The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 17, 2017 — 12:30 to 3:30 p.m.

MODEL RESPONSE SET

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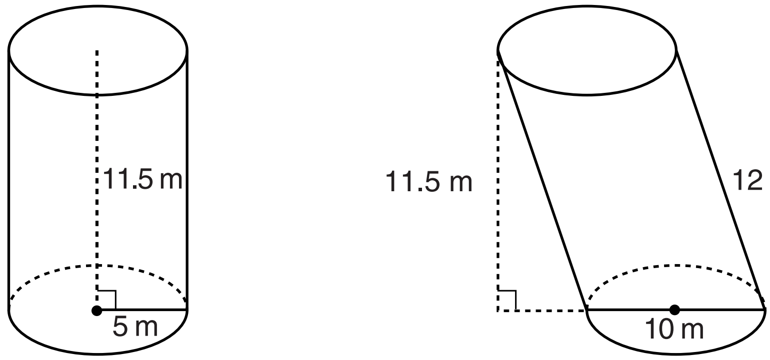
Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



$r = 5, \ h = 11.5$ $\quad$ $r = 5, \ h = 11.5$

Is Sue correct? Explain why.

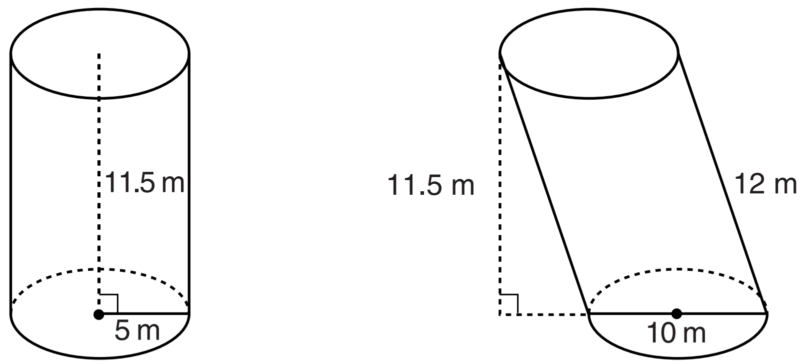
Yes, Sue is correct because when two cylinders have the same base areas and the same height, the two cylinders must have the same volume.

Score 2: The student gave a complete and correct response.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.

$V = \pi r^2 \cdot h$
$= \pi 5^2 \cdot 11.5$
$= 287.5\pi\, m^3$



$V = \pi r^2 \cdot h$
$= \pi 5^2 \cdot 11.5$
$= 287.5\pi\, m^3$

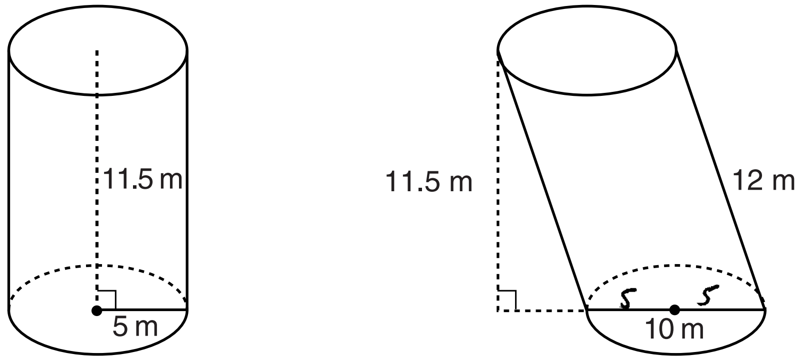
Is Sue correct? Explain why.

Sue is correct, Both cylinders' radii and hights are equal causing their volumes to be the same

Score 2: The student gave a complete and correct response.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



Is Sue correct? Explain why.

$V = \pi r^2 h$

$= \pi 5^2 \times 11.5$

$= 287.5\pi$

$V = \pi r^2 h$

$V = \pi 5^2 \times 11.5$

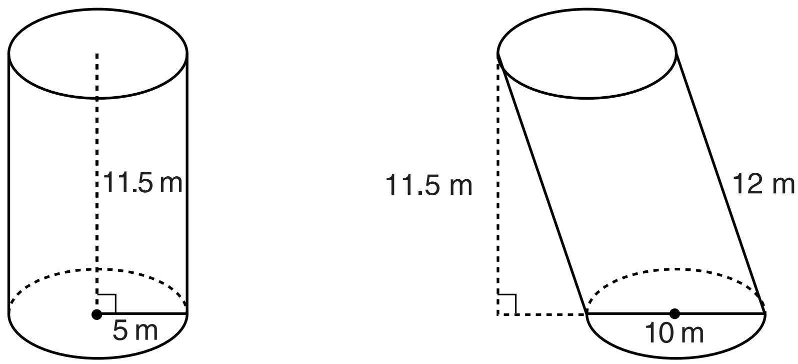
$= 287.5\pi$

Yes Sue is correct - the 2 cylinders have the same volume

Score 1: The student found the volumes of both cylinders, but did not write an explanation for why the volumes are the same.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



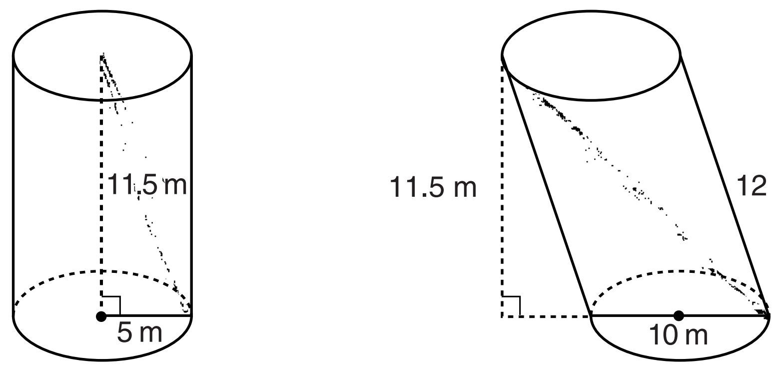
Is Sue correct? Explain why.

Yes because the cylinders have the same measurements one is just tilted

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 25

25 Sue believes that the two cylinders shown in the diagram below have equal volumes.



Is Sue correct? Explain why.

$V = \frac{1}{3} 10 \cdot 11.5$

$a^2 + b^2 = c^2$
$5^2 + b^2 = 11.5^2$
$25 + b^2 = 132.25$

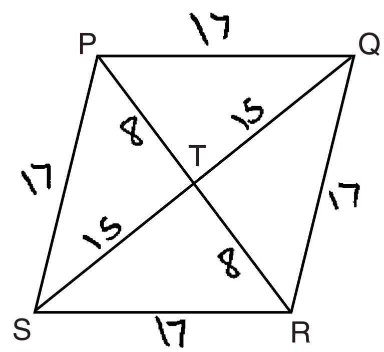
$V = \frac{1}{3} B \cdot h$
$V = \frac{1}{3} 5 \cdot 11.5$
$V = 19.16$

$V = \frac{1}{3}\ 10 \cdot 11.5$
$V = 40$

Sue is incorrect, their volumes are not equal.

Score 0: The student gave a completely incorrect response.

26 In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point T, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



$a^2 + b^2 = c^2$

$8^2 + 15^2 = c^2$

$64 + 225 = c^2$

$\sqrt{289} = \sqrt{c^2}$

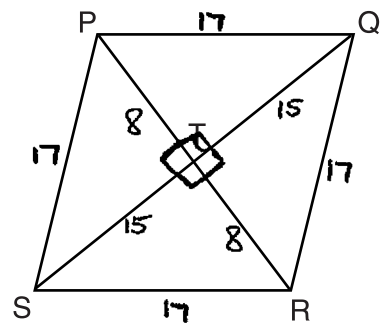
$17 = c$

perimeter = 68

Score 2: The student gave a complete and correct response.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point T, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



P = 68

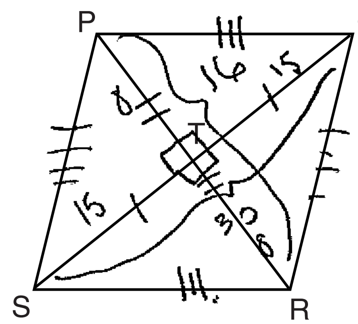
Score 2: The student gave a complete and correct response.

Question 26

26 In the diagram of rhombus *PQRS* below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point *T*, $PR = 16$, and $QS = 30$. Determine and state the perimeter of *PQRS*.

$8\sqrt{5}$

Diagonals bisect each other and are $\perp$.



$8\sqrt{5}$

Opposite sides are $\cong$

$\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$

$a^2+b^2=c^2$
$8^2+16^2=PQ^2$
$64+256=PQ^2$
$\sqrt{320}=\sqrt{PQ^2}$
$\sqrt{320}=PQ$
$\sqrt{64}\sqrt{5}=PQ$
$8\sqrt{5}=PQ$

$SP=PQ$
$SP=8\sqrt{5}$

$a^2+b^2=c^2$
$8^2+15^2=SP^2$
$64+225=SP^2$
$\sqrt{289}=\sqrt{SP^2}$
$17=SP$

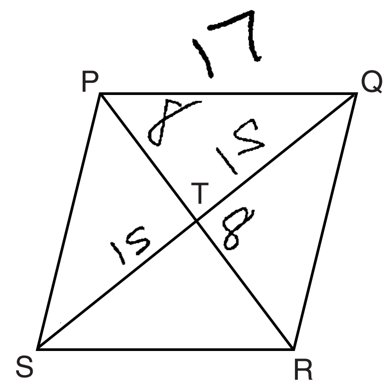
$QR=SP$
$QR=17$

$P=PQ+QR+RS+SP$
$=8\sqrt{5}+17+8\sqrt{5}+17$
$P=16\sqrt{5}+34$

Score 1: The student made an error in finding the lengths of sides $\overline{PQ}$ and $\overline{RS}$.

Question 26

26 In the diagram of rhombus *PQRS* below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point *T*, $PR = 16$, and $QS = 30$. Determine and state the perimeter of *PQRS*.



$8^2 + 15^2 = C^2$

$64 + 225 = C^2$

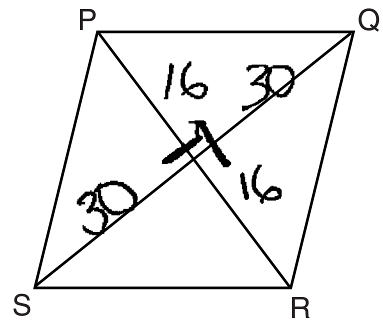
$\sqrt{289} = \sqrt{C^2}$

$C = 17$

Score 1: The student found the length of the side of the rhombus, but did not find the perimeter of the rhombus.

Question 26

26 In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point T, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



16 + 16
30 + 30
32
60

A = d' + d² ×

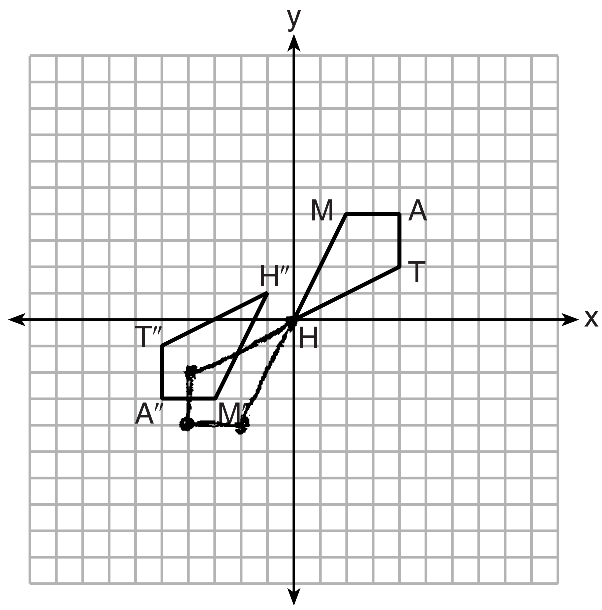
P = 92

$$\begin{array}{r} 32 \\ \times\ 60 \\ \hline 1{,}920 \end{array}$$

Score 0: The student gave a completely incorrect response.

Question 27

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.

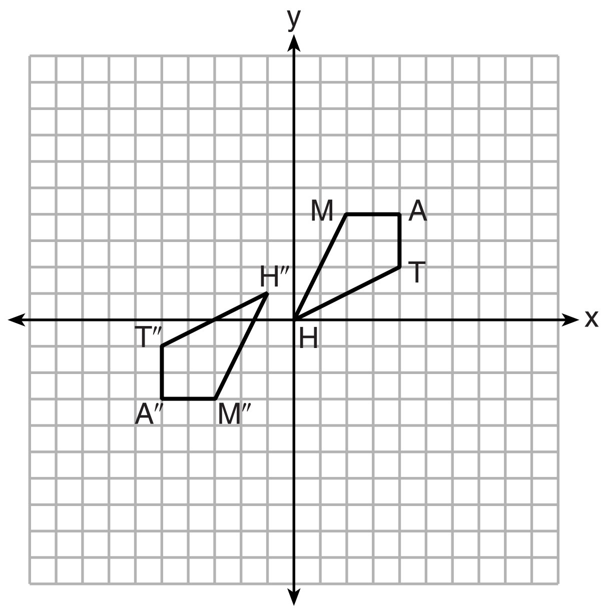


Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

a reflection over the origin → a translation of (x − 1, y + 1)

Score 2: The student gave a complete and correct response.

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

$\boxed{T_{-1,1} \circ D_{H,-1}}$

M (2,4)
A (4,4)
T (4,2)
H (0,0)

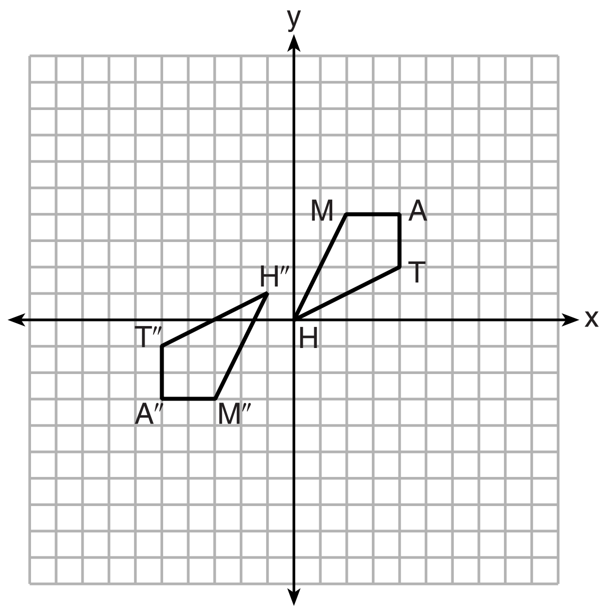
M' (-2,-4)
A' (-4,-4)
T' (-4,-2)
H' (0,0)

M'' (-3,-3)
A'' (-5,-3)
T'' (-5,-1)
H' (-1,1)

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.



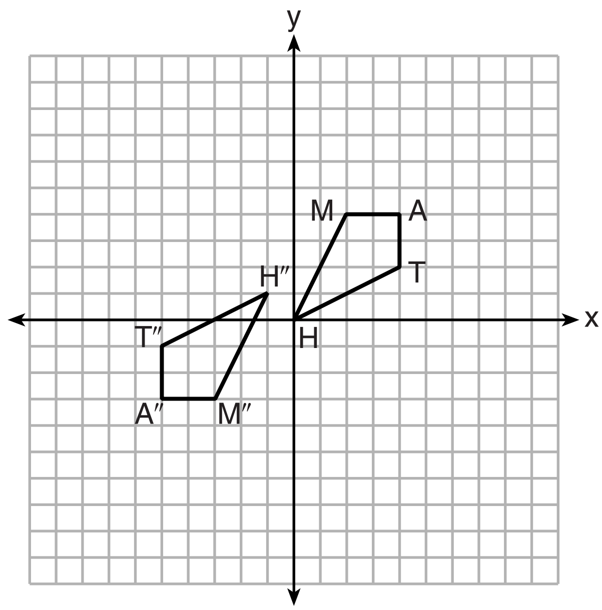
Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

Rotate quadrilateral MATH 180° about point $(-\frac{1}{2}, \frac{1}{2})$.

