The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (COMMON CORE)

Friday, June 17, 2016 — 1:15 to 4:15 p.m.

MODEL RESPONSE SET

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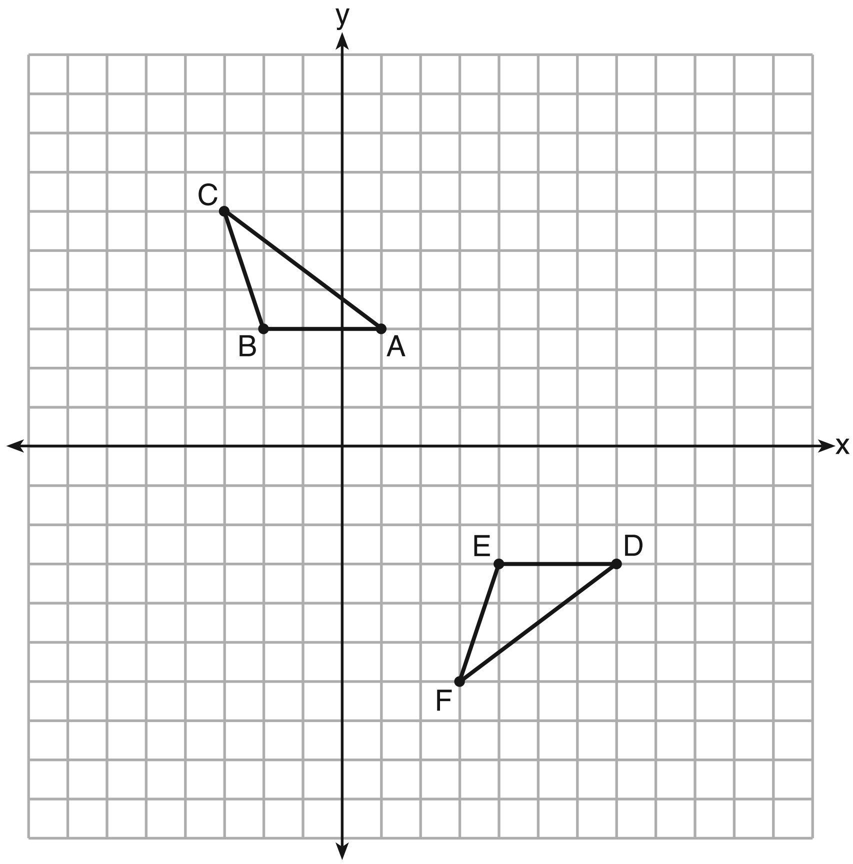
Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

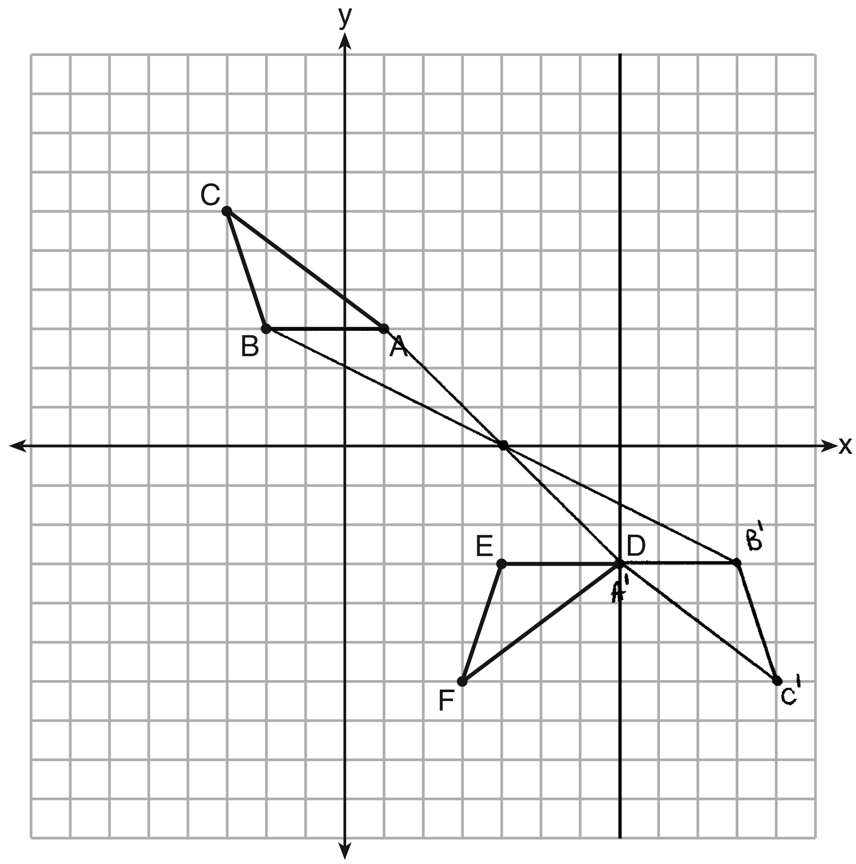


A reflection over the x-axis followed by a translation 6 units to the right.

Score 2: The student had a complete and correct response.

Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

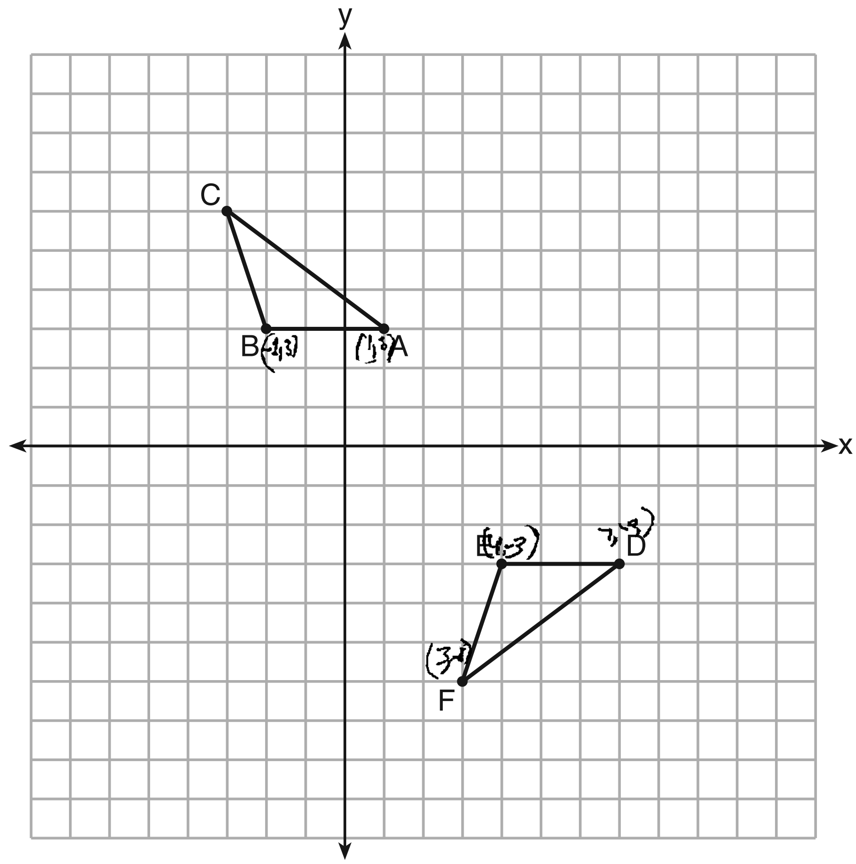


Rotate $\triangle ABC$ 180° about point (4,0) and then reflect $\triangle A'B'C'$ over the line $x=7$

Score 2: The student had a complete and correct response.

Question 25

25 Describe a sequence of transformations that will map △*ABC* onto △*DEF* as shown below.

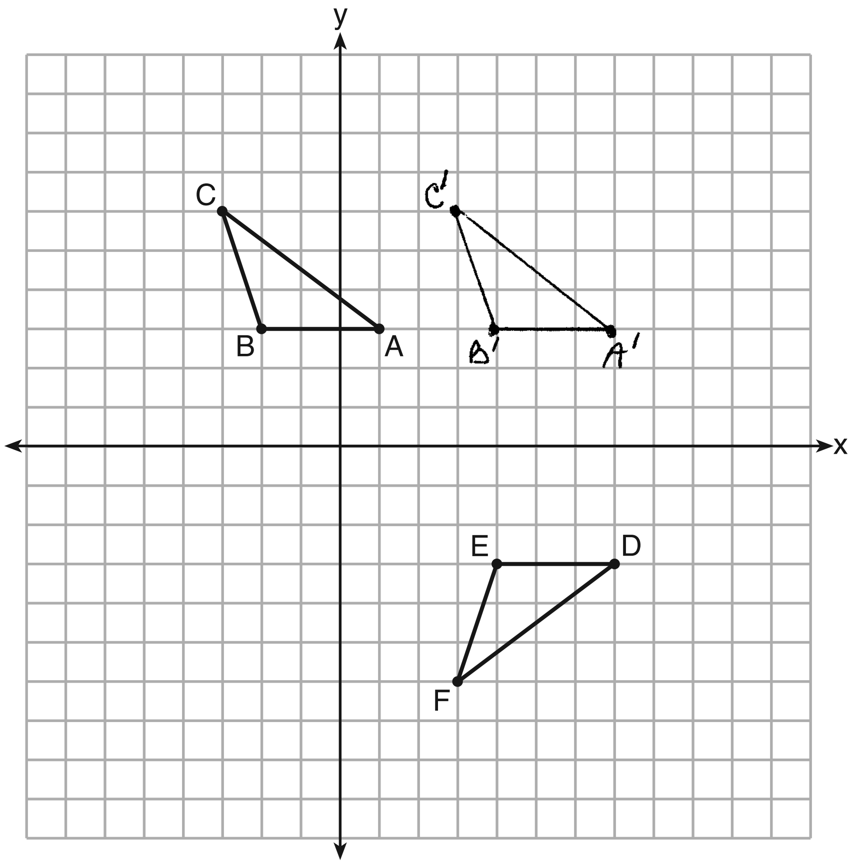


T(6,0) ∘ R x axis

Score 2: The student had a complete and correct response.

Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

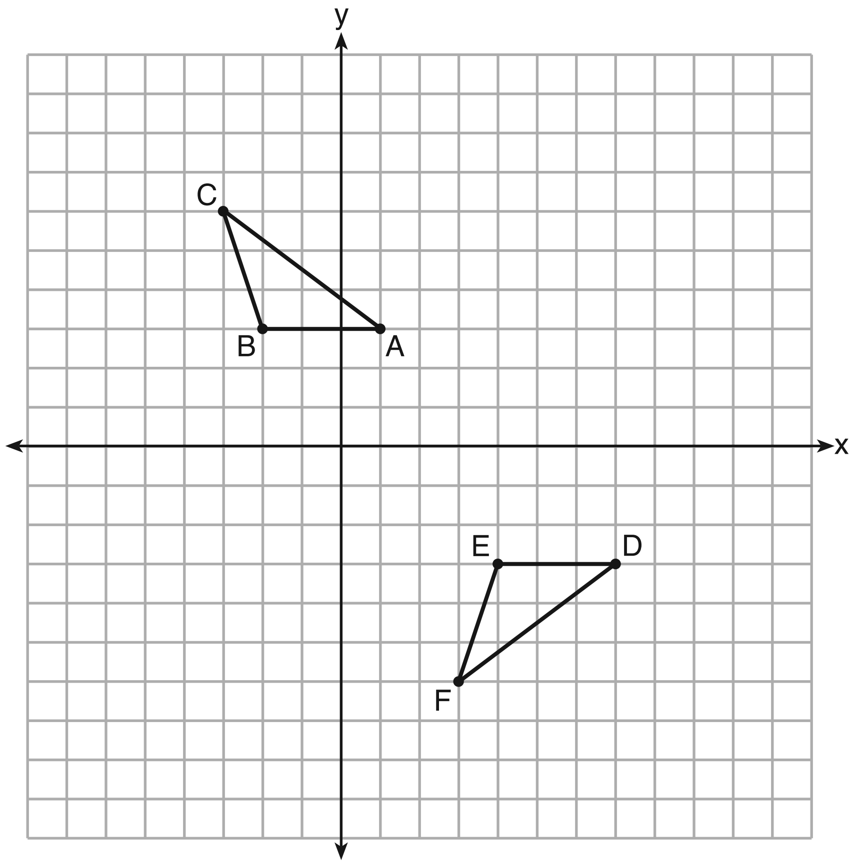


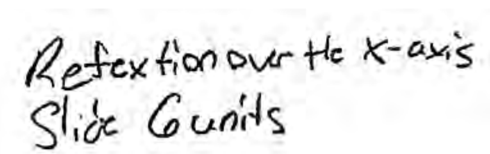
moved 6 units right
flipped over x axis

Score 2: The student had a complete and correct response.

Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

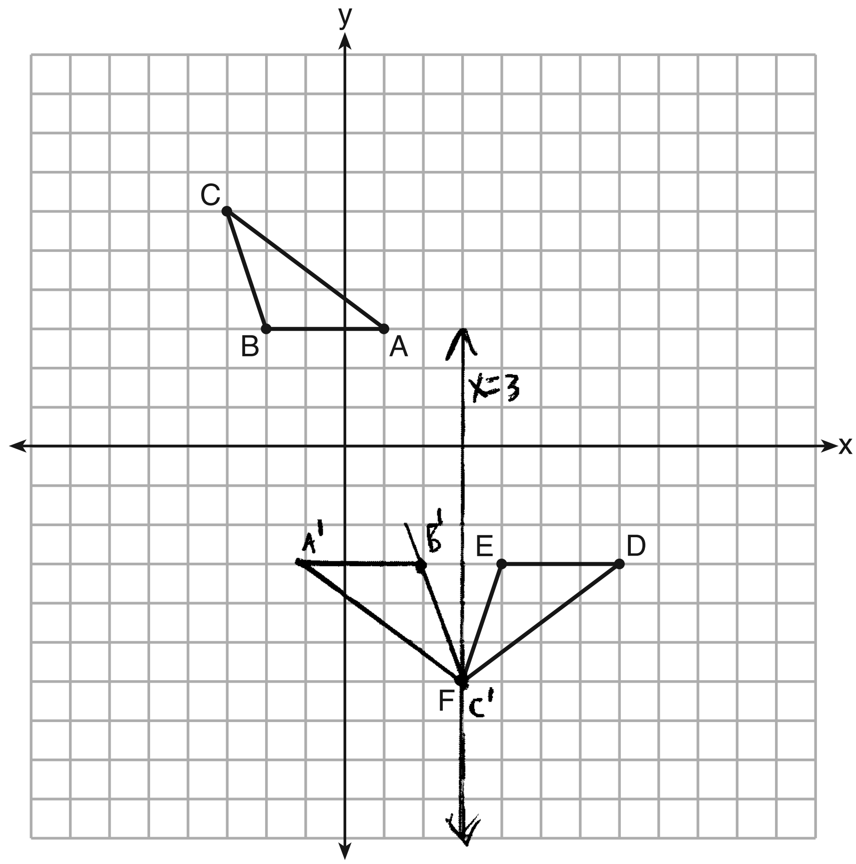




Score 1: The student gave a correct description of the reflection, but gave an incomplete description of the translation.

Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

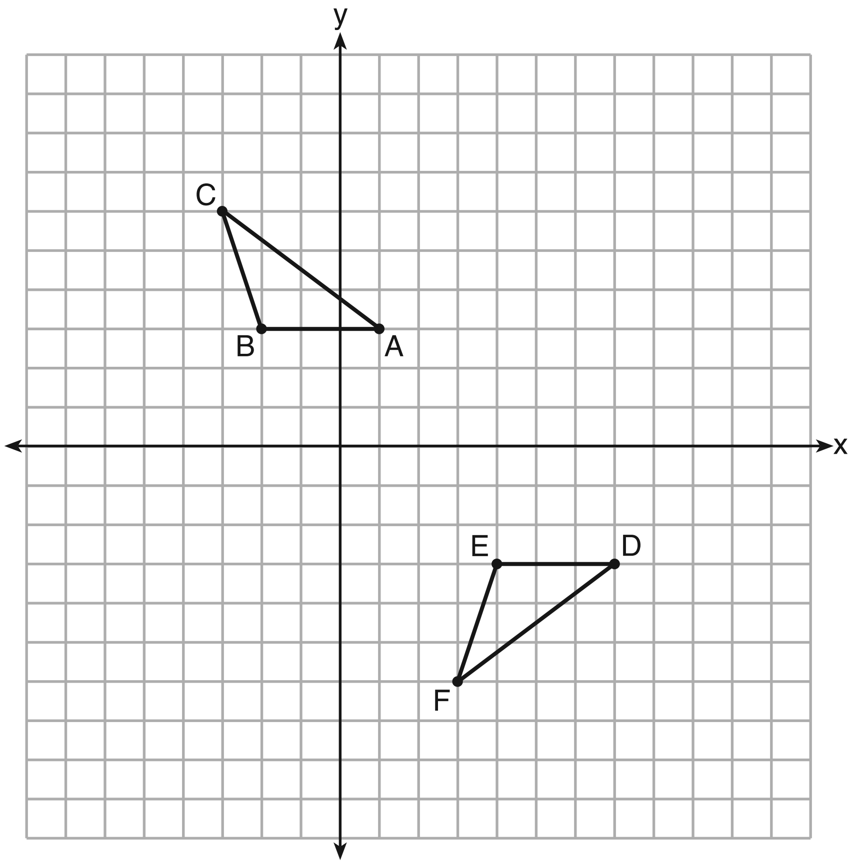


$r_{x=3} \circ R_{180°}$

Score 1: The student gave a correct description of the reflection, but the description of the rotation did not include the center.

