

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 24, 2024 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for
computations.

1 Which expression is equal to $\sin 30^\circ$?

(1) $\tan 30^\circ$

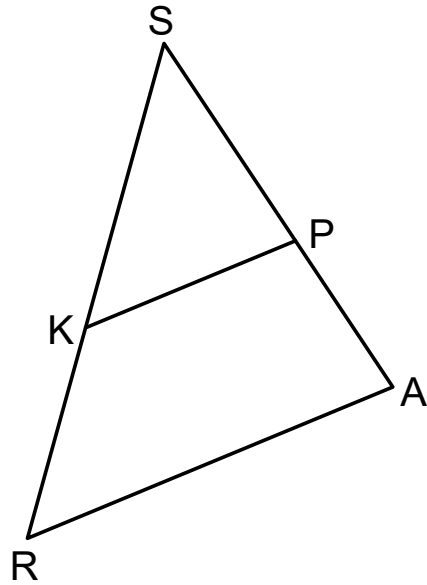
(3) $\cos 60^\circ$

(2) $\sin 60^\circ$

(4) $\cos 30^\circ$

2 In the diagram of $\triangle SRA$ below, \overline{KP} is drawn such that $\angle SKP \cong \angle SRA$.

Use this space for
computations.



If $SK = 10$, $SP = 8$, and $PA = 6$, what is the length of \overline{KR} , to the *nearest tenth*?

(1) 4.8

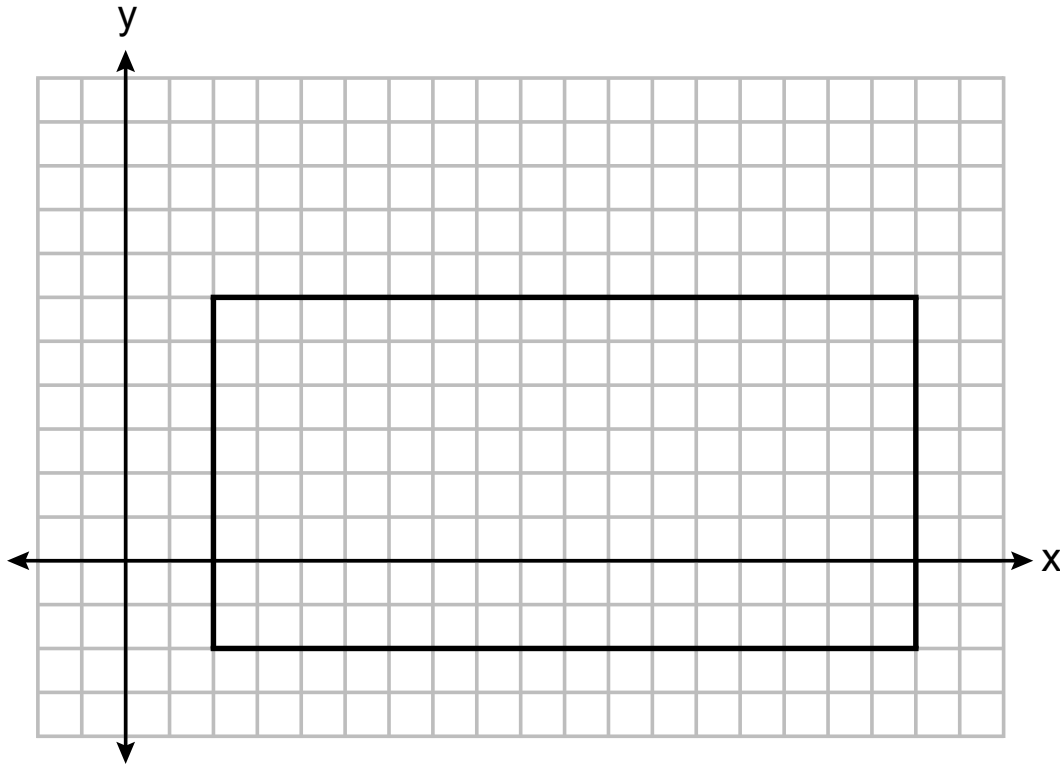
(3) 8.0

(2) 7.5

(4) 13.3

3 A rectangle is graphed on the set of axes below.

Use this space for computations.

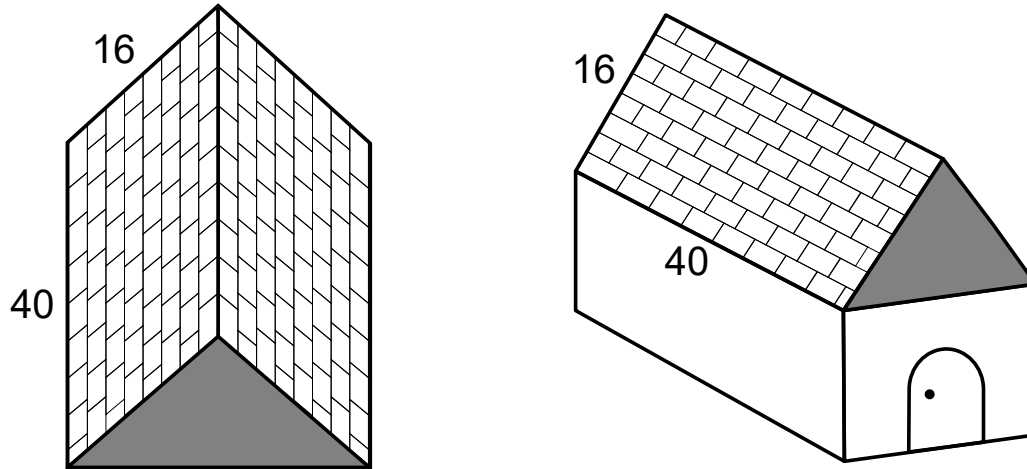


A reflection over which line would carry the rectangle onto itself?

- (1) $y = 2$ (3) $y = \frac{1}{2}x - 3$
(2) $y = 10$ (4) $y = -\frac{1}{2}x + 7$

**Use this space for
computations.**

- 4 The surface of the roof of a house is modeled by two congruent rectangles with dimensions 40 feet by 16 feet, as shown below.



Roofing shingles are sold in bundles. Each bundle covers $33\frac{1}{3}$ square feet. What is the minimum number of bundles that must be purchased to completely cover both rectangular sides of the roof?

- (1) 20 (3) 39
(2) 2 (4) 4

**Use this space for
computations.**

5 Which equation represents a line that is perpendicular to the line whose equation is $y - 3x = 4$?

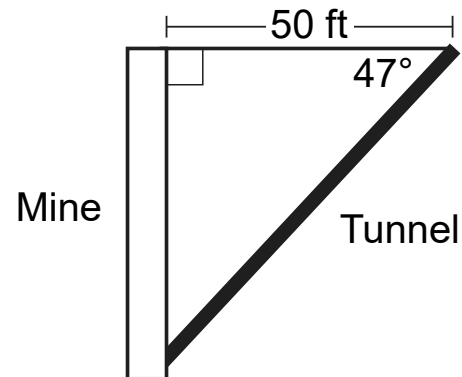
(1) $y = -\frac{1}{3}x - 4$

(3) $y = -3x + 4$

(2) $y = \frac{1}{3}x + 4$

(4) $y = 3x - 4$

6 A vertical mine shaft is modeled in the diagram below. At a point on the ground 50 feet from the top of the mine, a ventilation tunnel is dug at an angle of 47° .



