1}{\cancel{3x}}$

$3x - x = -1$

$2x = -1$

$x = \boxed{-\frac{1}{2}}$

Score 1: The student made an error reducing the first term.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{1}{x} - \frac{1}{3} = \frac{1}{3x}$$

$$\frac{0}{x-3} \times \frac{1}{3x}$$

$$0 = x - 3$$

$$x = 3$$

Score 0: The student made an error combining the fractions, and also made a transcription error by omitting the negative.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

Randomly separate 10 volunteers into two groups. Have 5 people try a toothpaste with ingredient x + have 5 people try one without it.

Score 2: The student wrote a correct description of a controlled experiment, including random assignment and a control group.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

I would collect two groups of individuals that are of equal age and sex to ensure accuracy and eliminate any other variables that can have an effect. I would use a large group of people say 40 in each. Then, I would give one random group an equal amount of toothpaste with the ingredient, where as the other random group will recieve toothpaste with no ingredient. It will be given in the morning at the same time. By the end of the day at the same time for a week, I will record the results to determine the impact of the ingredient

Score 2: The student gave a complete and correct response.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

One group of people will use the version with ingredient X and another will use the toothpaste without. Compare the results.

Score 1: The student wrote an incomplete description by omitting the random assignment of two groups.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

A controlled experiment can be used by distributing products with the ingredients to a group, while giving the control group to ~~[illegible]~~ a different group of people.

Score 0: The student's response lacked random assignment and had an insufficient explanation of a control group.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$$x - 5 = 0$$
$$x = 5$$
$$2(5)^3 - 4(5)^2 - 7(5) - 10 \stackrel{?}{=} 0$$
$$250 - 100 - 35 - 10 \stackrel{?}{=} 0$$
$$105 \neq 0$$

$x-5$ is not a factor of $2x^3-4x^2-7x-10$.
If $x-5$ is a factor of $2x^3-4x^2-7x-10$, then when $2x^3-4x^2-7x-10$ and 5 is substituted for x, the value of $2x^3-4x^2-7x-10$ should be 0.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$$\begin{array}{r|rrrr} & 2 & -4 & -7 & -10 \\ 5 & \downarrow & 10 & 30 & 115 \\ \hline & 2 & 6 & 23 & 105 \end{array}$$

(x-5) is not a factor because the last value (105) does not equal 0

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

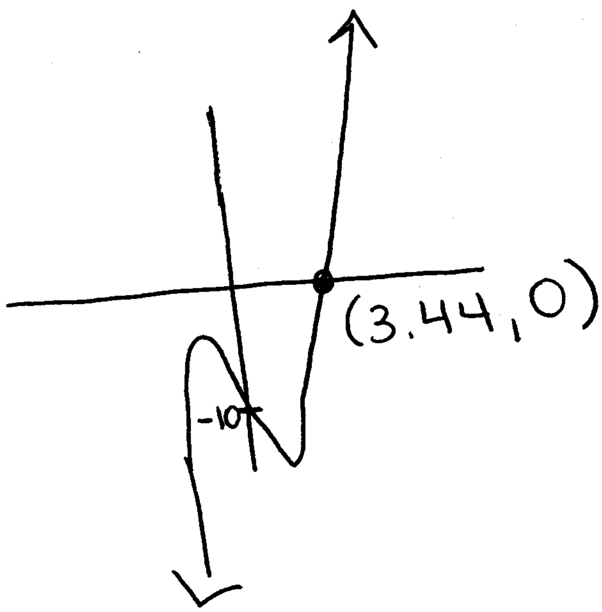
$$
\begin{array}{r}
2x^2 + 6x + 23 \\
x - 5 \enclose{longdiv}{2x^3 - 4x^2 - 7x - 10} \\
\underline{-2x^3 + 10x^2} \qquad\qquad \\
6x^2 - 7x \qquad \\
\underline{-6x^2 + 30x} \qquad \\
23x - 10 \\
\underline{-23x + 115} \\
105 = R
\end{array}
$$

x − 5 is not a factor because it did not divide evenly out of $2x^3 - 4x^2 - 7x - 10$.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.



There is one root @ 3.44.
Since there is no root @ 5, x-5
is not a factor.

Score 2: The student gave a complete and correct response.

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10 = 0$

$105 \neq 0$

x-5 is not a factor.

Score 1: The student wrote no explanation.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$$2(-5)^3 - 4(-5)^2 - 7(-5) - 10 = 0$$

$$-325 \neq 0$$

x-5 is not a factor because when you use the remainder theorem the remainder is -325 not 0.

Score 1: The student made one error by substituting -5 instead of 5.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

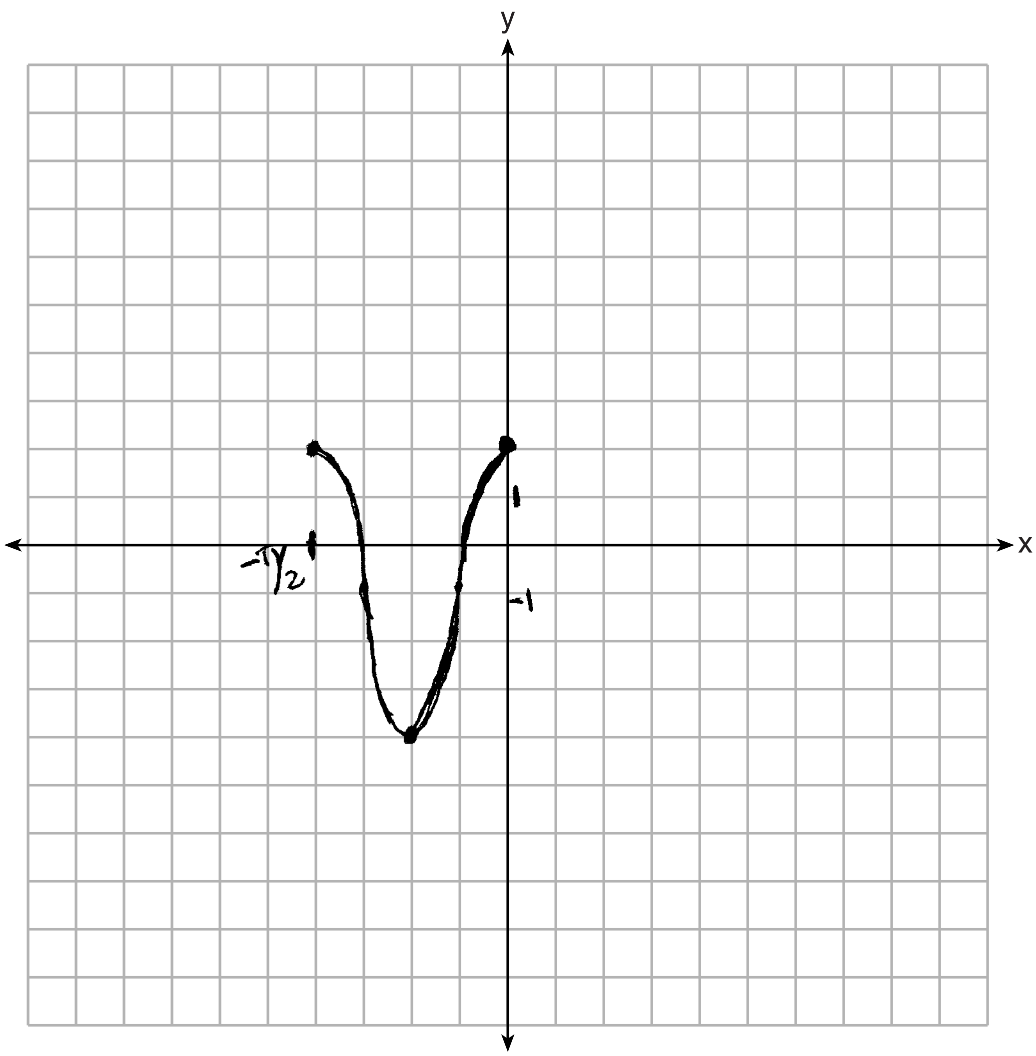
$$
\begin{array}{r}
2x^2 - 6x + 23 \\
x - 5 \enclose{longdiv}{2x^3 - 4x^2 - 7x - 10} \\
2x^3 - 10x^2 \\
6x^2 - 7x - 10 \\
6x^2 - 30x \\
23x - 10
\end{array}
$$

No

Score 0: The student made multiple errors dividing and did not provide the explanation.

Question 28

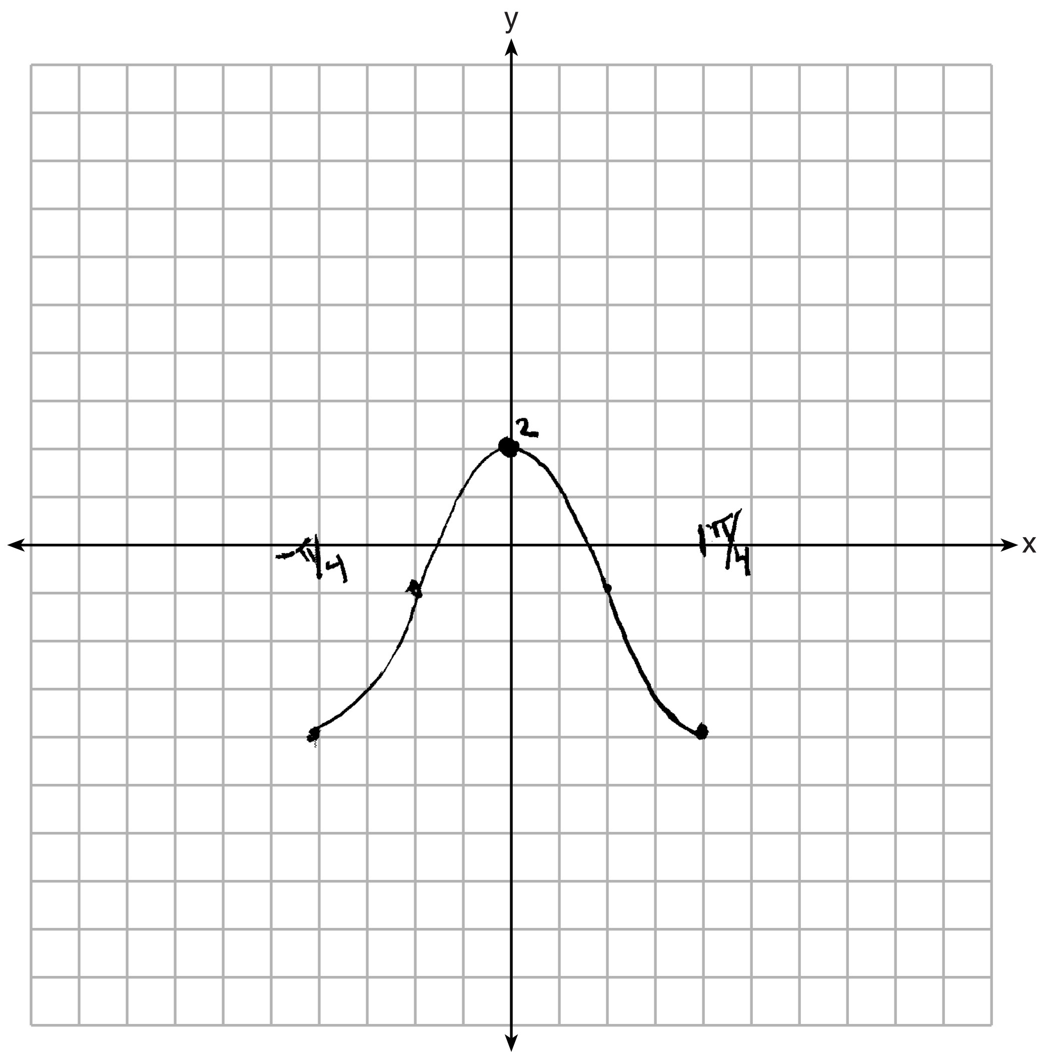
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

