The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, January 23, 2018 — 9:15 a.m. to 12:15 p.m.

MODEL RESPONSE SET

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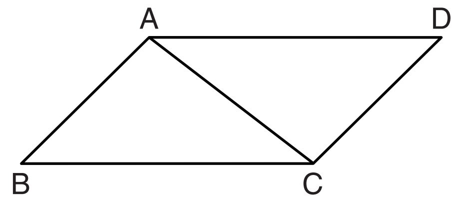
25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



Prove: $\triangle ABC \cong \triangle CDA$

① ▱Parallelogram ABCD, $\overline{AC}$ ① Given

② $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$ ② opposite sides of a parallelogram are $\cong$ and $\parallel$

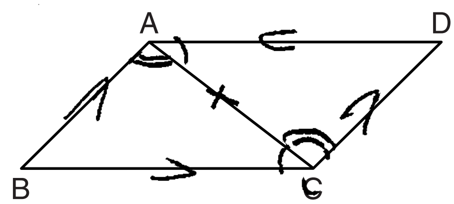
③ $\overline{AC} \cong \overline{AC}$ ③ reflexive p

④ $\triangle ABC \cong \triangle CDA$ ④ SSS $\cong$ SSS

Score 2: The student gave a complete and correct response.

Question 25

25 Given: Parallelogram *ABCD* with diagonal $\overline{AC}$ drawn

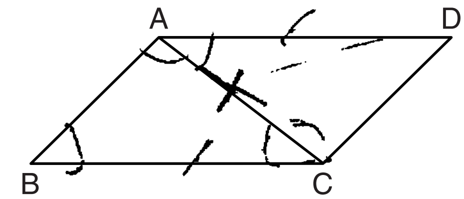


Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1. ABCD is a parallogam, $\overline{AC}$	1. Given
2. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$	2. Defintion of a parellogram
3. $m\angle DAC = m\angle ACB$ $m\angle BAC = m\angle ACD$	3. If two parralled lines are cut by a transversal, then alternate interior angles are equal.
4. $\overline{AC} = \overline{AC}$	4. Reflexive
5. $\triangle ABC \cong \triangle CDA$	5. ASA

Score 2: The student gave a complete and correct response.

25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



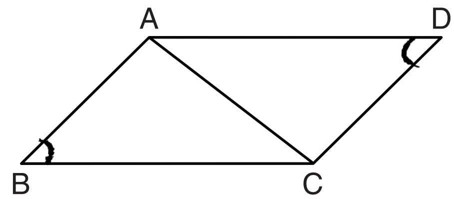
Prove: $\triangle ABC \cong \triangle CDA$

S	R
1. Parallelogram ABCD with Diagonal $\overline{AC}$ Dawn	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. Parallegram has 4 $\cong$ // Sides
3. $\angle DAC \cong \angle BCA$	3. 2 // lines cut by a trans alt interior $\angle$'s are $\cong$.
4. $\overline{AC} \cong \overline{AC}$	4. Refleexive
5. $\angle B = \angle C$ $\angle A = \angle D$	5. in a $\triangle$ $\cong$ sides of $\cong$ $\triangle$'s are $\cong$
6. $\triangle ABC \cong \triangle CDA$	6. ASA

Score 1: The student wrote a proof that demonstrates a good understanding of the method of proof, but some statements and/or reasons are missing or incorrect.

Question 25

25 Given: Parallelogram *ABCD* with diagonal $\overline{AC}$ drawn

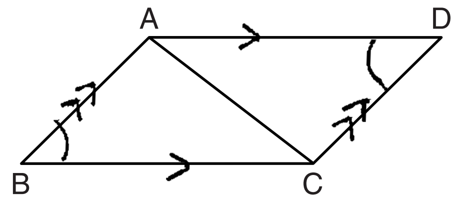


Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1) Para. ABCD 2) diagonal $\overline{AC}$	1) 2) Given
3) $\angle B \cong \angle D$	3) in a para. opp. $\angle$'s are $\cong$
4) $\overline{BC} \cong \overline{AD}$	4) in a para. opp. sides are $\cong$
5) $\overline{AB} \cong \overline{CD}$	5) in a para. opp. sides are $\cong$
$\triangle ABC \cong \triangle CDA$	By congruent parts

Score 1: The student did not state a correct reason of congruency in step 6.

25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



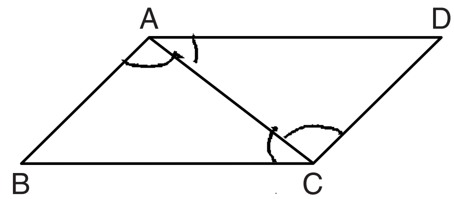
Prove: $\triangle ABC \cong \triangle CDA$

Statement	reason
parallelogram ABCD with diagonal $\overline{AC}$ drawn	given
$\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$	parallelograms have parallel sides
$\angle d \cong \angle C$	alternet interor angles are congruent
$\triangle ABC \cong \triangle CDA$	SAS

Score 0: The student did not state enough correct relevant statements and/or reasons to conclude the triangles are congruent by SAS.

Question 25

25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



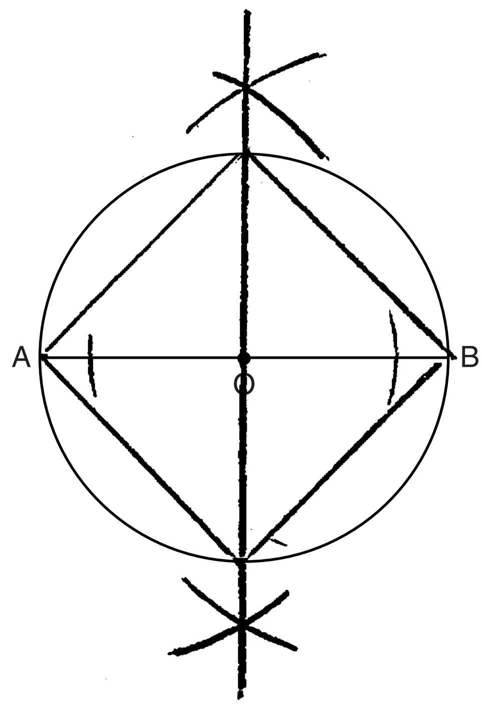
Prove: $\triangle ABC \cong \triangle CDA$

Statement	Reason
① $\angle A \cong \angle C$	① opp. $\angle$'s are $\cong$
② $\angle B \cong \angle D$	② opp. $\angle$'s are $\cong$
③ $\triangle ABC \cong \triangle CDA$	③ AAA

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 26

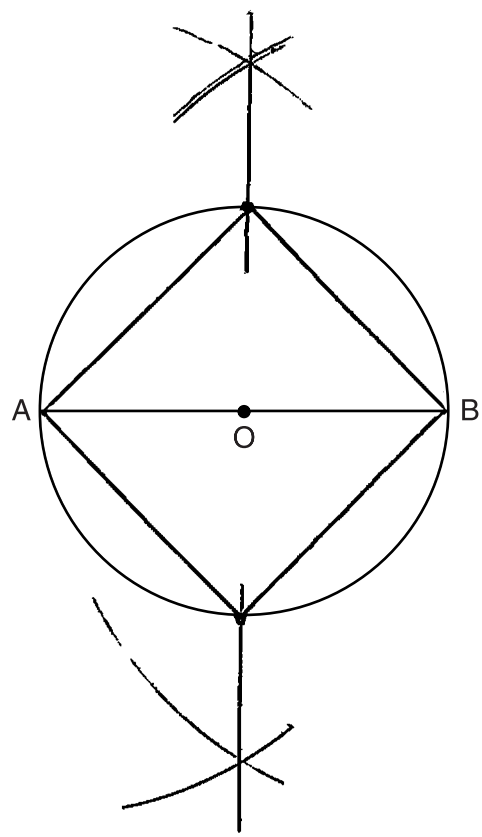
26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 26

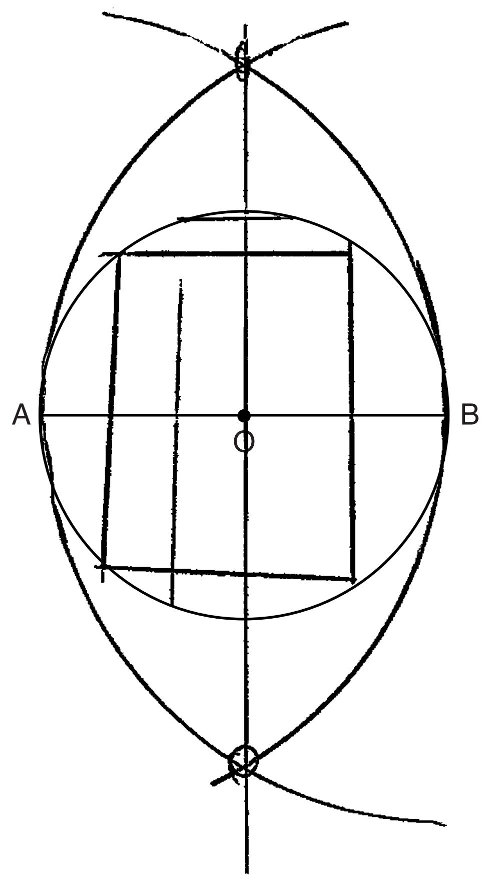
26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 26

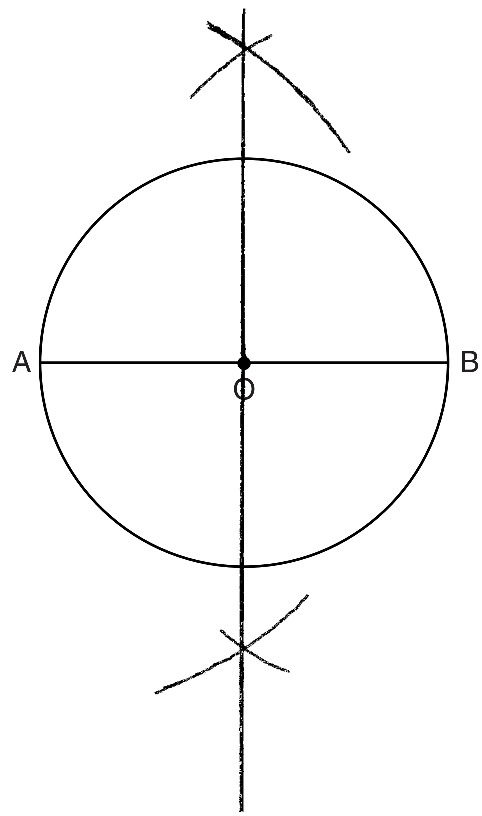
26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]



Score 1: The student drew an appropriate construction, but drew the square incorrectly.

Question 26

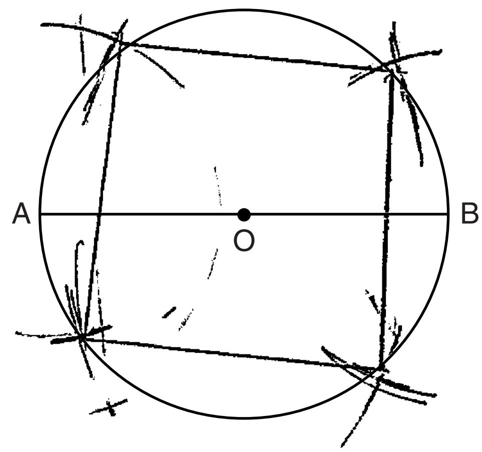
26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]



Score 1: The student drew an appropriate construction, but did not draw the square.