What is the length of the tunnel, to the *nearest foot*?

(1) 47

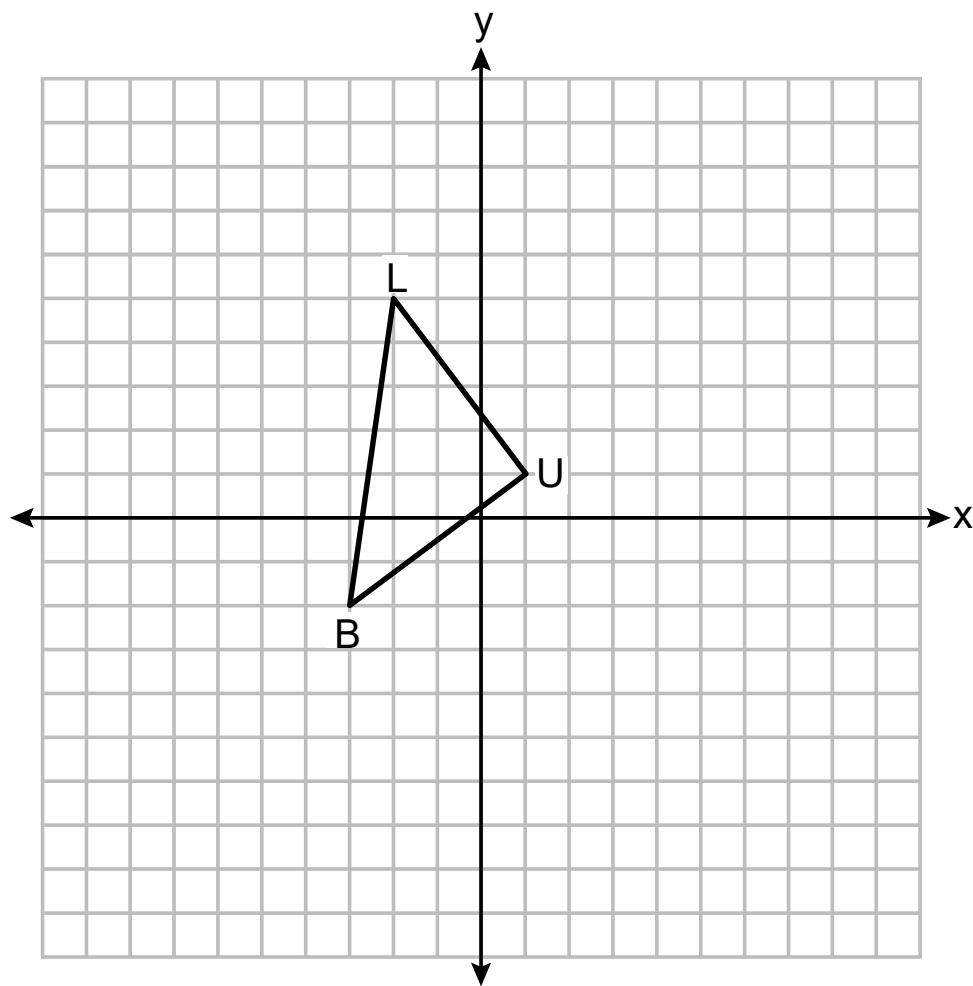
(3) 68

(2) 54

(4) 73

- 7 On the set of axes below, $\triangle BLU$ has vertices with coordinates $B(-3, -2)$, $L(-2, 5)$, and $U(1, 1)$.

Use this space for
computations.

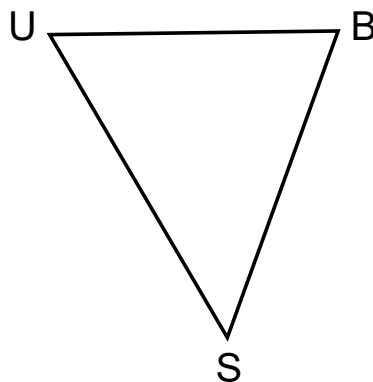
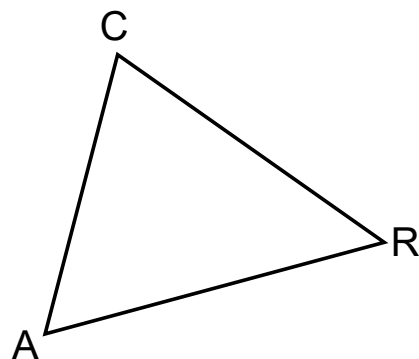


What is the area of $\triangle BLU$?

- (1) 11 (3) 14
(2) 12.5 (4) 17.1

8 In the diagram below, $\triangle CAR$ is mapped onto $\triangle BUS$ after a sequence of rigid motions.

Use this space for computations.

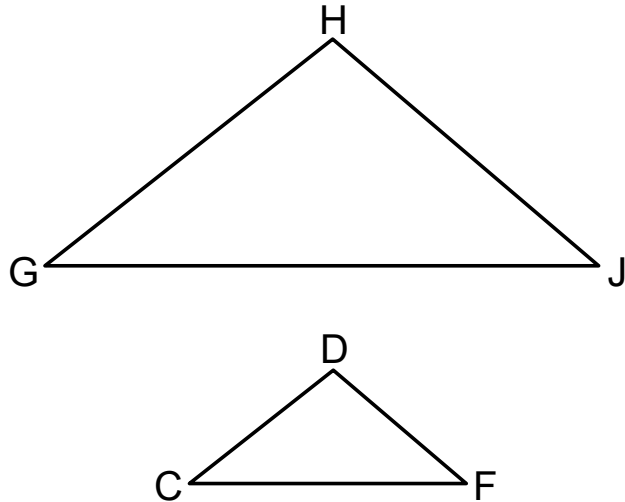


If $AR = 3x + 4$, $RC = 5x - 10$, $CA = 2x + 6$, and $SB = 4x - 4$, what is the length of \overline{SB} ?

- (1) 6
- (2) 16
- (3) 20
- (4) 28

- 9 In the diagram below, $\triangle GHJ$ is dilated by a scale factor of $\frac{1}{2}$ centered at point B to map onto $\triangle CDF$.

Use this space for computations.



B •

If $m\angle DFC = 40^\circ$, what is $m\angle HJG$?

- | | |
|----------------|----------------|
| (1) 20° | (3) 60° |
| (2) 40° | (4) 80° |

**Use this space for
computations.**

10 Directed line segment AJ has endpoints whose coordinates are $A(5,7)$ and $J(-10,-8)$. Point E is on \overline{AJ} such that $AE:EJ$ is $2:3$. What are the coordinates of point E ?

