Large-Type Edition

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, June 21, 2022 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

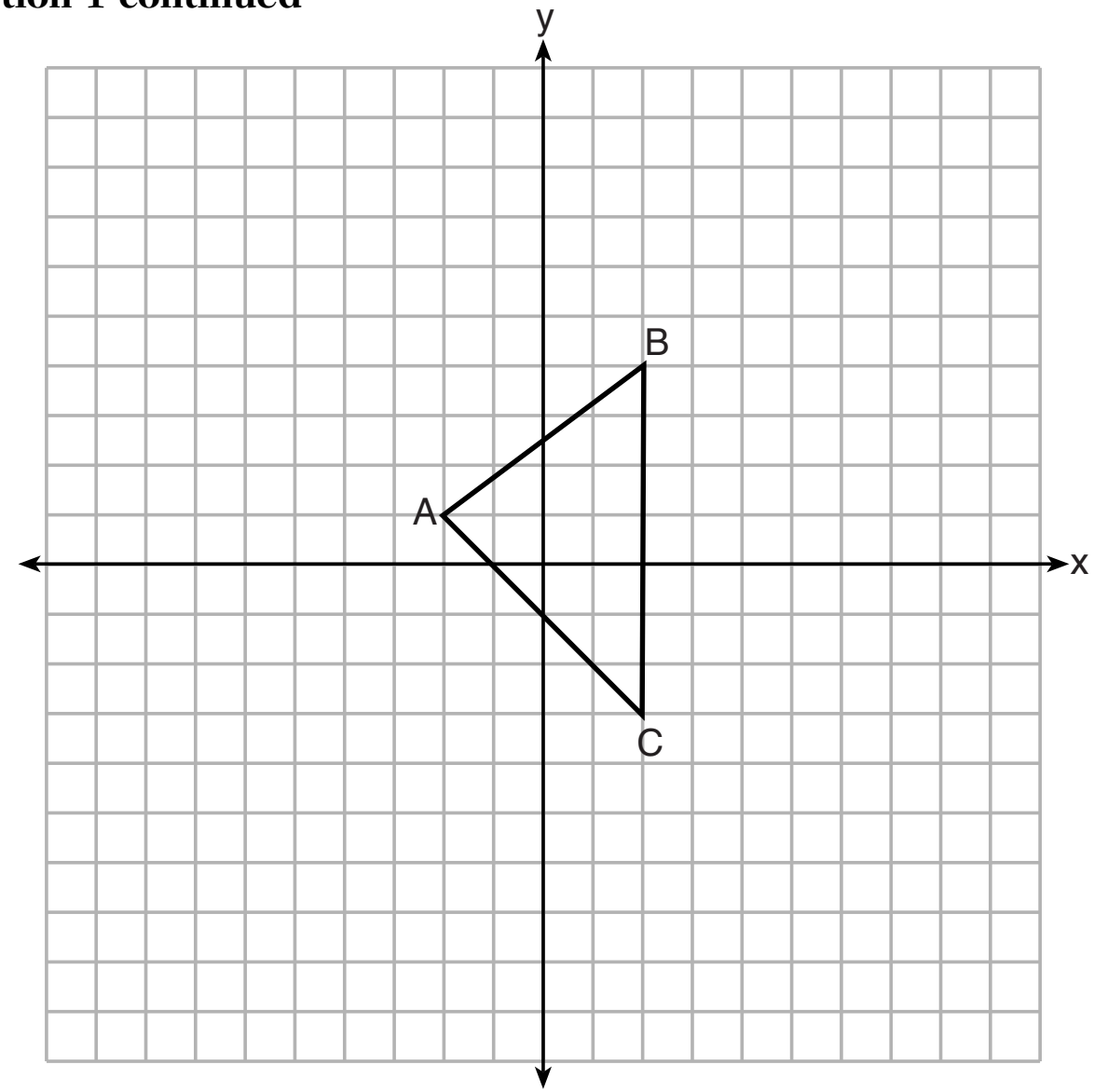
Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Triangle $A'B'C'$ is the image of $\triangle ABC$ after a dilation centered at the origin. The coordinates of the vertices of $\triangle ABC$ are $A(-2,1)$, $B(2,4)$, and $C(2,-3)$.

Question 1 is continued on the next page.

Use this space for computations.



If the coordinates of A' are $(-4,2)$, the coordinates of B' are

(1) $(8,4)$

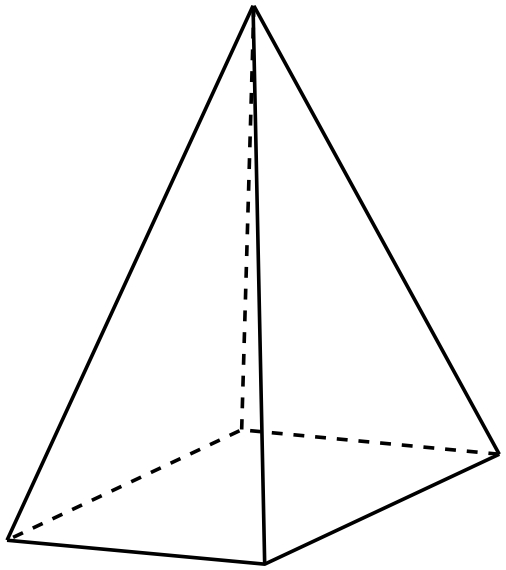
(2) $(4,8)$

(3) $(4,-6)$

(4) $(1,2)$

Use this space for computations.

2 In the diagram below, a plane intersects a square pyramid parallel to its base.



Which two-dimensional shape describes this cross section?

(1) circle

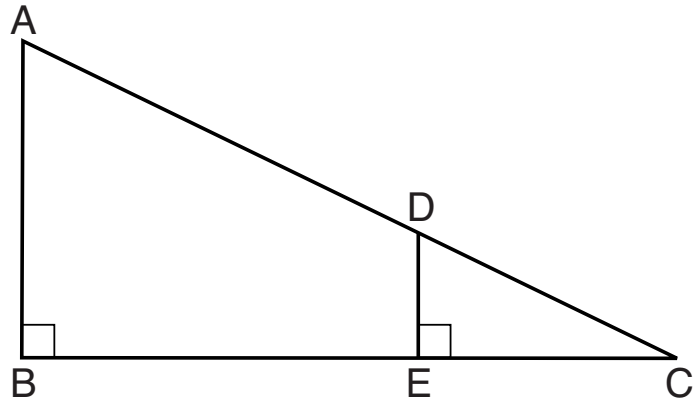
(2) square

(3) triangle

(4) pentagon

Use this space for computations.

3 In the diagram below, $\triangle CDE$ is the image of $\triangle CAB$ after a dilation of $\frac{DE}{AB}$ centered at C.



