ALGEBRA II

Large-Type Edition

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, June 14, 2018 — 1:15 to 4:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. One solution of this system is

(1) $(-5,-2)$

(3) $(1,4)$

(2) $(-1,-4)$

(4) $(-2,-1)$

2 Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?

(1) The graph is always increasing.

(2) The graph is always decreasing.

(3) The graph passes through $(1,0)$.

(4) The graph has an asymptote, $x = 0$.

3 For all values of x for which the expression is defined, $\dfrac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$, in simplest form, is equivalent to
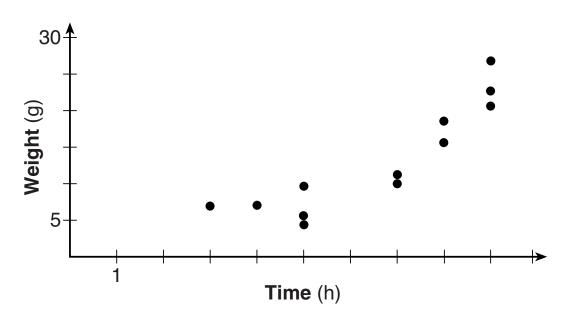
(1) 3

(2) $-\dfrac{17}{2}$

(3) $\dfrac{x + 3}{x}$

(4) $\dfrac{x^2 - 9}{x(x - 3)}$

4 A scatterplot showing the weight, w, in grams, of each crystal after growing t hours is shown below.

The relationship between weight, w, and time, t, is best modeled by

(1) $w = 4^t + 5$ (3) $w = 5(2.1)^t$

(2) $w = (1.4)^t + 2$ (4) $w = 8(.75)^t$

5 Where i is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to

(1) $-3x^2$

(3) $-3x^2 + 18xi$

(2) $-3x^2 - 18$

(4) $-3x^2 - 6xi - 18$

6 Which function is even?

(1) $f(x) = \sin x$

(3) $f(x) = |x - 2| + 5$

(2) $f(x) = x^2 - 4$

(4) $f(x) = x^4 + 3x^3 + 4$

7 The function $N(t) = 100e^{-0.023t}$ models the number of grams in a sample of cesium-137 that remain after t years. On which interval is the sample's average rate of decay the fastest?

(1) $[1,10]$

(3) $[15,25]$

(2) $[10,20]$

(4) $[1,30]$

8 Which expression can be rewritten as $(x + 7)(x - 1)$?

(1) $(x + 3)^2 - 16$

(2) $(x + 3)^2 - 10(x + 3) - 2(x + 3) + 20$

(3) $\dfrac{(x - 1)\left(x^2 - 6x - 7\right)}{(x + 1)}$

(4) $\dfrac{(x + 7)\left(x^2 + 4x + 3\right)}{(x + 3)}$

9 What is the solution set of the equation $\dfrac{2}{x} - \dfrac{3x}{x + 3} = \dfrac{x}{x + 3}$?

(1) $\{3\}$　　　　　　　　　　(3) $\{-2,3\}$

(2) $\left\{\dfrac{3}{2}\right\}$　　　　　　　　(4) $\left\{-1,\dfrac{3}{2}\right\}$

10 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.
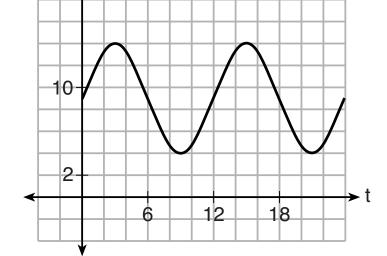