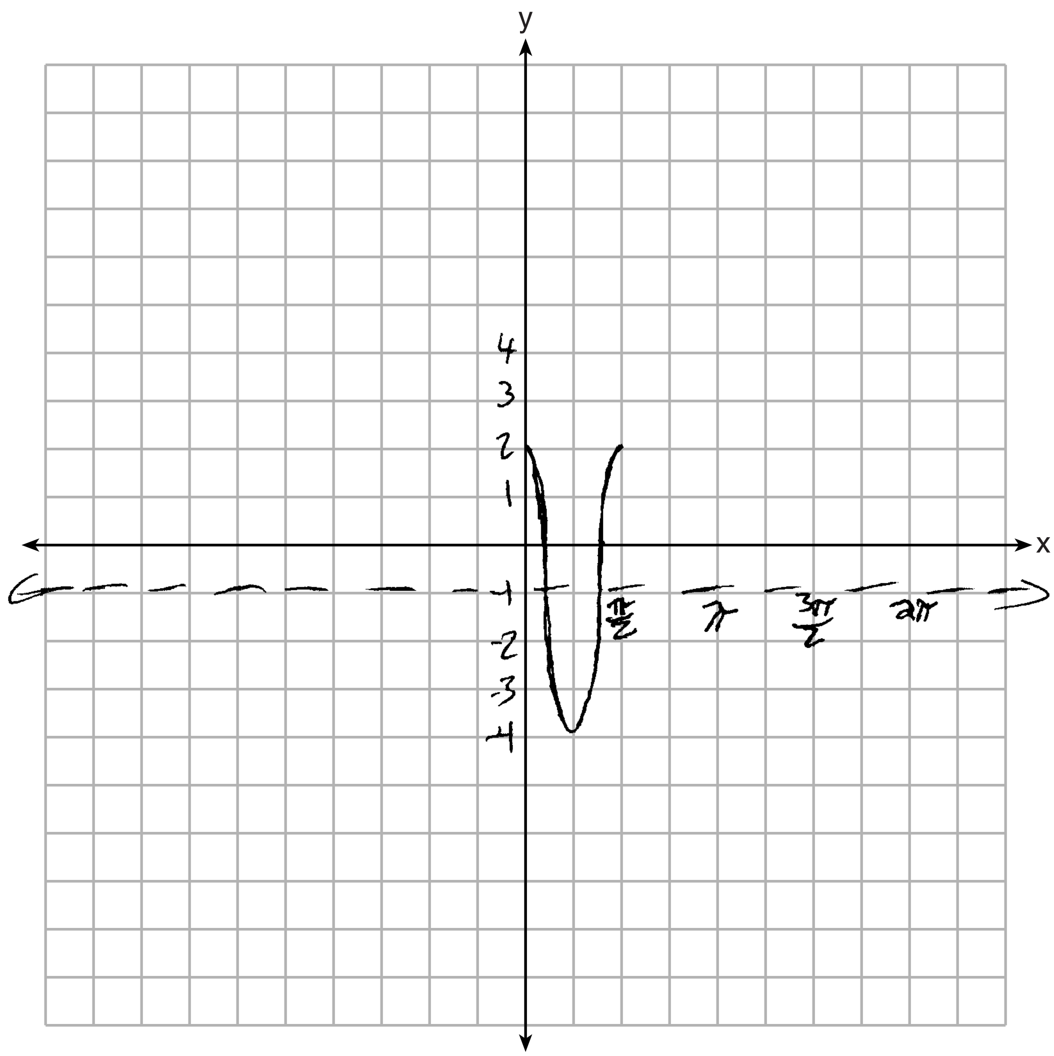
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

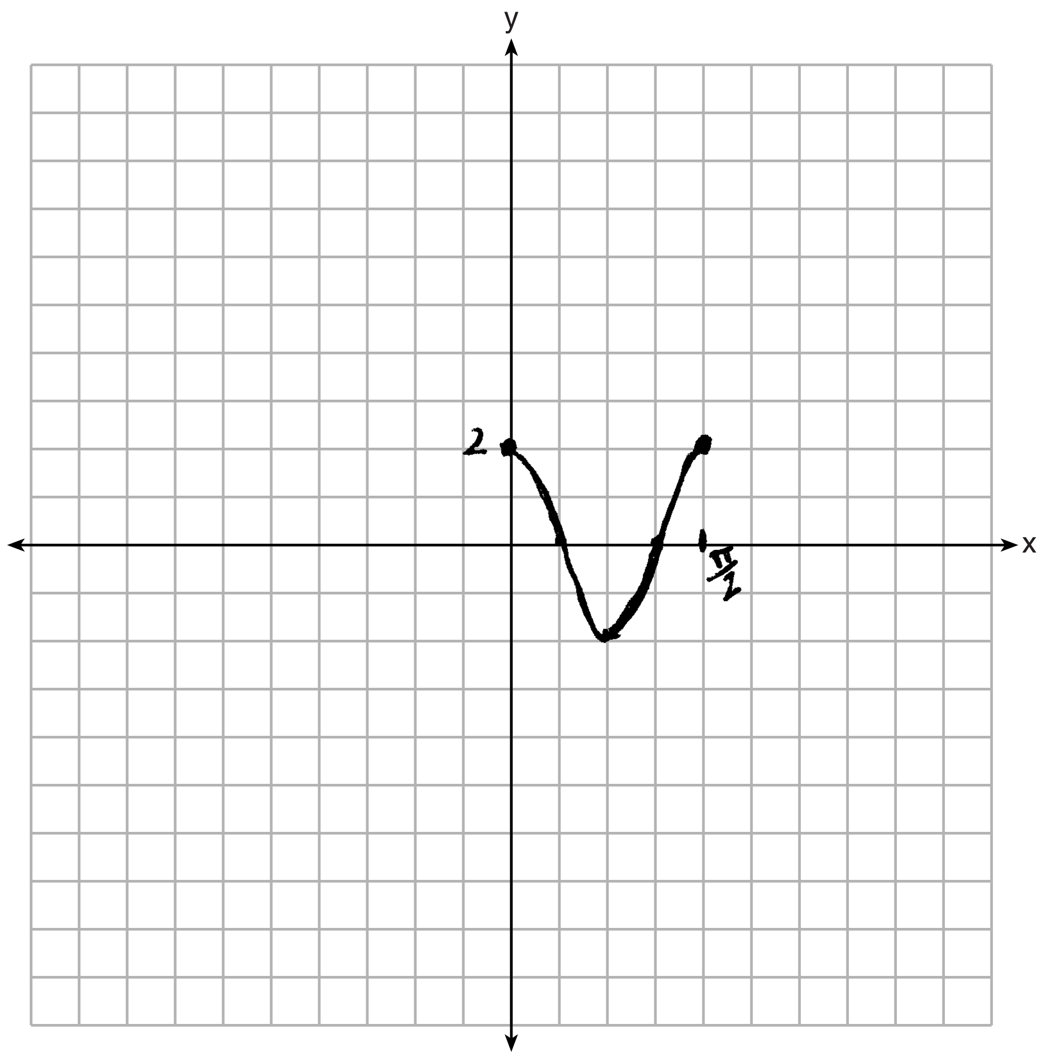
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

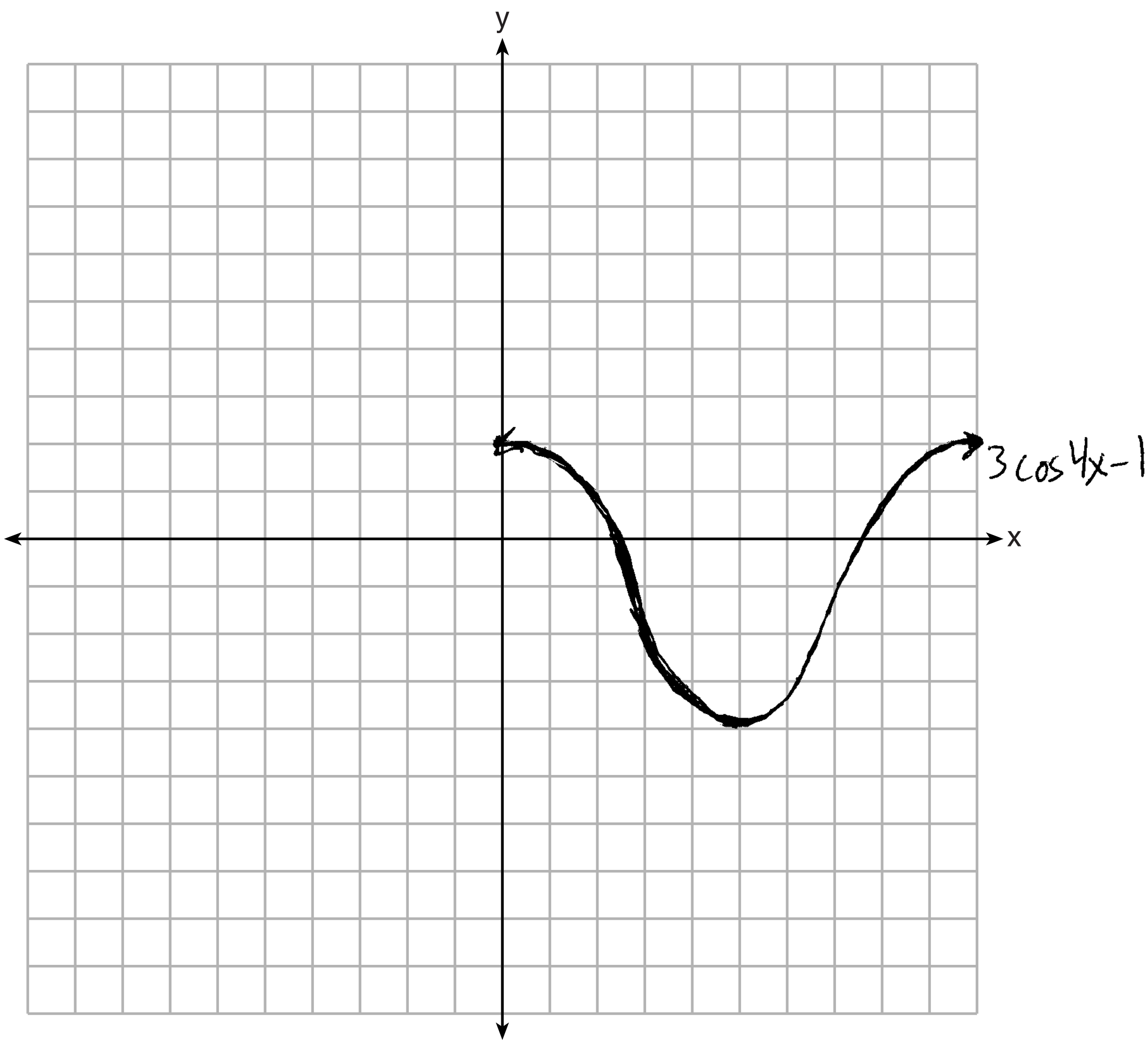
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student correctly graphed one cycle of a cosine function passing through (0,2) with period $\frac{\pi}{2}$, but used an incorrect amplitude that affected the midline.

