The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, January 30, 2014 — 9:15 a.m.

SAMPLE RESPONSE SET

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31 Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.

\[
\frac{\sqrt{84}}{2\sqrt{3}} = \frac{1}{2} \sqrt{\frac{28}{3}} = \frac{1}{2} \sqrt{4 \cdot \frac{7}{3}} = \frac{1}{2} \cdot 2 \sqrt{7} = \sqrt{7}
\]

**Score 2:** The student has a complete and correct response.
31 Express \( \frac{\sqrt{84}}{2\sqrt{3}} \) in simplest radical form.

\[
\begin{align*}
\frac{\sqrt{84}}{2\sqrt{3}} &= \frac{a\sqrt{21}}{2\sqrt{3}} \\
&= \frac{a\sqrt{21}}{2\sqrt{3}} \\
&= \frac{2a\sqrt{21}}{2\sqrt{3}} \\
&= \frac{2a\sqrt{21}}{2\sqrt{3}} \\
&= \sqrt{7}.
\end{align*}
\]

**Score 2:** The student has a complete and correct response.
31 Express \( \frac{\sqrt{84}}{2\sqrt{3}} \) in simplest radical form.

\[
\frac{\sqrt{84}}{2\sqrt{3}} = \frac{2\sqrt{21}}{2\sqrt{3}} = \frac{2\sqrt{7}}{2}\sqrt{3} = \sqrt{7}\sqrt{3} = 4.17
\]

**Score 1:** The student made one conceptual error by moving the 2 from the denominator to the numerator.
Question 31

31 Express \( \frac{\sqrt{84}}{2\sqrt{3}} \) in simplest radical form.

\[
\frac{\sqrt{28}}{2} = 2.645751311
\]

Score 1: The student showed appropriate work, but did not express the answer in simplest radical form.
31 Express $\frac{\sqrt{84}}{2\sqrt{3}}$ in simplest radical form.

Score 0: The student expressed the answer as a decimal and showed no work.
The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

<table>
<thead>
<tr>
<th>Text-Use Interval (minutes)</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>41–50</td>
<td>2</td>
</tr>
<tr>
<td>41–60</td>
<td>5</td>
</tr>
<tr>
<td>41–70</td>
<td>10</td>
</tr>
<tr>
<td>41–80</td>
<td>19</td>
</tr>
<tr>
<td>41–90</td>
<td>31</td>
</tr>
</tbody>
</table>

Determine which 10-minute interval contains the median. Justify your choice.

The Answer: 71–80, because when you list all of them out using variables to substitute them, and you cross both sides out evenly until there is only one variable left, you can find letter D only surviving.

Score 2: The student has a complete and correct response.
The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

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</tbody>
</table>

Determine which 10-minute interval contains the median. Justify your choice.

71–80 contains median

because it contains 10–14 or the middle frequency 10–27 frequencies

Score 2: The student has a complete and correct response.
The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

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Determine which 10-minute interval contains the median. Justify your choice.

Score 1: The student made one conceptual error by stating 41-80.
The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

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Determine which 10-minute interval contains the median. Justify your choice.

41–80 minutes has the median. Half of 31 is between 15 and 16, and this interval contains the data that is 15th and 16th in the list.

Score 1: The student made one conceptual error by stating 41-80 instead of 71-80. The student made a computational error in calculating the frequency of the 61-70 interval, but that value is not relevant to the answer.
32 The cumulative frequency table below shows the number of minutes 31 students spent text messaging on a weekend.

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<tr>
<td>41–50</td>
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</tr>
<tr>
<td>41–60</td>
<td></td>
</tr>
<tr>
<td>41–70</td>
<td>10</td>
</tr>
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<td>19</td>
</tr>
<tr>
<td>41–90</td>
<td>31</td>
</tr>
</tbody>
</table>

Determine which 10-minute interval contains the median. Justify your choice.

Score 0: The student made two conceptual errors. The student gave the cumulative interval (41-70) and chose the median of the cumulative frequency instead of the median student.
Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Find the balance in the account, to the nearest cent, at the end of 5 years.

\[ a = 1000 \left(1 + \frac{0.03}{1}\right)^5 \]

\[ \$1159.27 \]

**Score 2:** The student has a complete and correct response.
Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Find the balance in the account, to the nearest cent, at the end of 5 years.

\[
1000 \times 0.03 = 30
\]
\[
30 \times 5 = 150
\]
\[
1150.00
\]

**Score 1:** The student made a conceptual error by using simple interest. The student found an appropriate answer.
33 Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Find the balance in the account, to the nearest cent, at the end of 5 years.

\[
\begin{align*}
1 \quad (1000)(0.03) &= 300 \\
2 \quad (1300)(0.03) &= 390 \\
3 \quad (1690)(0.03) &= 50.7 \\
4 \quad (2197)(0.03) &= 65.91 \\
5 \quad (2856.1)(0.03) &= 85.683 \\
\end{align*}
\]

\[3712.93\]

**Score 1:** The student made one conceptual error by using 30%, but found an appropriate answer.
Kirsten invested $1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Find the balance in the account, to the nearest cent, at the end of 5 years.

