

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 17, 2017 — 12:30 to 3:30 p.m.

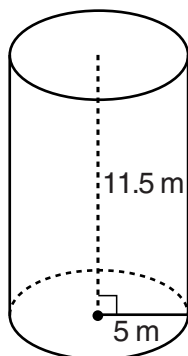
MODEL RESPONSE SET

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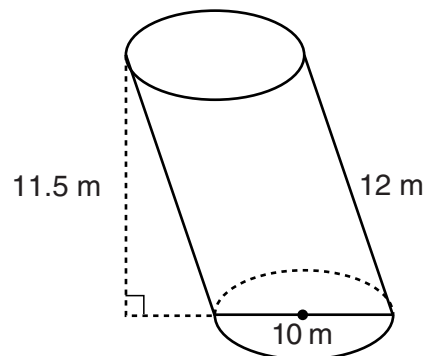
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Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



$$r=5, h=11.5$$



$$r=5, h=11.5$$

Is Sue correct? Explain why.

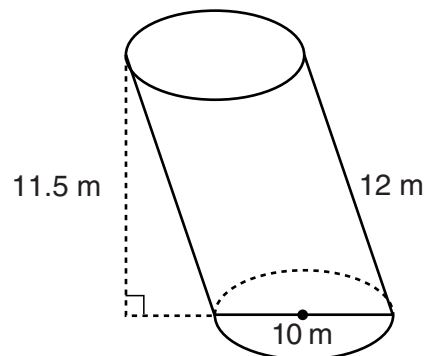
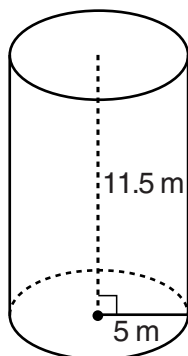
Yes, Sue is correct because when two cylinders have the same base areas and the same height, the two cylinders must have the same volume.

Score 2: The student gave a complete and correct response.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.

$$\begin{aligned} V &= \pi r^2 \cdot h \\ &= \pi 5^2 \cdot 11.5 \\ &= 287.5 \pi \text{ m}^3 \end{aligned}$$



$$\begin{aligned} V &= \pi r^2 \cdot h \\ &= \pi 5^2 \cdot 11.5 \\ &= 287.5 \pi \text{ m}^3 \end{aligned}$$

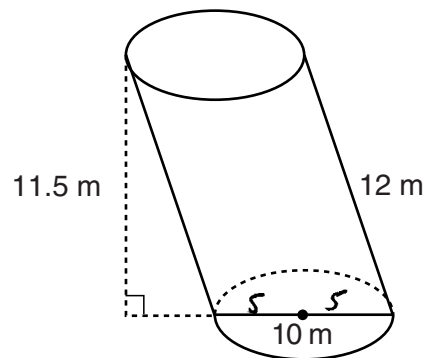
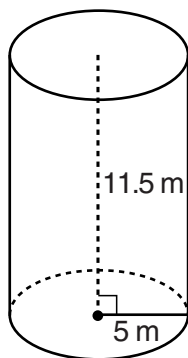
Is Sue correct? Explain why.

Sue is correct, Both cylinders' radii and heights are equal causing their volumes to be the same

Score 2: The student gave a complete and correct response.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



Is Sue correct? Explain why.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi 5^2 \times 11.5 \\ &= 287.5\pi \end{aligned}$$

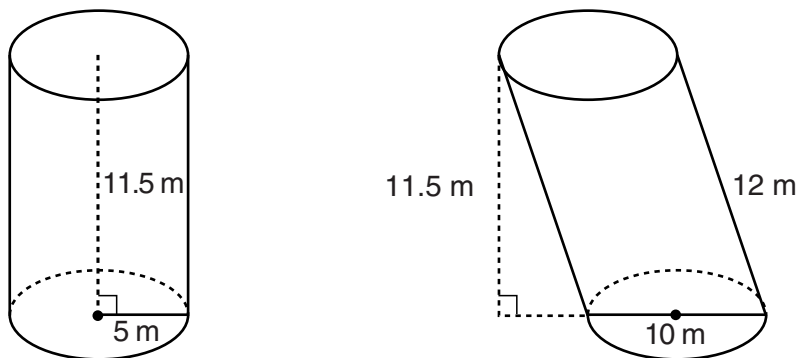
$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi 5^2 \times 11.5 \\ &= 287.5\pi \end{aligned}$$

Yes Sue is correct - the 2 cylinders have the same volume

Score 1: The student found the volumes of both cylinders, but did not write an explanation for why the volumes are the same.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



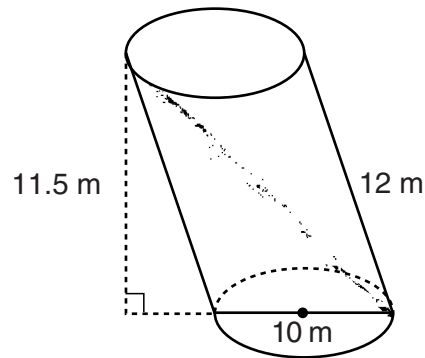
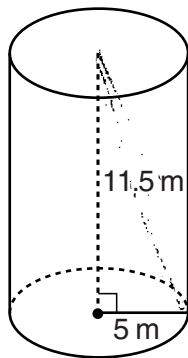
Is Sue correct? Explain why.

Yes because the cylinders
have the same
measurements one
is just tilted

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



Is Sue correct? Explain why.

$$V = \frac{1}{3} 10 \cdot 11.5$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + b^2 &= 11.5^2 \\ 25 + b^2 &= 132.25 \end{aligned}$$

$$V = \frac{1}{3} B \cdot h$$

$$V = \frac{1}{3} 15 \cdot 11.5$$

$$V = 19.16$$

$$V = \frac{1}{3} 10 \cdot 11.5$$

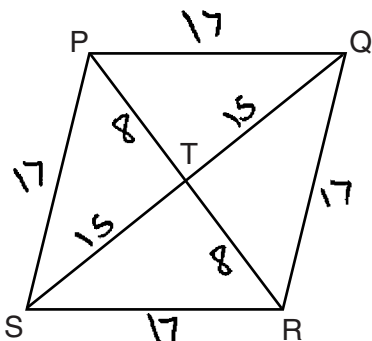
$$V = 40$$

Sue is incorrect, their volumes are not equal.

Score 0: The student gave a completely incorrect response.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PT = 8$, and $QT = 15$. Determine and state the perimeter of $PQRS$.



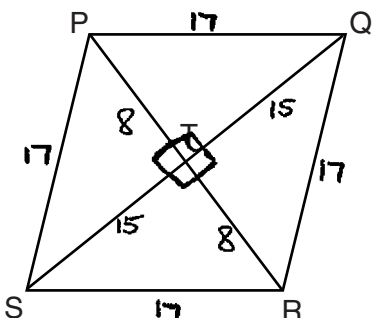
$$\begin{aligned} a^2 + b^2 &= c^2 \\ \downarrow \quad \downarrow & \\ 8^2 + 15^2 &= c^2 \\ \downarrow \quad \downarrow & \\ 64 + 225 &= c^2 \\ \sqrt{289} &= \sqrt{c^2} \\ \downarrow & \\ 17 &= c \end{aligned}$$

perimeter = 68

Score 2: The student gave a complete and correct response.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.

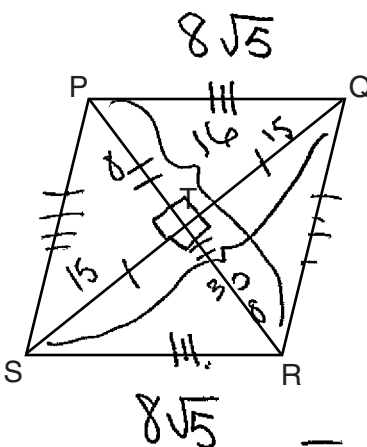


$$P = 68$$

Score 2: The student gave a complete and correct response.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PT = 8$, and $ST = 15$. Determine and state the perimeter of $PQRS$.



Diagonals bisect each other and are \perp .

Opposite sides are \cong

$\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 16^2 &= PQ^2 \\ 64 + 256 &= PQ^2 \\ \sqrt{320} &= \sqrt{PQ^2} \\ \sqrt{320} &= PQ \\ \sqrt{64} \sqrt{5} &= PQ \\ 8\sqrt{5} &= PQ \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 15^2 &= SP^2 \\ 64 + 225 &= SP^2 \\ \sqrt{289} &= \sqrt{SP^2} \\ 17 &= SP \end{aligned}$$

$$\begin{aligned} SP &= PQ \\ SP &= 8\sqrt{5} \end{aligned}$$

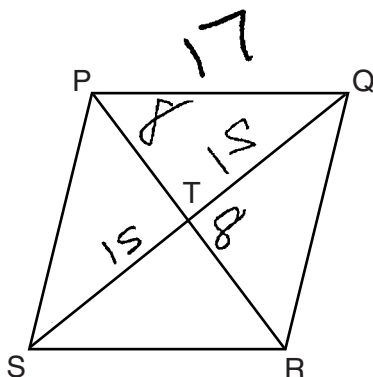
$$\begin{aligned} QR &= SP \\ QR &= 17 \end{aligned}$$

$$\begin{aligned} P &= PQ + QR + RS + SP \\ &= 8\sqrt{5} + 17 + 8\sqrt{5} + 17 \\ P &= 16\sqrt{5} + 34 \end{aligned}$$

Score 1: The student made an error in finding the lengths of sides \overline{PQ} and \overline{RS} .

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.

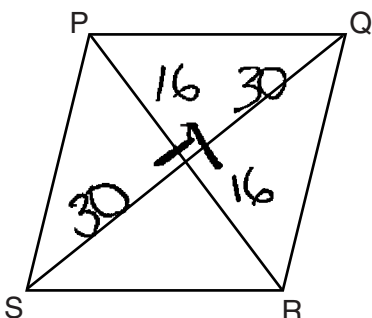


$$8^2 + 15^2 = C^2$$
$$64 + 225 = C^2$$
$$\sqrt{289} = C$$
$$C = 17$$

Score 1: The student found the length of the side of the rhombus, but did not find the perimeter of the rhombus.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals \overline{PR} and \overline{QS} intersect at point T , $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



$16 + 16$
 $30 + 30$
 32
 60
 $A = d^1 + d^2 \times$

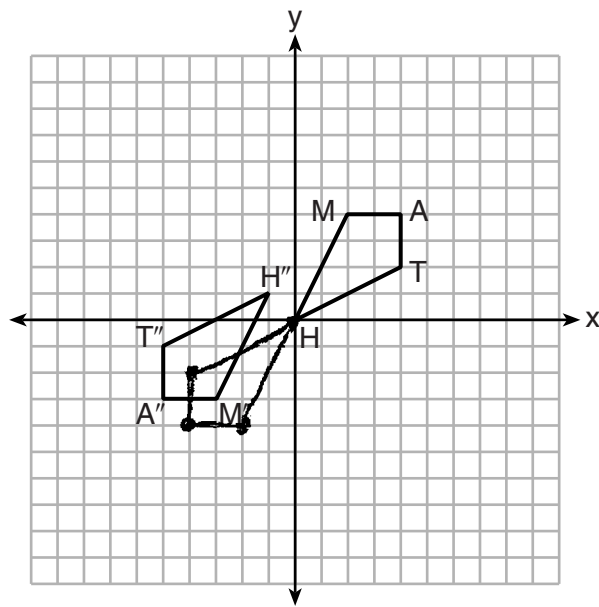
$P = 92$

$$\begin{array}{r}
 32 \\
 \times 60 \\
 \hline
 1,920
 \end{array}$$

Score 0: The student gave a completely incorrect response.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



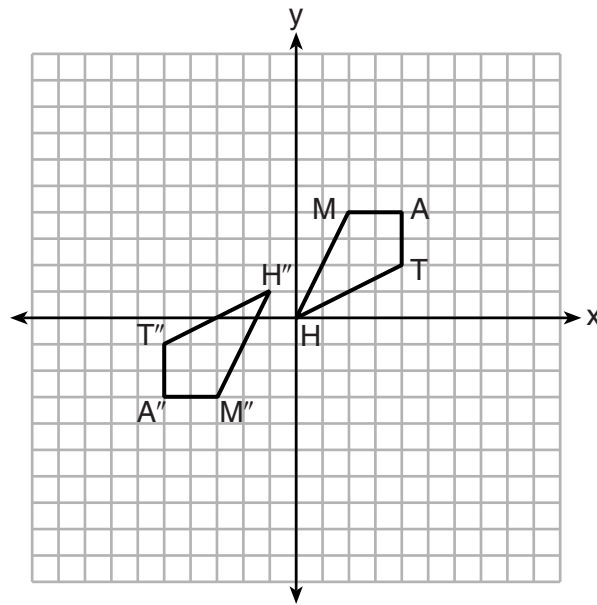
Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

a reflection over the origin \rightarrow a translation of $(x-1, y+1)$

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

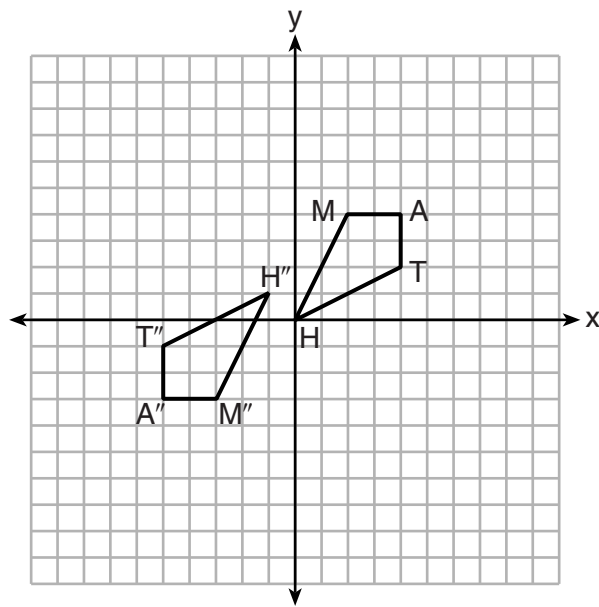
$$T_{-1,1} \circ D_{H,-1}$$

$M(2,4)$	$M'(-2,-4)$	$M''(-3,-3)$
$A(4,4)$	$A'(-4,-4)$	$A''(-5,-3)$
$T(4,2)$	$T'(-4,-2)$	$T''(-5,-1)$
$H(0,0)$	$H'(0,0)$	$H''(-1,1)$

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



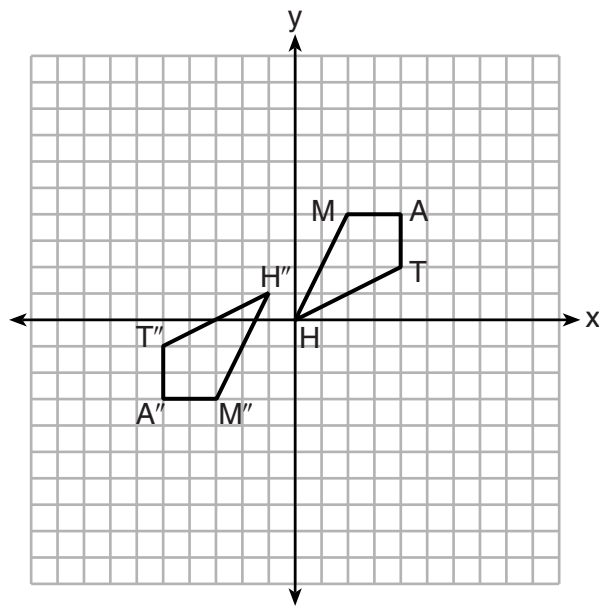
Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

Rotate quadrilateral $MATH$ 180° about point $(-\frac{1}{2}, \frac{1}{2})$.