Question 28

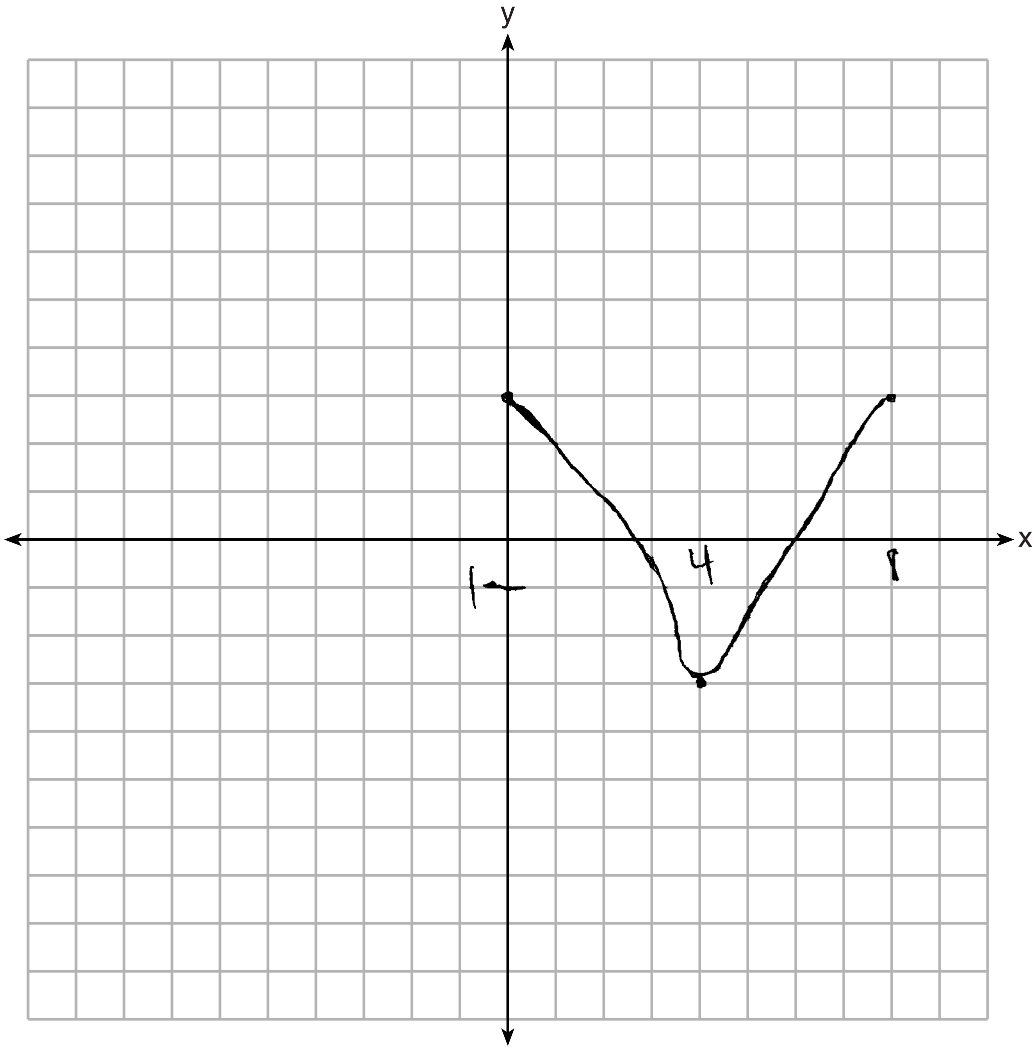
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student did not label the axes with appropriate values.

Question 28

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 0: The student made multiple errors.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$649 + 433$

$1082 - 974$

$\frac{108}{1376}$

Score 2: The student gave a complete and correct response.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$$\frac{649}{1376} + \frac{433}{1376}$$

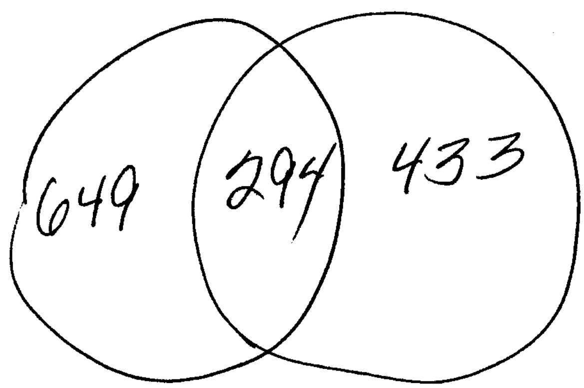
$$\begin{array}{r} 649 \\ +433 \\ \hline 1082 \end{array}$$

$$\frac{1082}{1376}$$

Score 1: The student made an error by not subtracting from $\frac{974}{1376}$.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?



$$\begin{array}{r} 1376 \\ -1082 \\ \hline 294 \end{array}$$

$$\frac{294}{974}$$

Score 0: The student made multiple errors.

Question 30

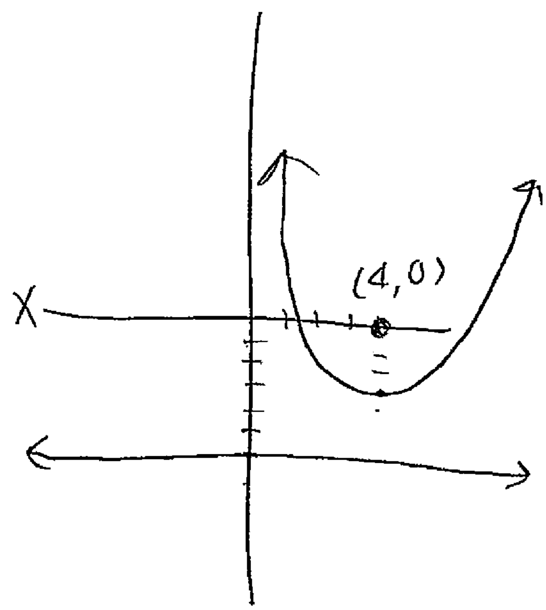
30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$$12y + 36 = x^2 - 8x + 16$$
$$-36 \qquad\qquad + 36$$
$$12y = x^2 - 8x + 20$$
$$y = \frac{x^2}{12} - \frac{2x}{3} - \frac{5}{3}$$

$$x = \frac{-b}{2a} = \frac{-\frac{2}{3}}{24} = \frac{2}{72} = \frac{1}{}$$

$$(x-4)^2 = \underset{\substack{\uparrow \\ 4(3)}}{12}(y+3)$$

$$\boxed{(4, 0)}$$



Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

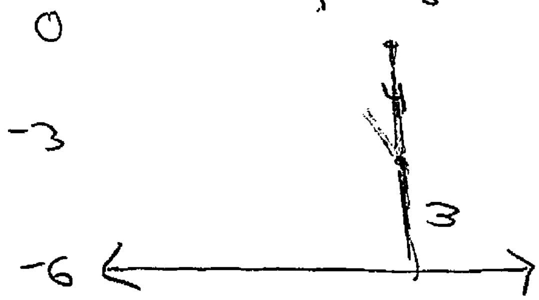
$y+3=\frac{1}{12}(x-4)^2$

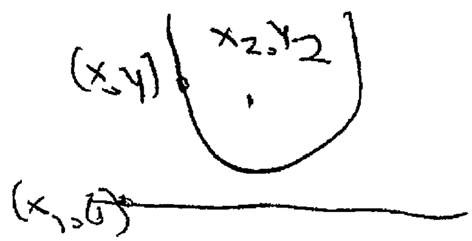
$y-k=a(x-h)^2$

vertex $(4,-3)$

directix $y=-6$

$\boxed{(4,0)}$

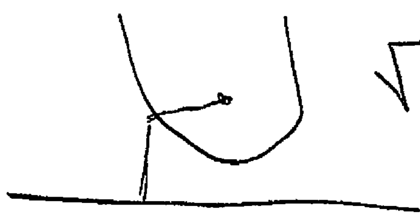




$(4,0)$

$d=\sqrt{(x-x_1)^2+(x^2+6)^2}$

$d=\sqrt{(x-x_2)^2+(x^2-y_2)}^2$



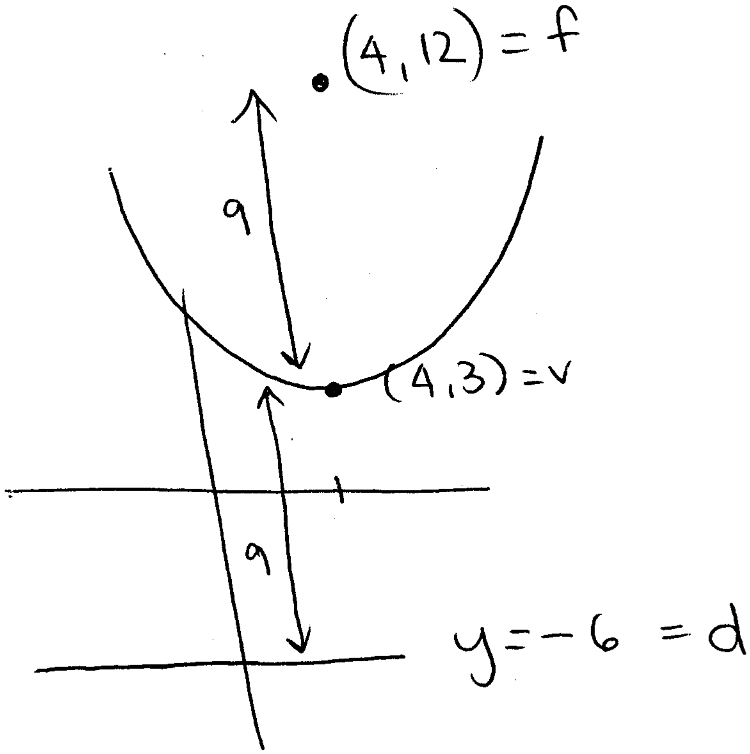
$\sqrt{\quad} = \sqrt{\quad}$

~~$(4,-3)$ $(4-x_2)^2+((-3)^2-y_2)^2$~~

Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.



$\boxed{(4, 12)}$

Score 1: The student found an incorrect vertex.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Vertex = (4, −3)

since directrix is $y = -6$

need to add 6 to y in vertex

Focus = (4, 3)

Score 1: The student misused the directrix.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$(x-4)(x-4)$

$x^2-4x-4x+16$

$x^2-8x+16$

$12(y+3)=(x-4)^2$

$12y+\cancel{36}=x^2-8x+16$

$\cancel{-36} \qquad -36$

$\frac{\cancel{12}y}{\cancel{12}}=\frac{x^2-8x-20}{12}$

$y=\frac{x^2-8x-20}{12}$

Focus : $(4,-3)$

Score 0: The student stated the vertex as the focus.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$$12y + 36 = x^2 - 8x + 16$$

$$\frac{\cancel{12}y}{\cancel{12}} = \frac{x^2 - 8x - 20}{12}$$

$$y = \frac{x^2 - 8x - 20}{12}$$

Focus: $(4, 0)$

$(-8, 0)$

Score 0: The student stated a partially correct answer that was obtained by an incorrect procedure.

Question 31

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+9-1+1}{x^3+8}$$

$$\frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$1 + \frac{1}{x^3+8} \quad \checkmark$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

$$1 + \frac{1}{x^3+8}$$

$$x^3 + 8 \overline{\smash{)}\, x^3 + 0x^2 + 0x + 9}$$

$$\underline{x^3 \qquad\qquad\qquad + 8}$$

$$1$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} \stackrel{?}{=} 1 + \frac{1}{x^3+8}$$

$$= (1)\left(\frac{x^3+8}{x^3+8}\right) + \frac{1}{x^3+8}$$

$$= \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{\cancel{x^3+8}} = \frac{(x^3+8)}{\cancel{(x^3+8)}1} + \frac{1}{\cancel{x^3+8}}$$

$$x^3+9 = x^3+8+1$$
$$-x^3 \quad -x^3$$

$$9 = 8+1$$

$$\boxed{9=9}$$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

MULTIPLY BY COMMON DENOMINATOR : x^3+8

$$(x^3+8)\left(\frac{x^3+9}{x^3+8}\right) = \left(1 + \frac{1}{x^3+8}\right)(x^3+8)$$

$$x^3+9 = x^3+8+1$$

$$x^3+9 = x^3+9$$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

let $x = 2$

$$\frac{2^3+9}{2^3+8} = \frac{8+9}{8+8} = \frac{17}{16}$$

$$1 + \frac{1}{2^3+8} = 1 + \frac{1}{8+8} = \frac{16}{16} + \frac{1}{16}$$

$$= \frac{17}{16}$$

$$\frac{17}{16} = \frac{17}{16}$$

Score 0: The student used an incorrect procedure by substituting a single value in for x.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$$\frac{135{,}000}{100{,}000} = \frac{100{,}000(1+x)^5}{100{,}000}$$

$$1.35 = (1+x)^5$$

$$\sqrt[5]{1.35} = \sqrt[5]{(1+x)^5}$$

$$1.061858759 = 1 + x$$

$$-1 \qquad\qquad -1$$

$$.061858759 = x$$

$$x = \boxed{6\%}$$

Score 2: The student gave a complete and correct response.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$$A = A_0 e^{k(t-t_0)} + B_0$$

$$A_t = A_0(1+r)^t$$

$$135{,}000 = 100{,}000(1+r)^5$$

$$\frac{27}{20} = (1+r)^5$$

$$\sqrt[5]{\frac{27}{20}} = 1+r$$

Score 1: The student wrote an incomplete solution.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$$(100{,}000)(x)^5 = 135000$$

$$x^5 = .\frac{27}{20}$$

$$x = \sqrt[5]{\frac{27}{20}}$$

$$x = 1.06$$

Growth Rate = $\boxed{1.06}$

Score 1: The student found the growth factor correctly, but incorrectly stated the annual growth rate percentage.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$$\frac{135000}{100000} = \frac{100000(x)^5}{100000}$$

$$\sqrt[5]{x^5} = \sqrt[5]{1.35}$$

$$x = 1.\underline{0}61858759$$

$$\boxed{1.1}$$

Score 1: The student found the growth factor correctly, but stated an incorrect annual growth rate percentage.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$$135000 = 100000(1+r)^5$$

$$35000 = (1+r)^5$$

$$1.54 = 5\log(1+r)$$

$$.31 = \log(1+r)$$

$$10^{.31} = 10^{\log(1+r)}$$

$$2.042 = 1+r$$

$$1.042 = r$$

Score 0: The student made an error by subtracting 100,000 and did not state a percentage.