If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

(1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$

(3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$

(2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$

(4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

11 On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

(1) 73%   (3) 23%

(2) 36%   (4) 12%

12 If $x - 1$ is a factor of $x^3 - kx^2 + 2x$, what is the value of k?

(1) 0   (3) 3

(2) 2   (4) −3

13 The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

(1) $c(x) = 0.2x^2 - 100x + 300$

(2) $c(x) = 0.2x^2 + 100x + 300$

(3) $c(x) = -0.2x^2 + 100x - 300$

(4) $c(x) = -0.8x^2 + 400x - 300$

14 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

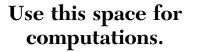| Town	Population	Annual Population Growth Rate
Jonesville	1240	6% increase
Williamstown	890	11% increase

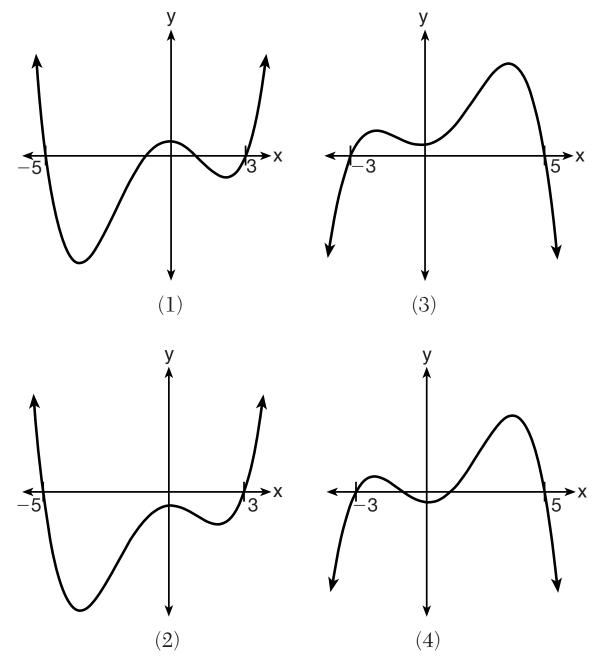Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

(1) 7   (3) 68

(2) 20   (4) 125

15 What is the inverse of $f(x) = x^3 - 2$?

(1) $f^{-1}(x) = \sqrt[3]{x} + 2$ (3) $f^{-1}(x) = \sqrt[3]{x + 2}$

(2) $f^{-1}(x) = \pm\sqrt[3]{x} + 2$ (4) $f^{-1}(x) = \pm\sqrt[3]{x + 2}$

16 A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

(1)

(3)

(2)

(4)

17 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

(1) 0.2257

(3) 0.7257

(2) 0.2743

(4) 0.7757

18 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body d days after being introduced is $I = 100\left(\dfrac{1}{2}\right)^{\frac{d}{8}}$.

When this equation is written in terms of the number e, the base of the natural logarithm, it is equivalent to $I = 100e^{kd}$. What is the approximate value of the constant, k?

(1) -0.087

(3) -11.542

(2) 0.087

(4) 11.542

19 The graph of $y = \log_2 x$ is translated to the right 1 unit and down 1 unit. The coordinates of the x-intercept of the translated graph are

(1) (0,0)   (3) (2,0)

(2) (1,0)   (4) (3,0)

20 For positive values of x, which expression is equivalent to $\sqrt{16x^2} \cdot x^{\frac{2}{3}} + \sqrt[3]{8x^5}$?

(1) $6\sqrt[5]{x^3}$ (3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$

(2) $6\sqrt[3]{x^5}$ (4) $4\sqrt{x^3} + 2\sqrt[5]{x^3}$

21 Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y = 9$?

(1) $(y - 7)^2 = 8(x + 2)$

(2) $(y - 7)^2 = -8(x + 2)$

(3) $(x + 2)^2 = 8(y - 7)$

(4) $(x + 2)^2 = -8(y - 7)$

22 Given the following polynomials

$$x = (a + b + c)^2$$
$$y = a^2 + b^2 + c^2$$
$$z = ab + bc + ac$$

Which identity is true?

