ALGEBRA

II

Large-Type Edition

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, August 14, 2019 — 12:30 to 3:30 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 When the expression $(x + 2)^2 + 4(x + 2) + 3$ is rewritten as the product of two binomials, the result is

(1) $(x + 3)(x + 1)$

(2) $(x + 5)(x + 3)$

(3) $(x + 2)(x + 2)$

(4) $(x + 6)(x + 1)$

2 The first term of a geometric sequence is 8 and the fourth term is 216. What is the sum of the first 12 terms of the corresponding series?

(1) 236,192

(2) 708,584

(3) 2,125,760

(4) 6,377,288

Use this space for computations.

3 Perry invested in property that cost him \$1500. Five years later it was worth \$3000, and 10 years from his original purchase, it was worth \$6000. Assuming the growth rate remains the same, which type of function could he create to find the value of his investment 30 years from his original purchase?

(1) exponential function
(2) linear function
(3) quadratic function
(4) trigonometric function

4 If $(a^3 + 27) = (a + 3)(a^2 + ma + 9)$, then m equals

(1) -9
(2) -3
(3) 3
(4) 6

Use this space for computations.

5 If $\cos\theta = -\frac{3}{4}$ and θ is in Quadrant III, then $\sin\theta$ is equivalent to

(1) $-\frac{\sqrt{7}}{4}$

(2) $\frac{\sqrt{7}}{4}$

(3) $-\frac{5}{4}$

(4) $\frac{5}{4}$

6 A veterinary pharmaceutical company plans to test a new drug to treat a common intestinal infection among puppies. The puppies are randomly assigned to two equal groups. Half of the puppies will receive the drug, and the other half will receive a placebo. The veterinarians monitor the puppies.

This is an example of which study method?

(1) census

(2) observational study

(3) survey

(4) controlled experiment

Use this space for computations.

7 The expression $2 - \frac{x-1}{x+2}$ is equivalent to

(1) $1 - \frac{3}{x+2}$

(2) $1 + \frac{3}{x+2}$

(3) $1 - \frac{1}{x+2}$

(4) $1 + \frac{1}{x+2}$

8 Which description could represent the graph of $f(x) = 4x^2(x + a) - x - a$, if a is an integer?

(1) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.

(2) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.

(3) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to -\infty$, and the graph has 4 x-intercepts.

(4) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 4 x-intercepts.

Use this space for computations.

9 After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

How can this sequence best be modeled recursively?

(1) $m_1 = 2000$
$m_n = m_{n-1} - 320$

(2) $m_n = 2000(0.84)^{n-1}$

(3) $m_1 = 2000$
$m_n = (0.84)m_{n-1}$

(4) $m_n = 2000(0.84)^{n+1}$

10 The expression $\frac{9x^2 - 2}{3x + 1}$ is equivalent to

(1) $3x - 1 - \frac{1}{3x + 1}$

(2) $3x - 1 + \frac{1}{3x + 1}$

(3) $3x + 1 - \frac{1}{3x + 1}$

(4) $3x + 1 + \frac{1}{3x + 1}$

Use this space for computations.

11 If $f(x)$ is an even function, which function must also be even?

(1) $f(x - 2)$

(2) $f(x) + 3$

(3) $f(x + 1)$

(4) $f(x + 1) + 3$

12 The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

(1) amplitude

(2) horizontal shift

(3) period

(4) midline

Use this space for computations.

13 Consider the probability statements regarding events A and B below.

$P(A \text{ or } B) = 0.3$;
$P(A \text{ and } B) = 0.2$, and
$P(A \mid B) = 0.8$

What is $P(B)$?

(1) 0.1
(2) 0.25
(3) 0.375
(4) 0.667

14 Given $y > 0$, the expression $\sqrt{3x^2y} \cdot \sqrt[3]{27x^3y^2}$ is equivalent to

(1) $81x^5y^3$
(2) $3^{1.5}x^2y$
(3) $3^{\frac{5}{2}}x^2y^{\frac{5}{3}}$
(4) $3^{\frac{3}{2}}x^2y^{\frac{7}{6}}$

Use this space for computations.

