Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

1. William is drawing pictures of cross sections of the right circular cone below.

Which drawing can not be a cross section of a cone?

(1) □
(2) △
(3) □
(4) □

2. An equation of a line perpendicular to the line represented by the equation \( y = -\frac{1}{2}x - 5 \) and passing through \((6, -4)\) is

(1) \( y = -\frac{1}{2}x + 4 \)  
(3) \( y = 2x + 14 \)  
(2) \( y = -\frac{1}{2}x - 1 \)  
(4) \( y = 2x - 16 \)
3 In parallelogram \( QRST \) shown below, diagonal \( TR \) is drawn, \( U \) and \( V \) are points on \( TS \) and \( QR \), respectively, and \( UV \) intersects \( TR \) at \( W \).

\[ \text{If } m\angle S = 60^\circ, m\angle SRT = 83^\circ, \text{ and } m\angle TWU = 35^\circ, \text{ what is } m\angle WVQ? \]

(1) 37º  (3) 72º
(2) 60º  (4) 83º

4 A fish tank in the shape of a rectangular prism has dimensions of 14 inches, 16 inches, and 10 inches. The tank contains 1680 cubic inches of water. What percent of the fish tank is empty?

(1) 10  (3) 50
(2) 25  (4) 75

5 Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?

(1) \((x,y) \rightarrow (y,x)\)  (3) \((x,y) \rightarrow (4x,4y)\)
(2) \((x,y) \rightarrow (x,-y)\)  (4) \((x,y) \rightarrow (x + 2, y - 5)\)
6 In the diagram below, $FE$ bisects $AC$ at $B$, and $GE$ bisects $BD$ at $C$.

Which statement is always true?

1. $AB \parallel DC$
2. $FB \parallel EB$
3. $BD$ bisects $GE$ at $C$.
4. $AC$ bisects $FE$ at $B$.

7 As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches.

If the altitude of the pyramid measures 12 inches, its volume, in cubic inches, is

1. 72
2. 144
3. 288
4. 432
8 Triangle $ABC$ and triangle $DEF$ are graphed on the set of axes below.

Which sequence of transformations maps triangle $ABC$ onto triangle $DEF$?

(1) a reflection over the $x$-axis followed by a reflection over the $y$-axis
(2) a $180^\circ$ rotation about the origin followed by a reflection over the line $y = x$
(3) a $90^\circ$ clockwise rotation about the origin followed by a reflection over the $y$-axis
(4) a translation 8 units to the right and 1 unit up followed by a $90^\circ$ counterclockwise rotation about the origin
9 In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?

(1) $\tan \angle A = \tan \angle B$

(2) $\sin \angle A = \sin \angle B$

(3) $\cos \angle A = \tan \angle B$

(4) $\sin \angle A = \cos \angle B$

10 A line that passes through the points whose coordinates are (1,1) and (5,7) is dilated by a scale factor of 3 and centered at the origin. The image of the line

(1) is perpendicular to the original line

(2) is parallel to the original line

(3) passes through the origin

(4) is the original line

11 Quadrilateral $ABCD$ is graphed on the set of axes below.

When $ABCD$ is rotated $90^\circ$ in a counterclockwise direction about the origin, its image is quadrilateral $A'B'C'D'$. Is distance preserved under this rotation, and which coordinates are correct for the given vertex?

(1) no and $C'(1,2)$

(2) no and $D'(2,4)$

(3) yes and $A'(6,2)$

(4) yes and $B'(-3,4)$
12 In the diagram below of circle $O$, the area of the shaded sector $LOM$ is $2\pi$ cm². If the length of $NL$ is 6 cm, what is $m\angle N$?

(1) 10°  (3) 40°
(2) 20°  (4) 80°

13 In the diagram below, $\triangle ABC \sim \triangle DEF$.

If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

(1) $DE = 9$, $DF = 12$, and $\angle A \equiv \angle D$
(2) $DE = 8$, $DF = 10$, and $\angle A \equiv \angle D$
(3) $DE = 36$, $DF = 64$, and $\angle C \equiv \angle F$
(4) $DE = 15$, $DF = 20$, and $\angle C \equiv \angle F$

14 The diameter of a basketball is approximately 9.5 inches and the diameter of a tennis ball is approximately 2.5 inches. The volume of the basketball is about how many times greater than the volume of the tennis ball?

