The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, January 26, 2012 — 9:15 a.m. to 12:15 p.m., only

Student Name: _________________________________________________________

School Name: _______________________________________________________________

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [56]

1. Line \( n \) intersects lines \( l \) and \( m \), forming the angles shown in the diagram below.

   \[
   \begin{align*}
   \angle n & = (6x + 42)^\circ \\
   \angle m & = (18x - 12)^\circ
   \end{align*}
   \]

   Which value of \( x \) would prove \( l \parallel m \)?
   (1) 2.5 (3) 6.25
   (2) 4.5 (4) 8.75

2. In a given triangle, the point of intersection of the three medians is the same as the point of intersection of the three altitudes. Which classification of the triangle is correct?
   (1) scalene triangle (3) equilateral triangle
   (2) isosceles triangle (4) right isosceles triangle

3. A circle has the equation \((x - 2)^2 + (y + 3)^2 = 36\). What are the coordinates of its center and the length of its radius?
   (1) \((-2,3)\) and 6 (3) \((-2,3)\) and 36
   (2) \((2,-3)\) and 6 (4) \((2,-3)\) and 36
4 In the diagram below, \( MATH \) is a rhombus with diagonals \( AH \) and \( MT \).

If \( \angle HAM = 12 \), what is \( \angle AMT \)?

(1) 12 \hspace{1cm} (3) 84
(2) 78 \hspace{1cm} (4) 156

5 A line segment has endpoints (4,7) and (1,11). What is the length of the segment?

(1) 5 \hspace{1cm} (3) 16
(2) 7 \hspace{1cm} (4) 25

6 In \( \triangle FGH \), \( \angle F = 42 \) and an exterior angle at vertex \( H \) has a measure of 104. What is \( \angle G \)?

(1) 34 \hspace{1cm} (3) 76
(2) 62 \hspace{1cm} (4) 146
7 Which diagram represents a correct construction of equilateral \( \triangle ABC \), given side \( AB \)?

(1) \( \triangle ABC \)

(2) \( \triangle ABC \)

(3) \( \triangle ABC \)

(4) \( \triangle ABC \)

8 In the diagram below, \( \triangle ABC \) is circumscribed about circle \( O \) and the sides of \( \triangle ABC \) are tangent to the circle at points \( D, E, \) and \( F \).

If \( AB = 20, AE = 12, \) and \( CF = 15 \), what is the length of \( AC \)?

(1) 8  
(2) 15  
(3) 23  
(4) 27
9 In $\triangle ABC$ and $\triangle DEF$, $\frac{AC}{DF} = \frac{CB}{FE}$. Which additional information would prove $\triangle ABC \sim \triangle DEF$?

(1) $AC = DF$  
(2) $CB = FE$  
(3) $\angle ACB \equiv \angle DFE$  
(4) $\angle BAC \equiv \angle EDF$

10 The angles of triangle $ABC$ are in the ratio of 8:3:4. What is the measure of the smallest angle?

(1) 12°  
(2) 24°  
(3) 36°  
(4) 72°

11 When a quadrilateral is reflected over the line $y = x$, which geometric relationship is not preserved?

(1) congruence  
(2) orientation  
(3) parallelism  
(4) perpendicularity

12 Which equation represents circle $O$ with center $(2, -8)$ and radius 9?

(1) $(x + 2)^2 + (y - 8)^2 = 9$  
(2) $(x - 2)^2 + (y + 8)^2 = 9$  
(3) $(x + 2)^2 + (y - 8)^2 = 81$  
(4) $(x - 2)^2 + (y + 8)^2 = 81$
13 Which statement is the negation of “Two is a prime number” and what is the truth value of the negation?

(1) Two is not a prime number; false
(2) Two is not a prime number; true
(3) A prime number is two; false
(4) A prime number is two; true

14 In the diagram below of circle $O$, chords $AB$ and $CD$ intersect at $E$.

If $m\angle AEC = 34$ and $m\overset{⏜}{AC} = 50$, what is $m\overset{⏜}{DB}$?

(1) 16  (2) 18  (3) 68  (4) 118

15 The volume of a rectangular prism is 144 cubic inches. The height of the prism is 8 inches. Which measurements, in inches, could be the dimensions of the base?

(1) 3.3 by 5.5  (2) 2.5 by 7.2  (3) 12 by 8  (4) 9 by 9
16 The diagram below shows a pair of congruent triangles, with \( \angle ADB \cong \angle CDB \) and \( \angle ABD \cong \angle CBD \).

Which statement must be true?

