ALGEBRA

II

Large-Type Edition

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 22, 2022 — 9:15 a.m. to 12:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 For all positive values of x, which expression is equivalent to $x^{\frac{3}{4}}$?

(1) $\sqrt[4]{x^3}$

(3) $(x^3)^4$

(2) $\sqrt[3]{x^4}$

(4) $3(x^4)$

2 Mrs. Favata's statistics class wants to conduct a survey to see how students feel about changing the school mascot's name. Which plan is the best process for gathering an appropriate sample?

(1) Survey students in a random sample of senior homerooms.

(2) Survey every tenth student entering art classes in the school.

(3) Survey every fourth student entering the cafeteria during each lunch period.

(4) Survey all members of the school's varsity sports teams.

3 Given $x \neq -3$, the expression $\dfrac{2x^3 + 7x^2 - 3x - 25}{x + 3}$ is equivalent to

(1) $2x^2 + x - 6 - \dfrac{7}{x + 3}$

(3) $2x^2 + x - 13$

(2) $2x^2 + 13x - 36 + \dfrac{83}{x + 3}$

(4) $x^2 + 4x - 15 + \dfrac{20}{x + 3}$

4 In a group of 40 people, 20 have brown hair, 22 have blue eyes, and 15 have both brown hair and blue eyes. How many people have neither brown hair nor blue eyes?

(1) 0

(3) 27

(2) 13

(4) 32

Algebra II – June '22

5

[OVER]

5 Consider the function $y = h(x)$, defined by the graph below.
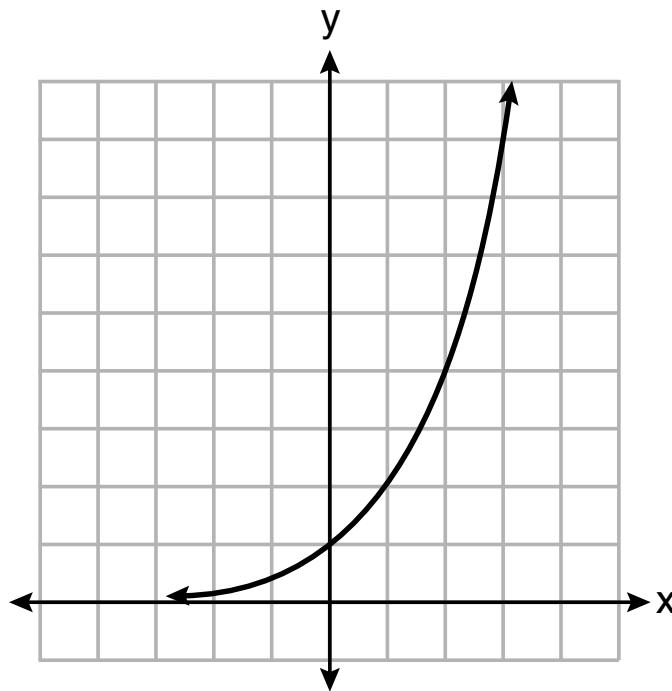
Question 5 continued

Which equation could be used to represent the graph shown below?

Use this space for computations.

(1) $y = h(x) - 2$ (3) $y = -h(x)$

(2) $y = h(x - 2)$ (4) $y = h(-x)$
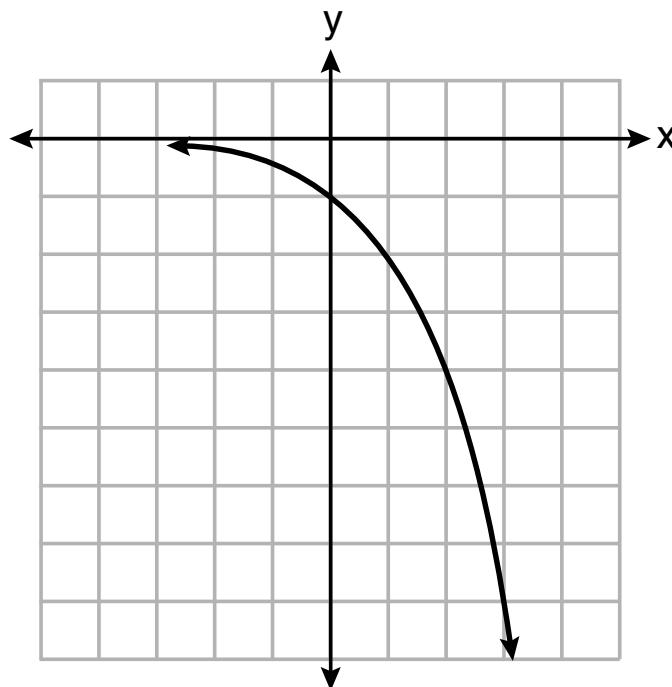
6 For the polynomial $p(x)$, if $p(3) = 0$, it can be concluded that