Question 26

26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]

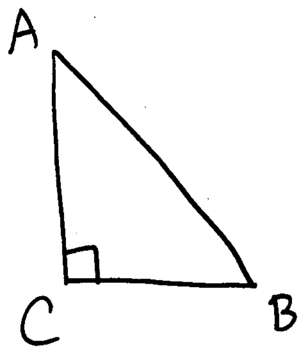


Score 0: The student had a completely incorrect response.

Question 27

27 Given: Right triangle ABC with right angle at C

If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.



Since sine and Cosine are cofunctions and $\angle A$ and $\angle B$ are Complementary, $\sin A = \cos B$.

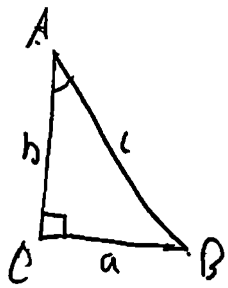
Therefore when $\sin A$ increases $\cos B$ increases.

Score 2: The student gave a complete and correct response.

Question 27

27 Given: Right triangle *ABC* with right angle at *C*

If sin *A* increases, does cos *B* increase or decrease? Explain why.



$\sin A = \frac{a}{c}$

$\cos B = \frac{a}{c}$

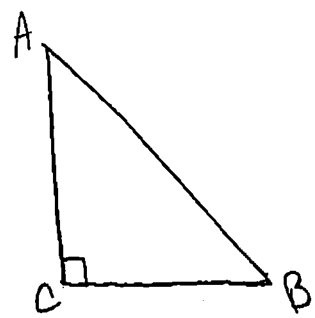
It also increases because the same ratio is used for sin A and cos B.

Score 2: The student gave a complete and correct response.

Question 27

27 Given: Right triangle *ABC* with right angle at *C*

If sin *A* increases, does cos *B* increase or decrease? Explain why.



$\sin A = \cos B$

So if A increases so does B

Score 1: The student wrote a partially correct explanation.

Question 27

27 Given: Right triangle *ABC* with right angle at *C*

If sin *A* increases, does cos *B* increase or decrease? Explain why.

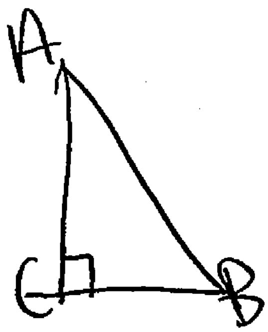
It increases because cos B and sin A are the same thing.

Score 1: The student wrote an incomplete explanation.

Question 27

27 Given: Right triangle ABC with right angle at C

If sin A increases, does cos B increase or decrease? Explain why.



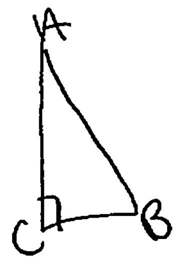
When Sin A increases cos B increases

Score 0: The student wrote increases, but no explanation was written.

Question 27

27 Given: Right triangle ABC with right angle at C

If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.

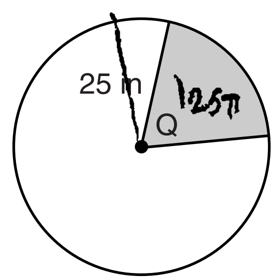


If sin A increases than cos B will decrease because there is only 180° in a triangle and if they both increase the degrees will go above 180.

Score 0: The student had a completely incorrect response.

Question 28

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$a = \pi r^2$ $\quad 360 \cdot .2 =$

$a = \pi\, 25^2$ $\quad 625\pi - 500\pi = 125\pi$

$a = \pi 625$

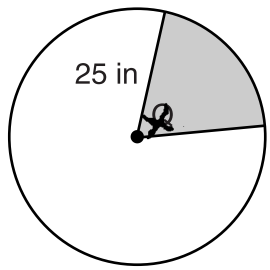
$\frac{125\pi}{625\pi}$ $\quad 20\%$

$\angle Q$ is $72°$

Score 2: The student gave a complete and correct response.

Question 28

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$(360)500\cancel{\pi} = \frac{x}{360}\cancel{\pi}(25)^2(360)$

$\frac{180000}{625} = \frac{625x}{625}$

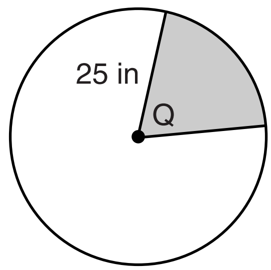
$288 = x$

$m\angle Q = 288°$

Score 1: The student calculated the measure of the central angle for the unshaded region.

Question 28

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$A = \pi r^2$

$A = \pi \cdot 25^2$

$A = 625\pi$

$625\pi - 500\pi = 125\pi$

Area of sector $= \frac{1}{2} r^2 \theta$

$(2) \cdot 125\pi = \frac{1}{2} \cdot 25^2 \theta \cdot (2)$

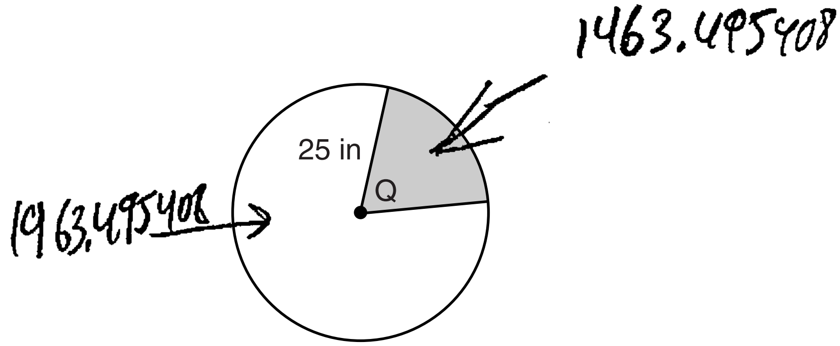
$\frac{250\pi}{625} = \frac{625\theta}{625}$

$\boxed{\frac{2\pi}{5} = \theta}$

Score 1: The student wrote the measure of the central angle in radian measure.

Question 28

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$$\frac{25}{500\pi \text{ in}^2} = \frac{x}{360}$$

$$A = \pi r^2$$
$$A = \pi(25)^2$$
$$A = 1963.495408$$

$$\frac{500x}{500} = \frac{9000}{500}$$

$x = 18°$ ← answer

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 29

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of 7.95 g/cm^3.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

$V = 1015 \text{ cm}^3$

\$ 0.29 per/kilo

density = 7.95 g/cm3

$D = \frac{m}{V}$

$7.95 = \frac{m}{1015}$ $m = 8069.25 \text{ g}$

m = 8.06925 kilograms

x 0.29

\$ 2.3400825

x 500

\$ 1,170.04125

$\boxed{= \$1,170}$

Score 2: The student gave a complete and correct response.

Question 29

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of 7.95 g/cm^3.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

1015 cm^3
$\times 500$
507500

507500
$\times\ 7.95$
$4{,}034{,}625$ grams

4034.625 kg
$\times$ \$.29 per kg

Roughly \$1170

Score 2: The student gave a complete and correct response.

Question 29

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of 7.95 g/cm^3.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

1015 · 7.95 = 8069.25 grams

80.6925 Kilograms

~~80.6925 · .29 = 23.400825~~

80 . 6925 · 500 = 40346.25

40346.25 · .29 = 11700.4125

It will cost ≈ 11,700 dollars

Score 1: The student did not correctly convert from grams to kilograms.

Question 29

29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of 7.95 g/cm^3.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

1 pound = 0.454 kilograms

$$\frac{1015}{7.95} = 127.672956$$

$$127.672956 \cdot 0.29 = 37.02515724$$

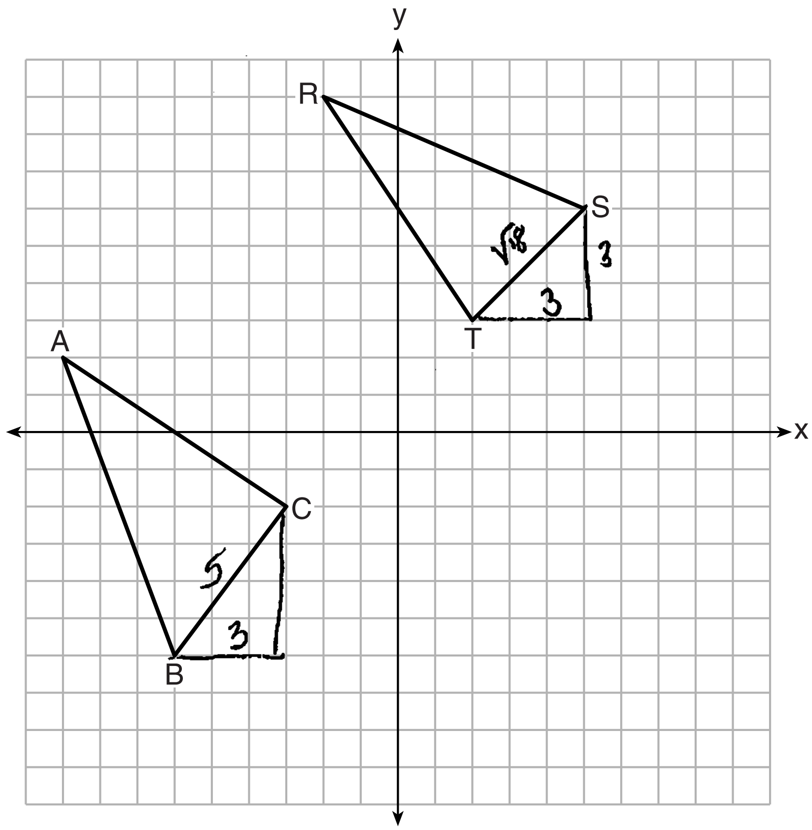
$$37.0251\!5724 \cdot 500 = 18512.57862$$

$\boxed{\therefore \text{The machinist will pay \$18,513 for the steel}}$

Score 0: The student did not convert from grams to kilograms and divided by the density instead of multiplying.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



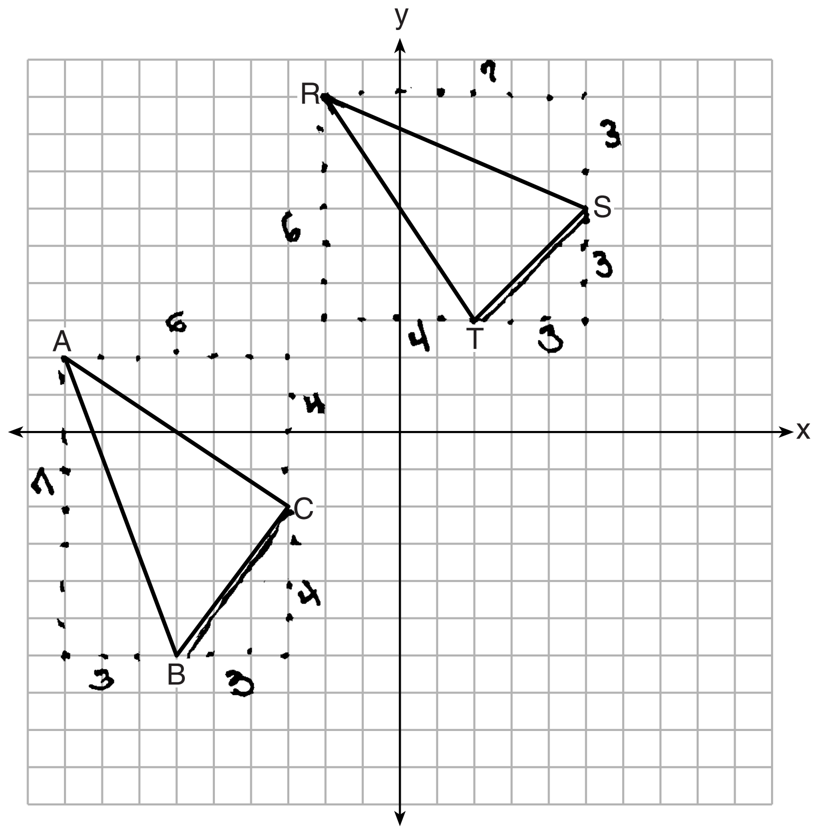
Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

No, $\overline{BC} \ncong \overline{ST}$ so $\triangle ABC \ncong \triangle RST$, there is no sequence of rigid motions that would map $\triangle ABC$ onto $\triangle RST$

Score 2: The student gave a complete and correct response.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

No

No. You could flip Δ ABC to map over Δ RST, but $\overline{BC}$ and $\overline{ST}$ would not match since they are not $\cong$.