Question 33

33 Solve the system of equations shown below algebraically.

$$\begin{cases} (x-3)^2 + (y+2)^2 = 16 & ① \\ 2x + 2y = 10 & ② \end{cases}$$

② : $2x+2y=10$

$2y=10-2x$

$y=5-x$ ③

put ③ to ① : $(x-3)^2+(5-x+2)^2=16$

$x^2-6x+9+49-14x+x^2=16$

$2x^2-20x+58=16$

$2x^2-20x+42=0$

$x^2-10x+21=0$

-3, -7

$(x-3)(x-7)=0$

$x_1=3 \quad x_2=7$

$y_1=2 \quad y_2=-2$

$2\times3+2y=16$

$2y=4$

$y=2$

$2\times7+2y=10$

$2y=-4$

$y=-2$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x-3)^2 + (y+2)^2 = 16$$
$$2x + 2y = 10 \rightarrow 2y = -2x + 10$$

$y = -1x + 5$

$(x-3)^2 + (y+2)^2 = 16$

$(x-3)(x-3) + (-1x+5+2)(-1x+5+2) = 16$

$x^2 \boxed{-6x} +9 + 1x^2 \boxed{-5x} \boxed{-2x} \boxed{-5x} +25 +10 \boxed{-2x} +10 +4 = 16$

$2x^2 - 20x + 58 = 16$
$\quad -16 \quad -16$

$2x^2 - 20x + 42 = 0$

$2x^2 - 14x - 6x + 42 = 0$

$2x(x-7) - 6(x-7) = 0$

$2x - 6 = 0 \quad x - 7 = 0$

$x = 3 \quad x = 7$

84; −6, −14; −20

$y = -1(3) + 5$
$y = -3 + 5$
$y = 2$
$(3, 2)$

$y = -1(7) + 5$
$y = -7 + 5$
$y = -2$
$(7, -2)$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x + 2y = 10$
$x + y = 5$
$y = 5 - x$

$(x-3)^2 + (y+2)^2 = 16$
$x^2 - 6x + 9 + y^2 + 4y + 4 = 16$
$x^2 - 6x + 9 + (5-x)^2 + 4(5-x) + 4 = 16$
$x^2 - 6x + 9 + 25 - 10x + x^2 + 20 - 4x + 4 = 16$
$2x^2 - 20x + 42 = 0$
$x^2 - 10x + 21 = 0$
$(x-7)(x-3) = 0$
$x = 7 \quad x = 3$

58
-16
42

Score 3: The student only found the correct x-values of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2(x+y)=10$

$x+y=5$

$y=5-x$

$x=5-y$

$\cancel{y=5-x}$

$\cancel{y=5-10}$

$\cancel{y=-5}$

$\cancel{(x-3)^2+(y+2)^2=16}$

$\cancel{(x-3)^2+(5-x+2)^2=16}$

$\cancel{(x-3)^2+(-x+7)^2=16}$

$\cancel{(x^2-6x+9)+(x^2-7x-7x+49)^2=16}$

$\cancel{(x^2-6x+9)+(x^2-14x+49)=16}$

$\cancel{2x^2-20x+58=16}$

$\cancel{x^2-20x+100=16-16+100}$

$\cancel{(x-10)^2=0}$

$\cancel{(x-10)(x-10)=0}$

$\cancel{x=10}$

$(-x+7)^2$

$(-x+7)(-x+7)$

$(5-y-3)^2$

$(-y+2)^2+(y+2)^2=16$

$(y^2-4y+4)+(y^2+4y+4)=16$

$2y^2+8=16$

$2y^2=8$

$y^2=4$

$\boxed{y=2}$

$2x+2(2)=10$

$2x+4=10$

$2x=6$

$\boxed{x=3}$

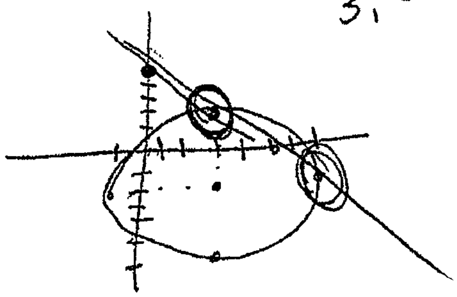
Score 3: The student found only one correct solution of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

3, -2



$$\begin{array}{l} -2x \quad -2x \\ \frac{2y}{2} = \frac{10 - 2x}{2} \\ y = 5 - x \end{array}$$

$$\boxed{\{(3,2),(7,-2)\}}$$

Score 2: The student obtained the correct solution, but used a method other than algebraic.

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2y=10-2x$

$y=5-x$

$(x-3)^2+(y+2)^2=16$

$x^2-3x-3x+9+y^2+2y+2y+4=16$

$x^2-6x+9+y^2+4y+4=16$

$x^2-6x+y^2+4y+13=16$

$y^2=-x^2+6x-4y+3$

$(5-x)^2=-x^2+6x-4(5-x)+3$

$5-x \quad 5-x$

$x^2-10x+25=-x^2+6x-20+4x+3$

$2x^2+45=10x+3$

$x^2-5x+21=0$

$x=\frac{5\pm\sqrt{25-4(1)(21)}}{2}$

$x=\frac{5\pm i\sqrt{59}}{2}$

Score 2: The student made a transcription error by losing a $-10x$, and did not find y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$(x-3)(x-3) + ((5-x)+2)((5-x)+2) = 16$

$x^2 - 3x - 3x + 9 + (3-x)(3-x) = 16$

$\cancel{x^2} \cancel{-6x} \cancel{+9} + 9 \cancel{-3x} \cancel{-3x} \cancel{+x^2} = 16$

$x^2 - 12x + 18 = 16$

$x^2 - 12x + 2 = 0$

$\frac{+12 \pm \sqrt{(-12)^2 - 4(2)(1)}}{2(1)} = \frac{12 \pm \overset{2\sqrt{35}}{\sqrt{140}}}{2} = 6 \pm \sqrt{35} = x$

$\frac{2y}{2} = \frac{10 - 2x}{2}$

$y = 5 - x$

$2(6 + \sqrt{35}) + 2y = 10$

$23.8 + 2y = 10$

$2y = -13.8$

$y = -6.92$

$2(6 - \sqrt{35}) + 2y = 10$

$.1678 + 2y = 10$

$2y = 9.8$

$y = 4.92$

$(6 + \sqrt{35}, -6.92), (6 - \sqrt{35}, 4.92)$

Score 2: The student made several computational errors.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2x=10-2y$

$x=\frac{10-2y}{2}$

$x=5-y$

$((5-y)-3)^2+(y+2)^2=16$

$(5-y)(5-y)-9+(y+2)^2=16$

$25 \cancel{-5y} \cancel{-5y} + y^2 - 9 + \cancel{y^2+4y} + 4 = 16$

$2y^2-6y+20=16$

$2y^2-6y+4=0$

$2(y^2-3y+2)=0$

$2(y-2)(y-1)=0$

$y-2=0$

$y=2$

$y-1=0$

$y=1$

$2x+2(2)=10$

$2x+4=10$

$2x=6$

$x=3$

$2x+2=10$

$2x=8$

$x=4$

$\boxed{\text{answer}= y=2,\ x=3}$

Check:

$(3-3)^2+(2+2)^2=16$

$(0^2)+(4)^2=16$

$16=16$ ✓

$(3-3)^2+(1+2)^2=16$

$(0^2)+(3^2)=16$

$9\neq16$

$(4-3)^2+(2+2)^2=16$

$(1)^2+(4^2)=16$

$17\neq16$

$(4-3)^2+(1+2)^2=16$

$(1)^2+(3)^2=16$

$1+9=16$

$10\neq16$

Score 1: The student made a conceptual error squaring the first term and did not express both ordered pairs.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x + 2y = 5$
$y = 5 + x$

$(x-3)^2 + (5+x+2)^2 = 16$
$x^2 - 6x + 9 + 49 + 14x + x^2 = 16$
$2x^2 + 8x + 42 = 0$
$x^2 + 4x + 21 = 0$
$(x+7)(x+3) = 0$
$x = 7 \mid x = 3$

Score 0: The student made several errors and did not find the y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$$x - 3 + y + 2 = 4$$
$$x + y - 1 = 4$$
$$x + y = 5$$
$$y = 5 - x$$

$$2x + 2(5 - x) = 10$$
$$2x + 10 - 2x = 10$$
$$10 = 10$$

Score 0: The student gave a completely incorrect response.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = \frac{33{,}000 - 33{,}000(1.80)}{-.04}$$

$$S_{15} = \frac{-26{,}431.14}{-.04}$$

$$S_{15} = 660{,}778.39$$

Score 4: The student gave a complete and correct response.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000 - 33000(1.04)^n}{1-1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$n = 15$

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1-1.04}$$

$$= \frac{-26431.14}{-.04}$$

$$= 660,778.50$$

Score 3: The student rounded too early.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$\boxed{S_n = \$-26432.18}$$

Score 3: The student failed to use parentheses when entering the expression into the calculator.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$Sn = \frac{33{,}000 - 33{,}000(0.04)^n}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(0.04)^{15}}{1-0.04}$$

$$\frac{33000}{0.96}$$

$$34375$$

Score 2: The student made a conceptual error interpreting the 4% increase.