15 What is the solution set of the equation $\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}$?

(1) {0,2}

(2) {0}

(3) {2}

(4) { }

16 What are the solution(s) to the system of equations shown below?

$$x^2 + y^2 = 5$$
$$y = 2x$$

(1) $x = 1$ and $x = -1$

(2) $x = 1$

(3) (1, 2) and (−1, −2)

(4) (1, 2), only

17 If \$5000 is put into a savings account that pays 3.5% interest compounded monthly, how much money, to the *nearest ten cents*, would be in that account after 6 years, assuming no money was added or withdrawn?

(1) \$5177.80

(2) \$5941.30

(3) \$6146.30

(4) \$6166.50

Use this space for computations.

18 The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?

(1) 7

(2) 10

(3) 11

(4) 18

Use this space for computations.

19 The mean intelligence quotient (IQ) score is 100, with a standard deviation of 15, and the scores are normally distributed. Given this information, the approximate percentage of the population with an IQ greater than 130 is closest to

(1) 2%
(2) 31%
(3) 48%
(4) 95%

20 After examining the functions $f(x) = \ln(x + 2)$ and $g(x) = e^{x-1}$ over the interval $(-2,3]$, Lexi determined that the correct number of solutions to the equation $f(x) = g(x)$ is

(1) 1
(2) 2
(3) 3
(4) 0

21 Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ± 1 and 2. What is the value of b, if $a = 1$?

(1) 1
(2) 2
(3) -1
(4) -2

Use this space for computations.

22 The equation $t = \frac{1}{0.0105}\ln\left(\frac{A}{5000}\right)$ relates time, t, in years, to the amount of money, A, earned by a \$5000 investment. Which statement accurately describes the relationship between the average rates of change of t on the intervals [6000, 8000] and [9000, 12,000]?

(1) A comparison cannot be made because the intervals are different sizes.

(2) The average rate of change is equal for both intervals.

(3) The average rate of change is larger for the interval [6000, 8000].

(4) The average rate of change is larger for the interval [9000, 12,000].

23 What is the inverse of $f(x) = \frac{x}{x+2}$, where $x \neq -2$?

(1) $f^{-1}(x) = \frac{2x}{x-1}$

(2) $f^{-1}(x) = \frac{-2x}{x-1}$

(3) $f^{-1}(x) = \frac{x}{x-2}$

(4) $f^{-1}(x) = \frac{-x}{x-2}$

Use this space for computations.

24 A study of black bears in the Adirondacks reveals that their population can be represented by the function $P(t) = 3500(1.025)^t$, where t is the number of years since the study began. Which function is correctly rewritten to reveal the monthly growth rate of the black bear population?

(1) $P(t) = 3500(1.00206)^{12t}$

(2) $P(t) = 3500(1.00206)^{\frac{t}{12}}$

(3) $P(t) = 3500(1.34489)^{12t}$

(4) $P(t) = 3500(1.34489)^{\frac{t}{12}}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 At Andrew Jackson High School, students are only allowed to enroll in AP U.S. History if they have already taken AP World History or AP European History. Out of 825 incoming seniors, 165 took AP World History, 66 took AP European History, and 33 took both. Given this information, determine the probability a randomly selected incoming senior is allowed to enroll in AP U.S. History.

Work space for question 25 is continued on the next page.

Question 25 continued

26 Explain what a rational exponent, such as $\frac{5}{2}$ means. Use this explanation to evaluate $9^{\frac{5}{2}}$.

27 Write $-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2$ in simplest $a+bi$ form.

28 A person's lung capacity can be modeled by the function $C(t) = 250\sin\left(\frac{2\pi}{5}t\right) + 2450$, where $C(t)$ represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

Work space for question 28 is continued on the next page.

Question 28 continued

29 Determine for which polynomial(s) $(x + 2)$ is a factor. Explain your answer.

$$P(x) = x^4 - 3x^3 - 16x - 12$$
$$Q(x) = x^3 - 3x^2 - 16x - 12$$

Work space for question 29 is continued on the next page.