$\overline{ST} \cong \sqrt{(5,-2)^2+(6-3)^2}$ $\overline{BC} = \sqrt{(-6+3)^2+(-6+2)^2}$ $d = \sqrt{(x_2-x_1)+(y_2-y_1)}$ distance formula

5,6) (2,3) $= \sqrt{9+9}$ (-6,-6) (-3,-2) $= \sqrt{9+16}$

$= \sqrt{18}$ $= \sqrt{25}$

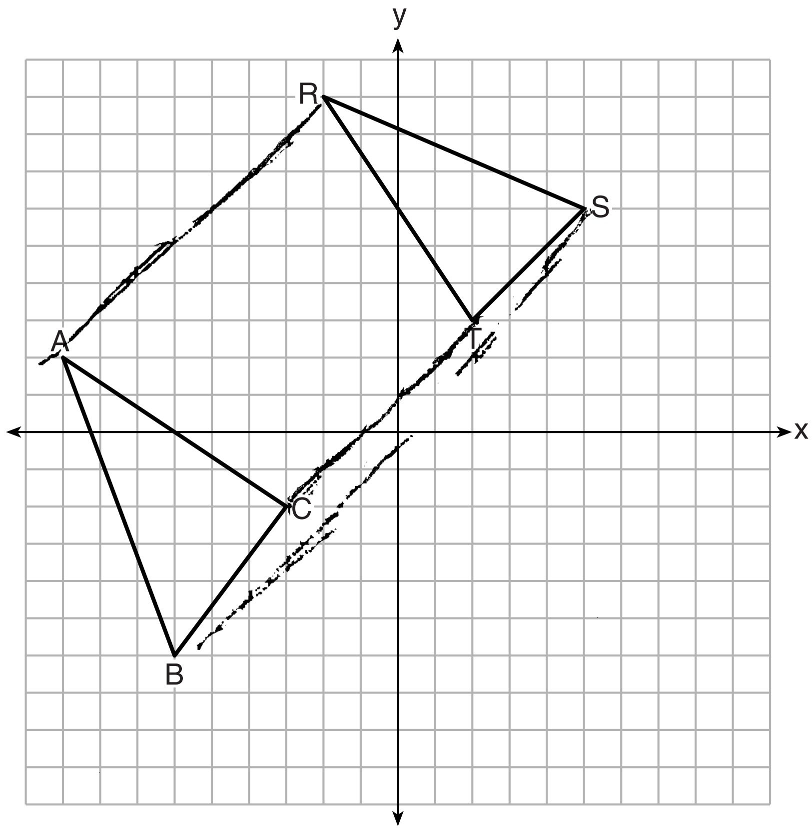
$= 3\sqrt{2}$ $= 5$

The distances are different therefore, triangles are not congruent.

Score 2: The student gave a complete and correct response.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



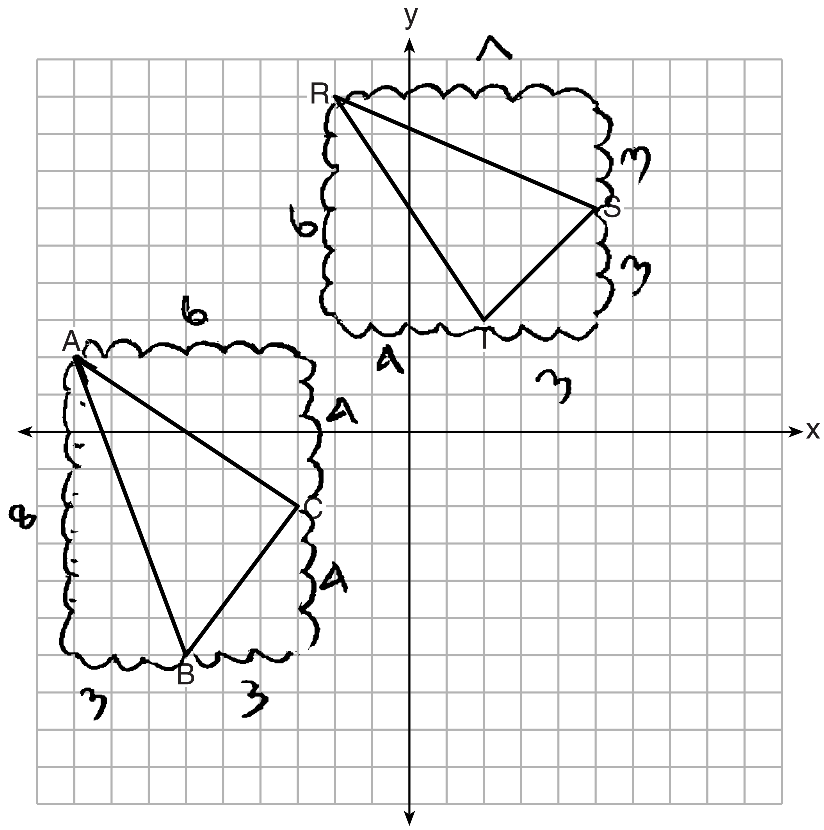
Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

$\triangle$ABC isn't congruent to $\triangle$RST because in order for the rigid motion to properly work each point would have to change in the (-y, -x) format. Point B to Point S doesn't follow this rule.

Score 1: The student wrote an incomplete explanation by not using the properties of rigid motions.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



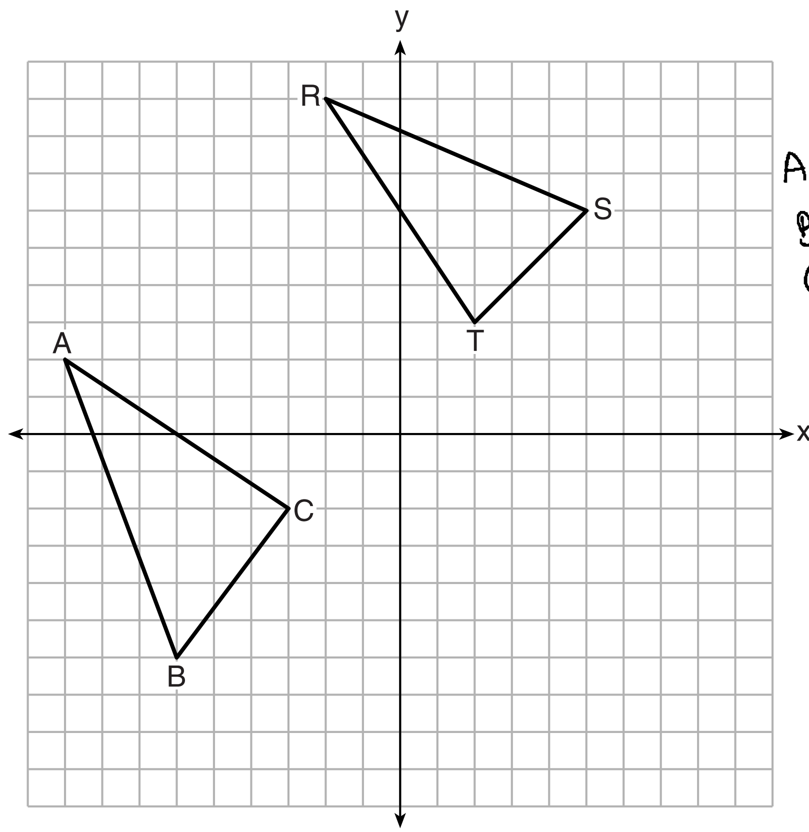
Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

$\triangle ABC$ is not congruent to $\triangle RST$ because the sides are not equal and it have different length of sides and the slope of the two triangle are different.

Score 1: The student wrote an incomplete explanation by not using the properties of rigid motions.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



A (-9,2) R (-2,9)
B (-6,-6) S (5,6)
C (-3,-2) T (2,3)

$\frac{y-y}{x-x}$ $\frac{-2-9}{-9--2} = \frac{-7}{-7}$

$\frac{-6-6}{-6-5}$ $\frac{-12}{-11}$

$\frac{-2-3}{-3-2}$ $\frac{-5}{-5}$

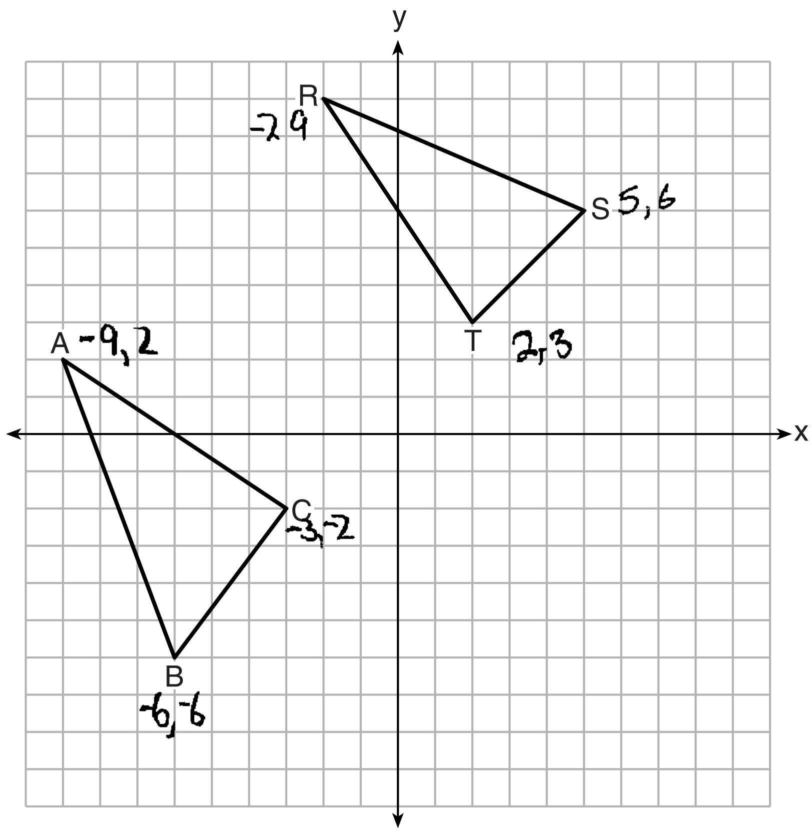
Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

no, because they have different slopes
So therefore they arent congruent.

Score 0: The student had a completely incorrect response. Preserving slope is not a property of rigid motions.

Question 30

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



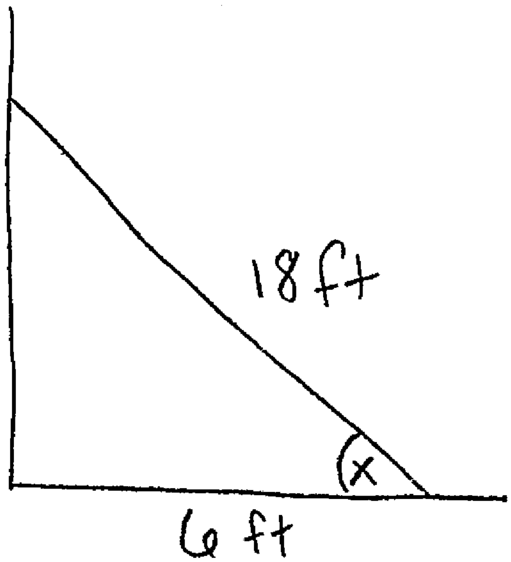
Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

No they are not congruent!