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



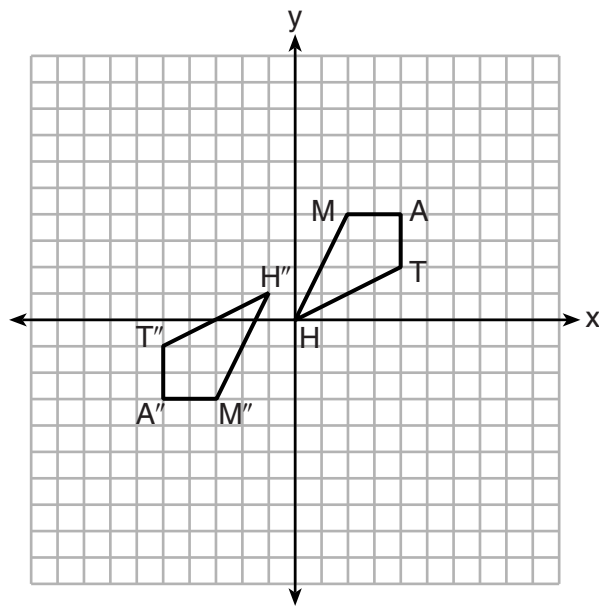
Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

- $MATH$ was translated up one unit.
- $MATH$ was translated to the left one unit.
- $MATH$ was rotated 180° clockwise.

Score 1: The student wrote an incomplete transformation by not stating the center of rotation.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



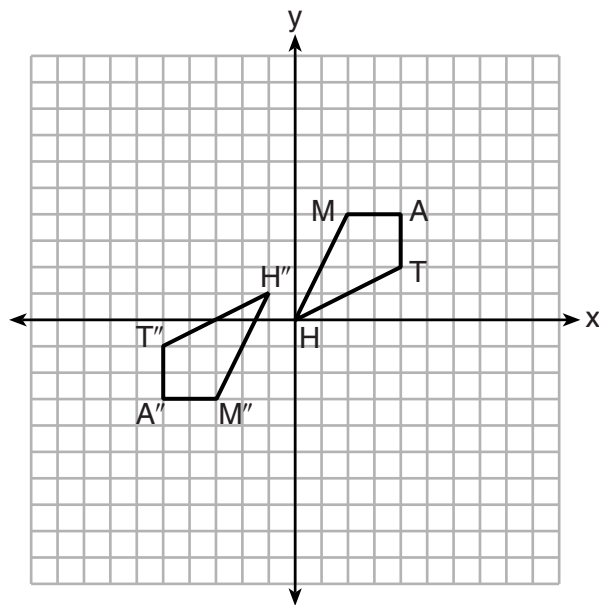
Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

Rotate 180° from point H. Then translate down 1 and right 1

Score 1: The student had a partially correct sequence of transformations.

Question 27

27 Quadrilateral $MATH$ and its image $M''A''T''H''$ are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral $MATH$ onto quadrilateral $M''A''T''H''$.

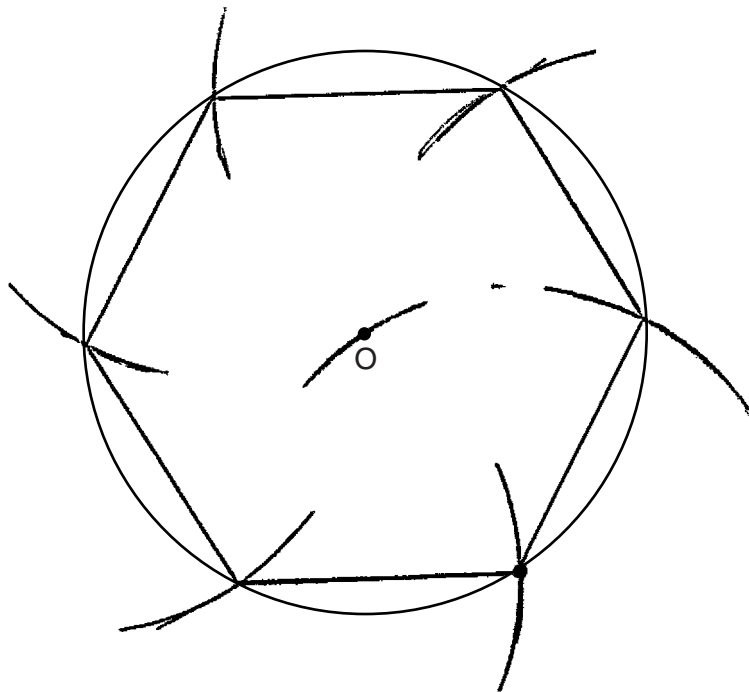
Spin it and move it over

Score 0: The student gave an incomplete description of the rotation (spin) and described the translation (move) incorrectly.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .

[Leave all construction marks.]

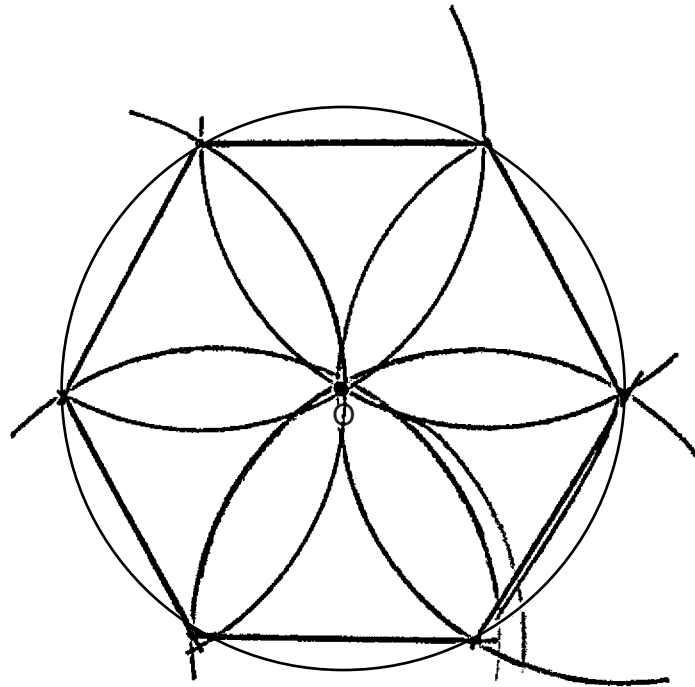


Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .

[Leave all construction marks.]

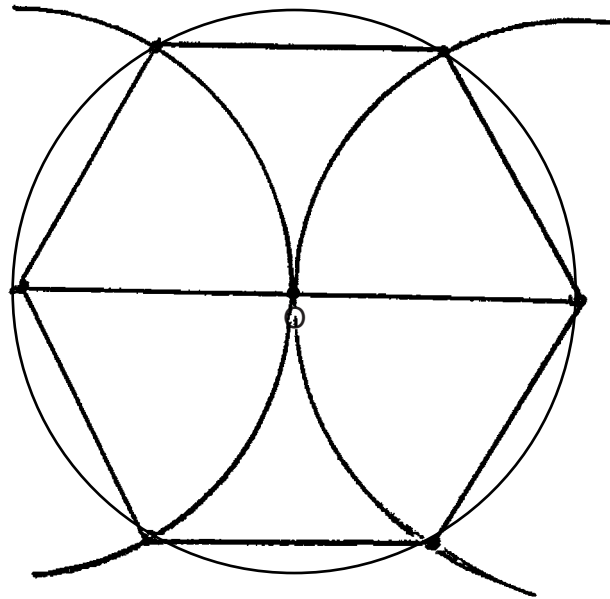


Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .

[Leave all construction marks.]

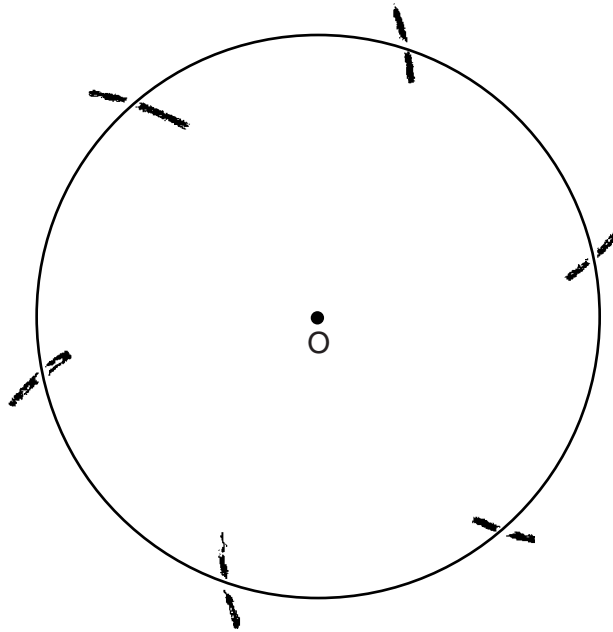


Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .

[Leave all construction marks.]

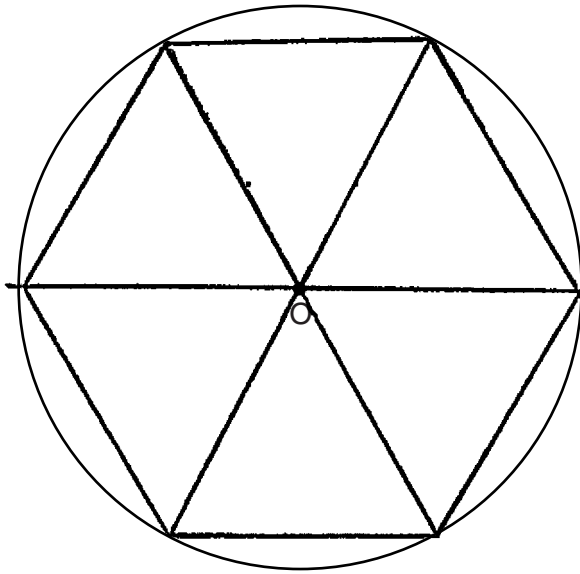


Score 1: The student drew an appropriate construction, but did not draw the hexagon.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O .

[Leave all construction marks.]



Score 0: The student had a drawing that is not a construction.

Question 29

29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

$$A' = (1, 1.5)$$
$$B' = (2.5, -1.5)$$

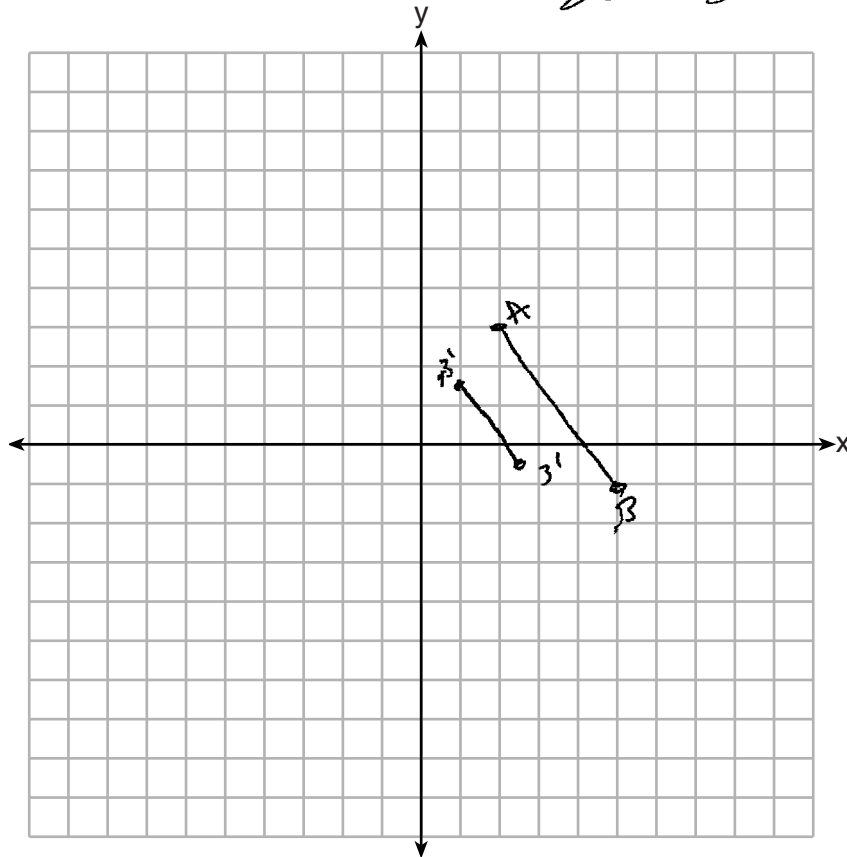
$$d = \sqrt{(2.5-0)^2 + (-1.5-1.5)^2}$$

$$d = \sqrt{(1.5)^2 + (-2)^2}$$

$$d = \sqrt{2.25 + 4}$$

~~$$d = 5.625$$~~

$$d = 2.5$$



Score 2: The student gave a complete and correct response.

Question 29

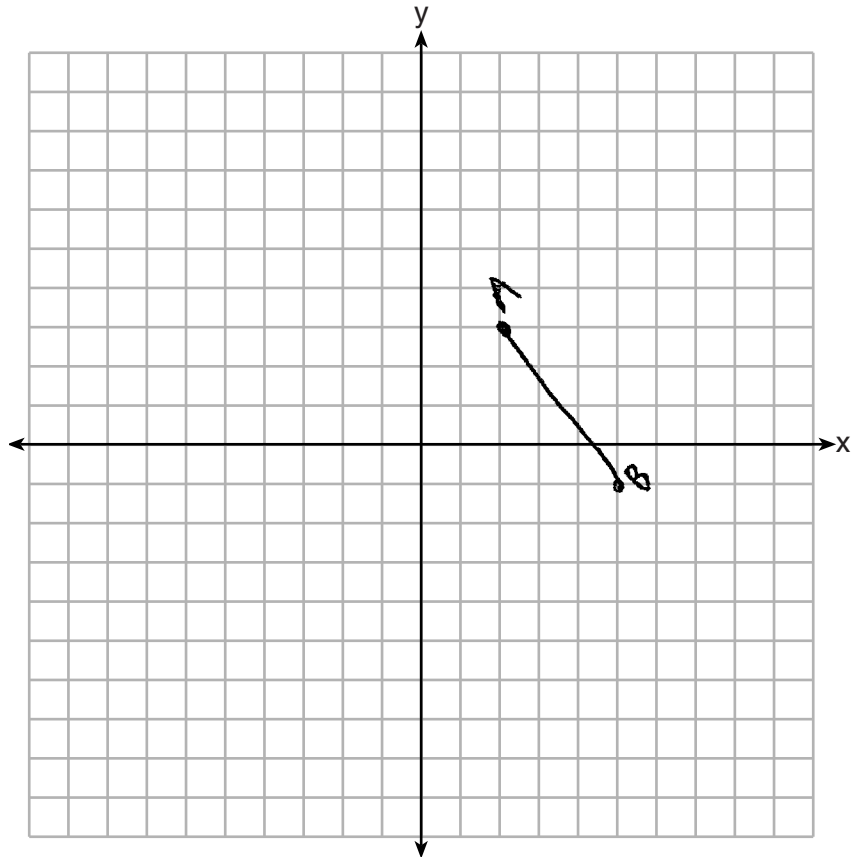
29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

$$\begin{aligned}d &= \sqrt{(x-x)^2 + (y-y)^2} \\ &= \sqrt{(5-2)^2 + (3-(-1))^2} \\ &= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}\end{aligned}$$

$$= 5 \cdot \frac{1}{2} = 2.5$$

Length of $\overline{A'B'}$ = 2.5 units



Score 2: The student gave a complete and correct response.

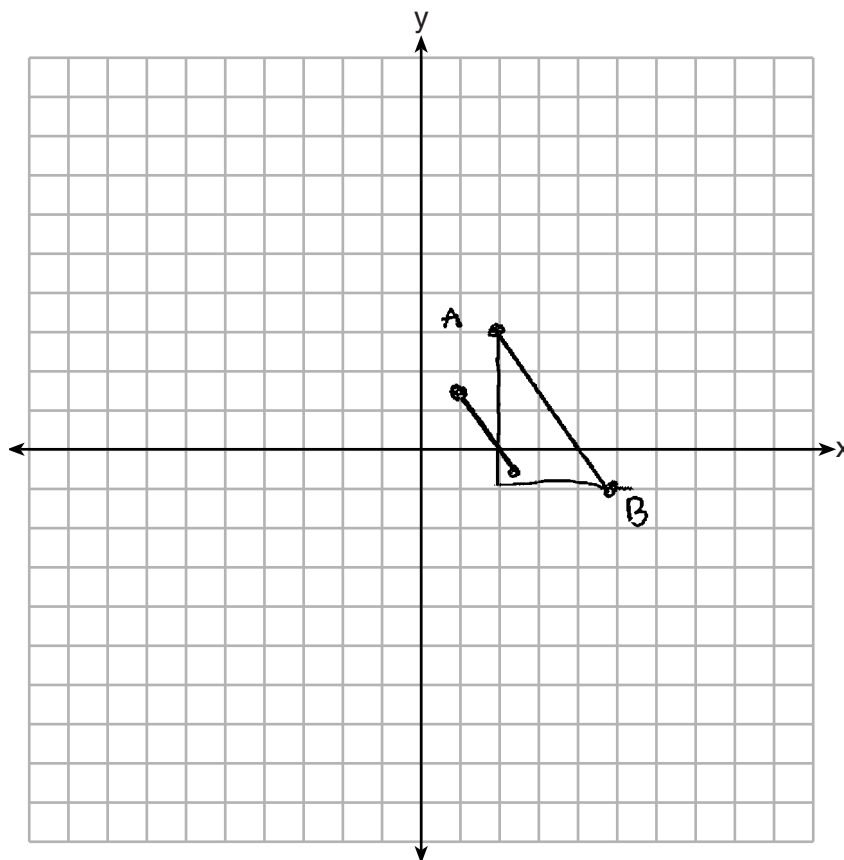
Question 29

29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

$$\overline{A'B'} = 2.5$$

$$\begin{aligned}4^2 + 3^2 &= c^2 \\16 + 9 &= c^2 \\ \sqrt{25} &= \sqrt{c^2} \\ c &= 5\end{aligned}$$



Score 2: The student gave a complete and correct response.