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000 \cdot 1.04^n}{1-1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33{,}000 - 33{,}000 \cdot 1.04^{15}}{1-1.04} = \frac{59{,}431}{} = 1{,}485{,}775$$

Score 2: The student only correctly wrote the geometric series formula.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000-(1.04)^N}{1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000-(1.04)^{15}}{1.04}$$

$$= \frac{33000 + 1.80094}{1.04}$$

$$= \$31732.50$$

Score 1: The student made a computational error in the second part, having received no credit for the first part.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$Sn = \frac{a_1 - a_1 r^n}{1 - r} \qquad S_n = \frac{a_1 - a_1^{(0.04)(t)}}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33{,}000^{(0.04)(15)}}{.96} =$$

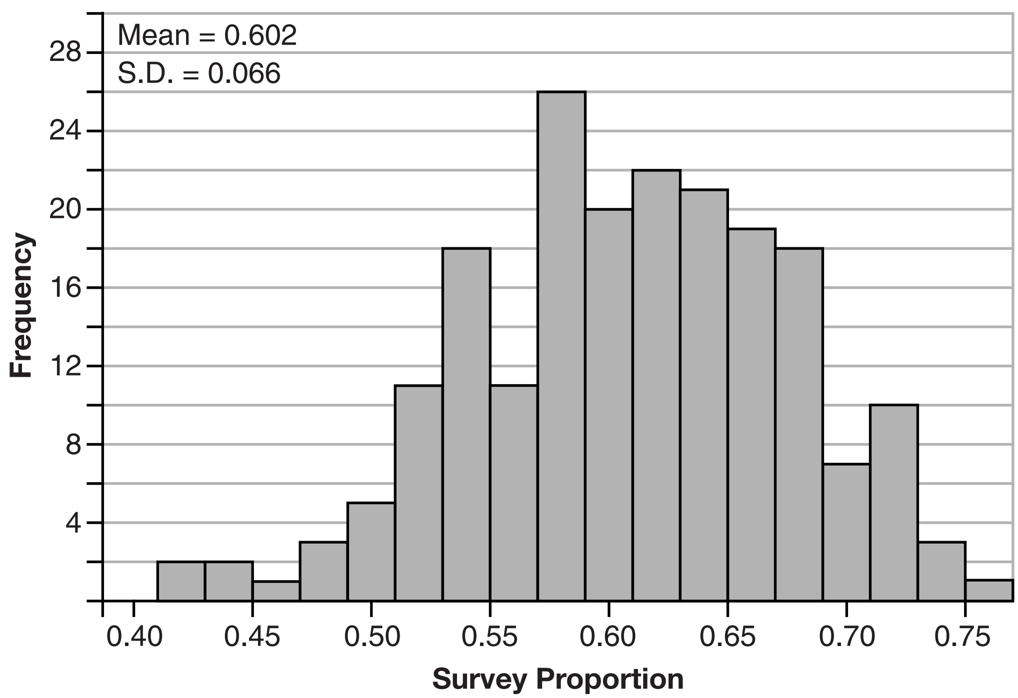
$$\boxed{S_n \approx \$535}$$

Score 0: The student made multiple errors.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

0.602 ± 0.132

From ~~0.60~~ 0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

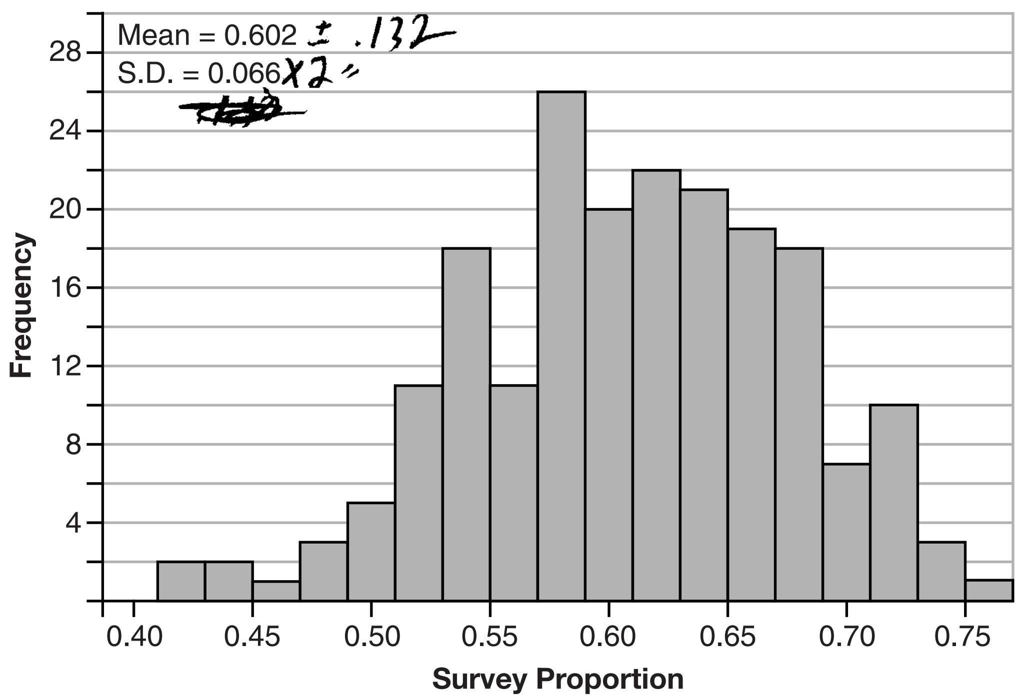
.50 or less occurs 13 out of 200 times which is possible

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

0.47 - 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

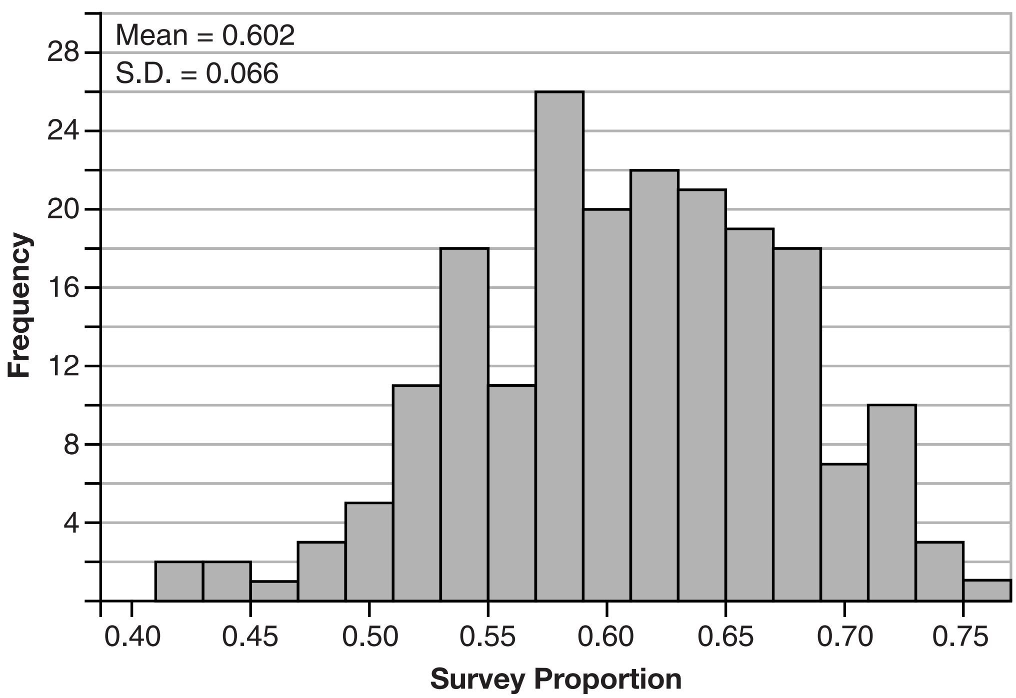
.50 is within this interval so its possible to get a split vote.

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$.602 + 2(0.066)$

$(.47 - .73)$

$.602 - 2(.066)$

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

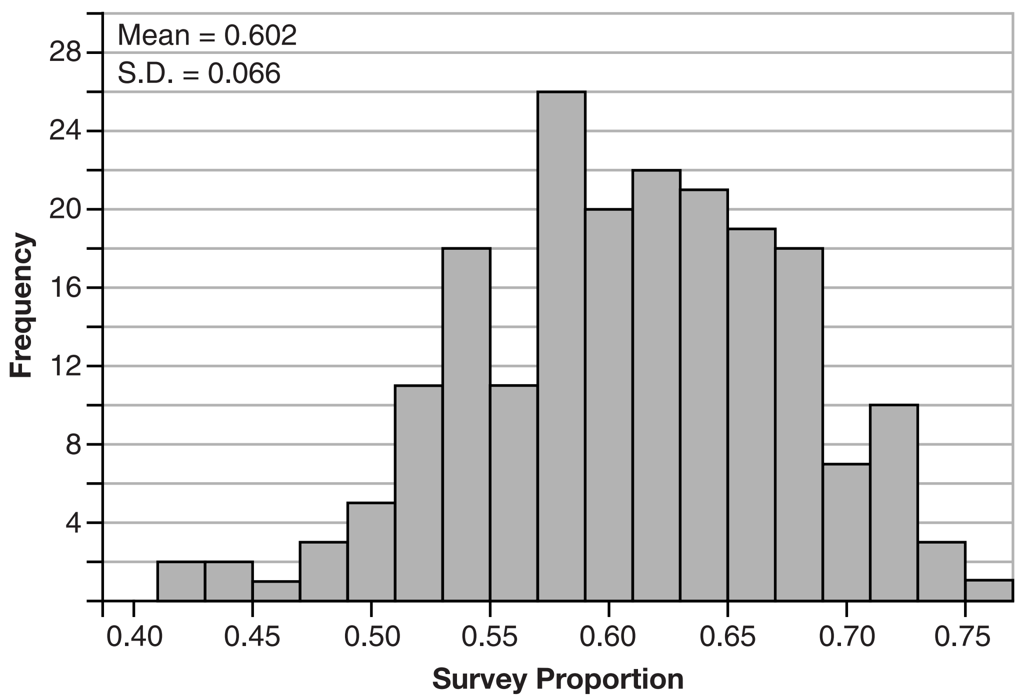
.50 is in the interval (.47 – .73) so even though its close to the end it could happen. .50 = 50% vote

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 + 2 \cdot 0.066 = 0.734$

$0.602 - 2 \cdot 0.066 = 0.47$

0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

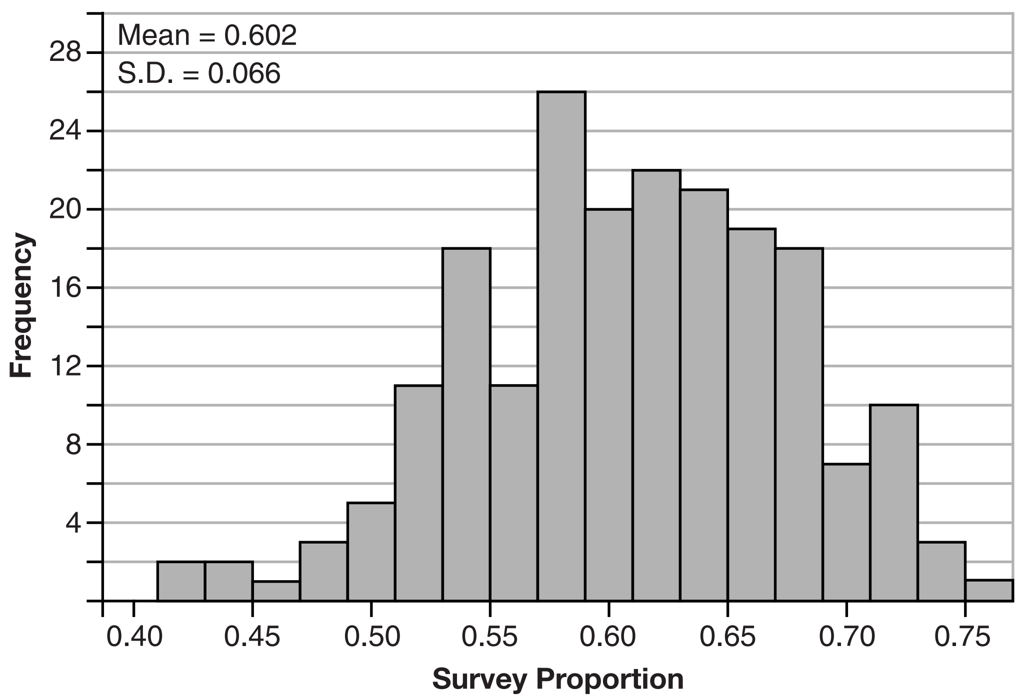
It is not a concern since .50 falls within the interval.