Score 0: The student did not write an explanation.

Question 31

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree,* the measure of the angle the bottom of the ladder makes with the ground.



$\cos \theta = \frac{6}{18}$

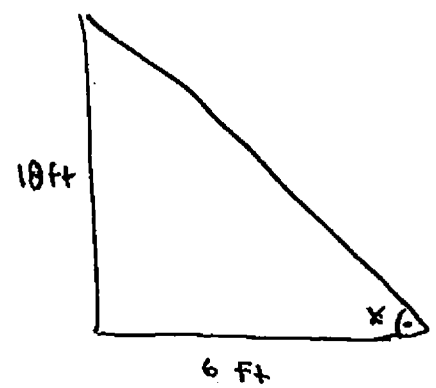
$x^\circ = 70.5$

$\boxed{\approx 71^\circ}$

Score 2: The student gave a complete and correct response.

Question 31

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree,* the measure of the angle the bottom of the ladder makes with the ground.



$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

$\tan x = \frac{18}{6}$

$\tan x = 3$

$\tan^{-1}(3)$

$x = 72^\circ$

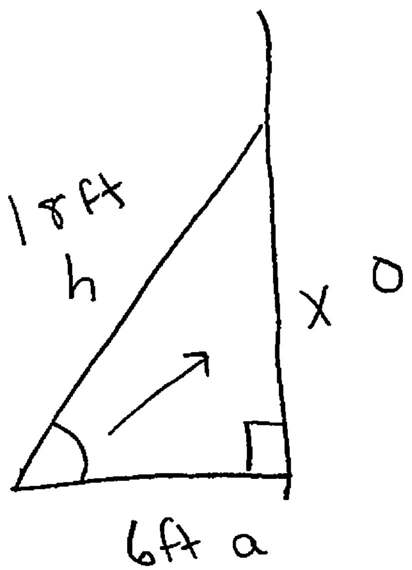
The measuere of angle is 72

Score 1: The student wrote an incorrect trigonometric equation, but solved the equation correctly.

Question 31

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree,* the measure of the angle the bottom of the ladder makes with the ground.

$S\frac{O}{H} \quad C\frac{a}{H} \quad T\frac{O}{a}$



$\cos = \frac{a}{H}$

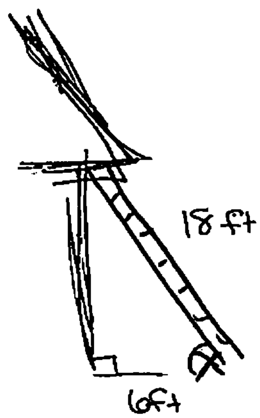
$\frac{\cos x}{1} = \frac{6}{18}$

$\frac{6}{18} = \frac{\cos x(18)}{18}$

Score 1: The student wrote a correct trigonometric equation, but no further correct work was shown.

Question 31

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.



$a^2 + b^2 = c^2$

$18^2 + b^2 = c^2$

$324 + 36 = c^2$

$\sqrt{360} = \sqrt{c^2}$

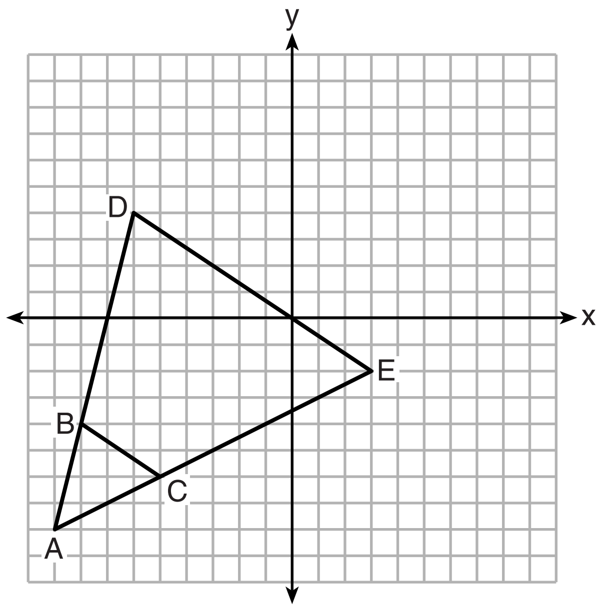
$c = 18.97366596$

$x = 19°$

Score 0: The student had a completely incorrect response.

Question 32

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

$\triangle$ABC is dilated by a scale factor of 3 ~~A~~ centered at pont A

Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

Dilations preserve angle measure, so $\angle A \cong \angle A$
$\angle ABC \cong \angle D$
$\angle ACB \cong \angle E$

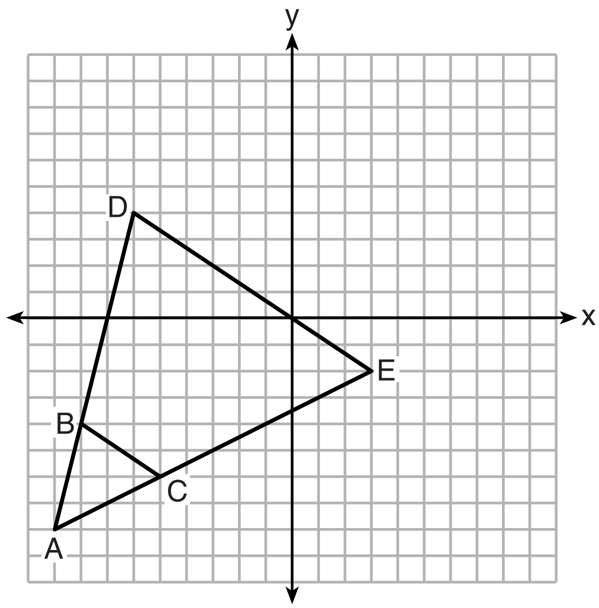
Using ans 2 pairs of angle

$\triangle ABC \sim \triangle ADE$ by AA

Score 4: The student gave a complete and correct response.

Question 32

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

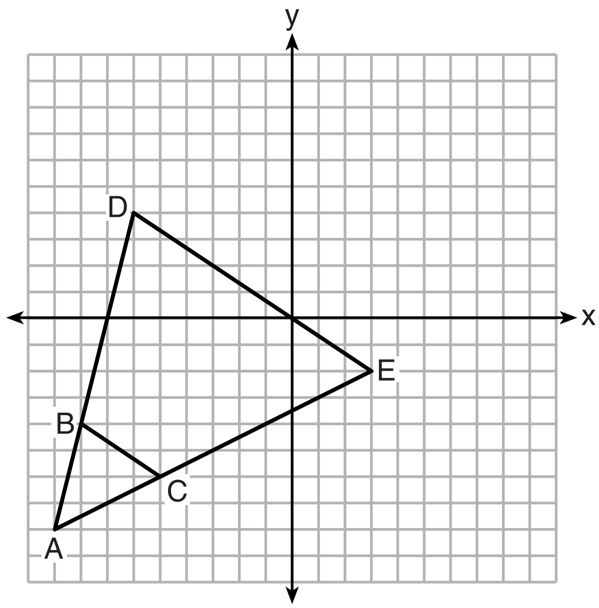
A dilation of 3 about point A would map $\triangle ABC$ onto $\triangle ADE$.

Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

A dilation makes two figures proportional, therefore $\triangle ADE$ is similar to $\triangle ABC$.

Score 3: The student made an incorrect statement that figures are proportional.

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

A transforation that maps triangle ABc onto ADE would be a dialation

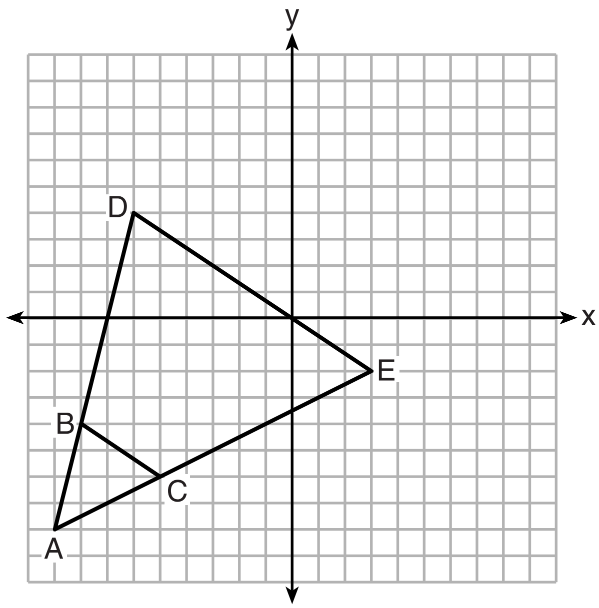
Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

This transformation makes triangle ADE similar to Δ ABC b/c the angles are the same.

Score 2: The student did not identify the center of dilation and the scale factor. The student did not provide a complete explanation connecting the transformation and the similarity.

Question 32

32 Triangle ABC and triangle ADE are graphed on the set of axes below.



Describe a transformation that maps triangle ABC onto triangle ADE.

A Dilation of 3 would map $\triangle ABC$ onto $\triangle ADE$.

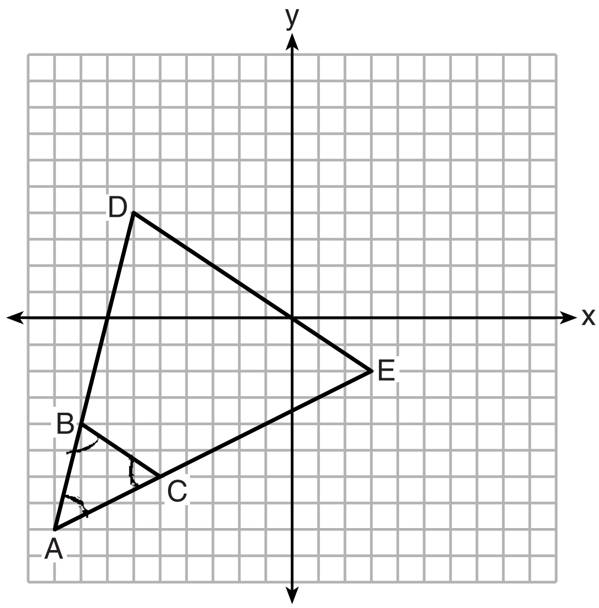
Explain why this transformation makes triangle ADE similar to triangle ABC.

The triangle is the same just dilated.

Score 1: The student wrote an incomplete description of the dilation by not stating the center of dilation. No further correct work was shown.

Question 32

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

Transformation :

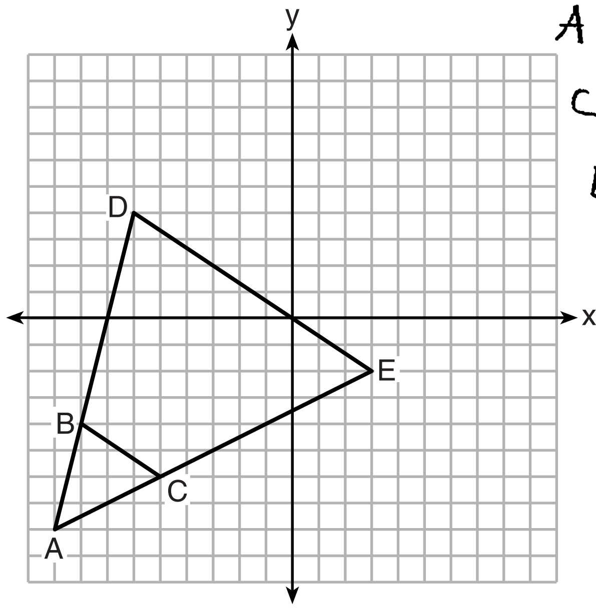
Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

because Transformation preserves the angle measurement

Score 1: The student did not describe the transformation. The student did not provide a complete explanation.