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.

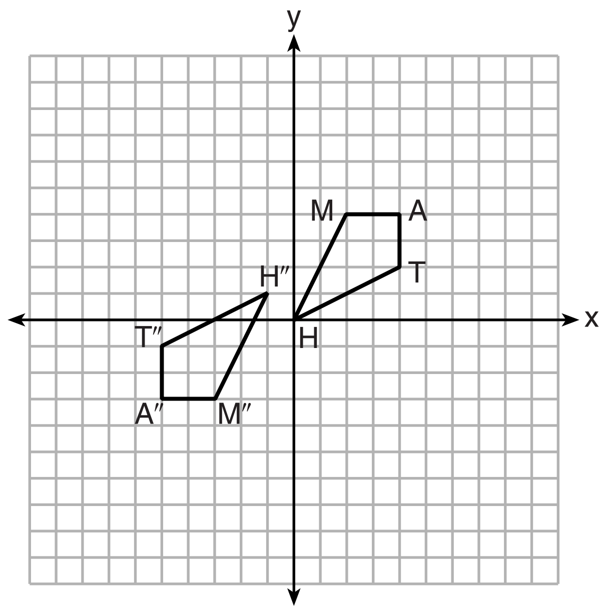


Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

- MATH was translated up one unit.
- MATH was translated to the left one unit.
- MATH was rotated 180° clockwise.

Score 1: The student wrote an incomplete transformation by not stating the center of rotation.

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.



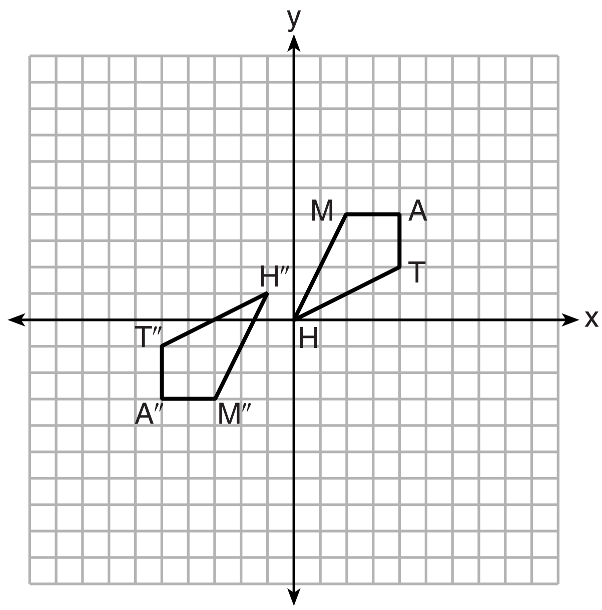
Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

Rotate 180° from Point H. Then translate down 1 and right 1

Score 1: The student had a partially correct sequence of transformations.

Question 27

27 Quadrilateral *MATH* and its image *M″A″T″H″* are graphed on the set of axes below.



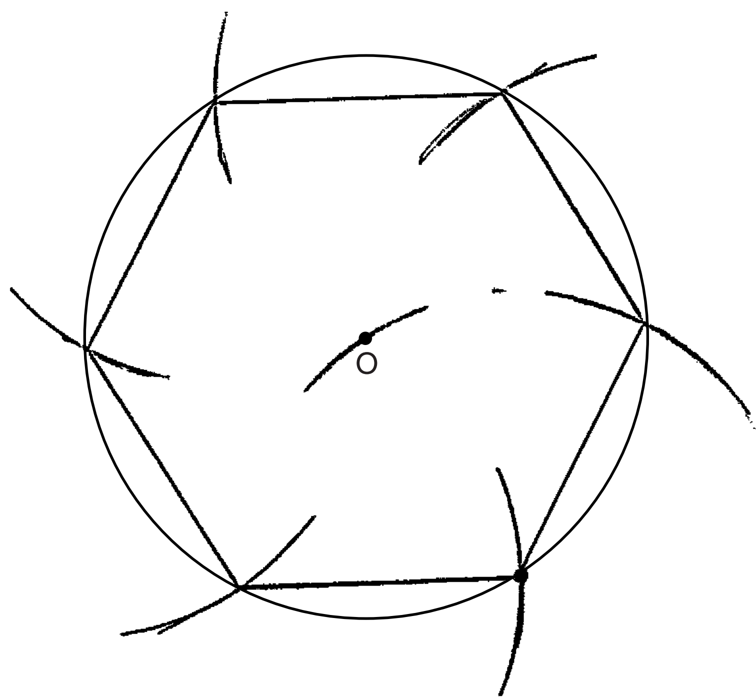
Describe a sequence of transformations that maps quadrilateral *MATH* onto quadrilateral *M″A″T″H″*.

Spin it and move it over

Score 0: The student gave an incomplete description of the rotation (spin) and described the translation (move) incorrectly.

Question 28

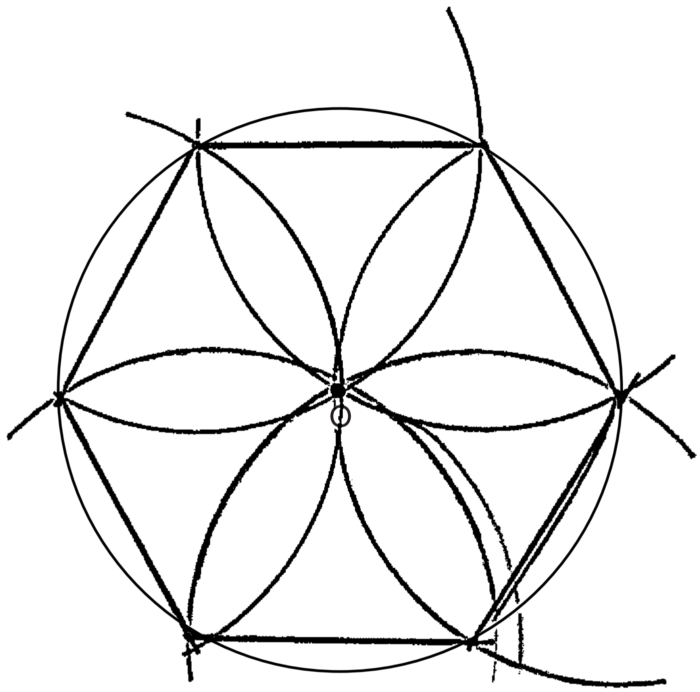
28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O. [Leave all construction marks.]



Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

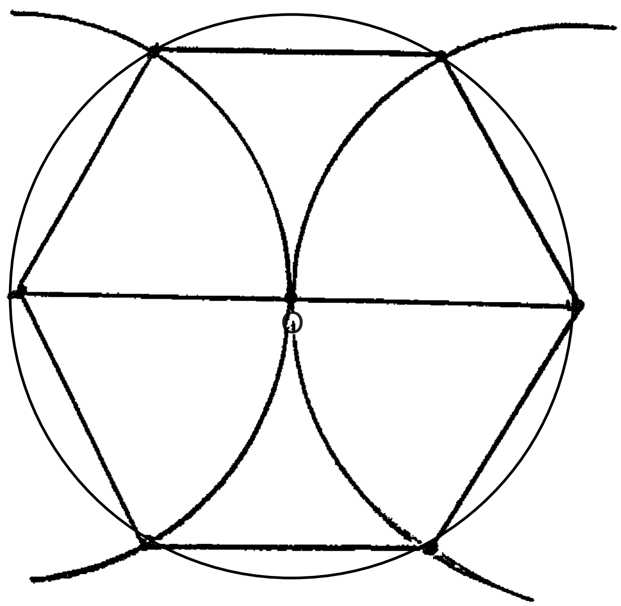
28 Using a compass and straightedge, construct a regular hexagon inscribed in circle O. [Leave all construction marks.]



Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

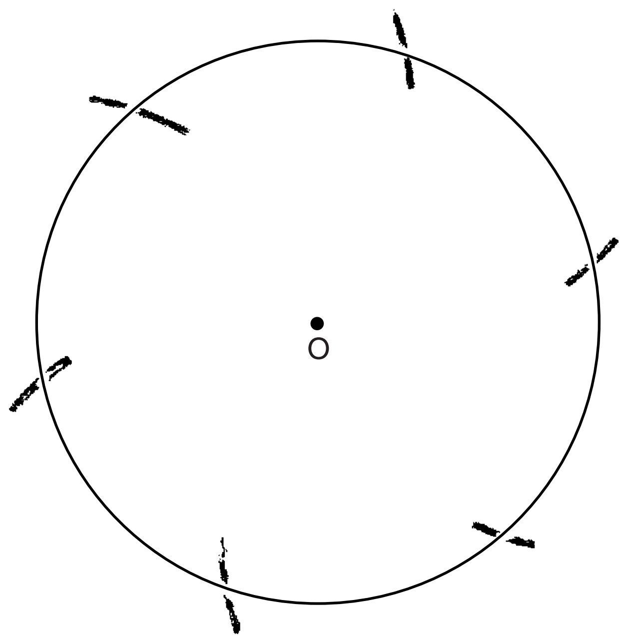
28 Using a compass and straightedge, construct a regular hexagon inscribed in circle *O*.
[Leave all construction marks.]



Score 2: A correct construction is drawn showing all appropriate arcs.

Question 28

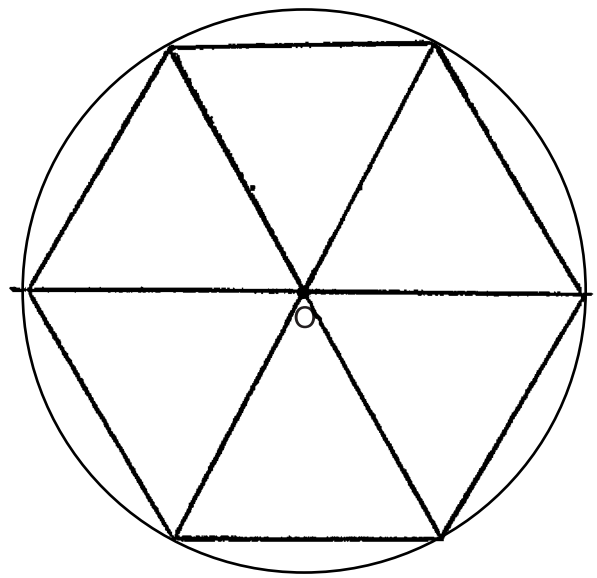
28 Using a compass and straightedge, construct a regular hexagon inscribed in circle *O*. [Leave all construction marks.]



Score 1: The student drew an appropriate construction, but did not draw the hexagon.

Question 28

28 Using a compass and straightedge, construct a regular hexagon inscribed in circle *O*.
[Leave all construction marks.]



Score 0: The student had a drawing that is not a construction.

Question 29

29 The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

$A' = (1, 1.5)$
$B' = (2.5, -.5)$

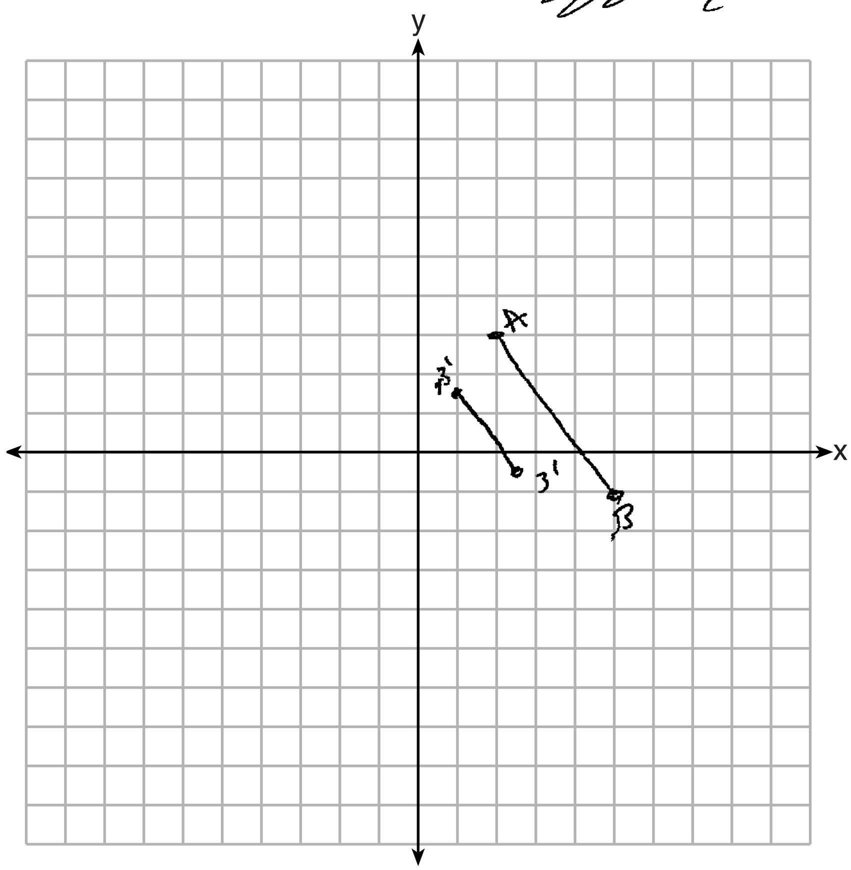
$d = \sqrt{(2.5-1)^2 + (-.5-1.5)^2}$

$d = \sqrt{(1.5)^2 + (-2)^2}$

$d = \sqrt{2.25+4}$

$\cancel{d = \sqrt{6.25}}$

$d = 2.5$



Score 2: The student gave a complete and correct response.

Question 29

29 The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

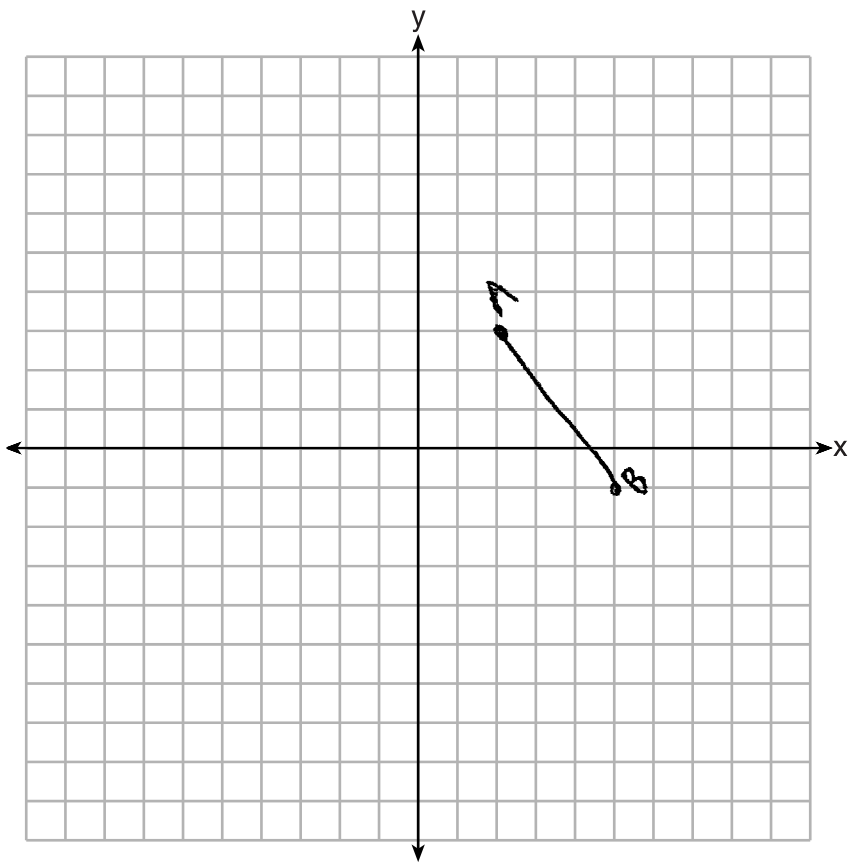
$d = \sqrt{(x-x)^2 + (y-y)^2}$

$= \sqrt{(5-2)^2 + (3--1)^2}$

$= \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\boxed{= 5} \cdot \frac{1}{2} = 2.5$

$\boxed{\text{Length of } \overline{A'B'} = 2.5 \text{ units}}$



Score 2: The student gave a complete and correct response.