Question 29

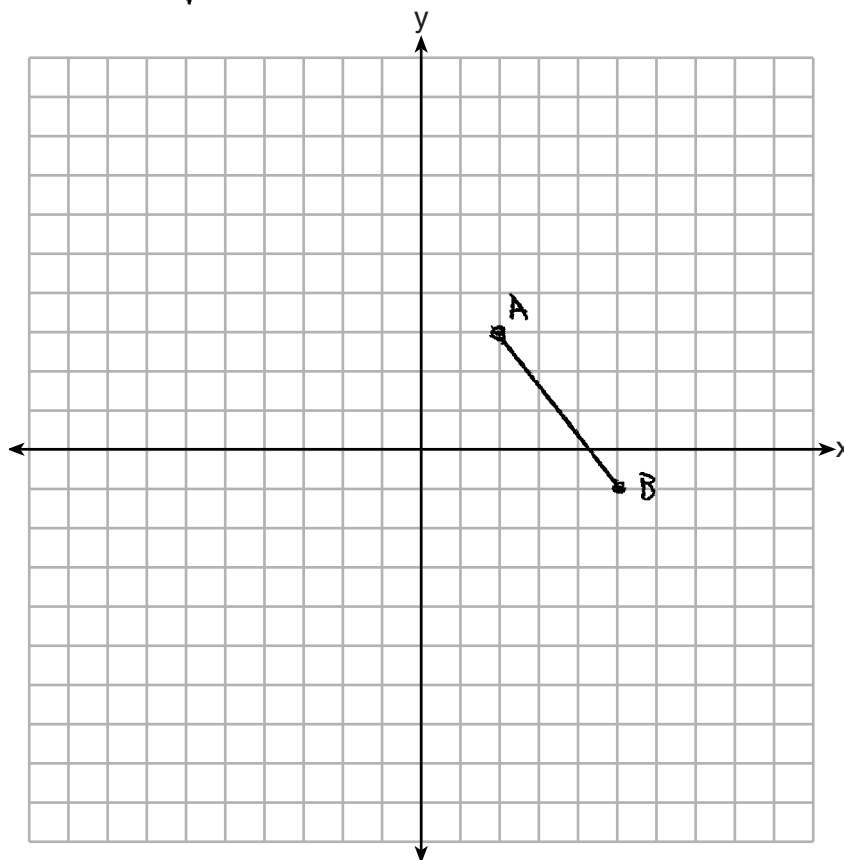
29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

$$d = \sqrt{(2+5)^2 + (3+1)^2}$$
$$\sqrt{(3)^2 + (4)^2}$$
$$\sqrt{9 + 16}$$

$$\sqrt{25}$$

$$d = 5$$



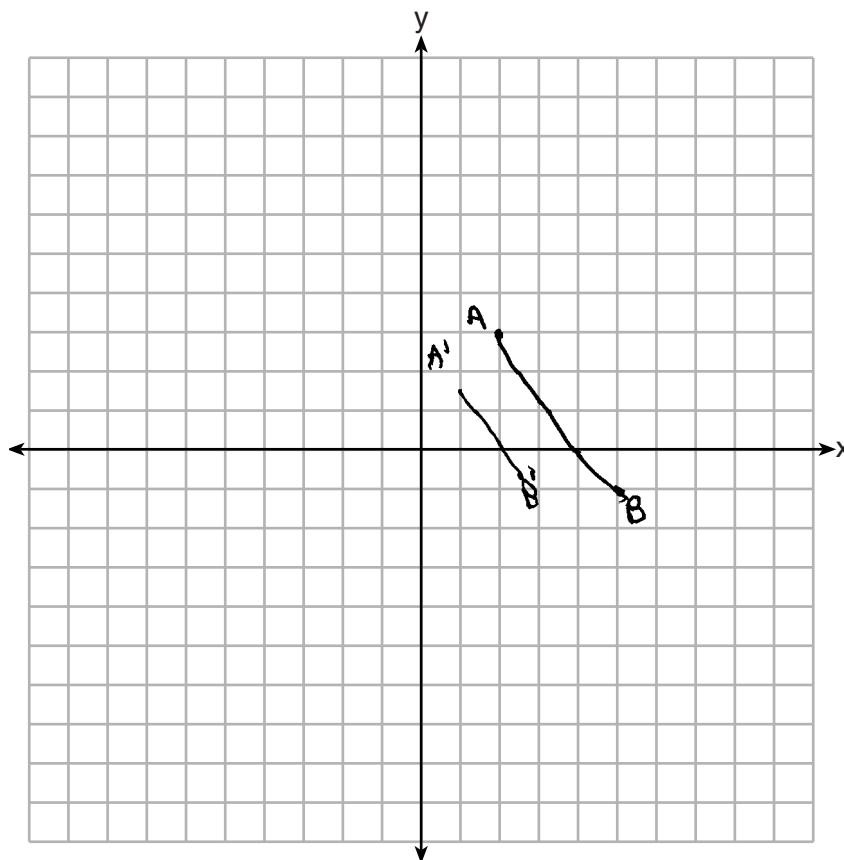
Score 1: The student found the length of \overline{AB} , but no further correct work is shown.

Question 29

29 The coordinates of the endpoints of \overline{AB} are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

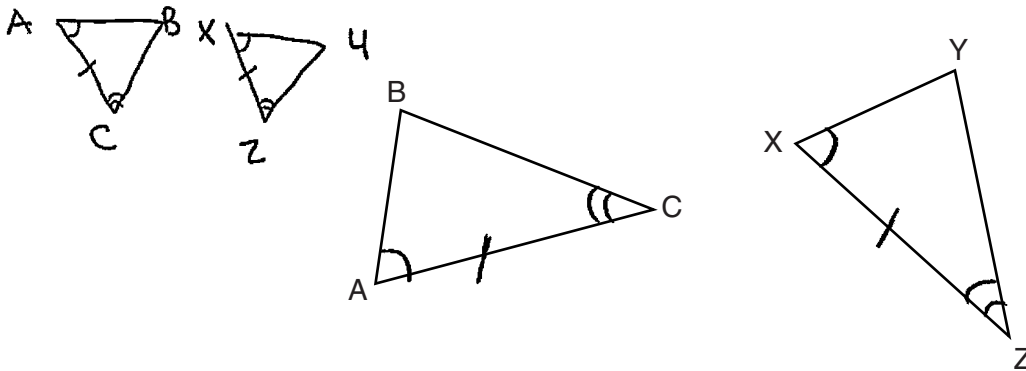
$\overline{A'B'}$ is shorter than \overline{AB}



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



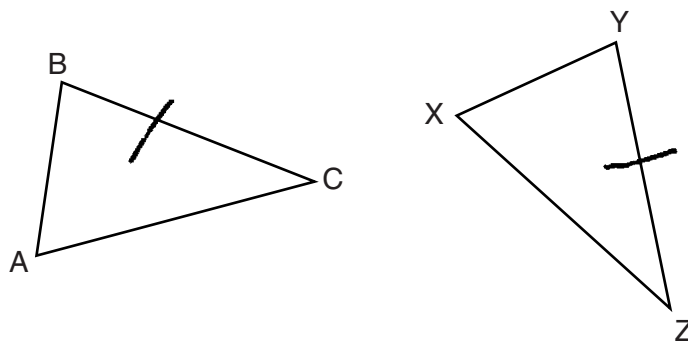
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ since the \triangle s are \cong by ASA since $\angle A \cong \angle X$, $\overline{AC} \cong \overline{XZ}$ and $\angle C \cong \angle Z$ because they can be mapped on to each other in a series of rigid motions which preserve side length + angle measure. So since $\triangle ABC \cong \triangle XYZ$, $\overline{BC} \cong \overline{YZ}$ because corresponding sides of \cong \triangle s are \cong .

Score 2: The student gave a complete and correct response.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



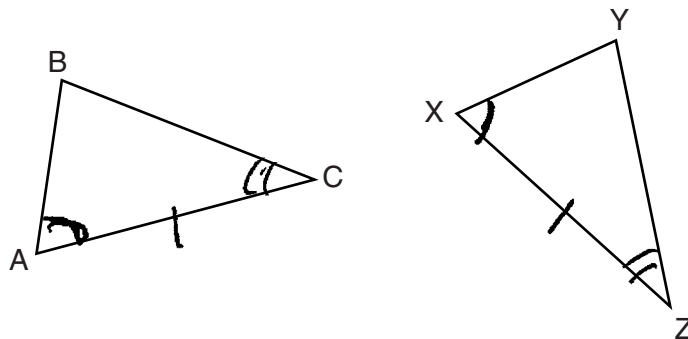
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

yes because basic rigid motions preserve segment length and angle measurement.

Score 1: The student gave an incomplete explanation by not stating the triangle congruency and not stating corresponding congruent sides.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



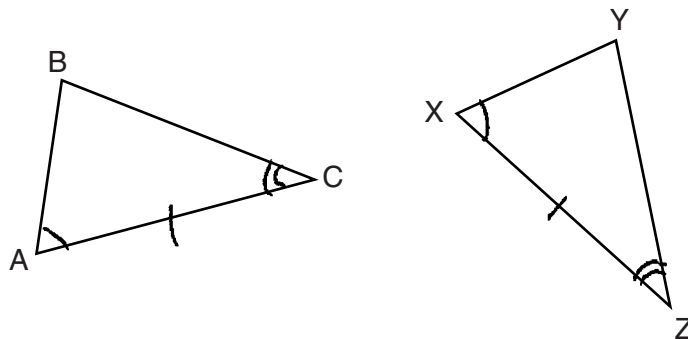
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ because corresponding parts of congruent triangles are congruent

Score 1: The student gave an incomplete explanation.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



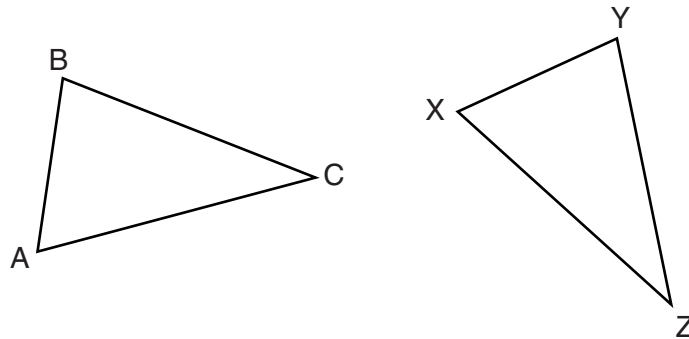
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

yes $\overline{BC} \cong \overline{YZ}$ because
of $ASA \cong ASA$.

Score 1: The student gave an incomplete explanation.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ because the triangles look the same.

Score 0: The student wrote an incorrect explanation.

Question 31

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$$\frac{6}{2} = 3^2 = 9$$

$$x^2 - 6x + y^2 + 8y = 56$$

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16$$

$$(x-3)^2 + (y+4)^2 = 81 \quad \sqrt{81} = 9$$

radius = 9
center = (3, -4)

Score 2: The student gave a complete and correct response.

Question 31

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

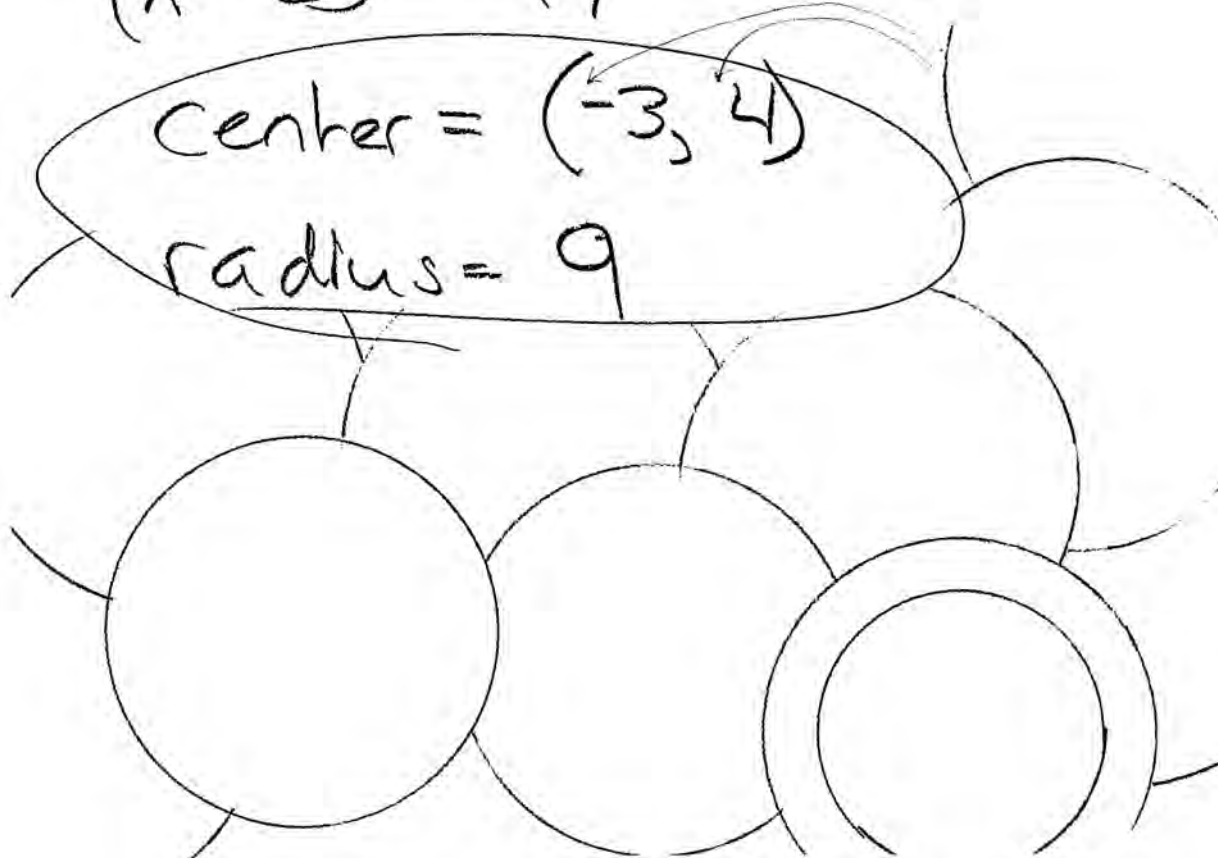
$$(x^2 - 6x) + (y^2 + 8y) = 56$$

$$(x^2 - 6x + 9) + (y^2 + 8y + 16) = 81$$

$$(x - 3)^2 + (y + 4)^2 = 81$$

Center = $(-3, 4)$

radius = 9



Score 1: The student had incorrect signs on the coordinates for the center of the circle.