(1) \( \angle ADB \cong \angle CBD \)  
(2) \( \angle ABC \cong \angle ADC \)  
(3) \( \overline{AB} \cong \overline{CD} \)  
(4) \( \overline{AD} \cong \overline{CD} \)

17 What is an equation of the line that is perpendicular to the line whose equation is \( y = \frac{3}{5}x - 2 \) and that passes through the point \((3, -6)\)?

(1) \( y = \frac{5}{3}x - 11 \)  
(2) \( y = -\frac{5}{3}x + 11 \)  
(3) \( y = -\frac{5}{3}x - 1 \)  
(4) \( y = \frac{5}{3}x + 1 \)

18 Point \( A \) lies in plane \( B \). How many lines can be drawn perpendicular to plane \( B \) through point \( A \)?

(1) one  
(2) two  
(3) zero  
(4) infinite
19 In the diagram below of isosceles trapezoid $ABCD$, $AB = CD = 25$, $AD = 26$, and $BC = 12$.

What is the length of an altitude of the trapezoid?

(1) 7  
(2) 14  
(3) 19  
(4) 24

20 What is an equation of circle $O$ shown in the graph below?

(1) $(x + 2)^2 + (y - 2)^2 = 9$  
(2) $(x + 2)^2 + (y - 2)^2 = 3$  
(3) $(x - 2)^2 + (y + 2)^2 = 9$  
(4) $(x - 2)^2 + (y + 2)^2 = 3$
21 The diagram below represents a rectangular solid.

Which statement must be true?
(1) \(EH\) and \(BC\) are coplanar.
(2) \(FG\) and \(AB\) are coplanar.
(3) \(EH\) and \(AD\) are skew.
(4) \(FG\) and \(CG\) are skew.

22 In \(\triangle RST\), \(m\angle R = 58\) and \(m\angle S = 73\). Which inequality is true?
(1) \(RT < TS < RS\)  (3) \(RT < RS < TS\)
(2) \(RS < RT < TS\)  (4) \(RS < TS < RT\)

23 The number of degrees in the sum of the interior angles of a pentagon is
(1) 72   (3) 540
(2) 360   (4) 720

24 What is the equation of a line passing through \((2, -1)\) and parallel to the line represented by the equation \(y = 2x + 1\)?
(1) \(y = -\frac{1}{2}x\)  (3) \(y = 2x - 5\)
(2) \(y = -\frac{1}{2}x + 1\)  (4) \(y = 2x - 1\)

Use this space for computations.
25 The coordinates of the endpoints of $\overline{AB}$ are $A(0,0)$ and $B(0,6)$. The equation of the perpendicular bisector of $\overline{AB}$ is

\begin{align*}
(1) & \quad x = 0 \\
(2) & \quad x = 3 \\
(3) & \quad y = 0 \\
(4) & \quad y = 3
\end{align*}

26 In the diagram below, point $P$ is the centroid of $\triangle ABC$.

If $PM = 2x + 5$ and $BP = 7x + 4$, what is the length of $\overline{PM}$?

\begin{align*}
(1) & \quad 9 \\
(2) & \quad 2 \\
(3) & \quad 18 \\
(4) & \quad 27
\end{align*}

27 In $\triangle PQR$, $\angle PRQ$ is a right angle and $\overline{RT}$ is drawn perpendicular to hypotenuse $\overline{PQ}$. If $PT = x$, $RT = 6$, and $TQ = 4x$, what is the length of $\overline{PQ}$?

\begin{align*}
(1) & \quad 9 \\
(2) & \quad 12 \\
(3) & \quad 3 \\
(4) & \quad 15
\end{align*}

28 In $\triangle ABC$, $AB = 5$ feet and $BC = 3$ feet. Which inequality represents all possible values for the length of $\overline{AC}$, in feet?

\begin{align*}
(1) & \quad 2 \leq AC \leq 8 \\
(2) & \quad 2 < AC < 8 \\
(3) & \quad 3 \leq AC \leq 7 \\
(4) & \quad 3 < AC < 7
\end{align*}
29 In the diagram below, two parallel lines intersect circle $O$ at points $A$, $B$, $C$, and $D$, with $m\overarc{AB} = x + 20$ and $m\overarc{DC} = 2x - 20$.

Find $m\overarc{AB}$. 

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]
30 In the diagram below, point $M$ is located on $\overline{AB}$.

Sketch the locus of points that are 1 unit from $\overline{AB}$ and the locus of points 2 units from point $M$.

Label with an $\textbf{X}$ all points that satisfy both conditions.
31 Determine whether the two lines represented by the equations \( y = 2x + 3 \) and \( 2y + x = 6 \) are parallel, perpendicular, or neither.