Question 29

29 The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

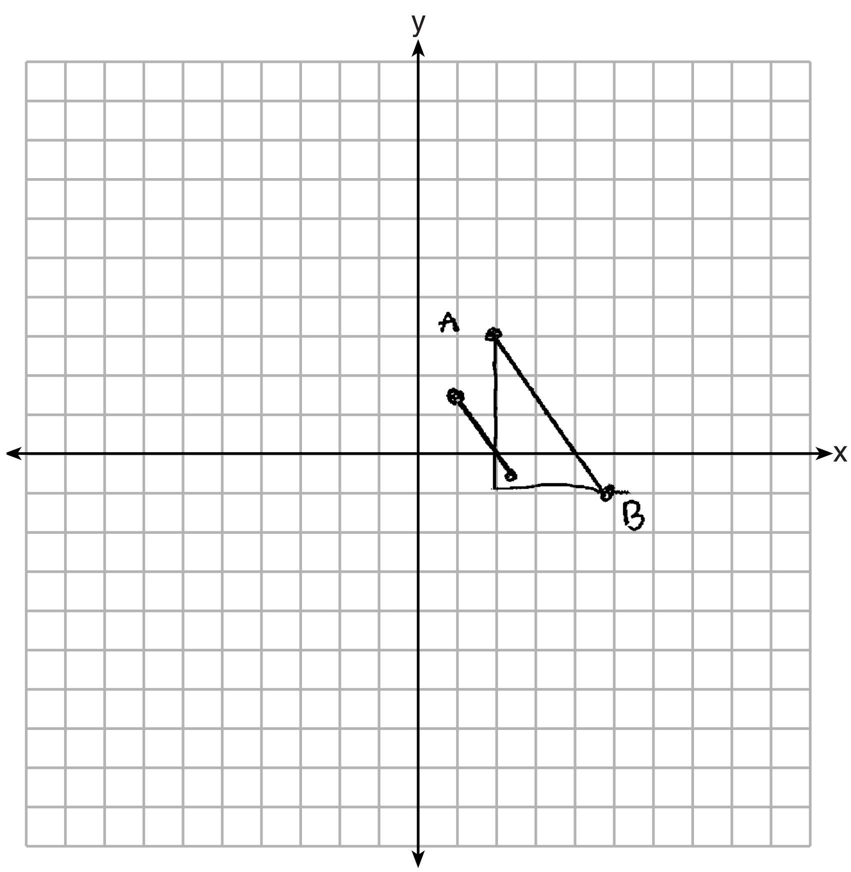
$\boxed{\overline{A'B'} = 2.5}$

$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

$\sqrt{25} = \sqrt{c^2}$

$c = 5$



Score 2: The student gave a complete and correct response.

29 The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

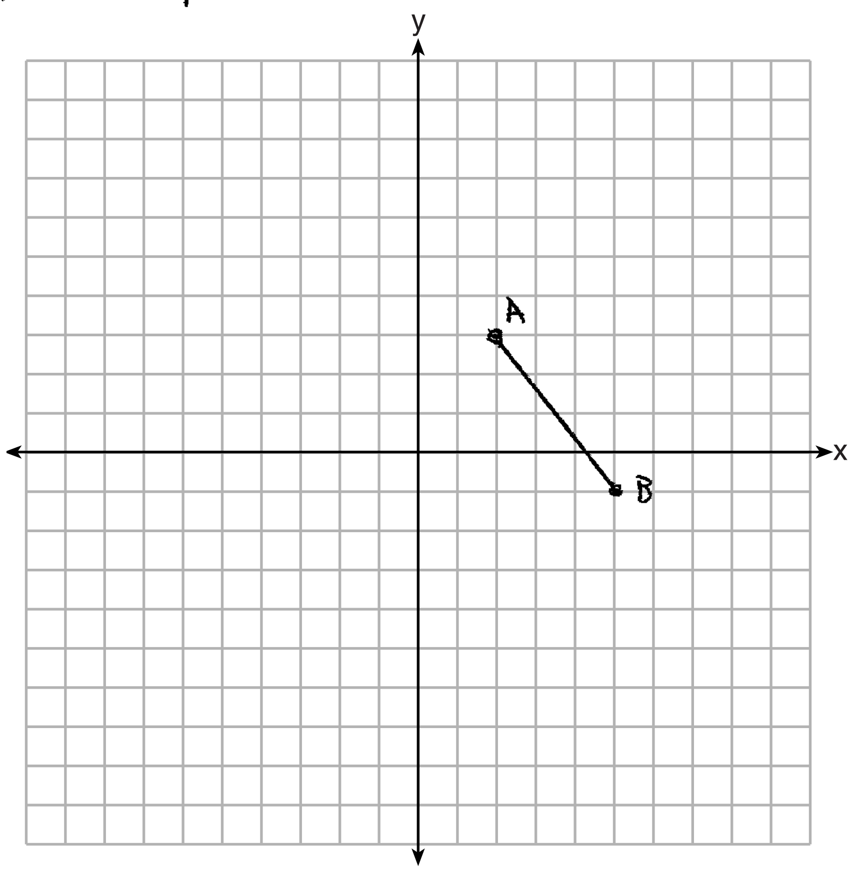
$d=\sqrt{(2-5)^2+(3-+1)^2}$

$\sqrt{(3)^2+(4)^2}$

$\sqrt{9+16}$

$\sqrt{25}$

$d=5$



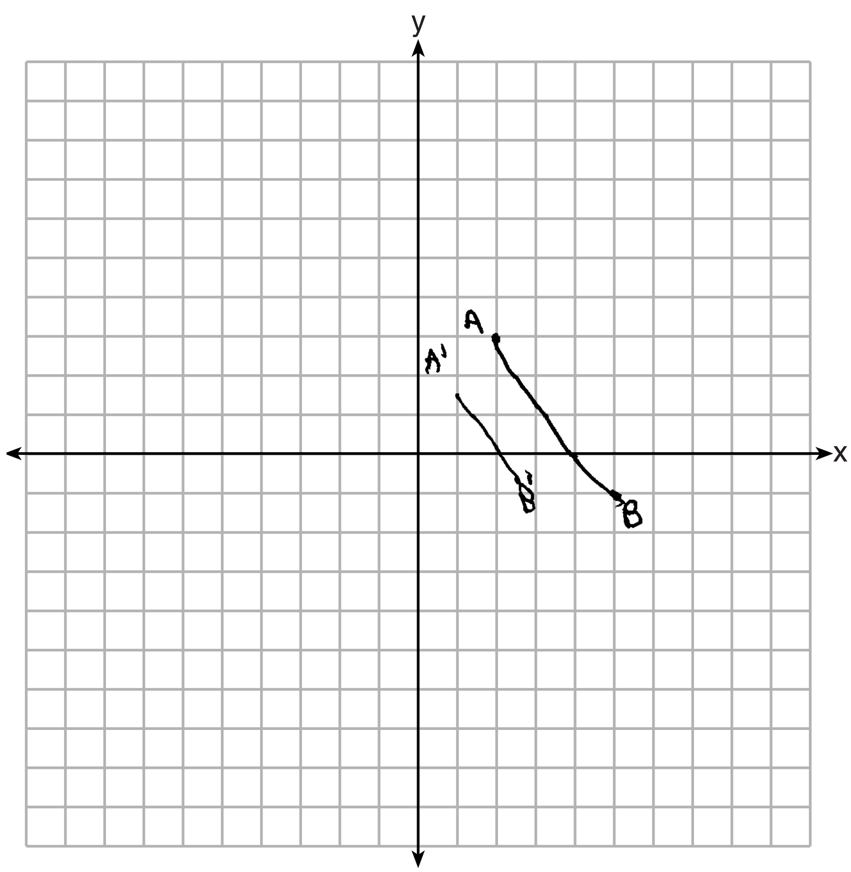
Score 1: The student found the length of $\overline{AB}$, but no further correct work is shown.

Question 29

29 The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

[The use of the set of axes below is optional.]

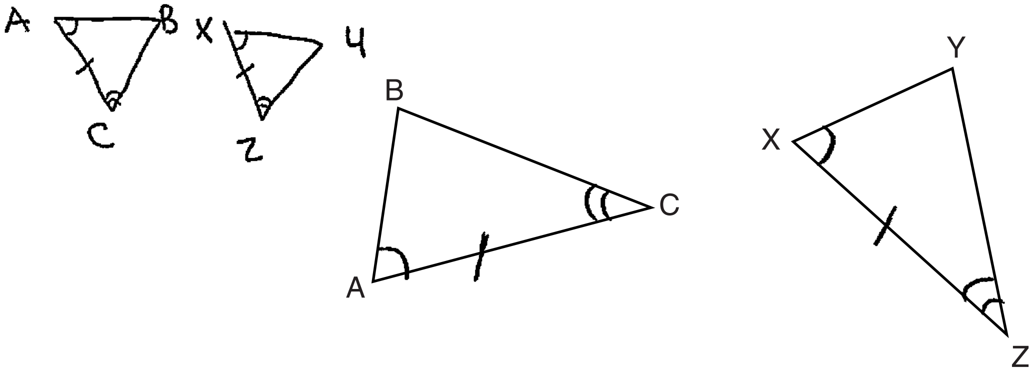
$\overline{A'B'}$ is shorter than $\overline{AB}$



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



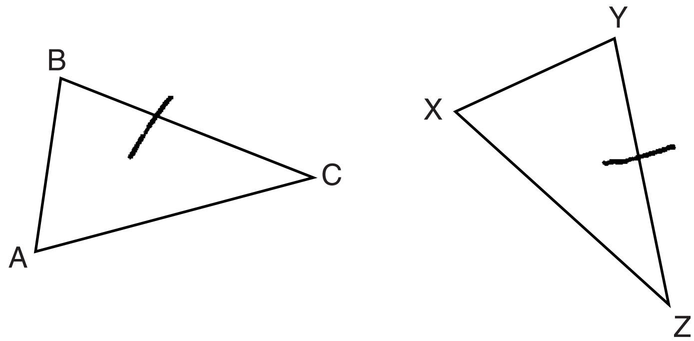
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ since the $\triangle$s are $\cong$ by ASA since $\angle A \cong \angle X$, $\overline{AC} \cong \overline{XZ}$ and $\angle C \cong \angle Z$ because they can be mapped on to eachothr in a series of rigid motions which preserve side length + angle measure. So since $\triangle ABC \cong \triangle XYZ$, $\overline{BC} \cong \overline{YZ}$ ~~b~~ because corresponding sides of $\cong$ $\triangle$s are $\cong$

Score 2: The student gave a complete and correct response.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



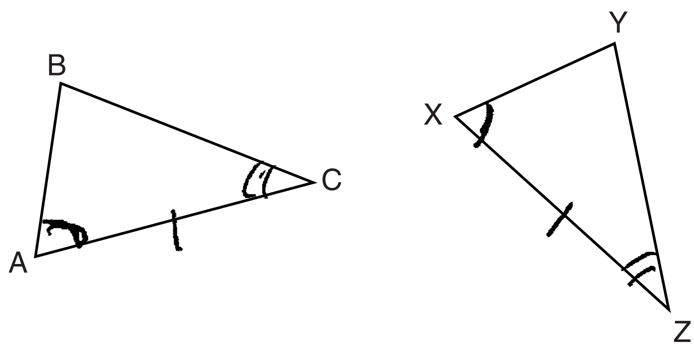
Determine and state whether $\boxed{\overline{BC} \cong \overline{YZ}}$. Explain why.

yes because basic rigid motions preserve segment length and angle measurement.

Score 1: The student gave an incomplete explanation by not stating the triangle congruency and not stating corresponding congruent sides.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



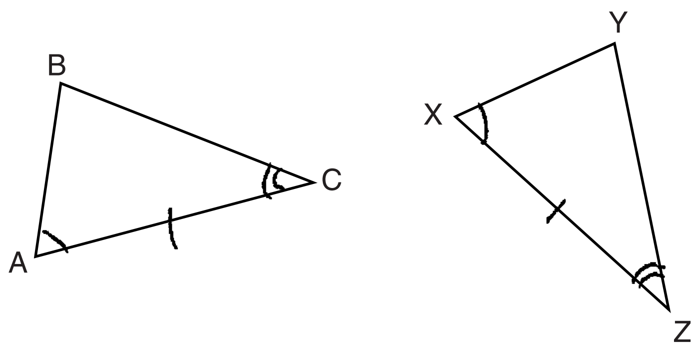
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong \overline{YZ}$ because corresponding parts of congruent triangles are congruent

Score 1: The student gave an incomplete explanation.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



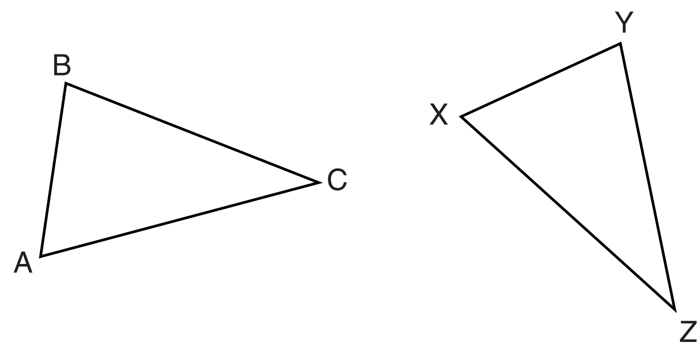
Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

yes $\overline{BC} \cong \overline{YZ}$ because of ASA $\cong$ ASA.

Score 1: The student gave an incomplete explanation.

Question 30

30 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

$\overline{BC} \cong YZ$ because the triangles look the same.

Score 0: The student wrote an incorrect explanation.

Question 31

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$\frac{6}{2} = 3^2 = 9$

$x^2 - 6x + y^2 + 8y = 56$

$x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16$

$(x-3)^2 + (x+4)^2 = 81$

$\sqrt{81} = 9$

$\boxed{\begin{array}{l} \text{radius} = 9 \\ \text{center} = (3, -4) \end{array}}$

Score 2: The student gave a complete and correct response.

Question 31

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$(x^2-6x)+(y^2+8y)=56$

$(x^2-6x+9)+(y^2+8y+16)=81$

$(x-3)^2+(y+4)^2=81$

center = $(-3, 4)$

radius = 9

Score 1: The student had incorrect signs on the coordinates for the center of the circle.