Question 25

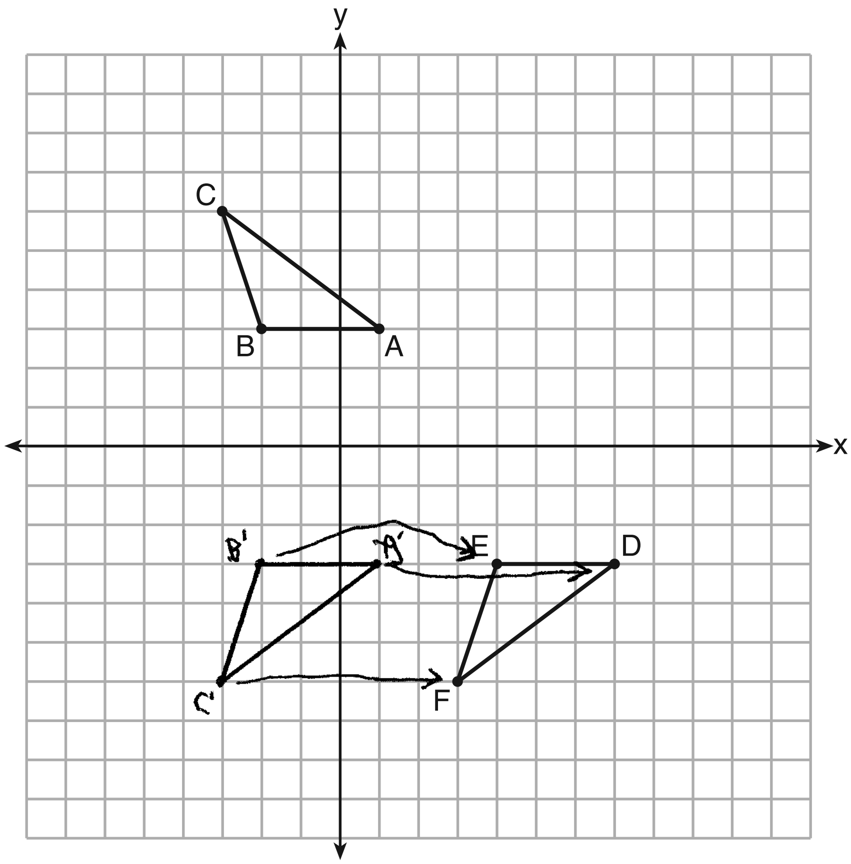
25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



It is a glide reflection

Score 1: The student described an appropriate sequence, but the description was incomplete.

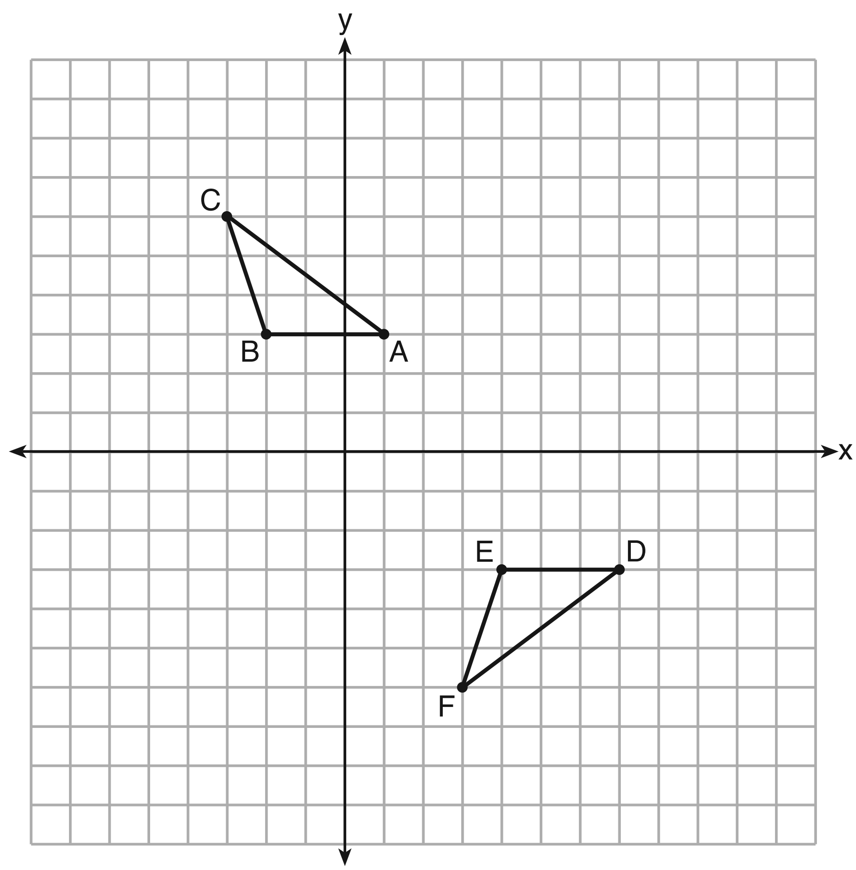
25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



Score 1: The student graphed the transformation correctly, but did not write a description.

Question 25

25 Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

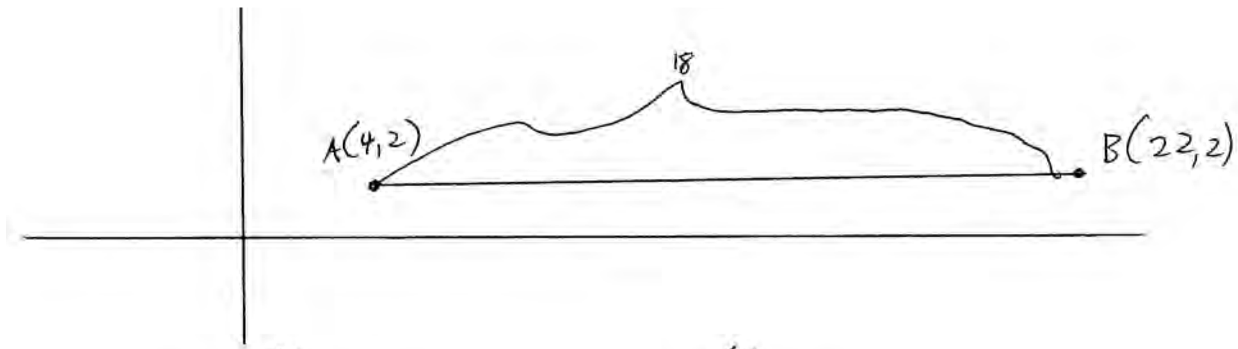


You Flip it and move it down

Score 0: The student gave an incomplete description of the reflection (flip) and described the translation (move) incorrectly.

Question 26

26 Point P is on segment AB such that AP:PB is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.



x	y
$\frac{4}{9}(22-4)$	$\frac{4}{9}(2-2)$
$= \frac{4}{9}(18)$	$= \frac{4}{9}(0)$
$= 8$	$= 0$

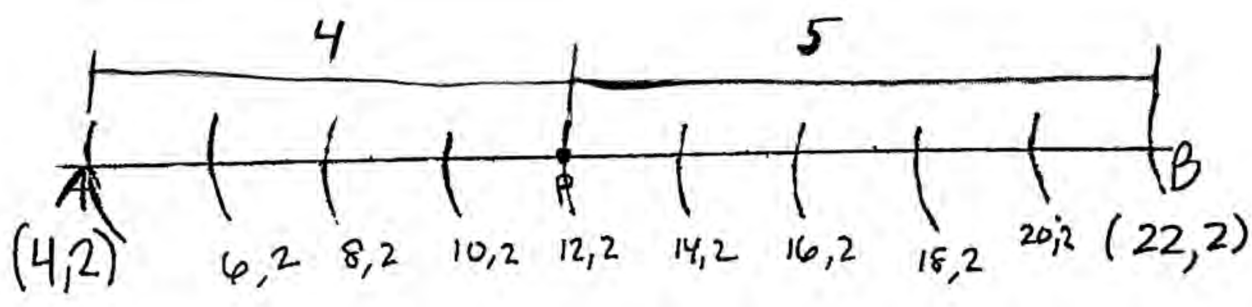
A (4, 2)
\+ 8 0

P (12, 2)

Score 2: The student had a complete and correct response.

Question 26

26 Point P is on segment AB such that $AP{:}PB$ is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.

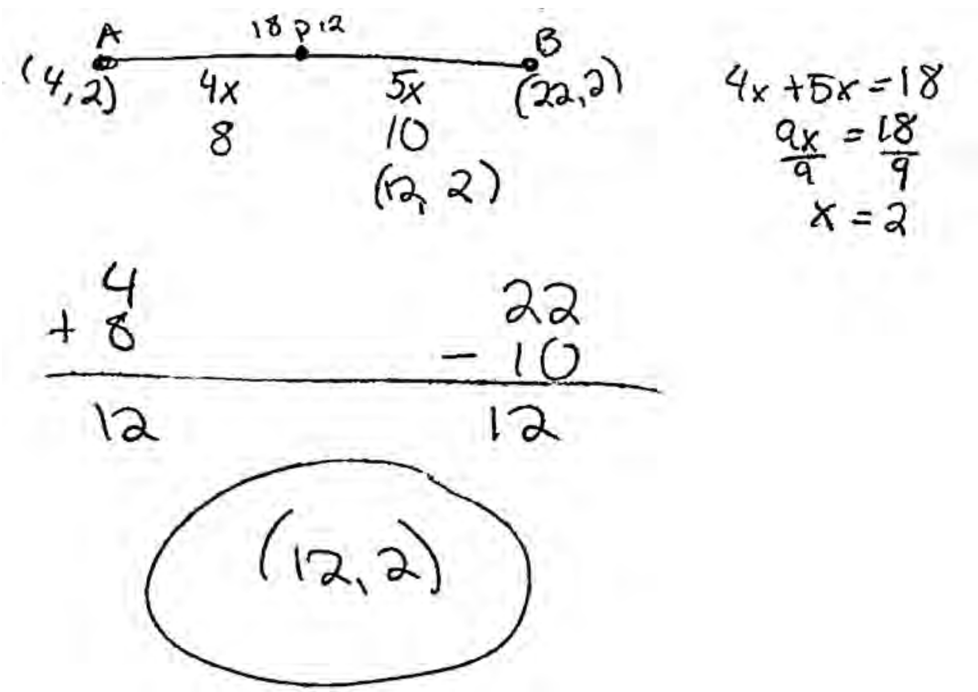


P = (12,2)

Score 2: The student had a complete and correct response.

Question 26

26 Point P is on segment AB such that $AP{:}PB$ is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.



Score 2: The student had a complete and correct response.

26 Point P is on segment AB such that $AP{:}PB$ is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.

$A(4,2)$ $B(22,2)$

Run $4 - 22 = -18$ Rise $2 - 2 = 0$ Scale Factor $= \frac{4}{4+5} = \frac{4}{9}$

$P = \left(22 + \frac{4}{9}(-18),\ 2 + \frac{4}{9}(0)\right)$

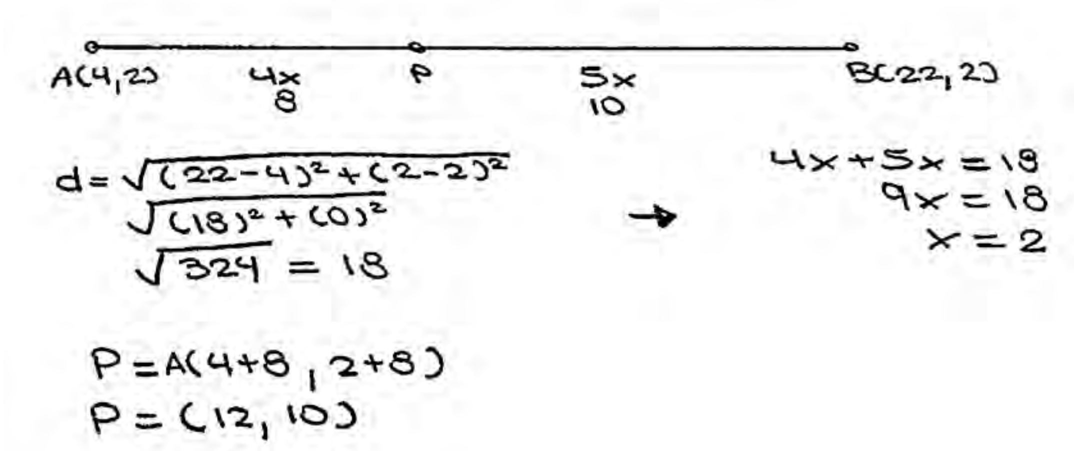
$P = (22 + -8,\ 2 + 0)$

$P = (14, 2)$

Score 1: The student showed correct work to partition the segment in a 5:4 ratio.

Question 26

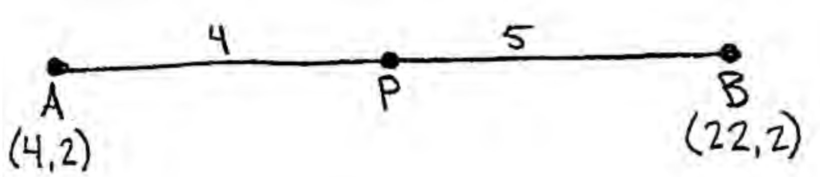
26 Point P is on segment AB such that AP:PB is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.



Score 1: The student showed correct work to determine the x-coordinate of P, but made an error in determining the y-coordinate.

Question 26

26 Point P is on segment AB such that $AP{:}PB$ is 4:5. If A has coordinates (4,2), and B has coordinates (22,2), determine and state the coordinates of P.

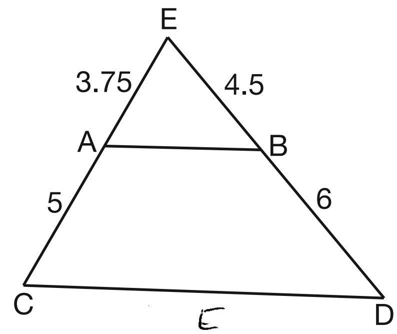


$$\left(\overset{x_1+x_2}{\frac{22+4}{2}}, \overset{y_1+y_2}{\frac{2+2}{2}}\right)$$

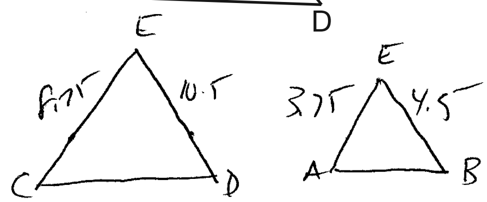
$$P = (13, 2)$$

Score 0: The student determined the correct y-coordinate by calculating the midpoint of $\overline{AB}$, but the midpoint was not relevant to the problem.

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why $\overline{AB}$ is parallel to $\overline{CD}$.



$\frac{EC}{EA} = \frac{ED}{EB}$ $\quad \frac{8.75}{3.75} = \frac{10.5}{4.5}$ $\quad 39.375 = 39.375$ $\quad \therefore \overline{EC} \sim \overline{EA}$ & $\overline{ED} \sim \overline{EB}$

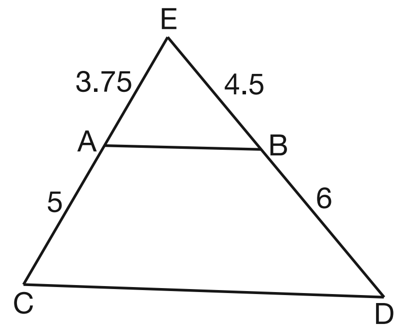
Since $\angle E \cong \angle E$ by the reflexive property and the corresponding sides shown above are in proportion $\triangle CED \sim \triangle AEB$.