Question 29 continued

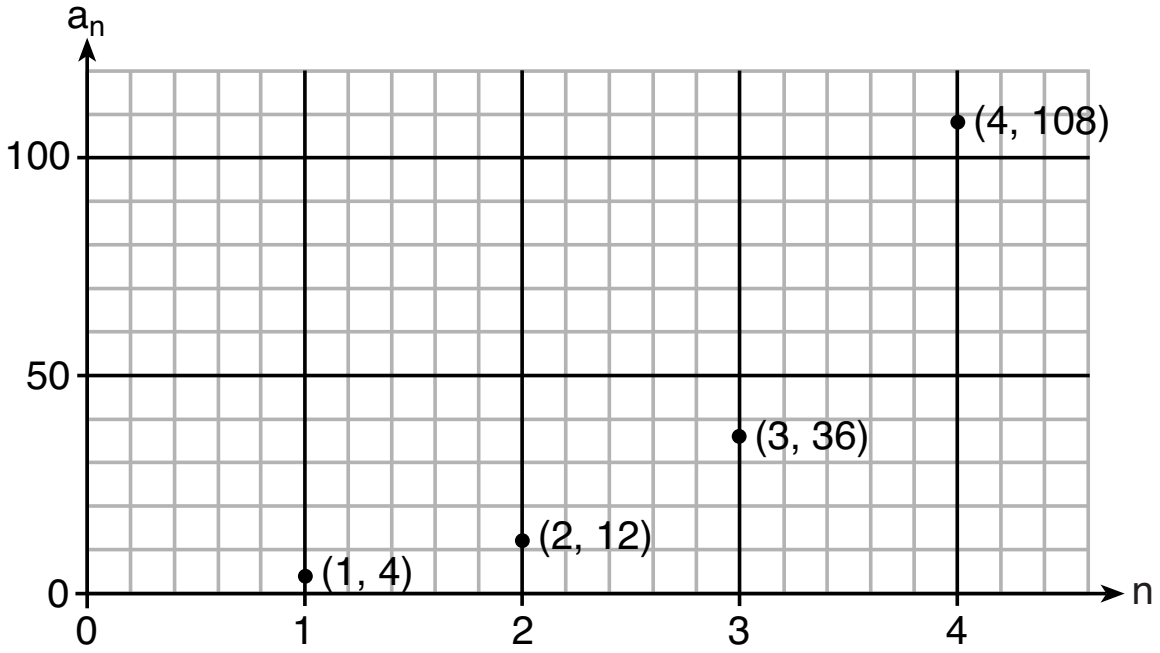
30 On July 21, 2016, the water level in Puget Sound, WA reached a high of 10.1 ft at 6 a.m. and a low of −2 ft at 12:30 p.m. Across the country in Long Island, NY, Shinnecock Bay's water level reached a high of 2.5 ft at 10:42 p.m. and a low of −0.1 ft at 5:31 a.m.

The water levels of both locations are affected by the tides and can be modeled by sinusoidal functions. Determine the difference in amplitudes, in feet, for these two locations.

Work space for question 30 is continued on the next page.

Question 30 continued

31 Write a recursive formula, a_n, to describe the sequence graphed below.



Work space for question 31 is continued on the next page.