31 Determine and state the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + y^2 - 6x = 56 - 8y$.

$x^2+y^2-6x=56-8y$
$+8y \quad +8y$

$x^2-6x+y^2+8y=56$

$x^2-6x+9+y^2+8y+16=56$

$(x-3)(x-3)+(y+4)(y+4)=56$

$(x-3)^2+(y+4)^2=56$

$x=-3 \quad y=+4$

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

Mid of $\overline{PQ} = \left(\frac{-3+-1}{-2}, \frac{-1+7}{2}\right)$

$= \left(\frac{-4}{2}, \frac{6}{2}\right)$

$A = (-2, 3)$

$B = \left(\frac{3+-1}{2}, \frac{7+3}{2}\right)$

$= \left(\frac{2}{2}, \frac{10}{2}\right)$

$B = (1, 5)$

Slope$_{\overline{AB}} = \frac{5-3}{1-(-2)}$

$= \frac{2}{3}$

Slope$_{\overline{PR}} = \frac{3-(-1)}{3-(-3)}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Since the slopes of $\overline{AB}$ and $\overline{PR}$ are the same, $\overline{AB} \parallel \overline{PR}$

$AB = \sqrt{(1-(-2))^2+(5-3)^2}$

$= \sqrt{(3)^2+(2)^2}$

$= \sqrt{9+4}$

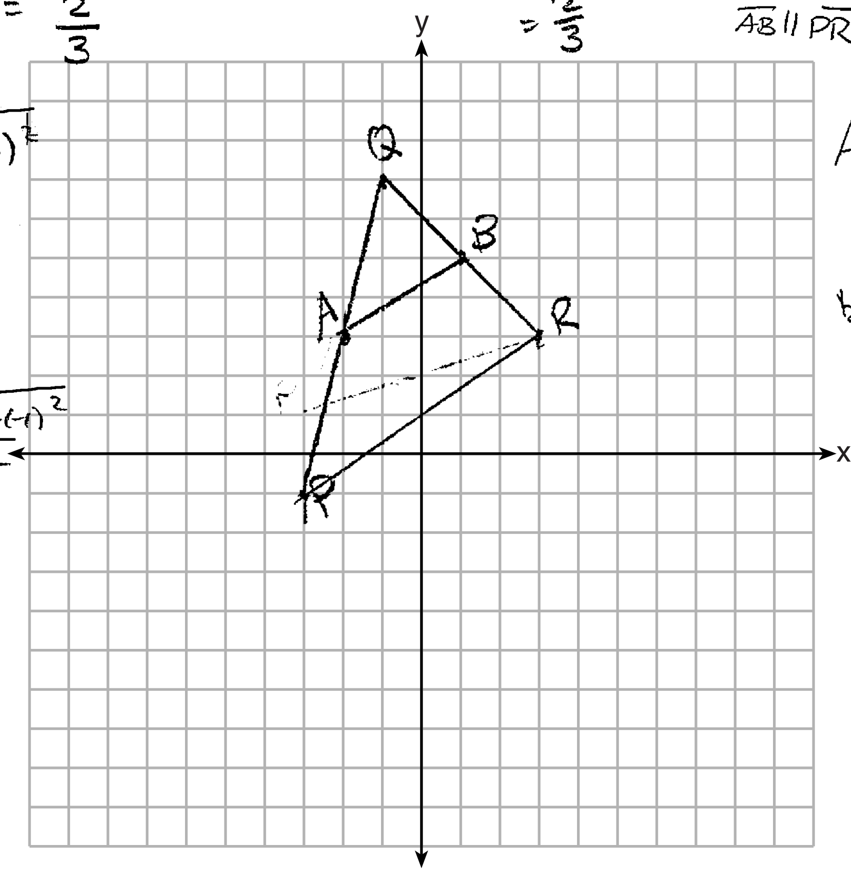
$= \sqrt{13}$

$PR = \sqrt{(3-(-3))^2+(3-(-1))^2}$

$= \sqrt{(6)^2+(4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$



AB is half the length of PR because $\sqrt{13}$ is half of $\sqrt{52}$.

Score 4: The student gave a complete and correct response.

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

$A(-1,3)$

$B(0,1)$

slope

$AB: \frac{-1}{1} = -1$

$QR: -\frac{1}{1}\ -1$

∴ $\overline{AB} \parallel \overline{QR}$ since both segments have equal slopes

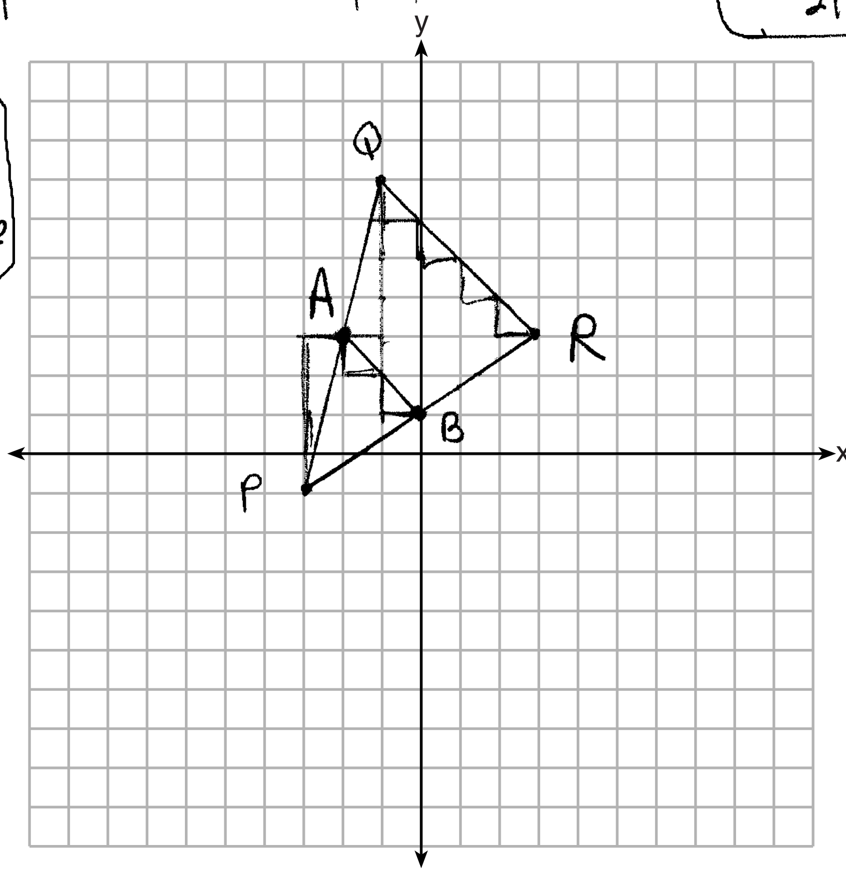
length

$|AB| = \sqrt{2^2+2^2} = \sqrt{8}$

$|QR| = \sqrt{4^2+4^2} = \sqrt{32}$

$\frac{1}{2}|QR| = \frac{1}{2}\sqrt{32} = \frac{1}{2}\cdot 4\sqrt{2} = 2\sqrt{2}$

$|AB| = \sqrt{8} = 2\sqrt{2}$ $\boxed{\therefore \frac{1}{2}|QR| = |AB|}$



Score 3: The student did correct work to show that the midsegment of a triangle is parallel and half the length to the third side of the triangle, but used the wrong midsegment.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

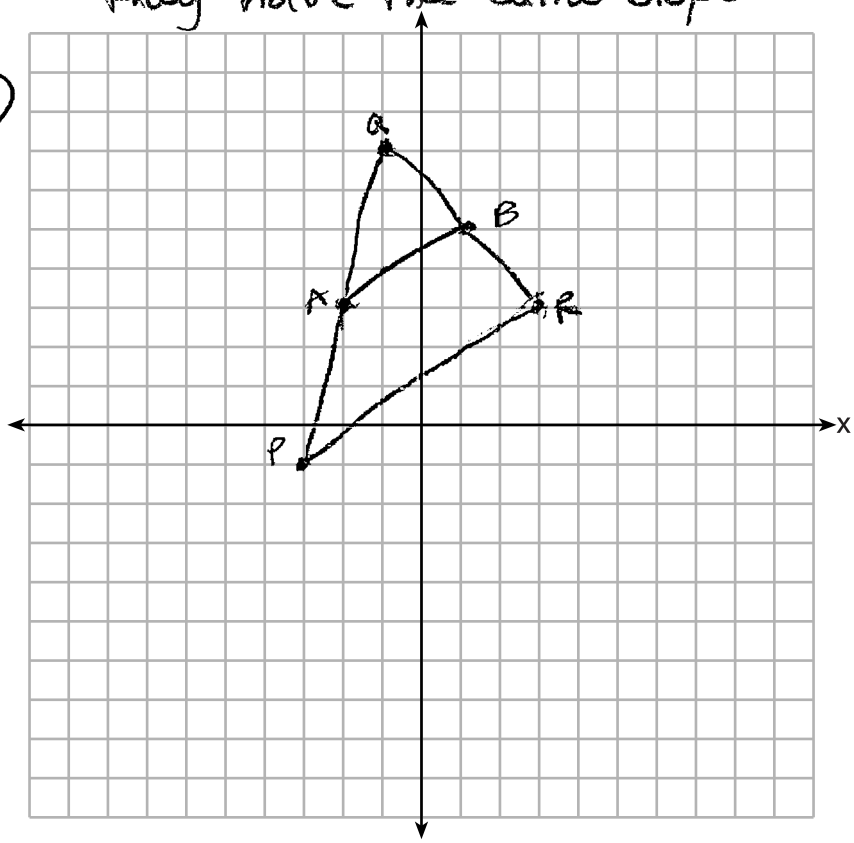
$$PR = \frac{3 - -1}{3 - -3} = \frac{3+1}{3+3} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

$$AB = \frac{5 - 3}{1 - -2} = \boxed{\frac{2}{3}}$$

$\overline{PR}$ and $\overline{AB}$ are ∥ because they have the same slope.

A(−2, 3)

B(1, 5)



Score 2: The student proved $\overline{AB} \parallel \overline{PR}$, but no further correct work is shown.

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

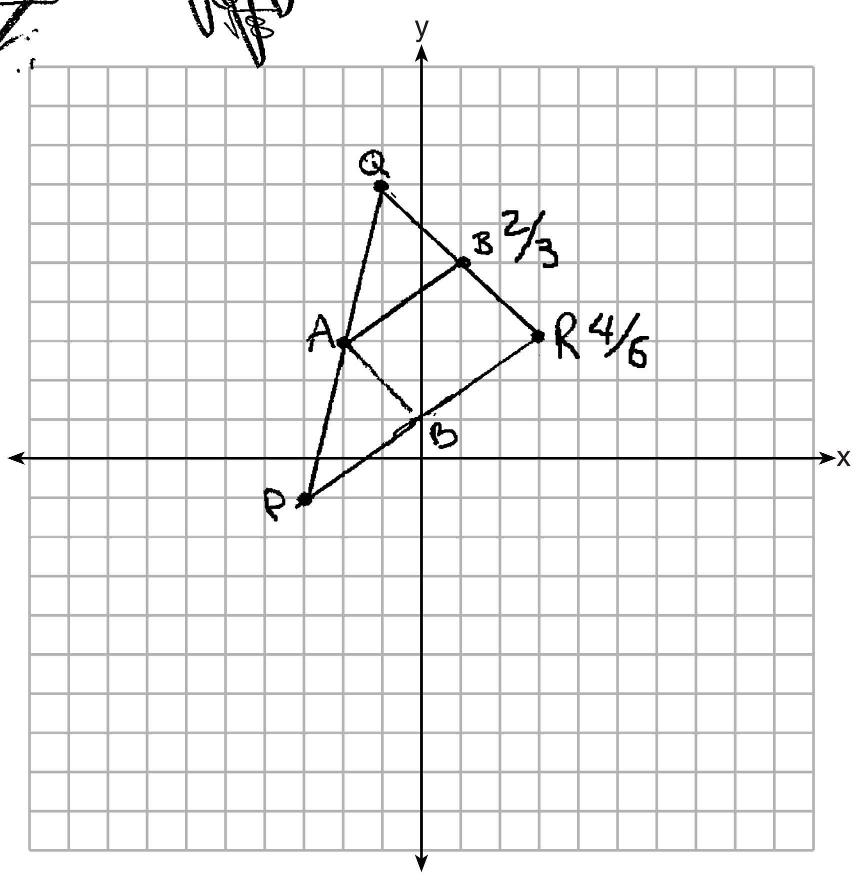
$M_{PQ} = \left(\frac{-3-1}{2}, \frac{-1+7}{2}\right)$

$M_{PQ} = (-2, 3)$

$M_{PR} = \left(\frac{-3+3}{2}, \frac{-1+3}{2}\right)$

$M_{PR} = (0, 1)$

$\overline{AB}$ is parallel to $\overline{PR}$ because their slopes are 2/3 for $\overline{AB}$ and 2/3 for $\overline{PR}$. $\overline{AB}$ is half of $\overline{PR}$ because $\overline{AB}$ is a midsegment.



Score 2: The student proved $\overline{AB} \parallel \overline{PR}$, but no further correct work is shown.

Question 32

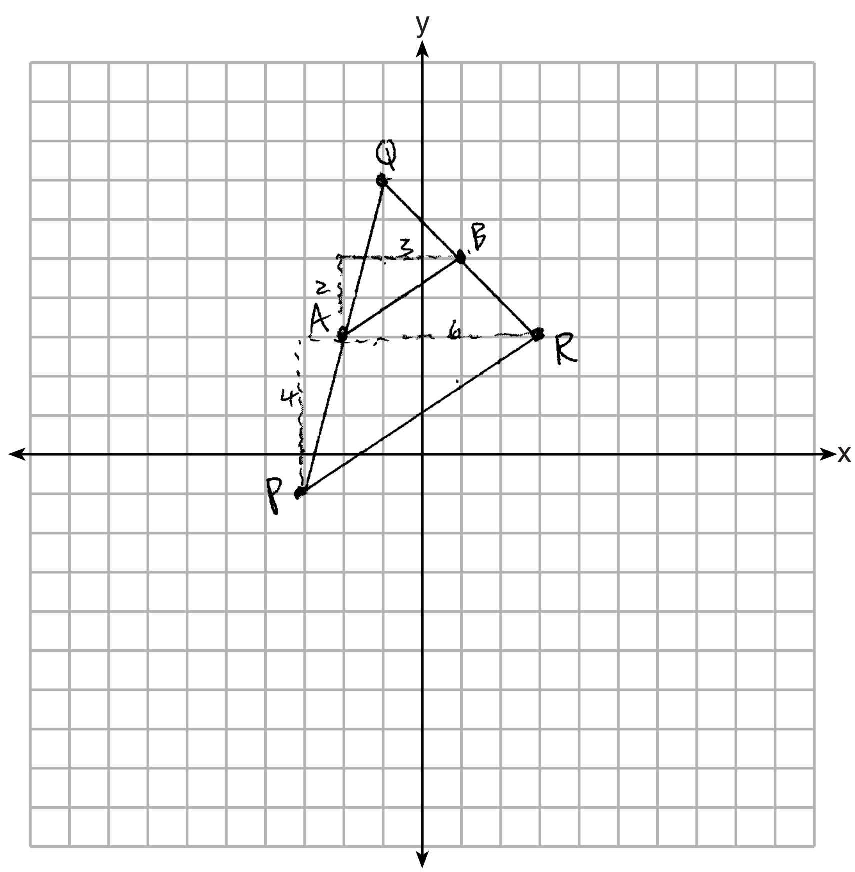
32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

Slope $\overline{AB} = \frac{2}{3}$

Slope $\overline{PR} = \frac{4}{6} = \frac{2}{3}$

$A(-2,3)$
$B(1,5)$



Score 1: The student found the slopes of $\overline{AB}$ and $\overline{PR}$, but no concluding statement is written.

Question 32

32 Triangle PQR has vertices $P(-3,-1)$, $Q(-1,7)$, and $R(3,3)$, and points A and B are midpoints of $\overline{PQ}$ and $\overline{RQ}$, respectively. Use coordinate geometry to prove that $\overline{AB}$ is parallel to $\overline{PR}$ and is half the length of $\overline{PR}$.

[The use of the set of axes below is optional.]