(1) $x + 3$ is a factor of $p(x)$

(2) $x - 3$ is a factor of $p(x)$

(3) when $p(x)$ is divided by 3, the remainder is zero

(4) when $p(x)$ is divided by -3, the remainder is zero

7 The solution to the equation $5e^{x + 2} = 7$ is

(1) $-2 + ln\left(\dfrac{7}{5}\right)$

(3) $\dfrac{-3}{5}$

(2) $\left(\dfrac{ln\ 7}{ln\ 5}\right) - 2$

(4) $-2 + ln(2)$

8 Consider the system of equations below.

$$x + 2y - z = 1$$
$$-x - 3y + 2z = 0$$
$$2x - 4y + z = 10$$

What is the solution to the given system of equations?

(1) $(1,1,2)$

(3) $(5,-1,2)$

(2) $(3,-1,0)$

(4) $(3,5,8)$

9 Monthly mortgage payments can be found using the formula below, where M is the monthly payment, P is the amount borrowed, r is the annual interest rate, and n is the total number of monthly payments.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

If Adam takes out a 15-year mortgage, borrowing $240,000 at an annual interest rate of 4.5%, his monthly payment will be

(1) $1379.09   (3) $1835.98

(2) $1604.80   (4) $9011.94

10 For all real values of x, if $f(x) = (x - 3)^2$ and $g(x) = (x + 3)^2$, what is $f(x) - g(x)$?

(1) -18 (3) $-12x$

(2) 0 (4) $2x^2 - 12x - 18$

11 If $f(t) = 50(.5)^{\frac{t}{5715}}$ represents a mass, in grams, of carbon-14 remaining after t years, which statement(s) must be true?

   I. The mass of the carbon-14 is decreasing by half each year.
   II. The mass of the original sample is 50 g.

(1) I, only   (3) I and II

(2) II, only   (4) neither I nor II

12 Consider the graph of g and the table representing t below.
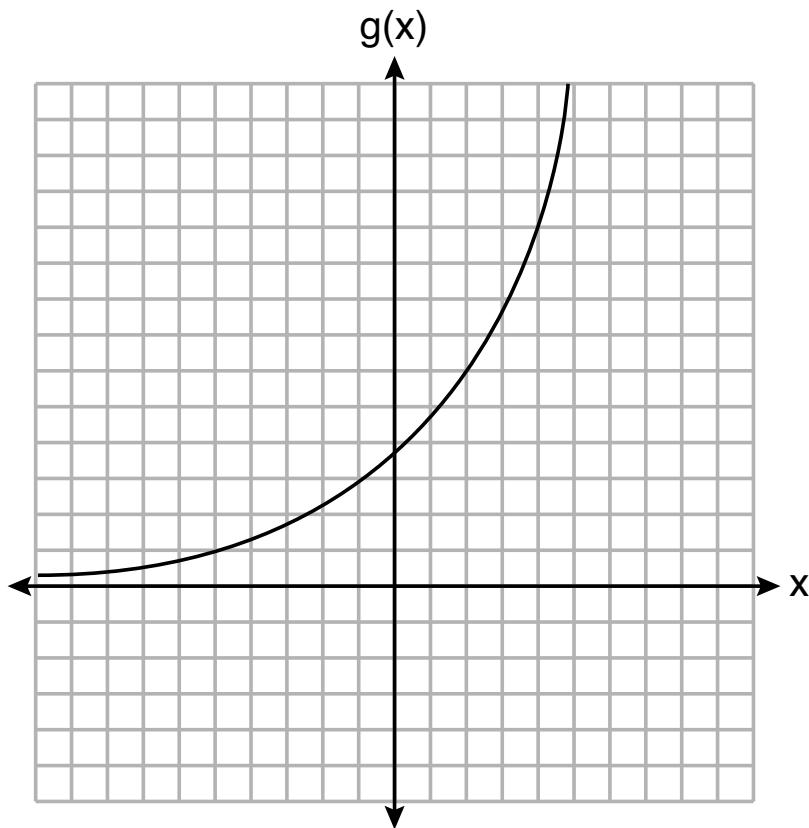
x	t(x)
−1	3
0	5
1	2
2	−5
3	−1
4	3

Over the interval $[2, 4]$, which statement regarding the average rate of change for g and t is true?