(1) $x = y - z$ (3) $x = y - 2z$

(2) $x = y + z$ (4) $x = y + 2z$

23 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29{,}400(1.068)^t$, where t is time in years. Which expression is equivalent to $29{,}400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

(1) $29{,}400\left(1.068^{\frac{1}{365}}\right)^t$

(3) $29{,}400\left(1 + \dfrac{0.068}{365}\right)^t$

(2) $29{,}400\left(\dfrac{1.068}{365}\right)^{365t}$

(4) $29{,}400\left(1.068^{\frac{1}{365}}\right)^{365t}$

24 A manufacturing plant produces two different-sized containers of peanuts. One container weighs x ounces and the other weighs y pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

(1) $\dfrac{124}{16x + y}$

(3) $\dfrac{124}{x + 16y}$

(2) $\dfrac{x + 16y}{124}$

(4) $\dfrac{16x + y}{124}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

Favorite Type of Program

	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshman	119	103	54

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.
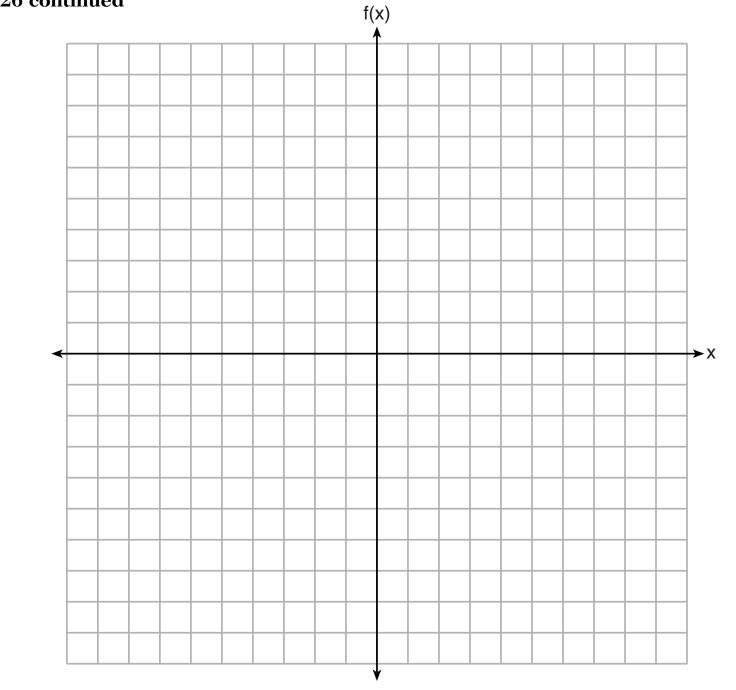
Work space for question 25 is continued on the next page.

Question 25 continued

26 On the grid on the next page, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \le x \le 4$.

The grid for question 26 is on the next page.

f(x)

x

27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

Question 27 continued

28 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

How's My Driving?

Call 1-555-DRIVING

The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

Question 28 continued

29 Determine the quotient and remainder when $(6a^3 + 11a^2 - 4a - 9)$ is divided by $(3a - 2)$.

Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$.

Question 29 continued

30 The recursive formula to describe a sequence is shown below.

$$a_1 = 3$$

$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence.

Can this sequence be represented using an explicit geometric formula? Justify your answer.

31 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = total number of monthly payments

Question 31 continued

32 An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin \theta$.

Question 32 continued

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve algebraically for all values of x:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

Work space for question 33 is continued on the next page.