In Similar $\triangle$'s, corresponding angles are $\cong$ so $\angle C \cong \angle EAB$. With $\overline{CE}$ as a transversal & $\angle C \cong \angle EAB$, $\overline{AB} \parallel \overline{CD}$ because the corresponding angles are $\cong$.

Score 2: The student had a complete and correct response.

Question 27

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why $\overline{AB}$ is parallel to $\overline{CD}$.

$$\frac{3.75}{8.75} = \frac{4.5}{10.5}$$

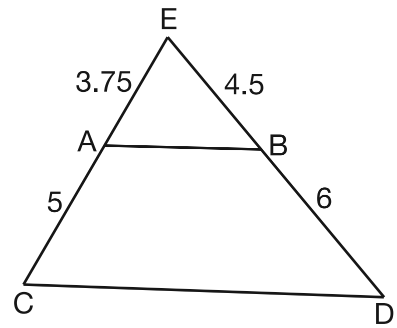
$$39.375 = 39.375$$

$\overline{AB}$ is parallel to $\overline{CD}$ b/c $\overline{AB}$ divides the sides proportionally

Score 2: The student had a complete and correct response.

Question 27

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why $\overline{AB}$ is parallel to $\overline{CD}$.

$$\frac{3.75}{5} = \frac{4.5}{6}$$

$$\frac{375}{500} = \frac{45}{60}$$

$$\frac{15}{20} = \frac{15}{20}$$

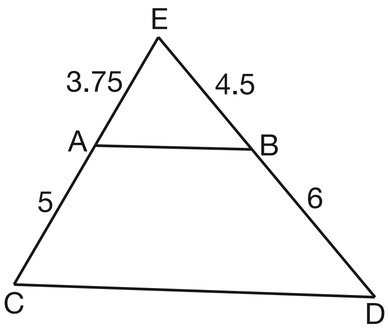
sides $\overline{AB}$ ~~and~~ divides sides $\overline{CE}$ and $\overline{DE}$ proportionally

so $\overline{AB} \parallel \overline{CD}$

Score 2: The student had a complete and correct response.

Question 27

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why $\overline{AB}$ is parallel to $\overline{CD}$.

$\overline{AB}$ is parallel to $\overline{CD}$ because $\overline{AB}$ is a midsegment of ΔCED. A midsegment is half the length of side its parallel to. Midsegment also makes congruent parts.

$\frac{CA}{AE} = \frac{DB}{BE}$ > proportional

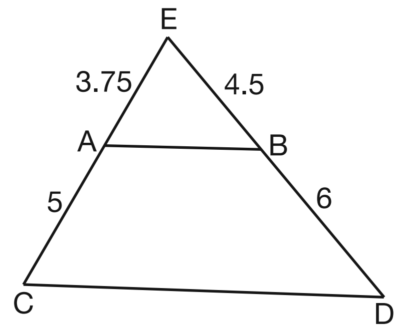
$\frac{5}{3.75} = \frac{6}{4.5}$ $\quad 5(4.5) = 6(3.75)$

$22.5 = 22.5$

Score 1: The student showed that the cross products of the proportion are equal, but the explanation was incorrect.

Question 27

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



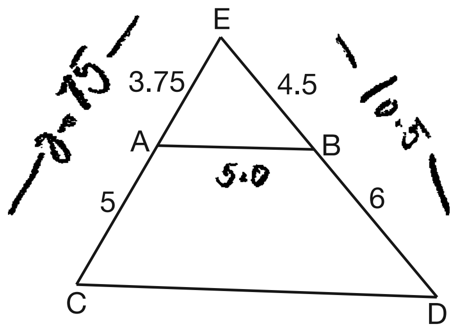
Explain why $\overline{AB}$ is parallel to $\overline{CD}$.

$$\frac{3.75}{5} = \frac{4.5}{6}$$

Score 0: The student only wrote a correct proportion.

Question 27

27 In $\triangle CED$ as shown below, points A and B are located on sides $\overline{CE}$ and $\overline{ED}$, respectively. Line segment AB is drawn such that $AE = 3.75$, $AC = 5$, $EB = 4.5$, and $BD = 6$.



Explain why $\overline{AB}$ is parallel to $\overline{CD}$.

They are parallel to each other cause these lines dont meet.

Score 0: The student had a completely incorrect response.

Question 28

28 Find the value of R that will make the equation $\sin 73° = \cos R$ true when $0° < R < 90°$. Explain your answer.

$90 - 73 = 17$

$\sin 73 = \cos 17$

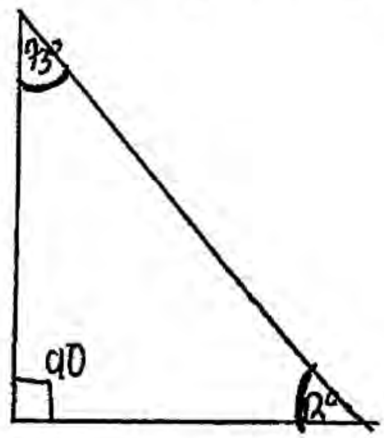
The sin and cos of complimentary angles are equal

Score 2: The student had a complete and correct response.

28 Find the value of R that will make the equation $\sin 73° = \cos R$ true when $0° < R < 90°$. Explain your answer.

SOH-CAH-TOA

$0° < R < 90°$



$$\begin{array}{r} 73 \\ +90 \\ \hline 163 \end{array} \qquad \begin{array}{r} 180 \\ -163 \\ \hline 17° \end{array}$$

$R = 17°$

Score 1: The student correctly determined the value of R, but the explanation was missing.

Question 28

28 Find the value of R that will make the equation $\sin 73° = \cos R$ true when $0° < R < 90°$. Explain your answer.

$73 + R = 90$
$-73 \quad -73$
$R = 17$

$R = 17°$

sine and cosine sign are complimantary, so they must equal 90° when added together

Score 1: The student correctly determined the value of R, but the explanation was incorrect.

Question 28

28 Find the value of R that will make the equation $\sin 73° = \cos R$ true when $0° < R < 90°$. Explain your answer.

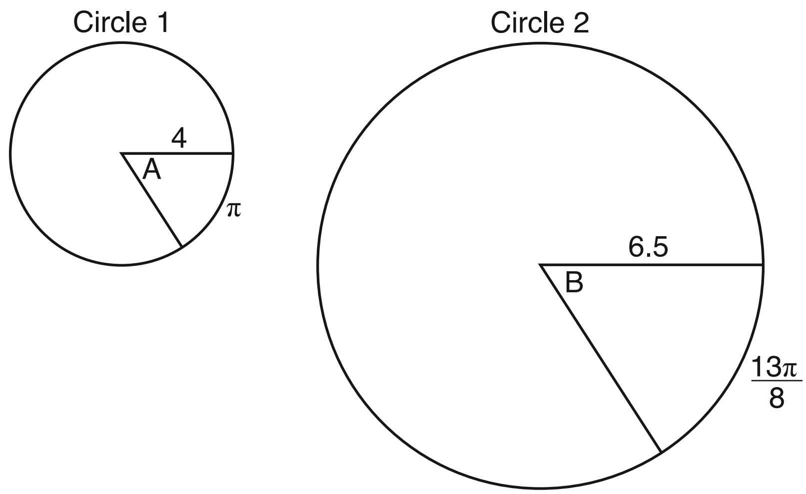
$.96 = \sin 73°$

$0° < .96 < 90°$

Score 0: The student had a completely incorrect response.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle *A* intercepts an arc of length π, and angle *B* intercepts an arc of length $\frac{13\pi}{8}$.



Dominic thinks that angles *A* and *B* have the same radian measure. State whether Dominic is correct or not. Explain why.

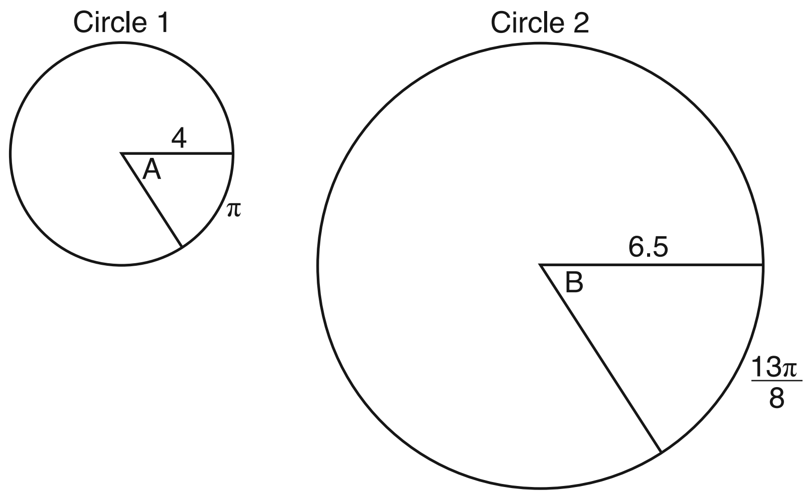
	circle 1	circle 2
radian	$\frac{\pi}{4}$	$\frac{\frac{13\pi}{8}}{6.5} = \frac{13\pi}{52} = \frac{\pi}{4}$

Dominic is correct because by caculating $\frac{\text{intercepted arc}}{\text{radius}}$ we will find the radian. As a result $\frac{\pi}{4}$ radian from angle A is equal to $\frac{(\frac{13\pi}{8})}{6.5}$ radian from angle B.

Score 2: The student had a complete and correct response.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.



Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

Circle 1

$s = \theta r$

$\pi = \theta \cdot 4$

$\frac{\pi}{4} = \frac{4\theta}{4}$

$\boxed{\theta = \frac{\pi}{4}}$

Circle 2

$s = \theta r$

$\frac{13\pi}{8} = \theta\, 6.5$

$\frac{13\pi}{8} \times \frac{6.5\theta}{1}$

$\frac{52\theta}{52} = \frac{13\pi}{52}$

$\theta = \frac{13\pi}{52}$

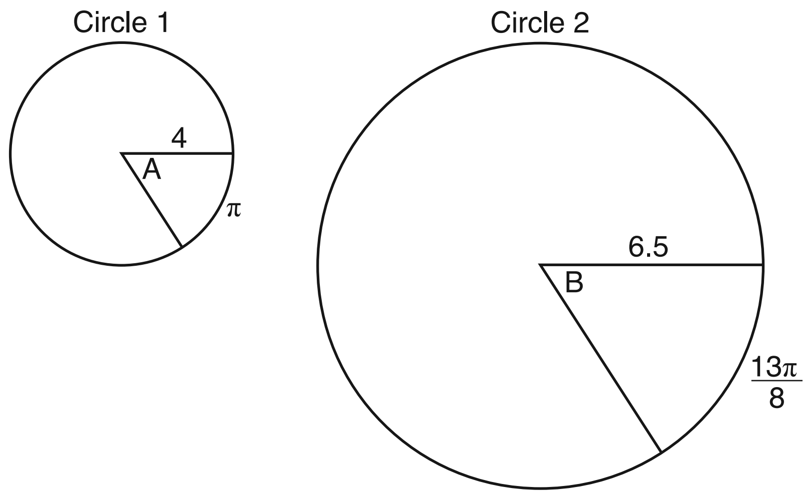
$\boxed{\theta = \frac{\pi}{4}}$

Dominic is correct. Using the formula for arc length, $s = \theta r$, both angles are equal.

Score 2: The student had a complete and correct response.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.



Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

$$\frac{4}{6.5} = \frac{\pi}{13\pi 8}$$

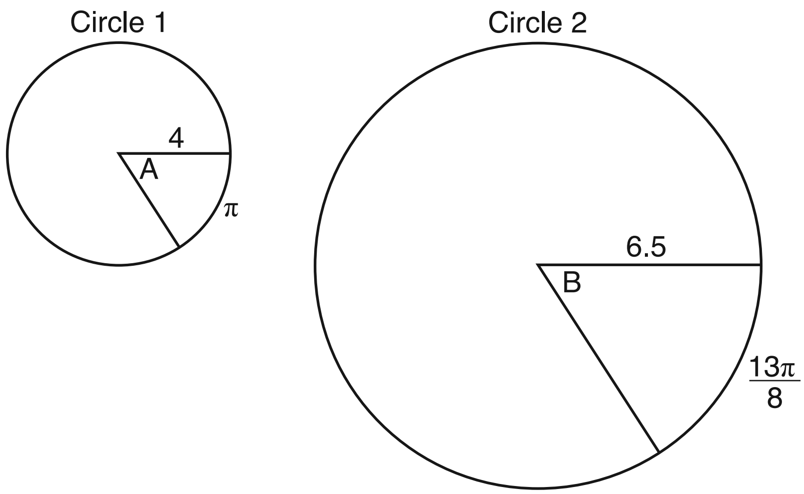
$$1306.9 = 20.4$$

No because the radii is not proportional to the arc length

Score 1: The student made an error in transcribing $\frac{13\pi}{8}$, but wrote a correct explanation based on the error.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.



Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

$\frac{4}{6.5} = \frac{\pi}{\frac{13\pi}{8}}$ $\qquad \frac{\pi}{1} \cdot \frac{8}{13\pi} = \frac{8}{13}$

$\frac{4}{6.5} \times \frac{8}{13}$

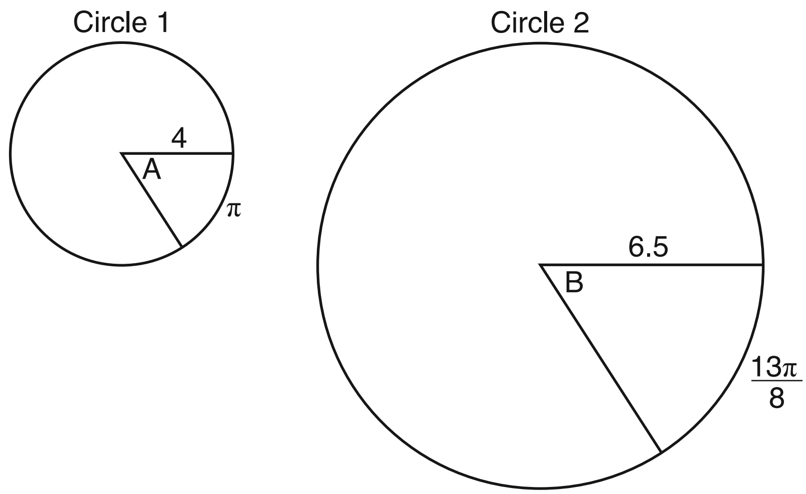
$52 = 52$

Dominic is correct

Score 1: The student wrote a correct proportion and showed work with a correct conclusion, but the explanation was missing.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle *A* intercepts an arc of length π, and angle *B* intercepts an arc of length $\frac{13\pi}{8}$.



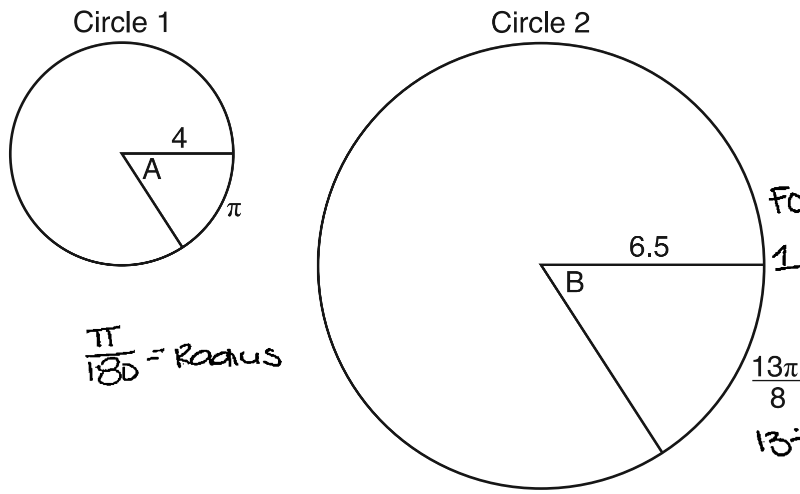
Dominic thinks that angles *A* and *B* have the same radian measure. State whether Dominic is correct or not. Explain why.

$$\frac{4}{\pi} = \frac{6.5}{\frac{13\pi}{8}}$$

Score 0: The student wrote a correct proportion, but no explanation was written.

Question 29

29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.



Formula: 1 degree = $\frac{\pi}{180}$ radius.