(1) g has a greater average rate of change.

(2) The average rates of change are equal.

(3) The average rate of change for g is twice the average rate of change for t.

(4) The average rate of change for g is half the average rate of change for t.

13 A parabola has a directrix of $y = 3$ and a vertex at $(2,1)$. Which ordered pair is the focus of the parabola?

(1) $(2,-1)$ (3) $(2,2)$

(2) $(2,0)$ (4) $(2,5)$

14 The heights of the 3300 students at Oceanview High School are approximately normally distributed with a mean of 65.5 inches and a standard deviation of 2.9 inches. The number of students at Oceanview who are between 64 and 68 inches tall is closest to

(1) 1660   (3) 2244

(2) 1070   (4) 1640

15 Which statement below about the graph of $f(x) = -\log(x + 4) + 2$ is true?

(1) $f(x)$ has a y-intercept at $(0,2)$.

(2) $-f(x)$ has a y-intercept at $(0,2)$.

(3) As $x \to \infty, f(x) \to \infty$.

(4) As $x \to -4, f(x) \to \infty$.

16 A researcher wants to determine if room-darkening shades cause people to sleep longer. Which method of data collection is most appropriate?

(1) census   (3) observation study

(2) survey   (4) controlled experiment

17 The inverse of $f(x) = -6x + \dfrac{1}{2}$ is

(1) $f^{-1}(x) = 6x - \dfrac{1}{2}$ (3) $f^{-1}(x) = -\dfrac{1}{6}x + \dfrac{1}{12}$

(2) $f^{-1}(x) = \dfrac{1}{-6x + \dfrac{1}{2}}$ (4) $f^{-1}(x) = -\dfrac{1}{6}x + 2$

18 The expression $\dfrac{x^2 + 12}{x^2 + 3}$ can be rewritten as

(1) $\dfrac{10}{x^2 + 3}$ (3) $x + 9$

(2) $1 + \dfrac{9}{x^2 + 3}$ (4) 4

19 An angle, θ, is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below.
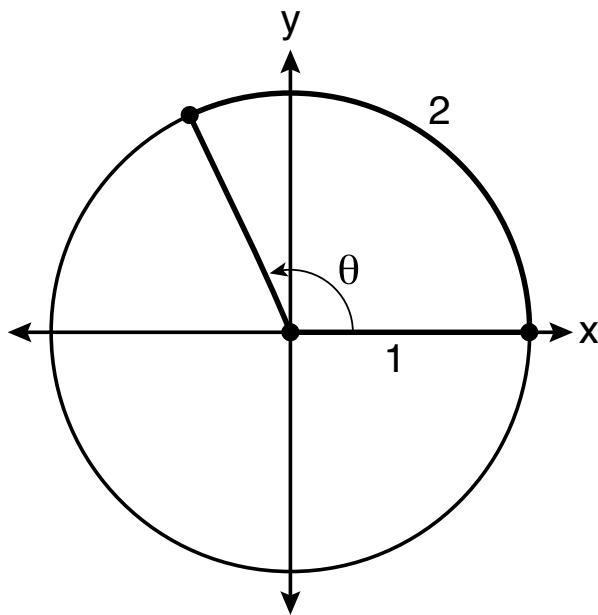
Which value represents the radian measure of angle θ?

(1) 1

(3) 65.4

(2) 2

(4) 114.6

20 The depth of the water, $d(t)$, in feet, on a given day at Thunder Bay, t hours after midnight is modeled by $d(t) = 5\sin\left(\frac{\pi}{6}(t - 5)\right) + 7$.

Which statement about the Thunder Bay tide is *false*?

(1) A low tide occurred at 2 a.m.

(2) The maximum depth of the water was 12 feet.

(3) The water depth at 9 a.m. was approximately 11 feet.

(4) The difference in water depth between high tide and low tide is 14 feet.