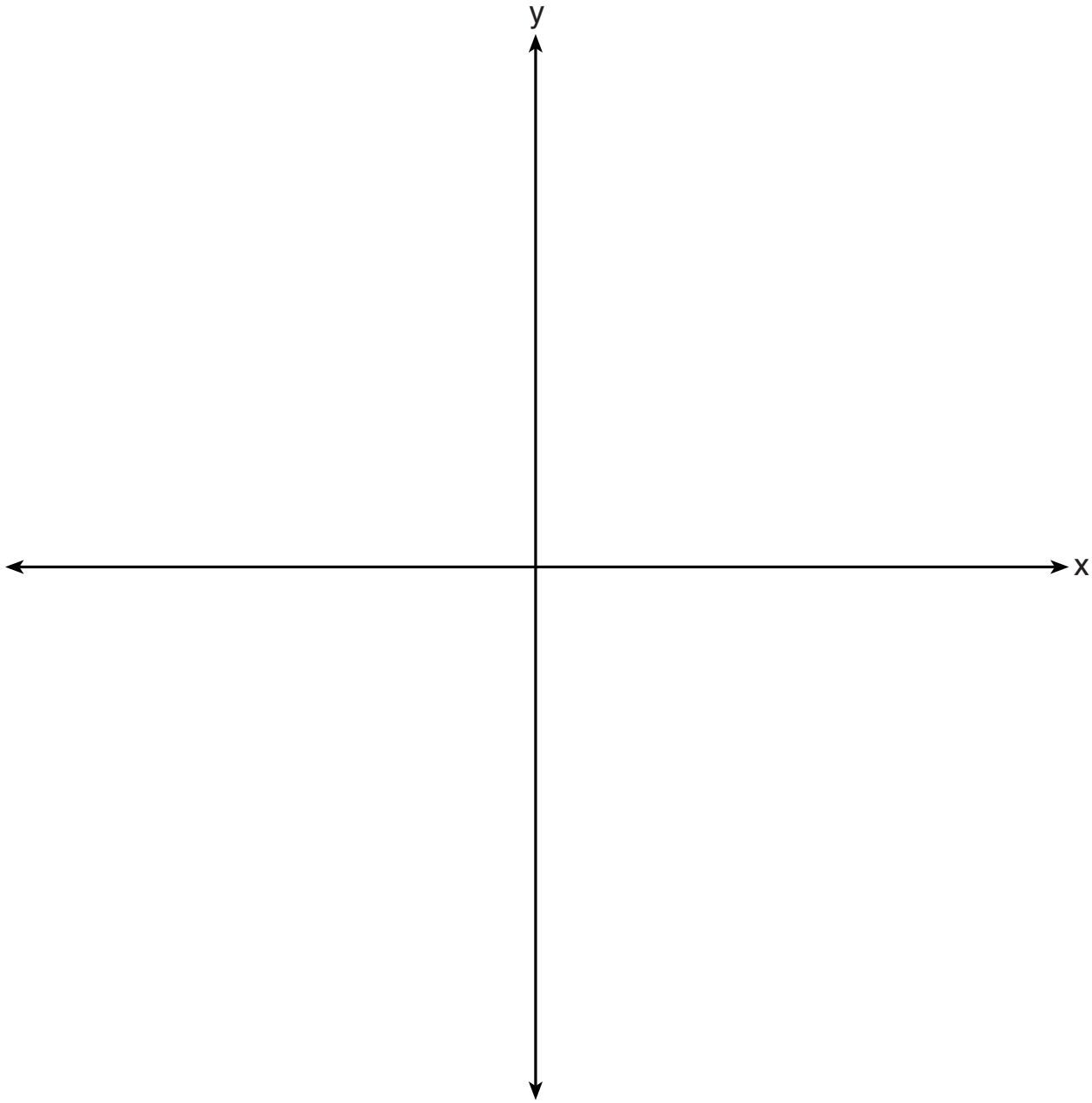
Question 31 continued

32 Sketch the graphs of $r(x) = \frac{1}{x}$ and $a(x) = |x| - 3$ on the set of axes on the next page.

Determine, to the *nearest tenth*, the positive solution of $r(x) = a(x)$.

The set of axes for question 32 is on the next page.

Question 32 continued



Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 A population of 950 bacteria grows continuously at a rate of 4.75% per day.

Write an exponential function, $N(t)$, that represents the bacterial population after t days and explain the reason for your choice of base.

Question 33 is continued on the next page.