(1) 3591  (3) 55
(2) 65  (4) 4

Use this space for computations.
15 The endpoints of one side of a regular pentagon are (−1, 4) and (2, 3). What is the perimeter of the pentagon?

(1) \(\sqrt{10}\)  
(2) \(5\sqrt{10}\)  
(3) \(5\sqrt{2}\)  
(4) \(25\sqrt{2}\)

16 In the diagram of right triangle ABC shown below, \(AB = 14\) and \(AC = 9\).

![Diagram of right triangle ABC with sides 9, 14, and hypotenuse AB]

What is the measure of \(\angle A\), to the nearest degree?

(1) 33  
(2) 40  
(3) 50  
(4) 57

17 What are the coordinates of the center and length of the radius of the circle whose equation is \(x^2 + 6x + y^2 - 4y = 23\)?

(1) (3, -2) and 36  
(2) (3, -2) and 6  
(3) (-3, 2) and 36  
(4) (-3, 2) and 6

18 The coordinates of the vertices of \(\triangle RST\) are \(R(-2, -3)\), \(S(8, 2)\), and \(T(4, 5)\). Which type of triangle is \(\triangle RST\)?

(1) right  
(2) acute  
(3) obtuse  
(4) equiangular
19 Molly wishes to make a lawn ornament in the form of a solid sphere. The clay being used to make the sphere weighs .075 pound per cubic inch. If the sphere’s radius is 4 inches, what is the weight of the sphere, to the nearest pound?

(1) 34 (3) 15
(2) 20 (4) 4

20 The ratio of similarity of \( \triangle BOY \) to \( \triangle GRL \) is 1:2. If \( BO = x + 3 \) and \( GR = 3x - 1 \), then the length of \( GR \) is

(1) 5 (3) 10
(2) 7 (4) 20

21 In the diagram below, \( \overline{DC}, \overline{AC}, \overline{DOB}, \overline{CB}, \) and \( \overline{AB} \) are chords of circle \( O \), \( \overline{FDE} \) is tangent at point \( D \), and radius \( \overline{AO} \) is drawn. Sam decides to apply this theorem to the diagram: “An angle inscribed in a semi-circle is a right angle.”

Which angle is Sam referring to?

(1) \( \angle AOB \) (3) \( \angle DCB \)
(2) \( \angle BAC \) (4) \( \angle FDB \)
22 In the diagram below, \( CD \) is the altitude drawn to the hypotenuse \( AB \) of right triangle \( ABC \).

Which lengths would not produce an altitude that measures \( 6\sqrt{2} \)?

(1) \( AD = 2 \) and \( DB = 36 \)  
(2) \( AD = 3 \) and \( AB = 24 \)  
(3) \( AD = 6 \) and \( DB = 12 \)  
(4) \( AD = 8 \) and \( AB = 17 \)

23 A designer needs to create perfectly circular necklaces. The necklaces each need to have a radius of 10 cm. What is the largest number of necklaces that can be made from 1000 cm of wire?

(1) 15  
(2) 16  
(3) 31  
(4) 32

24 In \( \triangle SCU \) shown below, points \( T \) and \( O \) are on \( SU \) and \( CU \), respectively. Segment \( OT \) is drawn so that \( \angle C \equiv \angle OTU \).

If \( TU = 4 \), \( OU = 5 \), and \( OC = 7 \), what is the length of \( ST \)?

(1) 5.6  
(2) 8.75  
(3) 11  
(4) 15
Triangle $ABC$ is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$. 
In the diagram below of circle $O$ with diameter $BC$ and radius $OA$, chord $DC$ is parallel to chord $BA$.

If $m\angle BCD = 30^\circ$, determine and state $m\angle AOB$. 

Geography (Common Core) – Jan. ’16
27 Directed line segment $PT$ has endpoints whose coordinates are $P(-2,1)$ and $T(4,7)$. Determine the coordinates of point $J$ that divides the segment in the ratio 2 to 1. [The use of the set of axes below is optional.]
28 As graphed on the set of axes below, \( \triangle A'B'C' \) is the image of \( \triangle ABC \) after a sequence of transformations.

