The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (COMMON CORE)

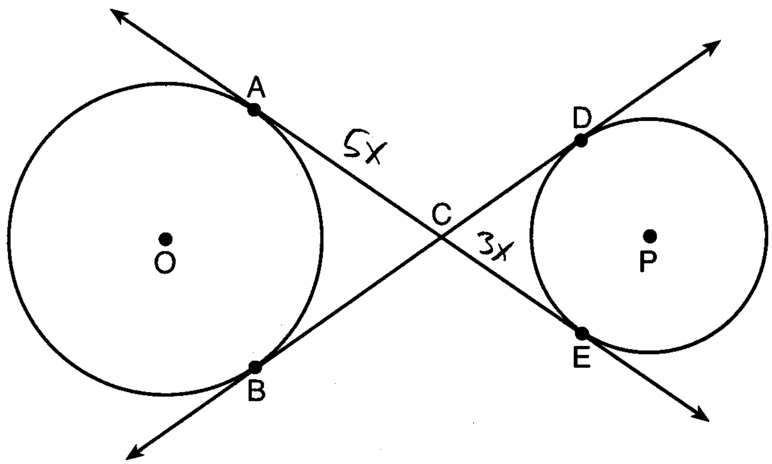
Wednesday, August 17, 2016 — 8:30 to 11:30 a.m.

MODEL RESPONSE SET

Table of Contents

Question 25

25 Lines AE and BD are tangent to circles O and P at A, E, B, and D, as shown in the diagram below. If AC:CE = 5:3, and BD = 56, determine and state the length of $\overline{CD}$.



$5x + 3x = 56$

$\frac{8x}{8} = \frac{56}{8}$

$x = 7$

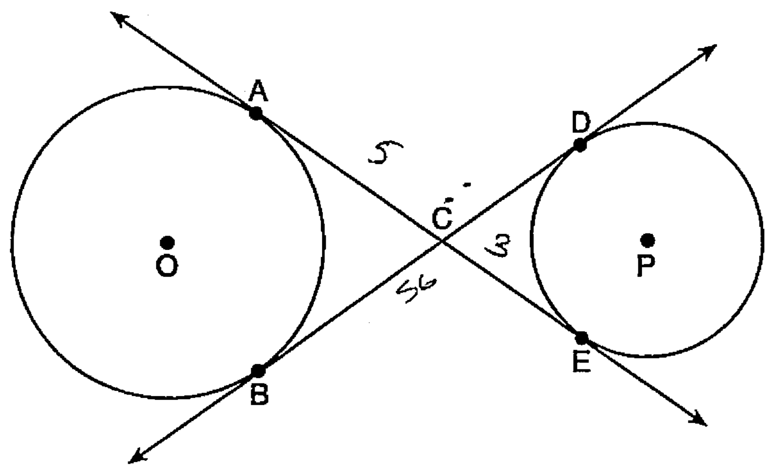
$CE = 3(7) = 21$

$CD = 21$

Score 2: The student had a complete and correct response.

Question 25

25 Lines AE and BD are tangent to circles O and P at A, E, B, and D, as shown in the diagram below. If $AC{:}CE = 5{:}3$, and $BD = 56$, determine and state the length of $\overline{CD}$.



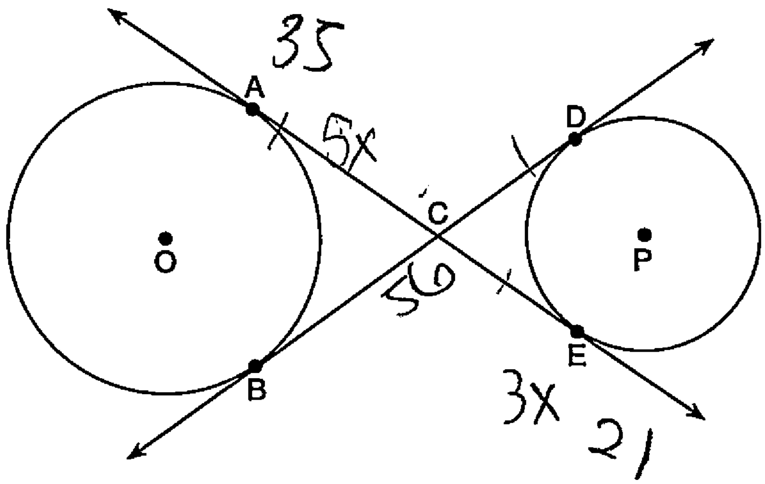
1 unit = 56 ÷ (5+3)
= 56 ÷ 8
= 7

; CD = 7(3)
= 21

Score 2: The student had a complete and correct response.

Question 25

25 Lines AE and BD are tangent to circles O and P at A, E, B, and D, as shown in the diagram below. If $AC:CE = 5:3$, and $BD = 56$, determine and state the length of $\overline{CD}$.



$5x + 3x = 56$

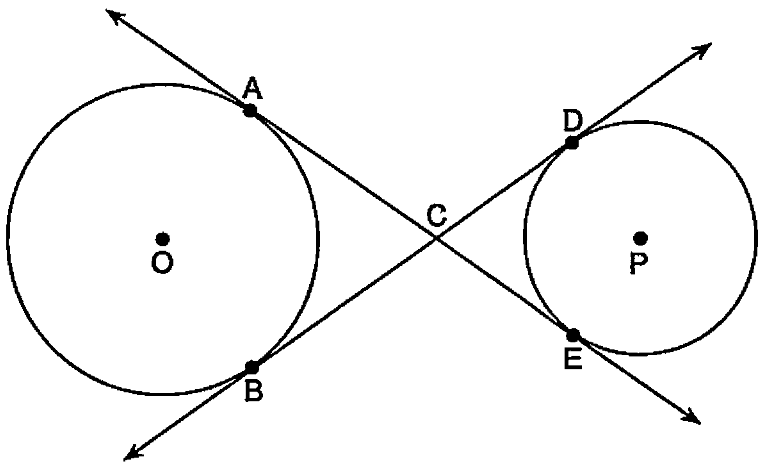
$\frac{\cancel{8}x}{\cancel{8}} = \frac{56}{8}$ $x = 7$

CD = 35

Score 1: The student substituted incorrectly and found the length of $\overline{CB}$.

Question 25

25 Lines *AE* and *BD* are tangent to circles *O* and *P* at *A*, *E*, *B*, and *D*, as shown in the diagram below. If $AC:CE = 5:3$, and $BD = 56$, determine and state the length of $\overline{CD}$.



$5x + 3x = 360$

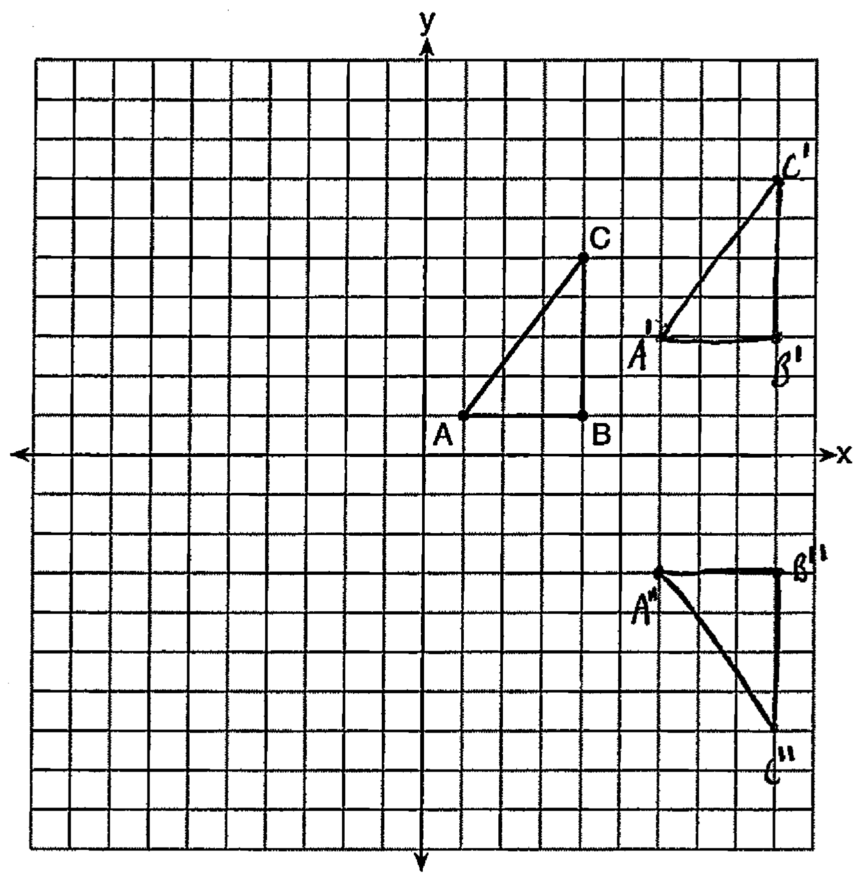
$\frac{\cancel{8}x}{\cancel{8}} = \frac{360}{8}$

$\boxed{x = 45}$

Score 0: The student did not show enough relevant correct work to receive any credit.

Question 26

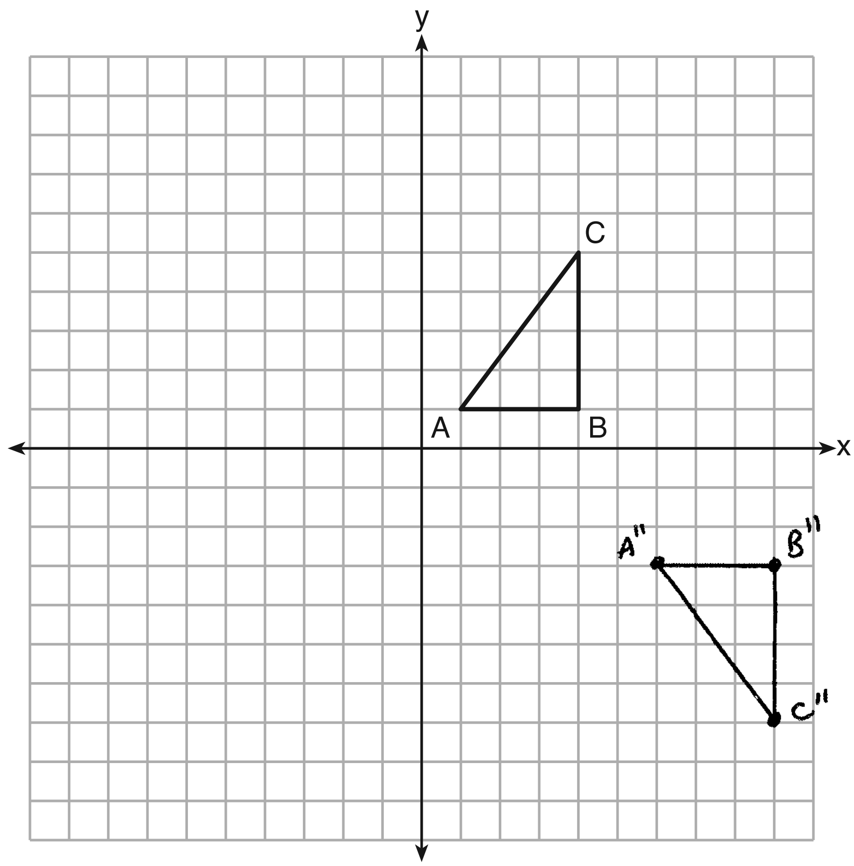
26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.



Score 2: The student had a complete and correct response.