21 A function is defined as $a_n = a_{n-1} + \log_{n+1}(n - 1)$, where $a_1 = 8$. What is the value of a_3?

(1) 8   (3) 9.2

(2) 8.5   (4) 10

22 Which function has a maximum y-value of 4 and a midline of $y = 1$?

f(x)

(1)

h(x)

(3)

$$g(x) = -3\cos(x) + 1$$

(2)

$$j(x) = 4\sin(x) + 1$$

(4)
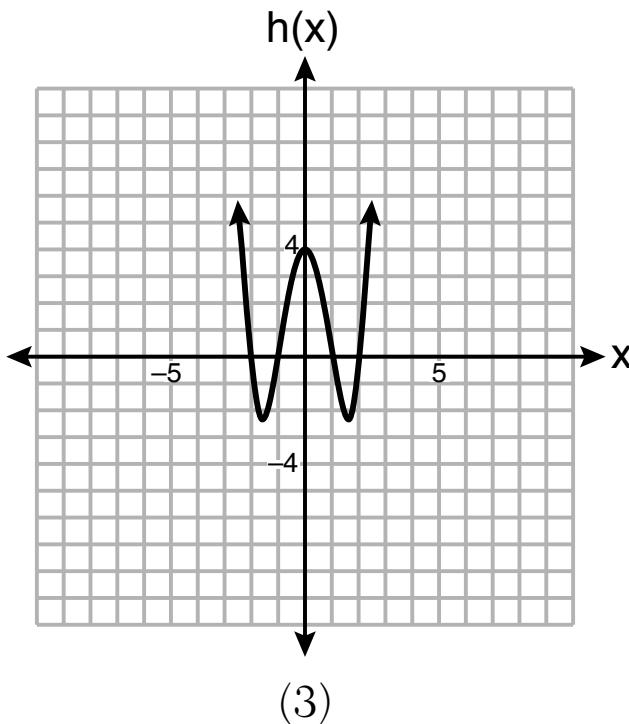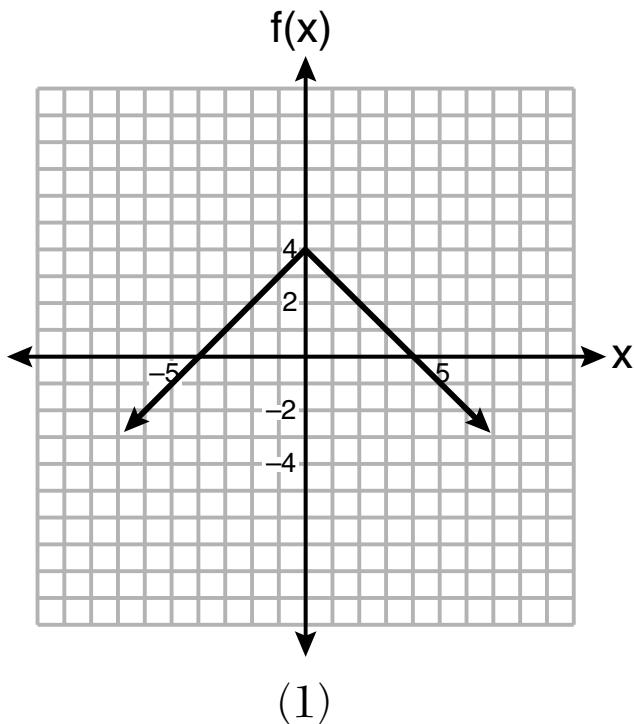
23 Which expression is equivalent to $(x + yi)(x^2 - xyi - y^2)$, where i is the imaginary unit?

(1) $x^3 + y^3i$

(2) $x^3 - xy^2 - (xy^2 + y^3)i$

(3) $x^3 - 2xy^2 - y^3i$

(4) $x^3 - y^3i$

24 The growth of a \$500 investment can be modeled by the function $P(t) = 500(1.03)^t$, where t represents time in years. In terms of the monthly rate of growth, the value of the investment can be best approximated by

(1) $P(t) = 500(1.00247)^{12t}$

(2) $P(t) = 500(1.00247)^t$

(3) $P(t) = 500(1.03)^{12t}$

(4) $P(t) = 500(1.03)^{\frac{t}{12}}$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

26 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the *nearest hundredth of a foot*, a child travels in the first five swings.

27 Solve algebraically for n: $\dfrac{2}{n^2} + \dfrac{3}{n} = \dfrac{4}{n^2}$.

28 Factor completely over the set of integers:

$$-2x^4 + x^3 + 18x^2 - 9x$$

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	Wear Glasses	Don't Wear Glasses
Blue Eyes	0.14	0.26
Brown Eyes	0.11	0.24
Green Eyes	0.10	0.15

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

Work space for question 29 is continued on the next page.