$\frac{\pi}{180}$ = Radius

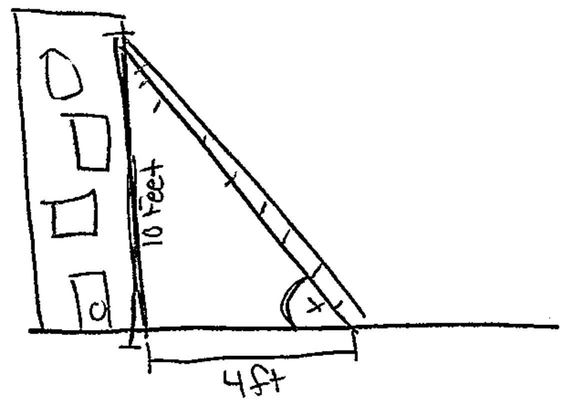
$13 \div 2 = 6.5$

Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

Dominic is incorrect because if you simply plug the numbers into the formula you would do (4)π over 180 and you would get two different radians which means Both wouldn't equal 6.5

Score 0: The student had a completely incorrect response.

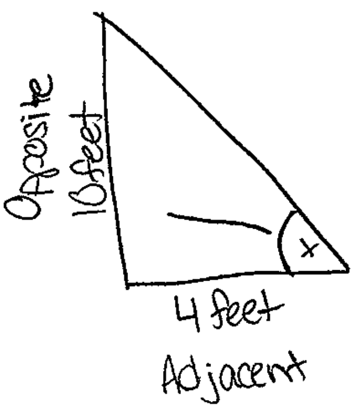
30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.



Sine
Opposite
Hypotenuse

Cosine
Adjacent
Hypotenuse

Tangent
Opposite
Adjacent



$\text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$

$\text{Tan} = \frac{10\text{ ft}}{4\text{ ft}}$ $\quad \text{Tan} = \frac{10}{4} = x$

$\text{Tan}^{-1}\left(\frac{10}{4}\right)$

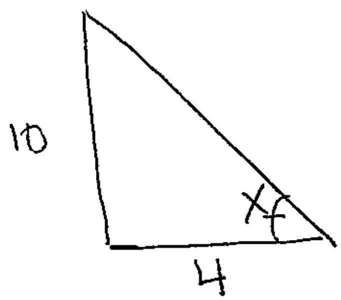
68°

$x = 68.19859051$

Score 2: The student had a complete and correct response.

Question 30

30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.



$\tan x = \frac{O}{A}$

$\tan x = \frac{10}{4}$

$\tan^{-1}(10/4) = x$

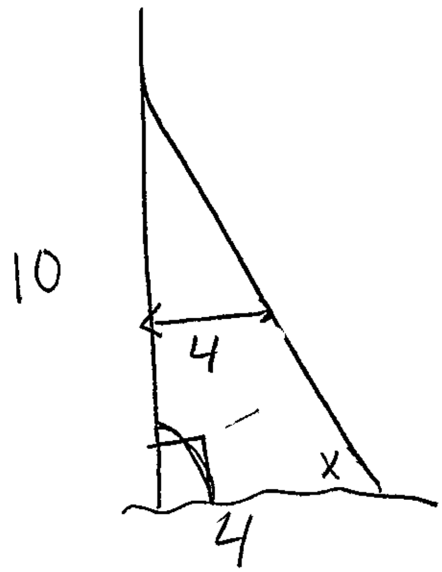
$68.19 = x$

$68° = x$

Score 2: The student had a complete and correct response.

Question 30

30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.



SOH CAH TOA

$$\frac{\tan x}{1} = \frac{10}{4}$$

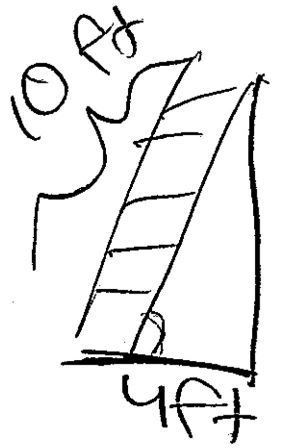
$$\frac{\tan x \cdot 4}{4} = \frac{10}{4}$$

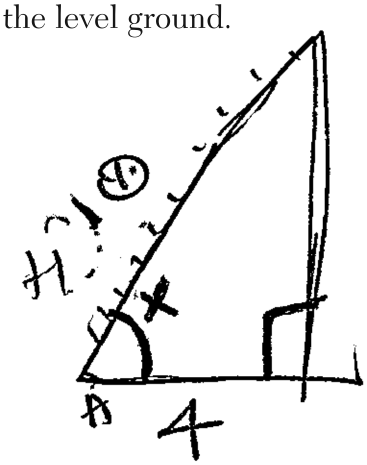
Tan

Score 1: The student wrote a correct trigonometric equation.

Question 30

30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.





$\cos x = \frac{4}{10}$

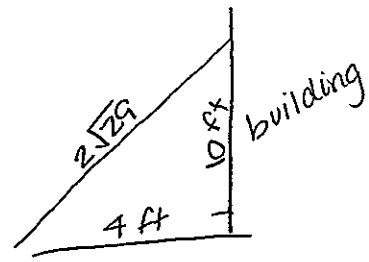
$x = 66.421$

$x = 66°$

Score 1: The student incorrectly labeled the height, but found an appropriate angle measure.

Question 30

30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the *nearest degree*, the angle that the ladder makes with the level ground.



$a^2+b^2=c^2$

$4^2+10^2=c^2$

$16+100=c^2$

$\sqrt{116}=\sqrt{c^2}$

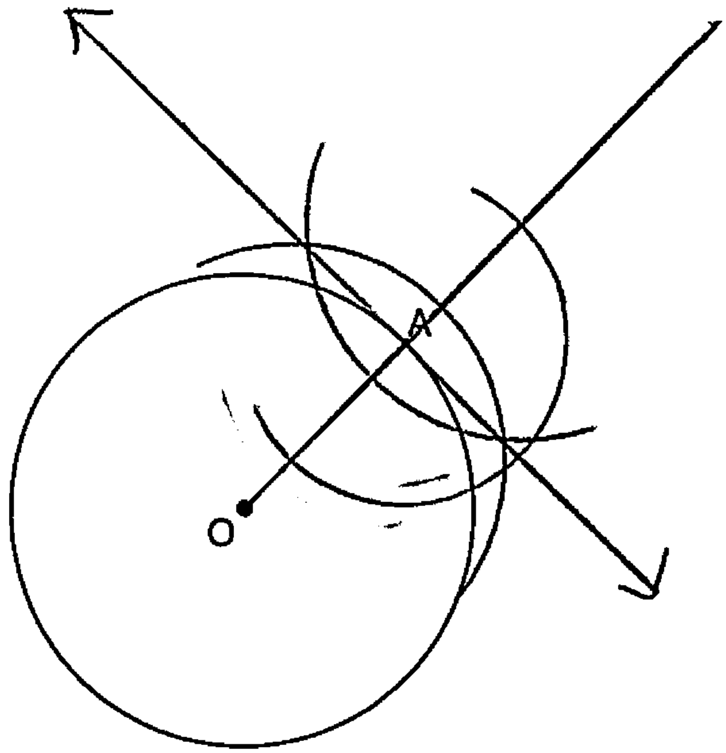
$\sqrt{4\sqrt{29}}=c^2$

$\boxed{2\sqrt{29}=c}$

Score 0: The student used the Pythagorean Theorem to find the length of the ladder and made no attempt to find the measure of the angle.

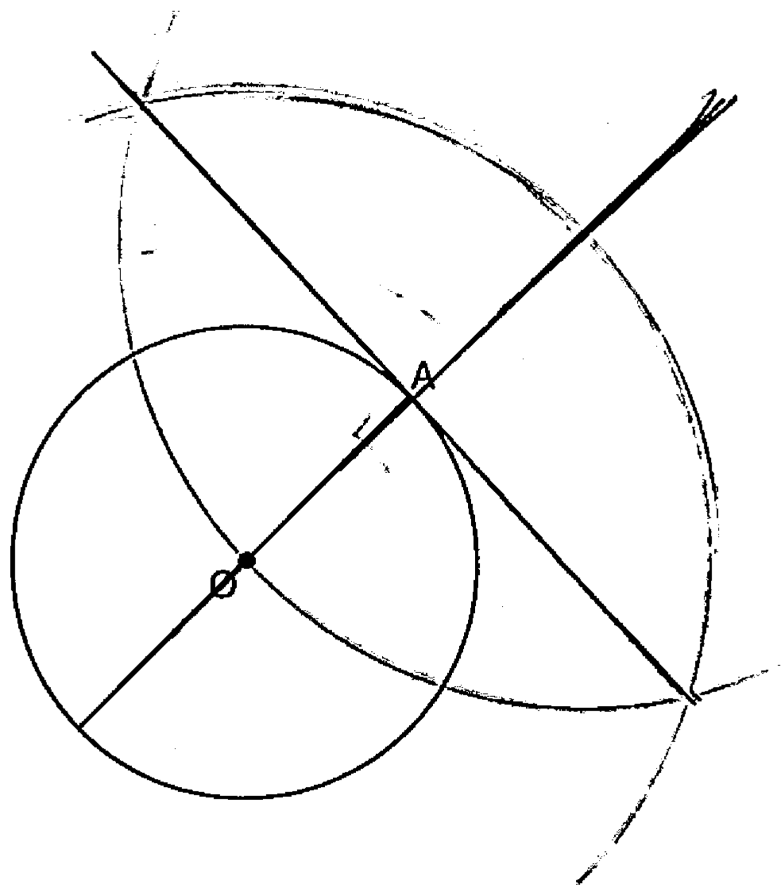
Question 31

31 In the diagram below, radius $\overline{OA}$ is drawn in circle O. Using a compass and a straightedge, construct a line tangent to circle O at point A. [Leave all construction marks.]



Score 2: The student had a complete and correct response.

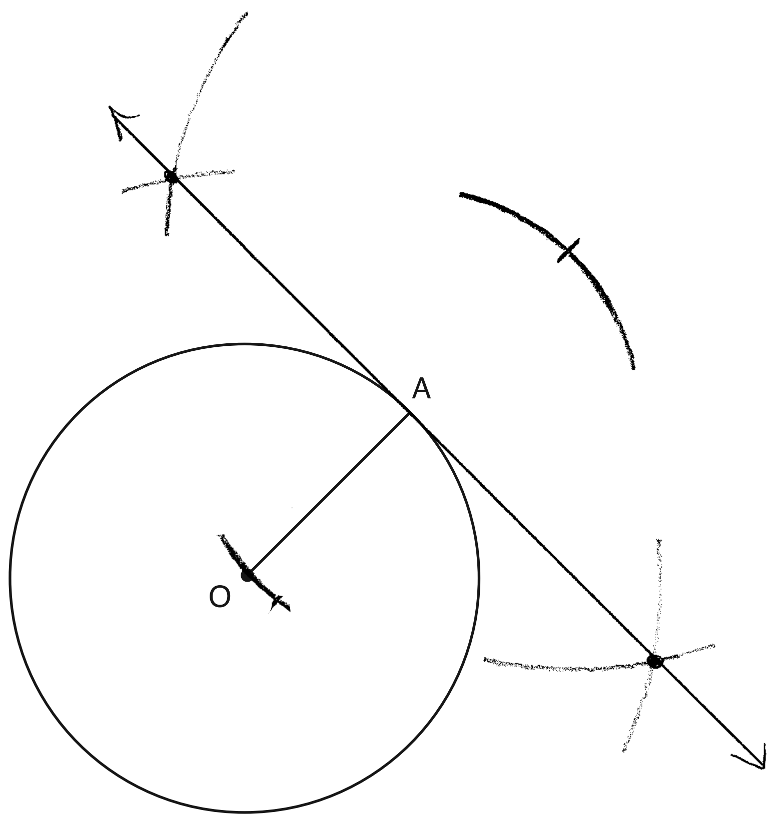
31 In the diagram below, radius $\overline{OA}$ is drawn in circle O. Using a compass and a straightedge, construct a line tangent to circle O at point A. [Leave all construction marks.]



Score 2: The student had a complete and correct response.

Question 31

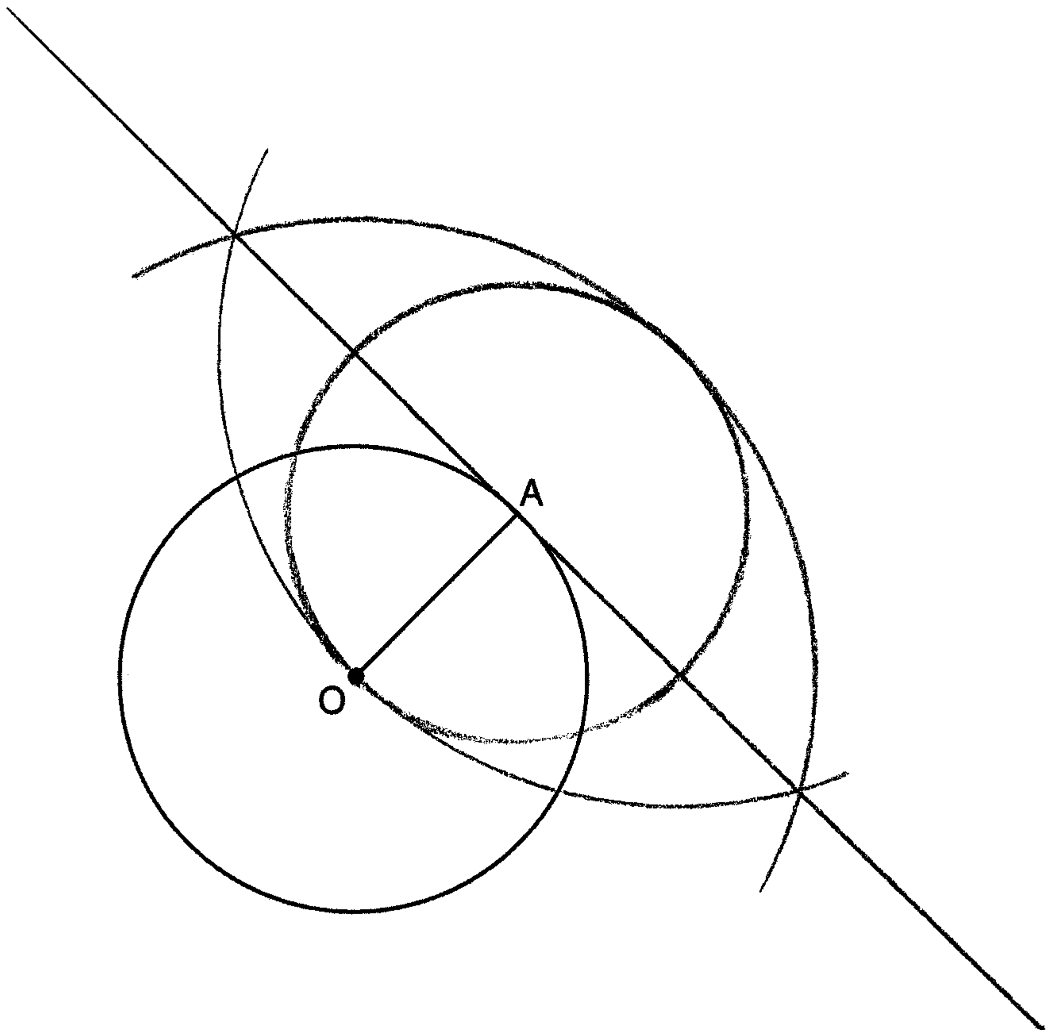
31 In the diagram below, radius $\overline{OA}$ is drawn in circle O. Using a compass and a straightedge, construct a line tangent to circle O at point A. [Leave all construction marks.]



Score 2: The student had a complete and correct response.

Question 31

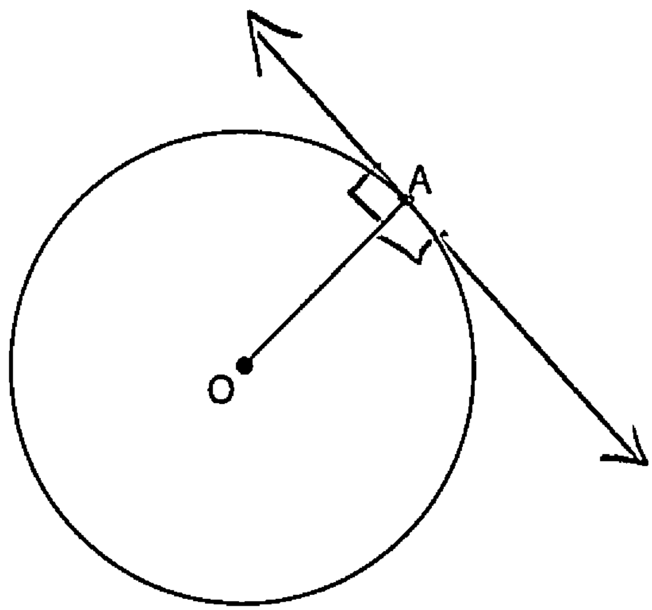
31 In the diagram below, radius $\overline{OA}$ is drawn in circle O. Using a compass and a straightedge, construct a line tangent to circle O at point A. [Leave all construction marks.]