$m=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$=\left(\frac{-3+-1}{2}, \frac{-1+7}{2}\right)$

$=\left(-\frac{4}{2}, \frac{6}{2}\right)$

$=(-2,3)$

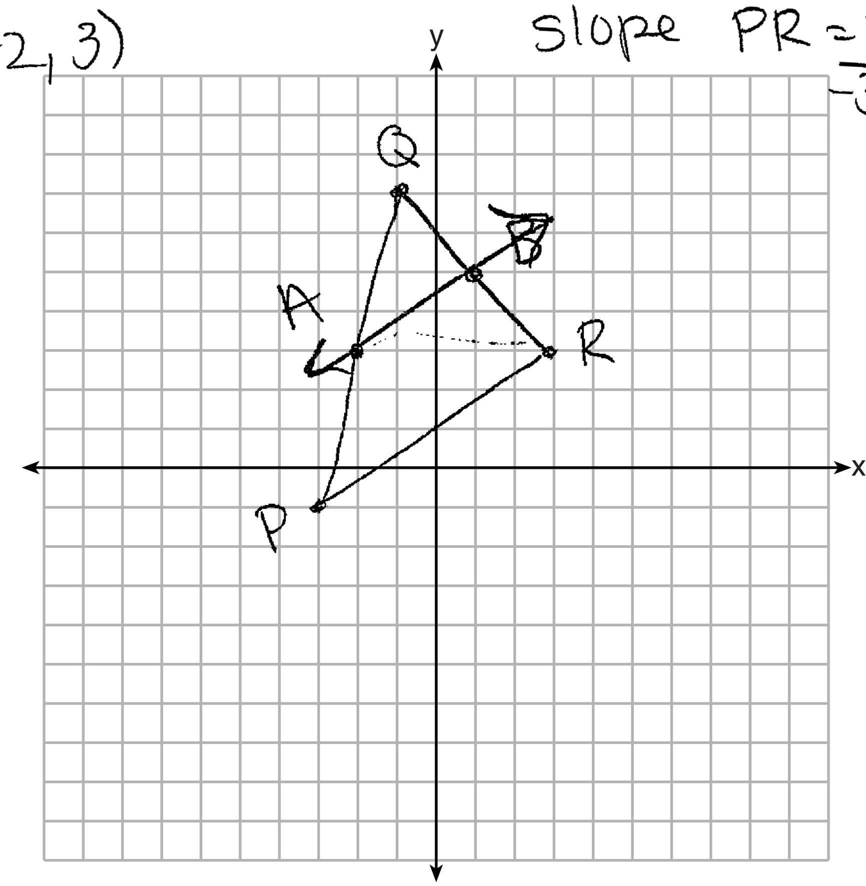
$m=\left(\frac{-1+3}{2}, \frac{7+3}{2}\right)$

$=\left(\frac{2}{2}, \frac{10}{2}\right)$

$=(1,5)$

slope $AB = \frac{5-3}{1+2} = \frac{2}{3}$

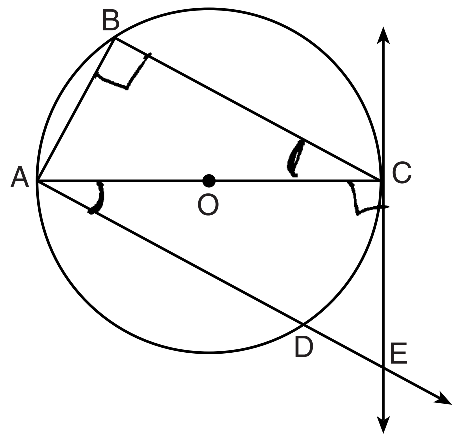
slope $PR = \frac{3-1}{-3+3} = \frac{2}{0}$?



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



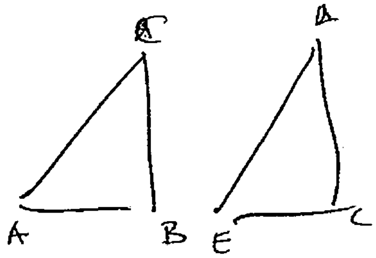
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

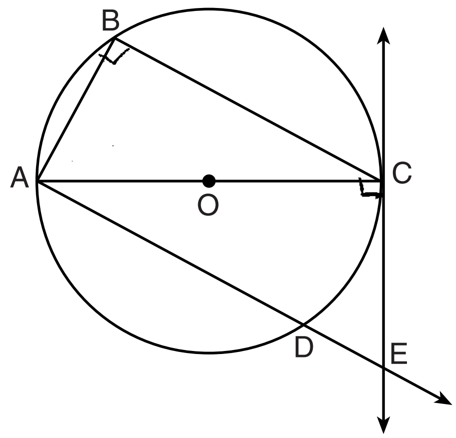
1. Circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$ Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$ Chord $\overline{AB}$ is drawn	1. Given
2. $\angle B$ is a right $\angle$	2. An angle inscribed in a semicircle is a right $\angle$.
3. $\overleftrightarrow{EC} \perp \overline{OC}$	3. A radius is perpendicular to a tangent at the point of contact.
4. $\angle ECA$ is a right $\angle$	4. Perpendicular lines form right angles
5. $\angle B \cong \angle ECA$	5. All right angles are $\cong$
6. $\angle BCA \cong \angle CAE$	6. If 2 parallel lines are cut by a transversal, the alternate interior angles are $\cong$
7. $\triangle ABC \sim \triangle ECA$	7. $AA \cong AA$
8. $\frac{BC}{CA} = \frac{AB}{EC}$	8. Corresponding sides of similar triangles are in proportion

Score 4: The student gave a complete and correct response.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overleftrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.





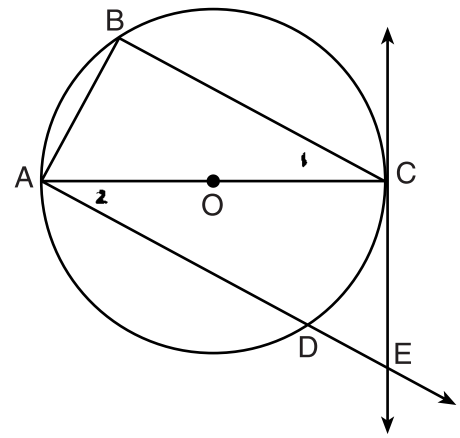
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statements	Reasons
1) In circle O, tangent $\overleftrightarrow{EC}$ is drawn to dia-meter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overleftrightarrow{ADE}$, and chord $\overline{AB}$ is drawn	1) Given.
2) $\angle ABC$ is a right angle (a)	2) Since $\widehat{AOC}$ is the diameter, it splits the circle into two congruent arcs measuring 180°, as a circle has 360°. Then $\angle ABC$ is the inscribed angle of $\widehat{AOC}$, which should measure 180°. Inscribed angles are half the measure of the arc, therefore, $\angle ABC$ is a right angle as it measures 90°.
3) $\angle ACE$ is a right angle.	3) A tangent intersecting with a diameter forms a 90° angle
4) $\angle ACE \cong \angle ABC$ (a)	4) Right angles are congruent.
5) $\angle BCA \cong \angle EAC$ (a)	5) Parallel lines intersected by a trans-versal forms congruent alternate interior angles
6) $\triangle BCA \sim \triangle CAE$	6) AA (4,5)
7) $\frac{BC}{CA} = \frac{AB}{EC}$	7) Similar triangles have proportional relationships with corresponding sides.

Score 4: The student gave a complete and correct response.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



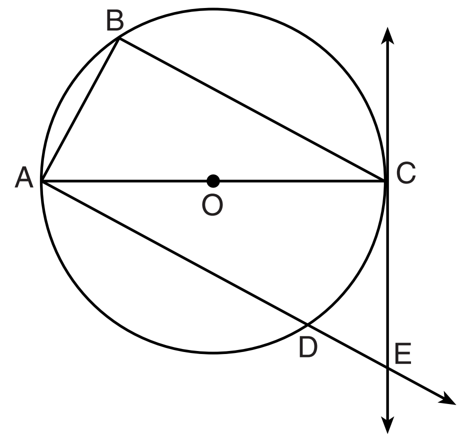
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statement	Reason
① In Circle O, tangent EC is drawn to diameter AC. Chord $\overline{BC}$ // secant $\overline{ADE}$	① Given
② ∡1 ≅ ∡2	② If 2 // lines are cut by a transversal, alt int ∡'s are ≅
③ ∡B is a rt. ∡	③ An angle inscribed in a semi-circle is a rt. ∡
④ $\overline{AC} \perp \overline{CE}$	④ A tangent is ⊥ to a diameter at its point of tangency
⑤ ∡ECA is a rt. ∡	⑤ ⊥ Lines form rt. ∡'s
⑥ ∡B ≅ ∡ECA	⑥ All rt. ∡'s are ≅
⑦ △ABC ~ △ECA	⑦ AA
✗ ⑧ BC × EC = AB × CA	⑧ Corr. sides of similar △'s are proportional
⑨ $\frac{BC}{CA} = \frac{AB}{EC}$	⑨ The product of the means equals the product of the extremes

Score 3: The student proved $\triangle ABC \sim \triangle ECA$, but no further correct work is shown.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

$\angle BCA \cong \angle EAC$ because $\overline{BC} \parallel \overline{AE}$ cut by a transversal makes $\cong$ alternate interior angles

$\angle ABC$ is a right angle because its an inscribed angle that intercepts a semi-circle of circle O.

$\overline{AC} \perp \overline{CE}$ because a diameter that intersects a tangent forms $\perp$ lines

$\angle ACE$ is a right angle because $\perp$ lines form right angles

$\angle\ BC \cong \angle ACE$ because all right angles are $\cong$

$\overline{AC} \cong \overline{AC}$ by reflexive property

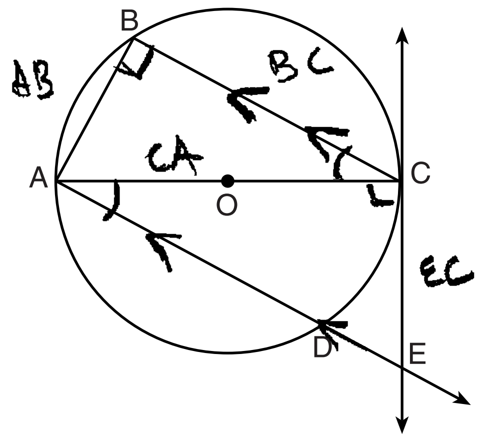
$\triangle ABC \cong \triangle ACE$ by AAS

$\frac{BC}{CA} = \frac{AB}{EC}$ because CPCTC

Score 2: The student proved $\angle BCA \cong \angle EAC$ and $\angle ABC \cong \angle ACE$, but no further correct work is shown.

Question 33

33 In the diagram below of circle O, tangent $\overrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overleftrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



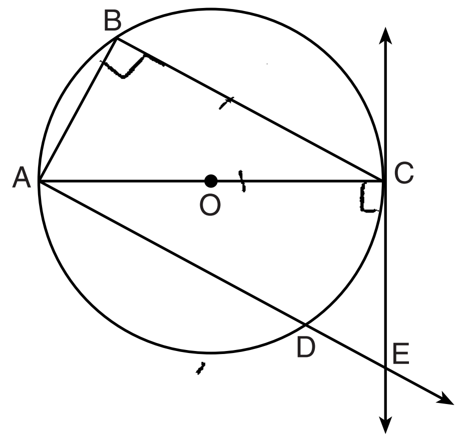
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

S	R
1.) $\angle BCA \cong \angle CAD$	1.) Alternate interior angles of parallel lines are equal
2.) $\angle ABC = 90°$	2.) any point on a circle connected to two end points of a diameter in the circle, creating a triangle, is a right angle in which it $= 90°$
3.) $\angle ACE = 90°$	3.) A radius to a line that passes through the circle once at a point makes two right angles.
4.) $\angle ABC = \angle ACE$	4.) Right angles are equal to Right angles
5.) $\triangle BCA \sim \triangle CAE$	5.) AA theorem.
6.) $\frac{BC}{CA} = \frac{AB}{EC}$	6.) proportions for sides lengths that apply to the same opposite congruent angles may be used in the same place of the lines

Score 2: The student did not include the given and had an incorrect reason in step 6.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

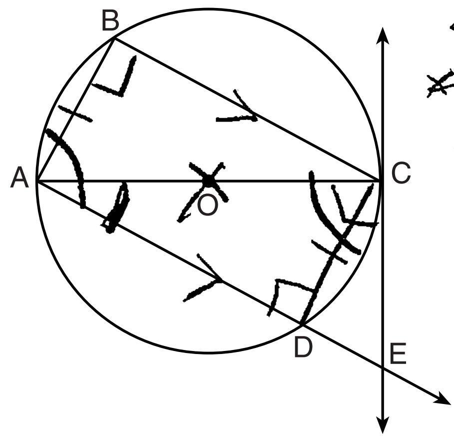
Statements	Reasons
① $\overline{EC}$ is drawn to diameter AC $\overline{BC} \parallel \overline{ADE}$, $\overline{AB}$ is drawn	① Given
② ∡ABC is a right angle	② If an angle is inscribed in the semicircle ~~...~~, it is a right angle
③ ∡ACE is a right angle	③ If a tangent is drawn to a circle, then the angle formed is a right angle
④ △ABC is a right triangle △ACE is a right triangle	④ Definition of a right triangle
⑤ △ABC ~ △ACE	⑤ AA~
⑥ $BC \cdot EC = CA \cdot AB$	⑥ Product of the means is equal to the product of the extremes
⑦ $\frac{BC}{CA} = \frac{AB}{EC}$	⑦ Substitution

Q.E.D

Score 1: The student had one correct relevant statement and reason in step 2.

Question 33

33 In the diagram below of circle O, tangent $\overleftrightarrow{EC}$ is drawn to diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, and chord $\overline{AB}$ is drawn.



$\measuredangle ABC = 90°$ b/c opp diamiter

$\measuredangle ACE = 90°$ b/c tangents are 90° from diameter

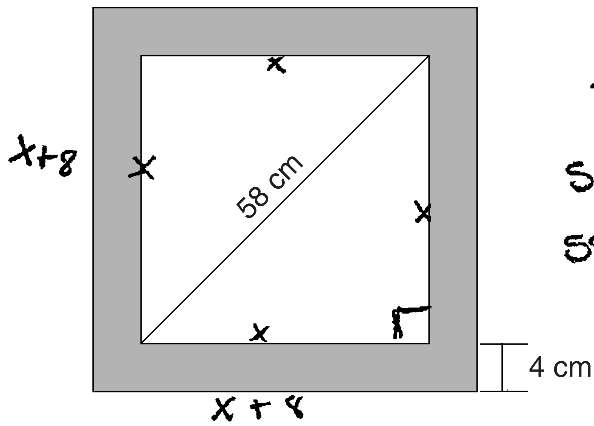
Prove: $\frac{BC}{CA} = \frac{AB}{EC}$

Statement	Reasons
1) $\overleftrightarrow{EC}$ is tangent to circle O's diameter $\overline{AC}$. Chord $\overline{BC}$ is parallel to secant $\overrightarrow{ADE}$, Chord $\overline{AB}$ is drawn	1) Givens
2) $\overline{AC} \cong \overline{AC}$	2) Reflexive
3) $\overline{CD} \cong \overline{AB}$	3) if 2 parralel lines are cut by a cord then that cord length is congruent to any other cord that are cut to the same parralel lines
4) $\measuredangle BAC \cong \measuredangle ACD$	4) if 2 parralel lines are cut by a transversal

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



$$\sin = \frac{O}{H}$$

$$\sin(45) = \frac{x}{58}$$

$$58(\sin(45)) = x$$

$$x = 41.0121\ldots$$

Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$$TA = (x+8)(x+8)$$

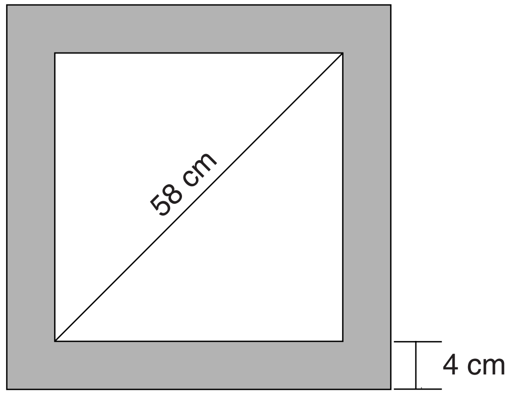
$$TA = (41.0121+8)(41.0121+8)$$

$$TA \approx \boxed{2402.2\text{ cm}}$$

Score 4: The student gave a complete and correct response.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$A=\frac{1}{2}d^2$