Which statement is always true?

(1) $\sin A = \frac{CE}{CD}$

(2) $\cos A = \frac{CD}{CE}$

(3) $\sin A = \frac{DE}{CD}$

(4) $\cos A = \frac{DE}{CE}$

4 A regular pentagon is rotated about its center. What is the minimum number of degrees needed to carry the pentagon onto itself?

(1) 72°

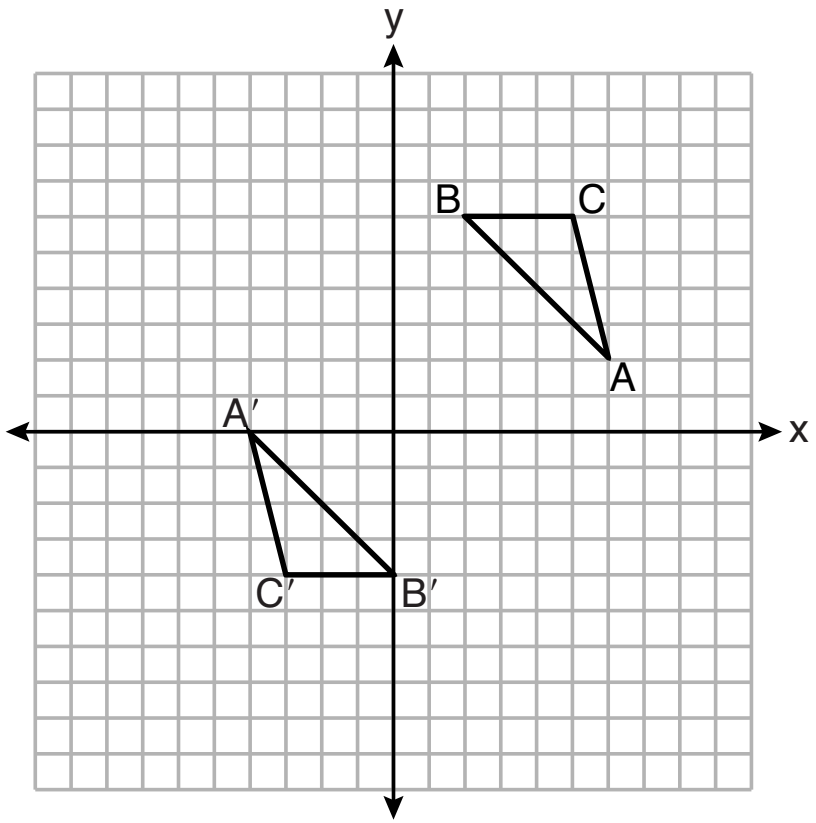
(2) 108°

(3) 144°

(4) 360°

Use this space for computations.

5 On the set of axes below, $\triangle ABC \cong \triangle A'B'C'$.



Triangle ABC maps onto $\triangle A'B'C'$ after a

(1) reflection over the line $y = -x$

(2) reflection over the line $y = -x + 2$

(3) rotation of 180° centered at (1,1)

(4) rotation of 180° centered at the origin

Use this space for computations.

6 Right triangle TMR is a scalene triangle with the right angle at M. Which equation is true?

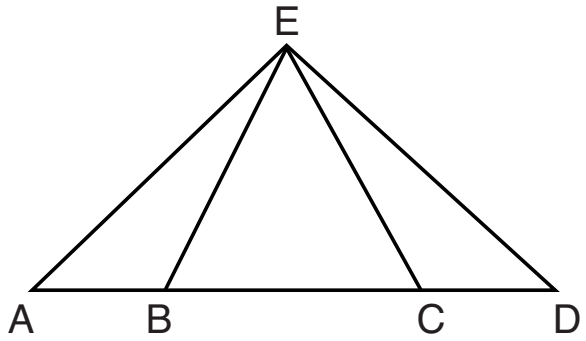
(1) $\sin M = \cos T$

(2) $\sin R = \cos R$

(3) $\sin T = \cos R$

(4) $\sin T = \cos M$

7 In the diagram below of $\triangle AED$ and $\overline{ABCD}$, $\overline{AE} \cong \overline{DE}$.



Which statement is always true?

(1) $\overline{EB} \cong \overline{EC}$

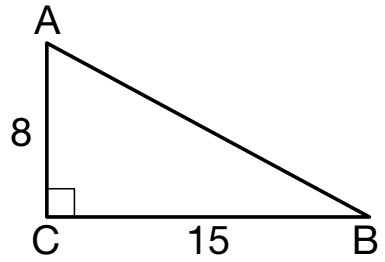
(2) $\overline{AC} \cong \overline{DB}$

(3) $\angle EBA \cong \angle ECD$

(4) $\angle EAC \cong \angle EDB$

Use this space for computations.

8 As shown in the diagram below, right triangle ABC has side lengths of 8 and 15.



If the triangle is continuously rotated about $\overline{AC}$, the resulting figure will be

(1) a right cone with a radius of 15 and a height of 8

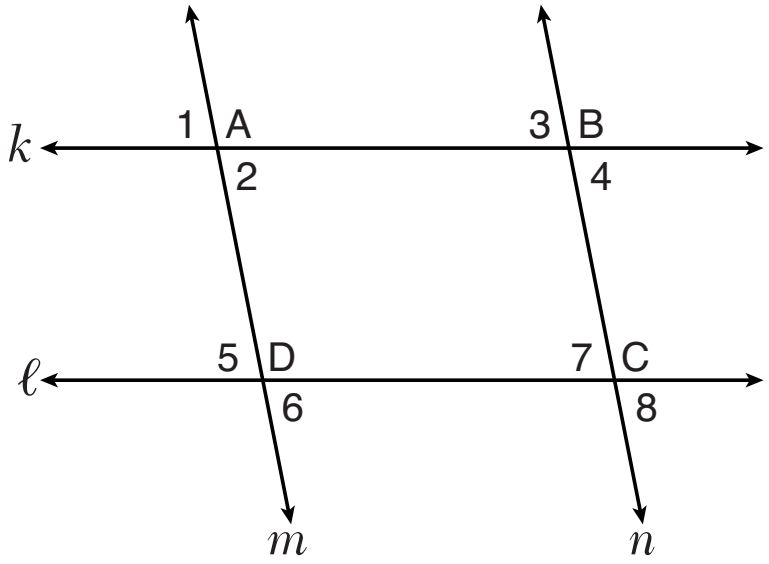
(2) a right cone with a radius of 8 and a height of 15

(3) a right cylinder with a radius of 15 and a height of 8

(4) a right cylinder with a radius of 8 and a height of 15

Use this space for computations.

9 In the diagram below, lines k and ℓ intersect lines m and n at points A, B, C, and D.



Which statement is sufficient to prove $ABCD$ is a parallelogram?