(1) $(1,-1)$

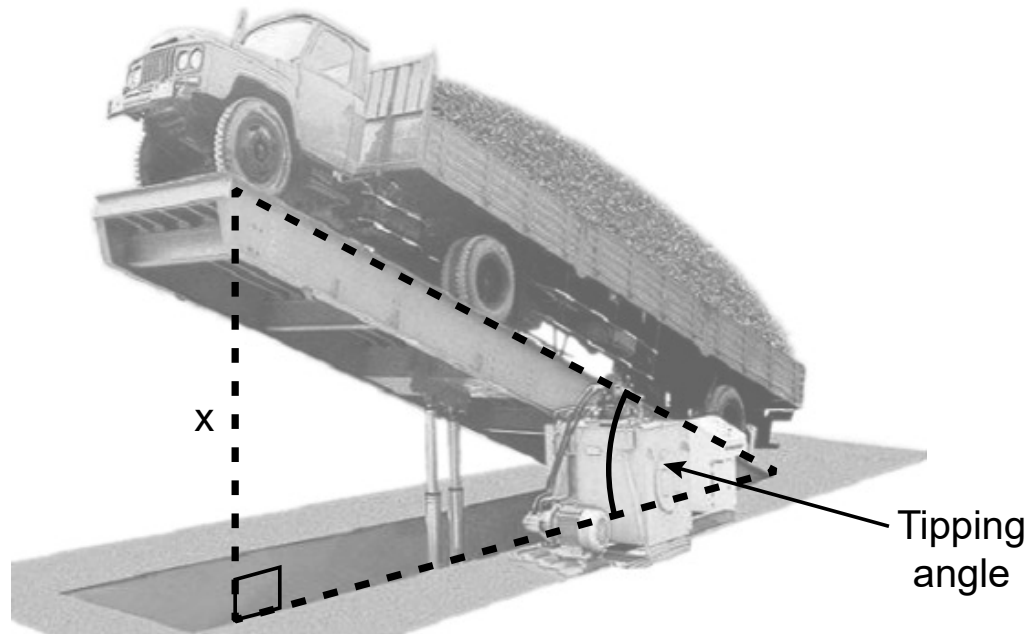
(3) $(-4,-2)$

(2) $(-5,-3)$

(4) $(-1,1)$

**Use this space for
computations.**

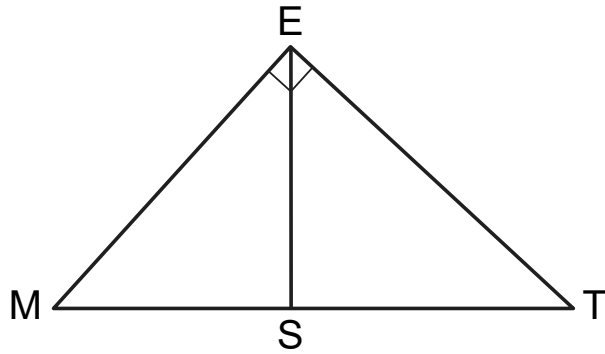
- 11** A tipping platform is a ramp used to unload trucks, as shown in the diagram below.



The truck is on a 75-foot-long ramp. The ramp is tipped at an angle of 30° . What is the height of the upper end of the ramp, x , to the *nearest tenth of a foot*?

- (1) 68.7 (3) 43.3
(2) 65.0 (4) 37.5

12 In the diagram below of right triangle MET , altitude \overline{ES} is drawn to hypotenuse \overline{MT} .



Use this space for
computations.

If $ME = 6$ and $SM = 4$, what is MT ?

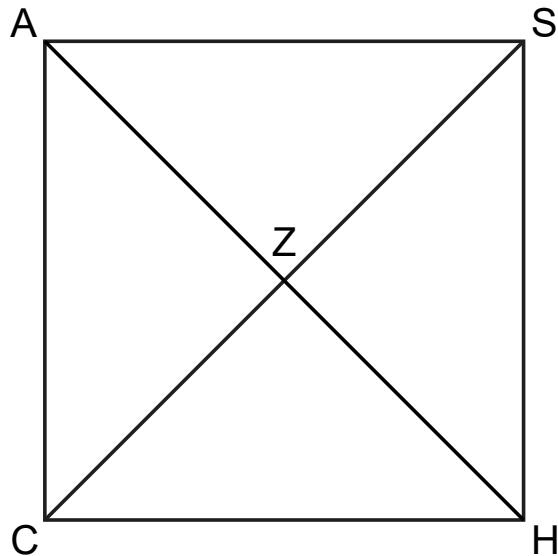
(1) 9

(3) 5

(2) 8

(4) 4

- 13 In the diagram below of square $CASH$, diagonals \overline{AH} and \overline{CS} intersect at Z .



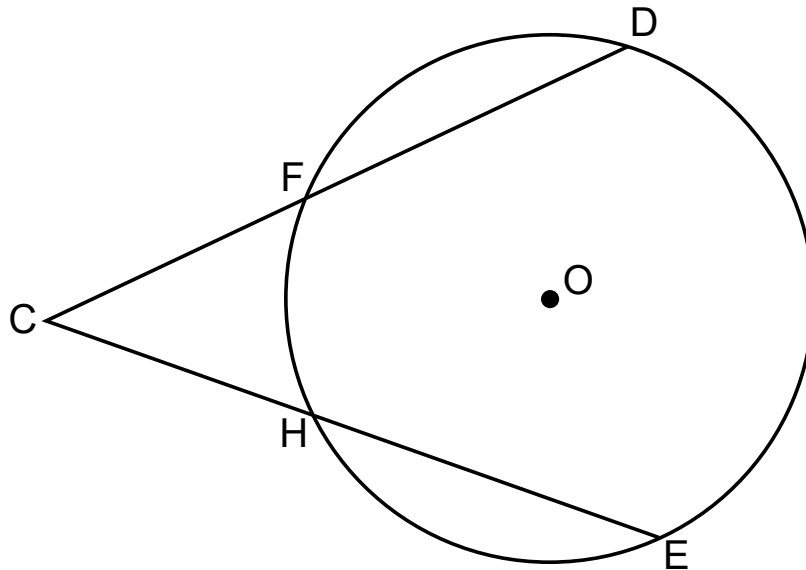
Use this space for
computations.

Which statement is true?

- (1) $m\angle ACZ > m\angle ZCH$ (3) $m\angle AZC = m\angle SHC$
(2) $m\angle ACZ < m\angle ASZ$ (4) $m\angle AZC = m\angle ZCH$

14 In the diagram below of circle O , secants \overline{CFD} and \overline{CHE} are drawn from external point C .

Use this space for computations.

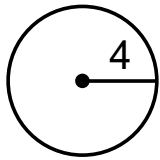


If $m\widehat{DE} = 136^\circ$ and $m\angle C = 44^\circ$, then $m\widehat{FH}$ is

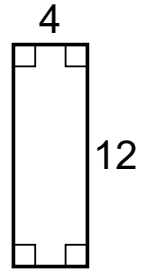
- | | |
|----------------|----------------|
| (1) 46° | (3) 68° |
| (2) 48° | (4) 88° |

**Use this space for
computations.**

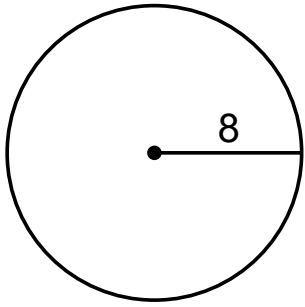
- 15** A right circular cylinder has a diameter of 8 inches and a height of 12 inches. Which two-dimensional figure shows a cross section that is perpendicular to the base and passes through the center of the base?