Question 26

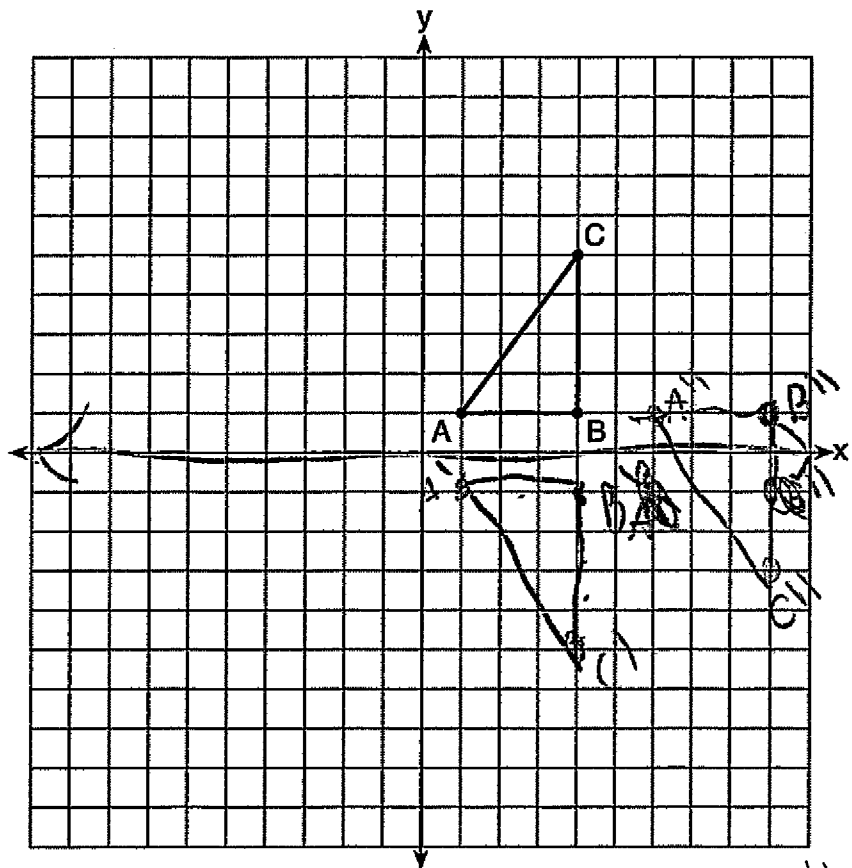
26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.



Score 2: The student had a complete and correct response.

Question 26

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.

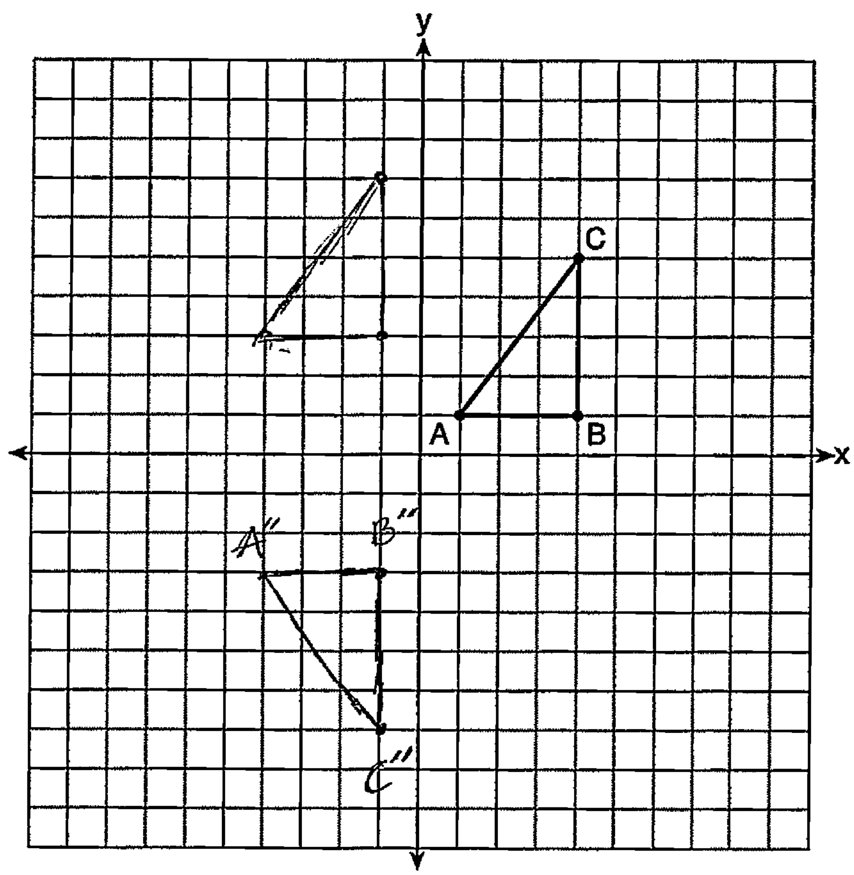


A''(6,1)
B''(9,1)
C''(9,-3)

Score 1: The student made an error by graphing the reflection and then the translation.

Question 26

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.

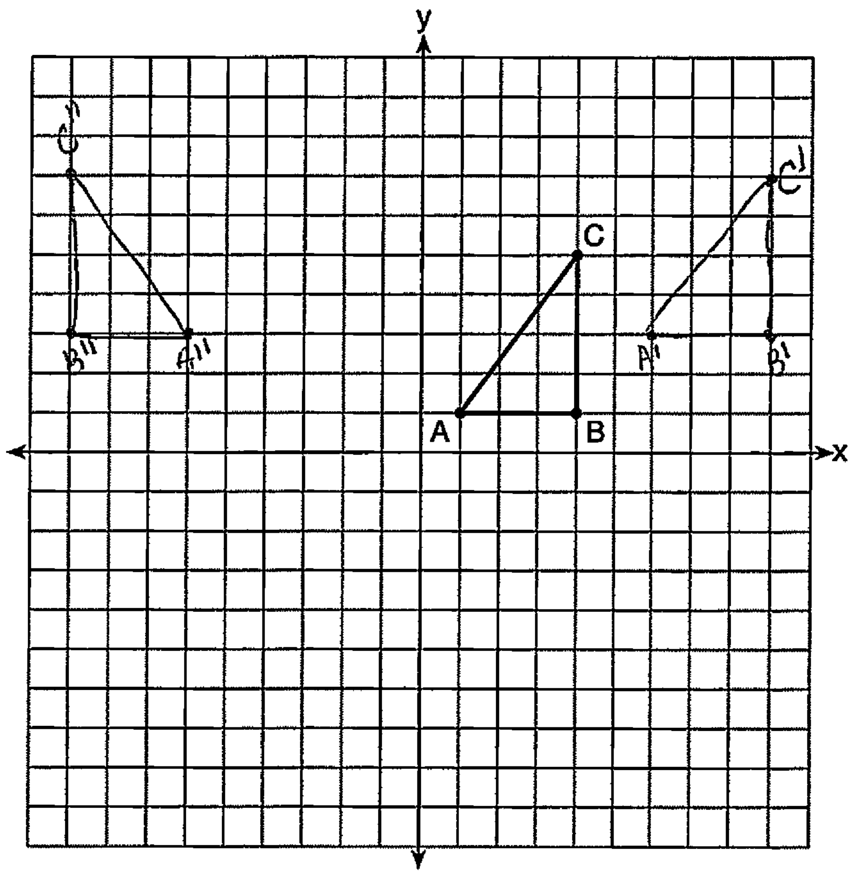


Score 1: The student made an error by translating five units to the left.

Question 26

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.

x+5 y+2

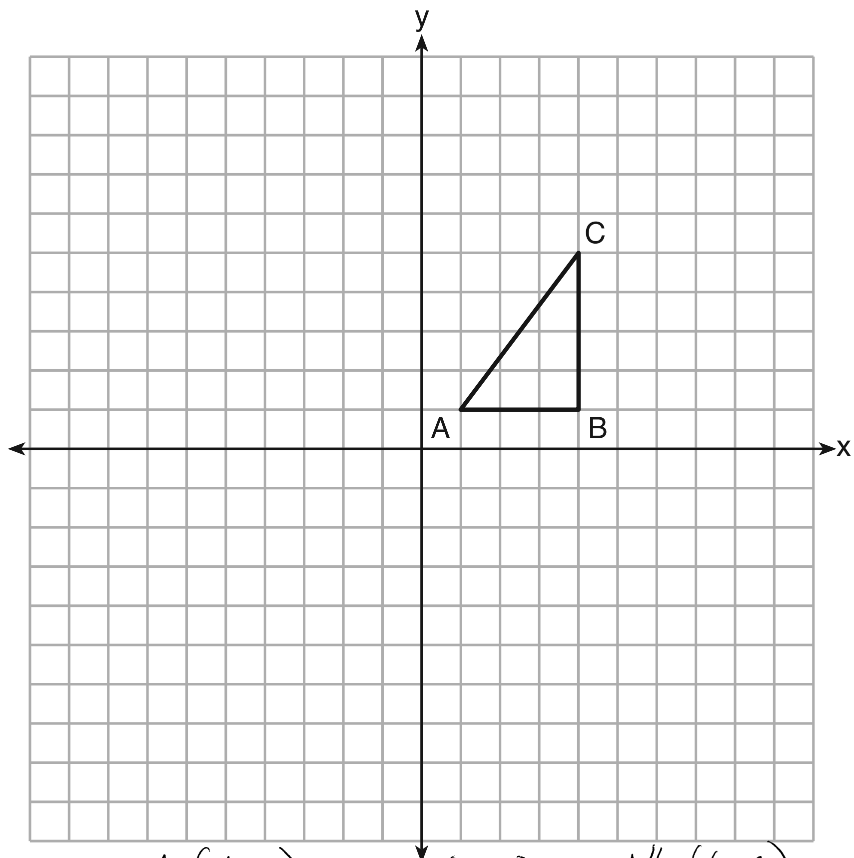


A(1,1) → A'(6,3) → A''(-6,3)
B(4,1) → B'(9,3) → B''(-9,3)
C(4,5) → C'(9,7) → C''(-9,7)

Score 1: The student made an error by reflecting over the y-axis.

Question 26

26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up followed by the reflection over the line $y = 0$.



$A(1,1) \longrightarrow (6,3) \rightarrow A''(6,-3)$

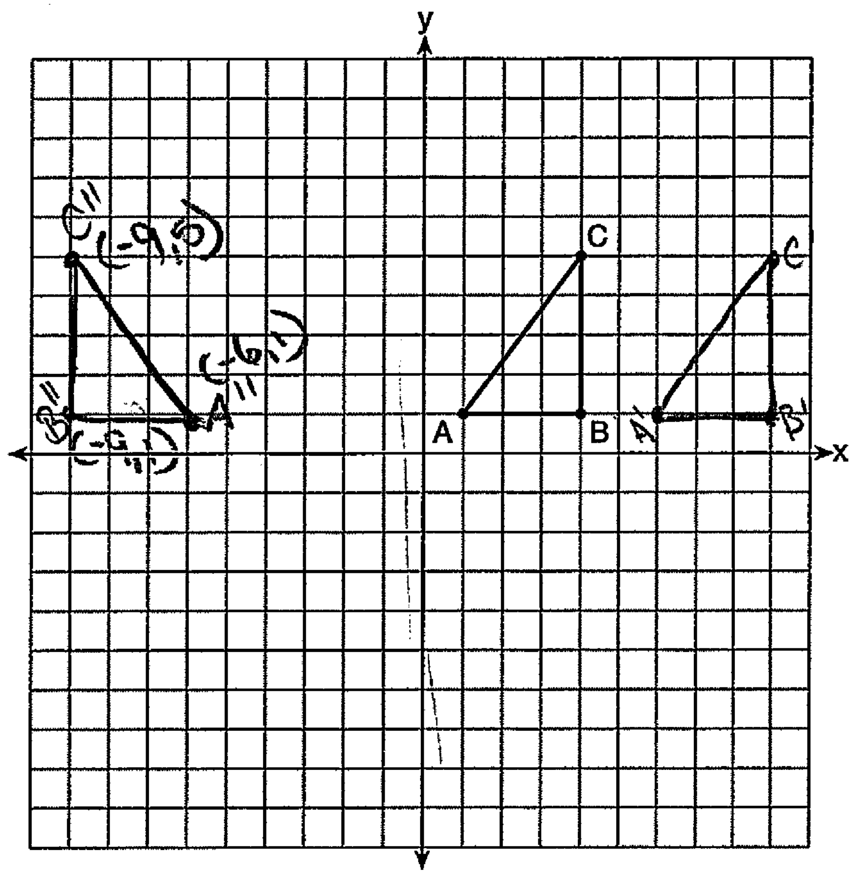
$B(4,1) \longrightarrow (9,3) \rightarrow B''(9,-3)$

$C(4,5) \longrightarrow (9,7) \longrightarrow C''(9,-7)$

Score 1: The student performed the sequence of transformations algebraically.