(1) $\angle 1 \cong \angle 3$

(2) $\angle 4 \cong \angle 7$

(3) $\angle 2 \cong \angle 5$ and $\angle 5 \cong \angle 7$

(4) $\angle 1 \cong \angle 3$ and $\angle 3 \cong \angle 4$

10 Which transformation does *not* always preserve distance?

(1) $(x,y) \rightarrow (x + 2, y)$

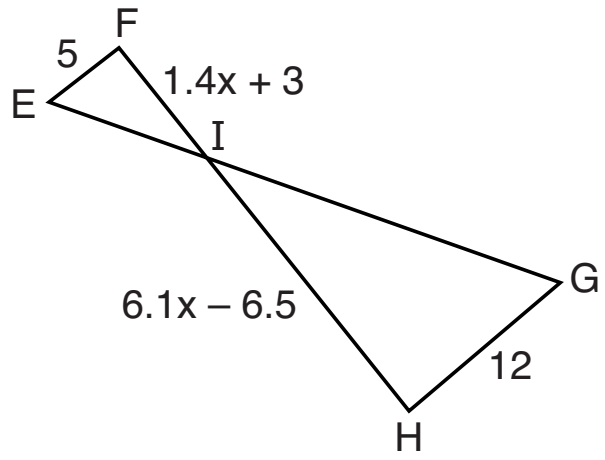
(2) $(x,y) \rightarrow (-y, -x)$

(3) $(x,y) \rightarrow (2x, y - 1)$

(4) $(x,y) \rightarrow (3 - x, 2 - y)$

Use this space for computations.

11 In the diagram below, $\overline{EF} \parallel \overline{HG}$, $EF = 5$, $HG = 12$, $FI = 1.4x + 3$, and $HI = 6.1x - 6.5$.



What is the length of $\overline{HI}$?

(1) 1

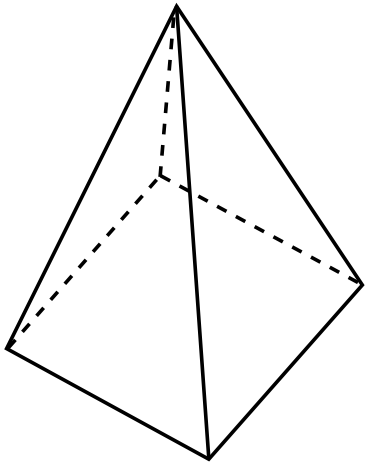
(2) 5

(3) 10

(4) 24

Use this space for computations.

12 The square pyramid below models a toy block made of maple wood.



Each side of the base measures 4.5 cm and the height of the pyramid is 10 cm. If the density of maple is 0.676 g/cm^3, what is the mass of the block, to the *nearest tenth of a gram*?

(1) 45.6

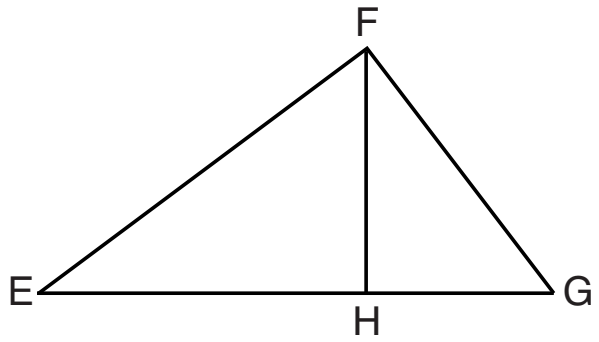
(2) 67.5

(3) 136.9

(4) 202.5

Use this space for computations.

13 In the diagram below of right triangle EFG, altitude $\overline{FH}$ intersects hypotenuse $\overline{EG}$ at H.



If $FH = 9$ and $EF = 15$, what is EG?

(1) 6.75

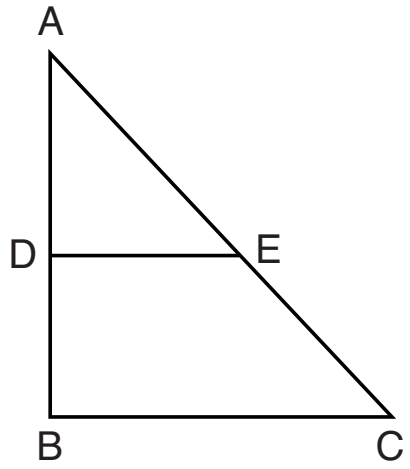
(2) 12

(3) 18.75

(4) 25

Use this space for computations.

14 In triangle ABC below, D is a point on $\overline{AB}$ and E is a point on $\overline{AC}$, such that $\overline{DE} \parallel \overline{BC}$.



Which statement is always true?

(1) $\angle ADE$ and $\angle ABC$ are right angles.

(2) $\triangle ADE \sim \triangle ABC$

(3) $DE = \frac{1}{2}BC$

(4) $\overline{AD} \cong \overline{DB}$

15 If one exterior angle of a triangle is acute, then the triangle must be

(1) right

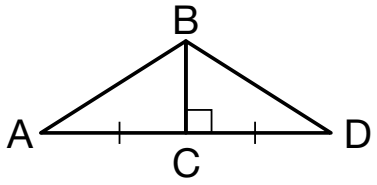
(2) acute

(3) obtuse

(4) equiangular

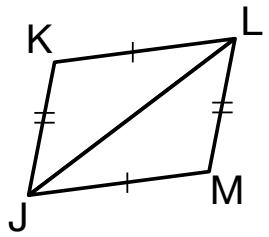
Use this space for computations.

16 Given the information marked on the diagrams below, which pair of triangles can *not* always be proven congruent?



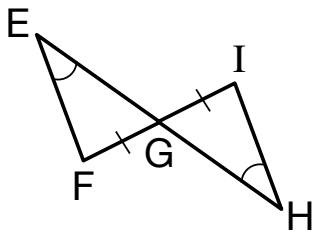
△*ABC* and △*DBC*

(1)



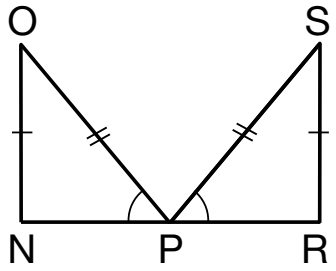
△*KLJ* and △*MJL*

(3)



△*EFG* and △*HIG*

(2)

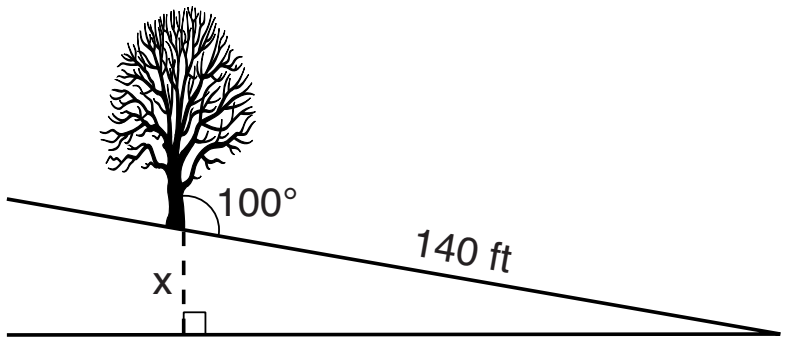


△*NOP* and △*RSP*

(4)

Use this space for computations.