Question 29 continued

30 For $x \neq 0$ and $y \neq 0$, $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$. Determine the value of a.

Question 30 continued

31 Graph $y = 2\cos\left(\dfrac{1}{2}x\right) + 5$ on the interval $[0, 2\pi]$, using the axes below.
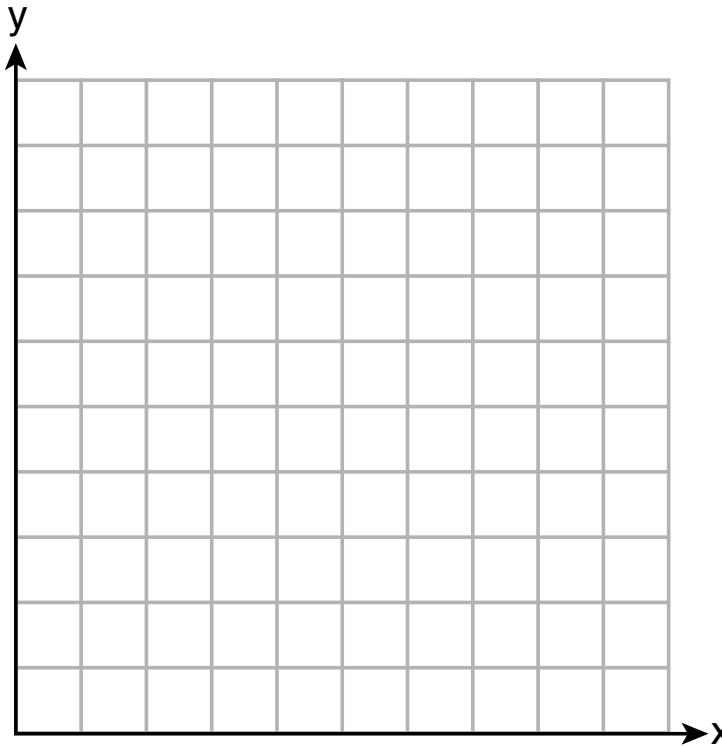
Question 31 continued

29

32 A cup of coffee is left out on a countertop to cool. The table below represents the temperature, $F(t)$, in degrees Fahrenheit, of the coffee after it is left out for t minutes.

t	0	5	10	15	20	25
F(t)	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation, $F(t)$, to model the temperature of the coffee. Round all values to the *nearest thousandth*.

Question 32 continued

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 On the set of axes on the next page, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

State the number of solutions to the equation $f(x) = g(x)$.

The set of axes for question 33 is on the next page.
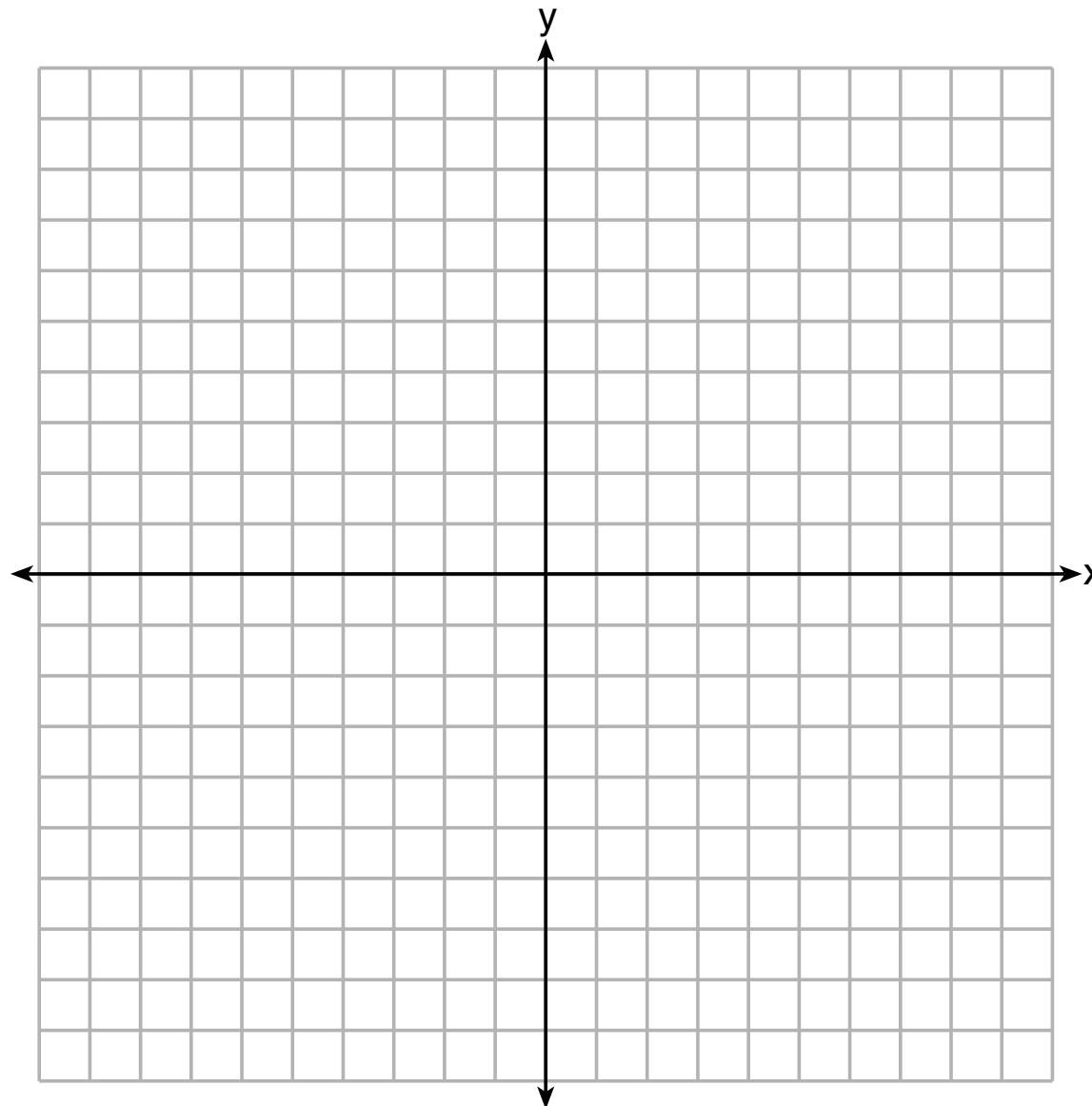
34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\dfrac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s^2.