Question 32

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



A(-9,-8) B(-8,-4)
C(-5,-6) D(-6,4)
E(3,-2)

Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

A dialation of (3, 12) would map △ABC onto △ADE.

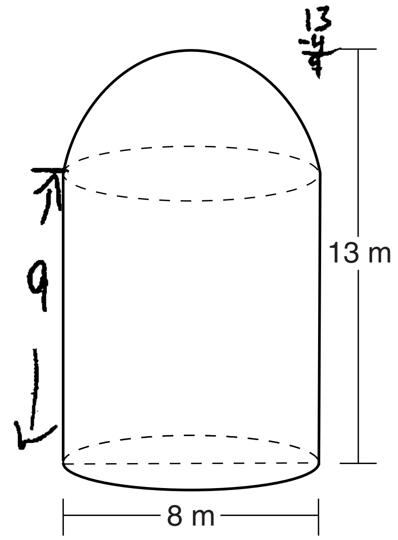
Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

It does because they would have the same coordinate points, making all of their sides congruent.

Score 0: The student had a completely incorrect response.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi 4^3$

$V = \frac{4}{3}\pi 64$

$V = \frac{268.0825731}{2}$

$V \approx 134.0412866$

$V = \pi r^2 h$

$V = \pi 4^2(9)$

$V = \pi 16(9)$

$V = \pi 144$

$V = 452.3893421$

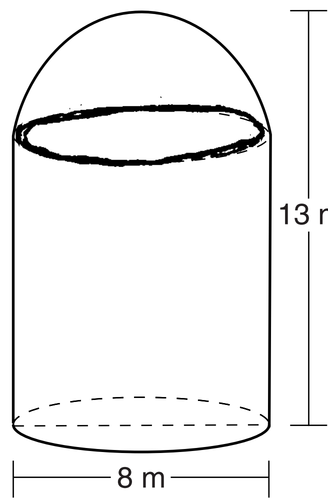
$$\begin{array}{r} 134.0412866 \\ +\ 452.3893421 \\ \hline V = 586.4306287 \end{array}$$

$\boxed{V \approx 586 m^3}$

Score 4: The student gave a complete and correct response.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(4)^3$

$V = \frac{4}{3}(64)\pi$

$V = \frac{268}{2}$ 134

$V = \pi r^2 h$

$V = \pi(4)^2(13)$

$V = 208\pi$

$V = 653$

$$\begin{array}{r} 134 \\ +653 \\ \hline \end{array}$$

$\boxed{V = 787\text{m}^3}$

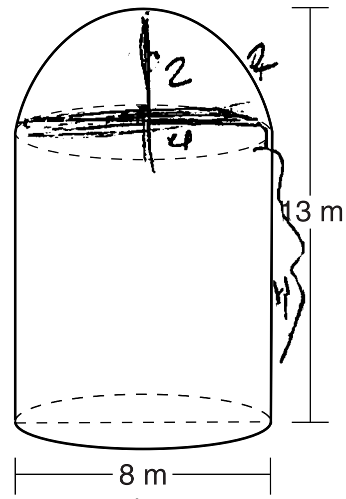
Score 3: The student used 13 instead of 9 for the height of the cylinder.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.

$V=\pi r^2 h$

$V=\dfrac{\frac{4}{3}\pi r^3}{2}$



$13-2=11$

$r=4$

Cylinder

$V=\pi\, 4^2(11)$

$V=16(11)\pi$

$V=176\pi$ +

Sphere

$V=\dfrac{\frac{4}{3}\pi(2)^3}{2}$

$V=\dfrac{\frac{4}{3}\left(\frac{8}{1}\right)\pi}{2}$

$V=\dfrac{10.6666667\pi}{2}$

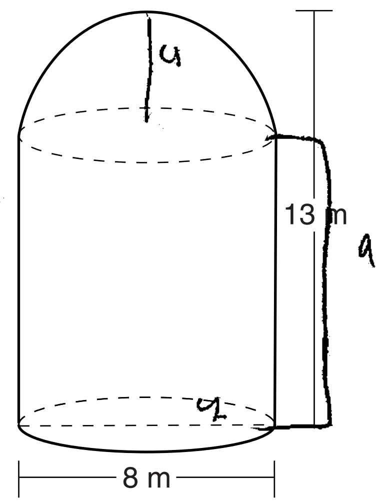
$\frac{4}{3}\times\frac{8}{1}=\frac{32}{3}$

569.7 m^3

Score 2: The student made one computational error in determining the radius and one rounding error.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \pi r^2 h$

$V = 4^2 \cdot 9 \cdot \pi$

$V = 144\pi$

$V = 4/3\, \pi r^3$

$V = 4/3\, \pi\, (4)^3$

$V = 4/3\, \pi\, (64)$

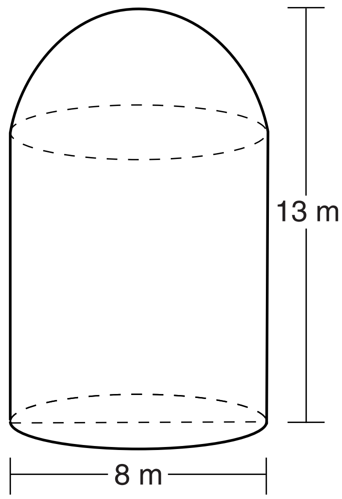
$V = (144\pi) + (4/3\, \pi\, (64))$

$\boxed{V = 720.5 \text{ m}^3}$

Score 2: The student did not divide the volume of a sphere by two and then rounded incorrectly.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \pi r^2 h$

$V = \pi 4^2 13$

$V = \pi(16)(13)$

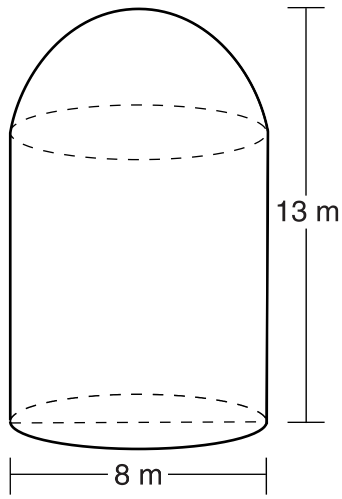
$V = 653.4571219$

$V = 653.5$

Score 1: The student made one conceptual error by assuming the entire tank is a cylinder and made one rounding error.

Question 33

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \pi r^2$

$V = \pi 4^2$

$V = 16\pi$

$V = 50$

$L = 2\pi r^2$

$L = 2\pi 4^2$

$L = 2\pi 16$

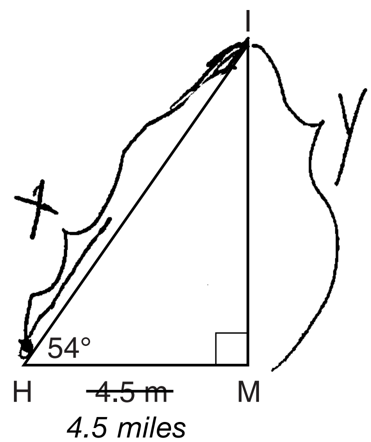
$L = 101$

151

Score 0: The student had a completely incorrect response.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$$\cos 54° = \frac{4.5}{x}$$

$$\frac{4.5}{\cos 54}$$

7.7 mi

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

TOA

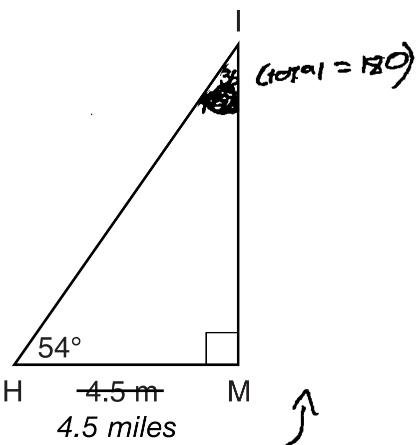
$$\tan 54° = \frac{y}{4.5}$$

$$4.5 \tan 54$$

6.2 mi

Score 4: The student gave a complete and correct response.

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



tan 54 =

$\tan 54 = \frac{x}{4.5}$; IM = 6.193

Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

Pythagorean theorem:

$4.5^2 + 6.2^2 = x^2$

$20.25 + 38.44 = \sqrt{58.69} = 7.66 =$ $\boxed{7.7 \text{ miles}}$

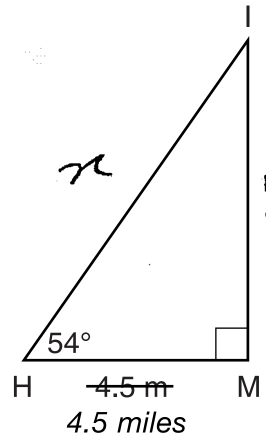
Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

$\boxed{6.2 \text{ miles}}$

Score 4: The student gave a complete and correct response.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Distance from boat house (H) to Island is = 7.7 mi

Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$$\frac{\cos 54°}{1} = \frac{4.5}{x}$$

$$\frac{x \cancel{\cos 54°}}{\cancel{\cos 54°}} = \frac{4.5}{\cos 45°}$$

$$x = 7.65$$

$\boxed{x = 7.7 \text{ mi}}$ answer

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

$$\frac{\tan 54°}{1} = \frac{y}{4.5}$$

$$y = 4.5(\tan 54°)$$

$$y = 0.35$$

$\boxed{y = 0.4 \text{ mi}}$ answer

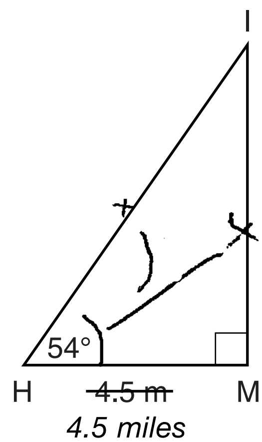
Distance from Island (I) to marina (M) is = 0.4mi

Score 3: The student made a computational error in finding IM.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.