Question 33 continued

34 Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.
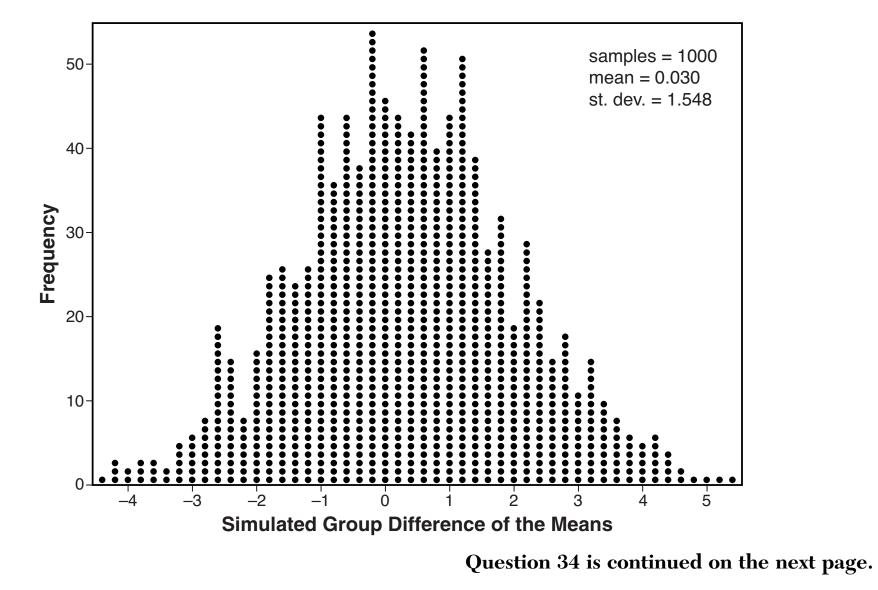
	Scented Paper	Unscented Paper
$\overline{x}$	23	18
s_x	2.898	2.408

Calculate the difference in means in the experimental test grades (scented – unscented).

A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

samples = 1000
mean = 0.030
st. dev. = 1.548

Simulated Group Difference of the Means

Question 34 is continued on the next page.

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*.

Question 34 continued

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account.

Calculate algebraically the number of years it will take for the account to reach $100,000, to the *nearest hundredth of a year*.

36 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$, where t represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \leq t \leq 2$.

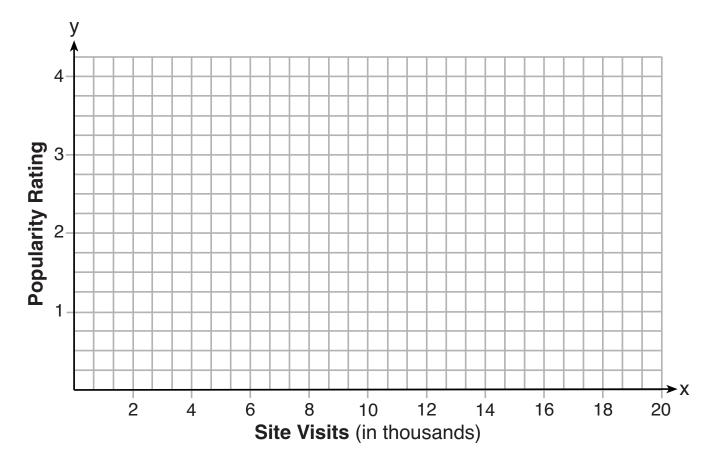At what value(s) of t, to the *nearest tenth of a second*, does $h(t) = 0$ in the interval $1 \le t \le 5$? Justify your answer.