Question 33 continued

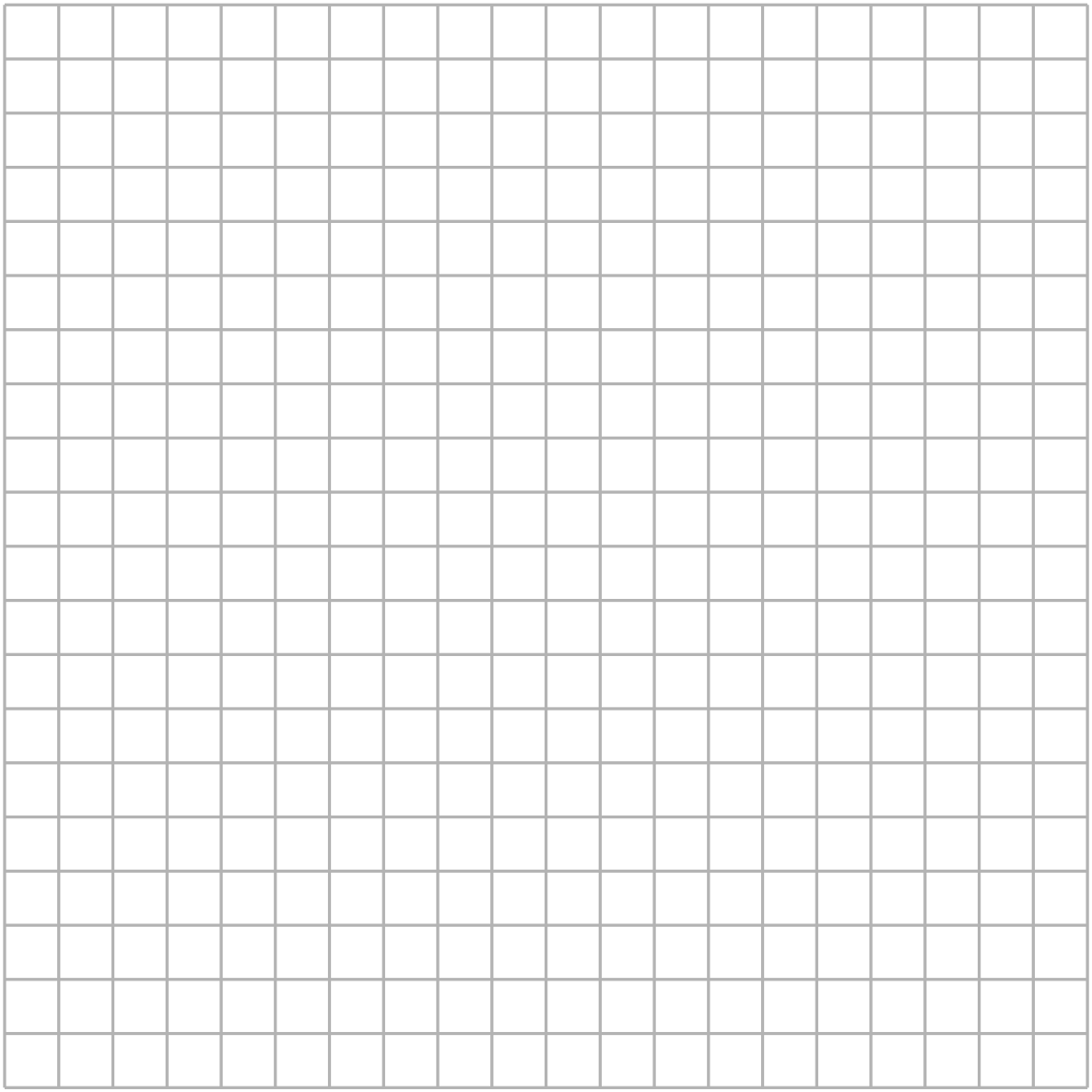
Determine the bacterial population after 36 hours, to the *nearest bacterium*.