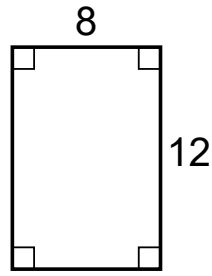
(1)



(3)



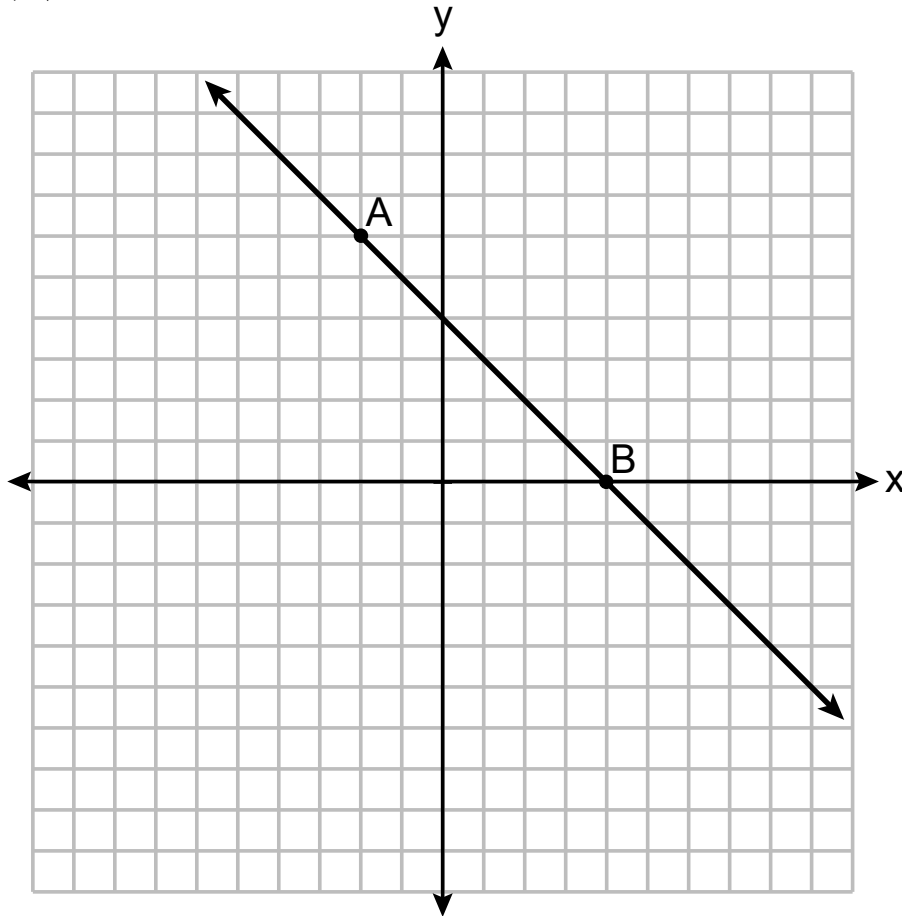
(2)



(4)

Use this space for
computations.

- 16 On the set of axes below, \overline{AB} is drawn and passes through $A(-2,6)$ and $B(4,0)$.



If \overline{CD} is the image of \overline{AB} after a dilation with a scale factor of $\frac{1}{2}$ centered at the origin, which equation represents \overline{CD} ?

- (1) $y = -x + 4$ (3) $y = -\frac{1}{2}x + 4$
(2) $y = -x + 2$ (4) $y = -\frac{1}{2}x + 2$

**Use this space for
computations.**

19 A sandbox in the shape of a rectangular prism has a length of 43 inches and a width of 30 inches. Jack uses bags of sand to fill the sandbox to a depth of 9 inches. Each bag of sand has a volume of 0.5 cubic foot. What is the minimum number of bags of sand that must be purchased to fill the sandbox?

(1) 14

(3) 7

(2) 13

(4) 4

20 Parallelogram $EATK$ has diagonals \overline{ET} and \overline{AK} . Which information is always sufficient to prove $EATK$ is a rhombus?

(1) $\overline{EA} \perp \overline{AT}$

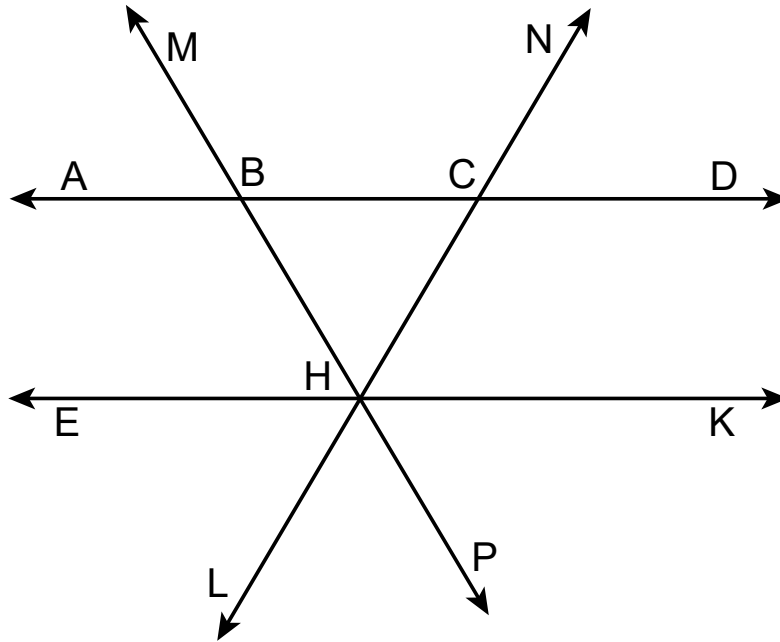
(3) $\overline{ET} \cong \overline{AK}$

(2) $\overline{EA} \cong \overline{AT}$

(4) $\overline{ET} \cong \overline{AT}$

21 In the diagram below, $\overleftrightarrow{ABCD} \parallel \overleftrightarrow{EHK}$, and $\overleftrightarrow{MBHP}$ and $\overleftrightarrow{NCHL}$ are drawn such that $\overline{BC} \cong \overline{BH}$.

Use this space for computations.



If $m\angle NCD = 62^\circ$, what is $m\angle PHK$?

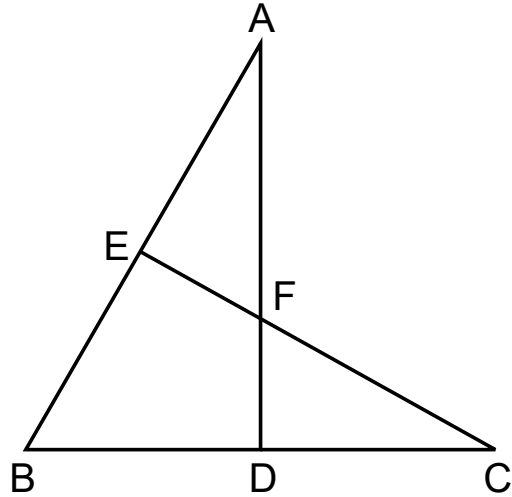
- | | |
|-----------------|----------------|
| (1) 118° | (3) 62° |
| (2) 68° | (4) 56° |

**Use this space for
computations.**

22 Triangles YEG and POM are two distinct non-right triangles such that $\angle G \cong \angle M$. Which statement is sufficient to prove $\triangle YEG$ is always congruent to $\triangle POM$?