$1,000.00

\[ \text{3\%} \times 5 \text{ yrs} = 15 \]

$10 \% \text{ of } 1,000 = $10

$5 \% \text{ of } 1,000 = $5

$1.5 \% \text{ of } 1,000 = $15

$1,015 \text{ after five years}$

**Score 0:** The student showed a completely incorrect response.
34 Graph and label the functions $y = |x|$ and $y = |2x|$ on the set of axes below.

Explain how increasing the coefficient of $x$ affects the graph of $y = |x|$.

The increasing coefficient makes the function more narrow.

Score 3: The student has a complete and correct response.
34 Graph and label the functions $y = |x|$ and $y = |2x|$ on the set of axes below.

Explain how increasing the coefficient of $x$ affects the graph of $y = |x|$.

*Increasing the coefficient makes the result on the graph come out narrower.*

**Score 2:** The student graphed both equations correctly and provided a correct explanation, but did not label either graph.
Question 34

34 Graph and label the functions $y = |x|$ and $y = |2x|$ on the set of axes below.

Explain how increasing the coefficient of $x$ affects the graph of $y = |x|$.

*Score 1:* The student graphed both equations correctly, but neither graph was labeled. The student provided an insufficient explanation.
34 Graph and label the functions $y = |x|$ and $y = |2x|$ on the set of axes below.

Explain how increasing the coefficient of $x$ affects the graph of $y = |x|$.

Score 0: The student made one conceptual error in graphing lines instead of absolute value functions. The student appropriately labeled the graphs. The student provided no explanation.
35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

\[
\text{SA} = 6(5^2) = 150
\]
\[
\text{SA} = 6(5.2)^2 = 162.24
\]
\[
\frac{162.24 - 150}{162.24} = \frac{12.24}{162.24} = 0.07544 \rightarrow 0.075
\]

Score 3: The student has a complete and correct response.
Question 35

35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

Score 2: The student showed correct work to find the expression, but did not find the relative error.
Question 35

35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

\[
\begin{align*}
5 \cdot 5 \cdot 6 &= 150 \\
5.2 \cdot 5.2 \cdot 5.2 &= 162.24 \\
\frac{162.24 - 150}{162.24} &= 0.074
\end{align*}
\]

Score 2: The student found the correct surface areas, but inappropriately rounded 162.24 to 162 before calculating the relative error.
35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

\[
SA = 2(5.2)(5.2) + 2(5.2)(5.2) + 2(5.2)(5.2)
\]
\[
SA = 162.240
\]
\[
SA = 2(5)(5) + 2(5)(5) + 2(5)(5)
\]
\[
SA = 150.000
\]
\[
\frac{162.240}{150.000}
\]
\[
SA = 1.085
\]

**Score 1:** The student showed correct work to find 162.24 and 150, but found the relative error incorrectly.
Question 35

35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

$$5^3 = 125$$
$$5.2^3 = 140.608$$

Score 1: The student made a conceptual error by calculating volume instead of surface area, but gave an appropriate answer.
35 Terry estimated the length of the edge of a cube to be 5 cm. The actual length of the side is 5.2 cm. Find the relative error of the surface area of the cube, to the nearest thousandth.

Score 0: The student made two conceptual errors.
36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\tan 52^\circ = \frac{80}{x} \\
x \tan 52^\circ = 80 \\
x \approx 62.5 \text{ ft}
\]

Score 3: The student has a complete and correct response.
36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\tan 38^\circ = \frac{x}{80}
\]

\[
80 \tan 38^\circ = x
\]

\[
x = 62.5 \text{ ft}
\]

**Score 3:** The student has a complete and correct response.
36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\tan 38^\circ = \frac{80}{x}
\]

\[
(x)(1.3) = \frac{80}{x}(x)
\]

\[
1.3x = \frac{80}{1.3}
\]

\[
x = 61.5\text{ ft}
\]

**Score 2:** The student made one rounding error by using 1.3 for \(\tan 52^\circ\) instead of 1.279941632. The rounding should be done at the final step, not in the first step.
Question 36