17 The diagram below shows a tree growing vertically on a hillside. The angle formed by the tree trunk and the hillside is 100°. The distance from the base of the tree to the bottom of the hill is 140 feet.



What is the vertical drop, x, to the base of the hill, to the *nearest foot*?

(1) 24

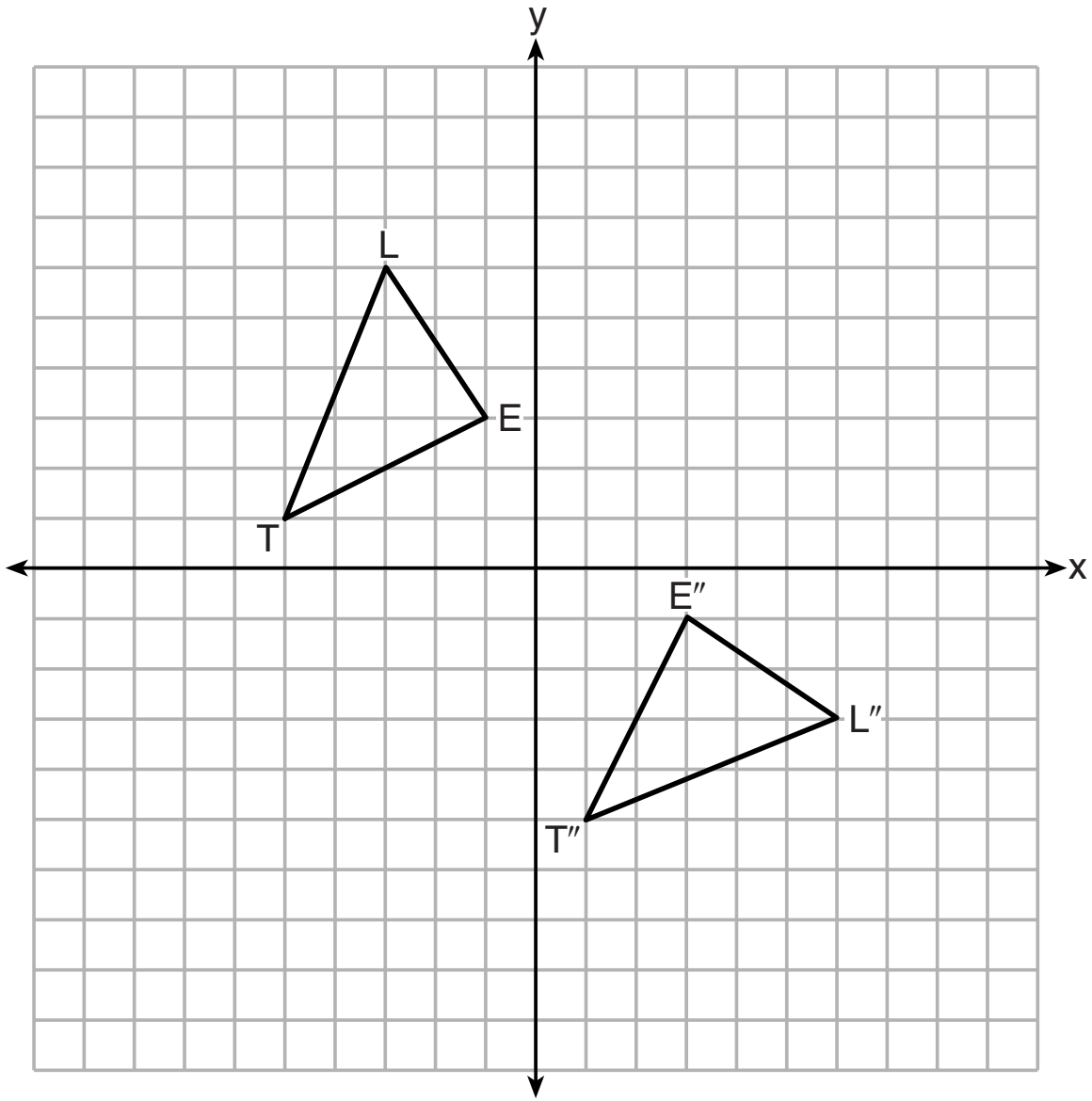
(2) 25

(3) 70

(4) 138

Use this space for computations.

18 On the set of axes below, $\triangle LET$ and $\triangle L''E''T''$ are graphed in the coordinate plane where $\triangle LET \cong \triangle L''E''T''$.



Question 18 is continued on the next page.

Use this space for computations.

Which sequence of rigid motions maps $\triangle LET$ onto $\triangle L''E''T''$?

(1) a reflection over the y-axis followed by a reflection over the x-axis

(2) a rotation of 180° about the origin

(3) a rotation of 90° counterclockwise about the origin followed by a reflection over the y-axis

(4) a reflection over the x-axis followed by a rotation of 90° clockwise about the origin

GO RIGHT ON TO THE NEXT PAGE ➡

Use this space for computations.

19 Diameter $\overline{ROQ}$ of circle O is extended through Q to point P, and tangent $\overline{PA}$ is drawn. If $m\widehat{RA} = 100°$, what is $m\angle P$?