Question 31

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$$\begin{array}{r} x^2 + y^2 - 6x = 56 - 8y \\ + 8y \\ \hline x^2 - 6x + y^2 + 8y = 56 \\ x^2 - 6x + 9 + y^2 + 8y + 16 = 56 \\ (x-3)(x-3) + (y+4)(y+4) = 56 \\ (x-3)^2 + (y+4)^2 = 56 \\ x = -3 \quad y = +4 \end{array}$$

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

[The use of the set of axes below is optional.]

$$\begin{aligned} \text{mid of } \overline{PQ} &= \left(\frac{-3+(-1)}{2}, \frac{-1+7}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{6}{2} \right) \end{aligned}$$

$$A = (-2, 3)$$

$$\begin{aligned} \text{slope } \overline{AB} &= \frac{5-3}{1-(-2)} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} B &= \left(\frac{3+(-1)}{2}, \frac{7+3}{2} \right) \\ &= \left(\frac{2}{2}, \frac{10}{2} \right) \end{aligned}$$

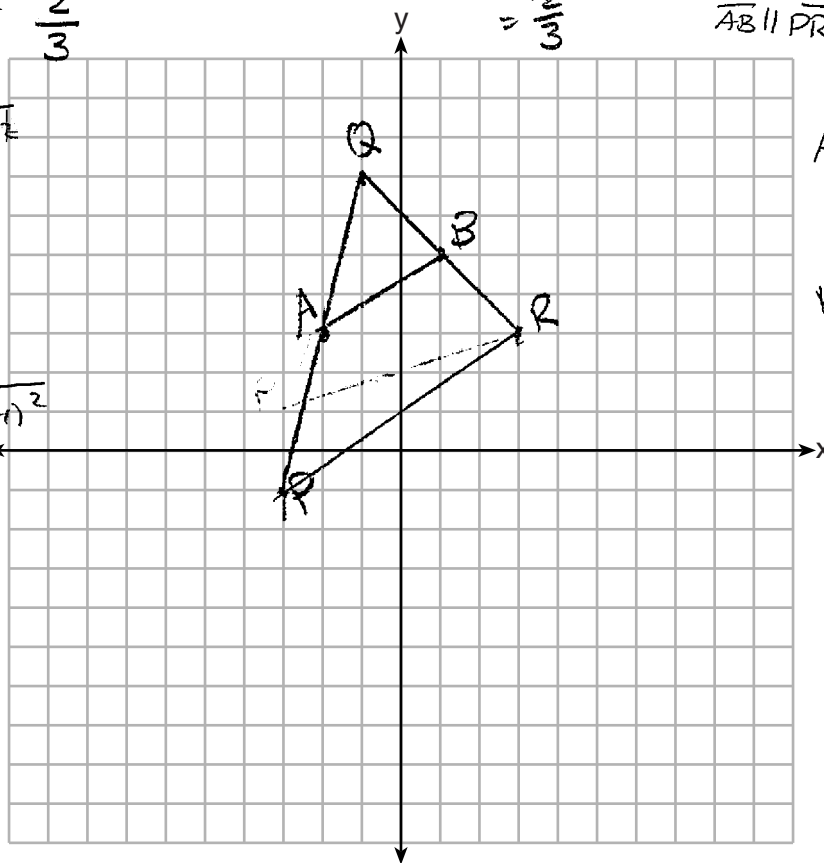
$$B = (1, 5)$$

$$\begin{aligned} \text{slope } \overline{PR} &= \frac{3-(-1)}{3-(-3)} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

Since the slopes of \overline{AB} and \overline{PR} are the same, $\overline{AB} \parallel \overline{PR}$

$$\begin{aligned} AB &= \sqrt{(1-(-2))^2 + (5-3)^2} \\ &= \sqrt{(3)^2 + (2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(3-(-3))^2 + (3-(-1))^2} \\ &= \sqrt{(6)^2 + (4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \end{aligned}$$



AB is half the length of PR because $\sqrt{13}$ is half of $\sqrt{52}$.

Score 4: The student gave a complete and correct response.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

[The use of the set of axes below is optional.]

length

$$A(-1, 3)$$

$$B(0, 1)$$

slope

$$AB : -\frac{1}{1} = -1$$

$$QR : -\frac{1}{1} = -1$$

$$|AB| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

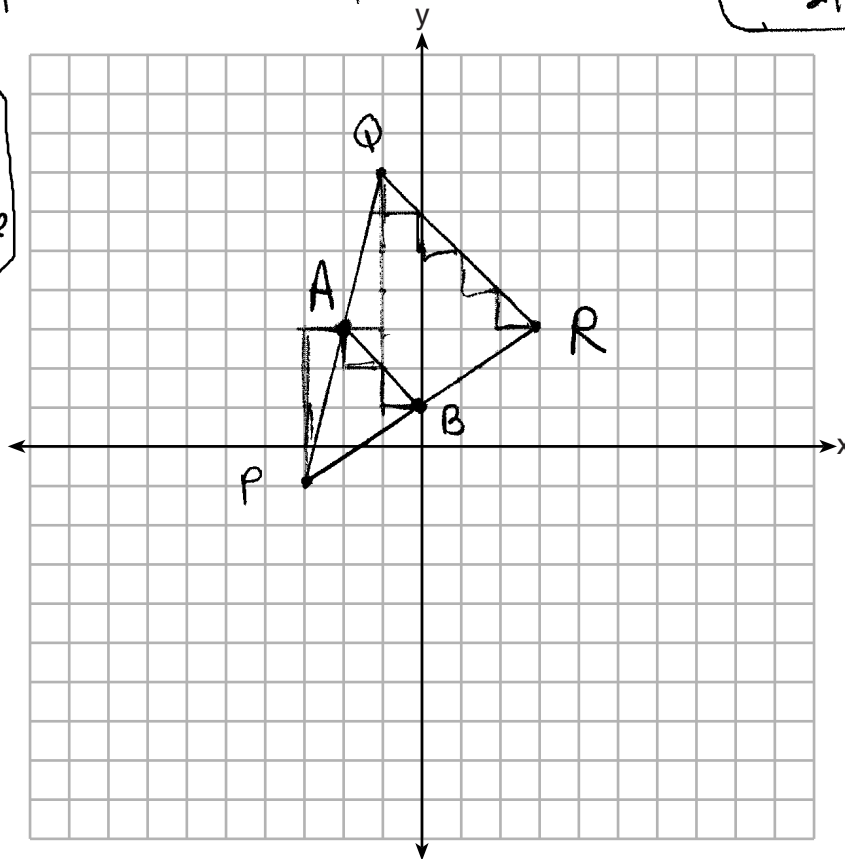
$$|QR| = \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$\frac{1}{2}|QR| = \frac{1}{2}\sqrt{32} = \frac{1}{2} \cdot 4\sqrt{2} = 2\sqrt{2}$$

$$|AB| = \sqrt{8} = 2\sqrt{2}$$

$$\therefore \frac{1}{2}|QR| = |AB|$$

$\therefore \overline{AB} \parallel \overline{QR}$
since both segments have equal slopes



Score 3: The student did correct work to show that the midsegment of a triangle is parallel and half the length to the third side of the triangle, but used the wrong midsegment.

Question 32

32 Triangle PQR has vertices $P(-3, -1)$, $Q(-1, 7)$, and $R(3, 3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

[The use of the set of axes below is optional.]

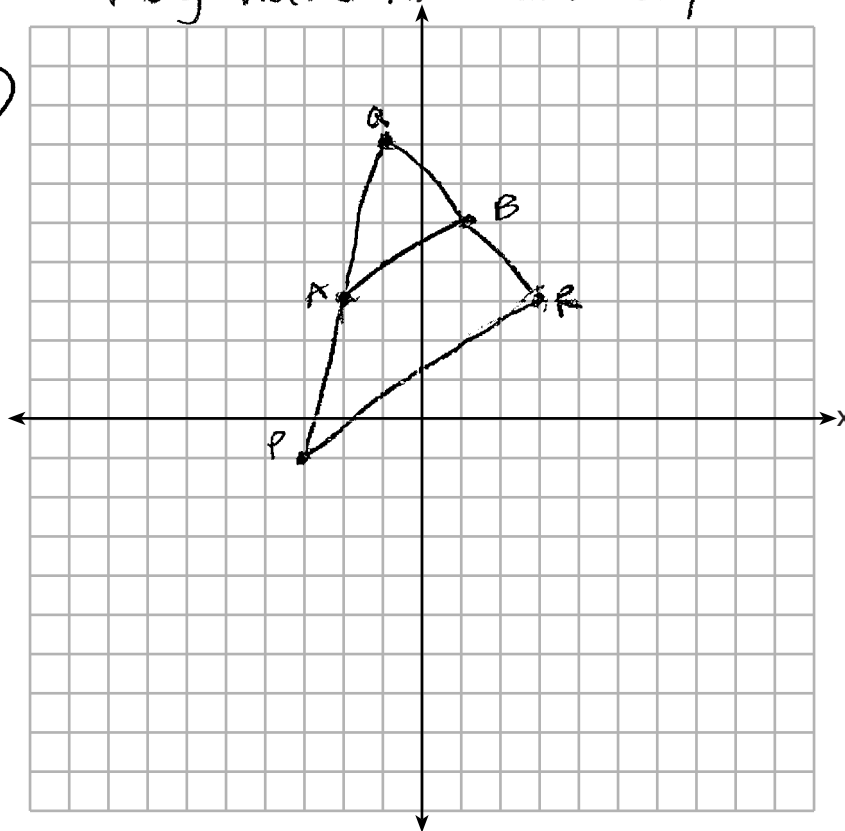
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$PR = \frac{3 - -1}{3 - -3} = \frac{3+1}{3+3} = \frac{4}{6} = \frac{2}{3}$$

$$AB = \frac{5 - 3}{1 - -2} = \frac{2}{3}$$

\overline{PR} and \overline{AB} are \parallel because they have the same slope.

$A(-2, 3)$
 $B(1, 5)$



Score 2: The student proved $\overline{AB} \parallel \overline{PR}$, but no further correct work is shown.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

[The use of the set of axes below is optional.]

$$M_{PQ} = \left(\frac{-3-1}{2}, \frac{-1+7}{2} \right)$$

$$M_{PQ} = (-2, 3)$$

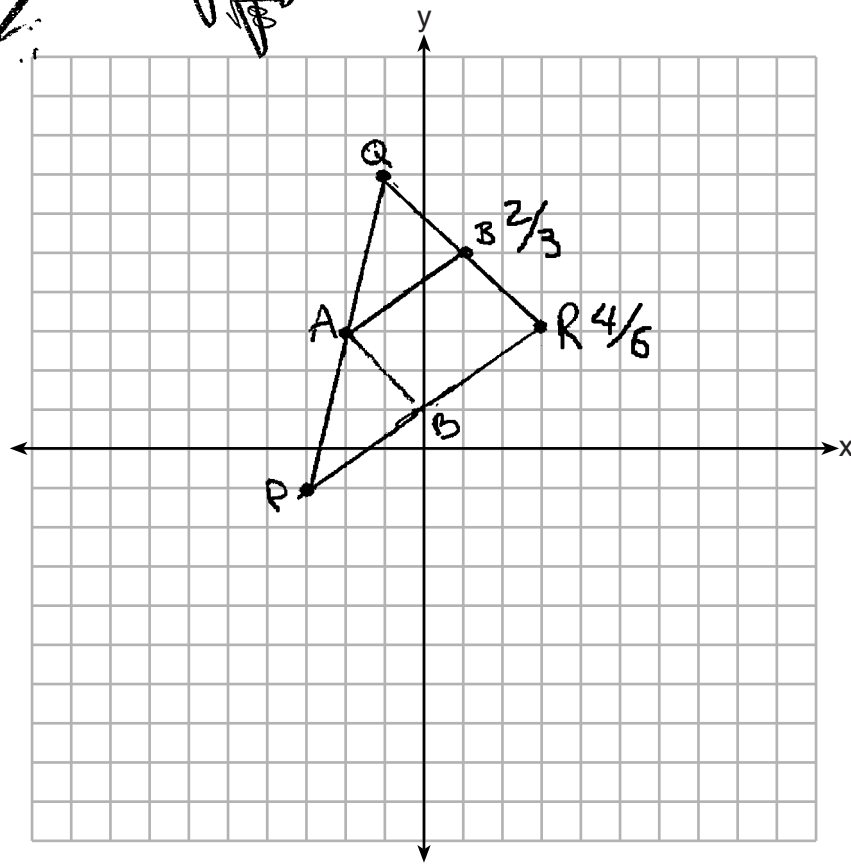
$$M_{RQ} = \left(\frac{-1+3}{2}, \frac{7+3}{2} \right)$$

$$M_{RQ} = (1, 5)$$

~~$M_{PR} = \left(\frac{-3+3}{2}, \frac{-1+3}{2} \right)$
 $M_{PR} = (0, 1)$
 $d_{AB} = \sqrt{(-2-1)^2 + (3-5)^2}$
 $d_{AB} = \sqrt{9+4}$
 $d_{AB} = \sqrt{13}$
 $d_{PR} = \sqrt{(-3-3)^2 + (-1-3)^2}$
 $d_{PR} = \sqrt{36+16}$
 $d_{PR} = \sqrt{52}$
 $d_{PR} = 2\sqrt{13}$~~

~~$d_{AB} = \sqrt{(-2-1)^2 + (3-5)^2}$
 $d_{AB} = \sqrt{9+4}$
 $d_{AB} = \sqrt{13}$
 $d_{PR} = \sqrt{(-3-3)^2 + (-1-3)^2}$
 $d_{PR} = \sqrt{36+16}$
 $d_{PR} = \sqrt{52}$
 $d_{PR} = 2\sqrt{13}$~~

\overline{AB} is parallel to \overline{PR} because their slopes are $\frac{2}{3}$ for \overline{AB} and $\frac{2}{3}$ for \overline{PR} . \overline{AB} is half of \overline{PR} because \overline{AB} is a midsegment.



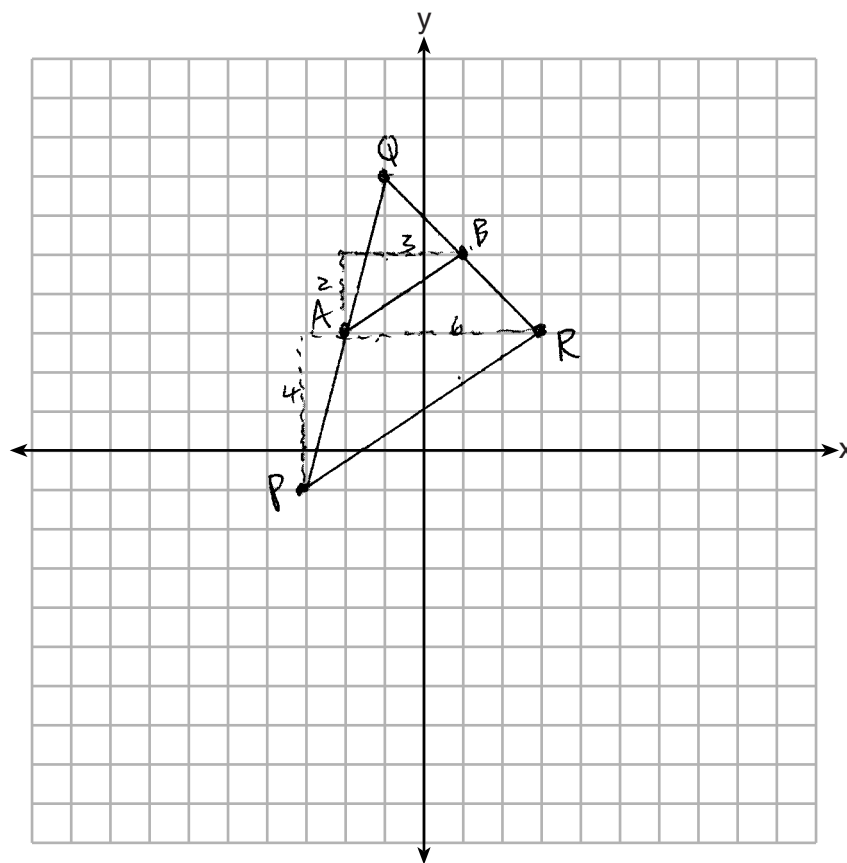
Score 2: The student proved $\overline{AB} \parallel \overline{PR}$, but no further correct work is shown.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

[The use of the set of axes below is optional.]

$$\begin{aligned} \text{Slope } \overline{AB} &= \frac{2}{3} & A(-2,3) \\ & & B(1,5) \\ \text{Slope } \overline{PR} &= \frac{4}{6} = \frac{2}{3} \end{aligned}$$



Score 1: The student found the slopes of \overline{AB} and \overline{PR} , but no concluding statement is written.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} .