$A=\frac{1}{2}(58)^2$

$A=1682\text{cm}^2$

$A=(\sqrt{1682}+8)^2$

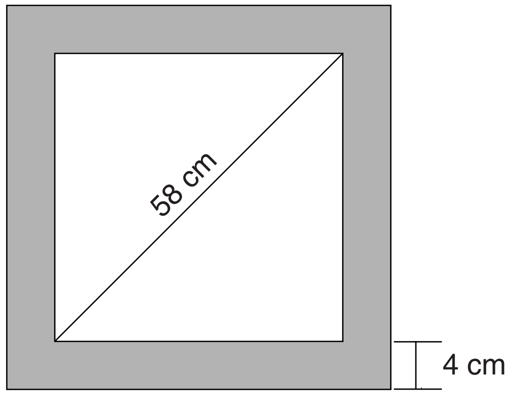
$A=2402.195093...$

$A=2402.2\text{cm}^2$

Score 4: The student gave a complete and correct response.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$A = l \cdot w$

$$\begin{array}{r} 41.01219 \\ +\ 8 \\ \hline 49.01219 \end{array}$$

$^2(58) = x^2 + x^2$

$\frac{3364}{2} = \frac{\cancel{2}x^2}{\cancel{2}}$

$\sqrt{1682} = \sqrt{x^2}$

$41.01219 = x$

$A = ls^2$

$A = 49.01219^2$

$A = 2402.2$

$\boxed{\begin{array}{l} \text{Poster Area} = 1682.0 \\ \text{Frame Area} = 720.2 \\ \hline \text{Total Area} = 2402.2 \end{array}}$

$A = s^2$

$A = 41.01219^2$

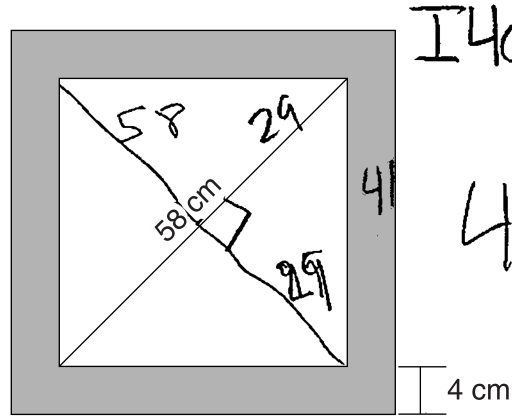
$A = 1682.0$

$$\begin{array}{r} 2402.2 \\ -1682.0 \\ \hline 720.2 \end{array}$$

Score 4: The student gave a complete and correct response.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



14cm

49

Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$a^2+b^2=c^2$

$29^2+29^2=c^2$

$841+841=c^2$

$\sqrt{1682}=\sqrt{c^2}$

$\sqrt{1681}=c$

$41=c$

$A=s^2$

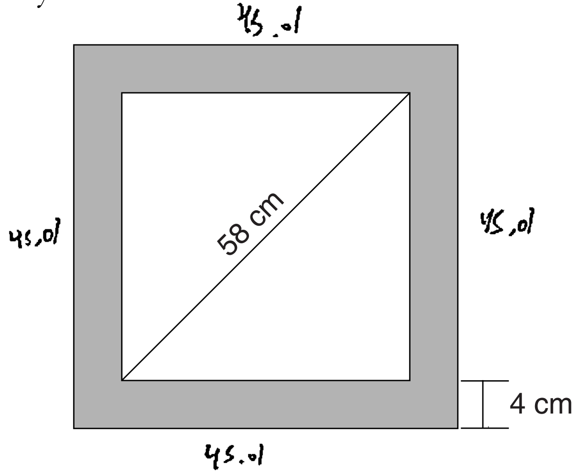
$A=49^2$

$\boxed{A=2401\text{ cm}^2}$

Score 3: The student made a transcription error by writing $\sqrt{1681}$.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$a^2+b^2=c^2$

$x^2+x^2=58^2$

$2x^2=58^2$

$2x^2=3364$

$x^2=1682$

$x\approx 41.01$

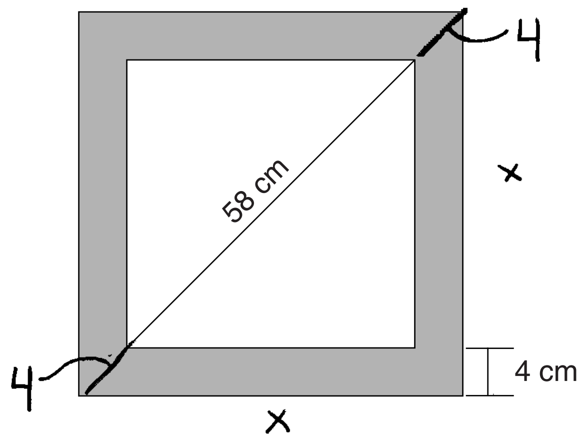
$A=45.01\cdot 45.01$

$A\approx 2025.9$ cm

Score 2: The student made an error in rounding $\sqrt{1682}$ early and another error by adding 4 rather than 8 to find the length of the frame.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

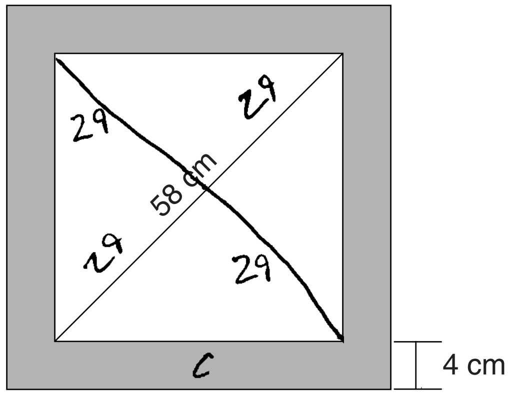
$$x^2 + x^2 = 66^2$$
$$2x^2 = 4356$$
$$x^2 = 2178 = \text{Area}$$

$$\begin{array}{r} 58 \\ +\ 8 \\ \hline 66 \end{array}$$

Score 2: The student made a conceptual error in finding the length of the diagonal.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

poster area = 1682 cm^2

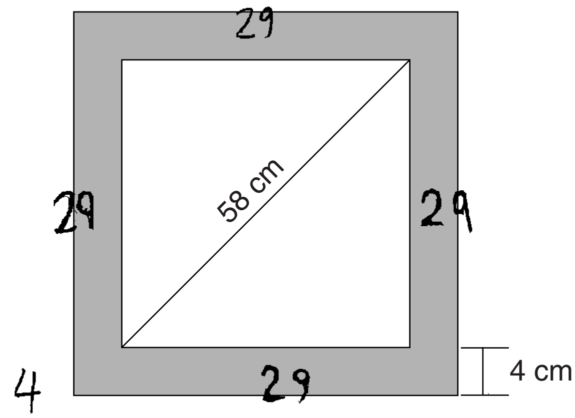
$29^2 + 29^2 = c^2 = \sqrt{1682} = 41.01\text{cm}$

41.01 cm

Score 1: The student found the area of the poster, but no further correct work is shown.

Question 34

34 Keira has a square poster that she is framing and placing on her wall. The poster has a diagonal 58 cm long and fits exactly inside the frame. The width of the frame around the picture is 4 cm.



Determine and state the total area of the poster and frame to the *nearest tenth of a square centimeter.*

$a^2+b^2=c^2$

$29^2+b^2=58^2$

$841+b^2=3364$

$-\cancel{841} \quad -841$

$b^2=2523$

$\sqrt{b^2}=\sqrt{2523}$

$b=50.23$

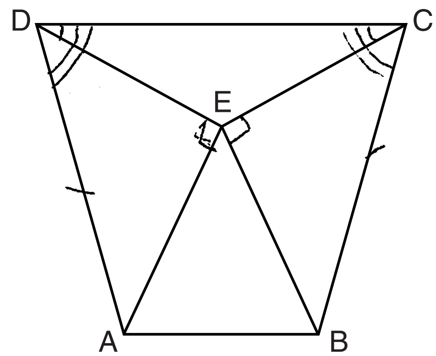
1456.67 —area of frame

3
29
× 4
116 ← area of poster

Score 0: The student gave a completely incorrect response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



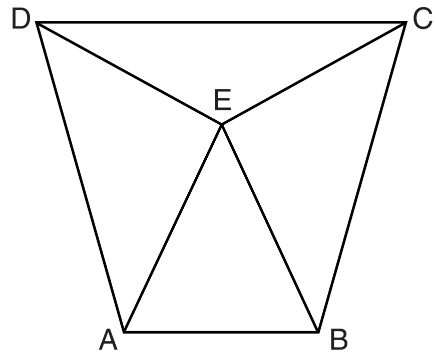
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

	1. ABDC is an isosceles trapezoid	1. given
S≅S	2. $\overline{AD} \cong \overline{BC}$	2. Legs in an isosceles trapezoid are =
	3. $\overline{AE} \perp \overline{DE}$ $\overline{BE} \perp \overline{CE}$	3 given
	4. ∠DEA is a right angle ∠CEB is a right angle	4. ⊥ lines form right angles.
A≅A	5. ∠DEA ≅ ∠CEB	5. All right angles are ≅
	6. ∠CDE ≅ ∠DCE	6 given
	7. ∠CDA ≅ ∠DCB	7. Base angles of isosceles trapezoid are ≅
A = A	8. ∠CDA − ∠CDE ≅ ∠DCB − ∠DCE ∠EDA ≅ ∠ECB	8. Subtraction Postulate
	9. △ADE ≅ △BCE	9. SAA ≅ SAA
	10. $\overline{EA} \cong \overline{EB}$	10. CPCTC
	11. △AEB is an isosceles △	11. isosceles △ has 2 ≅ sides.

Score 6: The student gave a complete and correct response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

$\overline{AE} \perp \overline{DE}$
$\overline{BE} \perp \overline{CE}$
Given

$\angle DEA$ is a rt $\angle$
$\angle CEB$ is a rt $\angle$
$\perp$ lines form rt $\angle$'s

$\angle DEA \cong \angle CEB$
All rt $\angle$s are $\cong$

$\triangle DEA$ & $\triangle CEB$ are rt $\triangle$'s
A rt $\triangle$ contains a rt $\angle$

Iso Trap $ABCD$ w/ base $\overline{DC}$ & $\overline{AB}$
Given

$\overline{AD} \cong \overline{BC}$
non-// sides of Iso Trap are $\cong$

$\angle CDE \cong \angle DCE$
Given

$\overline{DE} \cong \overline{CE}$
sides opp $\cong$ $\angle$s are $\cong$ in a $\triangle$

$\triangle DEA \cong \triangle CEB$
HL

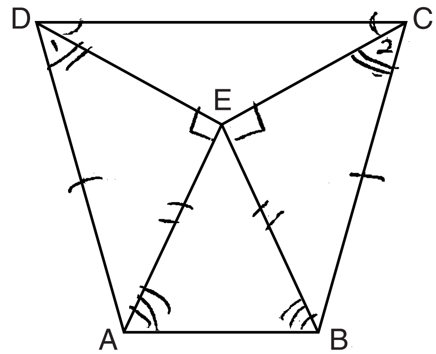
$\overline{EA} \cong \overline{EB}$
CPCTC

$\triangle AEB$ is Isosc
A $\triangle$ w/ 2 sides $\cong$ is Isosc.

Score 6: The student gave a complete and correct response.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



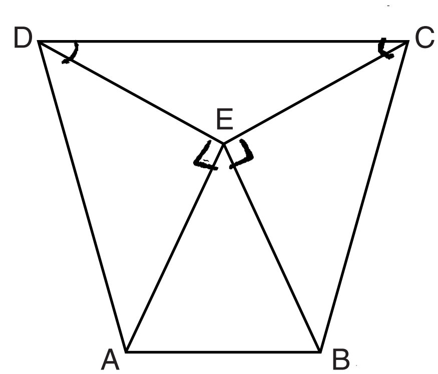
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statement	Reason
1. Isosceles trapezoid ABCD has bases $\overline{DC}$ + $\overline{AB}$	1. given
2. $\angle CDE \cong \angle DCE$	2. given
3. $\overline{AE} \perp \overline{DE}$	3. given
4. $\overline{BE} \perp \overline{CE}$	4. given
5. $\angle AED$ is a right $\angle$ $\angle BEC$ is a right $\angle$	5. perpendicular lines form right $\angle$'s.
6. $\angle AED \cong \angle BEC$	6. All right $\angle$'s are $\cong$
7. $\angle ADC \cong \angle BCD$	7. Angles opposite of two $\cong$ sides are also $\cong$.
8. $\angle 1 \cong \angle 2$	8. Subtraction
9. $\overline{AD} \cong \overline{BC}$	9. legs of an isosceles trapezoid are $\cong$.
10. $\triangle ADE \cong \triangle BCE$	10. SAA $\cong$ SAA
11. $\overline{AE} \cong \overline{BE}$	11. CPCTC
12. $\triangle AEB$ is an isosceles $\triangle$	12. A triangle w/ 2 $\cong$ sides is isosceles

Score 5: The student had an incorrect reason in step 7.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

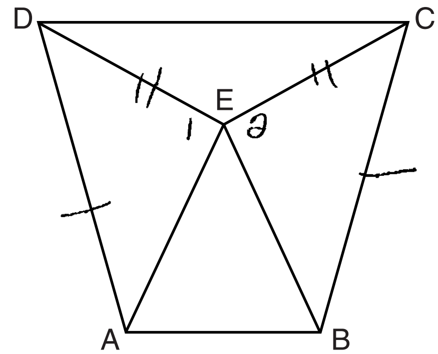


Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

S	R
Isosceles Trapezoid ABCD $\measuredangle CDE \cong \measuredangle DCE$ $AE \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	Given
$\measuredangle DEA$ and $\measuredangle CEB$ are right angles	Perpendicular lines form right angles
$\measuredangle DEA \cong \measuredangle CEB$	All right angles are congruent
$\overline{DE} \cong \overline{EC}$	If the base angles of a triangle are congruent then the sides opposite them are congruent
$\overline{DA} \cong \overline{CB}$	Properties of Isosceles trapezoid
$\triangle DEA \cong \triangle CEB$	$HL \cong HL$/High leg Theorm
$\overline{EA} \cong \overline{EB}$	CPCTC
$\triangle AEB$ is an Isosceles triangle	If the base angles of a triangle are congruent the triangle is Isosceles.

Score 4: The student did not prove $\triangle DEA$ and $\triangle CEB$ are right triangles and wrote an incorrect last reason by referencing base angles when the student proved congruent sides.

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

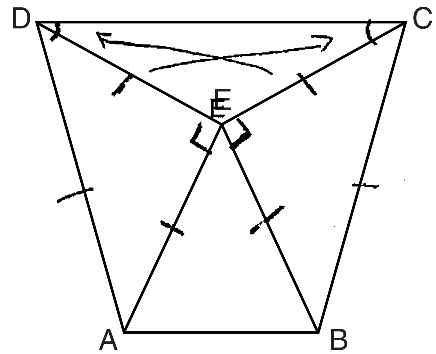


Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

statements	reasons
① isosceles trapezoid ABCD, $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	① given
② $\overline{DA} \cong \overline{CB}$	② An isosceles trapezoid has 2 ≅ legs
③ ∡1, ∡2 are Right ∡s	③ ⊥ lines intersect to form rt ∡s
④ ∡1 ≅ ∡2	④ Rt ∡s are ≅.
⑤ $\overline{DE} \cong \overline{CE}$	⑤ in a △, sides opposite ≅ ∡s are ≅
⑥ △ADE ≅ △BCE	⑥ SAS ≅ SAS
⑦ $\overline{AE} \cong \overline{BE}$	⑦ If 2 △s are ≅, corresponding parts are ≅
⑧ △AEB is isosceles	⑧ an isosceles △ is a △ w/ 2 ≅ sides.

Score 4: The student made one conceptual error in proving $\triangle ADE \cong \triangle BCE$ by SAS.