- (1) $\angle E \cong \angle O$ and $\angle Y \cong \angle P$
- (2) $\overline{YG} \cong \overline{PM}$ and $\overline{YE} \cong \overline{PO}$
- (3) There is a sequence of rigid motions that maps $\angle E$ onto $\angle O$ and \overline{YE} onto \overline{PO} .
- (4) There is a sequence of rigid motions that maps point Y onto point P and \overline{YG} onto \overline{PM} .

- 23 In the diagram of triangles ABD and CBE below, sides \overline{AD} and \overline{CE} intersect at F , and $\angle ADB \cong \angle CEB$.



Which statement can *not* be proven?

- (1) $\triangle ADB \cong \triangle CEB$ (3) $\triangle ADB \sim \triangle CEB$
(2) $\angle EAF \cong \angle DCF$ (4) $\triangle EAF \sim \triangle DCF$

**Use this space for
computations.**

**Use this space for
computations.**

24 A small town is installing a water storage tank in the shape of a cylinder. The tank must be able to hold at least 100,000 gallons of water. The tank must have a height of exactly 30 feet.

[1 cubic foot holds 7.48 gallons of water]

What should the minimum diameter of the tank be, to the *nearest foot*?

(1) 12

(3) 65

(2) 24

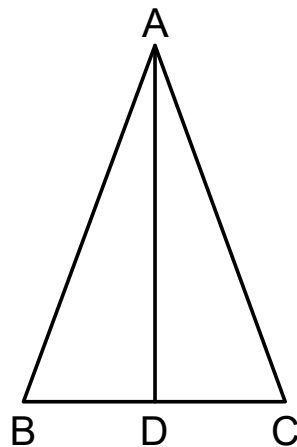
(4) 75

GO RIGHT ON TO THE NEXT PAGE ⇨

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 In isosceles triangle ABC shown below, $\overline{AB} \cong \overline{AC}$, and altitude \overline{AD} is drawn.



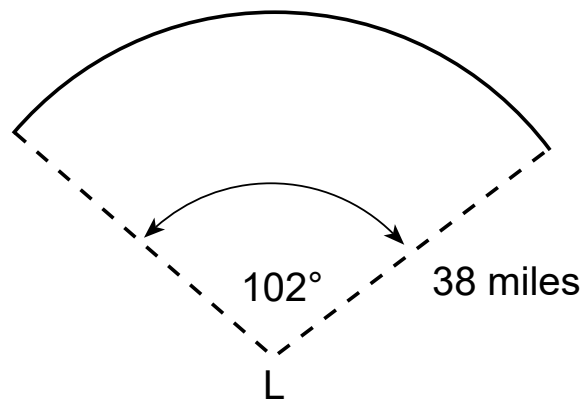
The length of \overline{AD} is 12 cm and the length of \overline{BC} is 10 cm.

Question 25 is continued on the next page.

Question 25 continued

Determine and state, to the *nearest cubic centimeter*, the volume of the solid formed by continuously rotating $\triangle ABC$ about \overline{AD} .

- 26** The diagram below models the projection of light from a lighthouse, L . The sector has a radius of 38 miles and spans 102° .



Determine and state the area of the sector, to the *nearest square mile*.

Work space for question 26 is continued on the next page.

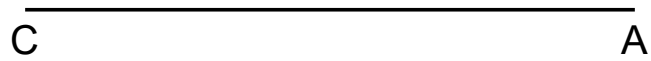
Question 26 continued

27 Segment CA is drawn on the next page. Using a compass and straightedge, construct isosceles right triangle CAT where $\overline{CA} \perp \overline{CT}$ and $\overline{CA} \cong \overline{CT}$.

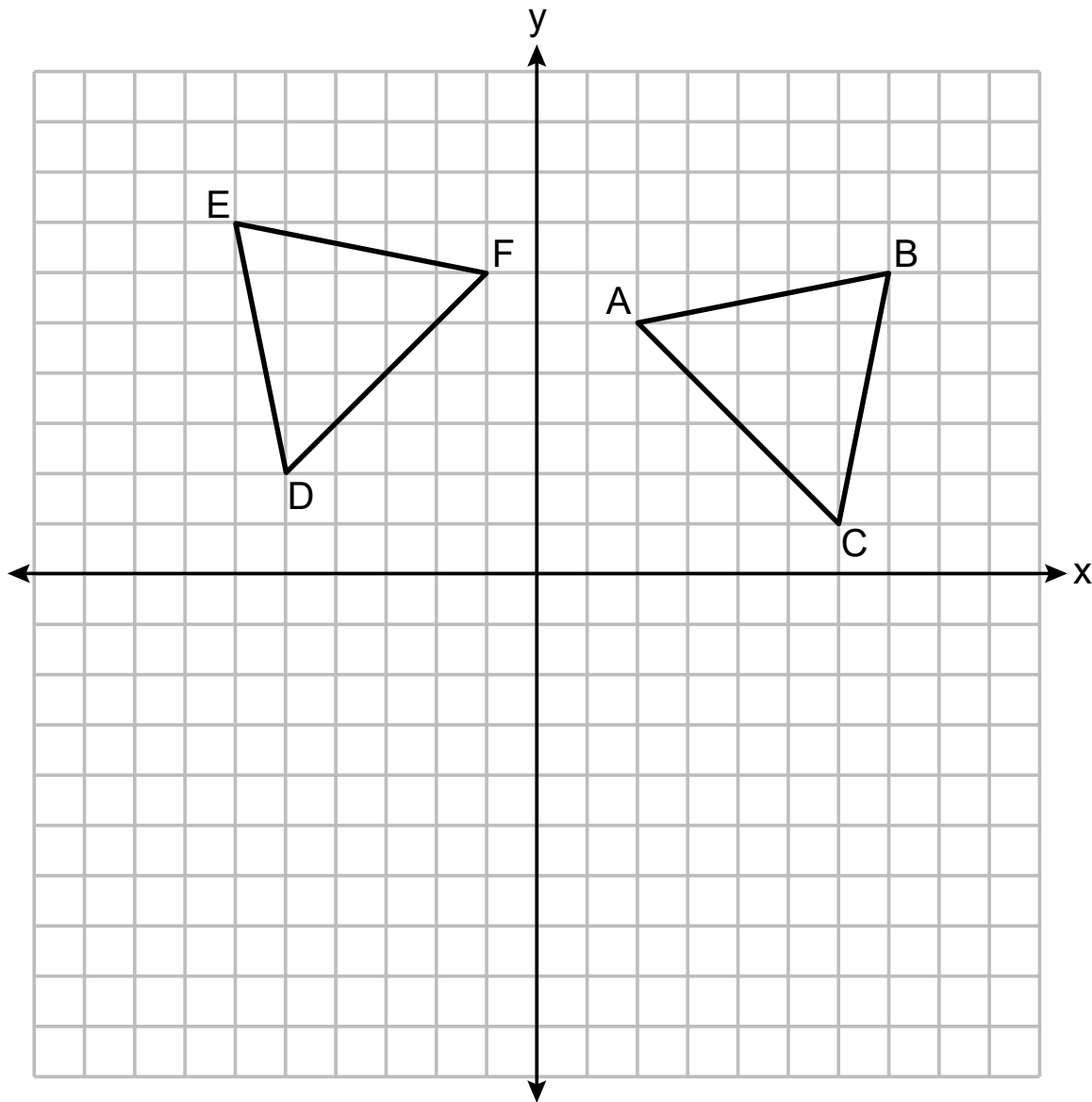
[Leave all construction marks.]