Justify your response.
The coordinates of the vertices of $\triangle RST$ are $R(-2, 3)$, $S(4, 4)$, and $T(2, -2)$. Triangle $R'S'T'$ is the image of $\triangle RST$ after a rotation of $90^\circ$ about the origin.

State the coordinates of the vertices of $\triangle R'S'T'$.

[The use of the set of axes below is optional.]
33 On the diagram below, use a compass and straightedge to construct the bisector of $\angle XYZ$. [Leave all construction marks.]
34 In the diagram below of circle $O$, diameter $AB$ is perpendicular to chord $CD$ at $E$.

If $AO = 10$ and $BE = 4$, find the length of $CE$. 

![Diagram of a circle with diameter AB perpendicular to chord CD at E. Points A, O, B, C, D, and E are labeled.]
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Triangle $ABC$ has coordinates $A(2, -2)$, $B(2,1)$, and $C(4, -2)$. Triangle $A'B'C'$ is the image of $\triangle ABC$ under $T_{5,-2}$.

On the set of axes below, graph and label $\triangle ABC$ and its image, $\triangle A'B'C'$.

Determine the relationship between the area of $\triangle ABC$ and the area of $\triangle A'B'C'$.

Justify your response.
A paint can is in the shape of a right circular cylinder. The volume of the paint can is $600\pi$ cubic inches and its altitude is 12 inches.

Find the radius, in inches, of the base of the paint can. Express the answer in simplest radical form.

Find, to the nearest tenth of a square inch, the lateral area of the paint can.
Triangle \( HKL \) has vertices \( H(-7,2), \ K(3,-4), \) and \( L(5,4) \). The midpoint of \( \overline{HL} \) is \( M \) and the midpoint of \( \overline{LK} \) is \( N \).

Determine and state the coordinates of points \( M \) and \( N \).

Justify the statement: \( MN \) is parallel to \( HK \).

[The use of the set of axes below is optional.]
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

38 In the diagram below of quadrilateral $ABCD$, $\overline{AD} \equiv \overline{BC}$ and $\angle DAE \equiv \angle BCE$.

Line segments $AC$, $DB$, and $FG$ intersect at $E$.

Prove: $\triangle AEF \equiv \triangle CEG$
# Reference Sheet

<table>
<thead>
<tr>
<th>Volume</th>
<th>Cylinder</th>
<th>$V = Bh$ where $B$ is the area of the base</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pyramid</td>
<td>$V = \frac{1}{3}Bh$ where $B$ is the area of the base</td>
</tr>
<tr>
<td></td>
<td>Right Circular Cone</td>
<td>$V = \frac{1}{3}Bh$ where $B$ is the area of the base</td>
</tr>
<tr>
<td></td>
<td>Sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lateral Area ($L$)</th>
<th>Right Circular Cylinder</th>
<th>$L = 2\pi rh$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Right Circular Cone</td>
<td>$L = \pi rl$ where $l$ is the slant height</td>
</tr>
</tbody>
</table>

| Surface Area       | Sphere                             | $SA = 4\pi r^2$ |
Scrap Graph Paper — This sheet will *not* be scored.
Scrap Graph Paper — This sheet will not be scored.
The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION  

GEOMETRY  

Thursday, January 26, 2012 — 9:15 a.m. to 12:15 p.m., only  

ANSWER SHEET

Student ........................................... Sex: □ Male □ Female Grade .....  
Teacher ................................. School .................................  

Your answers to Part I should be recorded on this answer sheet.  

Part I  
Answer all 28 questions in this part.  

1 ............... 8 ............... 15 ............... 22 ...............  
2 ............... 9 ............... 16 ............... 23 ...............  
3 ............... 10 ............... 17 ............... 24 ...............  
4 ............... 11 ............... 18 ............... 25 ...............  
5 ............... 12 ............... 19 ............... 26 ...............  
6 ............... 13 ............... 20 ............... 27 ...............  
7 ............... 14 ............... 21 ............... 28 ...............  

Your answers for Parts II, III, and IV should be written in the test booklet.  

The declaration below must be signed when you have completed the examination.  

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.  

Signature  

Geometry – January ’12
<table>
<thead>
<tr>
<th>Question</th>
<th>Maximum Credit</th>
<th>Credits Earned</th>
<th>Rater's/Scorer's Initials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I 1–28</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part II 29</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part III 35</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part IV 38</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Total</td>
<td>86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Raw Score</th>
<th>Checked by</th>
<th>Scale Score (from conversion chart)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>