Question 26

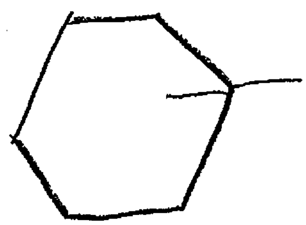
26 In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the translation five units to the right and two units up



Score 0: The student graphed the sequence of transformations incorrectly.

Question 27

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.



$$\frac{180(n-2)}{n} = \frac{180(6-2)}{6}$$

$$\frac{720}{6} = 120 \qquad \begin{array}{r} 180 \\ -120 \\ \hline \end{array}$$

60°

Score 2: The student had a complete and correct response.

Question 27

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.

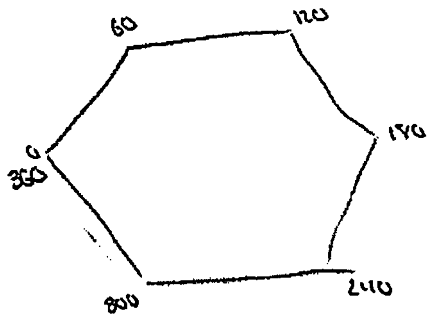
$$\frac{360}{6} = \boxed{60^\circ}$$

Score 2: The student had a complete and correct response.

Question 27

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.

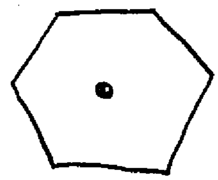
60°



Score 2: The student had a complete and correct response.

Question 27

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.



$180(n-2)$

$180(6-2)$

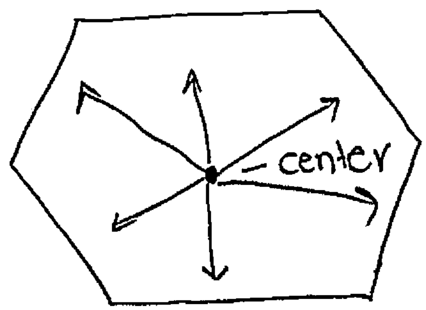
$180(4)$

$\frac{720}{6} = 120°$

Score 1: The student found the measure of one interior angle of the hexagon.

Question 27

27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.



$n+6=180$

$n+6=180$

$6+6=180$

$12=180$

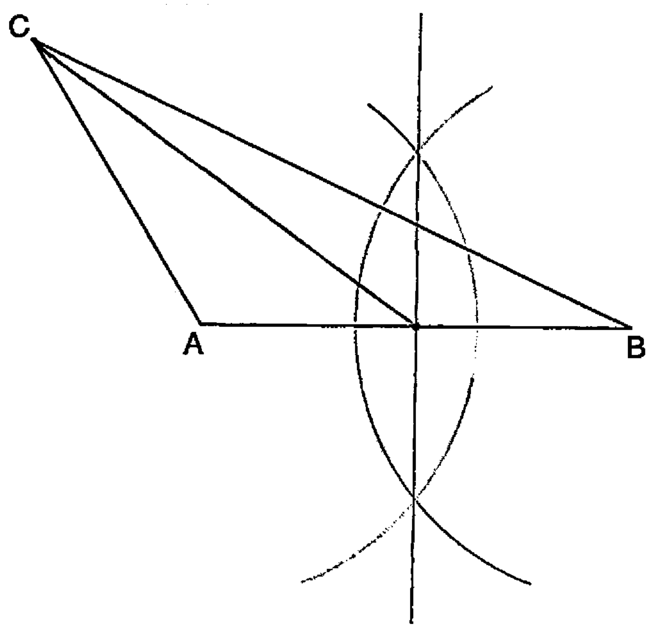
$x=15$

Minimun number of degrees in a rotation is 15°

Score 0: The student had a completely incorrect response.

Question 28

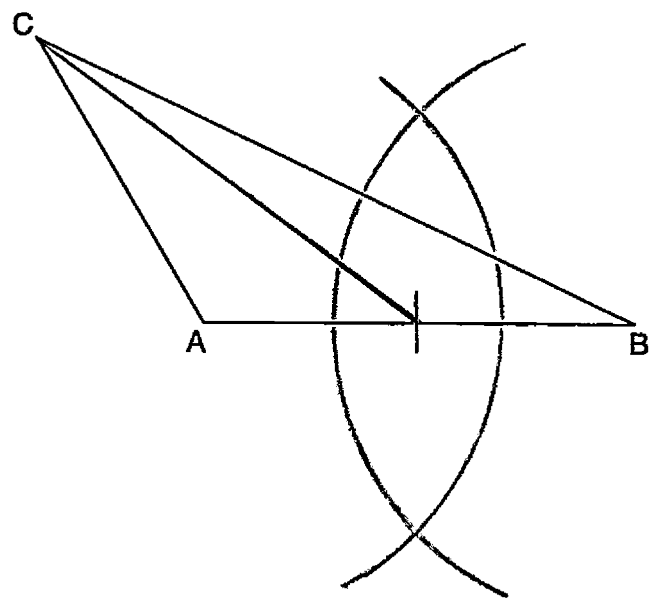
28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $\overline{AB}$. [Leave all construction marks.]



Score 2: The student had a complete and correct response.

Question 28

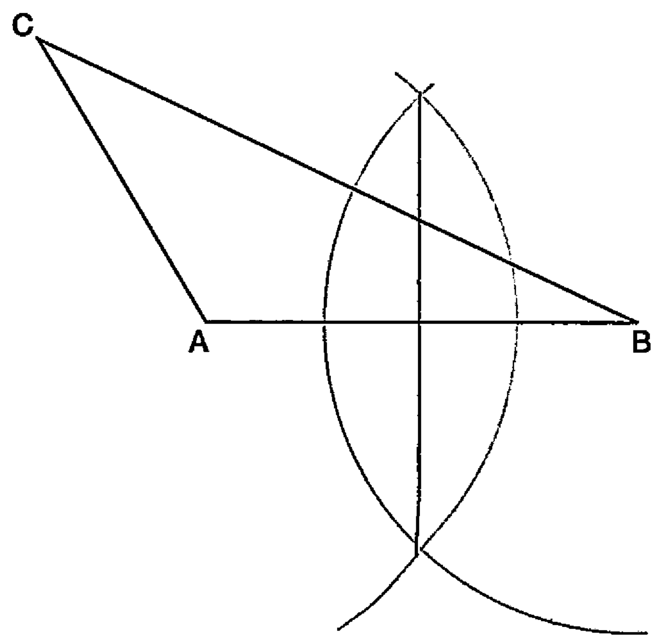
28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $\overline{AB}$. [Leave all construction marks.]



Score 2: The student had a complete and correct response.

Question 28

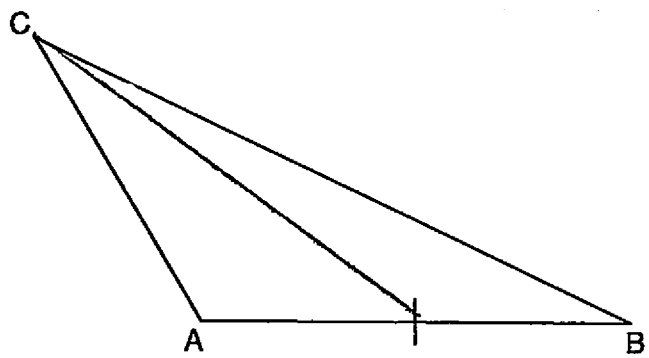
28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $\overline{AB}$. [Leave all construction marks.]



Score 1: The student had a correct construction of a perpendicular bisector, but did not draw the median.

Question 28

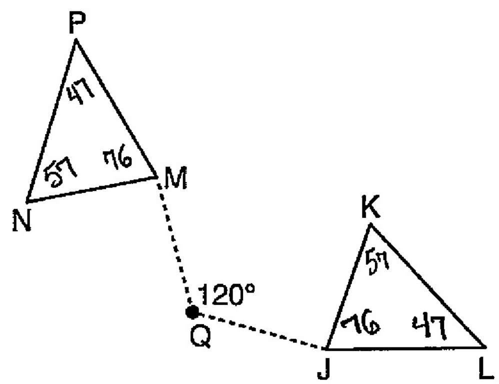
28 In the diagram of $\triangle ABC$ shown below, use a compass and straightedge to construct the median to $\overline{AB}$. [Leave all construction marks.]



Score 0: The student made a drawing that was not a construction.

Question 29

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.



180
−57
−47
76

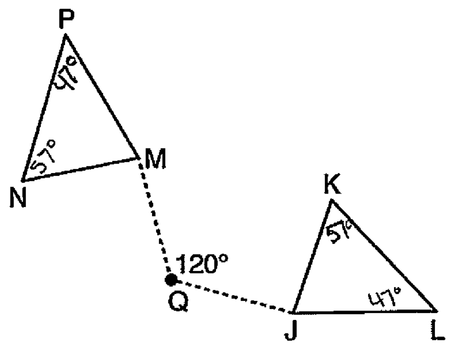
$\boxed{m\angle M \text{ is. } 76°}$ because angle K is angle N after the rotation, and angle L becomes angle P and angle J becomes angle M. Rotation is a rigid motion, so their measures will be the same.
On ΔMNP $m\angle P = 47°$ and $m\angle N = 57°$. A triangle is 180° so 180° − 57° − 47° = 76°, the measure of $\angle M$.

Score 2: The student had a complete and correct response.

Question 29

---

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.



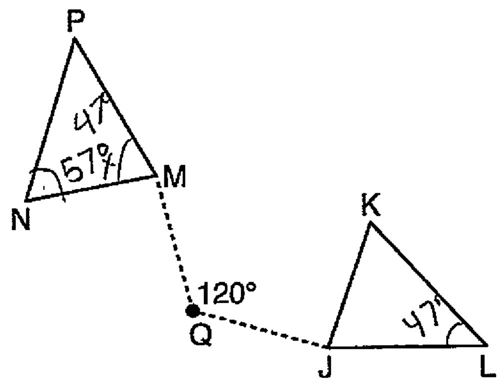
$$\begin{array}{r} 180 \\ -47 \\ -57 \\ \hline \boxed{76^\circ} \end{array}$$

A rotation is an isometry so the triangles must be congruent and have corresp. congruent angles. So that means $\angle P \cong \angle L$ and $\angle P = 47°$ by substitution. Then 47° and 57° can be subtracted from 180° to find $\angle M = 76°$

Score 2: The student had a complete and correct response.

Question 29

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.



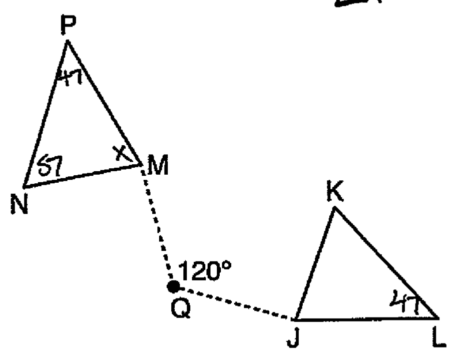
angle M = 47° because rotating the triangle doesn't change angle measurements.

Score 1: The student wrote a correct explanation, but the angle measure was incorrect.

Question 29

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.

$\angle P \cong \angle L$



$$\begin{array}{r} 47 \\ +57 \\ \hline 104 \end{array} \qquad \begin{array}{r} 180 \\ -104 \\ \hline 76 \end{array} \qquad \boxed{m = 76^\circ}$$

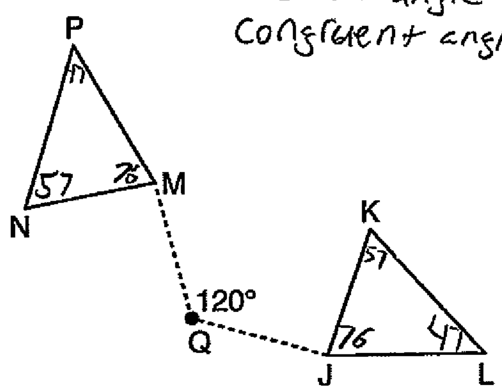
180° in a $\underline{\Delta}$

Score 1: The student did not write an explanation.

Question 29

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.