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

Question 34 continued

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Mean = 0.651
SD = 0.034

Proportion of residents who drive to work
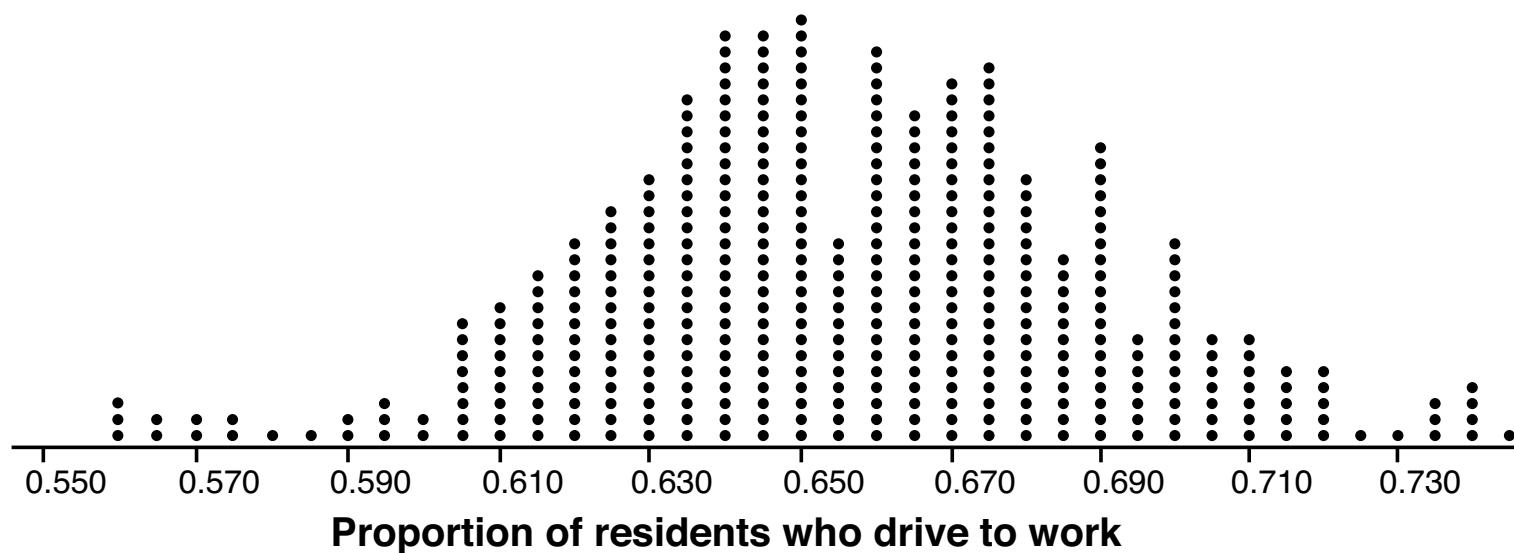
Question 35 continued

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$
$$y + 5 = 2x$$

Question 36 continued

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

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37 The population, in millions of people, of the United States can be represented by the recursive formula below, where a_0 represents the population in 1910 and n represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

Question 37 is continued on the next page.