Score 1: The student did not indicate the endpoint of the diameter of circle A, which was necessary to construct the other arcs.

Question 31

31 In the diagram below, radius $\overline{OA}$ is drawn in circle O. Using a compass and a straightedge, construct a line tangent to circle O at point A. [Leave all construction marks.]



Score 0: The student made a drawing that was not a construction.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$d = 22.5 \quad h = 33.5$

$V = \pi r^2 h$

$V = \pi (11.25)^2 \cdot 33.5$

$V = 13319.86198 \text{ in}^3$

$$\frac{13319.86198}{231}$$

$= 57.66174016$ gallons

$= 57.7$ gallons

Score 4: The student had a complete and correct response.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

of cylinder

$V = \pi r^2 h$

D = 22.5
H = 33.5

$V = \pi(11.25)^2(33.5)$

$V = 13220.5$

[illegible]

$13220.5 \div 231 = 57.2$

There are 57.2 gallons of fuel

Score 3: The student made an error in calculating the volume.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$V = \pi r^2 h$

$V = \pi(11.25)^2(33.5)$

$V = \pi\, 126.5625(33.5)$

$V = \frac{4239.84375}{231} = \boxed{18.3 \text{ gallons.}}$

Score 2: The student did not multiply by π and made a rounding error.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$V = \pi r^2 h$

$V = \pi (22.5^2)(33.5)$

$V = 53{,}279.44791$

$\boxed{V = 53{,}279.4}$

Score 1: The student made an error in using the diameter to find the volume of the barrel, and did not find the number of gallons.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$V = \frac{1}{3}\pi r^2 h$ $\qquad r = \frac{22.5}{2} = 11.25$

$\frac{1}{3}\pi(11.25)^2(33.5)$

$V = \frac{4439.954}{231} =$ 19 gallons

Score 1: The student used an incorrect volume formula and made a rounding error.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$$\frac{22.5}{2} = 11.25$$

$$V = \pi(11.25^2)(33.5)$$

Score 1: The student made correct substitutions into the volume formula of a cylinder, but no further correct work was shown.

Question 32

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the *nearest tenth*, the gallons of fuel that are in a barrel of fuel oil.

$V = \frac{1}{2}\pi r^2(h)$

$V = \frac{1}{2}\pi 11.25^2(33.5)$

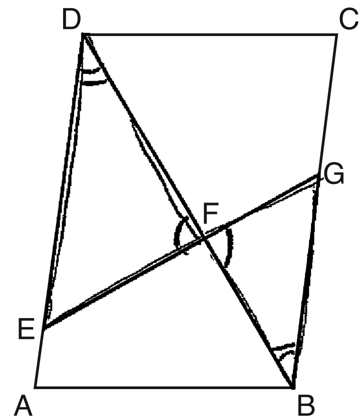
$V = \frac{1}{2}\pi 126.5625(33.5)$

$V = \frac{1}{2}\pi 4201.875$

$\boxed{6600.294816}$

Score 0: The student used an incorrect formula, made a computational error, and did not determine the number of gallons of fuel.

33 Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$



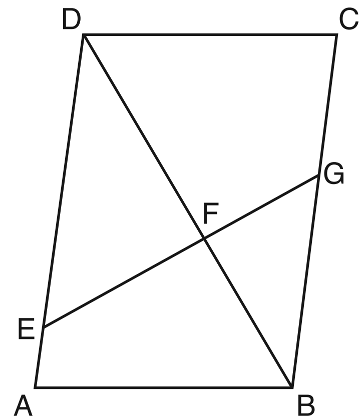
Prove: $\triangle DEF \sim \triangle BGF$

statements	reasons
1) Parallelogram ABCD, $\overline{EFG}$, and diagonal $\overline{DFB}$	1) Given
2) ∠DFE ≅ ∠GFB	2) Intersecting lines form ≅ vertical angles
3) $\overline{DA} \parallel \overline{CB}$	3) In a parallelogram, opposite sides are parallel
4) ∠ADB ≅ ∠DBC	4) Parallel lines cut by a transversal forms ≅ alternate interior angles
5) △DEF ~ △BGF	5) A.A. ≅ A.A.

Score 4: The student had a complete and correct proof.

Question 33

33 Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$



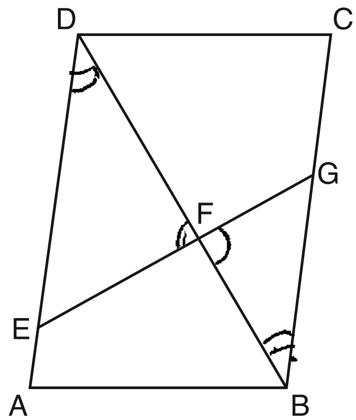
Prove: $\triangle DEF \sim \triangle BGF$

In parallelogram ABCD, the opposite sides $\overline{AD}$ and $\overline{CB}$ are parallel. Since $\overline{AD} \parallel \overline{CB}$ are cut by transversals $\overline{EFG}$ and $\overline{DFB}$, then you have congruent alternate interior angles, $\angle EDF \cong \angle GBF$ and $\angle DEF \cong \angle BGF$. Since two pairs of angles in the triangles are congruent, then $\triangle DEF \sim \triangle BGF$ by AA triangle similarity.

Score 4: The student had a complete and correct response.

Question 33

33 Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$



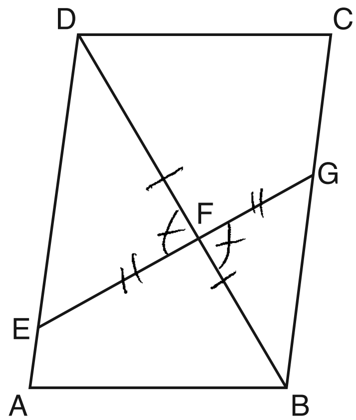
Prove: $\triangle DEF \sim \triangle BGF$

Statement.	Reason.
1. Parallelogram ABCD, $\overline{EFG}$, and diagonal $\overline{DFB}$	1. Given
2. ∠DFE and ∠BFG are vertical angles.	2. opposite and share a vertex.
3. ∠DFE ≅ ∠BFG (A)	3. vertical angles are ≅.
4. ∠ADB ≅ ∠CBD (A)	4. ∥ lines cut by transversal, ∴ alternate interior angles are ≅
5. △DEF ~ △BGF	5. AA

Score 3: The student omitted one statement and reason.

Question 33

33 Given: Parallelogram *ABCD*, $\overline{EFG}$, and diagonal $\overline{DFB}$



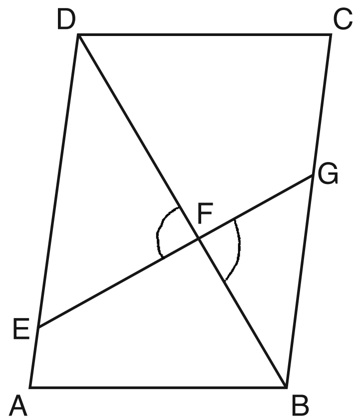
Prove: $\triangle DEF \sim \triangle BGF$

STATEMENTS	REASONS
① Parallelogram ABCD, $\overline{EFG}$ + Diagonal $\overline{DFB}$	① Given
② $\overline{DF} \cong \overline{FB}$, $\overline{EF} \cong \overline{FG}$	② Diagonals of a Parallelogram bisect each other.
③ $\angle DFE \cong \angle BFG$	③ Vertical Angles are congruent
④ $\triangle DEF \cong \triangle BGF$	④ SAS ≅ SAS
⑤ $\triangle DEF \sim \triangle BGF$	⑤ All congruent triangles are Similar

Score 2: The student made an error in assuming that $\overline{DFB}$ and $\overline{EFG}$ are both diagonals, which significantly reduced the difficulty of the proof.

Question 33

33 Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$



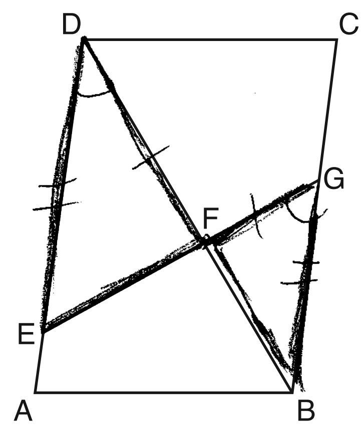
Prove: $\triangle DEF \sim \triangle BGF$

1. ▱ABCD, $\overline{EFG}$, diagonal $\overline{DFB}$	1. Given
2. ∡DFE ≅ ∡GFB	2. Vertical angles are ≅.
3. ∡D ≅ ∡B.	3. opposite angles ≅.
4. △DEF ~ △BGF	4. Because they are similar

Score 1: The student had only one correct relevant statement and reason.

Question 33

33 Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$



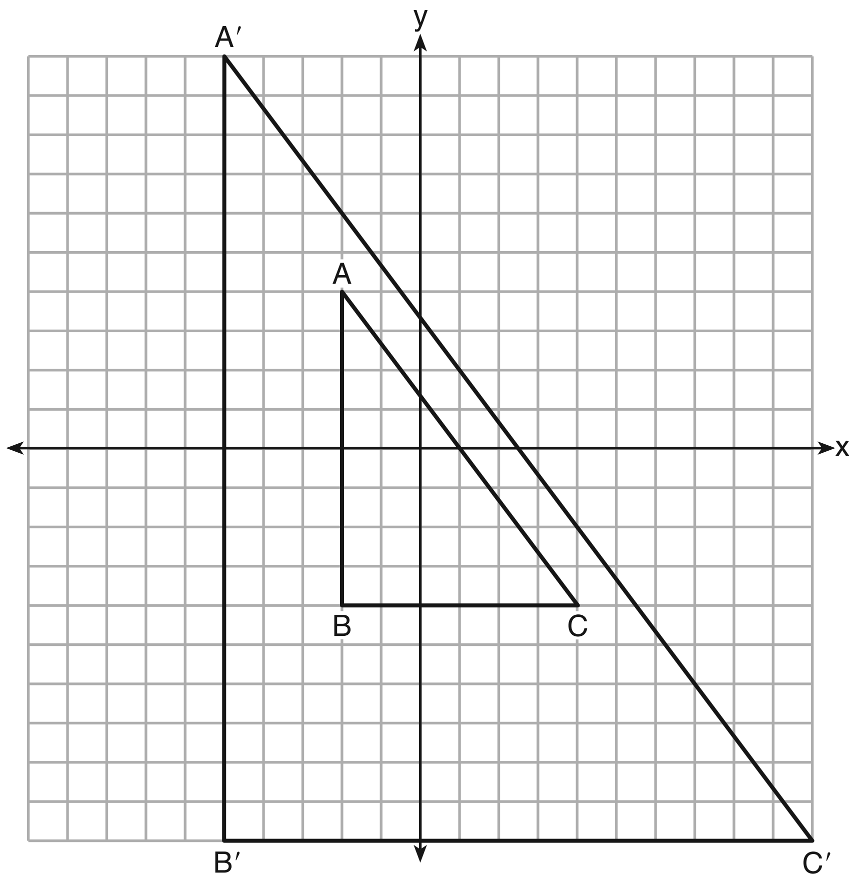
Prove: $\triangle DEF \sim \triangle BGF$

1. ▱ABCD, $\overline{EFG}$, diagonal $\overline{DFB}$	1. Given
2. $\overline{DE} \cong \overline{BG}$	2. opposite sides are ≅ in a parallelogram.
3. ∡D ≅ ∡G	3. Angles are ≅.
4. $\overline{DF} \cong \overline{GF}$	4. Bisector
5. $\triangle DEF \sim \triangle BGF$	5. SAS

Score 0: The student had no correct reasons.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

A dilation of 5/2 about the origin.

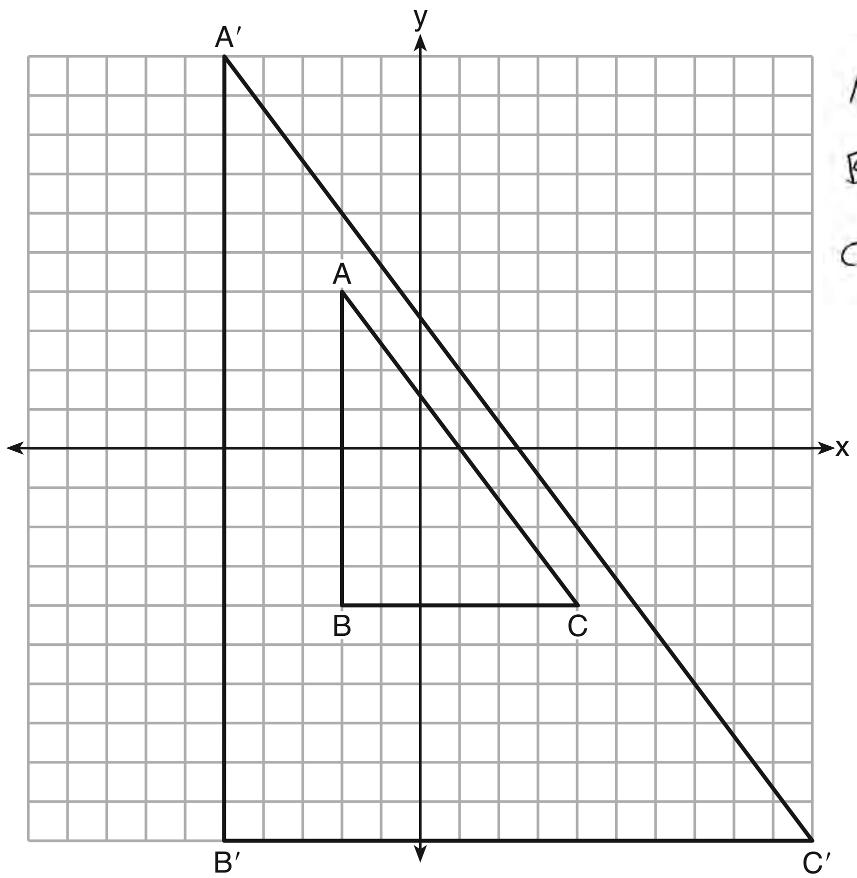
Explain why $\triangle A'B'C' \sim \triangle ABC$.

Dilations preserve angle measure so $\triangle ABC \sim \triangle A'B'C'$ by AA similarity.

Score 4: The student had a complete and correct response.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



$A(-2,4) \rightarrow A'(-5,10)$

$B(-2,-4) \rightarrow B'(-5,-10)$

$C(4,4) \rightarrow C'(10,-10)$

Describe the transformation that was performed.

$\frac{A'}{A}$ $\frac{-5}{-2} = 2.5$, $\frac{10}{4} = 2.5$

Dilation of 2.5, centered about the origin.

Explain why $\triangle A'B'C' \sim \triangle ABC$.

Under a dilation, angle measure is preserved.