SoHcAHToa



Tan

Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$\cos 54 = \frac{4.5}{x}$

$4.5 = \frac{\cos 54x}{10}$

7.7 miles

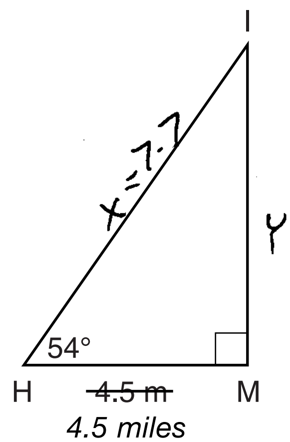
Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

6.2 miles

Score 3: The student showed no work to determine IM.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$$\cos 54^\circ = \frac{4.5}{x}$$

$$\frac{x \cos 54^\circ}{\cos 54^\circ} = \frac{4.5}{\cos 54^\circ}$$

$$x = 7.655857$$

$$\boxed{x = 7.7}$$

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

$$\tan 54^\circ = \frac{4.5}{y}$$

$$\frac{y \tan 54^\circ}{\tan 54^\circ} = \frac{4.5}{\tan 54^\circ}$$

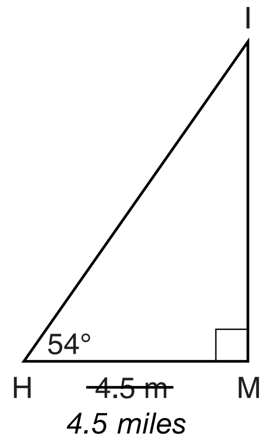
$$y = 3.2694$$

$$\boxed{y = 3.2}$$

Score 2: The student found HI correctly, but wrote an incorrect trigonometric equation and rounded incorrectly.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$$\cos°54 = \frac{4.5}{X}$$

$$\overline{HI} = 2.6 \text{ miles}$$

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

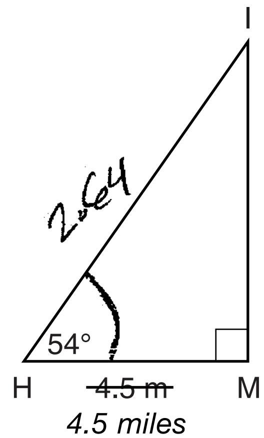
$$\text{Tan}°54 = \frac{2.6}{X}$$

$$\overline{IM} = 3.6 \text{ miles}$$

Score 1: The student wrote a correct trigonometric equation to find HI, but no further correct work was shown.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



$\frac{\cos 54}{1} = \frac{x}{4.5}$

$x = 4.5 \cos 54$

$x = 2.64$

$\frac{\tan 54}{1} = \frac{x}{4.5}$

$x = 4.5 \tan 54$ $x = 6.19$

Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

From H to I = 2.64

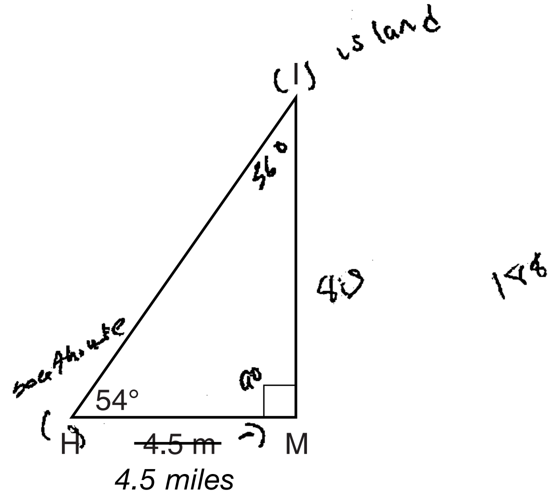
Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

From I to M = 6.19

Score 1: The student made two rounding errors and wrote an incorrect trigonometric equation to find HI.

Question 34

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

The nearest distance from the boat

$a^2+b^2=c^2$

$4.5(4.9) = 54°$

$(4.5)^2 (20.25 + 20.25 = 20.25$

$= 6.4$

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

The nearest tenth of a mile is

4.5

Score 0: The student did not show enough correct relevant work to receive any credit.

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$AT = \sqrt{(-4-5)^2 + (5--2)^2} = \sqrt{130}$

$PA = \sqrt{(-4--1)^2 + (5--6)^2} = \sqrt{130}$

$\triangle PAT$ is isosceles b/c $AT = PA$.

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

(2,9)

Question 35 is continued on the next page.

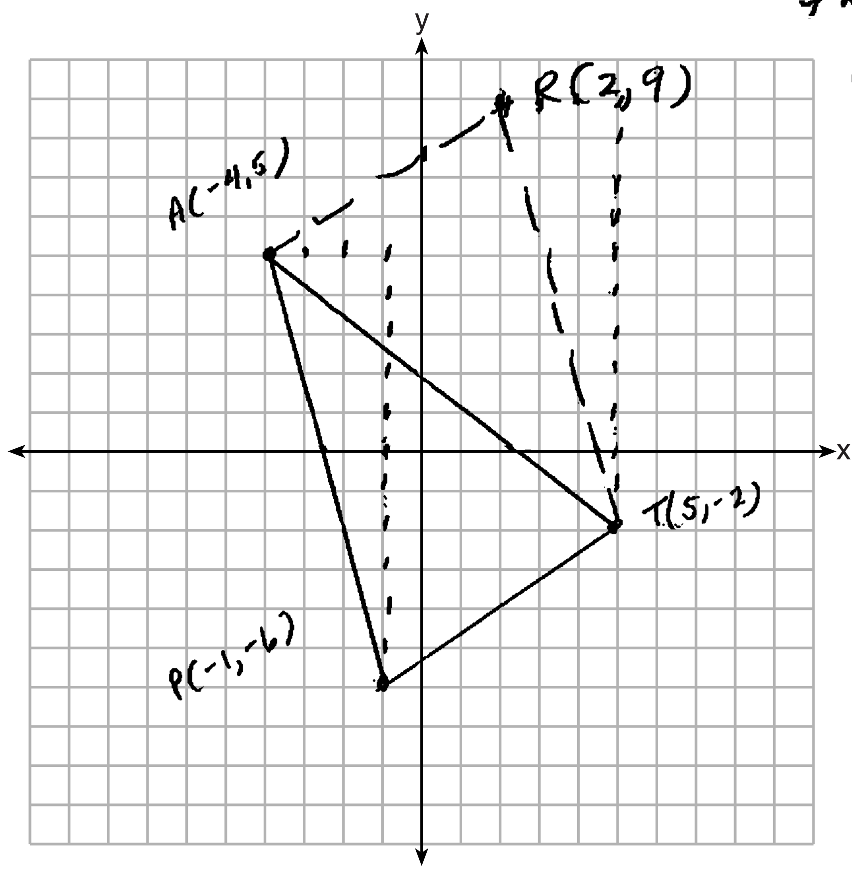
Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

Slope of AP = $-11/3$ PA = $\sqrt{130}$

Slope of RT = $-11/3$ RT = $\sqrt{(2-5)^2+(9- -2)^2} = \sqrt{130}$

PART is a parallelogram b/c one pair of opp. sides are both ∥ as demonstrated by the equal slopes, and ≅, as demonstrated by the distance formula



Score 6: The student gave a complete and correct response.

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$\overline{PA} = \sqrt{3^2+11^2} = \sqrt{130}$

$TA = \sqrt{9^2+7^2} = \sqrt{130}$

$\therefore$ $\triangle PAT$ is isoscelos b/c 2 sides are $\cong$

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

R(2,9)

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

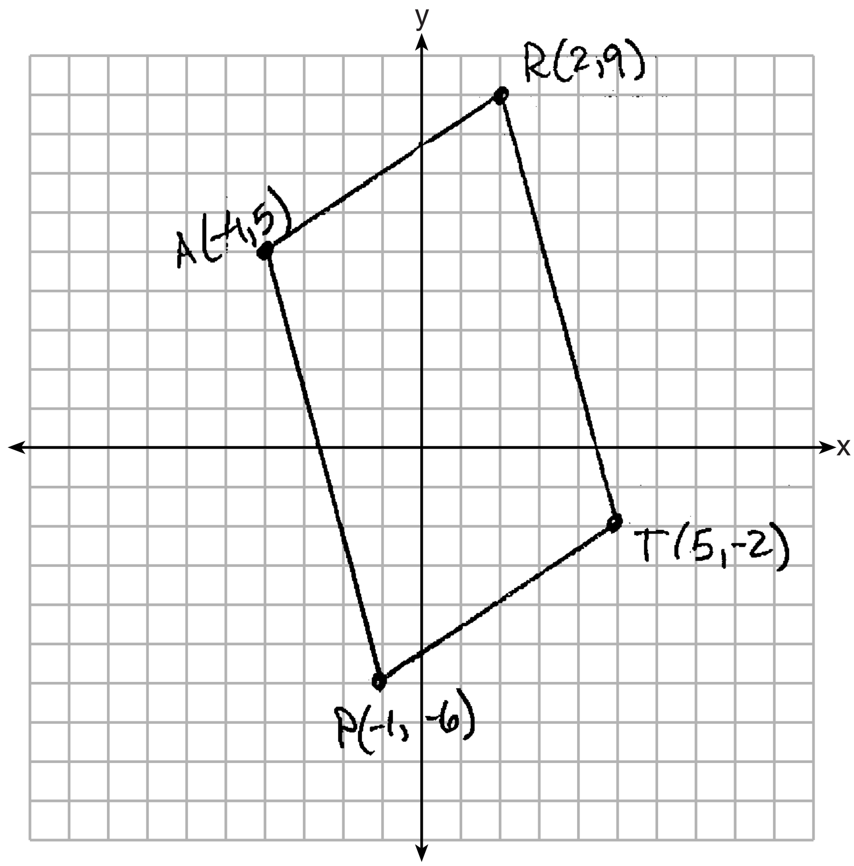
Slope of $\overline{AR} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4}{-6} = \frac{2}{3}$

slope of $\overline{PT} = \frac{y_2-y_1}{x_2-x_1} = \frac{-4}{-6} = \frac{2}{3}$

Slope of $\overline{PA} = \frac{-6-5}{-1+4} = -\frac{11}{3}$

Slope of $TR = \frac{9+2}{2-5} = -\frac{11}{3}$

$\therefore$ PART is a parallelogram



Score 5: The student wrote an incomplete conclusion when proving *PART* is a parallelogram.

35 In the coordinate plane, the vertices of triangle *PAT* are *P*(−1,−6), *A*(−4,5), and *T*(5,−2). Prove that △*PAT* is an isosceles triangle. [The use of the set of axes on the next page is optional.]

△PAT is an isos △ because $\overline{AT}$ and $\overline{PA}$ are = lengths.

Distance of:

$\overline{PA} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(-4--1)^2+(5--6)^2}$

$\sqrt{9+121}$ $\sqrt{130}$

$\overline{TA} =$

$\sqrt{(5--4)^2+(-2-5)^2}$

$\sqrt{81+49}$

$\sqrt{130}$

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

R (2,9)

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

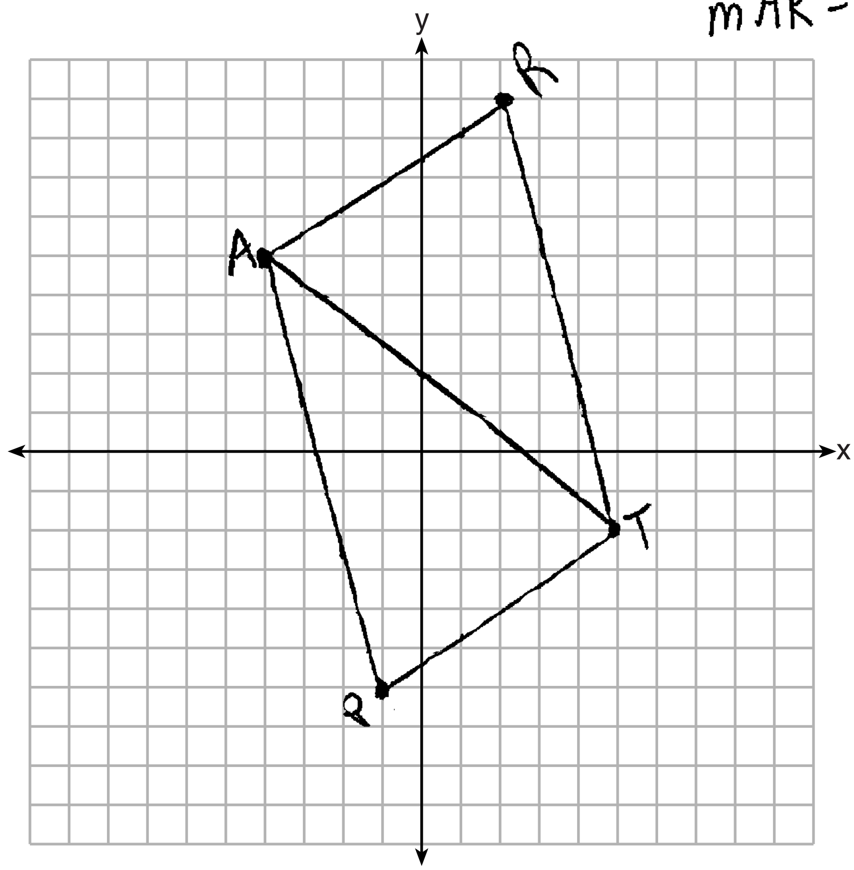
PART is a parallelogram because it has 2 sets of ∥ sides.