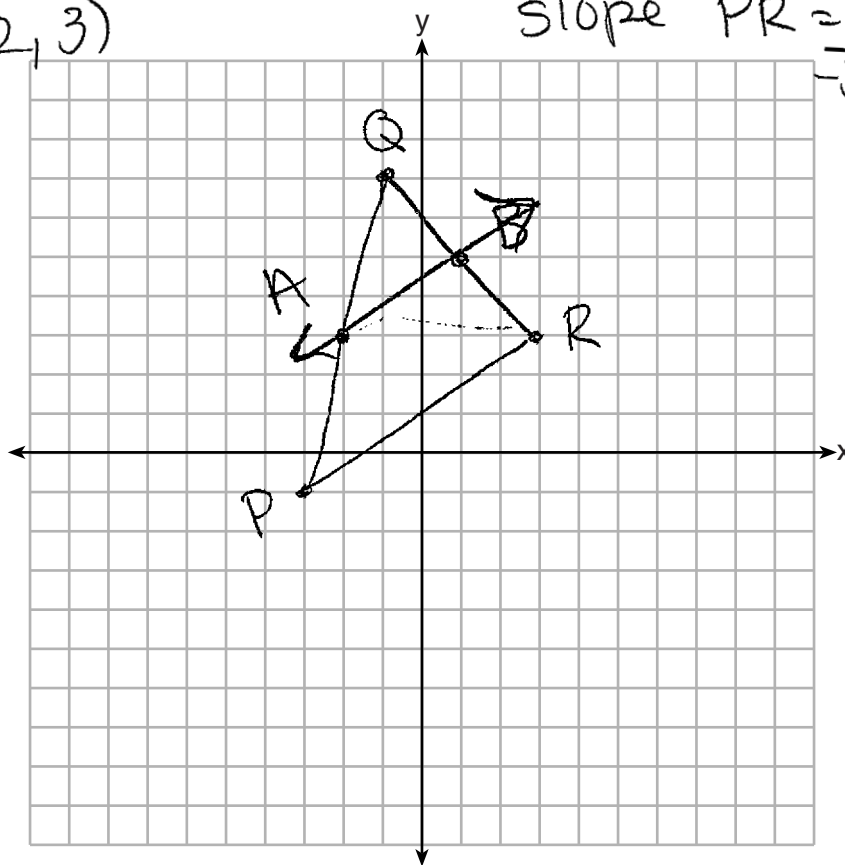
[The use of the set of axes below is optional.]

$$\begin{aligned}
 m &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-3 + -1}{2}, \frac{-1 + 7}{2} \right) \\
 &= \left(\frac{-4}{2}, \frac{6}{2} \right) \\
 &= (-2, 3)
 \end{aligned}$$

$$\begin{aligned}
 m &= \left(\frac{-1 + 3}{2}, \frac{7 + 3}{2} \right) \\
 &= \left(\frac{2}{2}, \frac{10}{2} \right) \\
 &= (1, 5)
 \end{aligned}$$

$$\text{slope } AB = \frac{5 - 3}{1 - (-2)} = \frac{2}{3}$$

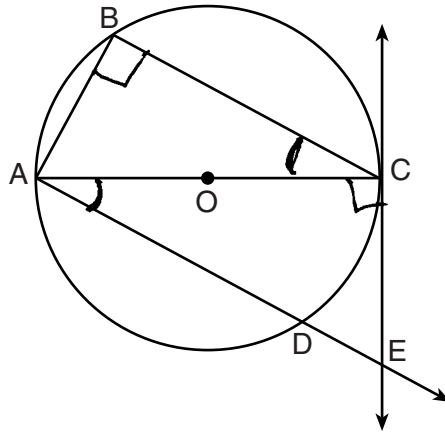
$$\text{slope } PR = \frac{3 - (-1)}{-3 - 3} = \frac{2}{-6} = -\frac{1}{3}$$



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

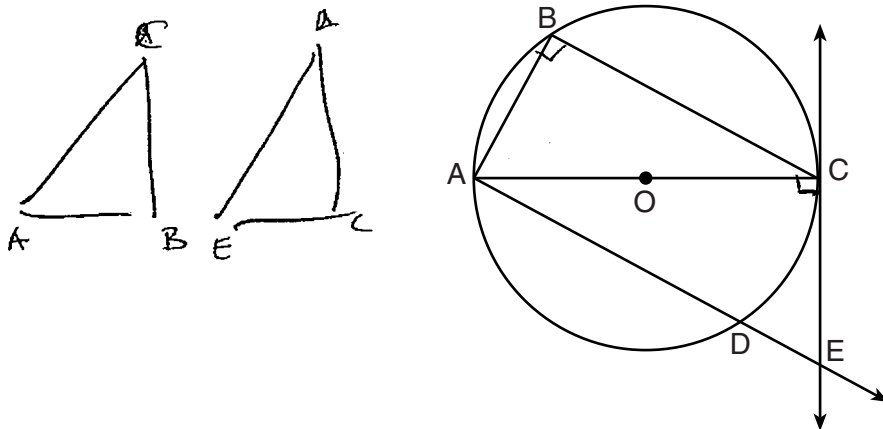
1. Circle O , tangent \overleftrightarrow{EC} is drawn to diameter \overline{AC}
Chord \overline{BC} is parallel to secant \overline{ADE}
Chord \overline{AB} is drawn
2. $\angle B$ is a right \angle
3. $\overleftrightarrow{EC} \perp \overline{OC}$
4. $\angle ECA$ is a right \angle
5. $\angle B \cong \angle ECA$
6. $\angle BCA \cong \angle CAE$
7. $\triangle ABC \sim \triangle ECA$
8. $\frac{BC}{CA} = \frac{AB}{EC}$

1. Given
2. An angle inscribed in a semicircle is a right \angle .
3. A radius is perpendicular to a tangent at the point of contact.
4. Perpendicular lines form right angles
5. All right angles are \cong .
6. If 2 parallel lines are cut by a transversal, the alternate interior angles are \cong .
7. $\angle A \cong \angle A$
8. Corresponding sides of similar triangles are in proportion

Score 4: The student gave a complete and correct response.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



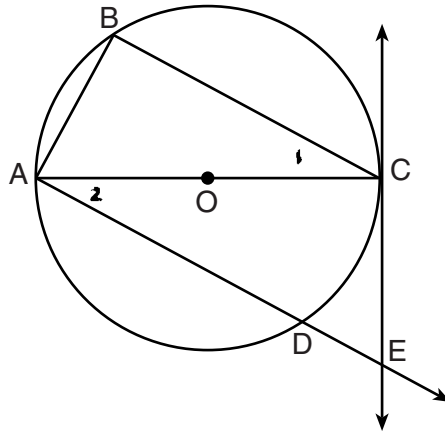
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statements	Reasons
1) In circle O , tangent \overleftrightarrow{EC} is drawn to diameter \overline{AC} , chord \overline{BC} is parallel to secant \overleftrightarrow{ADE} , and chord \overline{AB} is drawn.	1) Given.
2) $\angle ABC$ is a right angle (a)	2) Since \overline{AOC} is the diameter, it splits the circle into two congruent arcs measuring 180° . Then $\angle ABC$ is the inscribed angle of \overline{AC} , which should measure 180° . Inscribed angles are half the measure of the arc, therefore, $\angle ABC$ is a right angle as it measures 90° . (as a circle has 360°.)
3) $\angle ACE$ is a right angle.	3) A tangent intersecting with a diameter forms a 90° angle.
4) $\angle ACE \cong \angle ABC$ (a)	4) Right angles are congruent.
5) $\angle BCA \cong \angle EAC$ (a)	5) Parallel lines intersected by a transversal forms congruent alternate interior angles.
6) $\triangle BCA \sim \triangle CAE$	6) AA (4,5)
7) $\frac{BC}{CA} = \frac{AB}{EC}$	7) Similar triangles have proportional relationships with corresponding sides.

Score 4: The student gave a complete and correct response.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



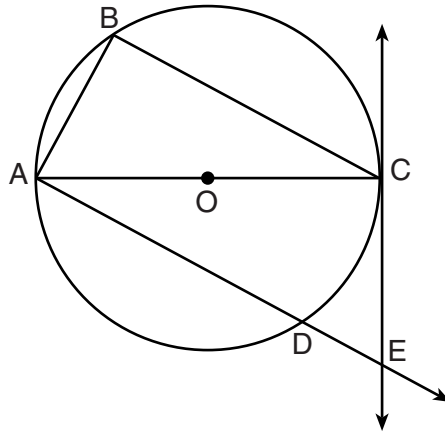
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statement	Reason
① In Circle O , tangent EC is drawn to diameter AC Chord $\overline{BC} \parallel$ secant \overline{ADE}	① Given ② If 2 // lines are cut by a transversal, all int \angle 's are \cong
② $\angle 1 \cong \angle 2$	③ An angle inscribed in a semi-circle is a rt. \angle
③ $\angle B$ is a rt. \angle	④ A tangent is \perp to a diameter at its point of tangency
④ $\overline{AC} \perp \overline{CE}$	⑤ \perp Lines form rt. \angle 's
⑤ $\angle ECA$ is a rt. \angle	⑥ All rt. \angle 's are \cong
⑥ $\angle B \cong \angle ECA$	⑦ AA
⑦ $\triangle ABC \sim \triangle ECA$	⑧ Corr. sides of similar \triangle 's are proportional
⑧ $BC \times EC = AB \times CA$	⑨ The product of the means equals the product of the extremes
⑨ $\frac{BC}{CA} = \frac{AB}{EC}$	

Score 3: The student proved $\triangle ABC \sim \triangle ECA$, but no further correct work is shown.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



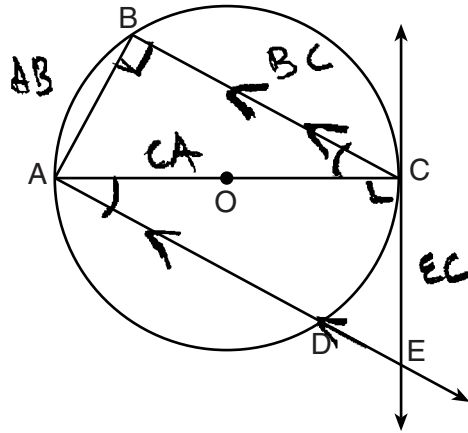
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

$\angle BCA \cong \angle EAC$ because $\overline{BC} \parallel \overline{AE}$ cut by a transversal makes \cong alternate interior angles
 $\angle ABC$ is a right angle because its an inscribed angle that intercepts a semi-circle of circle O .
 $\overline{AC} \perp \overline{CE}$ because a diameter that intersects a tangent forms \perp lines
 $\angle ACE$ is a right angle because \perp lines form right angles
 $\angle BC \cong \angle ACE$ because all right angles are \cong
 $\overline{AC} \cong \overline{AC}$ by reflexive property
 $\triangle ABC \cong \triangle ACE$ by ~~AAS~~ AAS
 $\frac{BC}{CA} = \frac{AB}{EC}$ because CACTC

Score 2: The student proved $\angle BCA \cong \angle EAC$ and $\angle ABC \cong \angle ACE$, but no further correct work is shown.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

S

R

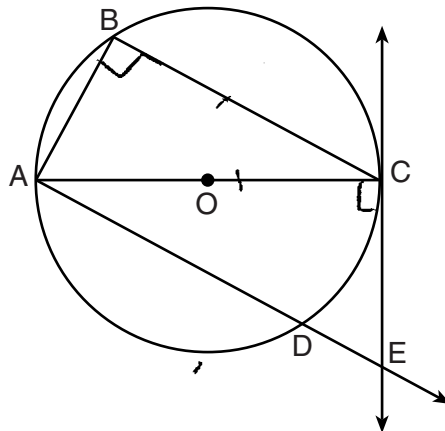
- 1.) $\angle BCA \cong \angle CAD$
- 2.) $\angle ABC = 90^\circ$
- 3.) $\angle ACE = 90^\circ$
- 4.) $\angle ABC = \angle ACE$
- 5.) $\triangle BCA \sim \triangle CAE$
- 6.) $\frac{BC}{CA} = \frac{AB}{EC}$

- 1.) Alternate interior angles of parallel lines are equal
- 2.) any point on a circle connected to two end points of a diameter in the circle, creating a triangle, is a right angle in which it = 90°
- 3.) A radius to a line that passes through the circle once at a point makes two right angles.
- 4.) Right angles are equal to right angles
- 5.) AA theorem.
- 6.) proportions for sides lengths that apply to the same opposite congruent angles may be used in the same place of the lines

Score 2: The student did not include the given and had an incorrect reason in step 6.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



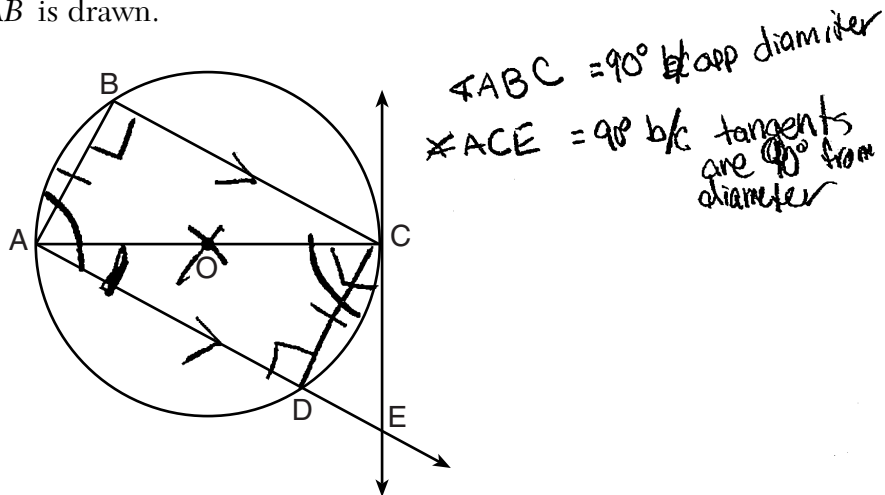
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

- | | |
|--|---|
| ① \overline{EC} is drawn to diameter AC
$\overline{BC} \parallel \overline{ADE}$, \overline{AB} is drawn | ① Given |
| ② $\angle ABC$ is a right angle | ② If an angle is inscribed in the triangle ^{semicircle} , it is a right angle |
| ③ $\angle ACE$ is a right angle | ③ If a tangent is drawn to a circle, then the angle formed is a right angle |
| ④ $\triangle ABC$ is a right triangle
$\triangle ACE$ is a right triangle | ④ Definition of a right triangle |
| ⑤ $\triangle ABC \sim \triangle ACE$ | ⑤ AA~ |
| ⑥ $BC \cdot EC = CA \cdot AB$ | ⑥ Product of the means is equal to the product of the extremes |
| ⑦ $\frac{BC}{CA} = \frac{AB}{EC}$ | ⑦ Substitution |
- Q.E.D

Score 1: The student had one correct relevant statement and reason in step 2.

Question 33

33 In the diagram below of circle O , tangent \overline{EC} is drawn to diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overline{ADE} , and chord \overline{AB} is drawn.



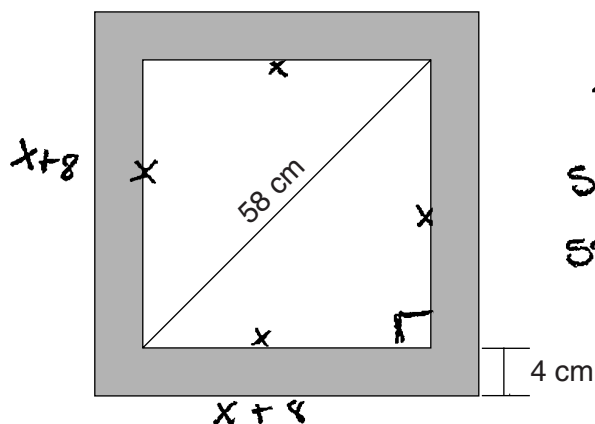
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statement	Reasons
1) \overleftrightarrow{EC} is tangent to circle O 's diameter \overline{AC} . Chord \overline{BC} is parallel to secant \overleftrightarrow{ADE} , chord \overline{AB} is drawn	1) Given
2) $\overline{AC} \cong \overline{AC}$	2) Reflexive
3) $\overline{CD} \cong \overline{AB}$	3) if 2 parallel lines are cut by a cord then that cord length is congruent to any other cord that are cut to the same parallel lines
4) $\angle BAC \cong \angle ACD$	4) if 2 parallel lines are cut by a transversal

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



$$\sin = \frac{O}{H}$$
$$\sin(45) = \frac{x}{58}$$
$$58(\sin(45)) = x$$
$$x = 41.0121\dots$$

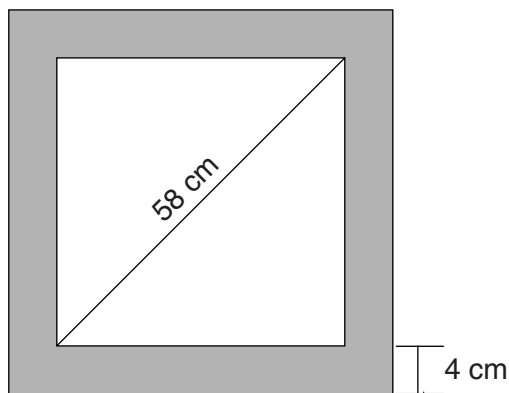
Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

$$TA = (x+8)(x+8)$$
$$TA = (41.0121+8)(41.0121+8)$$
$$TA \approx \boxed{2402.2 \text{ cm}^2}$$

Score 4: The student gave a complete and correct response.