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*?
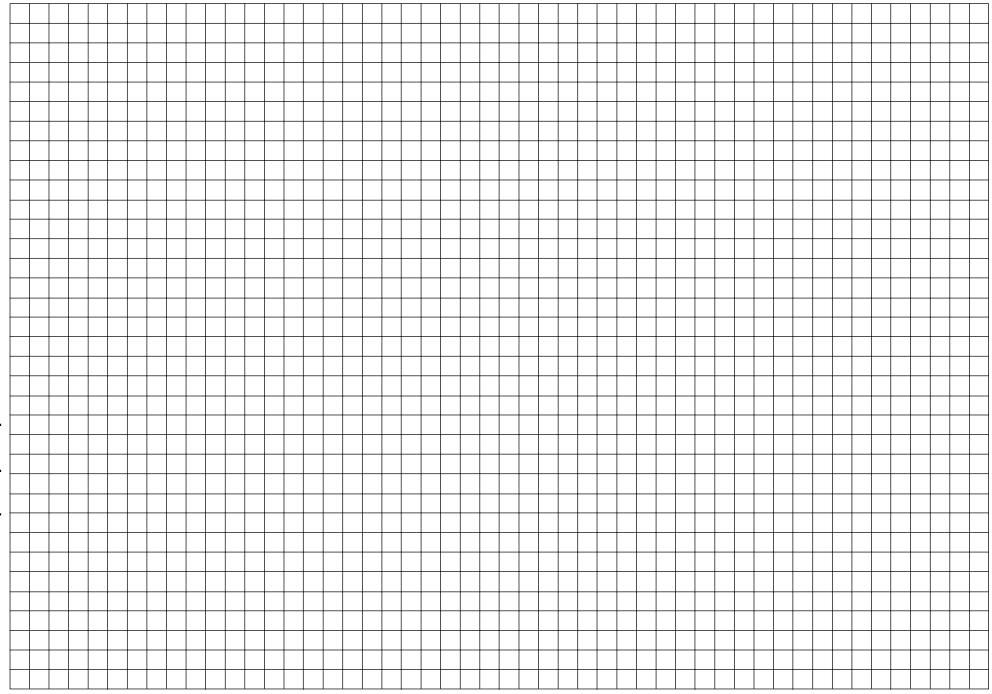
Graph $y = P(x)$ on the axes below.
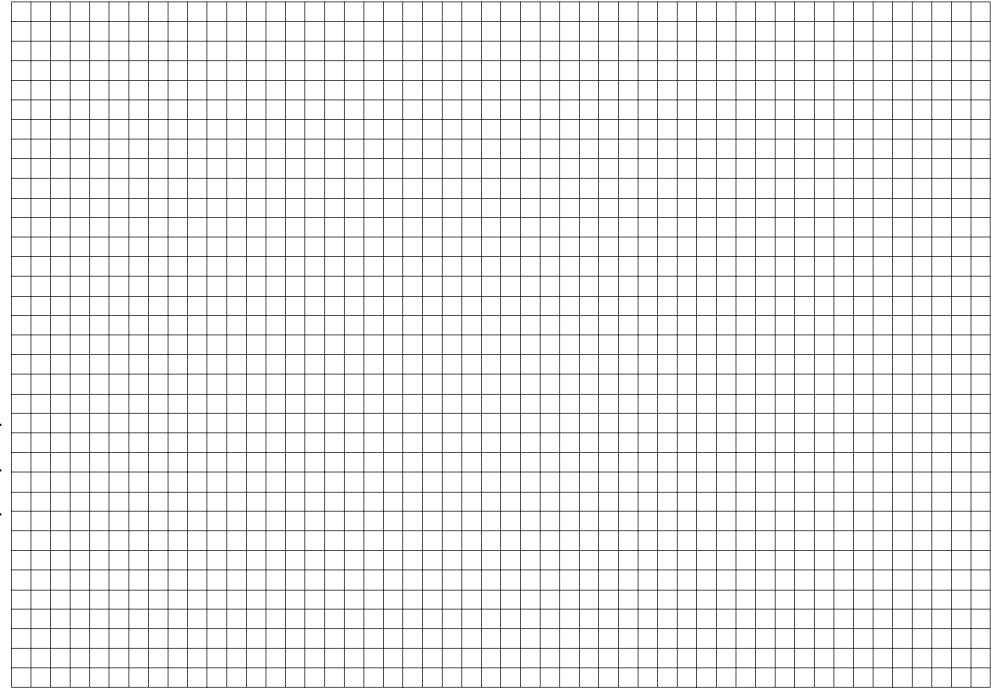
An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian $= \dfrac{180}{\pi}$ degrees
Degrees	1 degree $= \dfrac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$