$m\overline{AP} = \frac{11}{3}$

$m\overline{RT} = \frac{-11}{3}$

$m\overline{PT} = \frac{4}{6} = \frac{2}{3}$

$m\overline{AR} = \frac{4}{6} = \frac{2}{3}$



Score 5: The student did not connect the equal slopes to parallelism in proving *PART* as a parallelogram, therefore the concluding statement is incomplete.

Question 35

35 In the coordinate plane, the vertices of triangle *PAT* are *P*(−1,−6), *A*(−4,5), and *T*(5,−2). Prove that △*PAT* is an isosceles triangle. [The use of the set of axes on the next page is optional.]

PA

$d=\sqrt{3^2+11^2}$

$d=\sqrt{9+121}$

$\boxed{d=\sqrt{130}}$

TA

$d=\sqrt{7^2+9^2}$

$d=\sqrt{49+81}$

$\boxed{d=\sqrt{130}}$

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

$\boxed{R=(2,9)}$

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

AR = $y = mx + b$ PT = $y = mx + b$

$\boxed{y = \frac{2}{3}x + b}$ $\boxed{y = \frac{2}{3}x + b}$

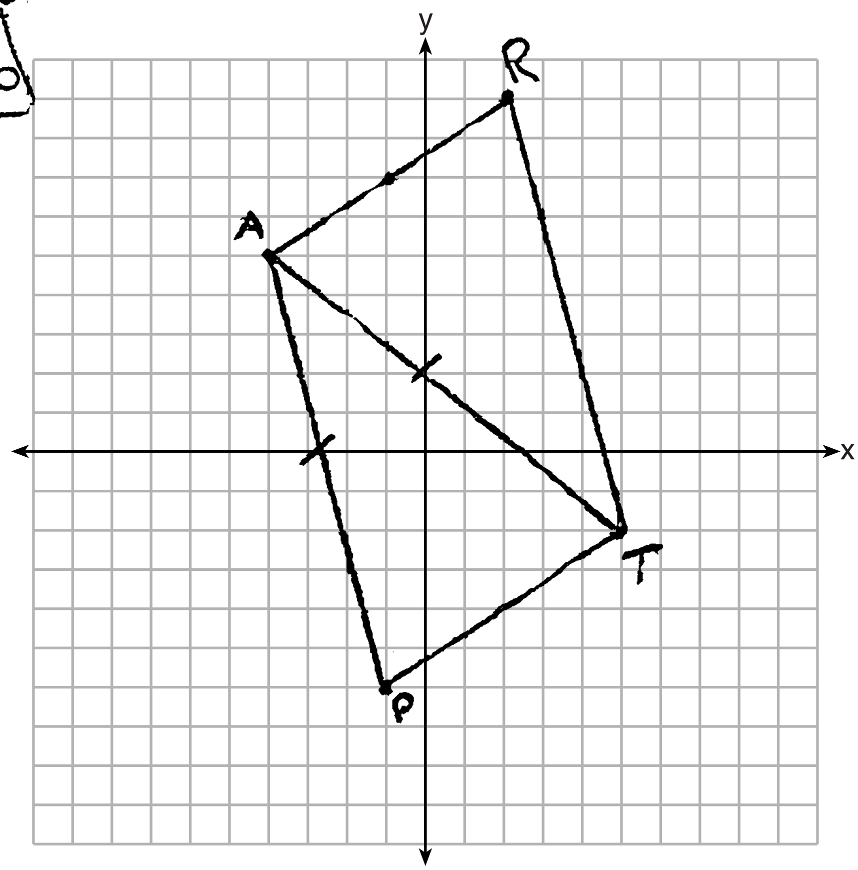
RT = $y = mx + b$

$\boxed{y = \frac{-11}{3x} + b}$

PA = $y = mx + b$

$\boxed{y = \frac{-11}{3}x + b}$

PART is a parallelogram because opposite sides are parallel



Score 4: In proving $\triangle PAT$ is isosceles, no conclusion was written. In proving *PART* as a parallelogram, the student did not connect the equal slopes to parallelism.

Question 35

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

R(2,9)

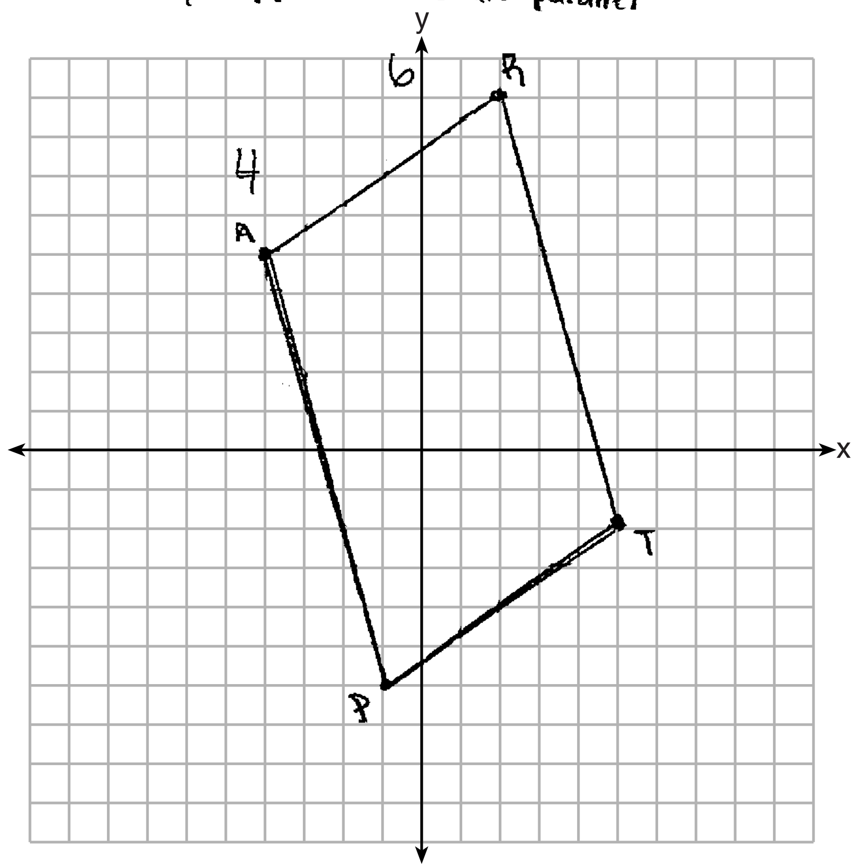
Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

PA $\frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{5+6}{-4+1} \rightarrow \frac{11}{-3} ? -\frac{11}{3}$

Statements	Reasons
1) $\overline{PA}$ has a slope of $-\frac{11}{3}$	1) slope formula $\left(\frac{y_2-y_1}{x_2-x_1}\right)$
2) $\overline{RT}$ has a slope of $-\frac{11}{3}$	2) slope formula $\left(\frac{y_2-y_1}{y_2-x_1}\right)$
3) $\overline{PA} \parallel \overline{RT}$	3) $\overline{PA}$ + $\overline{RT}$ have the same slope
4) $\overline{AR}$ has a slope of $\frac{2}{3}$	4) slope formula $\left(\frac{y_2-y_1}{x_2-x_1}\right)$
5) $\overline{PT}$ has a slope of $\frac{2}{3}$	5) slope formula $\left(\frac{y_2-y_1}{x_2-x_1}\right)$
6) $\overline{AR} \parallel \overline{PT}$	6) $\overline{AR}$ + $\overline{PT}$ have the same slope
7) quadrilateral PART	7) opposite sides are parallel



Score 3: The student did not prove $\triangle PAT$ is an isosceles triangle. The student wrote an incomplete statement in proving *PART* is a parallelogram (step 7).

Question 35

35 In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

AP $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$D = \sqrt{(-4 + (1))^2 + (5 + (+6))^2}$

$D = \sqrt{(-3)^2 + 11^2}$

$D = \sqrt{9 + 121}$

$D = \sqrt{130}$

AT $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$D = \sqrt{(5 + (+4))^2 + (-2 + -5)^2}$

$D = \sqrt{9^2 + (-7)^2}$

$D = \sqrt{81 + 49}$

$D = \sqrt{130}$

Using the distance formula

$\overline{AP} = \overline{AT} = \sqrt{130}$

$\therefore \overline{AP} \cong \overline{AT}$

If two legs of a triangle are congruent, then the triangle is isosceles.

$\therefore$ PAT is an isosceles triangle

State the coordinates of R so that quadrilateral $PART$ is a parallelogram.

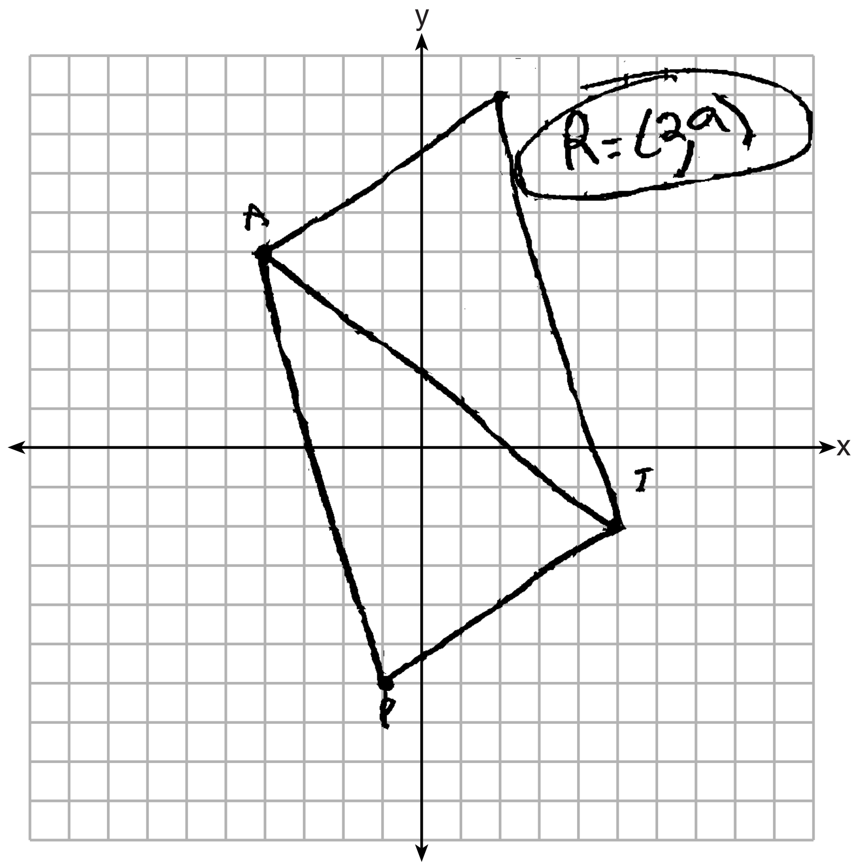
$R = (2, 9)$

Question 35 is continued on the next page.

Question 35

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.



Score 3: The student correctly proved that $\triangle PAT$ is isosceles and correctly identified point (2,9). No further correct work was shown.