34 Write an equation for a sine function with an amplitude of 2 and a period of $\frac{\pi}{2}$.

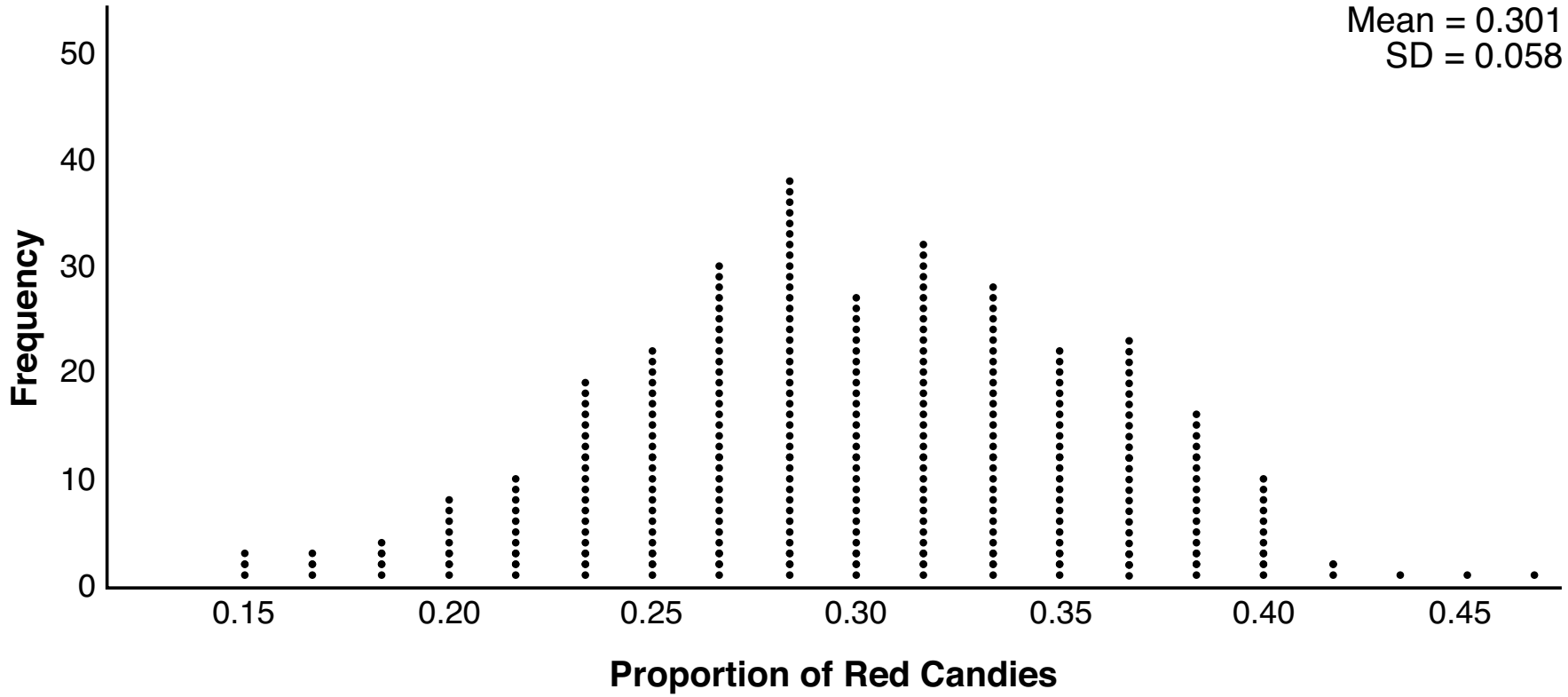
Question 34 is continued on the next page.

Question 34 continued

On the grid below, sketch the graph of the equation in the interval 0 to 2π.



35 Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



Question 35 is continued on the next page.

Question 35 continued

Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within.

Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

36 a) Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.

$$x^2 - 2x + 7 = 4x - 10$$

Question 36 is continued on the next page.

Question 36 continued

b) Consider the system of equations below.

$$y = x^2 - 2x + 7$$
$$y = 4x - 10$$

The graph of this system confirms the solution from part *a* is imaginary. Explain why.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation $B = 1.69\sqrt{s + 4.45} - 3.49$, where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

Question 37 is continued on the next page.

Beaufort Wind Scale

Beaufort Number	Force of Wind
0	Calm
1	Light air
2	Light breeze
3	Gentle breeze
4	Moderate breeze
5	Fresh breeze
6	Steady breeze
7	Moderate gale
8	Fresh gale
9	Strong gale
10	Whole gale
11	Storm
12	Hurricane

Question 37 is continued on the next page.

Question 37 continued

Using the table on page 39, classify the force of wind at a speed of 30 mph. Justify your answer.

In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a B value of exactly 15. Algebraically determine the value of s, to the *nearest mph*.

Question 37 is continued on the next page.