(1) $10°$

(2) $20°$

(3) $40°$

(4) $50°$

20 Segment JM has endpoints $J(-5,1)$ and $M(7,-9)$. An equation of the perpendicular bisector of $\overline{JM}$ is

(1) $y - 4 = \frac{5}{6}(x + 1)$

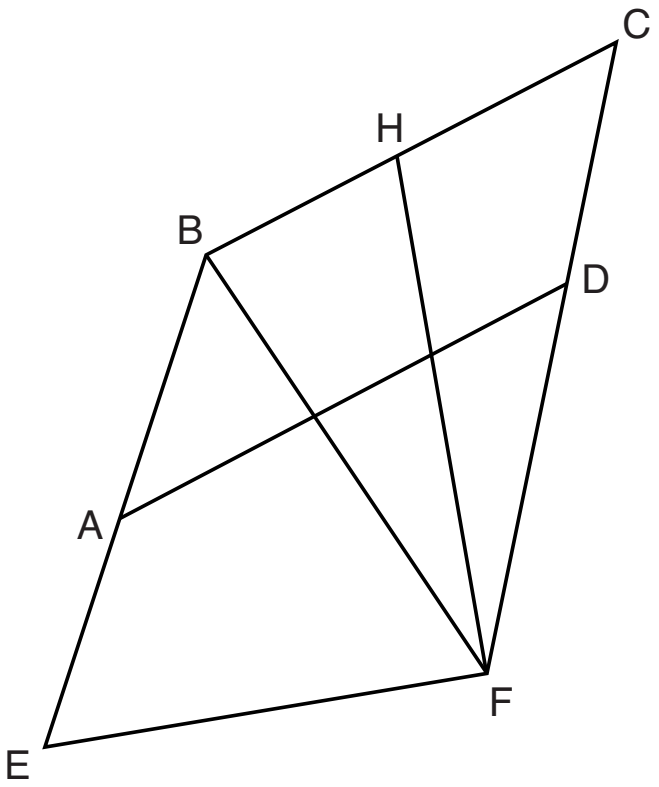
(2) $y + 4 = \frac{5}{6}(x - 1)$

(3) $y - 4 = \frac{6}{5}(x + 1)$

(4) $y + 4 = \frac{6}{5}(x - 1)$

Use this space for computations.

21 Quadrilateral $EBCF$ and $\overline{AD}$ are drawn below, such that $ABCD$ is a parallelogram, $\overline{EB} \cong \overline{FB}$, and $\overline{EF} \perp \overline{FH}$.



If $m\angle E = 62°$ and $m\angle C = 51°$, what is $m\angle FHB$?

(1) 79°

(2) 76°

(3) 73°

(4) 62°

Use this space for computations.

22 Point P divides the directed line segment from point $A(-4,-1)$ to point $B(6,4)$ in the ratio 2:3. The coordinates of point P are

(1) $(-1,1)$

(2) $(0,1)$

(3) $(1,0)$

(4) $(2,2)$

23 A line is dilated by a scale factor of $\frac{1}{3}$ centered at a point on the line. Which statement is correct about the image of the line?

(1) Its slope is changed by a scale factor of $\frac{1}{3}$.

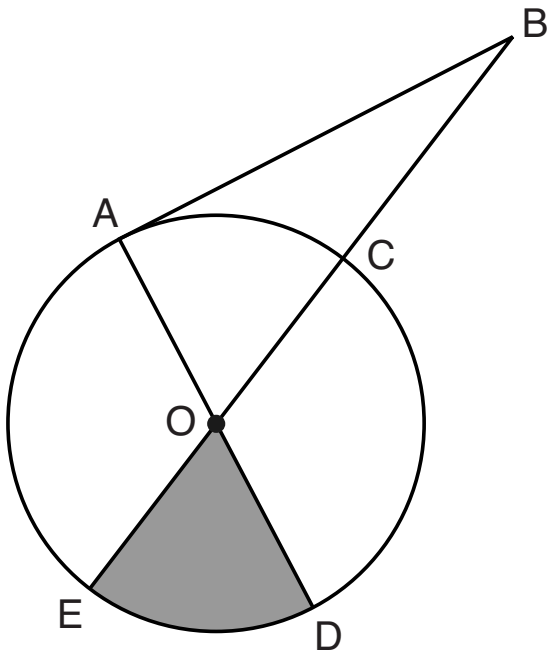
(2) Its y-intercept is changed by a scale factor of $\frac{1}{3}$.

(3) Its slope and y-intercept are changed by a scale factor of $\frac{1}{3}$.

(4) The image of the line and the pre-image are the same line.

Use this space for computations.

24 In the diagram below of circle O, tangent $\overline{AB}$ is drawn from external point B, and secant $\overline{BCOE}$ and diameter $\overline{AOD}$ are drawn.



If $m\angle OBA = 36°$ and $OC = 10$, what is the area of shaded sector DOE?

(1) $\frac{3\pi}{10}$

(2) 3π

(3) 10π

(4) 15π

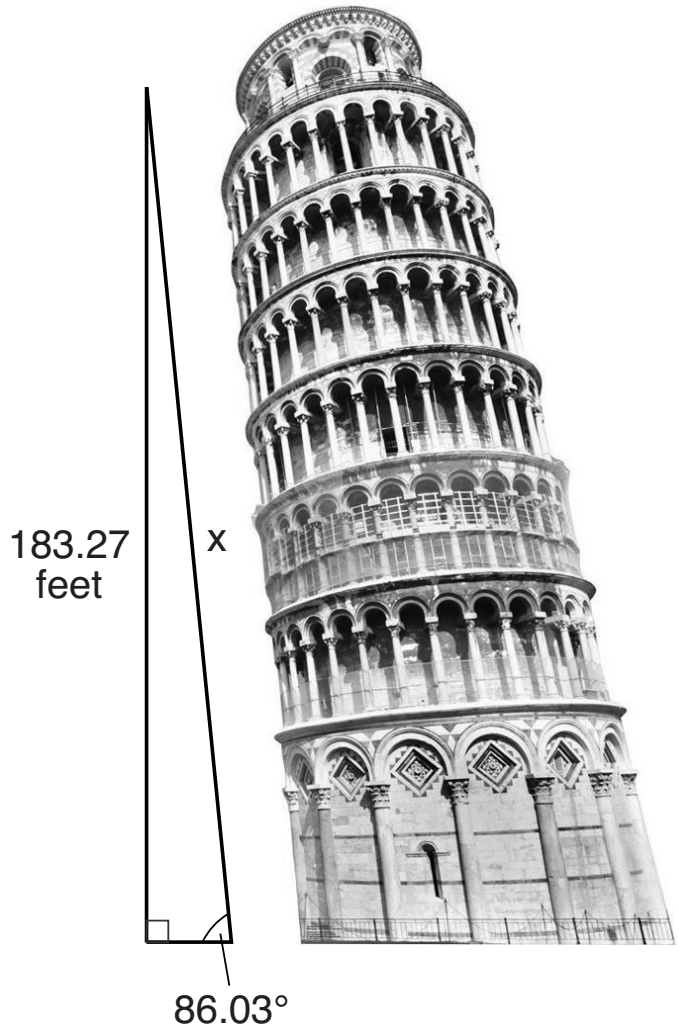
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 The Leaning Tower of Pisa in Italy is known for its slant, which occurred after its construction began. The angle of the slant is 86.03° from the ground. The low side of the tower reaches a height of 183.27 feet from the ground.

Question 25 is continued on the next page.