Score 3: The student determined a correct interval, but provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 \pm 2 \cdot 0.066$ → .47, .734

.47 to .73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

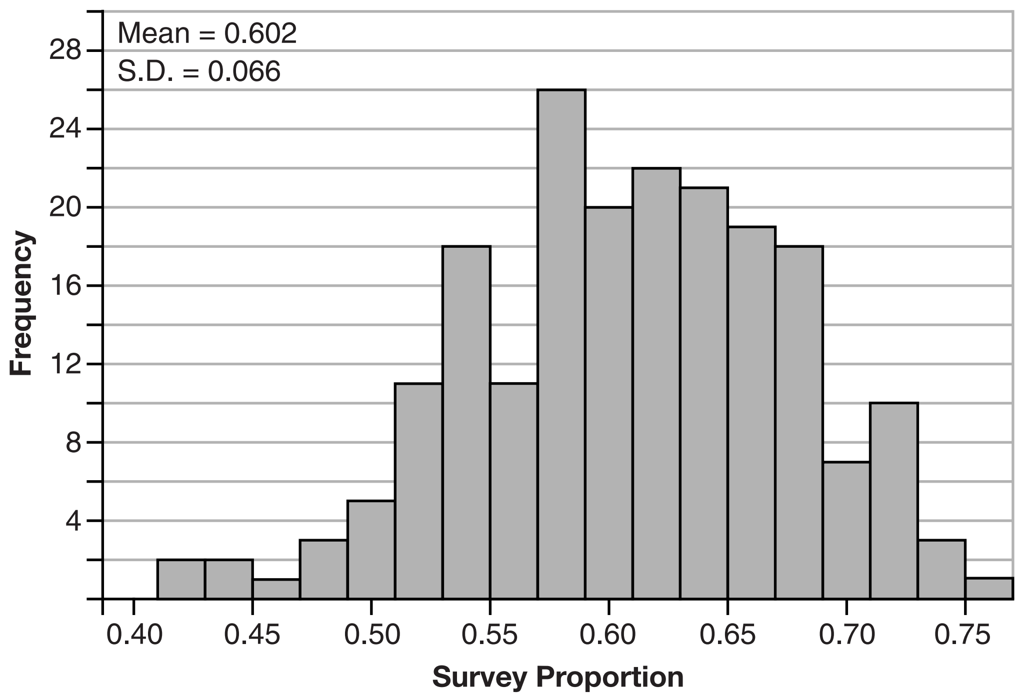
The graph shows 0.50

Score 2: The student gave no statistical explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

Interval is $.602 \pm .066$ or $.536 - .668$

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

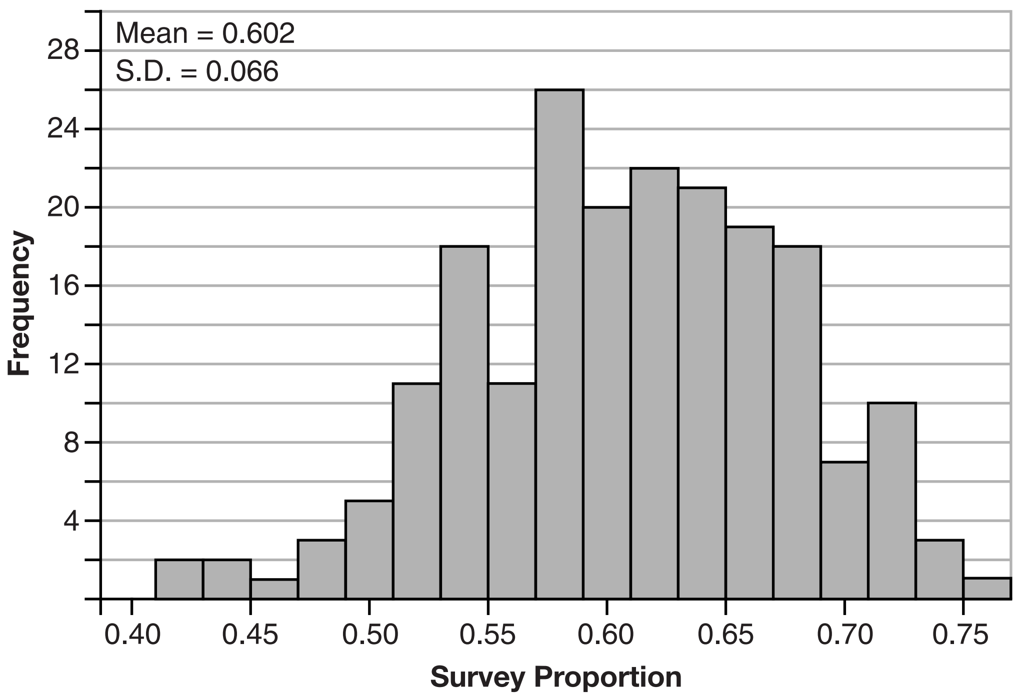
There is concern because .50 is not within this interval

Score 1: The student used only one standard deviation in the interval, rounded incorrectly, and provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

.066 +2.602 = 1.27

0.66 −2.602 = −1.138

−1.138 to 1.27

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

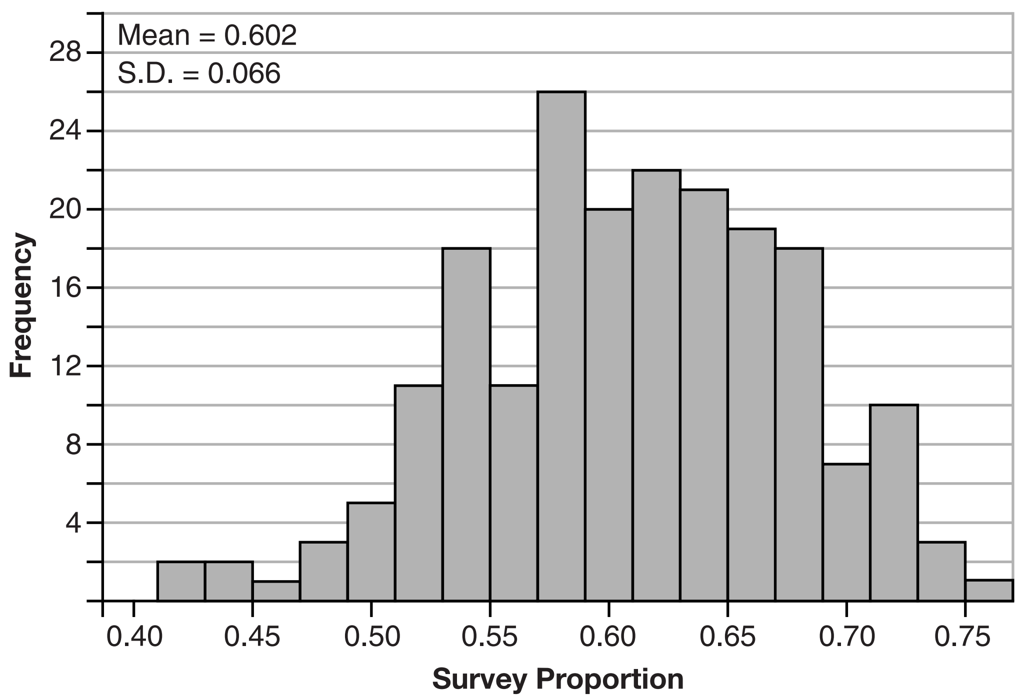
Since 60% of the students voted and 50% is close to 60%, they could be concerned that 50% is possible because voting can vary.

Score 1: The student used the standard deviation as the center and rounded incorrectly. The student gave an incomplete explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

.602 + .132 = .734 (.602, .734)

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.5 is in the interval.

Score 0: The student made multiple conceptual and computational errors.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$$\frac{80-1.25}{4-(-2)} = \frac{78.75}{6}$$

$$= 13.125$$

$$= \frac{105}{8}.$$

$g(-2) = -32-20+3$

$= -49$

$g(4) = 179$

$$\frac{179-(-49)}{4-(-2)}$$

$$= \frac{228}{6}$$

$$= 38$$

g(x) b/c its rate of change over [−2, 4] is 38, and f(x)'s rate of change is 13.125.

Score 4: The student gave a complete and correct response.

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

g(x) has a greater avg rate of change on the interval [-2,4] because g(x) went from -49 to 179 which is a greater change than from 1.25 to 80.

Score 4: The student gave a complete and correct response.

Question 36

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
x_1 −2	y_1 1.25
−1	2.5
0	5
1	10
2	20
3	40
x_2 4	y_2 80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

X	Y
−2	−49
−1	−26
0	3
1	2
2	15
3	66
4	179

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{80 - 1.25}{4 - (-2)}$$

$$\frac{78.75}{7} = 11.25 \text{ rate}$$

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{179 - (-49)}{4 - (-2)}$$

$$\frac{228}{7} = 32.57142857 \text{ rate}$$

$\boxed{g(x) = 4x^3 - 5x^2 + 3 \text{ has the greater average rate of change on the interval } [-2, 4]}$

Score 3: The student made a computational error when calculating the denominators.

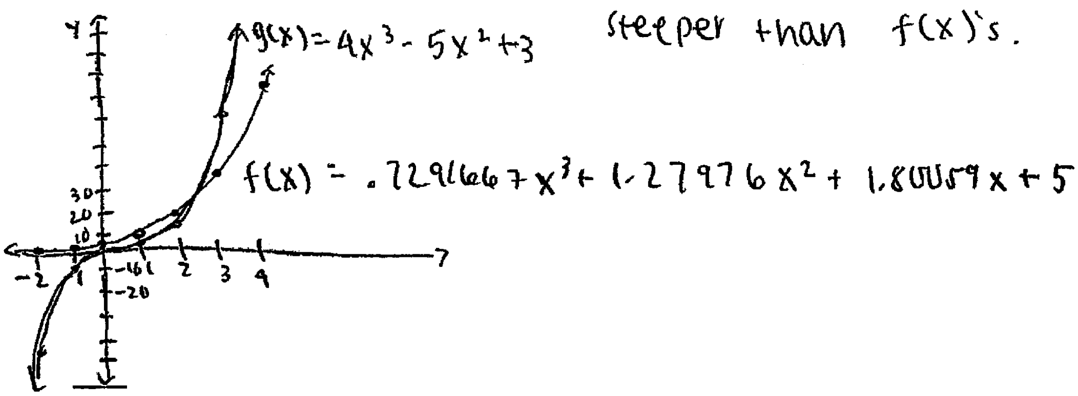
36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

x	g(x)
−2	−49
−1	−6
0	3
1	2
2	15
3	66
4	179

Function $g(x) = 4x^3 - 5x^2 + 3$ has a greater average rate of change in the interval $[-2, 4]$. This is because when both functions are graphed, $g(x)$'s slope is steeper than $f(x)$'s.



Score 2: The student made a conceptual error by creating an appropriate model for $f(x)$, but wrote an appropriate explanation for that model.

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$y = 5 \cdot 2^x$

$g(x)=4x^3-5x^2+3$ has a greater average rate of change because between the interval [-2,4] it went from (-2,-49) to (4,179) for the chart, the function would be $f(x)=5\cdot 2^x$. In this case, it went from (-2,1.25) to (4,80). Between the two functions, $g(x)=4x^3-5x^2+3$ had the greater average rate of change.

Score 2: The student found $g(-2)$ and $g(4)$ correctly, but made no comparison of the average rates of change.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

78.75

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -8(4) - 5(4) + 3$
$= -32 - 20 + 3$
$= -52 + 3$
$= -49$

$g(4) = 4(64) - 5(16) + 3$
$= 252 - 80 + 3$
$= 169$

$\boxed{169 > 78.75}$

$g(x) = 4x^3 - 5x^2 + 3$ has a greater change.

Score 1: The student made an error finding the average rates of change by not dividing by Δx, and made one computational error.

Question 36

36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

↑ thes Function has a greater average change on the interval [-2, 4] because this Function is a geometric sequence which doubles its x-values.

Score 0: The student did not calculate an average rate of change and wrote an irrelevant explanation.

Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

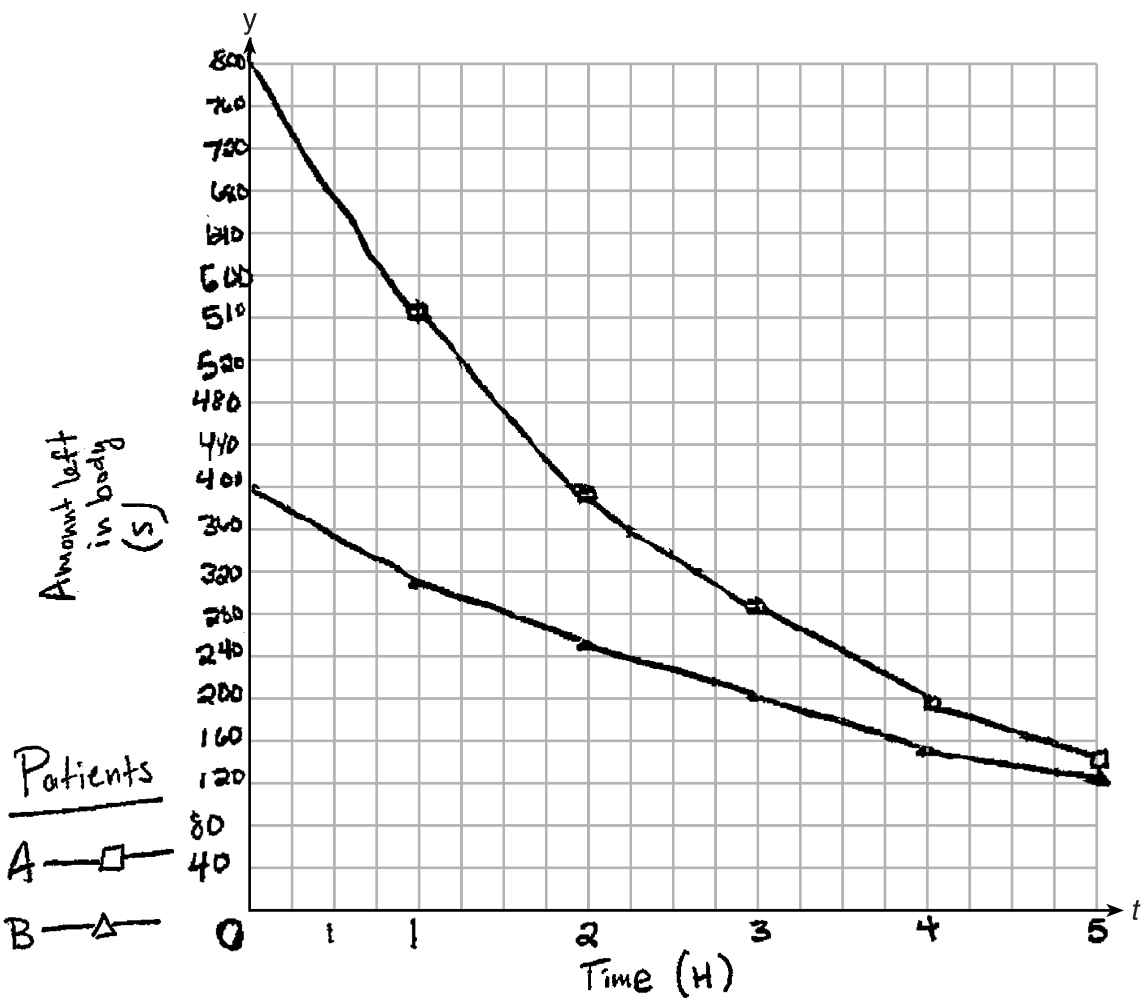
P_A

$A(t) = 800\text{mg}(e)^{-.347t}$

P_B

$B(t) = 400\text{mg}(e)^{-.231t}$

Graph each function on the set of axes below.



Score 6: The student gave a complete and correct response.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 6 because when the equation is solved the amount left in B is 100 while A has about 99.7.

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$800(.15) = 120\text{mg}$

$H_5 = 141$

$\frac{120}{800} = \frac{800\text{mg}}{800}(e)^{-.347(t)}$

$.15 = (e)^{-.347(t)}$

$\log_e .15 = \log_e e^{-.347(t)}$

$\frac{-1.897}{-.347} = \frac{-.347(t)}{-.347}$

$5.5 \text{ hrs} = t$

Question 37

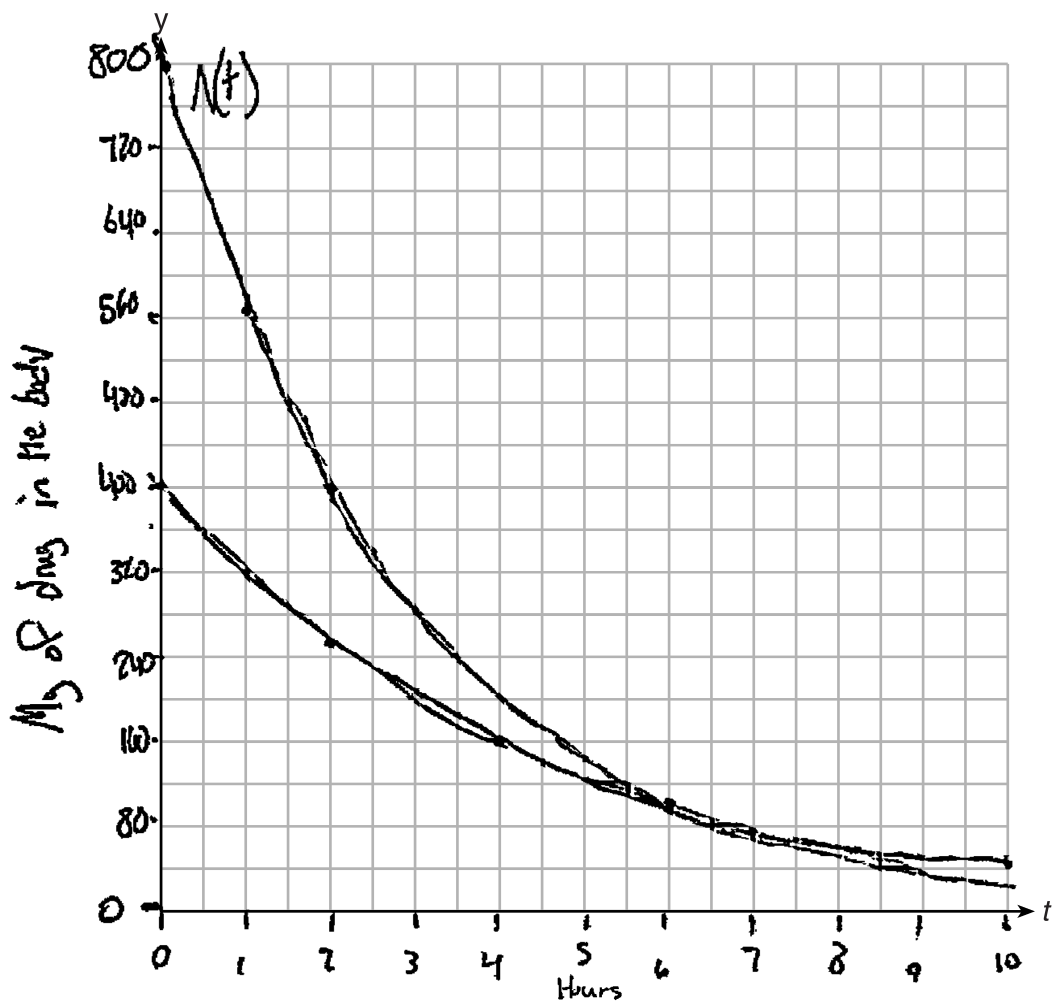
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$N(t) = 800(e)^{-.347t}$

$400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 5: The student did not indicate which function models which patient.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 6

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$\frac{120 = 800(e)^{-.347t}}{800}$$

$$.15 = e^{-.347t}$$

$$\log_e .15 = -.347t$$

$$\frac{\ln .15 = -.347t}{-.347}$$

$$5.5 = t$$

5.5 hours

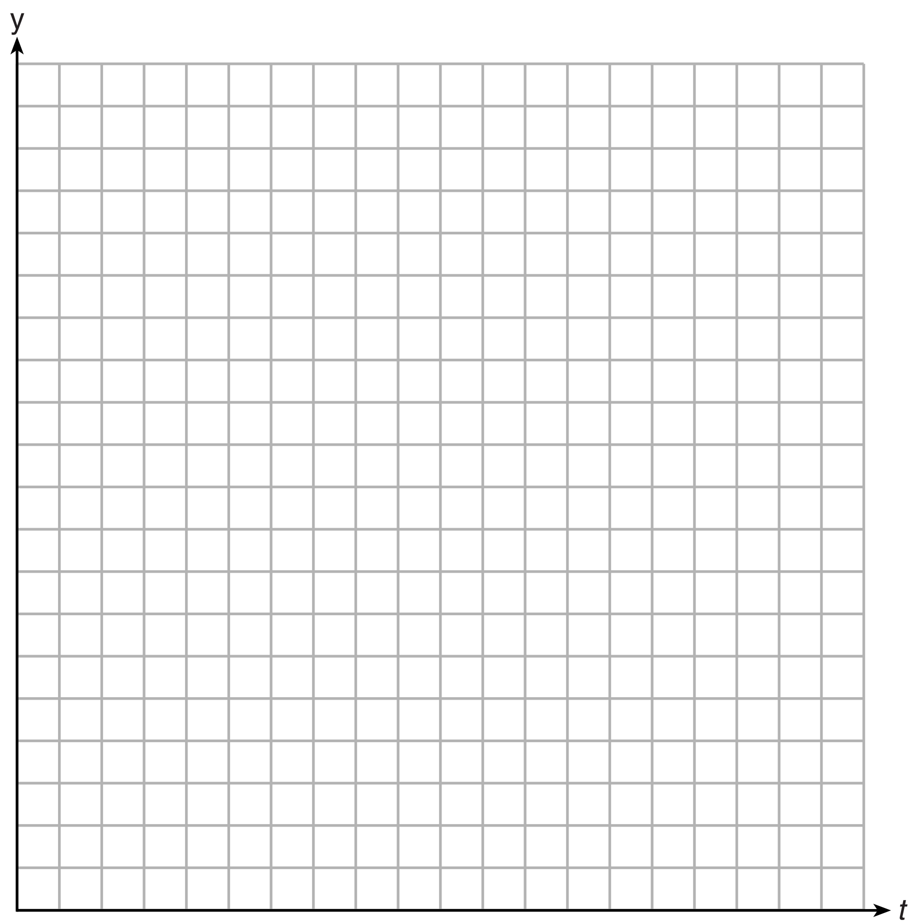
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800mg(e)^{-.347t}$

$B(t) = 400mg(e)^{-.231t}$

Graph each function on the set of axes below.



Score 4: The student did not graph either function.

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

After 6 hours. I see this after graphing both functions on my calculator and looking at the table. I could then see that at 6 hours Patient a would have 99.74 mg of drug, while patient b would have 100.03. This is probably because, despite starting with more drug, Patient A's decay rate is also greater.

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$\frac{15}{100} = \frac{x}{800}$$

$$\frac{15 \cdot 800 = \cancel{100}x}{100 \quad \cancel{100}}$$

$$120 = x$$

120 mg left

Using my graphing calculator...

I know that Patient A would have to wait approximately 5.5 hours or 5 hours and 30 minutes.

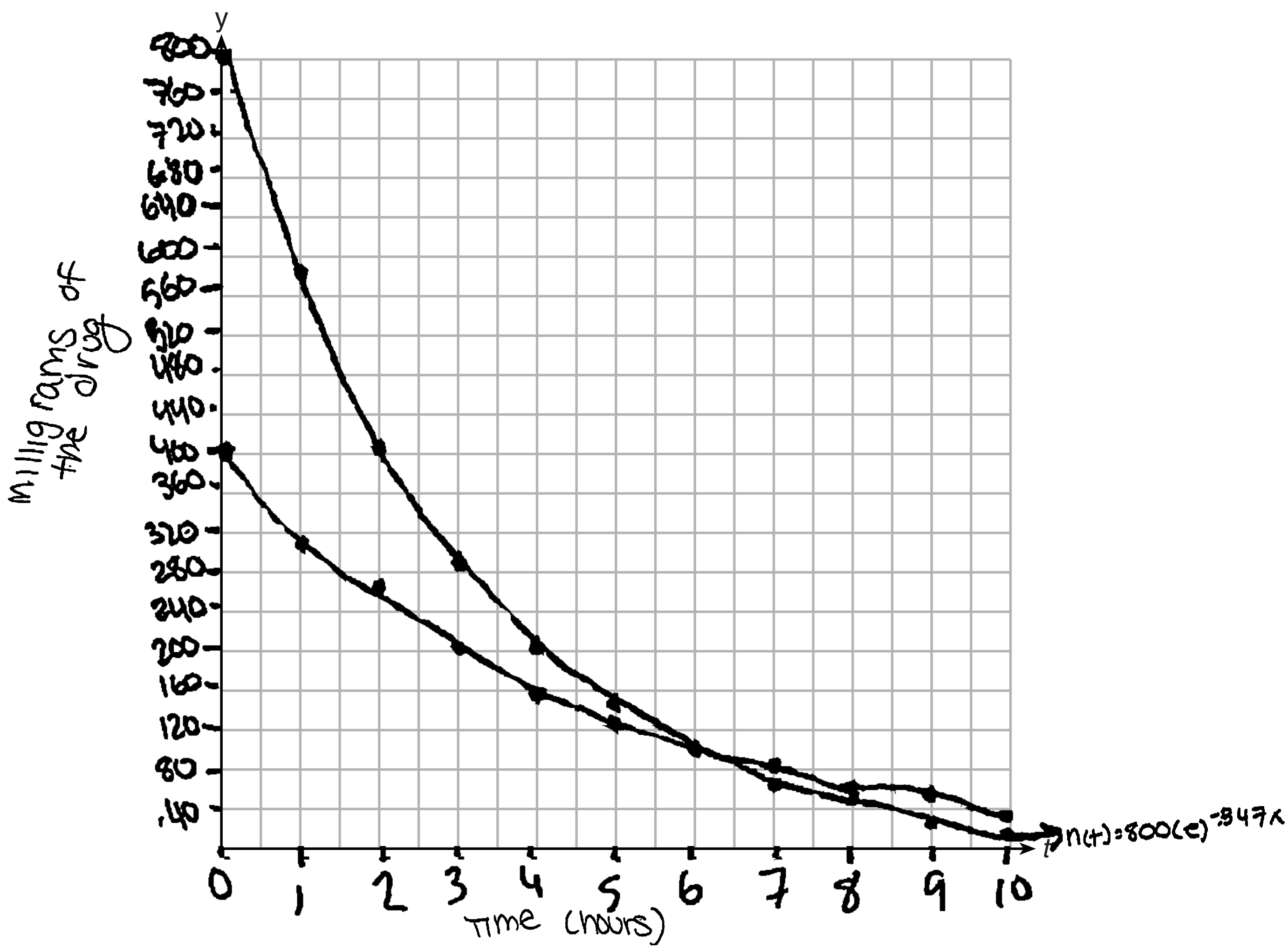
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347x}$