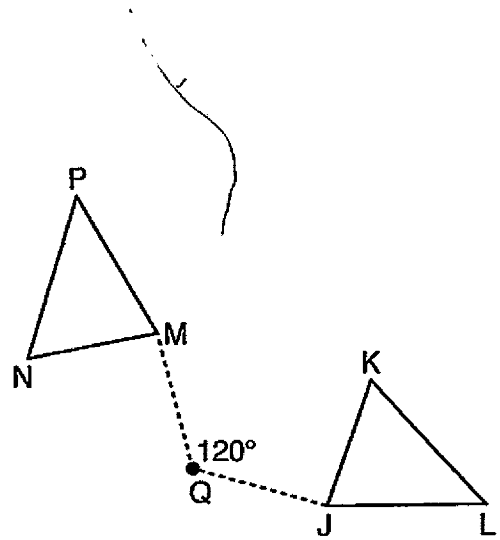
The measure of angle M is 76°
because angle J and angle M are
congruent angle.



Score 1: The student had an incomplete explanation.

Question 29

29 Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.



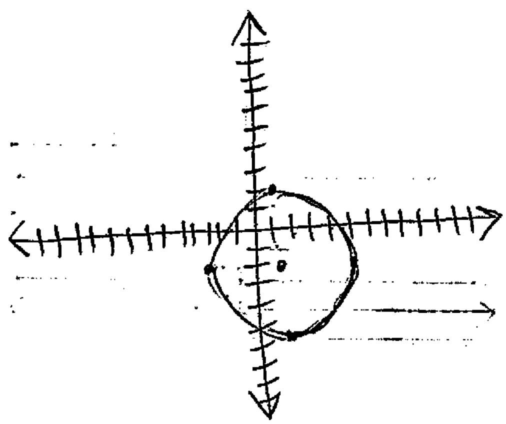
$120 + 47 + 57 = 224$

$224 - 180 = 44^{\circ}$

Score 0: The student had a completely incorrect response.

Question 30

30 A circle has a center at (1,–2) and radius of 4. Does the point (3.4,1.2) lie on the circle? Justify your answer.



$$(x-1)^2+(x+2)^2=4^2$$

$$(3.4-1)^2+(1.2+2)^2=16$$

$$\downarrow$$

$$5.76+10.24=16$$

$$16=16 \checkmark$$

$\boxed{\text{yes}}$ → The point (3.4, 1.2) lie on the circle. By using the equation of the circle, $(x-1)^2+(x+2)^2=4^2$, you plug in the x and the y. First you get $(3.4-1)^2+(1.2+2)^2=16$. When you solve + simplify everything, you get $16=16$.

Score 2: The student had a complete and correct response.

Question 30

30 A circle has a center at (1,–2) and radius of 4. Does the point (3.4,1.2) lie on the circle? Justify your answer.

Yes.

$r = \sqrt{(3.4-1)^2 + (1.2+2)^2}$

$r = \sqrt{5.76 + 10.24}$

$r = \sqrt{16} = 4$

Score 2: The student had a complete and correct response.

Question 30

30 A circle has a center at (1,–2) and radius of 4. Does the point (3.4,1.2) lie on the circle? Justify your answer.

$$(x+1)^2+(y-2)^2=16$$

$$(3.4+1)^2+(1.2-2)^2=16$$

$$19.36+0.64\stackrel{?}{=}16$$

$$20\neq 16$$

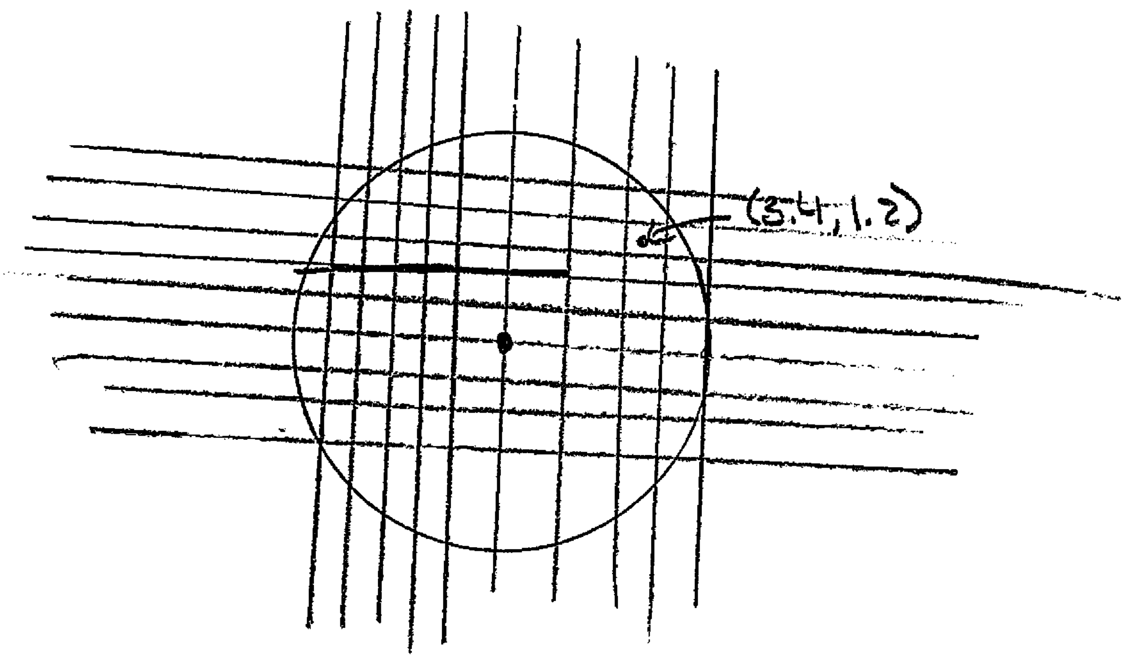
no

Score 1: The student made a substitution error, but wrote an appropriate conclusion.

Question 30

30 A circle has a center at (1,–2) and radius of 4. Does the point (3.4,1.2) lie on the circle? Justify your answer.

No it doesnot.



Score 0: The student had a completely incorrect response.

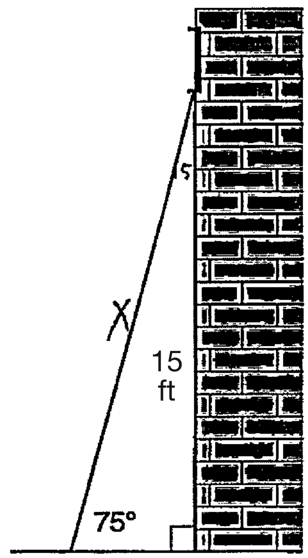
Question 31

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the *nearest tenth of a foot*.

$\frac{\text{Sine}(75)}{1} = \frac{15}{x}$

$x = \frac{15}{\text{Sine}(75)}$

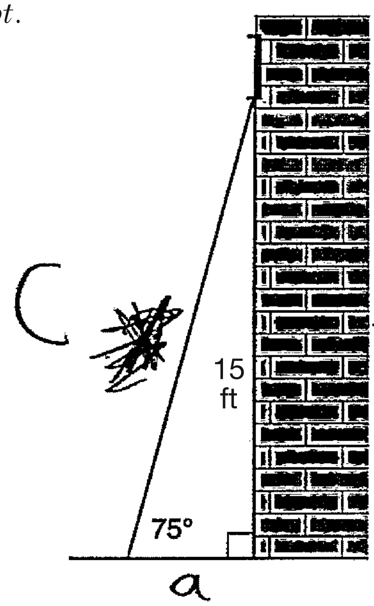
$\boxed{x = 15.5}$



Score 2: The student had a complete and correct response.

Question 31

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the *nearest tenth of a foot*.



$$a \cdot \tan(75) = \frac{15}{\cancel{a}} \cdot a$$

$$a = \frac{15}{\tan(75)}$$

$$a^2 + b^2 = c^2$$

$$\sqrt{\left(\frac{15}{\tan(75)}\right)^2 + 225} = \sqrt{c^2}$$

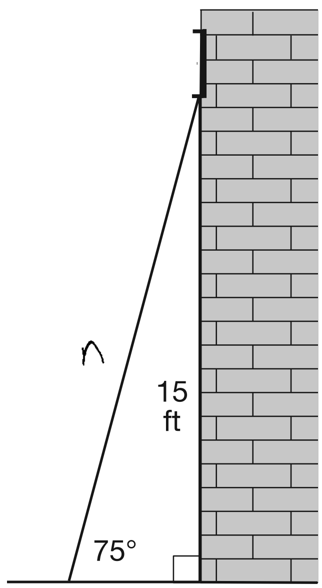
$$15.5 = c$$

Score 2: The student had a complete and correct response.

Question 31

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the *nearest tenth of a foot*.

$\sin(75) = \frac{15}{n}$



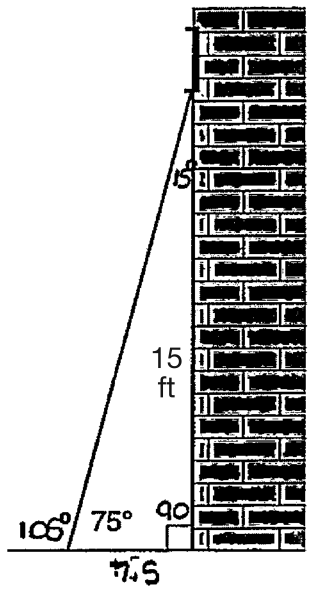
$n = 15 \cdot \sin(75)$

$n = 14.5$

Score 1: The student had a correct equation, but solved it incorrectly.

Question 31

31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the *nearest tenth of a foot*.



$90 + 75 = 165$

$180 - 165 = 15$

$15/2 = 7.5$

$a^2 + b^2 = c^2$

$(7.5)^2 + (15)^2 = c^2$

$56.25 + 225 = c^2$

$281.25 = c^2$

$\sqrt{281.25} = \sqrt{c^2}$

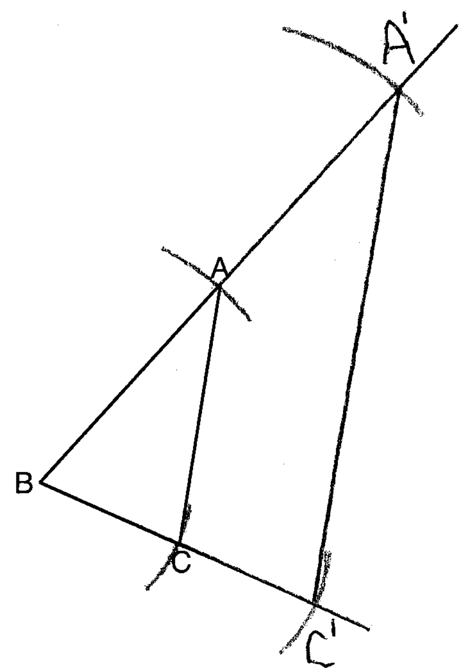
$16.770509831 = c$

$\boxed{\text{Length of Ladder} = 16.8 \text{ Ft.}}$

Score 0: The student did not show enough relevant correct work to receive any credit.

Question 32

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]

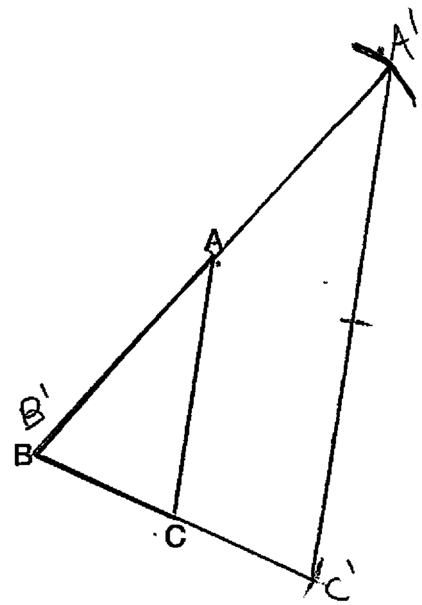


Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

the length of $\overline{A'C'}$ is twice the length of $\overline{AC}$

Score 4: The student had a complete and correct response.