Question 25 continued



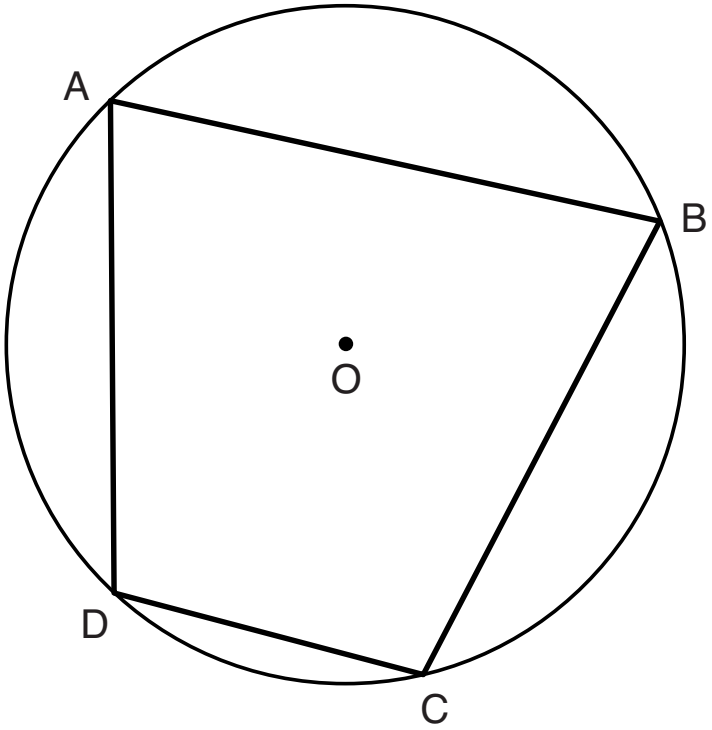
Determine and state the slant height, x, of the low side of the tower, to the *nearest hundredth of a foot*.

Work space for question 25 is continued on the next page.

Question 25 continued

GO RIGHT ON TO THE NEXT PAGE ➡

26 In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $\text{m}\overset{\frown}{CD} : \text{m}\overset{\frown}{DA} : \text{m}\overset{\frown}{AB} : \text{m}\overset{\frown}{BC} = 2:3:5:5$.

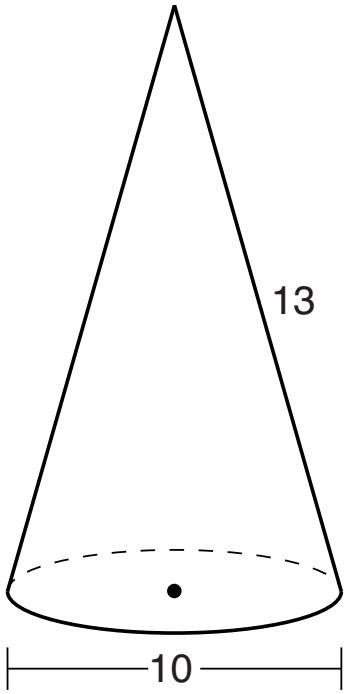


Determine and state $\text{m}\angle B$.

Work space for question 26 is continued on the next page.

Question 26 continued

27 In the diagram below, a right circular cone has a diameter of 10 and a slant height of 13.

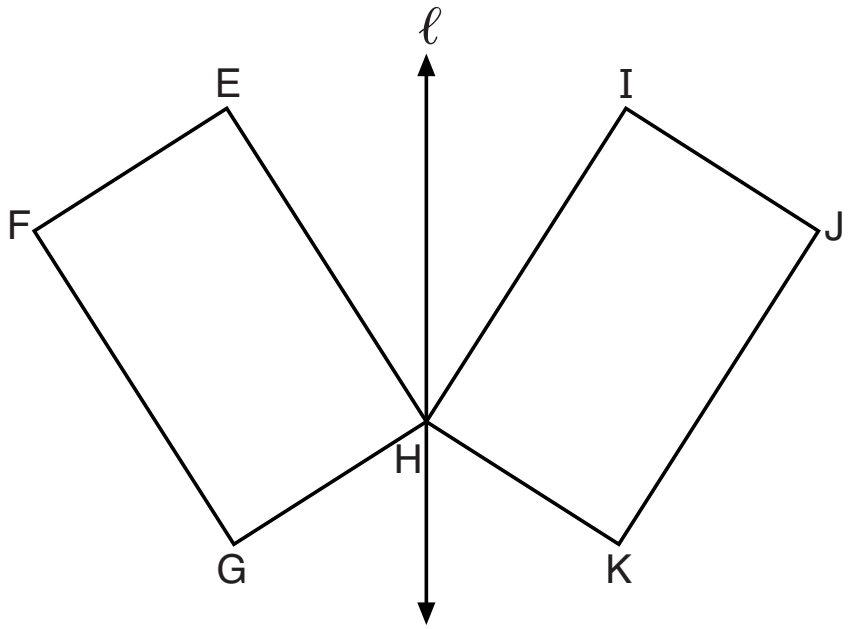


Determine and state the volume of the cone, in terms of π.

Work space for question 27 is continued on the next page.

Question 27 continued

28 In the diagram below, parallelogram *EFGH* is mapped onto parallelogram *IJKH* after a reflection over line ℓ.

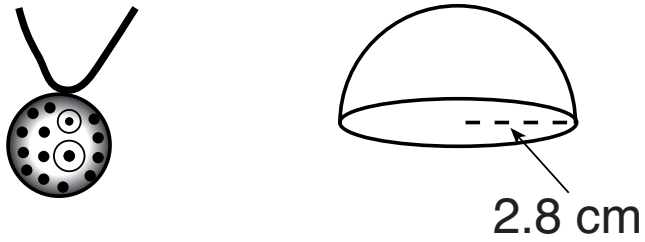


Use the properties of rigid motions to explain why parallelogram *EFGH* is congruent to parallelogram *IJKH*.

Work space for question 28 is continued on the next page.

Question 28 continued

29 Izzy is making homemade clay pendants in the shape of a solid hemisphere, as modeled below. Each pendant has a radius of 2.8 cm.



How much clay, to the *nearest cubic centimeter*, does Izzy need to make 100 pendants?

Work space for question 29 is continued on the next page.

Question 29 continued

30 Determine and state the coordinates of the center and the length of the radius of the circle whose equation is $x^2 + y^2 + 6x = 6y + 63$.