$400(e)^{-.281x}$

Graph each function on the set of axes below.



Score 3: The student drew a correct graph and gave a correct answer of 5.5 hours.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 7

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient A will have to wait to take another 800 milligram dose of the drug.

5.5 hours

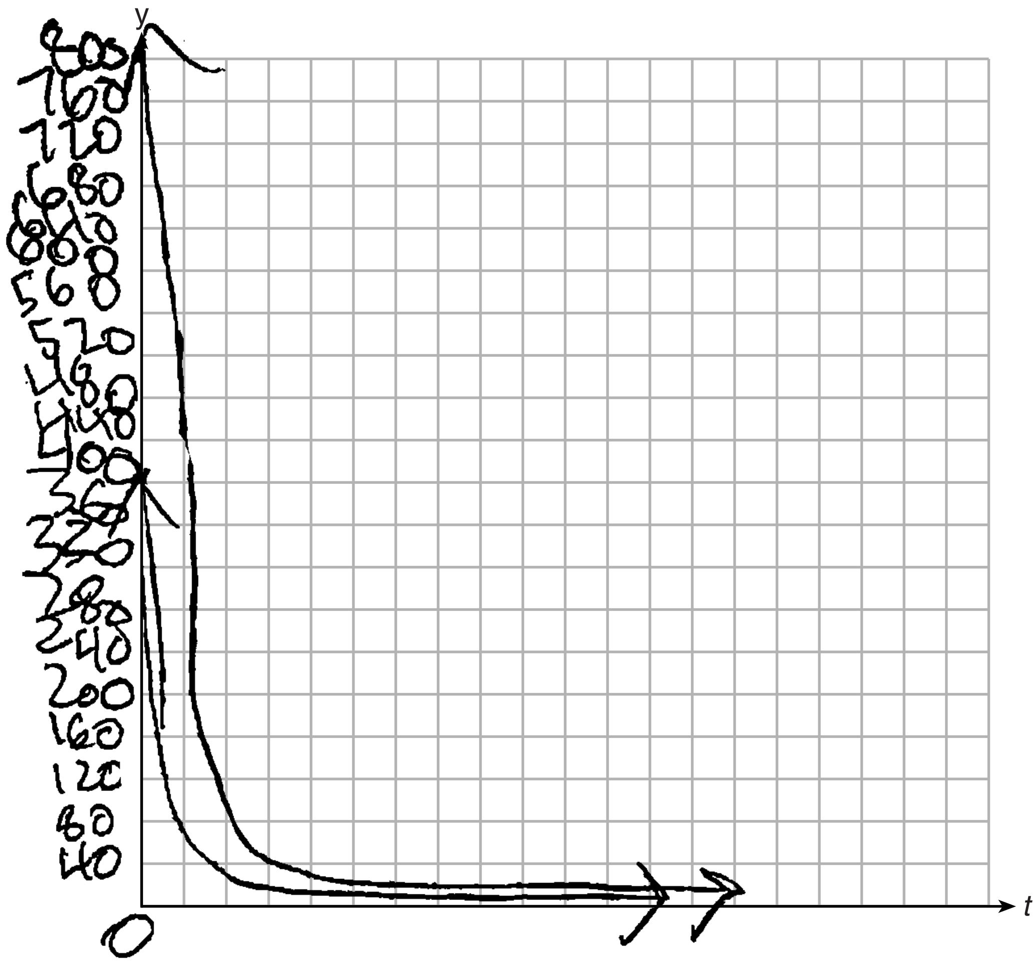
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-0.347t}$

$400(e)^{-0.231t}$

Graph each function on the set of axes below.



Score 2: The student correctly identified 6 hours and 5.5 hours.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

6 hours because that is what N(t) = when you plug 6 in for t

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

5.5 hours

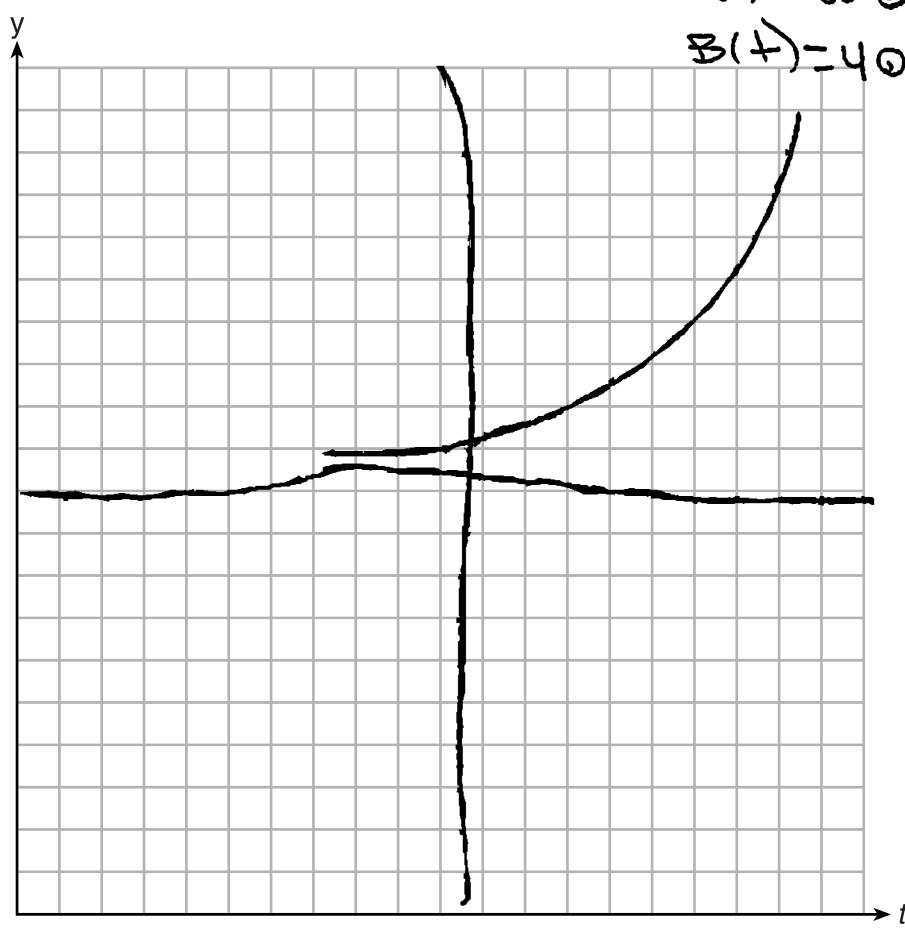
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800(e)^{-.347t}$

$B(t) = 400(e)^{.231t}$

Graph each function on the set of axes below.



Score 1: The student created and labeled correct functions.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$.15 = 800(e)^{-.347t}$

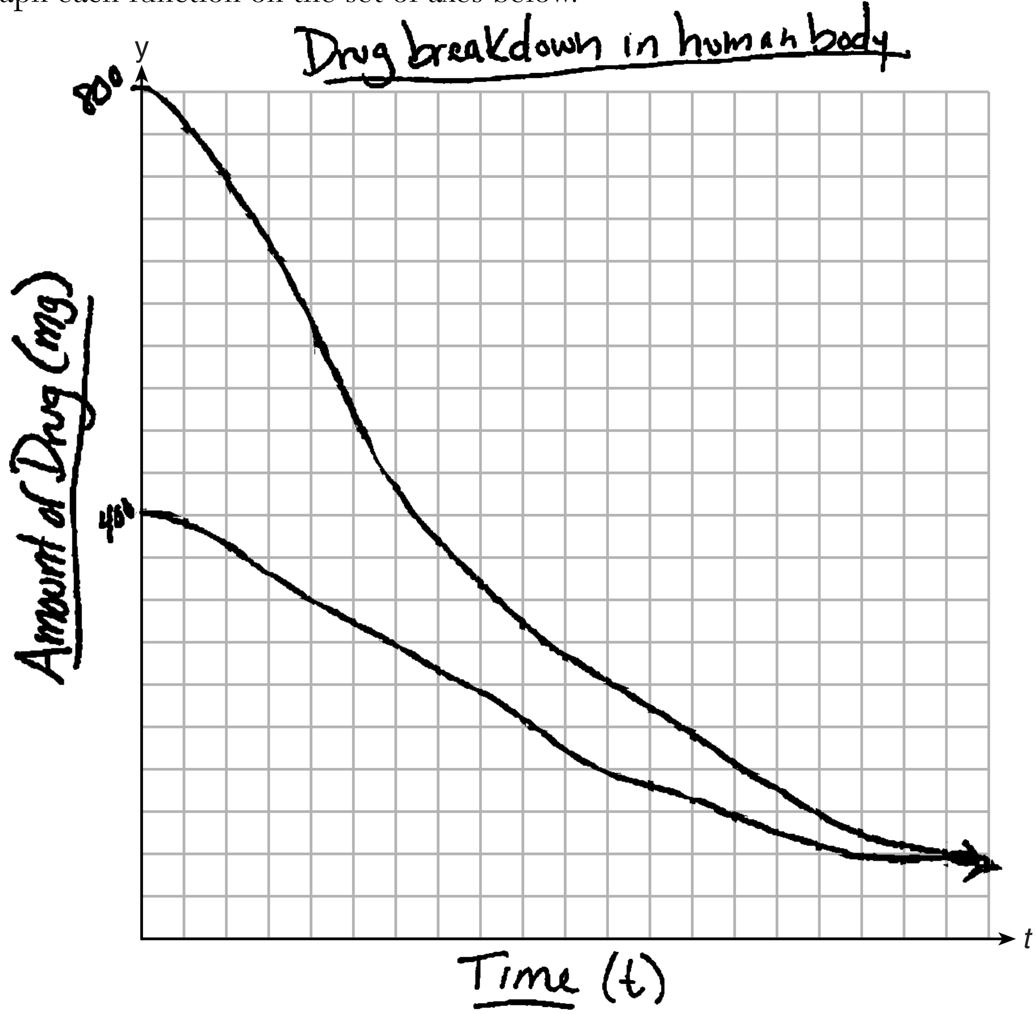
$.15 = 800(e) - .347 \ln t$

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347t}$
400

Graph each function on the set of axes below.



Score 0: The student did not complete enough correct work in any part to receive credit.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 5

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$120 = 800(e)^{-.347t}$$