Question 34

- 34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

$$A = \frac{1}{2}d^2$$

$$A = \frac{1}{2}(58)^2$$

$$A = 1682 \text{ cm}^2$$

$$A = (\sqrt{1682} + 8)^2$$

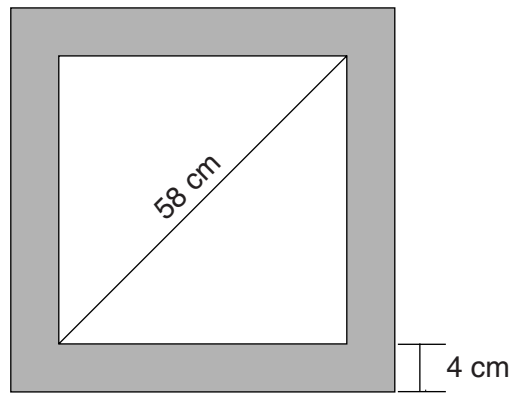
$$A = 2402.195093, \dots$$

$$A = 2402.2 \text{ cm}^2$$

Score 4: The student gave a complete and correct response.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

$$A = l \cdot w$$

$$\begin{array}{r} 41.01219 \\ + 8 \\ \hline 49.01219 \end{array}$$

$$2(58) = x^2 + x^2$$

$$\frac{3364}{2} = \frac{2x^2}{2}$$

$$\sqrt{1682} = \sqrt{x^2}$$

$$41.01219 = x$$

$$A = s^2$$

$$A = 49.01219^2$$

$$A = 2402.2$$

Poster Area = 1682.0
Frame Area = 720.2
Total Area = 2402.2

$$A = s^2$$

$$A = 41.01219^2$$

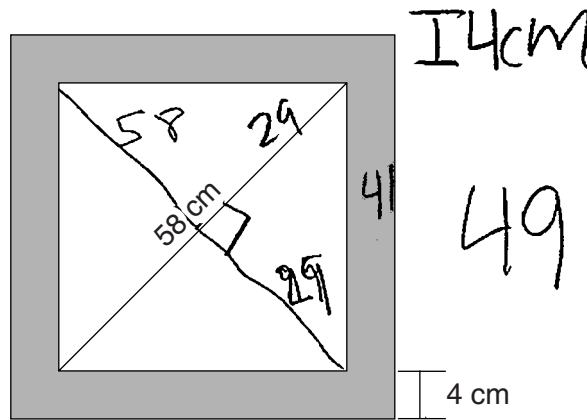
$$A = 1682.0$$

$$\begin{array}{r} 2402.2 \\ - 1682.0 \\ \hline 720.2 \end{array}$$

Score 4: The student gave a complete and correct response.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth* of a square centimeter.

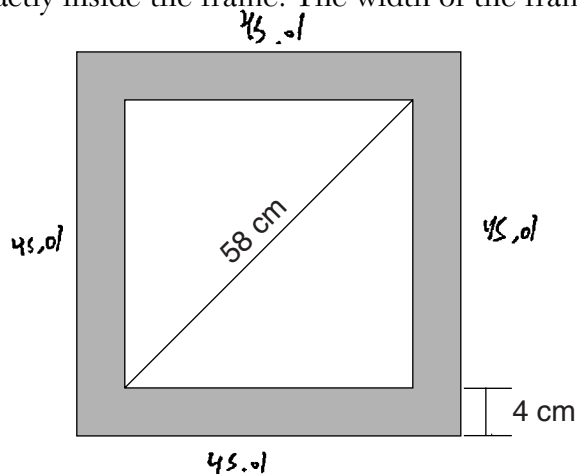
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 29^2 + 29^2 &= c^2 \\
 841 + 841 &= c^2 \\
 \sqrt{1682} &= \sqrt{c^2} \\
 \sqrt{1681} &= c \\
 41 &= c
 \end{aligned}$$

$$\begin{aligned}
 A &= s^2 \\
 A &= 49^2 \\
 A &= 2401 \text{ cm}^2
 \end{aligned}$$

Score 3: The student made a transcription error by writing $\sqrt{1681}$.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

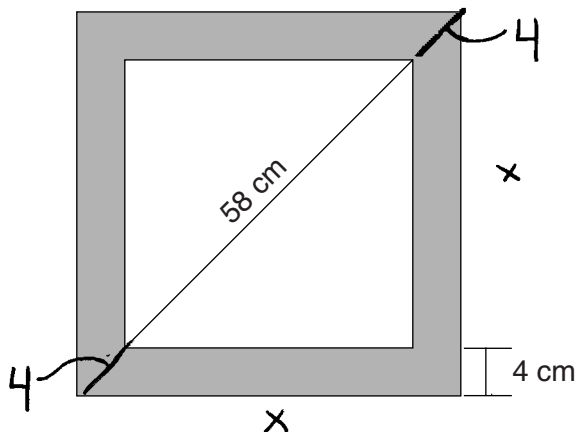
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 x^2 + x^2 &= 58^2 \\
 2x^2 &= 58^2 \\
 2x^2 &= 3364 \\
 x^2 &= 1682 \\
 x &= 41.01
 \end{aligned}$$

$$\begin{aligned}
 A &= 45.01 \cdot 45.01 \\
 A &= 2025.9 \text{ cm}^2
 \end{aligned}$$

Score 2: The student made an error in rounding $\sqrt{1682}$ early and another error by adding 4 rather than 8 to find the length of the frame.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

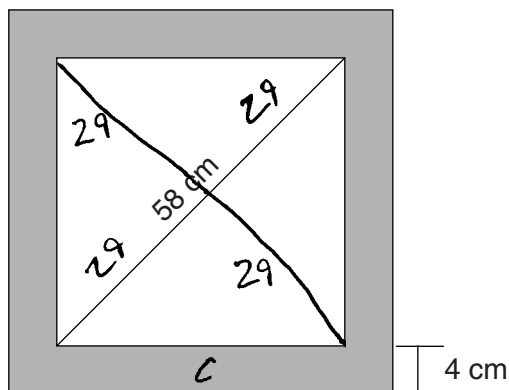
$$\begin{aligned}x^2 + x^2 &= 66^2 \\2x^2 &= 4356 \\x^2 &= 2178 = \text{Area}\end{aligned}$$

$$\begin{array}{r}58 \\+ 8 \\ \hline 66\end{array}$$

Score 2: The student made a conceptual error in finding the length of the diagonal.

Question 34

- 34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



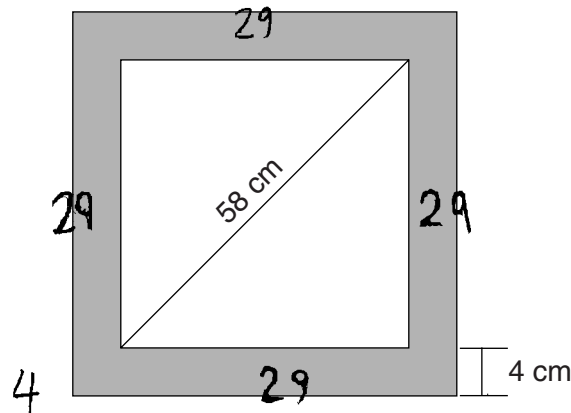
Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

$$\text{poster area} = 1682 \text{ cm}^2$$
$$29^2 + 29^2 = c^2 = \sqrt{1682} = 41.01 \text{ cm}$$
$$41.01 \text{ cm}$$

Score 1: The student found the area of the poster, but no further correct work is shown.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter*.

$$\begin{array}{r}
 3 \\
 29 \\
 \times 4 \\
 \hline
 116
 \end{array}
 \leftarrow \text{area of poster}$$

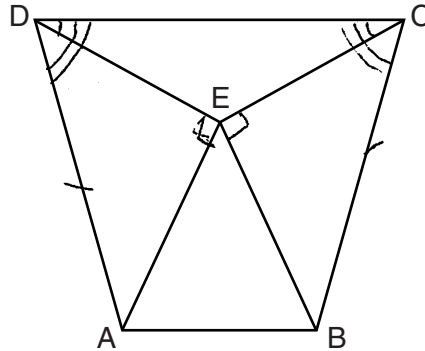
$$\begin{array}{r}
 a^2 + b^2 = c^2 \\
 29^2 + b^2 = 58^2 \\
 841 + b^2 = 3364 \\
 \underline{-841} \qquad \underline{-841} \\
 b^2 = 2523 \\
 \sqrt{b^2} = \sqrt{2523} \\
 b = 50.23
 \end{array}$$

$$\textcircled{1456.67} \text{ — area of frame}$$

Score 0: The student gave a completely incorrect response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



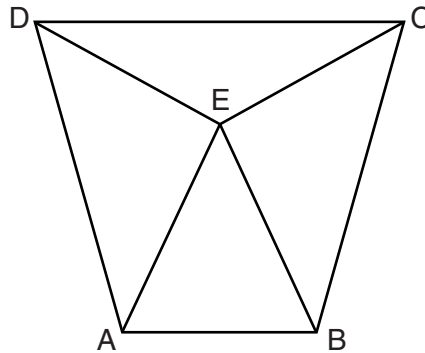
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

- | | |
|---|--|
| <p>1. $ABCD$ is an isosceles trapezoid</p> <p>2. $\overline{AD} \cong \overline{BC}$</p> <p>3. $\overline{AE} \perp \overline{DE}$
$\overline{BE} \perp \overline{CE}$</p> <p>4. $\angle DEA$ is a right angle
$\angle CEB$ is a right angle</p> <p>5. $\angle DEA \cong \angle CEB$</p> <p>6. $\angle CDE \cong \angle DCE$</p> <p>7. $\angle CDA \cong \angle DCB$</p> <p>8. $\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$
$\angle EDA \cong \angle ECB$</p> <p>9. $\triangle ADE \cong \triangle BCE$</p> <p>10. $\overline{EA} \cong \overline{EB}$</p> <p>11. $\triangle AEB$ is an isosceles \triangle</p> | <p>1. given</p> <p>2. Legs in an isosceles trapezoid are =</p> <p>3 given</p> <p>4. \perp lines form right angles.</p> <p>5. All right angles are \cong</p> <p>6 given</p> <p>7. Base angles of isosceles trapezoid are \cong</p> <p>8. Subtraction Postulate</p> <p>9. \cong AA \cong SA</p> <p>10. \angle PCTC</p> <p>11. isosceles \triangle has 2 \cong sides.</p> |
|---|--|

Score 6: The student gave a complete and correct response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

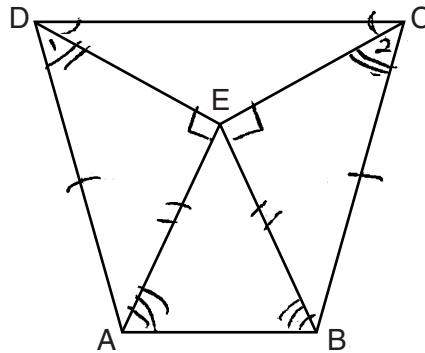
Handwritten student proof:

- Given: $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$
- Given: $\angle DEA$ is a rt \angle , $\angle CEB$ is a rt \angle
- Given: $\angle CDE \cong \angle DCE$
- Given: Iso Trap $ABCD$ w/ base \overline{DC} & \overline{AB}
- Given: $\overline{AD} \cong \overline{BC}$ (non- \parallel sides of Iso Trap are \cong)
- Given: $\angle CDE \cong \angle DCE$
- From $\angle CDE \cong \angle DCE$: $\overline{DE} \cong \overline{CE}$ (Sides opp \cong \angle s are \cong in a Δ)
- From $\angle DEA$ and $\angle CEB$ are rt \angle 's: All rt \angle 's are \cong
- From $\angle DEA \cong \angle CEB$ and $\overline{DE} \cong \overline{CE}$: $\triangle DEA \cong \triangle CEB$ (HL)
- From $\triangle DEA \cong \triangle CEB$: $\overline{EA} \cong \overline{EB}$ (CPCTC)
- Conclusion: $\triangle AEB$ is isosceles (If a Δ has 2 sides \cong , it is isosc.)

Score 6: The student gave a complete and correct response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



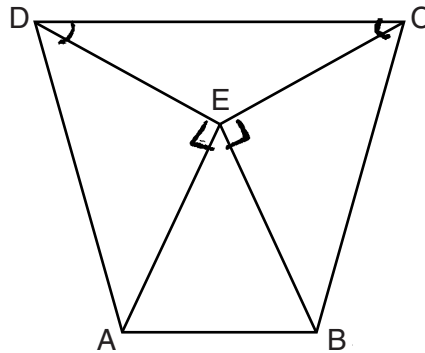
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statement	Reason
1. Isosceles trapezoid $ABCD$ has bases \overline{DC} & \overline{AB}	1. given
2. $\angle CDE \cong \angle DCE$	2. given
3. $\overline{AE} \perp \overline{DE}$	3. given
4. $\overline{BE} \perp \overline{CE}$	4. given
5. $\angle AED$ is a right \angle $\angle BEC$ is a right \angle	5. perpendicular lines form right \angle 's.
6. $\angle AED \cong \angle BEC$	6. All right \angle 's are \cong
7. $\angle ADC \cong \angle BCD$	7. Angles opposite of two \cong sides are also \cong .
8. $\angle 1 \cong \angle 2$	8. Subtraction
9. $\overline{AD} \cong \overline{BC}$	9. legs of an isosceles trapezoid are \cong .
10. $\triangle ADE \cong \triangle BCE$	10. SAA \cong SAA
11. $\overline{AE} \cong \overline{BE}$	11. CPCTC
12. $\triangle AEB$ is an isosceles \triangle	12. A triangle w/ 2 \cong sides is isosceles

Score 5: The student had an incorrect reason in step 7.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



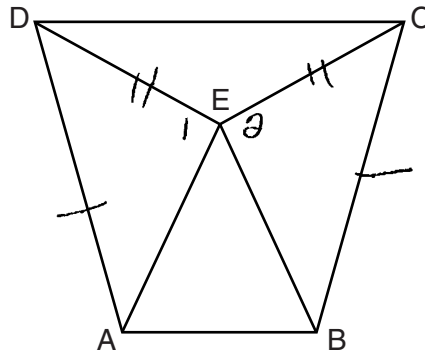
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

S	R
Isosceles Trapezoid $ABCD$ $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	Given
$\angle DEA$ and $\angle CEB$ are right angles	Perpendicular lines form right angles
$\angle DEA \cong \angle CEB$	All right angles are congruent
$\overline{DE} \cong \overline{CE}$	If the base angles of a triangle are congruent, then the sides opposite them are congruent
$\overline{DA} \cong \overline{CB}$	Properties of isosceles trapezoid
$\triangle DEA \cong \triangle CEB$	HL \cong HL / High Leg Theorem
$\overline{EA} \cong \overline{EB}$	C.P.C.T.C.
$\triangle AEB$ is an isosceles triangle	If the base angles of a triangle are congruent, the triangle is isosceles.