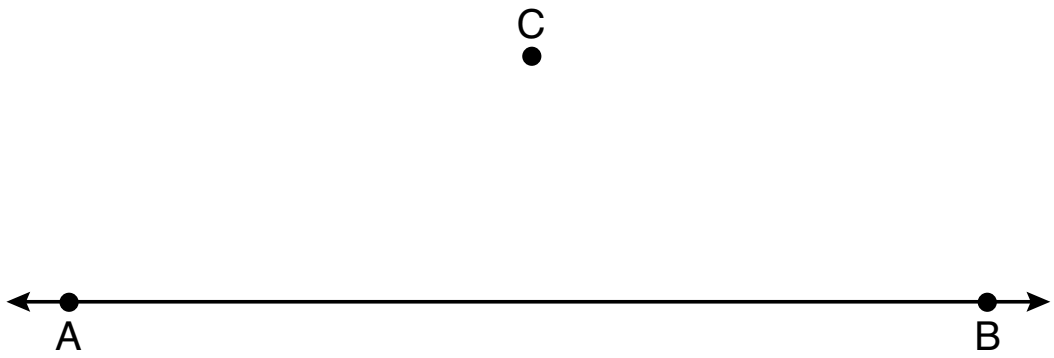
Work space for question 30 is continued on the next page.

Question 30 continued

31 Use a compass and straightedge to construct a line parallel to $\overleftrightarrow{AB}$ through point C, shown on the next page.
[Leave all construction marks.]

The diagram for question 31 is on the next page.

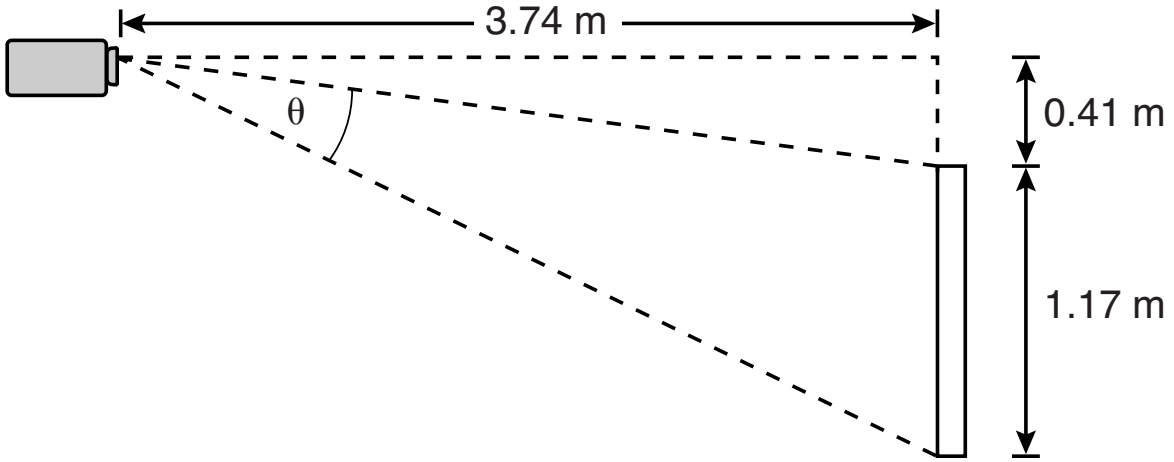
Question 31 continued



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 As modeled below, a projector mounted on a ceiling is 3.74 m from a wall, where a whiteboard is displayed. The vertical distance from the ceiling to the top of the whiteboard is 0.41 m, and the height of the whiteboard is 1.17 m.

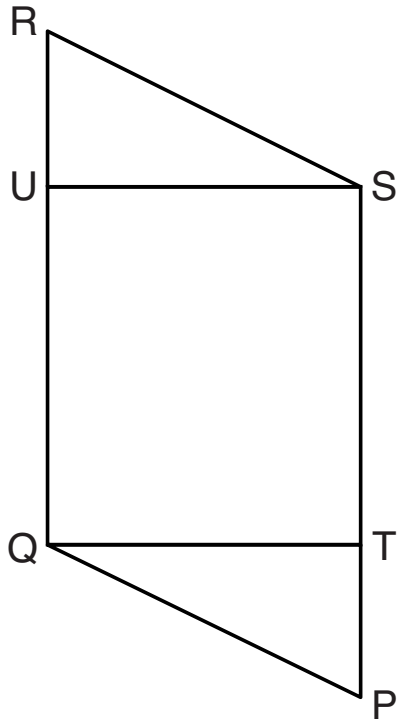


Determine and state the projection angle, θ, to the *nearest tenth of a degree*.

Work space for question 32 is continued on the next page.

Question 32 continued

33 Given: Parallelogram $PQRS$, $\overline{QT} \perp \overline{PS}$, $\overline{SU} \perp \overline{QR}$

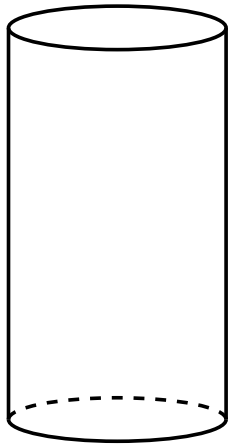


Prove: $\overline{PT} \cong \overline{RU}$

Work space for question 33 is continued on the next page.

Question 33 continued

34 A concrete footing is a cylinder that is placed in the ground to support a building structure. The cylinder is 4 feet tall and 12 inches in diameter. A contractor is installing 10 footings.



If a bag of concrete mix makes $\frac{2}{3}$ of a cubic foot of concrete, determine and state the minimum number of bags of concrete mix needed to make all 10 footings.

Work space for question 34 is continued on the next page.

Question 34 continued

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 The coordinates of the vertices of $\triangle ABC$ are $A(-2,4)$, $B(-7,-1)$, and $C(-3,-3)$.

Prove that $\triangle ABC$ is isosceles.

[The use of the set of axes on page 49 is optional.]

Question 35 is continued on the next page.

Question 35 continued

State the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$, after a translation 5 units to the right and 5 units down.