Question 27 is continued on the next page.

Question 27 continued



28 On the set of axes below, congruent triangles ABC and DEF are graphed.

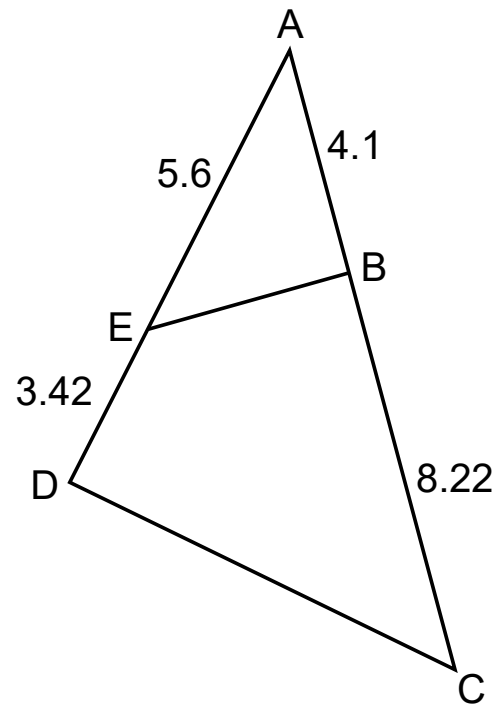


Question 28 is continued on the next page.

Question 28 continued

Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.

29 In $\triangle ADC$ below, \overline{EB} is drawn such that $AB = 4.1$, $AE = 5.6$, $BC = 8.22$, and $ED = 3.42$.



Is $\triangle ABE$ similar to $\triangle ADC$? Explain why.

Work space for question 29 is continued on the next page.

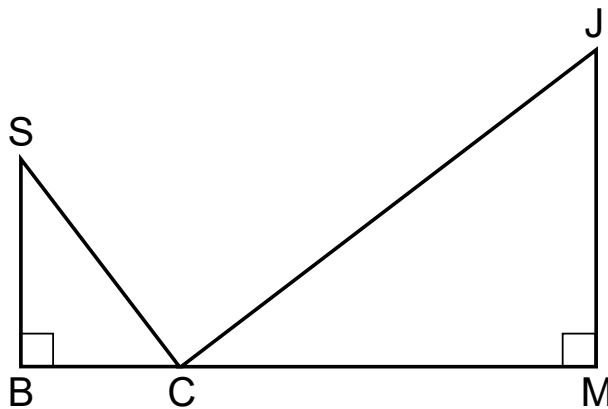
Question 29 continued

30 Determine and state the coordinates of the center and the length of the radius of the circle represented by the equation $x^2 + 16x + y^2 + 12y - 44 = 0$.

Work space for question 30 is continued on the next page.

Question 30 continued

31 In the diagram below, $\triangle SBC \sim \triangle CMJ$ and $\cos J = \frac{3}{5}$.



Determine and state $m\angle S$, to the *nearest degree*.

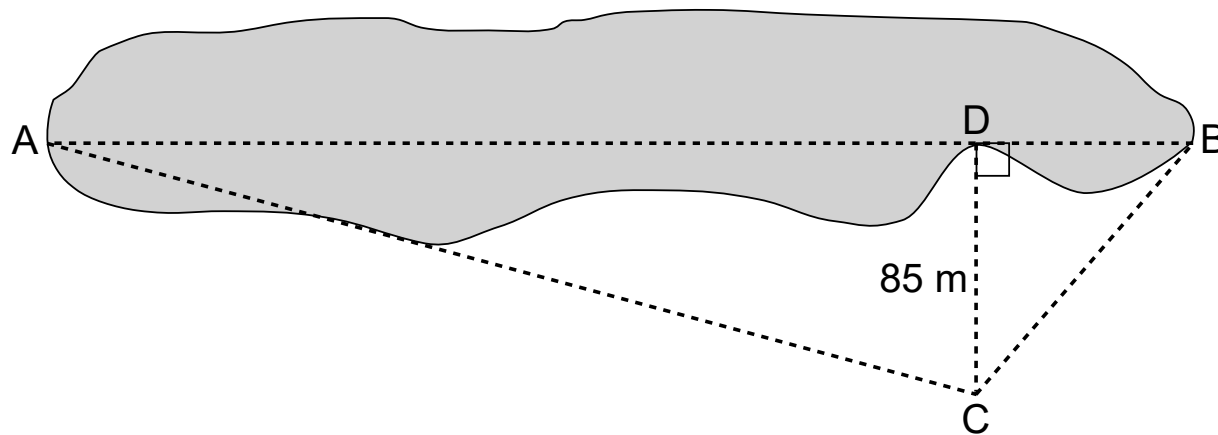
Work space for question 31 is continued on the next page.

Question 31 continued

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32** Trish is a surveyor who was asked to estimate the distance across a pond. She stands at point C , 85 meters from point D , and locates points A and B on either side of the pond such that A , D , and B are collinear.



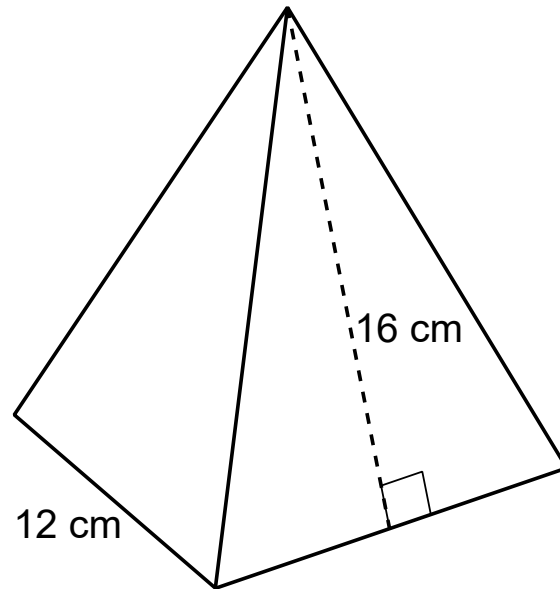
Trish approximates the measure of angle DCB to be 35° and the measure of angle ACD to be 75° .

Question 32 is continued on the next page.

Question 32 continued

Determine and state the distance across the pond, \overline{AB} , to the *nearest meter*.