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]

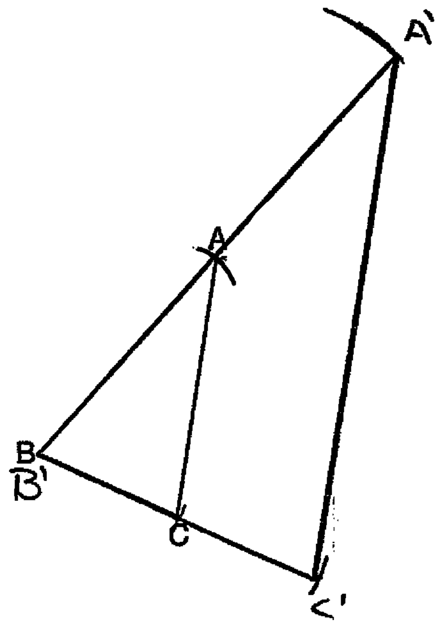


Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

The ratio of the lengths of $\overline{A'C'}$ to $\overline{AC}$ is 2:1

Score 4: The student had a complete and correct response.

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]



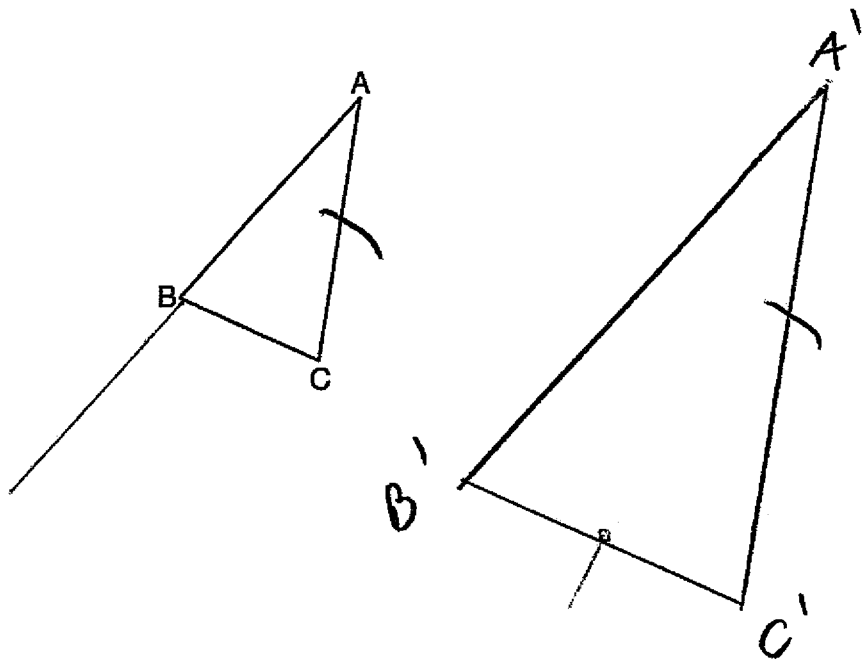
Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

$\overline{AC}$ and $\overline{A'C'}$ are both parallel to each other.

Score 3: The student had a correct construction, but the description was of a correct relationship other than length.

Question 32

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]

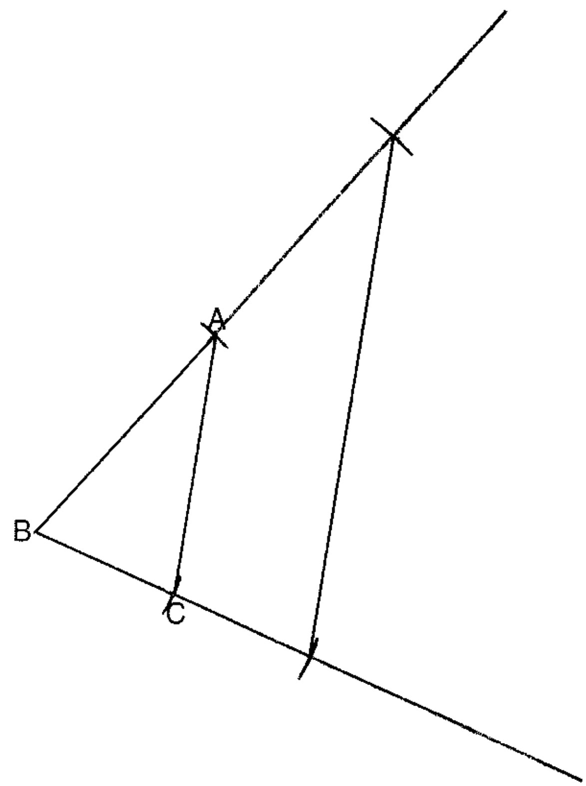


Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

AC is half A' C'

Score 2: The student had a correct description, but no further correct work was shown.

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]



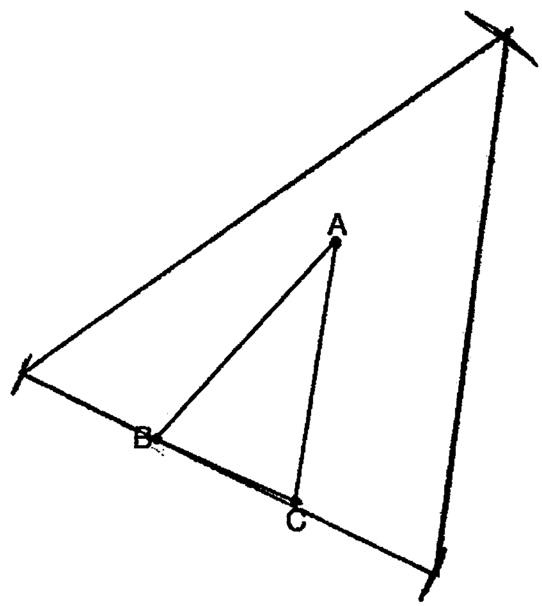
Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

They are in a ratio of 2:1

Score 2: The student did not label A' and C' on the construction. The description was incomplete.

Question 32

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]



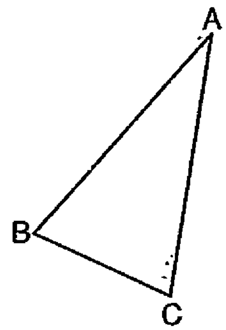
Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

it is doubled

Score 1: The student made an incorrect construction. The description was incomplete.

Question 32

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]

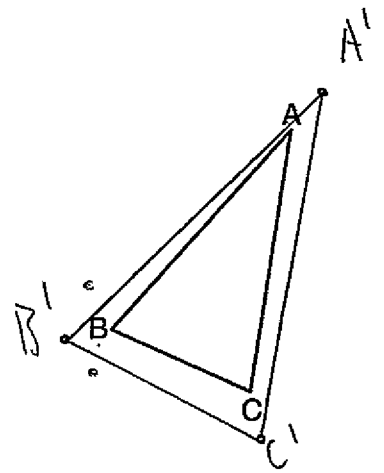


Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

is 2x bigger.

Score 1: The student wrote an incomplete description and the construction was missing.

32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at B. [Leave all construction marks.]



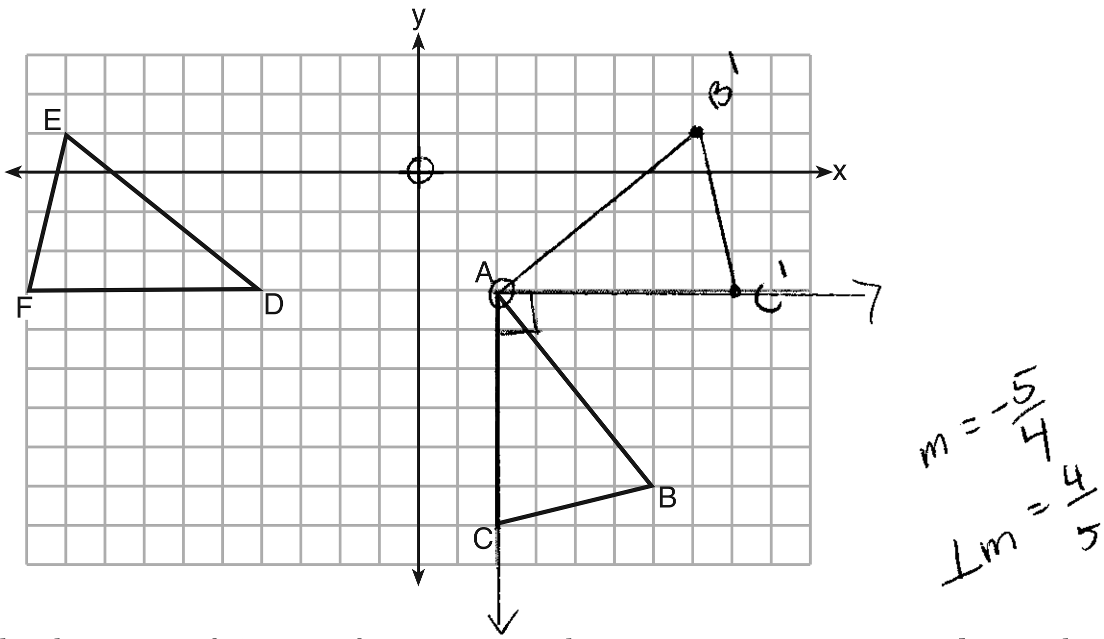
Describe the relationship between the lengths of $\overline{AC}$ and $\overline{A'C'}$.

$\overline{A'C}$ is a dialated line Segment of $\overline{AC}$

Score 0: The construction and description were completely incorrect.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

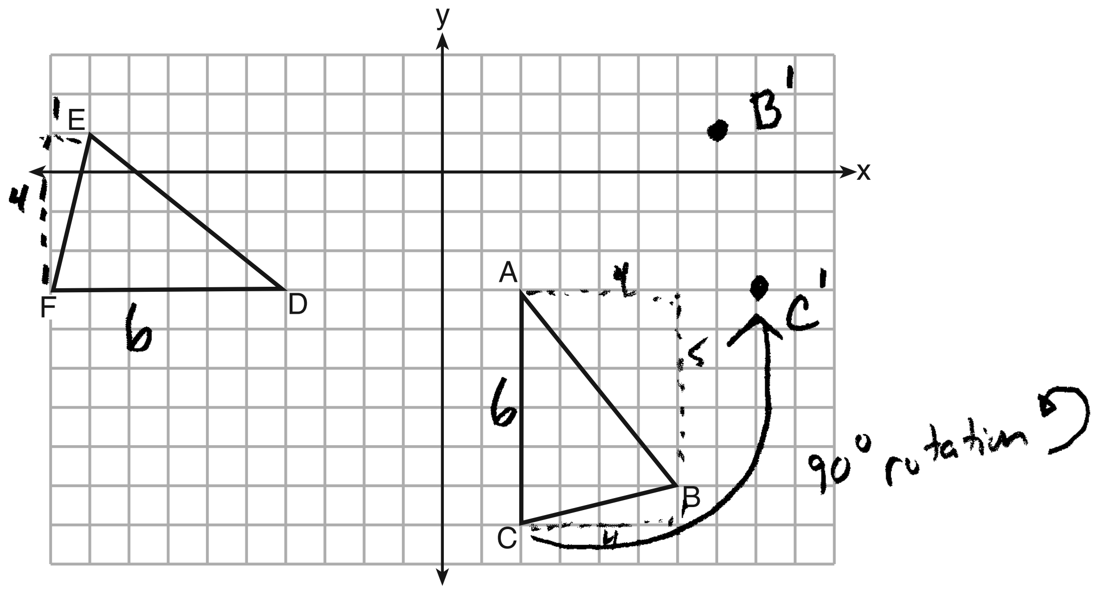
B'(7,1) The angle of Rotation that took C to C' was 90° counterclockwise, so take B to B' by finding the point using the same rotation about point A.

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

Yes, when $\triangle A'B'C'$ is reflected over the line $x=-1$, it will map to $\triangle DEF$. Since a reflection is a rigid motion that preserves distance, $\triangle DEF \cong \triangle A'B'C'$

Score 4: The student had a complete and correct response.

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

Rotate B counterclockwise 90° about A(2,-3)

$B(6,-8) \xrightarrow{\text{Subtract }(2,-3)} (4,-5) \xrightarrow{\text{Rotate}} (5,4) \xrightarrow{\text{Add }(2,-3)} B'(7,1)$

B'(7,1), was found by applying the rotation algorithm from a non-origin point.