Question 37 continued

Any B values that round to 10 receive a Beaufort number of 10. Using technology, find an approximate range of wind speeds, to the *nearest mph*, associated with a Beaufort number of 10.

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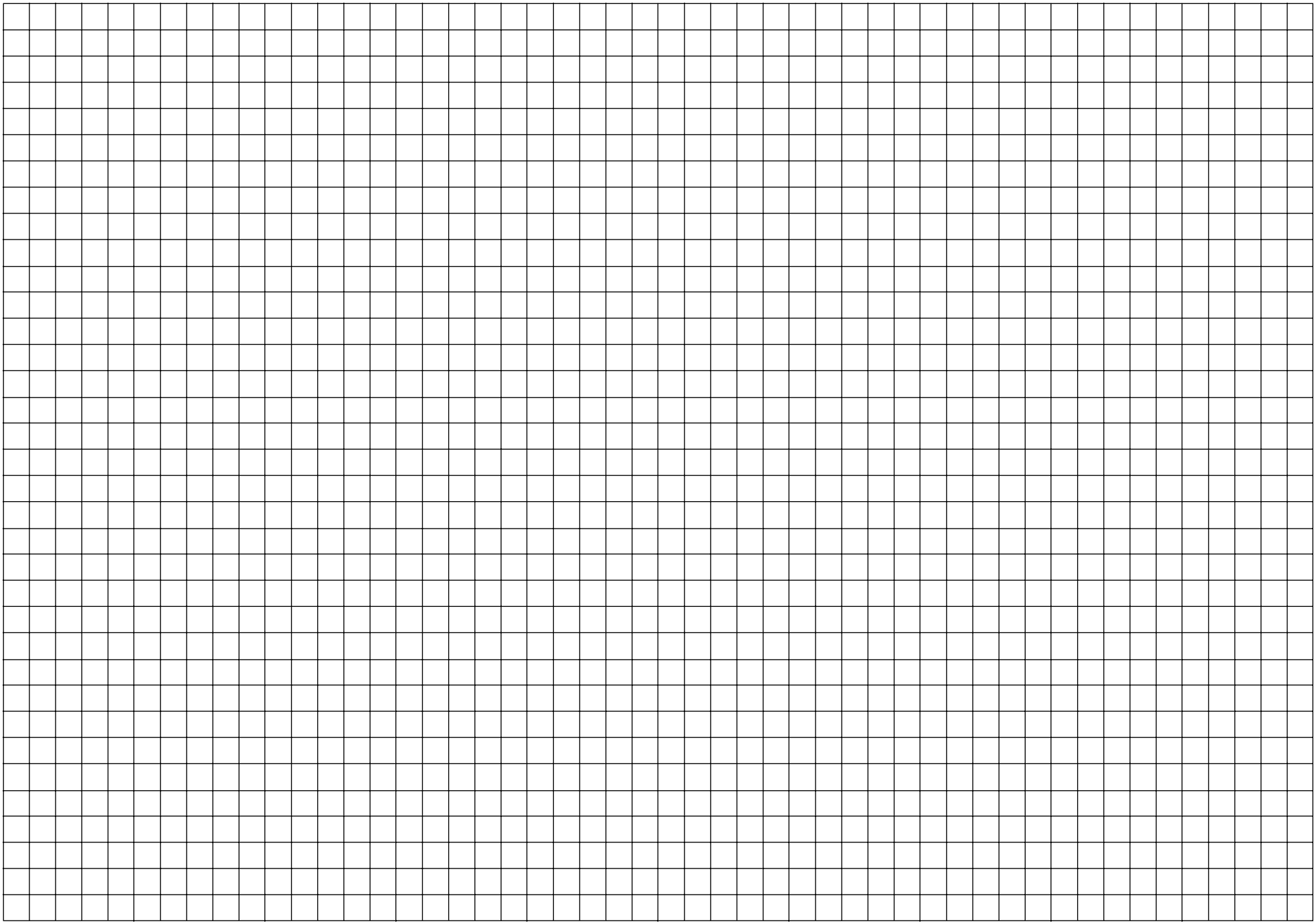

High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$