- 33** A candle in the shape of a right pyramid is modeled below. Each side of the square base measures 12 centimeters. The slant height of the pyramid measures 16 centimeters.



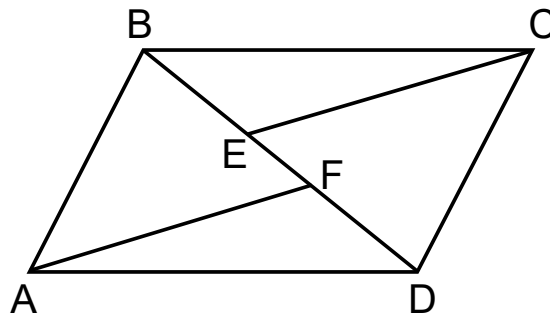
Determine and state the volume of the candle, to the *nearest cubic centimeter*.

Question 33 is continued on the next page.

Question 33 continued

The wax used to make the candle weighs 0.032 ounce per cubic centimeter. Determine and state the weight of the candle, to the *nearest ounce*.

- 34 In the diagram of quadrilateral $ABCD$ below, $\overline{AB} \cong \overline{CD}$, and $\overline{AB} \parallel \overline{CD}$. Segments CE and AF are drawn to diagonal \overline{BD} such that $\overline{BE} \cong \overline{DF}$.



Prove: $\overline{CE} \cong \overline{AF}$

Work space for question 34 is continued on the next page.

Question 34 continued

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Quadrilateral $MATH$ has vertices with coordinates $M(-1,7)$, $A(3,5)$, $T(2,-7)$, and $H(-6,-3)$.

Prove that quadrilateral $MATH$ is a trapezoid.

[The use of the set of axes on page 49 is optional.]

Question 35 is continued on the next page.

Question 35 continued

State the coordinates of point Y such that point A is the midpoint of \overline{MY} .

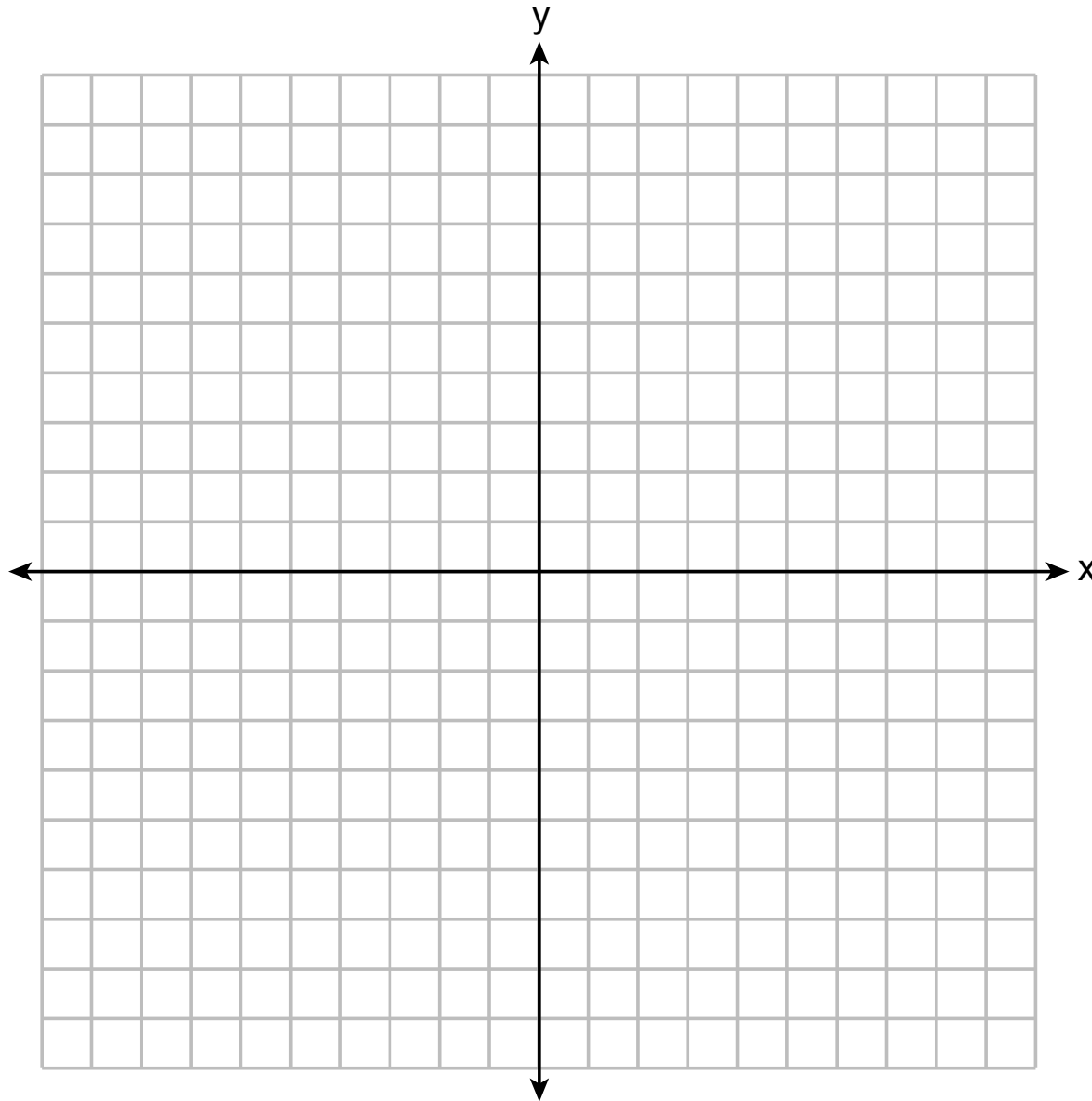
Question 35 is continued on the next page.