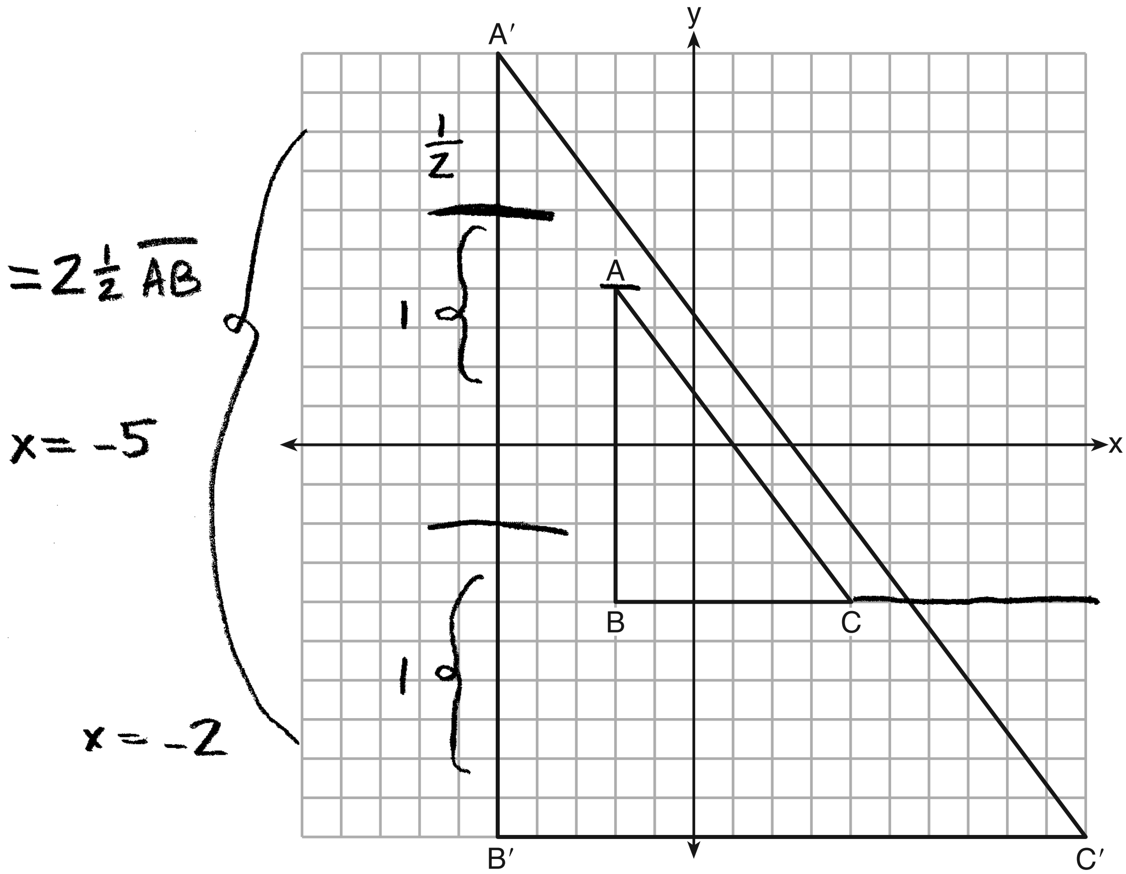
Therefore, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, $\angle C \cong \angle C'$

$\triangle A'B'C' \sim \triangle ABC$ by AAA

Score 4: The student had a complete and correct response.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

a dialation of 2.5 (Scale factor)

Explain why $\triangle A'B'C' \sim \triangle ABC$.

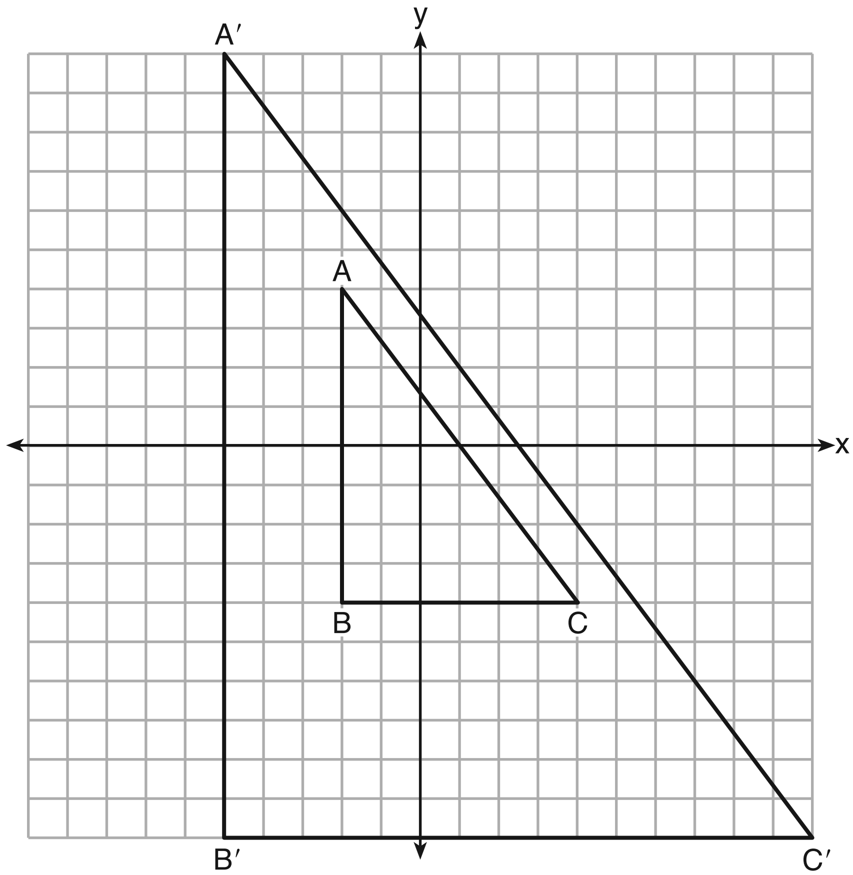
In a dialation the angles remain the same however the lenghts of the sides change (in proportion to their original lenghts) (AAA property of similarity)

definition of similarity

Score 3: The student did not state the center of dilation.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

$\triangle ABC$ was dilated by $\frac{5}{2}$ to get $A'B'C'$

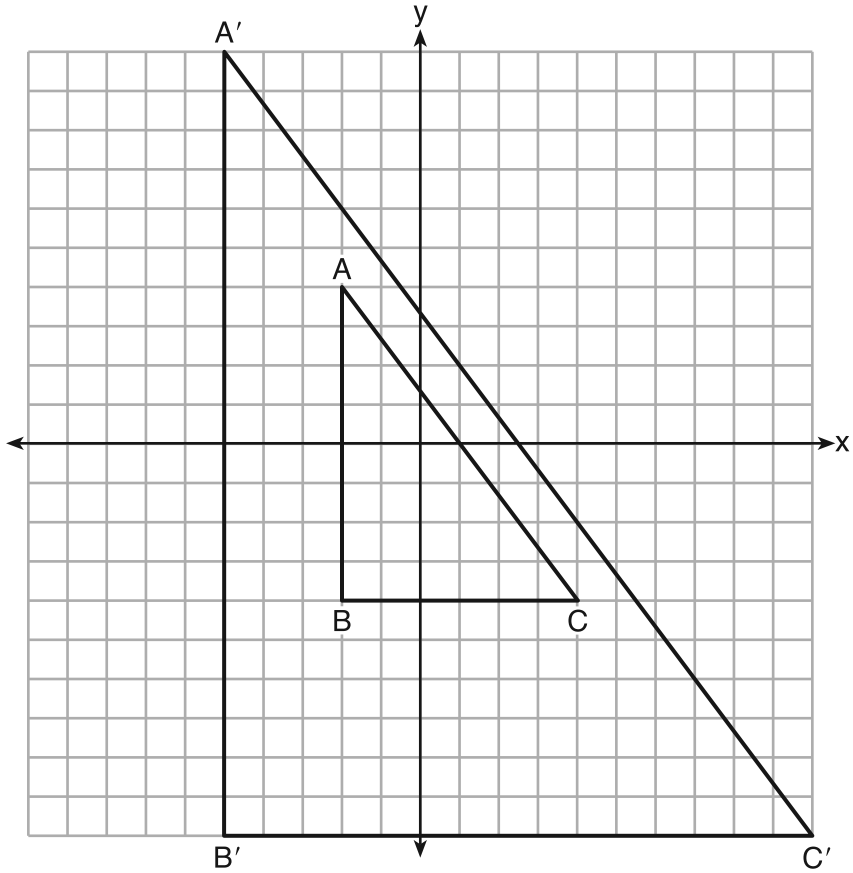
Explain why $\triangle A'B'C' \sim \triangle ABC$.

$\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$ because angle measure is preserved because its a dilation

Score 2: The student did not state the center of dilation. The student explained why the angles are congruent, but did not explain why the triangles are similar.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

Δ ABC has been dilated with a center at origin.

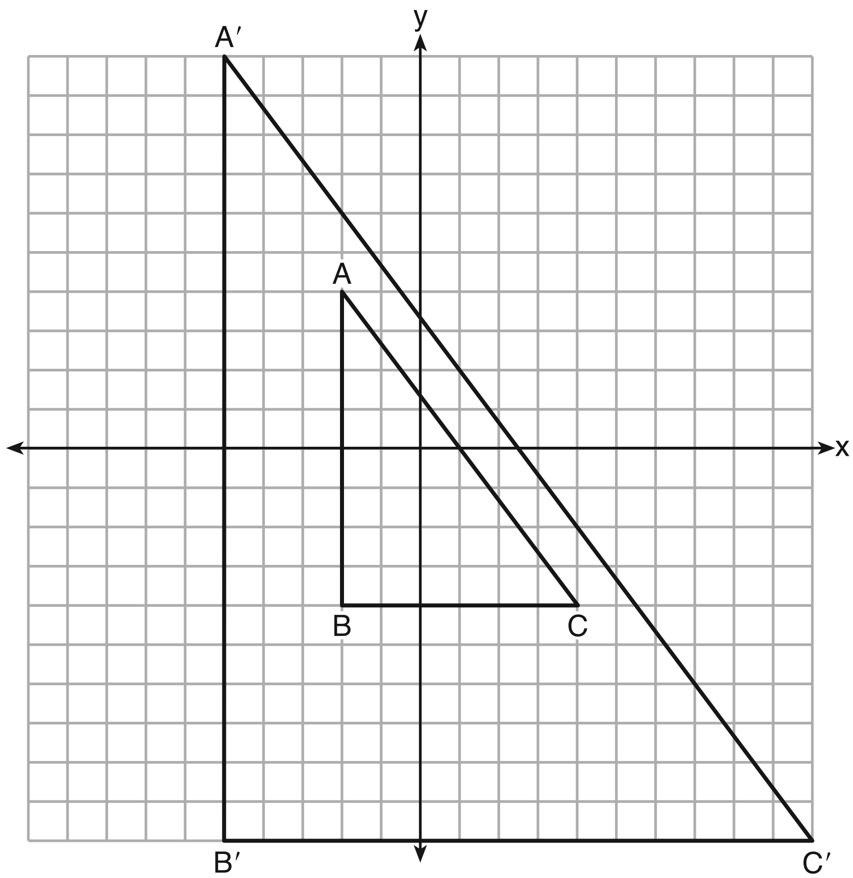
Explain why $\triangle A'B'C' \sim \triangle ABC$.

All similar triangles have congruent angles.

Score 1: The student did not state the scale factor of the dilation and did not write a correct explanation.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

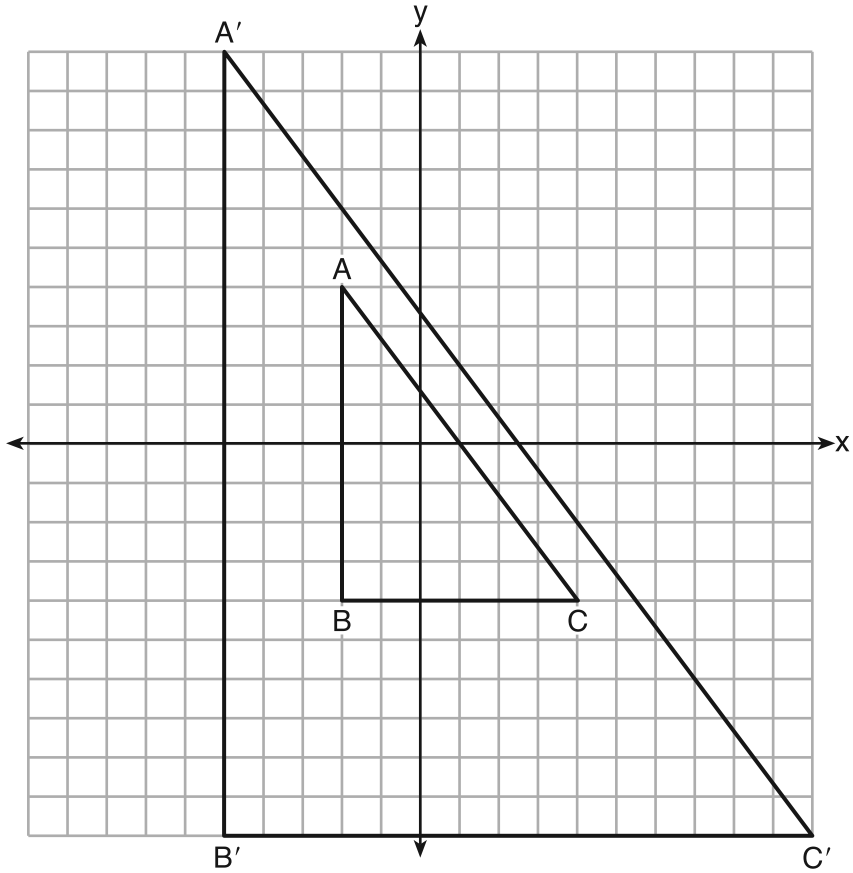
Dilation of $\frac{5}{2}$

Explain why $\triangle A'B'C' \sim \triangle ABC$.

They have the same shape

Score 1: The student had an incomplete description of the dilation and an incorrect explanation of the similar triangles.

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

Triangle ABC was dialated to make it larger.

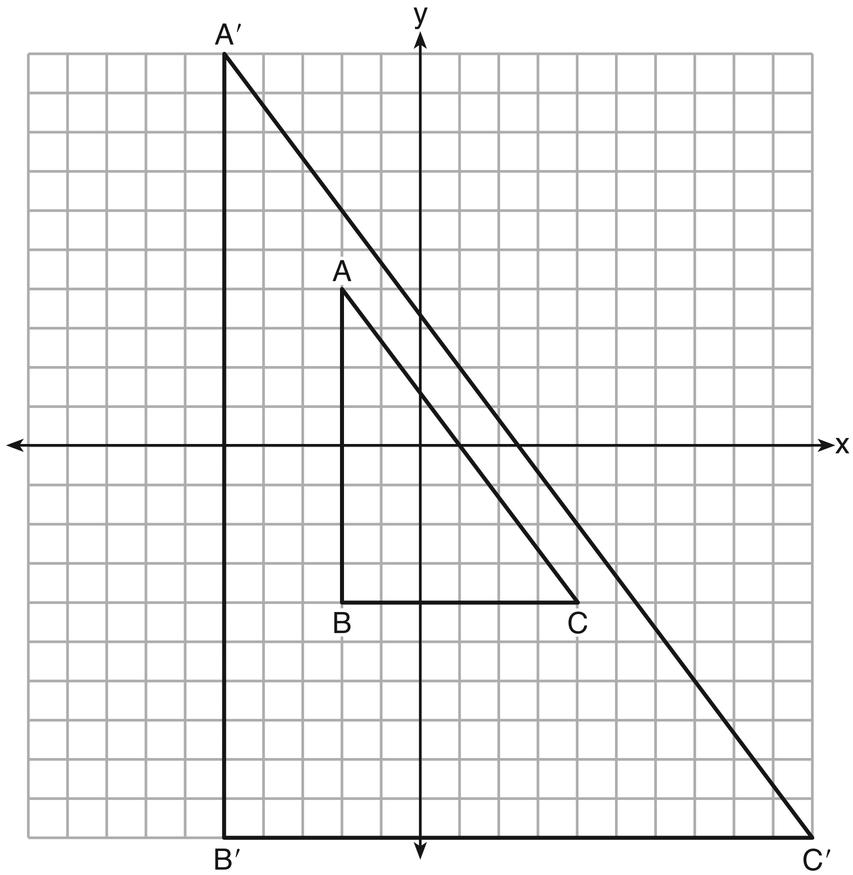
Explain why $\triangle A'B'C' \sim \triangle ABC$.

Because it was dialated

Score 0: The student did not describe the dilation, and had an incorrect explanation of the similar triangles.

Question 34

34 In the diagram below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a transformation.



Describe the transformation that was performed.

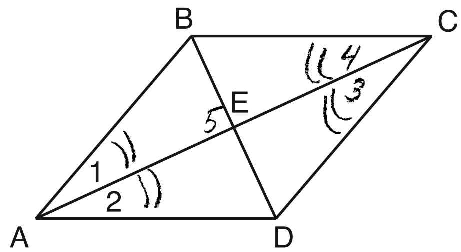
Translation took place.

Explain why $\triangle A'B'C' \sim \triangle ABC$.

because they are the same shape.

Score 0: The student had a completely incorrect response.