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer. YES

$AC = 6 \quad DF = 6$ Counted

$BC = \sqrt{4^2+1^2} = \sqrt{17}, \quad EF = \sqrt{4^2+1^2} = \sqrt{17}$

$AB = \sqrt{5^2+4^2} = \sqrt{41}, \quad DE = \sqrt{5^2+4^2} = \sqrt{41}$

$\left.\right\}$ $\triangle ABC \cong \triangle DEF$ by SSS

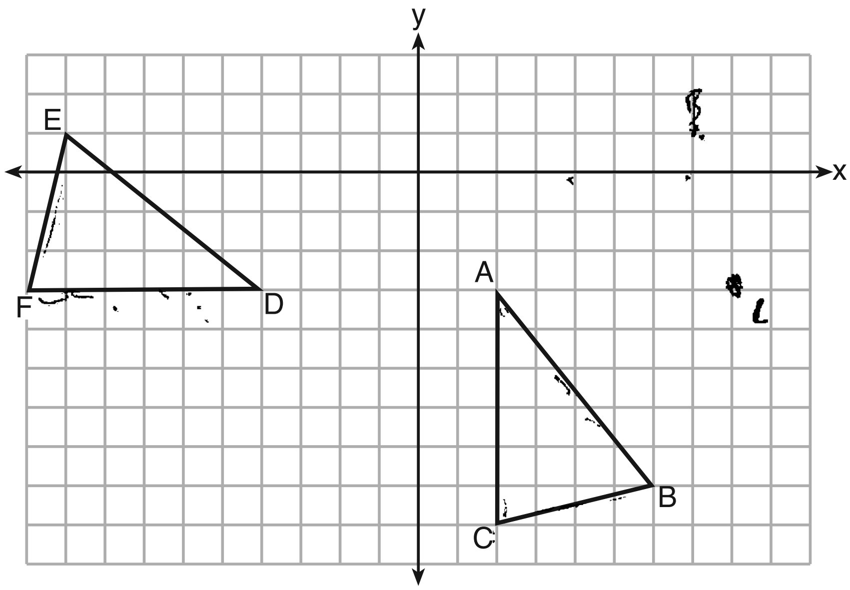
Since a rotation preserves distance, $\triangle ABC \cong \triangle A'B'C'$

So by substitution $\triangle DEF \cong \triangle A'B'C'$

Score 4: The student had a complete and correct response.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

It rotated 90° counter clockwise around A so
Point B' would be (7,1)

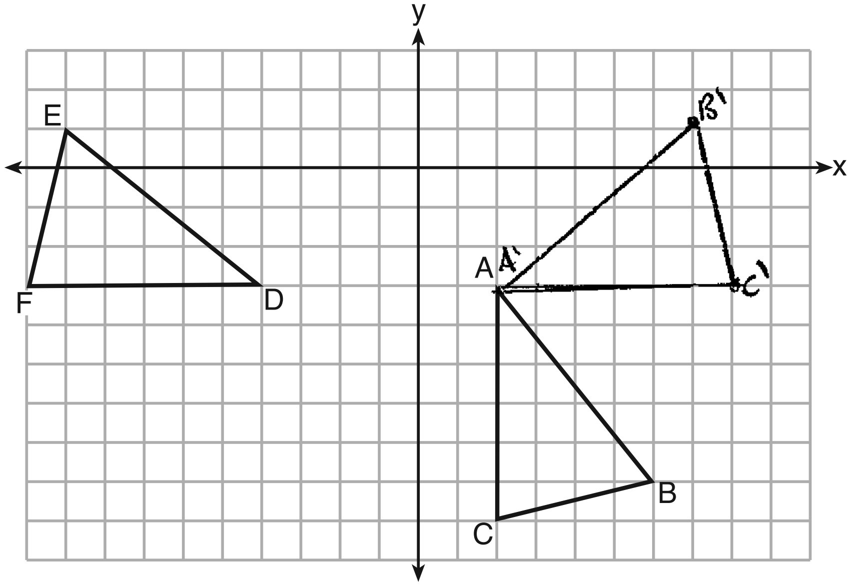
Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

$\triangle$DEF and $\triangle$ABC are congruent
by SSS so $\triangle$DEF is congruent to $\triangle$A'B'C'
by SSS.

Score 3: The student wrote an incomplete explanation for why $\triangle DEF$ is congruent to $\triangle A'B'C'$.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

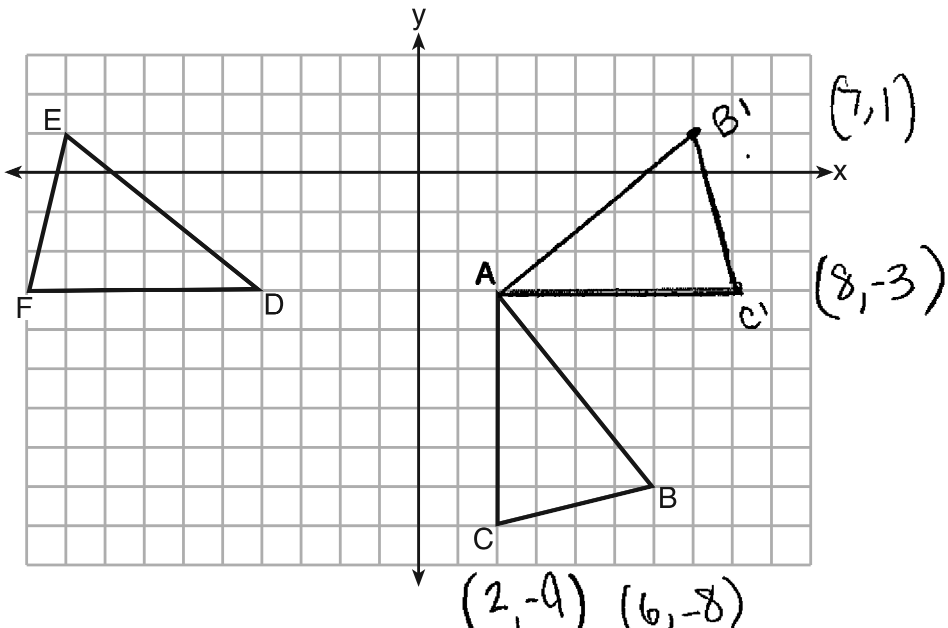
B`(7,1) C rotated 90°, so I rotated B 90°

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

Yes, if you reflect ΔA`B`C` over x=-1, it matches up perfectly, so ΔDEF is ≅ to ΔA`B`C`

Score 3: The student wrote an incomplete explanation for why $\triangle DEF$ is congruent to $\triangle A'B'C'$.

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

(7,1) B' had to be the same distance away from C', as B and C were from eachother.

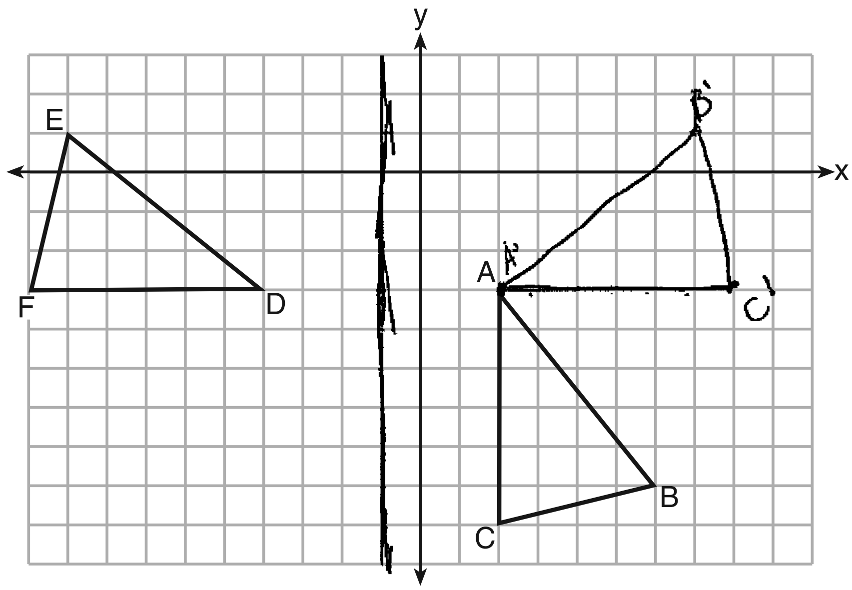
Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

Yes, each point is the same distance apart from one-another.

Score 2: The student wrote two incomplete explanations.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

Counted the slopes and ploted points based on C''s location

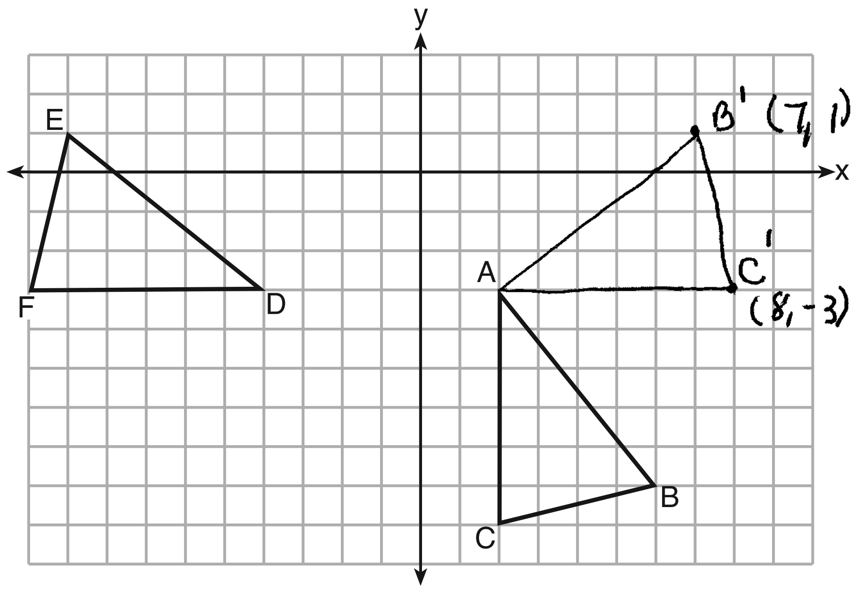
Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

Yes all the slopes are the same they just reflexted over $x=-1$

Score 1: The student wrote yes, but the explanation was incorrect. No further correct work was shown.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.



Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

Yes

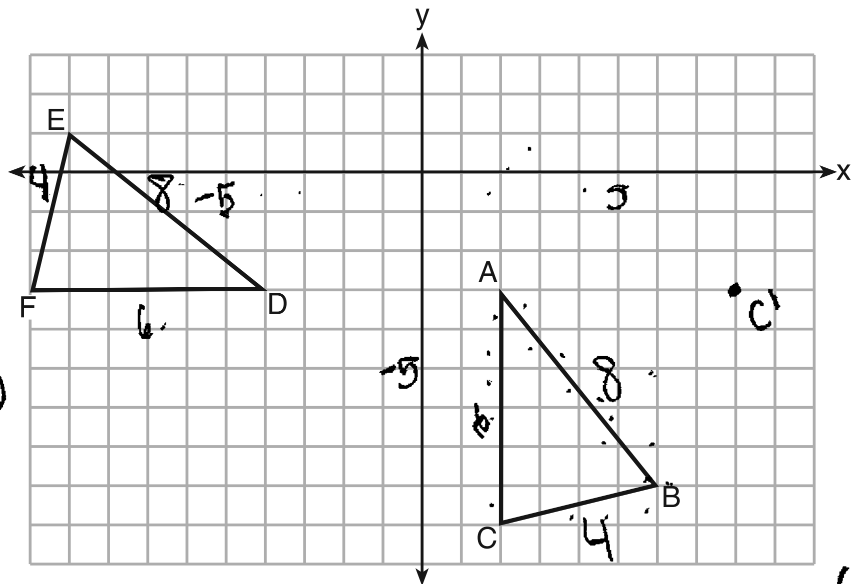
Score 1: The student showed work to find (7,1), and wrote yes, but did not write any explanations.

Question 33

33 The grid below shows $\triangle ABC$ and $\triangle DEF$.

$A = (2,-3)$
$B = (6,-8)$
$C = (2,-9)$

$D = (-4,-3)$
$E = (9,-1)$
$F = (10,-3)$



$(2,-9)$ $(6,-8)$

Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point A. Determine and state the location of B' if the location of point C' is (8,–3). Explain your answer.