Is \( \triangle A'B'C' \) congruent to \( \triangle ABC \)? Use the properties of rigid motion to explain your answer.
A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70º angle with the ground. To the nearest foot, determine and state the length of the ladder.
During an experiment, the same type of bacteria is grown in two petri dishes. Petri dish A has a diameter of 51 mm and has approximately 40,000 bacteria after 1 hour. Petri dish B has a diameter of 75 mm and has approximately 72,000 bacteria after 1 hour.

Determine and state which petri dish has the greater population density of bacteria at the end of the first hour.
31 Line $\ell$ is mapped onto line $m$ by a dilation centered at the origin with a scale factor of 2. The equation of line $\ell$ is $3x - y = 4$. Determine and state an equation for line $m$. 
32 The aspect ratio (the ratio of screen width to height) of a rectangular flat-screen television is 16:9. The length of the diagonal of the screen is the television’s screen size. Determine and state, to the nearest inch, the screen size (diagonal) of this flat-screen television with a screen height of 20.6 inches.
Given the theorem, “The sum of the measures of the interior angles of a triangle is $180^\circ$,” complete the proof for this theorem.

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

Fill in the missing reasons below.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\triangle ABC$</td>
<td>(1) Given</td>
</tr>
<tr>
<td>(2) Through point $C$, draw $\overline{DCE}$ parallel to $\overline{AB}$.</td>
<td>(2) _______________</td>
</tr>
<tr>
<td>(3) $m\angle 1 = m\angle ACD$, $m\angle 3 = m\angle BCE$</td>
<td>(3) _______________</td>
</tr>
<tr>
<td>(4) $m\angle ACD + m\angle 2 + m\angle BCE = 180^\circ$</td>
<td>(4) _______________</td>
</tr>
<tr>
<td>(5) $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$</td>
<td>(5) _______________</td>
</tr>
</tbody>
</table>
Triangle XYZ is shown below. Using a compass and straightedge, on the line below, construct and label \( \triangle ABC \), such that \( \triangle ABC \cong \triangle XYZ \). [Leave all construction marks.]

Based on your construction, state the theorem that justifies why \( \triangle ABC \) is congruent to \( \triangle XYZ \).
Given: Parallelogram $\text{ANDR}$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points $W$ and $E$, respectively

Prove that $\triangle ANW \cong \triangle DRE$.

Prove that quadrilateral $\text{AWDE}$ is a parallelogram.
Cathy wants to determine the height of the flagpole shown in the diagram below. She uses a survey instrument to measure the angle of elevation to the top of the flagpole, and determines it to be $34.9^\circ$. She walks 8 meters closer and determines the new measure of the angle of elevation to be $52.8^\circ$. At each measurement, the survey instrument is 1.7 meters above the ground.

Determine and state, to the nearest tenth of a meter, the height of the flagpole.
## High School Math Reference Sheet

1 inch = 2.54 centimeters  
1 meter = 39.37 inches  
1 mile = 5280 feet  
1 mile = 1760 yards  
1 mile = 1.609 kilometers  
1 kilometer = 0.62 mile  
1 pound = 16 ounces  
1 pound = 0.454 kilogram  
1 cup = 8 fluid ounces  
1 pint = 2 cups  
1 quart = 2 pints  
1 gallon = 4 quarts  
1 pound = 0.454 kilogram  
1 liter = 0.264 gallon  
1 liter = 1000 cubic centimeters

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Pythagorean Theorem</th>
<th>Quadratic Formula</th>
<th>Arithmetic Sequence</th>
<th>Geometric Sequence</th>
<th>Geometric Series</th>
<th>Radians</th>
<th>Degrees</th>
<th>Exponential Growth/Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$A = A_0 e^{k(t - t_0)} + B_0$</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$A = bh$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$C = \pi d$ or $C = 2\pi r$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Prisms</td>
<td>$V = Bh$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder</td>
<td>$V = \pi r^2h$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cone</td>
<td>$V = \frac{1}{3}\pi r^2h$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pyramid</td>
<td>$V = \frac{1}{3}Bh$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Scrap Graph Paper — This sheet will *not* be scored.
Scrap Graph Paper — This sheet will *not* be scored.