36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

Score 2: The student wrote the correct tangent ratio, but used radian mode instead of degree mode.
36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\tan 38^\circ = \frac{80}{x}
\]

\[
80 = \frac{\tan 38^\circ \times x}{\tan 38^\circ}
\]

\[
x = 102.4 \text{ ft}
\]

Score 1: The student showed appropriate work, but made one conceptual error by using an incorrect trigonometric equation.
Question 36

36 From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the nearest tenth of a foot.

\[
\frac{38}{80} = x
\]

Score 0: The student gave a completely incorrect response.
37 On the set of axes below, solve the following system of equations graphically for all values of $x$ and $y$. State the coordinates of all the solutions.

\[ y = x^2 + 4x - 5 \]
\[ y = 2x + 3 \]

Score 4: The student has a complete and correct response.
On the set of axes below, solve the following system of equations graphically for all values of $x$ and $y$. State the coordinates of all the solutions.

$y = x^2 + 4x - 5$

$y = 2x + 3$

Score 3: The student graphed both equations correctly, but stated only one point.
37 On the set of axes below, solve the following system of equations graphically for all values of $x$ and $y$. State the coordinates of all the solutions.

\[ y = x^2 + 4x - 5 \]
\[ y = 2x + 3 \]

Score 2: The student graphed both equations correctly, but stated neither point.
37 On the set of axes below, solve the following system of equations graphically for all values of $x$ and $y$. State the coordinates of all the solutions.

$$y = x^2 + 4x - 5$$

$$y = 2x + 3$$

**Score 2:** The student made a conceptual error by graphing a line instead of a parabola, but stated an appropriate solution.
37 On the set of axes below, solve the following system of equations graphically for all values of $x$ and $y$. State the coordinates of all the solutions.

$$y = x^2 + 4x - 5$$
$$y = 2x + 3$$

**Score 1:** The student graphed $y = 2x + 3$ correctly, but showed no further correct work.
37 On the set of axes below, solve the following system of equations graphically for all values of \( x \) and \( y \). State the coordinates of all the solutions.

\[
\begin{align*}
  y &= x^2 + 4x - 5 \\
  y &= 2x + 3
\end{align*}
\]

**Score 0:** The student showed completely incorrect work.
38 Solve algebraically for all values of $x$: \[
\frac{3}{x+5} = \frac{2x}{x^2-8}
\]

\[
3(x^2-8) = 2x(x+5)
\]

\[
3x^2-24 = 2x^2+10x
\]

\[
x^2-2x-24 = 10x
\]

\[
x^2-12x-24 = 0
\]

\[
(x-12)(x+2)
\]

$X=12, \quad x=-2$

Score 4: The student has a complete and correct response.
38 Solve algebraically for all values of $x$: \[ \frac{3}{x+5} = \frac{2x}{x^2-8} \]

\[
3(x^2-8) = 2x(x+5) \\
3x^2 - 24 = 2x^2 + 10x \\
-2x^2 \\
\frac{3x^2 - 24}{-2x^2} = \frac{2x^2 + 10x}{-2x^2} \\
\frac{x^2 - 24}{x^2} = \frac{10x}{-2x} \\
\frac{x^2 - 24}{x^2} = -\frac{10x}{-2x} \\
x^2 - 24 - 10x = 0 \\
(x-12)(x+2) \\
x = 12 \quad x = 2
\]

Score 3: The student made one computational error.
Question 38

38 Solve algebraically for all values of $x$: \[
\frac{3}{x+5} \leq \frac{2x}{x^2-8}
\]

\[
3(x^2-8) = 2x(x+5)
\]

\[
\begin{align*}
3x^2 - 24 & = 2x^2 + 10x \\
-2x^2 & = -2x^2
\end{align*}
\]

\[
x^2 - 24 = 10x
\]

\[
x^2 - 10x - 24 = 0
\]

\[
(x+6)(x-4) = 0
\]

\[
\begin{align*}
x+6 & = 0 \\
x & = -6
\end{align*}
\]

\[
\begin{align*}
x-4 & = 0 \\
x & = 4
\end{align*}
\]

Score 3: The student made one factoring error.
Question 38

38 Solve algebraically for all values of \( x \):

\[
\frac{3}{x+5} = \frac{2x}{x^2-8} \\
3x^2 - 24 = 2x^2 + 10x \\
9x - 24 = 4x + 10x \\
5x - 24 = 14x \\
-9x \\
\frac{-24}{5} = \frac{5x}{8} \\
\frac{-14.4}{8} = x
\]

Score 2: The student made one conceptual error (\( 3x^2 \rightarrow 9x \), \( 2x^2 \rightarrow 4x \)), but followed through and correctly solved \( 9x - 24 = 4x + 10x \) for an appropriate answer.
Question 38