35 Given: Quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



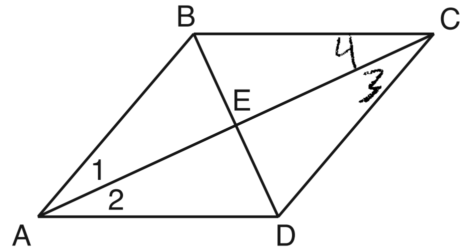
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

① Quad ABCD, Diagonal $\overline{AC}$ + $\overline{BC}$ Bisect $\angle 1 \cong \angle 2$	① Given
② Quad ABCD is A parallelogram	② a parallelogram has diagonal that bisect each other
③ $\overline{AB} \parallel \overline{CD}$	③ opposite sides of a parallelogram are parallel
④ $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	④ $\angle 1$ + $\angle 3$ and $\angle 2$ + $\angle 4$ are alt interior $\angle$'s and ALT interior $\angle$'s are $\cong$ when lines are parallel
⑤ $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$	⑤ Substitution
⑥ $\triangle$ ACD is Isoceles	⑥ An Isosceles $\triangle$ has 2 $\cong$ base $\angle$'s
⑦ $\overline{AD} \cong \overline{DC}$	⑦ The sides of an Isosceles $\triangle$ are $\cong$
⑧ Quad ABCD is a Rhombus	⑧ IF two consecutive sides of a parallelogram are congruent it is a Rhombus
⑨ $\overline{AE} \perp \overline{BE}$	⑨ THE Diagonals of a Rhombus form $\perp$ lines
⑩ $\angle 5$ is a RIGHT $\angle$	⑩ $\perp$ segments form RIGHT $\angle$'s
⑪ $\triangle$ AEB is a Right $\triangle$	⑪ A Right $\triangle$ has a RIGHT angle

Score 6: The student had a complete and correct proof.

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



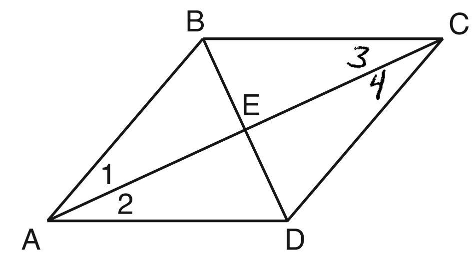
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

- quad ABCD is a parallelogram ($\square$) because the diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other
- $\overline{AB} \parallel \overline{CD}$ because opp sides of $\square$ are $\parallel$
- $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$ because $\parallel$ lines and a transversal form $\cong$ alt int angles
- Since $\angle 1 \cong \angle 2$ then $\angle 2 \cong \angle 3$ by substitution.
- $\overline{AD} \cong \overline{CD}$ because the sides opp $\cong$ $\angle$'s of a $\triangle$ are $\cong$
- $\triangle ACD$ is an isosc triangle because any $\triangle$ with $\cong$ legs is isosc.
- $\square$ ABCD is a rhombus because it has 2 consecutive $\cong$ sides
- $\angle BEA$ is a right $\angle$ because ABCD is a rhombus and $\overline{AC} \perp \overline{BD}$
- $\triangle AEB$ is a right $\triangle$ because it contains a right $\angle$

Score 6: The student had a complete and correct response.

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



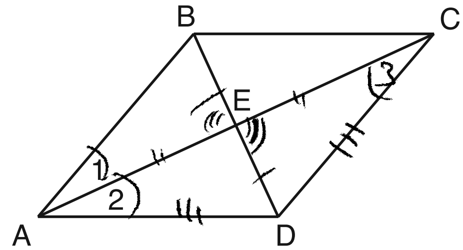
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statements	Reasons
① Quad ABCD w/ diagonal $\overline{AC}, \overline{BD}$ that bisect each other, $\angle 1 \cong \angle 2$	① Given
② Quad ABCD is a parallelogram	② If the diagonals of a quad bisect each other its a parallelogram
③ $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$	③ Opposite sides of a parallelogram are $\parallel$
④ $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$	④ $\parallel$ lines cut by a transversal make $\cong$ alt. int. $\angle$'s
⑤ $\angle 2 \cong \angle 4$	⑤ substitution
⑥ $\overline{AD} \cong \overline{CD}$	⑥ If 2 ~~angle opposite~~ angles of a Δ are $\cong$ then the 2 sides opposite the angles are $\cong$.
⑦ ΔACD is an isosceles Δ	⑦ An isosceles Δ has 2 $\cong$ sides
⑧ ABCD is a rhombus	⑧ a parallelogram that has 2 consecutive sides $\cong$ is a rhombus
⑨ $\overline{BD} \perp \overline{CA}$	⑨ A rhombus diagonals are $\perp$
⑩ ΔAEB is a RT Δ	⑩ A Δ w/ a RT $\angle$ is a RT Δ.

Score 5: The student had a statement and reason missing between steps 9 and 10.

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



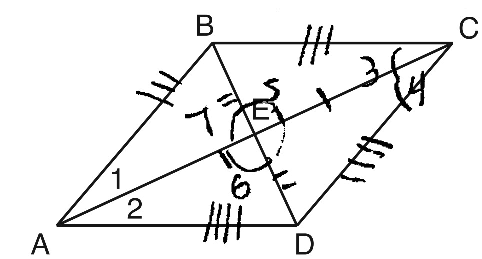
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statements	Reasons
1. Quad ABCD with $\overline{AC}$ and $\overline{BD}$ that bisect each other, $\angle 1 \cong \angle 2$	1. Given
2. $\angle BEA \cong \angle DEC$	2. Vertical $\angle$s are $\cong$.
3. $\triangle BEA \cong \triangle DEC$	3. SAS
4. $\angle 1 \cong \angle 3$	4. CPCTC
5. $\angle 2 \cong \angle 3$	5. Substitution
6. $\overline{AD} \cong \overline{DC}$	6. If 2 $\angle$s of a $\triangle$ are $\cong$, their opp sides are $\cong$.
7. $\triangle ACD$ is isosceles	7. Def of isos $\triangle$
8. $\triangle AED \cong \triangle CED$	& SSS
9. $\angle AED \cong \angle CED$	9. CPCTC
10. $\angle AED$, $\angle CED$ form a linear pair	10. Def of linear pair
11. $\angle AED$, $\angle CED$ are rt $\angle$s	11. If two angles are $\cong$ and form a linear pair, they are right
12. $\angle AEB$ is a rt $\angle$	12. Substitution
13. $\triangle AEB$ is a rt $\triangle$	13. Def of rt $\triangle$

Score 4: The student had a statement and reason missing to prove step 3 and a statement and reason missing to prove step 8.

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$

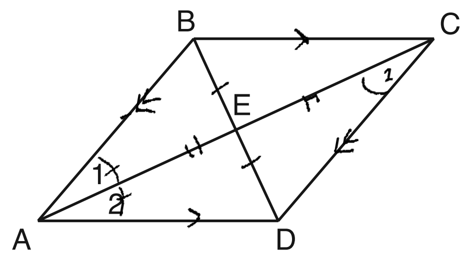


Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statements	Reasons
① AC + BD bisect eachother quad ABCD ∠1 ≅ ∠2	① Givens
② BE ≅ ED AE ≅ EC	② def. of a bisector
③ BC ∥ AD AB ∥ DC	③ in a quad, op. sides are ∥
④ ∠1 ≅ ∠4 ∠2 ≅ ∠3	④ alternate interior ∠
⑤ ∠2 ≅ ∠4 ∠1 ≅ ∠3	⑤ substitution
⑥ AD ≅ DC AB ≅ BC	⑥ in a Δ, if two ∡s are ≅ then the sides opp. the ∡s are ≅
⑦ ΔACD is an isosceles Δ	⑦ in a Δ, if two sides are ≅, and two base ∡s ≅, it is a isosceles Δ
⑧ ∠5 ≅ ∠6	⑧ vertical ∡s ≅s
⑨ ΔBEC ≅ ΔAED	⑨ SAS
⑩ BC ≅ AD	⑩ cpctc
⑪ AB ≅ BC ≅ CD ≅ AD	⑪ substitution
⑫ quad ABCD is a rhombus	⑫ in a quad, if all sides are ≅, it is a rhombus.
⑬ BD ⊥ AC	⑬ in a rhombus, diagonals are ⊥
⑭ ∠7 is a rt. ∡	⑭ ⊥ forms rt ∡s
⑮ ΔAEB is a rt. ∡	⑮ if a triangle has a rt. ∠, then it is a rt. Δ.

Score 4: The student had an incorrect reason in step 3 and an incomplete reason in step 4.

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$

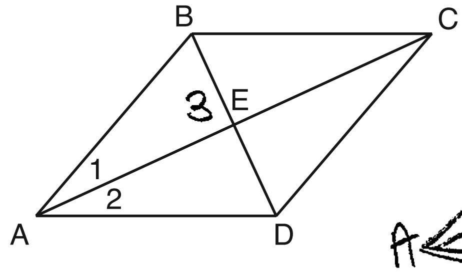


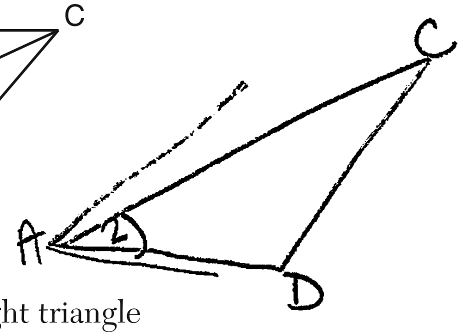
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statement	Reason
1) Quadrilateral ABCD diagonals $\overline{AC}$, $\overline{BD}$ bisect each other $\angle 1 \cong \angle 2$	1) Given
2) $\overline{BE} \cong \overline{ED}$, $\overline{AE} \cong \overline{EC}$	2) Definition of Segment Bisector
3) ▱ABCD is a parallelogram	3) Diagonals bisect each other in a parallelogram
4) $\overline{BC} \parallel \overline{AD}$, $\overline{BA} \parallel \overline{CD}$	4) Definition of parallelogram
5) $\angle DCE \cong \angle BAE$	5) Opposite interior angle theorem
6) $\angle DCE \cong \angle DAE$	6) Substitution Property of Equality
7) $\triangle ACD$ is isosceles	7) $\triangle$ has 2 $\cong$ $\angle$'s
8) $\angle AEB$ is a right angle	8) Definition of parallelogram
9) $\triangle AEB$ is right	9) Definition of right triangle

Score 3: The student had an incorrect reason in proving the isosceles triangle, and no further correct work was shown.

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$





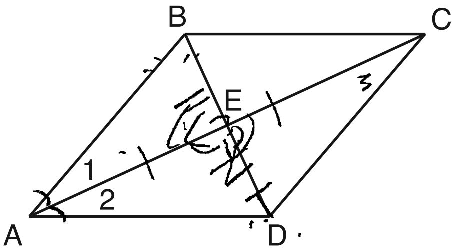
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

St's	R's
① Quad ABCD, Diag's $\overline{AC}$: $\overline{BD}$ bisect each other, ∠1 ≅ ∠2	① Given
2. $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{ED}$	2. Bisector cuts a segm. into 2 ≅ pa
3. ABCD rhombus	3. In a rhombus, the diag's bisect each other.
4. $\overline{AD} \cong \overline{DC}$	4. In a rhombus consecutive sic are ≅.
✱ 5. △ACD isos.	5. Iso. △'s have 2 ≅ sides
6. ∠3 is a rt. ∠	6. In a rhombus the diag's make rt. ∠'s
7. △AEB is a rt. △	7. Rt. △'s have 1 rt. ∠.

Score 2: The student made one conceptual error in step 3 and had one missing statement and reason to prove step 6.

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$

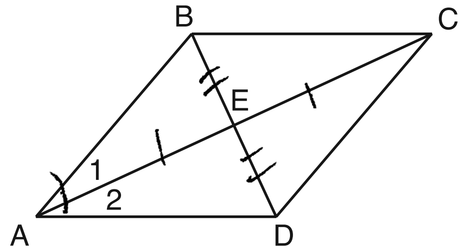


Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

S	R
1. $\overline{AC}$ and $\overline{BD}$ bisect each other Quadrilateral ABCD $\angle 1 \cong \angle 2$	1. Given
2. Quadrilateral ABCD is a parallelogram	2. A quadrilateral with diagonals that bisect each other is a parallelogram
3. $\overline{AD} \parallel \overline{BC}$	3. A parallelogram has opposite parallel sides
4. $\angle 1 \cong \angle 3$	4. alternate interior $\angle$s are $\cong$
5. $\angle 2 \cong \angle 3$	5. substitution
6. $\triangle ACD$ is isosceles	6. An isosceles $\triangle$ has two $\cong$ base $\angle$s
7.	

Score 2: The student used the incorrect parallel sides to conclude $\angle 1 \cong \angle 3$, had an incomplete reason in step 4, and did not prove the right triangle.

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



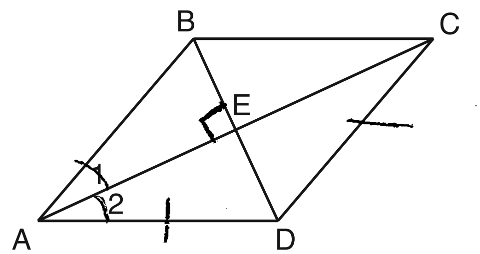
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statements	Reasons
1.) $\overline{AC}$ bisects $\overline{BD}$	1.) Given
2.) $\overline{BE} \cong \overline{ED}$	2.) Definition of bisector
3.) $\overline{BD}$ bisects $\overline{AC}$	3.) Given
4.) $\overline{AE} \cong \overline{EC}$	4.) Definition of bisector
5.) $\angle 1 \cong \angle 2$	5.) Given
6.) $\angle BEA$ is a right angle	6.) Bisectors of a rhombus form right
7.) $\triangle AEB$ is a right triangle	7.) It has a right angle
8.) $\triangle ABE \cong \triangle ADE$	8.) HL $\cong$ HL
9.) $\triangle CDE \cong \triangle BCE$	9.) HL $\cong$ HL
10.) $\triangle ACD$ is isosceles	10.) $\overline{AD} \cong \overline{CD}$

Score 1: The student had only two correct statements and reasons. (Steps 2 and 4 can be combined.)

Question 35

35 Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$



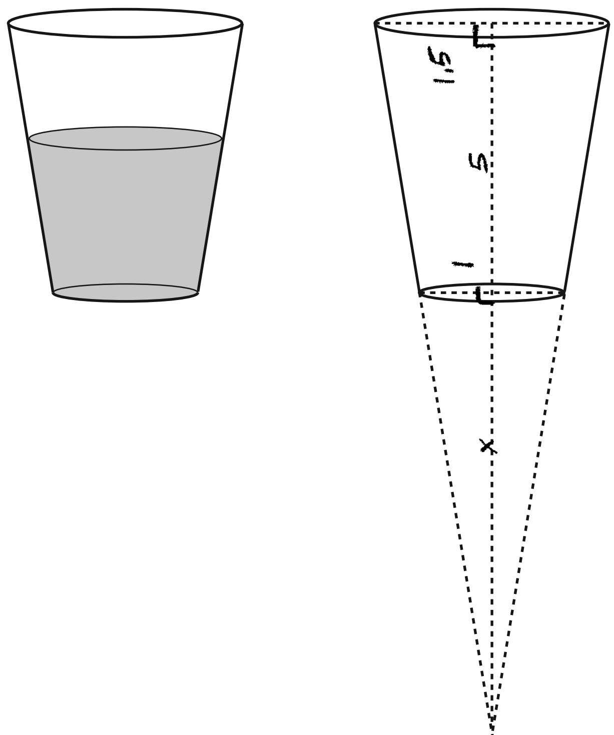
Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle

Statement	Reasoning
① Quadrilateral ABCD with diagnals $\overline{AC}$ and $\overline{BD}$ bisect each other	① Given
② ∠BAE ≅ ∠DAE	② Given
③ $\overline{AD} \cong \overline{DC}$	③ Definition of Isosceles
④ $\overline{AEC} \perp \overline{BD}$ at E	④ Definition of bisector (90°)
⑤ ∠A has congruent angles	⑤ Complementary angles which form a right triangle
⑥ △ACD is isoseceles and △AEB is right	⑥ ASA + HL
⑦ Isosceles + Right	⑦ CPCTC

Score 0: The student had no correct work.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

Because you need similar triangles in order to set up and solve a proportion.

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

From diagram above

$$\frac{x+5}{1.5} = \frac{x}{1}$$

$$x + 5 = 1.5x$$

$$5 = 0.5x$$

$$x = 10$$

The height of the cone is 15 inches.