35 In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$PA = \sqrt{(-4-{-1})^2+(5-{-6})^2}$

$= \sqrt{(-3)^2+11^2}$

$= \sqrt{9+121}$

$= \sqrt{130}$

$AT = \sqrt{(5-{-4})^2+(-2-5)^2}$

$= \sqrt{9^2+(-7)^2}$

$= \sqrt{81+49}$

$= \sqrt{130}$

$\triangle PAT$ is isosceles because two of its sides are congruent

$\sqrt{130} = \sqrt{(x-{-4})^2+(y-5)^3}$

$\sqrt{130} = \sqrt{x^2+16+y^2+25}$

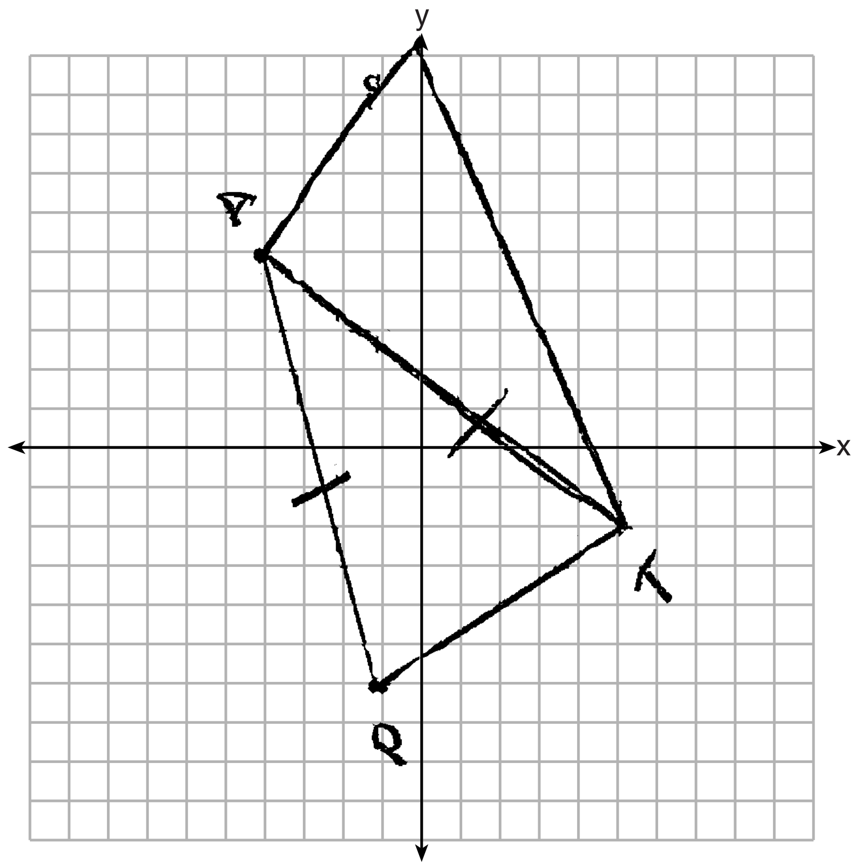
$\sqrt{130} = \sqrt{x^2+y^2+41}$

State the coordinates of R so that quadrilateral $PART$ is a parallelogram.

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.



Score 2: Isosceles triangle *PAT* was proven, but no further correct work was shown.

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$9^2+7^2=d^2$

You can tell
ΔPAT is
iscocelese
be cause
$\overline{PA}$ is parallel
to $\overline{TA}$.

$11^2+3^2=c^2$

State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

R= 2,9

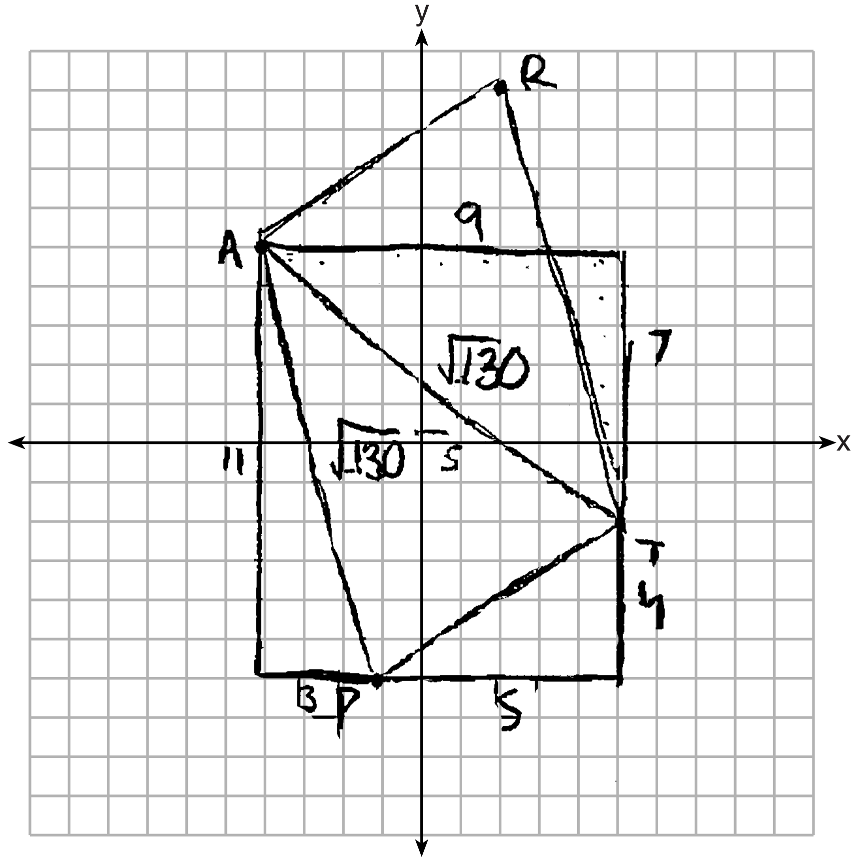
Question 35 is continued on the next page.

Question 35

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

Part is a parallelogram because $\overline{AP}$ and $\overline{RT}$ have the same slopes and $\overline{AR}$ and $\overline{PT}$ have the same slope meaning opposite sides have the same slope.



Score 1: The student correctly found the lengths of $\overline{AP}$ and $\overline{AT}$, but no further correct work was shown. Point *R* was not written as coordinates.

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

Statements	Reasons
① $\angle RAP \cong PTR$ $\angle APT \cong ART$	① Opposite angles are congruent
② $\overline{AR} \parallel \overline{PT}$ $\overline{AP} \parallel \overline{RT}$	② Parallel sides are congruent
③ $\overline{AT} \cong \overline{AT}$	③ Reflexive Postulate
④ $\triangle ART \cong \triangle TRR$	④ Triangles reflected over the reflexive postulate are congruent

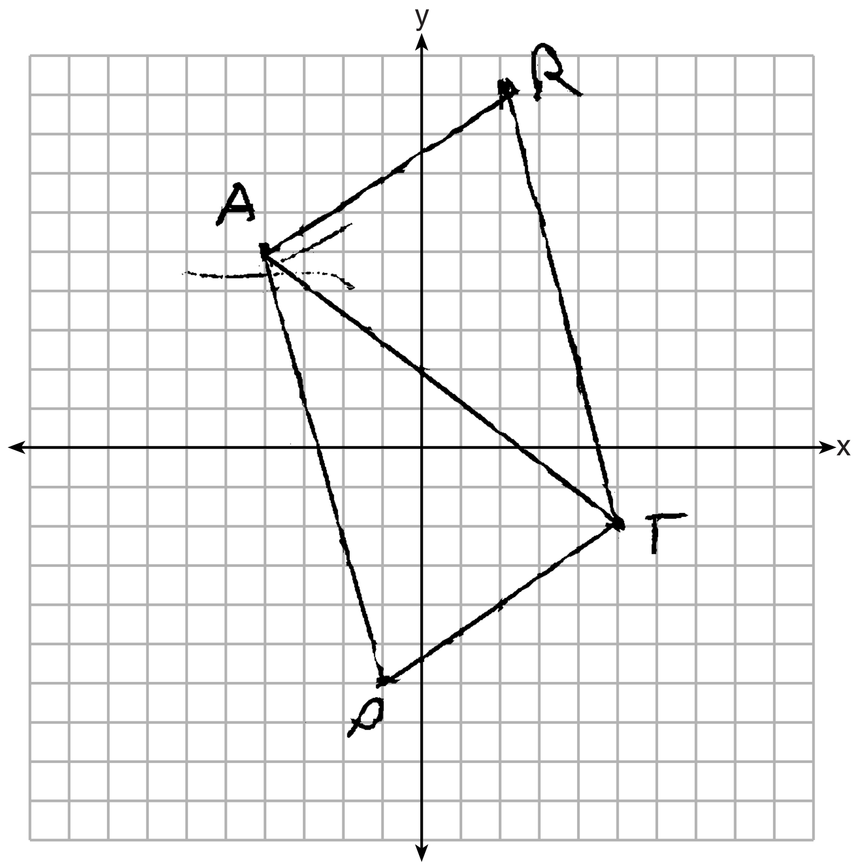
State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

R(2, 9)

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.



Score 1: The student found the correct coordinates of point *R*, but no further correct work was shown.

Question 35

35 In the coordinate plane, the vertices of triangle *PAT* are *P*(−1,−6), *A*(−4,5), and *T*(5,−2). Prove that △*PAT* is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$m\overline{PA} = \frac{-11}{3}$ $m\overline{AT} = \frac{-7}{9}$

not = slopes ∴ △PAT isn't isosceles

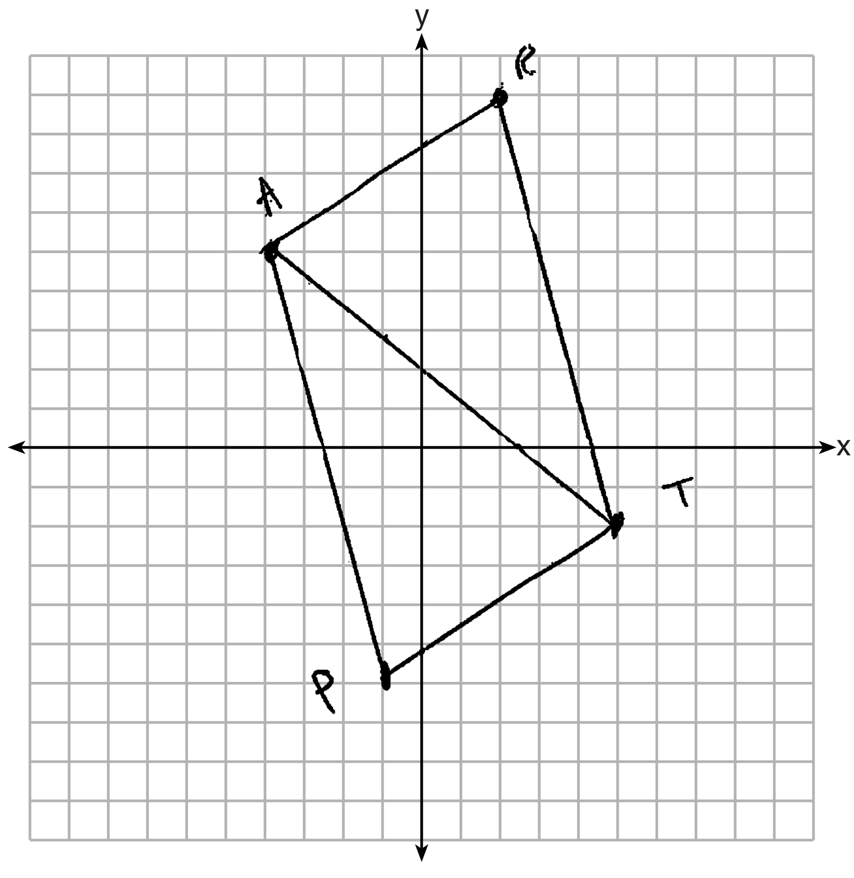
State the coordinates of *R* so that quadrilateral *PART* is a parallelogram.

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

PART is a □ because
it has 2 sets of opposite
sides that intersect.



Score 0: The student had a completely incorrect response.