Score 4: The student did not prove $\triangle DEA$ and $\triangle CEB$ are right triangles and wrote an incorrect last reason by referencing base angles when the student proved congruent sides.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



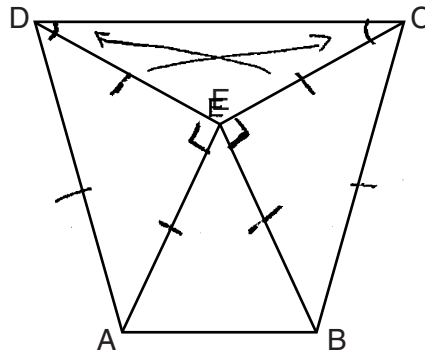
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
① Isosceles trapezoid $ABCD$ $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	① given
② $\overline{AD} \cong \overline{BC}$	② An isosceles trapezoid has 2 \cong legs
③ $\angle 1$, $\angle 2$ are right \angle s	③ \perp lines intersect to form rt \angle s
④ $\angle 1 \cong \angle 2$	④ rt \angle s are \cong .
⑤ $\overline{DE} \cong \overline{CE}$	⑤ In a \triangle , sides opposite \cong \angle s are \cong
⑥ $\triangle ADE \cong \triangle BCE$	⑥ SAS \cong SAS
⑦ $\overline{AE} \cong \overline{BE}$	⑦ If 2 \triangle s are \cong , corresponding parts are \cong
⑧ $\triangle AEB$ is isosceles	⑧ an isosceles \triangle w a \triangle w/ 2 \cong sides.

Score 4: The student made one conceptual error in proving $\triangle ADE \cong \triangle BCE$ by SAS.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



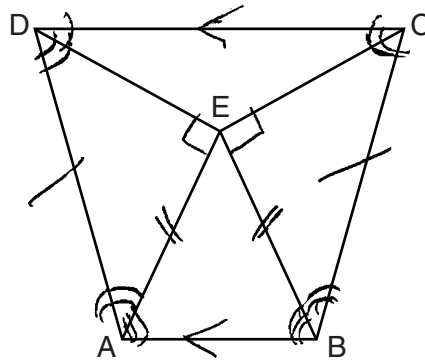
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
① trapezoid $ABCD$ $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE}$ $\overline{BE} \perp \overline{CE}$	① given
② $\overline{DA} \cong \overline{CB}$	② def of trapezoid
③ $\triangle DEC$ is an isosceles \triangle	③ has 2 congruent sides
④ $\triangle ADE$ and $\triangle BCE$ are right \triangle s	④ have right \angle s
⑤ $\triangle ADE \cong \triangle BCE$	⑤ HL \cong HL
⑥ $\overline{AE} \cong \overline{BE}$	⑥ CPCTC
⑦ $\triangle AEB$ is an isosceles \triangle	⑦ has 2 congruent sides

Score 3: The student did not prove $\overline{DE} \cong \overline{CE}$ and that $\angle DEA$ and $\angle CEB$ are right angles. The student also had an incorrect reason in step 2.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



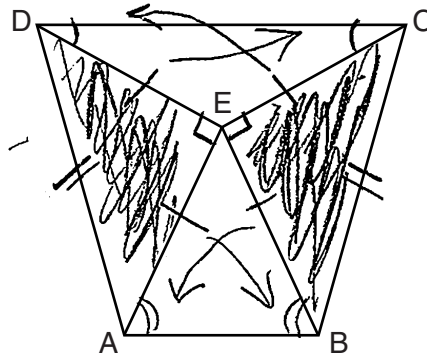
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statement	Reason
① Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} , $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$	① Given
② $\overline{DC} \parallel \overline{AB}$	② Isosceles trapezoids have one pair of \parallel lines
③ $\angle AED$ & $\angle BEC$ are rt \angle 's	③ \perp lines form rt \angle 's
④ $\angle AED \cong \angle BEC$	④ All rt \angle 's are \cong .
⑤ $\angle ECB$ and $\angle CBE$, $\angle EDA$ and $\angle ADE$ are corresponding \angle 's.	⑤ When \parallel lines are cut by a transversal corresponding \angle 's are formed.
⑥ $\triangle ADE \cong \triangle BCE$	⑥ S.A.S
⑦ $\triangle AEB$ is an isosceles \triangle .	⑦ \cong base \angle 's create an isosceles \triangle .

Score 2: The student proved $\angle AED \cong \angle BEC$, but no further correct relevant work is shown.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



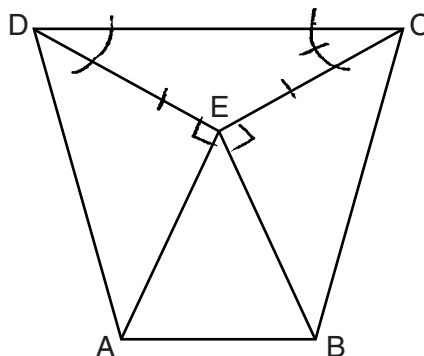
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

statements	Proofs
1) Isosceles trapezoid $ABCD$ bases \overline{DC} , \overline{AB} $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE}$ $\overline{BE} \perp \overline{CE}$	1) Given
2) $\angle DEA$ and $\angle CEB$ are right \angle s	2) perpendicular bisector form right angles
3) $\angle DEA \cong \angle CEB$	3) Right angles are always congruent
4) $\overline{DA} \cong \overline{CB}$	4) Given
5) $\triangle DEC$ is an isosceles \triangle	5) Isosceles triangles have 2 congruent angles.
6) $\overline{DE} \cong \overline{CE}$	6) Isosceles triangles are triangles with 2 sides with equal length.
7) $\triangle DEA \cong \triangle CEB$	7) SAS = SAS
8) $\overline{AE} \cong \overline{BE}$	8) corresponding parts of congruent triangles are congruent
9) $\triangle AEB$ is an isosceles triangle	9) If two sides are congruent, then the angles opposite of the sides in the triangle are congruent.

Score 2: Some correct relevant statements about the proof are made in steps 3, 6, and 8, but four or more statements and/or reasons are missing or incorrect.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



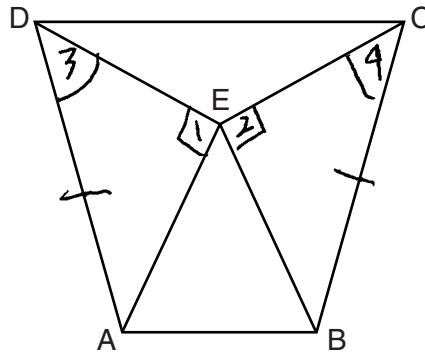
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
1. $\angle CDE \cong \angle DCE$	1. Given
2. $\overline{DE} \cong \overline{CE}$	2. In a triangle, \cong line lay opposite \cong sides.
3. $\overline{AE} \perp \overline{DE}$; $\overline{BE} \perp \overline{CE}$	3. Given
4. $\angle DEA$ and $\angle CEB$ are right angles	4. \perp lines intersect to form right angles.
5.	

Score 1: The student had one correct statement and reason in step 4.

Question 35

35 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



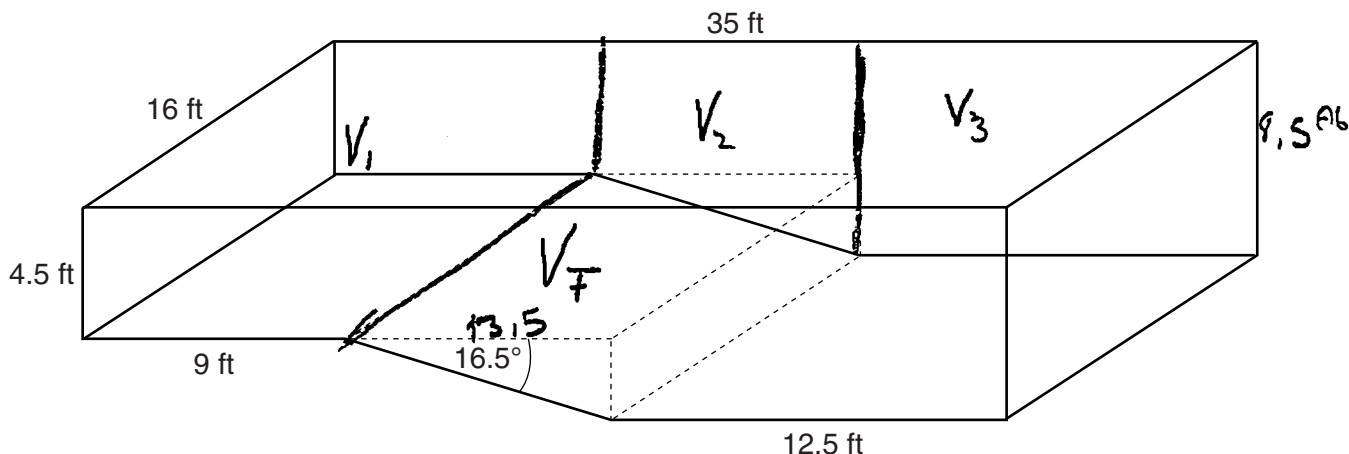
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
① Isoc trap $ABCD$ has bases \overline{DC} and \overline{AB} with non parallel legs \overline{AD} and \overline{BC}	① given
② $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE} \ \& \ \overline{BE} \perp \overline{CE}$	② given
③ $\angle 1 \cong \angle 2$	③ verticle angles are congruent
④ $\overline{DA} \cong \overline{CB}$	④ opp. sides of a trap.
⑤ $\angle 3 \cong \angle 4$	⑤ substitution
⑥ $\triangle ADE \cong \triangle BCE$	⑥ ASA
⑦ $\triangle AEB$ is an isoc \triangle	⑦ CPCTC

Score 0: The student gave a completely incorrect response.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the nearest tenth of a foot?

$$35 - 9 - 12.5 = h$$

$$13.5 = h$$

$$4.5 + x = 9.5$$

$$\tan(16.5) = \frac{x}{13.5}$$

$$13.5 \tan(16.5) = x$$

$$x = 3.99882182$$

$$8.5 \text{ ft}$$

Find the volume of the inside of the pool to the nearest cubic foot.

$$V_1 = 4.5 \cdot 16 \cdot 9 = 648$$

$$V_3 = 12.5 \cdot 16 \cdot 8.5 = 1700$$

$$V_2 = 13.5 \cdot 4.5 \cdot 16 = 972$$

$$V_T = \frac{13.5 \cdot 4}{2} \cdot 16$$

$$V_T = 432$$

$$V_1 + V_2 + V_3 + V_T = 3752$$

$$3752 \text{ ft}^3$$

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? [1 ft³ = 7.48 gallons]

$$A_{\text{air}} = 16.35 \cdot 15 = \boxed{245}$$

$$3752.280 = 3472 \text{ ft}^3$$

$$3472 \cdot 7.48 = \boxed{25970.56 \text{ gallons}}$$

$$25970.56 / 10.5 = 2473.386667$$

$$\frac{2473.386667 \text{ minutes}}{60}$$

"

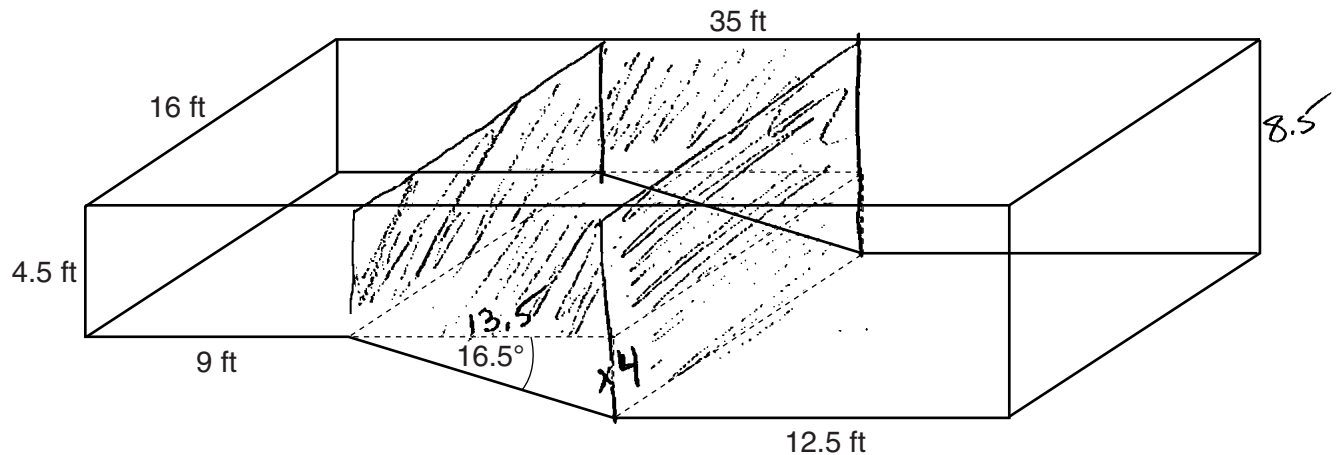
$$41.2231111$$

$$\boxed{41 \text{ hours}}$$

Score 6: The student gave a complete and correct response.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$$\tan(16.5) = \frac{x}{13.5} = 3.99 = 4$$

$$4 + 4.5 = 8.5$$

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$(16)(4.5)(9) = 648$$

$$(12.5)(8.5)(16) = 1700$$

$$(13.5)(4.5)(16) = 972$$

$$\frac{1}{2}(4)(13.5)(16) = 432$$

Volume $\boxed{3752}$

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? [1 ft³ = 7.48 gallons]

3752 volume of pool

volume of air
 $(.5)(16)(35) = 280$

$$\begin{array}{r} 3752 \\ - 280 \\ \hline \end{array}$$

$$3472 \cdot 7.48$$

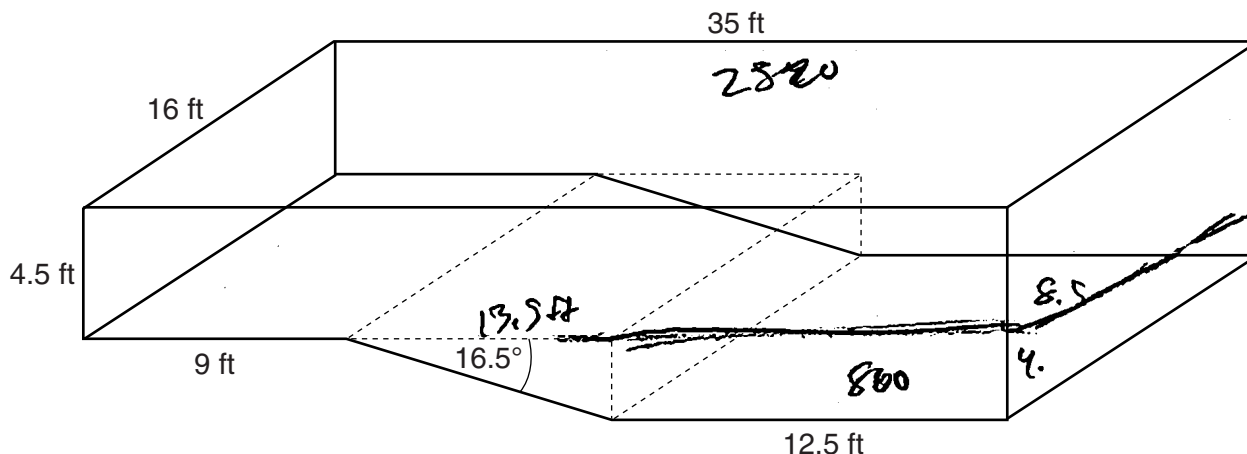
$$= \frac{25970.5}{(10.5)(60)} = 41.223$$

41.2 hours

Score 5: The student made a rounding error when finding the time.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the nearest tenth of a foot?

~~$4.5 \cdot 16 = 72$~~ $\tan(16.5) = \frac{x}{13.5}$

$\tan = 3.99885718149 + 4.5$

The depth is 8.5 ft

Find the volume of the inside of the pool to the nearest cubic foot.

$4.5 \cdot 16 = 72$

~~$72 \cdot 35 = 2520$~~

$12.5 \cdot 16 = 200 \cdot 4 = 800$

$13.5 \cdot 16 = 216 \cdot 4 = \frac{864}{2} = 432$

$2520 + 800 + 432 = 3752 \text{ ft}^3$

Question 36 is continued on the next page.