+6, −6

$\boxed{B' = (12,-14)}$ because to get C to C' you had to +6 to the x and −6 from the y.

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.

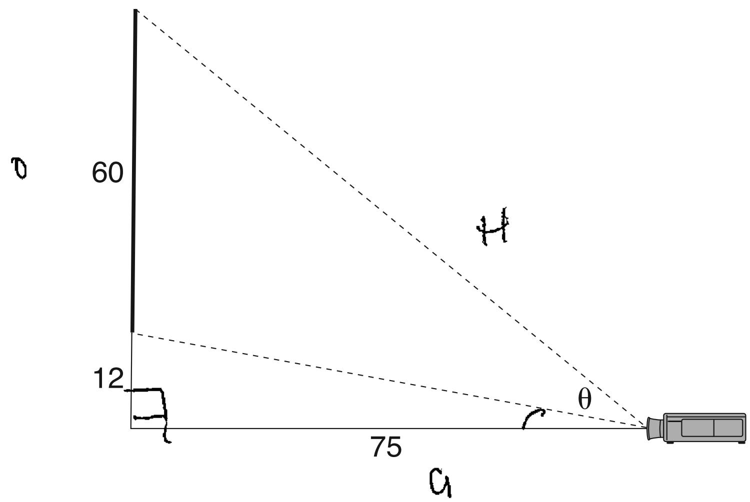
$$\overline{AB} = \sqrt{(6-2)^2 + (-8+3)^2} = \sqrt{16+25} = \sqrt{41}$$

$$\overline{ED} = \sqrt{(9+4)^2 + (-1+3)^2} = \sqrt{169+4} = \sqrt{173}$$

No, because not all of the sides are equal in measure.

Score 0: The student had a completely incorrect response.

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

SOHCAHTOA

$\tan x = \frac{12}{75}$

$(\tan^{-1})\tan x = \frac{12}{75}(\tan^{-1})$

$x = 9.0903$

$\tan y = \frac{72}{75}$

$(\cancel{\tan^{-1}})\cancel{\tan}\, y = \frac{72}{75}(\tan^{-1})$

$y = 43.8308$

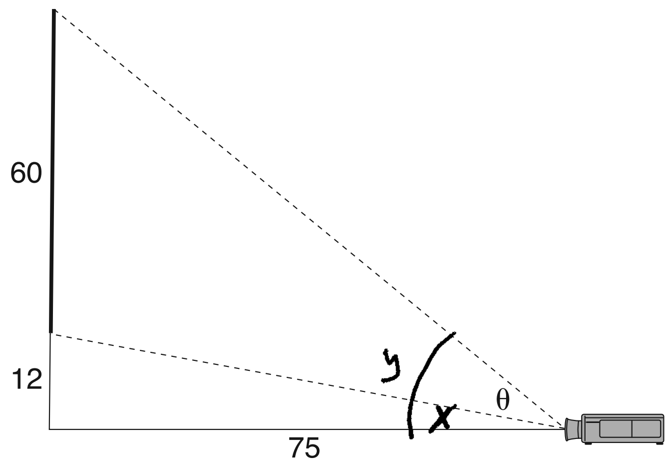
$43.8308 - 9.0903 = 34.7405$

$\boxed{\text{The measure of } \theta \text{ is } 34.7°}$

Score 4: The student had a complete and correct response.

Question 34

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

$\theta = y - x$

$T = \frac{O}{A}$

2ND Tan $\frac{72}{75}$ 2ND Tan $\frac{12}{75}$

$\theta = 43.83 - 9.09$

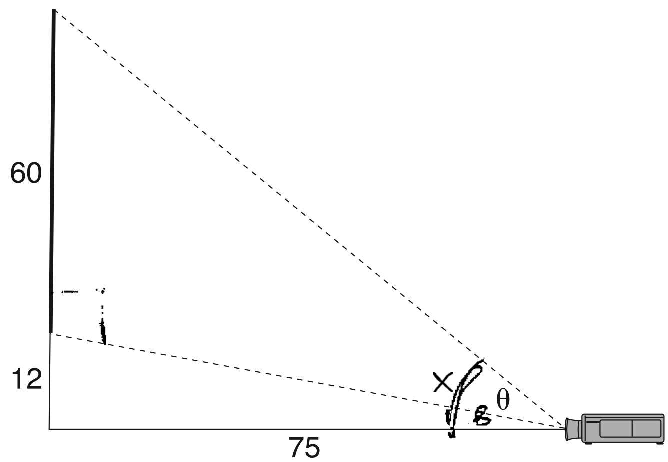
34.74

34.7

Score 4: The student had a complete and correct response.

Question 34

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

$\text{Tan } B = \frac{12}{75}$

$\text{Tan}^{-1}\left(\frac{12}{17}\right) = 35.21759297$

$\begin{array}{r} 60 \\ +12 \\ \hline 72 \end{array}$

$\text{Tan } x = \frac{72}{75}$

$\text{Tan}^{-1}\left(\frac{72}{75}\right) = 43.83086067$

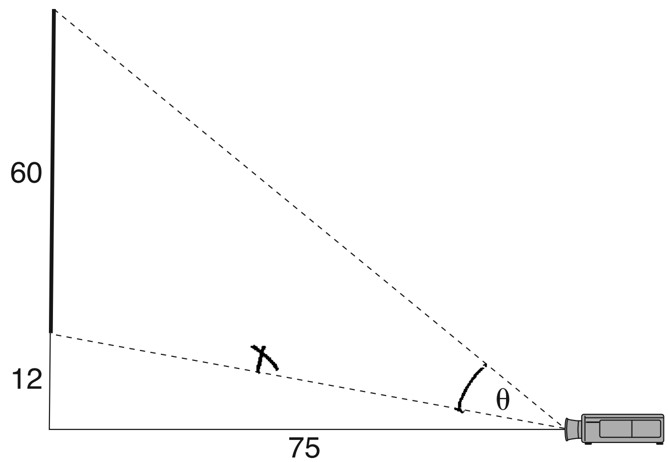
$\begin{array}{r} 43.83086067 \\ -35.21759297 \\ \hline 8.6132677 \end{array}$

$\boxed{\text{The measure of } \theta \text{ is } 8.6}$

Score 3: The student made a transcription error.

Question 34

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

$75^2 + 12^2 = x^2$

$5625 + 144 = x^2$

$\sqrt{5769} = \sqrt{x^2}$

$75.953\ldots \approx x$

$\tan(\theta) = \frac{60}{\sqrt{5769}}$

$\tan^{-1}(\tan(\theta)) = \left(\frac{60}{\sqrt{5769}}\right) \tan^{-1}$

$\theta = 38.307\ldots$

$\boxed{\theta \approx 38.3}$

Score 2: The student made a conceptual error in using an obtuse triangle for right triangle trigonometry.

Question 34

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.

72

$\tan x = \frac{72}{75}$

43.8°

x

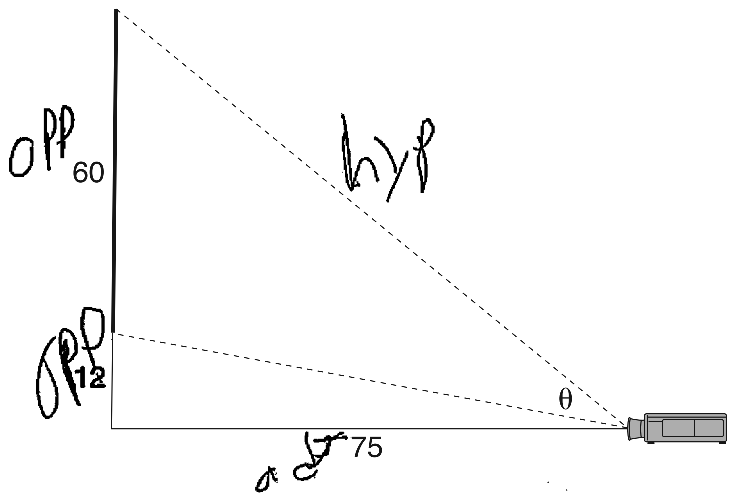
Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

43.8°

Score 1: The student determined only one angle of elevation.

Question 34

34 As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.



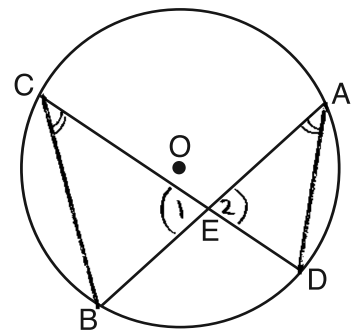
Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

$\tan^{-1}(72/75)$

43.9

Score 0: The student did not show enough correct work to receive any credit.

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

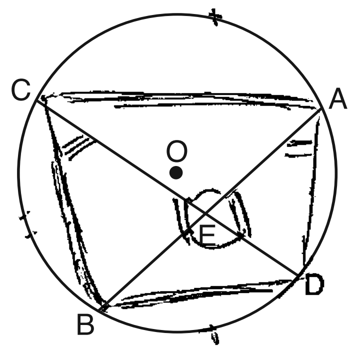
Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

Statement	Reason
① Circle O, chords $\overline{AB}$ + $\overline{CD}$	① Given
② $\overline{CB}$ and $\overline{AD}$ are drawn	② auxiliary lines can be drawn
③ $\angle 1 \cong \angle 2$	③ Vertical ∡'s are ≅
④ $\angle C \cong \angle A$	④ Inscribed ∡'s that intercept the same arc are ≅
⑤ $\triangle BCE \sim \triangle DAE$	⑤ AA
⑥ $\frac{AE}{CE} = \frac{ED}{EB}$	⑥ Corresponding sides of Similar Δ's are proportional
⑦ $AE \cdot EB = CE \cdot ED$	⑦ In a proportion, the product of the means equals the product of the extremes

Score 6: The student had a complete and correct response.

Question 35

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

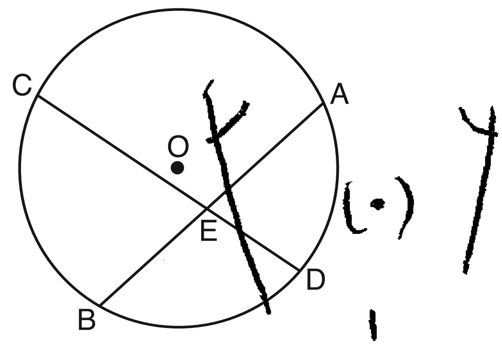
Prove this theorem by proving $AE \bullet EB = CE \bullet ED$.

Statements	reasons
1. Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E	1. Given
2. $\angle CEA \cong \angle BED$, $\angle AED \cong \angle CEB$	2. vertical $\angle$'s $\cong$
3. $\angle BCE \cong \angle DAB$	3. Inscribed $\angle$'s that intercept the same arc are $\cong$
4. $\triangle ECB \sim \triangle EAD$	4. AA Sim Thm.
5 $\frac{AE}{CE} = \frac{ED}{EB}$	5. Corres. sides of $\sim$ $\triangle$'s are in prop.
6. $AE \cdot EB = CE \cdot ED$	In a proporation, 6. The product of the means is equal to the product of the extremes

Score 5: The student did not include drawing chords $\overline{AC}$, $\overline{CB}$, $\overline{BD}$, and $\overline{AD}$ in the proof.

Question 35

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

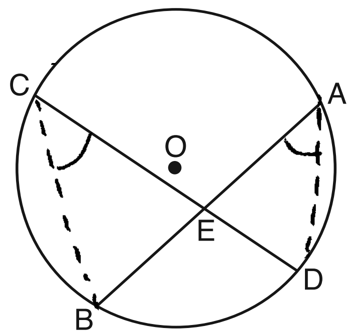
Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

$\triangle CBE \sim \triangle AED$

S	r
① Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E	① given
② ∠CEB and ∠AED are vertical angles	② intersecting lines form vertical angles
③ ∠CEB ≅ ∠AED	③ vertical angles are congruent
④ ∠C ≅ ∠A	④ inscribed angles of the same arc are congruent
⑤ $\triangle CBE \sim \triangle ADE$	⑤ two angles in the triangles are congruent
⑥ $\frac{EB}{ED} = \frac{CE}{AE}$	⑥ In two similar triangles, corresponding sides are in proportion
⑦ $AE \cdot EB = CE \cdot ED$	⑦ In a proportion, product of means equals product of extremes

Score 4: The student omitted one statement and reason, and another reason was incomplete.