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



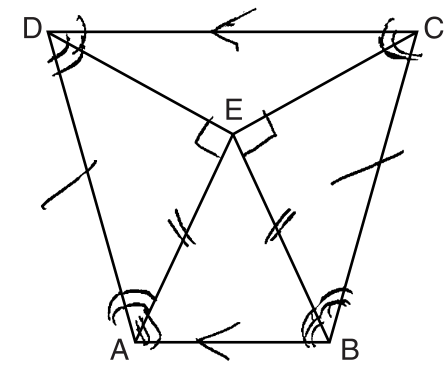
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
① trapezoid ABCD $\measuredangle CDE \cong \measuredangle DCE$ $\overline{AE} \perp \overline{DE}$ $\overline{BE} \perp CE$	① given
② $\overline{DA} \cong \overline{CB}$	② def of trapazoid
③ $\triangle DEC$ is an iscosola $\triangle$	③ has 2 congruent sides
④ $\triangle ADE$ and $\triangle BCE$ are right $\triangle$s	④ have right $\measuredangle$s
⑤ $\triangle ADE \cong \triangle BCE$	⑤ HL $\cong$ HL
⑥ $\overline{AE} \cong \overline{BE}$	⑥ CPCTC
⑦ $\triangle AEB$ is an iscosceles $\triangle$	⑦ has 2 congruent sides

Score 3: The student did not prove $\overline{DE} \cong \overline{CE}$ and that $\angle DEA$ and $\angle CEB$ are right angles. The student also had an incorrect reason in step 2.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



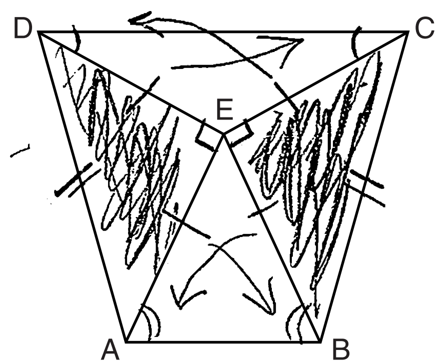
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statement	Reason
① Isosceles trapazoid ABCD has bases $\overline{DC}$ and $\overline{AB}$, $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$	① Given
② $\overline{DC} \parallel \overline{AB}$	② Isosceles trapazoids have one pair of $\parallel$ lines
③ $\angle AED$ & $\angle BEC$ are rt $\angle$'s	③ $\perp$ lines form rt $\angle$'s
④ $\angle AED \cong \angle BEC$	④ All rt $\angle$'s are $\cong$.
⑤ $\angle ECB$ and $\angle CBE$, $\angle EDA$ and $\angle ADE$ are corresponding $\angle$'s.	⑤ when $\parallel$ lines are cut by a transversal corresponding $\angle$'s are formed.
⑥ $\triangle ADE \cong \triangle BCE$	⑥ S.A.S
⑦ $\triangle AEB$ is an isosceles $\triangle$.	⑦ $\cong$ base $\angle$'s create an isosceles $\triangle$.

Score 2: The student proved $\angle AED \cong \angle BEC$, but no further correct relevant work is shown.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



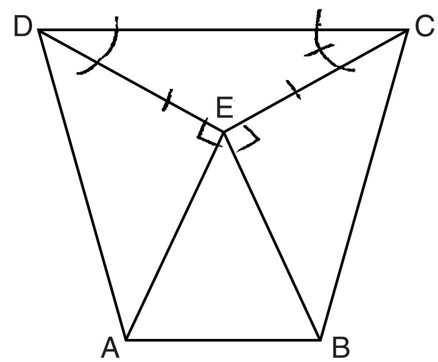
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Proofs
1) Isosceles trapezoid ABCD bases $\overline{DC}, \overline{AB}$ $\angle CDE \cong \angle DCE$ $\overline{AE} \perp \overline{DE}$ $\overline{BE} \perp \overline{CE}$	1) Given
2) $\angle DEA$ and $\angle CEB$ are right $\angle$s	2) perpendicular bisector form right angles
3) $\angle DEA \cong \angle CEB$	3) right angles are always congruent
4) $\overline{DA} \cong \overline{CB}$	4) Given
5) $\triangle DEC$ is an isosceles $\triangle$	5) Isosceles triangles have 2 congruent angles.
6) $\overline{DE} \cong \overline{EC}$	6) Isosceles triangles are triangles with 2 sides with equal length.
7) $\triangle DEA \cong \triangle CEB$	7) SAS = SAS
8) $\overline{AE} \cong \overline{BE}$	8) corresponding parts of congruent triangles are congruent
9) $\triangle AEB$ is an isosceles triangle	9) If two sides are congruent, then the angles opposite of the sides in the triangle are congruent.

Score 2: Some correct relevant statements about the proof are made in steps 3, 6, and 8, but four or more statements and/or reasons are missing or incorrect.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



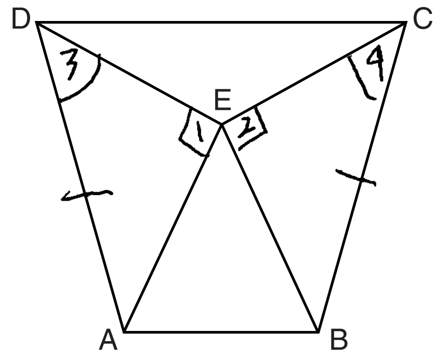
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
1. $\measuredangle CDE \cong \measuredangle DCE$	1. Given
2. $\overline{DE} \cong \overline{EC}$	2. In a triangle, $\cong$ line lay opposite $\cong$ sides.
3. $\overline{AE} \perp \overline{DE}$; $\overline{BE} \perp \overline{CE}$	3. Given
4. $\measuredangle DEA$ and $\measuredangle CEB$ are right angles	4. $\perp$ lines intersect to form right angles.
5.	

Score 1: The student had one correct statement and reason in step 4.

Question 35

35 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



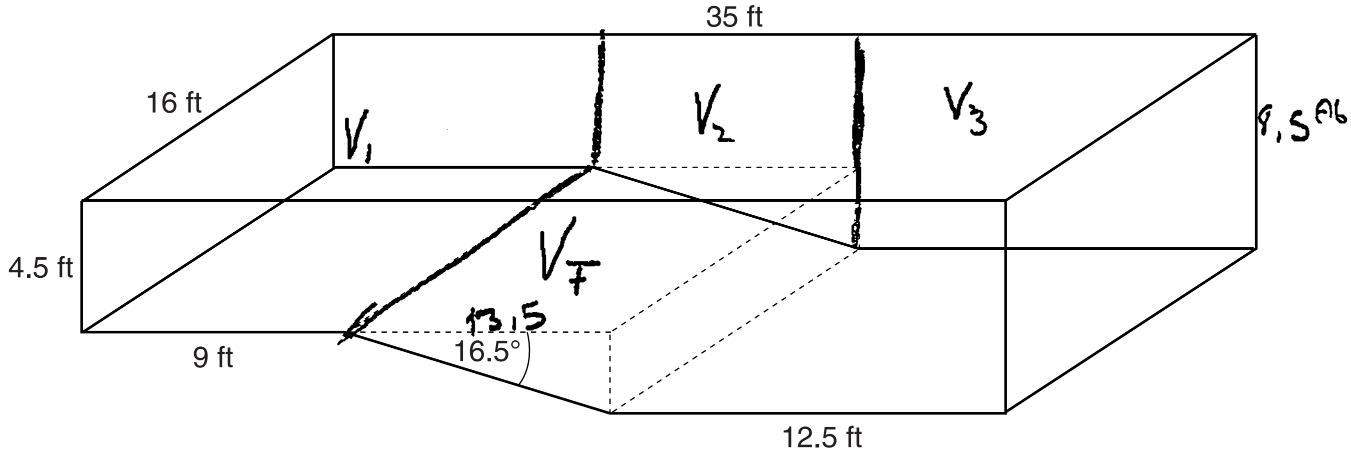
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

Statements	Reasons
① isoc trap ABCD has bases $\overline{DC}$ and $\overline{AB}$ with non parallel legs $\overline{AD}$ and $\overline{BC}$	① given
② $\angle CDE \cong \angle DCE$, $AE \perp DE$ & $BE \perp CE$	② given
③ $\angle 1 \cong \angle 2$	③ verticle angles are congruent
④ $\overline{DA} \cong \overline{CB}$	④ opp. sides of a trap.
⑤ $\angle 3 \cong \angle 4$	⑤ substitution
⑥ $\triangle ADE \cong \triangle BCE$	⑥ ASA
⑦ $\triangle AEB$ is an isoc $\triangle$	⑦ CPCTC

Score 0: The student gave a completely incorrect response.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

13.5
16.5 x

$35-9-12.5=h$

$13.5=h$

$4.5+x=\boxed{8.5}$

$\tan(16.5)=\frac{x}{13.5}$

$13.5\tan(16.5)=x$

$x=3.99882182$

$\boxed{8.5\text{ ft}}$

Find the volume of the inside of the pool to the *nearest cubic foot.*

$\boxed{V_1=4.5\cdot16\cdot9=\boxed{648}}$

$\boxed{V_3=12.5\cdot16\cdot8.5=1700}$

$\boxed{V_2=13.5\cdot4.5\cdot16=\boxed{972}}$

$V_T=\frac{13.5\cdot4}{2}\cdot16$

$V_T=432$

$V_1+V_2+V_3+V_T=3752$

3752 ~~ft³~~

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

$A_{air} = 16 \cdot 35 \cdot .5 = \boxed{280}$

$3752 - 280 = 3472\ ft^3$

$3472 \cdot 7.48 = \boxed{25970.56 \text{ gallons}}$

$25970.56 / 10.5 = 2473.386666\ 7$

$\dfrac{2473.386667 \text{ minutes}}{60}$

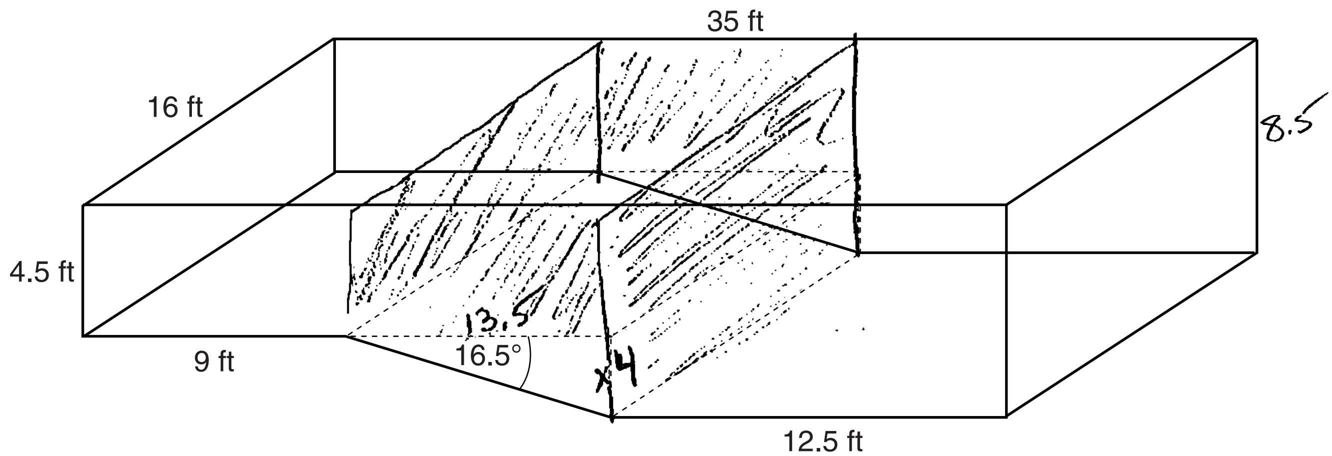
$=$

41.22311111

$\boxed{41 \text{ hours}}$

Score 6: The student gave a complete and correct response.

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$\text{Tan}(16.5) = \frac{x}{13.5} = 3.99 = 4$

$4 + 4.5 = 8.5$

Find the volume of the inside of the pool to the *nearest cubic foot.*

$(16)(4.5)(9) = 648$

$(12.5)(8.5)(16) = 1700$

$(13.5)(4.5)(16) = 972$

$\frac{1}{2}(4)(13.5)(16) = 432$

volume 3752

Question 36 is continued on the next page.

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

3752 volume of pool

volume of air

$(.5)(16)(35) = 280$

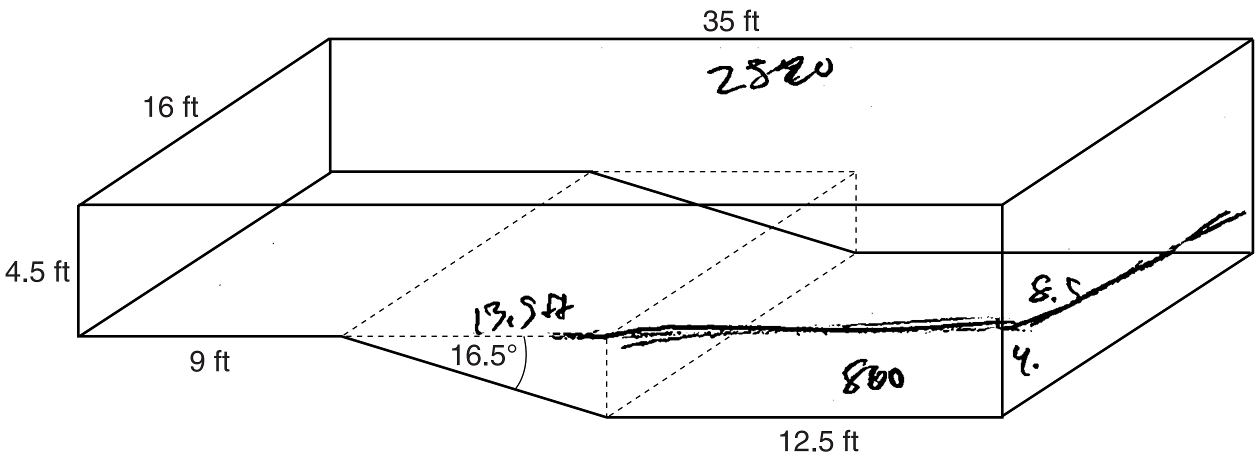
$$\begin{array}{r} 3752 \\ -\ 280 \\ \hline 3472 \end{array} \cdot 7.48 = \frac{25970.5}{(10.5)(60)} = 41.223$$

41.2 hours

Score 5: The student made a rounding error when finding the time.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

~~4.5·16=72~~ $\quad \text{Tan.}(16.5) \frac{x}{13.5}$

Tan = 3.99885218129 + 4.5

The depth is 8.5 ft

Find the volume of the inside of the pool to the *nearest cubic foot.*

12.5·16 = 200 ×4 = 800

4.5·16=72

72·35=2520

13.5·16=216·4=864
2 = 432

2520+800+432 = 3752 ft

Question 36 is continued on the next page.

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

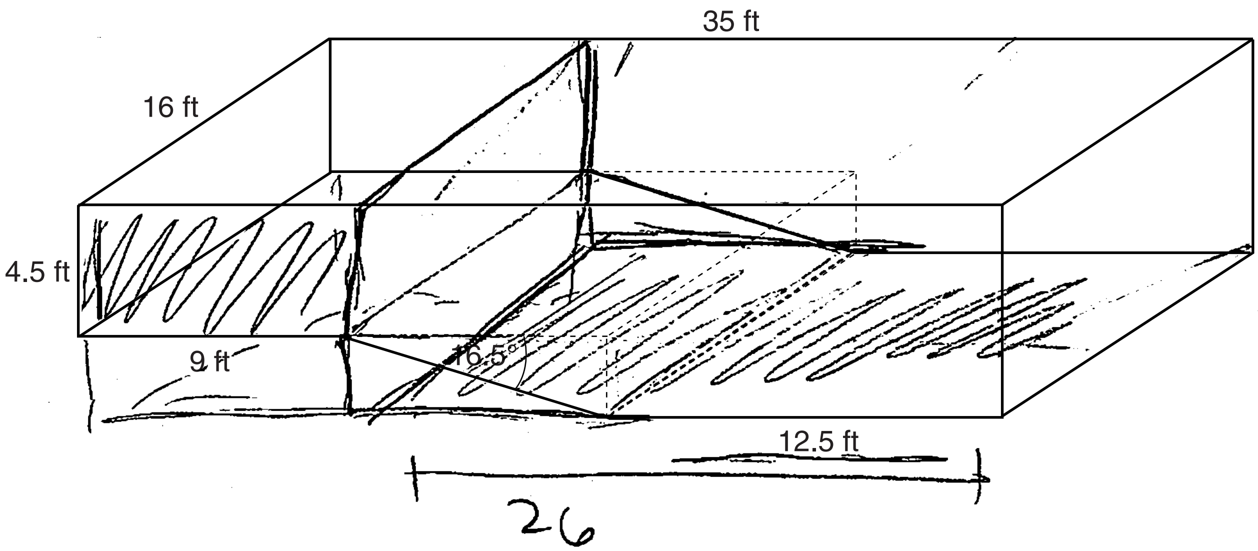
$$\frac{3752}{7.48} = \frac{501.6042780\ldots \text{ gallons}}{10.5}$$

47.7

Score 4: The student found 8.5 and 3752, but no further correct work is shown.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.