Question 36

Question 36 continued

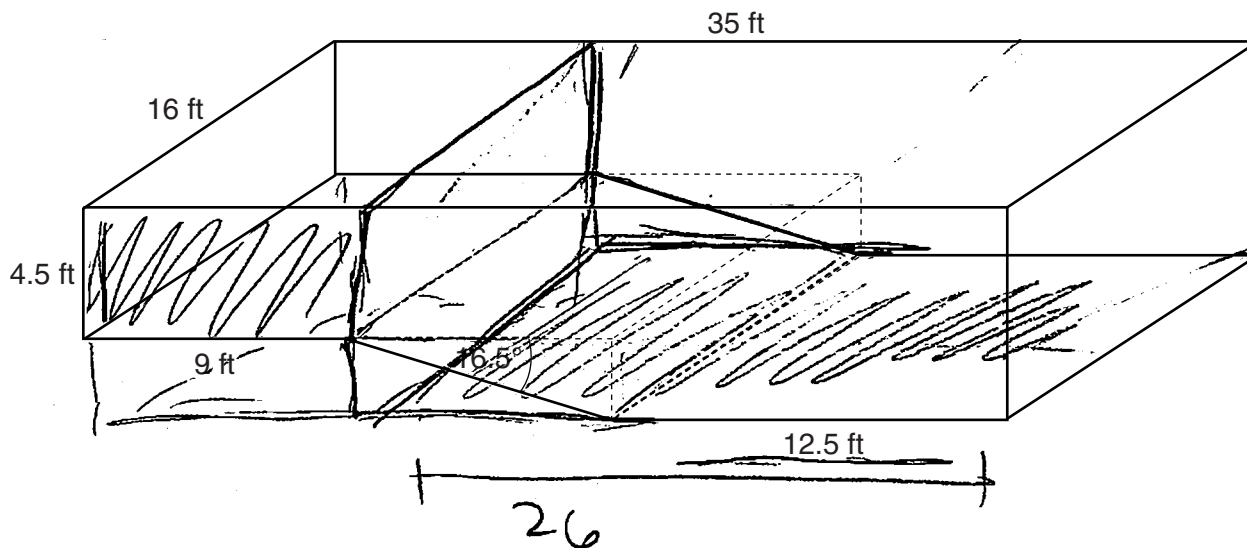
A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

$$\frac{3752}{7.48} = 501.604278073 \text{ gallons}$$
$$\frac{501.604278073}{10.5}$$
$$47.7$$

Score 4: The student found 8.5 and 3752, but no further correct work is shown.

Question 36

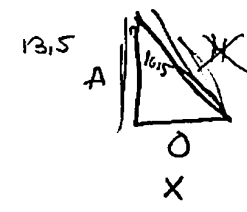
36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the nearest tenth of a foot?

$$4.5 + 3.9988821$$

$$13.5 (\tan 16.5 = \frac{x}{13.5})$$



$$3.9988821 = x$$

Depth of pool = 8.5 ft

Find the volume of the inside of the pool to the nearest cubic foot.

$\square \times \text{height}$

$$4.5 \times 9 \times 16$$

$$648$$

$$3536 - 431.8793$$

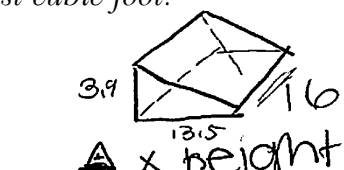
$$3104.1207 + 648$$

$$V = 3752$$

$\square \times \text{height}$

$$16 \times 26 \times 8.5$$

$$3536$$



$\square \times \text{height}$

$$(3.9988821 \times 13.5) \div 2 \times 16$$

$$26.99245418 \times 16$$

$$431.8$$

Question 36 is continued on the next page.

Question 36

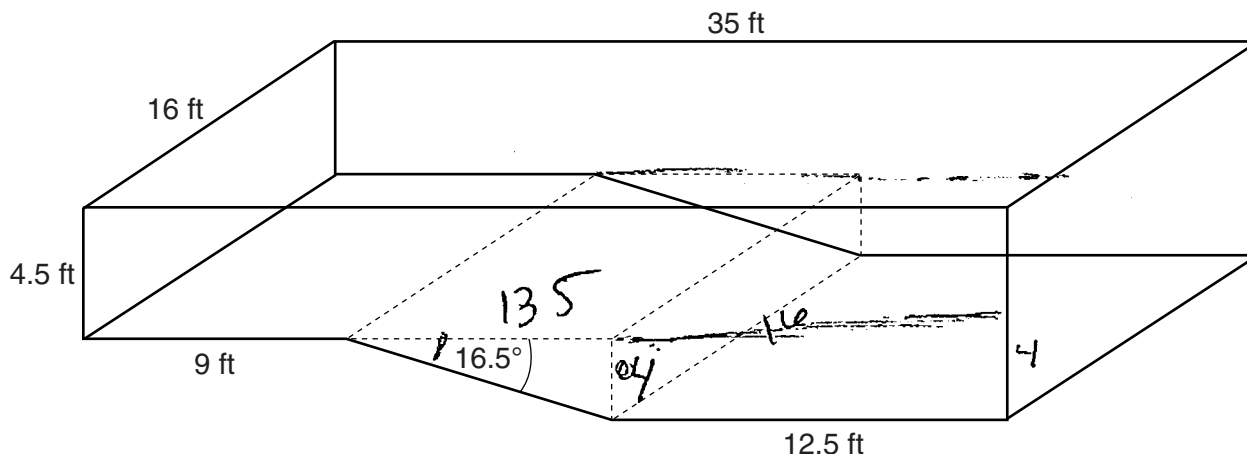
Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

Score 4: The student did not find the time.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$$\tan(6.5) = \frac{0}{13.5}$$

$$13.5 \tan(6.5) = 0$$

$$0 = 4ft$$

The depth of the pool is 4ft.

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$V = Bh$$

$$V = (16)(4.5)(35)$$

$$V = 2520$$

$$+ 800$$

$$432$$

$$V = (4)(16)(12.5)$$

$$V = 800$$

$$V = 3752 + 3$$

$$V = \frac{1}{2} Bh$$

$$V = \frac{1}{2} (16)(4)(13.5)$$

$$V = 432$$

Question 36 is continued on the next page.

Question 36

Question 36 continued

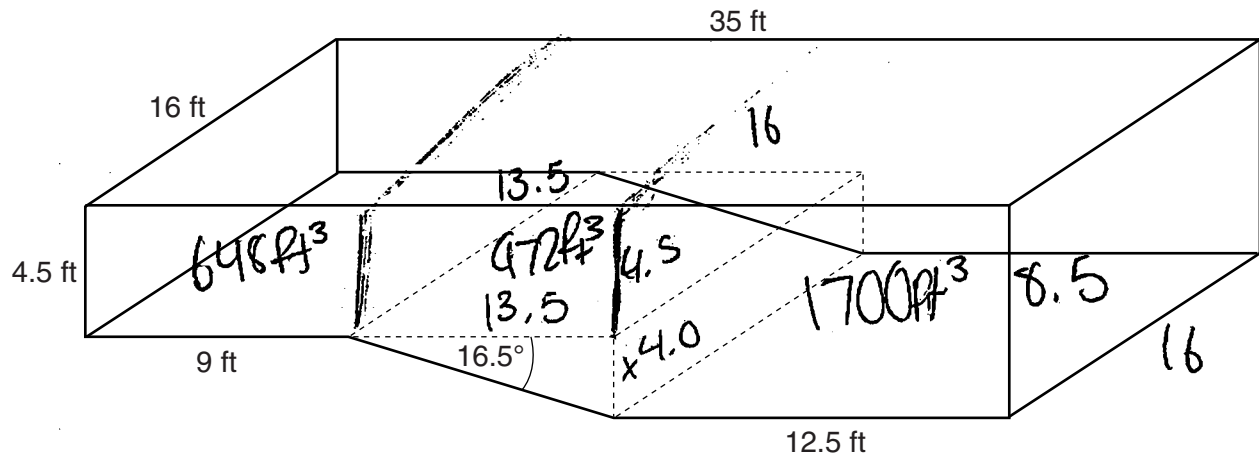
A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

$$\frac{10.5}{7.48} = \frac{6}{1}$$
$$\cancel{44.88 = 10.5}$$

Score 3: The student correctly found the volume of the pool, but did not add 4.5 when finding the depth, and did not find the time correctly.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$$35 - (9 + 12.5) = 13.5$$

$$\left(\tan 16.5 = \frac{x}{13.5} \right) 13.5$$

$$13.5 \tan 16.5 = x$$

$$3.99 = x$$

$$4.0 = x$$

$$4.0 + 4.5 = \boxed{8.5 \text{ ft}}$$

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$V = \frac{1}{2}bh$$

$$V = \frac{1}{2}(13.5 \cdot 4)$$

$$V = \frac{1}{2}(54)$$

$$V = 27 \text{ ft}^3$$

$$648 + 172 + 27 + 1700 =$$

$$\boxed{3347 \text{ ft}^3}$$

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

$$3347 \cdot 7.48 = 25,035.56 \text{ gallons}$$

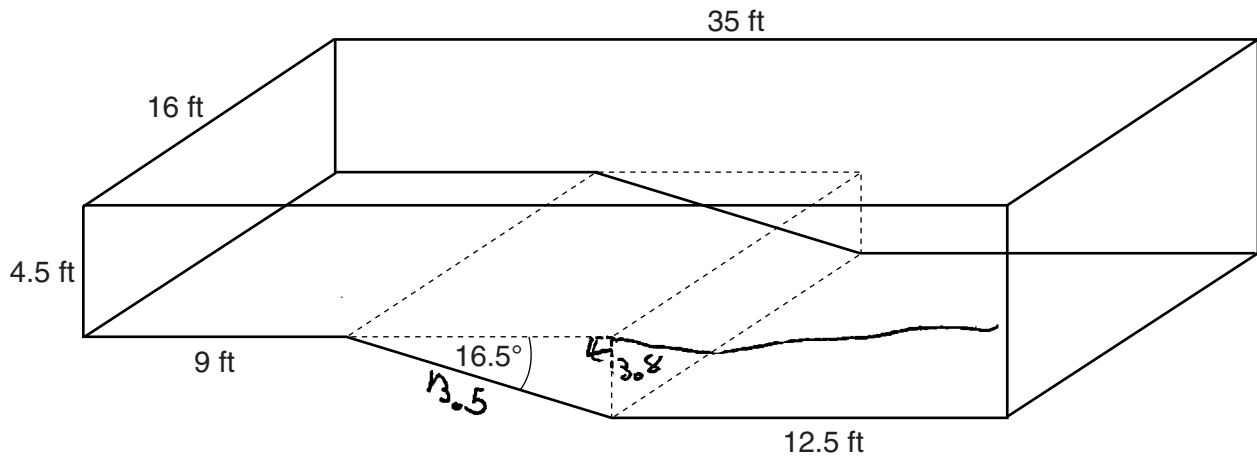
$$10.5 \cdot 25,003.5 = 262,873.38$$

$$262,873 \text{ minutes}$$

Score 3: The student did not multiply by 16 when finding the volume of the triangular prism and did not find the time correctly.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$$\sin 16.5 = \frac{d}{13.5}$$

$$\frac{3.8347071594}{4.5}$$

$$8.33$$

8.3 feet

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$4.5(16)(35) = 2520$$

$$(3.8)(16)(12.5) = 766$$

$$18.99(16) = 207$$

3487

Question 36 is continued on the next page.

Question 36

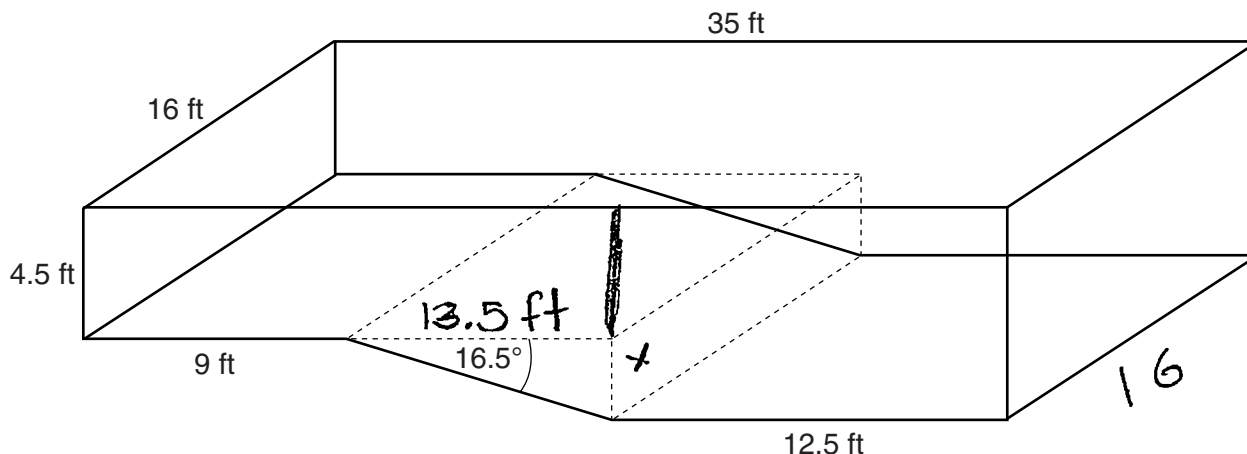
Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

Score 2: The student made an error when labeling 13.5 in the diagram, made an error when finding the volume of the triangular prism, and did not find the time.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth* of a foot?

SOH CHH ~~TOA~~ $\tan 16.5 = \frac{x}{13.5}$

$x = 18.2396$
 $x \approx 18.2 \text{ ft}$

Find the volume of the inside of the pool to the *nearest cubic foot*.

$A = \frac{bh}{2}$
 $A = \frac{13.5 \times 18.2}{2}$
 $A = \frac{245.2}{2}$
 $V = 122.85 \approx 123$

$A = lwh$
 $A = 4.5 \times 9 \times 16$
 $V = 648$

$A = 12.5 \times 16 \times 22.7$
 $A = 4540$

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? [1 ft³ = 7.48 gallons]

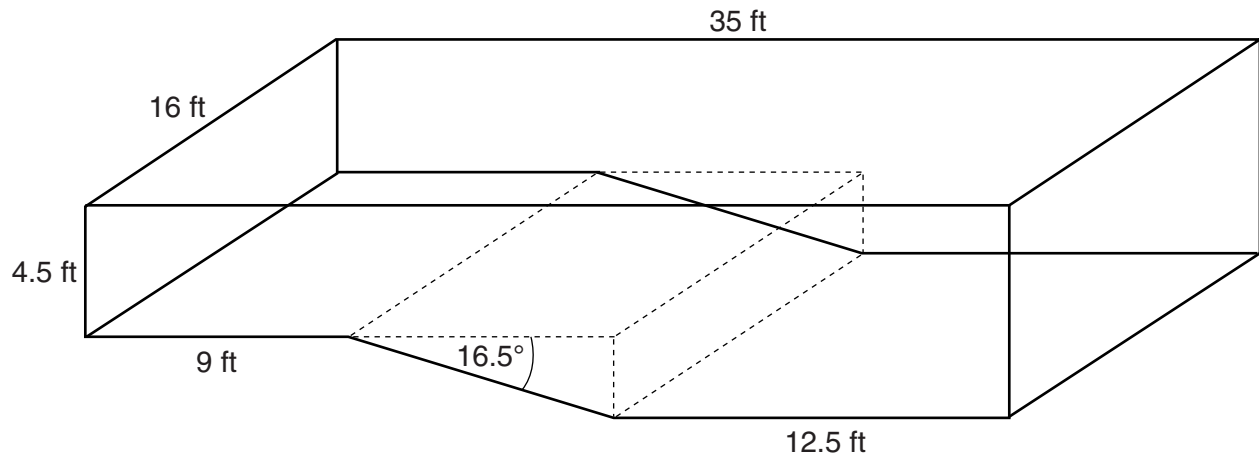
$$\frac{10.5 \text{ g}}{1 \text{ min}} = \frac{x}{60 \text{ min}}$$

$$x = 630 \text{ gallons in an hour}$$

Score 1: The student wrote a correct trigonometric equation to find the depth of the pool. The student did not show enough correct work to find the total volume of the pool. The student did not find the time to fill the pool.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

4.5

Find the volume of the inside of the pool to the *nearest cubic foot*.

$$\begin{aligned} V &= lwh \\ &= 9 \cdot 16 \cdot 35 \\ &= 5040 \text{ ft}^3 \end{aligned}$$

$$\frac{5040}{10.5} = 480$$

ANS 480 HRS

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top?
[1 ft³ = 7.48 gallons]

Score 0: The student showed no correct relevant work.