Question 37 continued

Write an exponential function, P, where $P(t)$ represents the United States population in millions of people, and t is the number of years since 1910.

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.
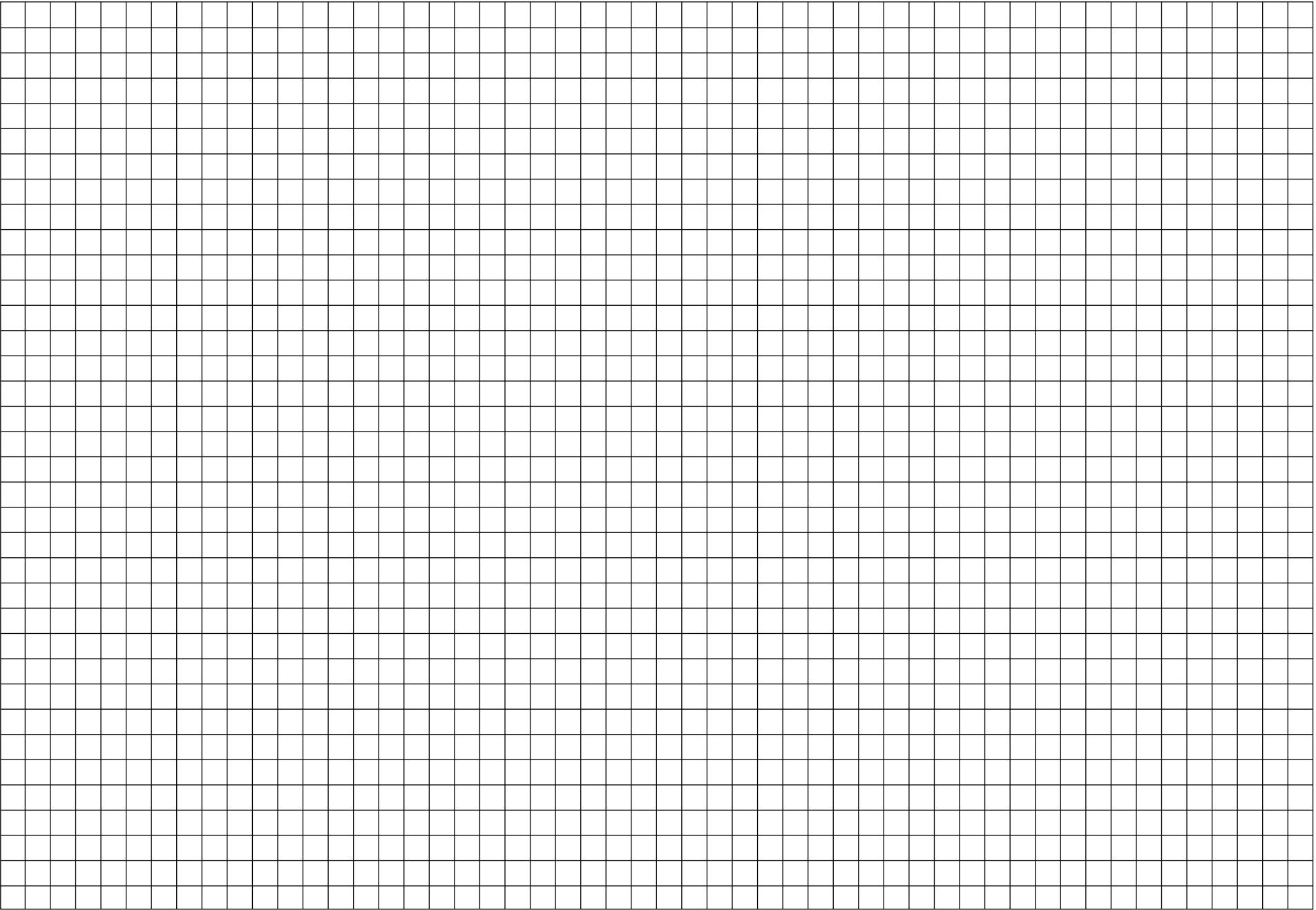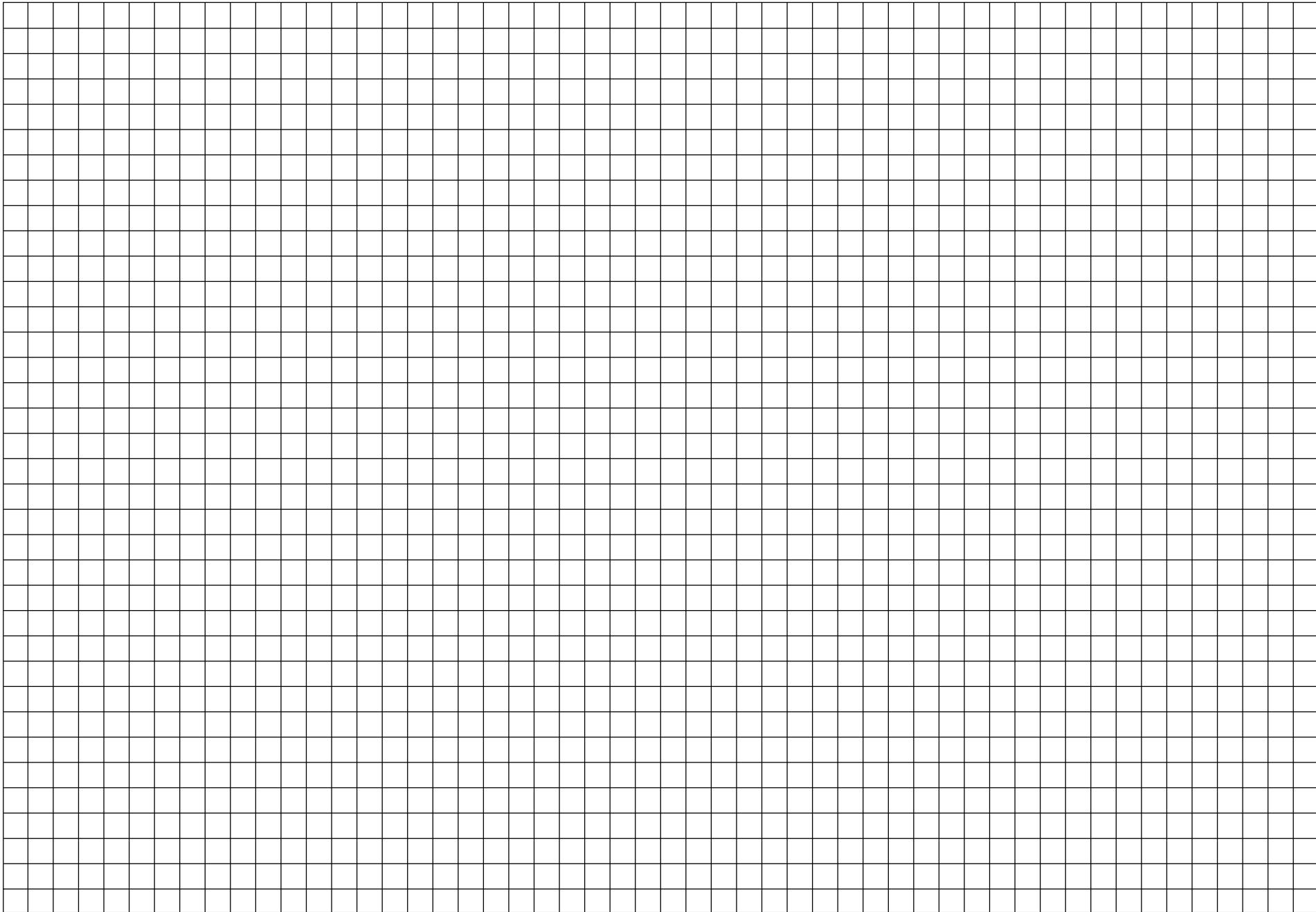
High School Math Reference Sheet

1 inch = 2.54 centimeters 1 kilometer = 0.62 mile   1 cup = 8 fluid ounces
1 meter = 39.37 inches   1 pound = 16 ounces   1 pint = 2 cups
1 mile = 5280 feet   1 pound = 0.454 kilogram 1 quart = 2 pints
1 mile = 1760 yards   1 kilogram = 2.2 pounds   1 gallon = 4 quarts
1 mile = 1.609 kilometers 1 ton = 2000 pounds   1 gallon = 3.785 liters
                                                     1 liter = 0.264 gallon
                                                     1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Reference Sheet — concluded

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$