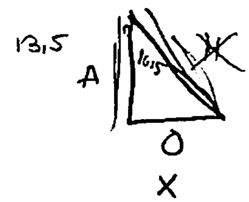


If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$4.5 + 3.9988821$

$13.5\left(\tan 16.5 = \frac{x}{13.5}\right)$

$3.9988821 = x$



Depth of pool = 8.5 ft

Find the volume of the inside of the pool to the *nearest cubic foot.*

$\boxed{A} \times$ height

$4.5 \times 9 \times 16$

648

$\boxed{A} \times$ height

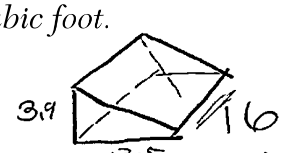
$16 \times 26 \times 8.5$

3536

$3536 - 431.8793$

$3104.1207 + 648$

$V = 3752$



A x height

$(3.9988821 \times 13.5) \div 2 \times 16$

26.99245418×16

431.8

Question 36 is continued on the next page.

Question 36

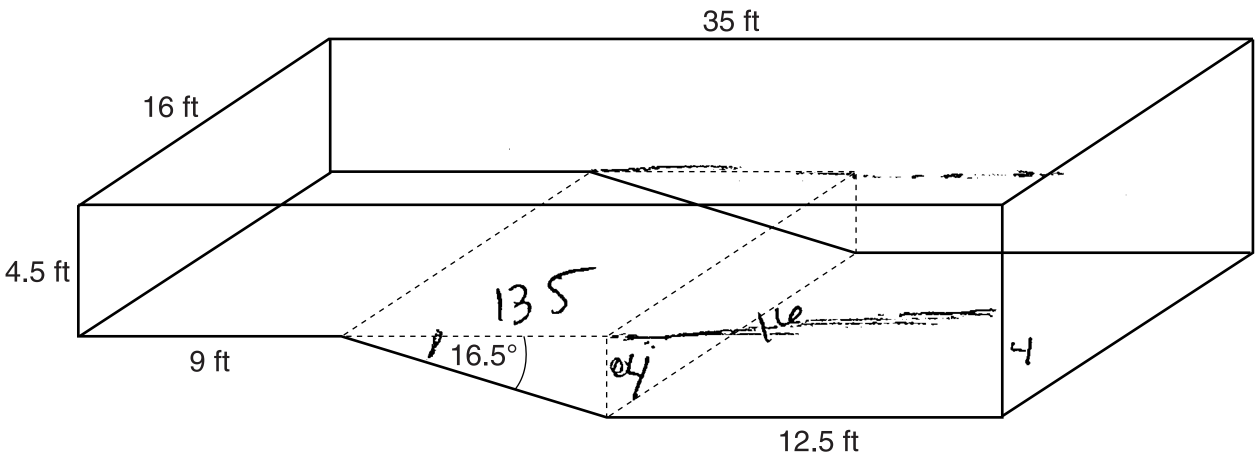
Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

Score 4: The student did not find the time.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$\tan 16.5 = \frac{O}{13.5}$

$13.5 \tan 16.5 = O$

$O = 4 ft$

the depth of the pool is 4ft.

Find the volume of the inside of the pool to the *nearest cubic foot*.

$V = Bh$

$V = (16)(4.5)(35)$

$V = 2520$

$+ 800$

432

$\boxed{V = 3752 ft^3}$

$V = (4)(16)(12.5)$

$V = 800$

$V = \frac{1}{2}Bh$

$V = \frac{1}{2}(16)(4)(13.5)$

$V = 432$

Question 36 is continued on the next page.

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

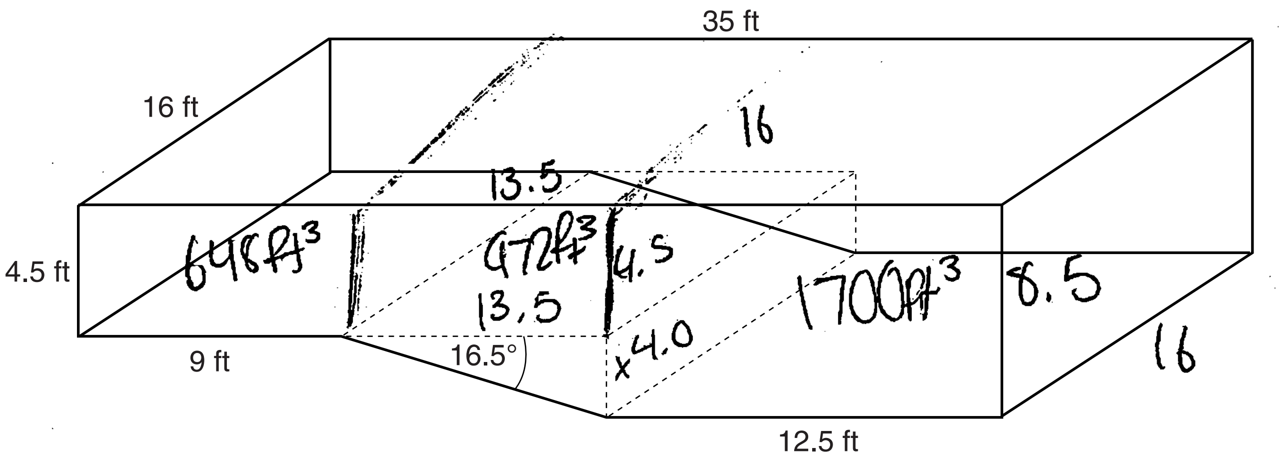
$$\frac{10.5}{7.48} = \frac{6}{1}$$

$$\cancel{44.88 = 10.5}$$

Score 3: The student correctly found the volume of the pool, but did not add 4.5 when finding the depth, and did not find the time correctly.

Question 36

36 A rectangular in-ground pool is modeled by the prism below. The inside of the pool is 16 feet wide and 35 feet long. The pool has a shallow end and a deep end, with a sloped floor connecting the two ends. Without water, the shallow end is 9 feet long and 4.5 feet deep, and the deep end of the pool is 12.5 feet long.



If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$35-(9+12.5)=13.5$

$\left(\tan 16.5=\frac{x}{13.5}\right)13.5$

$13.5\tan 16.5=x$

$3.99=x$

$4.0=x$

$4.0+4.5=\boxed{8.5\text{ ft}}$

Find the volume of the inside of the pool to the *nearest cubic foot.*

$V=\frac{1}{2}bh$

$V=\frac{1}{2}(13.5\cdot 4)$

$V=\frac{1}{2}(54)$

$V=27\text{ft}^3$

$648+472+27+1700=$

$\boxed{3347\text{ ft}^3}$

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour*, will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

$3347 \cdot 7.48 = 25,035.56$ gallons

$10.5 \cdot 25,0035 = 262,873.38$

$\boxed{262,873 \text{ minutes}}$

Score 3: The student did not multiply by 16 when finding the volume of the triangular prism and did not find the time correctly.

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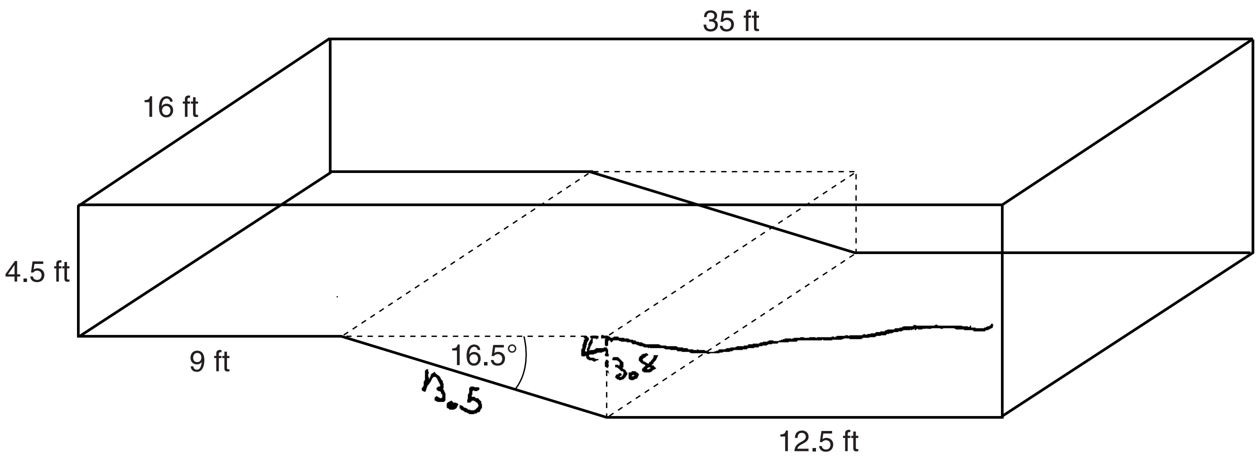

If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

$\sin 16.5 = \frac{a}{13.5}$

3.834207159

+ 4.5

8.33

8.3 feet

Find the volume of the inside of the pool to the *nearest cubic foot.*

2

4.5(16)(35) = 2520

(3.8)(16)(12.5) = 760

18.99 (16) = 207

3487

Question 36 is continued on the next page.

Question 36 continued

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Score 2: The student made an error when labeling 13.5 in the diagram, made an error when finding the volume of the triangular prism, and did not find the time.

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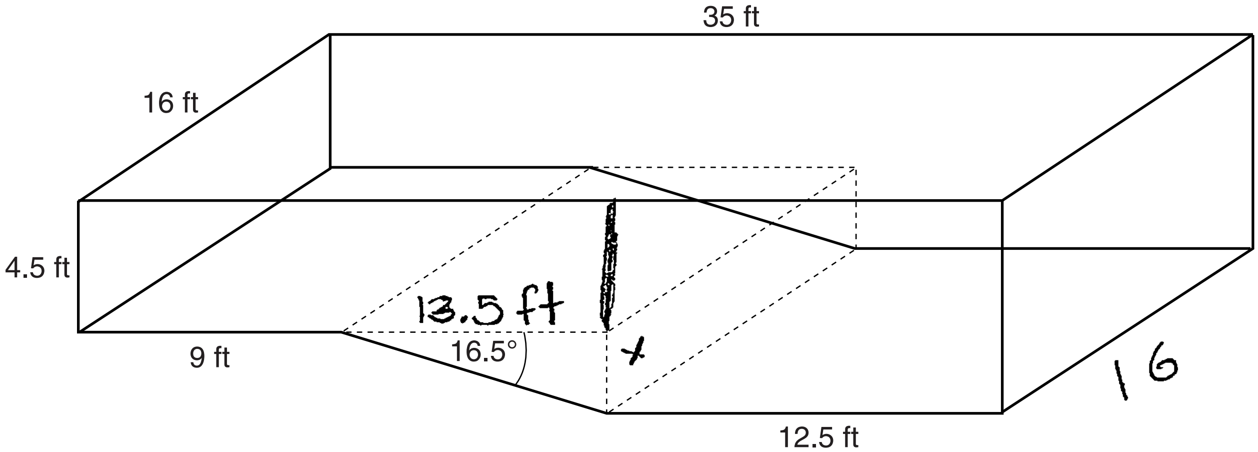
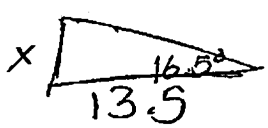


If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

SOH CHH TOA



$\tan \frac{16.5}{1} = \frac{x}{13.5}$

$x = 18.2396$

$\boxed{x \approx 18.2 \text{ ft}}$

Find the volume of the inside of the pool to the *nearest cubic foot.*

$A = \frac{bh}{2}$

$A = \frac{13.5 \times 18.2}{2}$

$A = \frac{245.2}{2}$

$\boxed{V = 122.85 \approx 123}$

$A = lwh$

$A = 4.5 \times 9 \times 16$

$\boxed{V = 648}$

$\cancel{A = 12.5 \times 16 \times 22.7}$

$\boxed{A = 4540}$

Question 36 is continued on the next page.

Question 36 continued

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$$\frac{10.5\text{g}}{1\text{ min}} = \frac{x}{60\text{ min}}$$

$x = 630$ gallons in an hour

Score 1: The student wrote a correct trigonometric equation to find the depth of the pool. The student did not show enough correct work to find the total volume of the pool. The student did not find the time to fill the pool.

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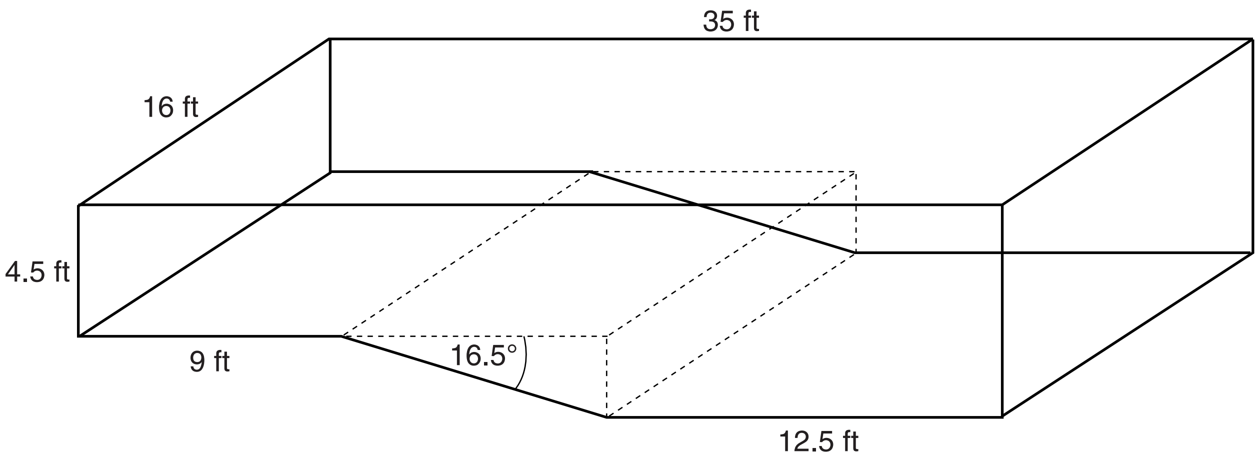

If the sloped floor has an angle of depression of 16.5 degrees, what is the depth of the pool at the deep end, to the *nearest tenth of a foot*?

4.5

Find the volume of the inside of the pool to the *nearest cubic foot.*

V = lwh

= 9 · 16 · 35

= 5040 ft^3

$\frac{5040}{10.5} = 480$

ANS 480 hrs

Question 36 is continued on the next page.

Question 36

Question 36 continued

A garden hose is used to fill the pool. Water comes out of the hose at a rate of 10.5 gallons per minute. How much time, to the *nearest hour,* will it take to fill the pool 6 inches from the top? [1 ft^3 = 7.48 gallons]

Score 0: The student showed no correct relevant work.