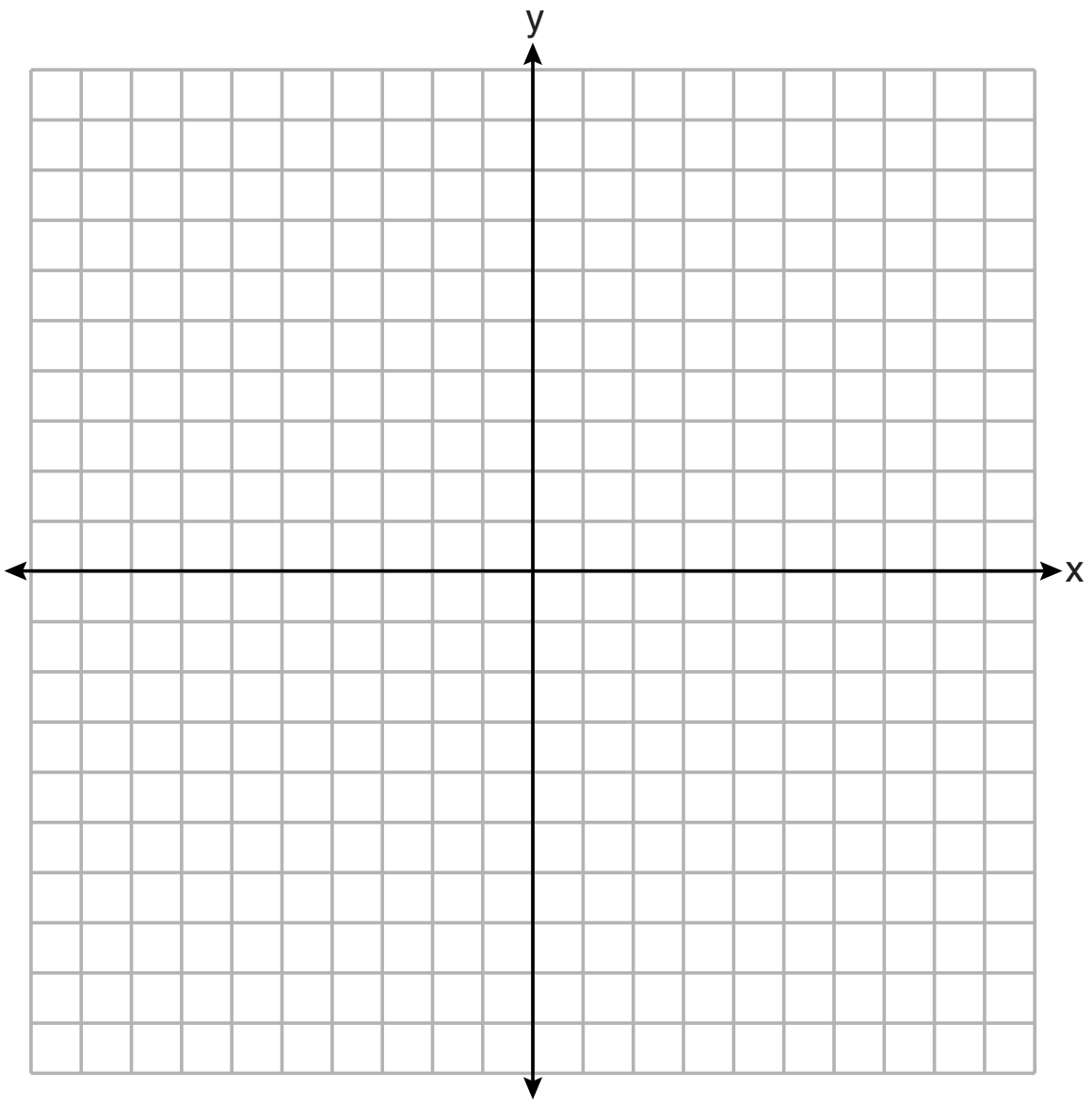
Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral $AA'C'C$ is a rhombus.
[The use of the set of axes on page 49 is optional.]

The set of axes for question 35 is on the next page.

Question 35 continued



Scrap Graph Paper — This sheet will *not* be scored.

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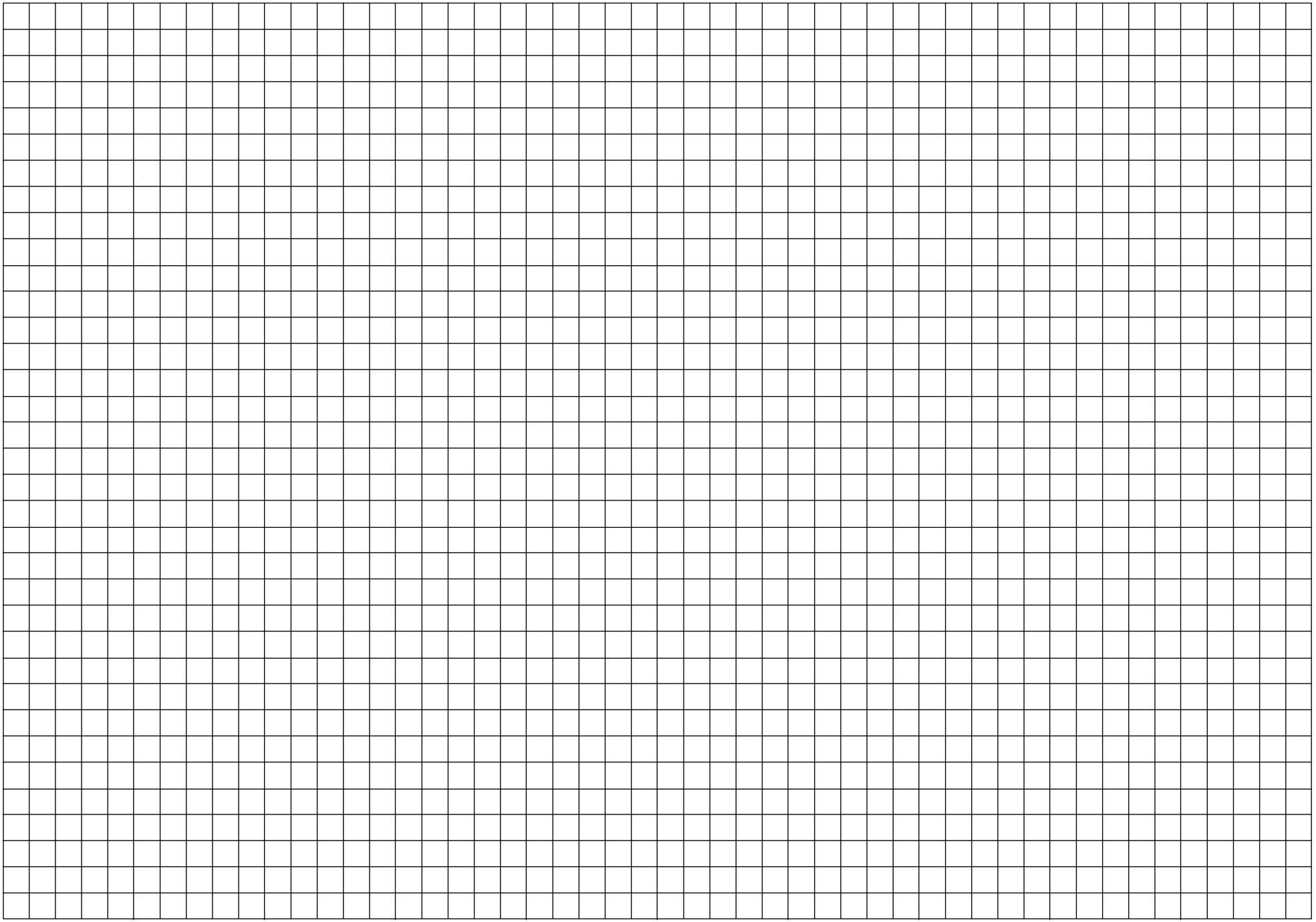

High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$