35 Given: Circle *O*, chords $\overline{AB}$ and $\overline{CD}$ intersect at *E*



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

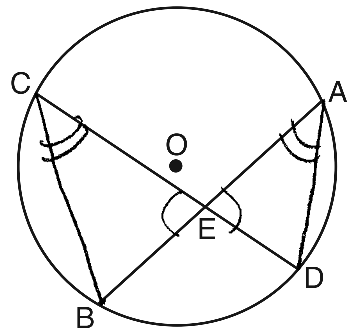
Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

1) $\overline{AB}$ and $\overline{CD}$ are chords in Circle O. Chords intersect at E	1) Given
2) Draw auxiliary lines $\overline{BC}$ and $\overline{AD}$	2) Between 2 pts there exists a line segment.
3) $\angle C \cong \angle A$ $\angle B \cong \angle D$	3) Angles inscribed in the same arc are $\cong$
4) $\triangle BCE \cong \triangle DAE$	4) AA ~ Thm
5) $AE \cdot EB = CE \cdot ED$	5) CPCTC

Score 3: The student had three missing or incomplete statements.

Question 35

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



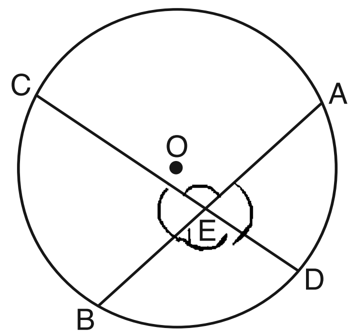
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

Given circle O and chords $\overline{AB}$ and $\overline{CD}$ intersect at E. Since vertical angles are congruent, $\angle CEB \cong \angle AED$. $\angle C$ and $\angle A$ are inscribed in $\overset{\frown}{BD}$, so $\angle C \cong \angle A$. So $\triangle CEB \cong \triangle AED$. By the means extremes property, $AE \cdot EB = CE \cdot ED$.

Score 2: The student gave two correct relevant statements and reasons.

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

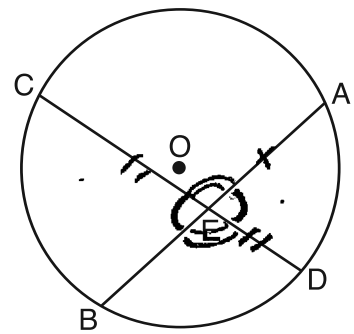
Prove this theorem by proving $AE \bullet EB = CE \bullet ED$.

Statements	Reason
chords $\overline{AB}$ and $\overline{CD}$ intersect at E	Given
$\angle CEA \cong \angle BED$	vertical angles are congruent
$\angle CEB \cong \angle AED$	vertical angles are congruent
$AE \cdot EB = CE \cdot ED$	if two cords intersect in a circle the product of the lengths of the segments of one chod is equal to the product of the lengths of the segments of the other chord.

Score 1: The student correctly stated the vertical angles were congruent.

Question 35

35 Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E



Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

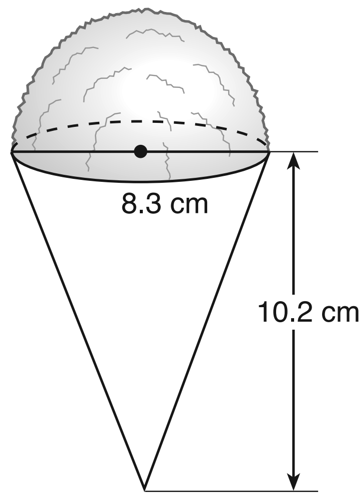
Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

Statement	Reason
1. $\overline{AB}$ and $\overline{CD}$ intersect at E	1. Given
2. $\angle CEB \cong \angle AED$	3. opposite exterior angles are congruent
3. $\angle CEA \cong \angle BED$	3. " "
4. $\overline{CD}$ bisect AB	4. $\overline{AB}$: $\overline{CD}$ intersect at E

Score 0: The student had a completely incorrect response.

Question 36

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

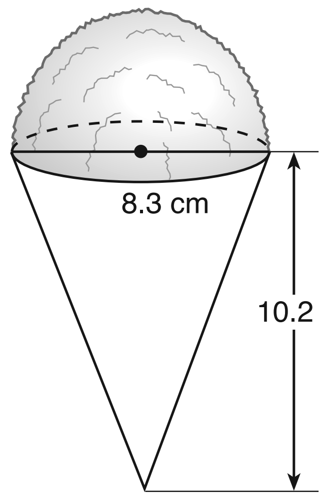


The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

Score 6: The student had a complete and correct response.

Question 36

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$V_{cone} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(4.15)^2(10.2)$

$= 58.5565\pi$

$V_{Sphere} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(4.15)^3$

$= 95.29783333\pi$

$V = 153.8543333\pi = \boxed{483.3476433 \text{ cm}^3}$

$\frac{m}{D \mid V}$

$m = (483.3476433)(.697) = 336.8933074$ g for 1 snowcone

$\times 50$

16844.66537 g

$m = 16.84466537$

$\times 3.83$

Cost = \$64.52

Score 5: The student found the volume of a sphere and not a hemisphere.

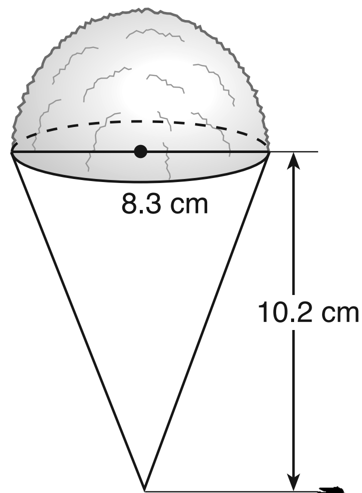
36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

V of cone $= \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\, 4.15^2 \cdot 10.2$

$V = \frac{1}{3}\pi\, 17.2225 \cdot 10.2$

$V = 183.96$



V of sphere $= \dfrac{\frac{4}{3}\pi r^3}{2}$

$V = \dfrac{\frac{4}{3}\pi\, 71.473375}{2}$

$V = 149.69$

total Volume $= 333.7\,cm^3$

The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

232.5889 grams per cone
$\times 50$

$\dfrac{11629.445 \text{ grams}}{1000} = 11.63$

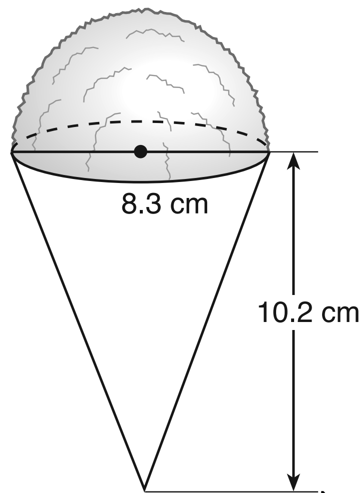
Kilogram = 1000 grams

\$44.54

Score 5: The student used an incorrectly rounded total volume of one snow cone when computing the mass.

Question 36

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



$8.3 = d \quad 8.3/2 = 4.15 = r$

The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$V_{CONE} = \frac{1}{3}\pi r^2 h$

$V_{CONE} = \frac{1}{3}\pi * 4.15^2 * 10.2$

$V_{CONE} = \frac{1}{3} * \pi * 17.2225 * 10.2$

$V_{CONE} = 183.961 \text{ cm}^3$

\$3.83 ⇒ 1,000 g

0.00383 → 1 g

183.961 * 0.697 = 128.221 g/cm^3 → 1 snow cone

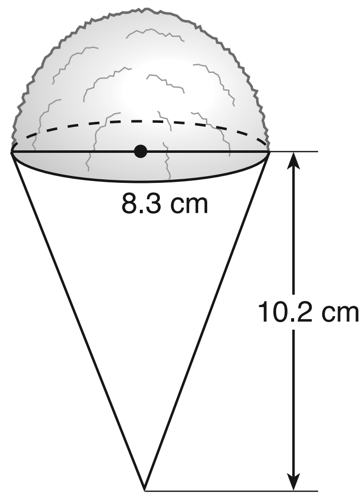
128.221 * 50 = 6411.05

6411.05 * 0.00383 = 24.55

\$24.55

Score 4: The student determined the cost of the cone without the hemisphere.

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$\text{Vol} = \frac{1}{2} \cdot \frac{4}{3} \cdot \pi \cdot 4.15^3 = 149.693$

$\text{Vol} = \frac{1}{3} \cdot \pi \cdot 4.15^2 \cdot 10.2 = 183.961$

Total volume = 333.654

50 snow cones: vol = 16682.7 cm^3

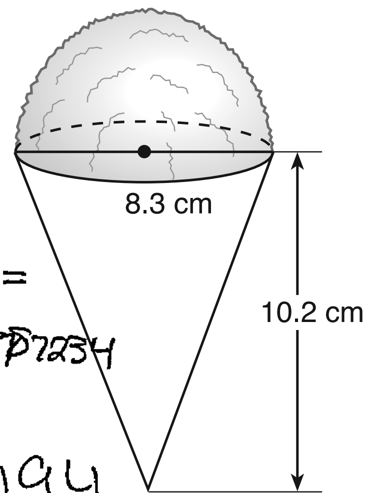
kg : $\frac{16682.7}{697} = 23.935$ kg

cost : $23.935 \times 3.83 = 91.67105$

Score 3: The student found the volume of fifty snow cones, but no further correct work was shown.

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

$r = 4.15$



hemisphere:

$V = \frac{2}{3}\pi r^3$

$\frac{2}{3}\pi \cdot 4.15^3$

$\frac{3}{3}\pi \cdot 71.473375$

149.6934866

$\frac{1}{3}\pi \cdot 4.15 \cdot 10.2 =$

44.32~~87234~~

Cone:

$V = \frac{1}{3}\pi r^2 h$

194

The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$0.697 \cdot 194 = 135.218$

$\times\ 50$

6760.9

M / D | V

3.83

$\times\ 50$

191.5

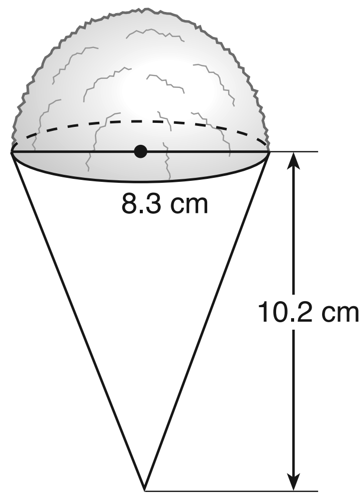
$\frac{6760.9}{191.5} = 35.30$

\$35.30

Score 2: The student made an error in determining the volume of the cone, but found an appropriate mass of fifty snow cones. No further correct work was shown.

Question 36

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$D = \frac{m}{V}$ $\quad 0.697\ g/cm^3$ $\quad V = \frac{1}{3}\pi r^2 h$

$\underline{50(3.83)}$ $\quad V = \frac{1}{3}\pi(4.15)^2 10.2$

191.5 $\quad V = 183.9606702\ cm^3$

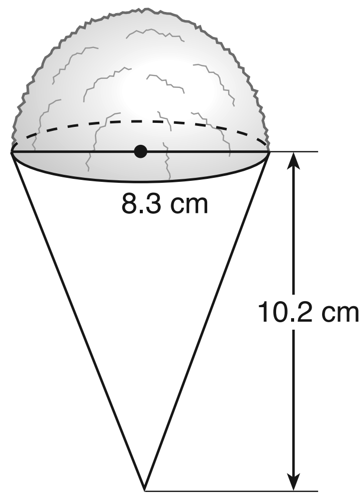
$50(183.9606702)$

$\boxed{\$\ 9198.03 \text{ for 50 snow cones}}$

Score 1: The student determined the volume of the cone.

Question 36

36 A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \cdot 4.15^2 \cdot 10.2$

$V = \frac{1}{3}\pi \cdot 17.22 \cdot 10.2$

$V = \frac{1}{3}\pi \cdot 175.644$

$V = 58.548$

Score 0: The student did not show enough work to receive any credit.