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

Volume of large cone $= \frac{1}{3}\pi(1.5)^2 15 = 35.343 \text{ in}^3$

Volume of small cone $= \frac{1}{3}\pi(1)^2(10) = 10.472 \text{ in}^3$

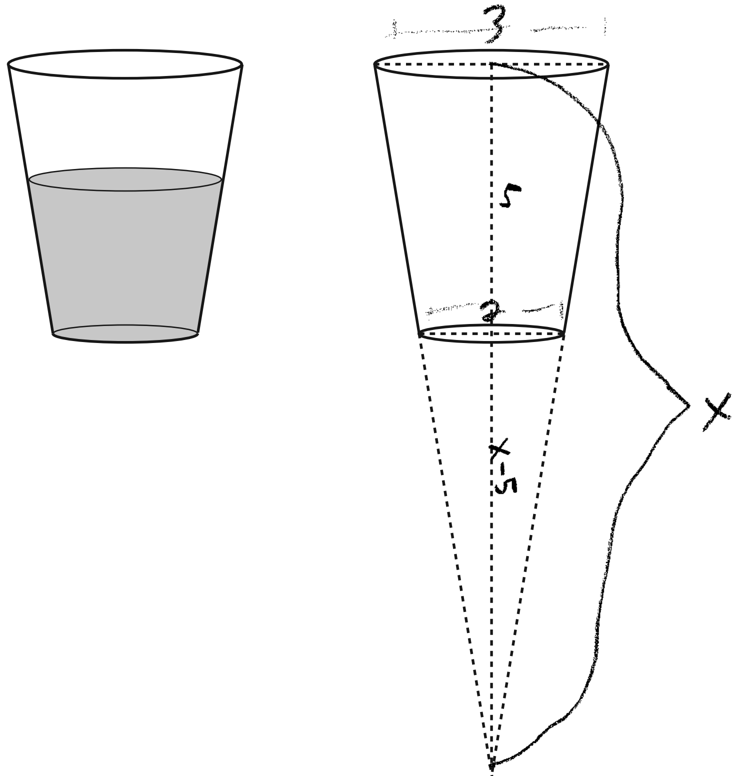
Volume of glass = (Volume of large cone) - (Volume of small cone)

$= 24.9 \text{ in}^3$

Score 6: The student had a complete and correct response.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

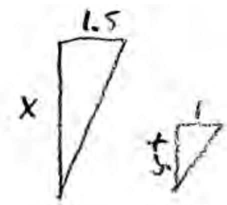
Parallel lines form ≅ corresponding angles resulting in similar triangles.

Corresponding sides of similar triangles are in proportion. The proportion can be used to find the height.

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.



$$\frac{1.5}{x} = \frac{1}{x-5}$$

$$x = 1.5x - 7.5$$

$$-1.5x \quad -1.5x$$

$$\frac{-0.5x}{-0.5} = \frac{-7.5}{-0.5}$$

$x = 15$ height of the larger cone.

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

Volume larger cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (1.5^2)(15)$

$V = 35.3429$

Volume smaller cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (1^2)(10)$

$V = 10.4719$

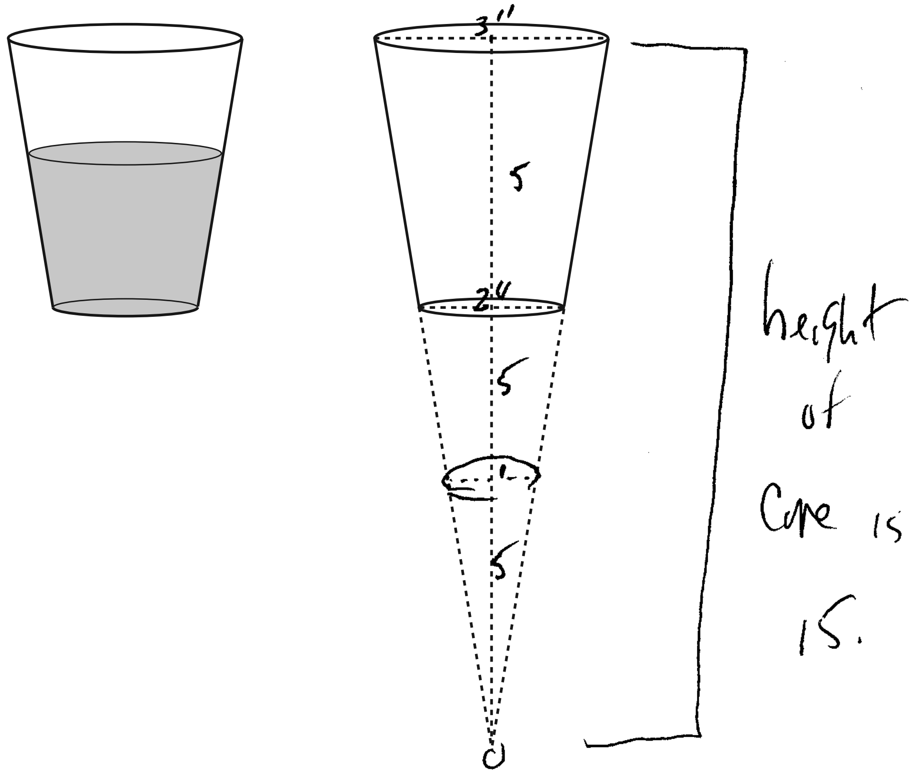
Volume Glass = 24.871

$V = 24.9$

Score 6: The student had a complete and correct response.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

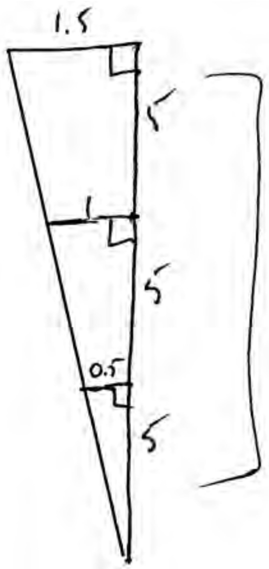
The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

The height of the cone is perpendicular to the bases of every cone in the figure. This forces the planes that contain the bases to be parallel, and a series of similar right triangles are formed

Question 36 is continued on the next page.

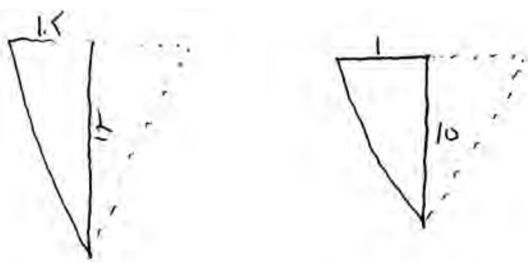
Question 36 continued

Determine and state, in inches, the height of the larger cone.



height of cone = 15

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.



$V = \frac{1}{3}\pi r^2 h$ $\quad$ $V = 1/3\pi r^2 h$

$V = \frac{1}{3}\pi(1.5)^2(15)$ $\quad$ $V = \frac{1}{3}(1)^2(10)$

$V = \frac{1}{3}\pi(2.25)(15)$

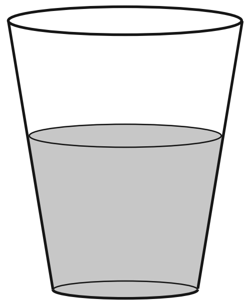
$V = 11.25\pi$ $\quad$ $V = 10/3\pi$

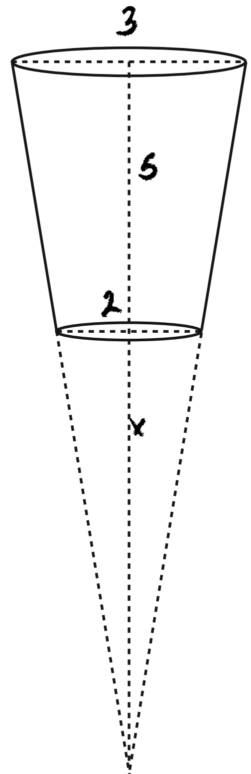
Volume of Glass $= \boxed{11.25\pi - 10/3\pi} \approx 25 \text{ in}^3$

Score 5: The student made one rounding error.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.





The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

The bases are both ⊥ to the height, so they are ∥ to each other. This lets us use similar triangles to find the height of the cone

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

$$\frac{3}{x+5} = \frac{2}{x}$$

$$3x = 2x + 10$$

$$x = 10$$

height = 5 + 10

15

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

$$V_B - V_S$$

$$V = \pi r^2 h - \pi r^2 h$$

$$V = \pi(1.5)^2(15) - \pi(1)^2(10)$$

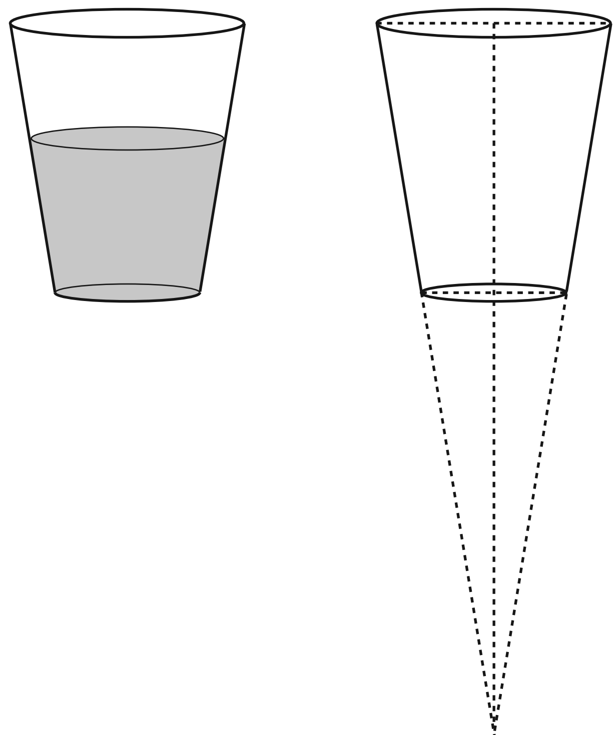
$$V = 33.75\pi - 10\pi$$

$$V = 23.75\pi$$

$$V = 74.6$$

Score 4: The student made a conceptual error in using the wrong formula in determine the volume of the water glass.

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

If the bases were not parallel, the centers would be skew and the cylinder could be oblique. – also if they weren't // then we couldn't find the ratio of Δ in diameter to Δ height

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

Change in 1" diameter = 5" hight

3" → 2" = 5" hight
2" → 1" = 5" hight
1" → 0" = 5" hight

15"

3 : 2 $\frac{2}{3}$ total hight

$\frac{2}{3}(5) = 3\frac{1}{3}$

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

$V = \frac{1}{3}\pi r^2 h$

$= \frac{\pi 1.5^2(15)}{3}$

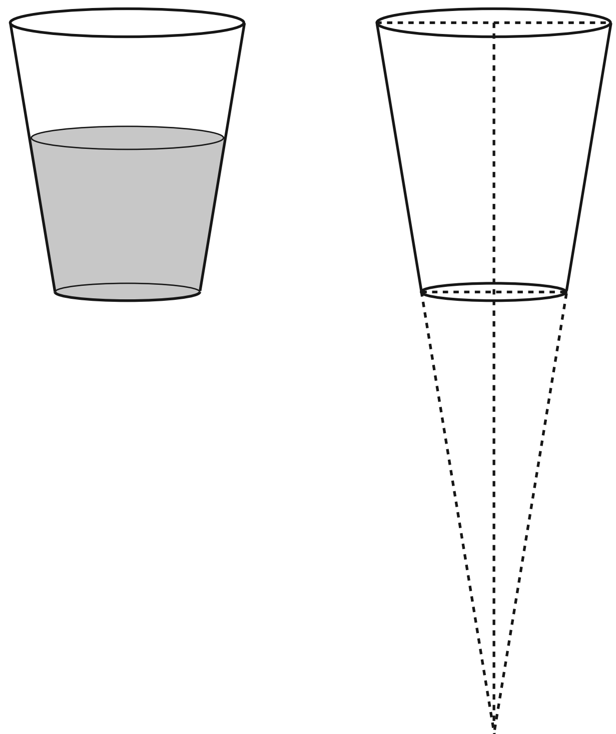
$= \frac{33.75\pi}{3}$

$\approx 35.3 in^3$

Score 3: The student correctly determined the height and the volume of the larger cone.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

Because ... ?

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

Every time the diameter decreases by 1 the length increases by 5. So going from a diameter of 3 down to 0 makes the length 15.

15 in

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

$V = \pi r^2 h$

$= \pi (1.5)^2 (5)$

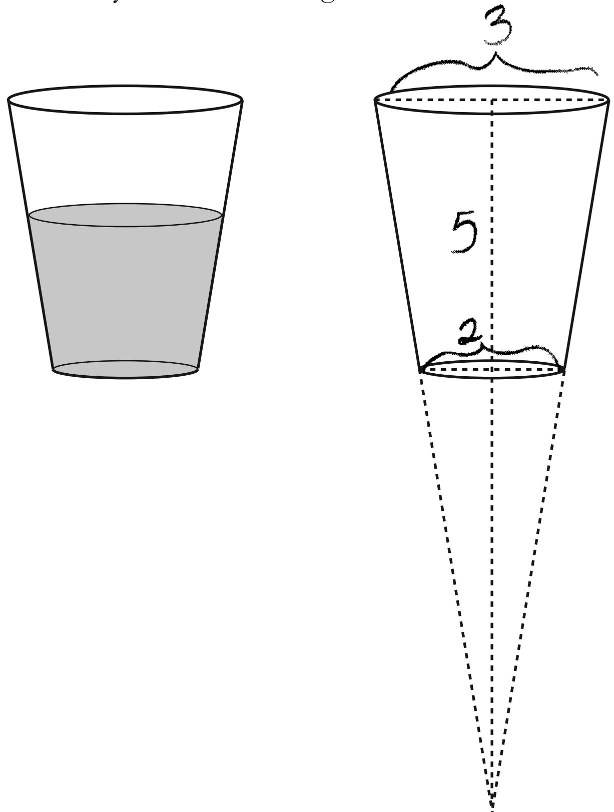
$= 35.34$

35.3 in^3

Score 2: The student only found the correct value of the height.

Question 36

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

The base with the 2" diameter must be parallel to the base with the 3" diameter to create right triangles that are similar so the sides are in proportion.

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(1.5)^2(5)$

$V = 3.75\pi$

$V = 11.8$

$V = 12$

$12 = \frac{1}{3}\pi r^2 h$

$12 = \frac{1}{3}\pi(1)^2(h)$

$12 = \frac{1}{3}\pi h$

$12 = 1.05h$

$11.4 = h$

$\boxed{h = 11}$

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

$V = \frac{1}{3}\pi r^2 h$

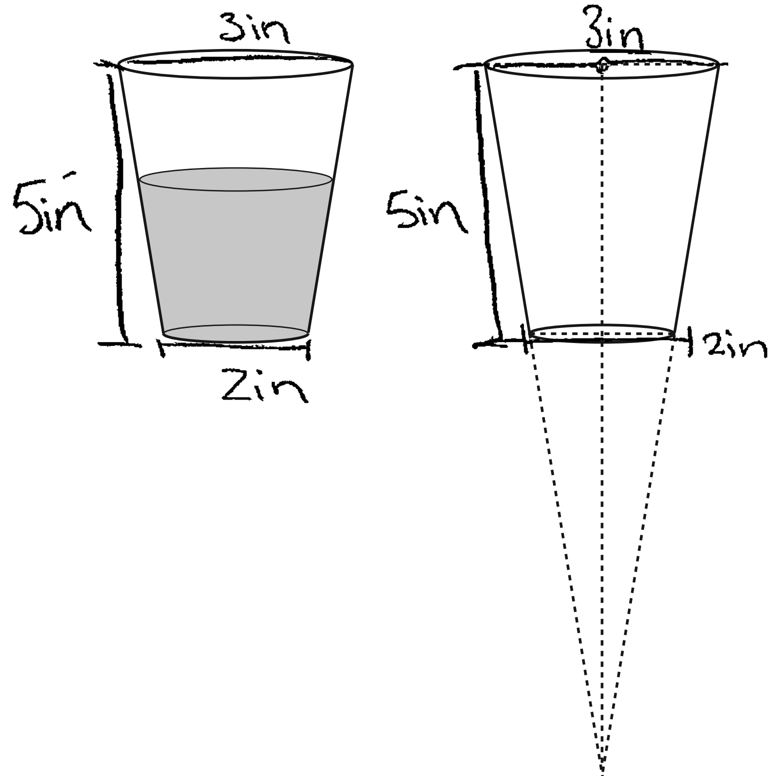
$V = \frac{1}{3}\pi(1.5)^2(5)$

$V = 3.75\pi$

$\boxed{V = 11.8}$

Score 1: The student had a correct explanation.

36 A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.



The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

If they are not parrellel then they are not a cone because the two tops shout not intersect.

Question 36 is continued on the next page.

Question 36 continued

Determine and state, in inches, the height of the larger cone.

$V = 1/3\pi r^2h + 2\pi r^2$

$V = 1/3\pi(1.5)^2 5 + 2\pi(1.5)^2$

$V = 11.78097245 + 14.13716694$

$\boxed{V = 26 \text{ in}^2}$

Determine and state, to the *nearest tenth of a cubic inch*, the volume of the water glass.

$V = \frac{1}{3}\pi r^2h$

$V = 1/3\pi(1.5)^2(5)$

$V = 11.7809$

$\boxed{V = 12.0 \text{ in}^3}$

Score 0: The student had no correct work.