38 Solve algebraically for all values of \( x \):

\[
\frac{3}{x+5} = \frac{2x}{x^2-8}
\]

Score 2: The student found both 12 and \(-2\), but used a method other than algebraic.
Question 38

38 Solve algebraically for all values of $x$: \( \frac{3}{x+5} = \frac{2x}{x^2-8} \)

\[
3(x^2-8) = 2x(x+5) \\
3x^2 - 24 = 2x^2 + 10x \\
+2x^2 \\
5x^2 - 24 = 10x \\
-10x \\
-10x \\
5x^2 - 34x = 0 \\
x(5x - 34) = 0 \\
x = 0 \\
5x = 34 \\
\frac{5x}{5} = \frac{34}{5} \\
x = 6.8
\]

**Score 1:** The student made one computational error (adds $2x^2$) and one conceptual error ($-24 + -10x = -34x$), but stated appropriate values.
38 Solve algebraically for all values of $x$: \[ \frac{3}{x+5} = \frac{2x}{x^2-8} \]

\[ 2x(x+5) - 3(x^2-8) \]

\[ 2x^2 + 10x - 3x^2 + 24 \]

\[ -x^2 + 14x - 24 \]

\[ -14x + 24 \]

\[ 8 + \sqrt{13} \]

\[ \sqrt{13} = \sqrt{13} \]

\[ x = \sqrt{13} \]

**Score 1:** The student made one conceptual error by using distribution incorrectly and one computational error by not writing $\pm \sqrt{13}$. 
38 Solve algebraically for all values of \( x \): \[
\frac{3}{x+5} = \frac{2x}{x^2-8}
\]

Score 0: The student cross-multiplied, but expressed the result as a quotient, not an equation. The student showed no further work.
38 Solve algebraically for all values of \( x \):

\[
\frac{3}{x + 5} = \frac{2x}{x^2 - 8}
\]

\[
x^2 + 5 = 3x^2 - 24
\]

\[
x^2 = 29
\]

\[
x = \pm \sqrt{29}
\]

**Score 0:** The student made two conceptual errors. The student did not distribute correctly and solved \( x^2 = 29 \) by dividing by 2.
39 Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug’s outfits that consist of a cap and a jacket that are different colors.

On Spirit Day, Doug wants to wear either green or white, his school’s colors. Find the number of his outfits from which he can choose.

Score 4: The student has a complete and correct response.
Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug's outfits that consist of a cap and a jacket that are different colors.

\[
\begin{array}{c|c|c|c|c}
\text{Hat Color} & \text{Jacket Color} \\
\hline
\text{tan} & \text{white} & \text{tan} & \text{blue} \\
\text{red} & \text{red} & \text{red} & \text{red} \\
\text{blue} & \text{blue} & \text{blue} & \text{blue} \\
\text{green} & \text{white} & \text{green} & \text{red} \\
\text{green} & \text{white} & \text{green} & \text{blue} \\
\text{green} & \text{white} & \text{green} & \text{blue} \\
\end{array}
\]

\[
\frac{10}{12}
\]

On Spirit Day, Doug wants to wear either green or white, his school's colors. Find the number of his outfits from which he can choose.

\[
\frac{6}{12}
\]

**Score 3:** The student showed a correct sample space, but \[\frac{10}{12}\] and \[\frac{6}{12}\] are given instead of 10 and 6.
39 Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug’s outfits that consist of a cap and a jacket that are different colors.

On Spirit Day, Doug wants to wear either green or white, his school’s colors. Find the number of his outfits from which he can choose.

Score 3: The student drew a correct tree diagram, but only 6 is stated correctly.
Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug’s outfits that consist of a cap and a jacket that are different colors.

On Spirit Day, Doug wants to wear either green or white, his school’s colors. Find the number of his outfits from which he can choose.

Score 2: The student drew a partially correct tree diagram and only 10 is stated correctly.
Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug’s outfits that consist of a cap and a jacket that are different colors.

On Spirit Day, Doug wants to wear either green or white, his school's colors. Find the number of his outfits from which he can choose.

Score 1: The student drew a partially correct tree diagram. The student showed no further correct work.
39 Doug has four baseball caps: one tan, one blue, one red, and one green. He also has three jackets: one blue, one red, and one white. Draw a tree diagram or list a sample space to show all possible outfits consisting of one baseball cap and one jacket.

Find the number of Doug’s outfits that consist of a cap and a jacket that are different colors.

On Spirit Day, Doug wants to wear either green or white, his school’s colors. Find the number of his outfits from which he can choose.

**Score 0:** The student drew an incorrect tree diagram, and gave one correct response based on an obviously incorrect procedure.