Question 35 continued

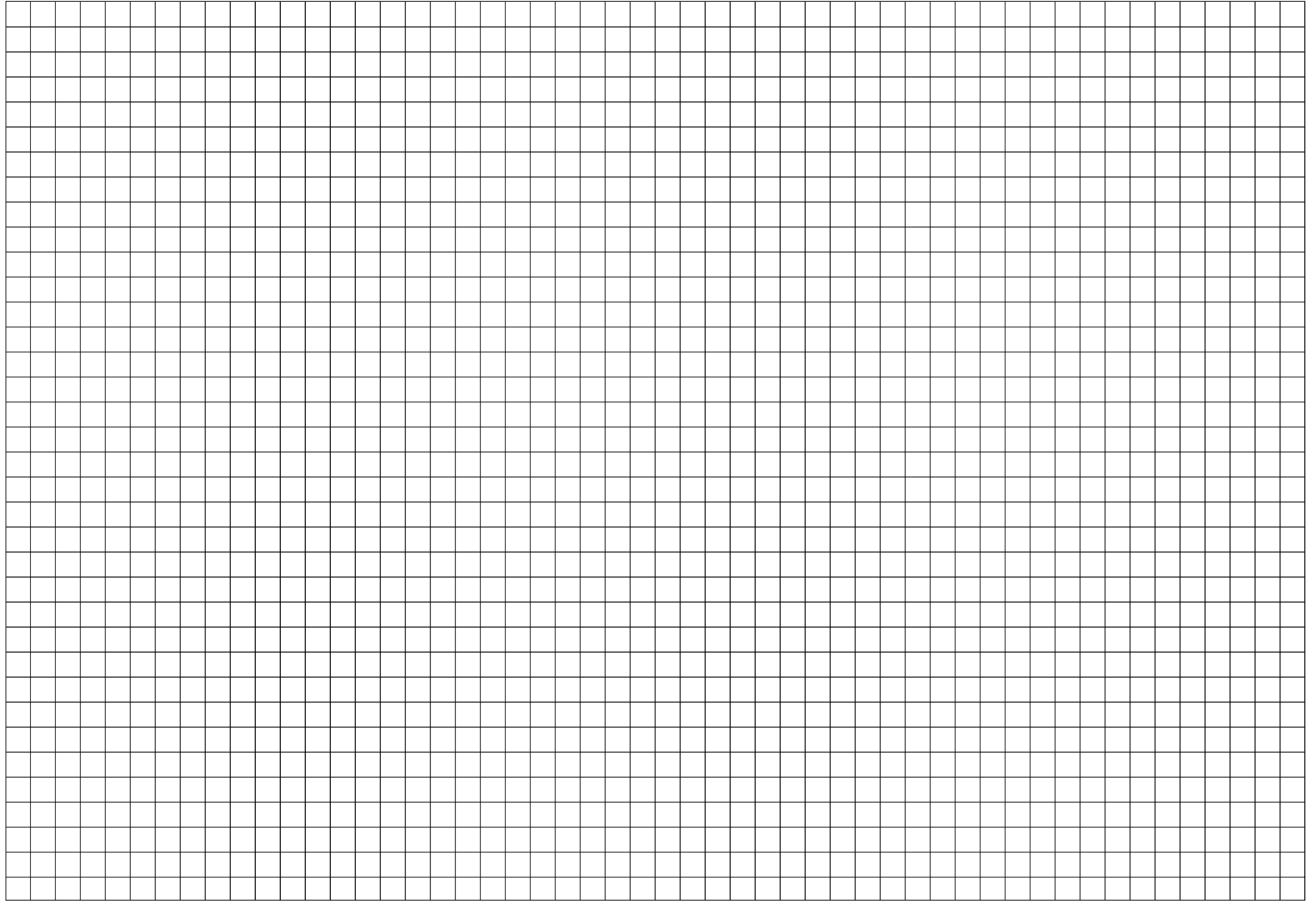
Prove that quadrilateral *MYTH* is a rectangle. [The use of the set of axes on the next page is optional.]

The set of axes for question 35 is on the next page.

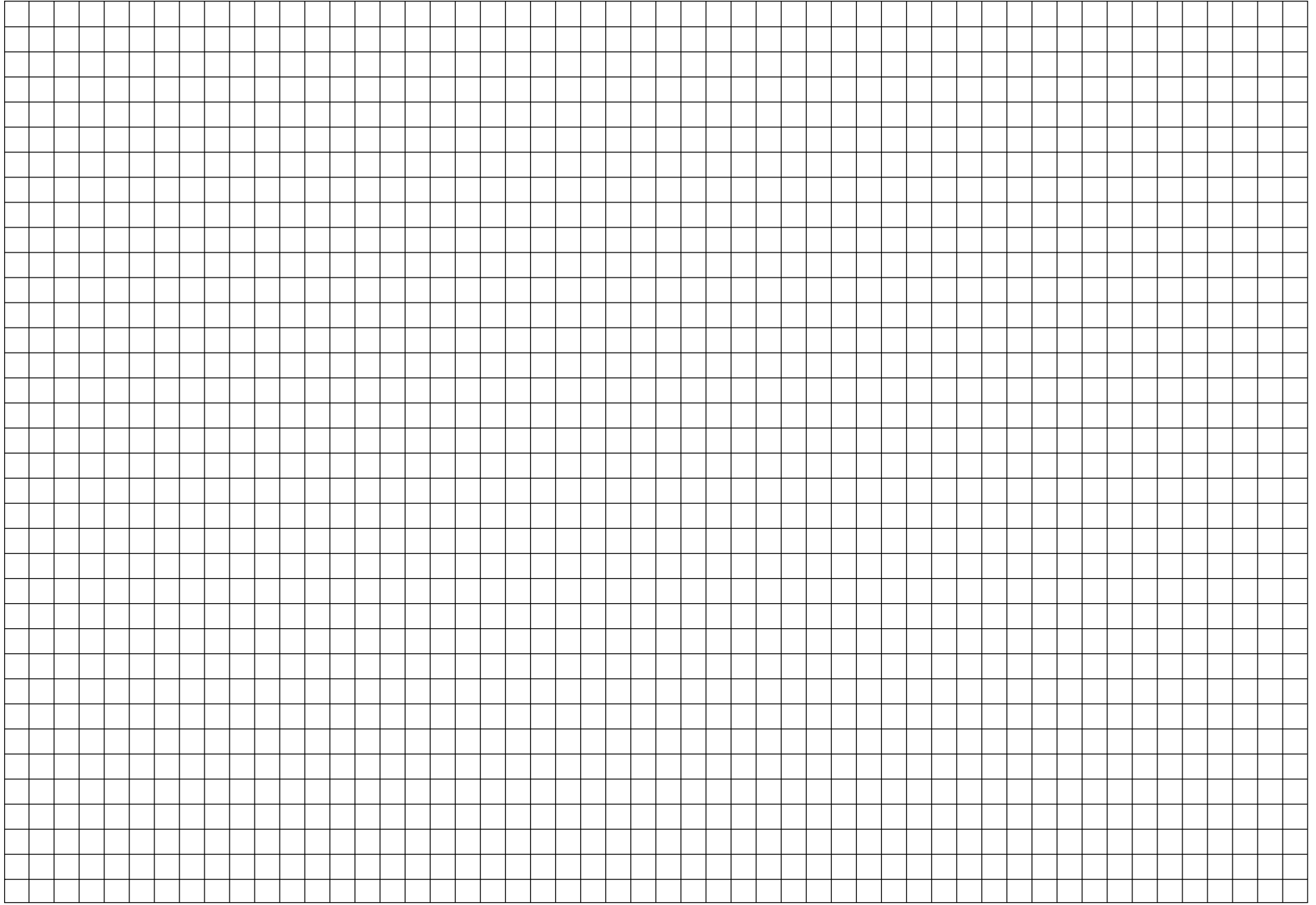
Question 35 continued



Scrap Graph Paper — this sheet will *not* be scored.



Scrap Graph Paper — this sheet will *not* be scored.



High School Math Reference Sheet

1 inch = 2.54 centimeters

1 meter = 39.37 inches

1 mile = 5280 feet

1 mile = 1760 yards

1 mile = 1.609 kilometers

1 kilometer = 0.62 mile

1 pound = 16 ounces

1 pound = 0.454 kilogram

1 kilogram = 2.2 pounds

1 ton = 2000 pounds

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

1 gallon = 3.785 liters

1 liter = 0.264 gallon

1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Reference Sheet — concluded

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$