The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.
1. In the diagram below, lines \( \ell, m, n, \) and \( p \) intersect line \( r \).

Which statement is true?

(1) \( \ell \parallel n \)  
(2) \( \ell \parallel p \)  
(3) \( m \parallel p \)  
(4) \( m \parallel n \)

2. Which transformation would not always produce an image that would be congruent to the original figure?

(1) translation  
(2) dilation  
(3) rotation  
(4) reflection

3. If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?

(1) cone  
(2) pyramid  
(3) prism  
(4) sphere
4 In the diagram below, \( \angle BDC = 100^\circ \), \( \angle A = 50^\circ \), and \( \angle DBC = 30^\circ \).

Which statement is true?

1. \( \triangle ABD \) is obtuse.  
2. \( \triangle ABC \) is isosceles.  
3. \( m\angle ABD = 80^\circ \)  
4. \( \triangle ABD \) is scalene.

5 Which point shown in the graph below is the image of point \( P \) after a counterclockwise rotation of 90° about the origin?

1. \( A \)  
2. \( B \)  
3. \( C \)  
4. \( D \)
6 In $\triangle ABC$, where $\angle C$ is a right angle, $\cos A = \frac{\sqrt{21}}{5}$. What is $\sin B$?

(1) $\frac{\sqrt{21}}{5}$  
(2) $\frac{\sqrt{21}}{2}$  
(3) $\frac{2}{5}$  
(4) $\frac{5}{\sqrt{21}}$

7 Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.

Which information is not enough to prove $ABCD$ is a parallelogram?

(1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$
(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
(3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
(4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

8 An equilateral triangle has sides of length 20. To the nearest tenth, what is the height of the equilateral triangle?

(1) 10.0  
(2) 11.5  
(3) 17.3  
(4) 23.1
9 Given: \(\triangle AEC, \triangle DEF\), and \(FE \perp CE\)

What is a correct sequence of similarity transformations that shows \(\triangle AEC \sim \triangle DEF\)?

1. A rotation of 180 degrees about point \(E\) followed by a horizontal translation
2. A counterclockwise rotation of 90 degrees about point \(E\) followed by a horizontal translation
3. A rotation of 180 degrees about point \(E\) followed by a dilation with a scale factor of 2 centered at point \(E\)
4. A counterclockwise rotation of 90 degrees about point \(E\) followed by a dilation with a scale factor of 2 centered at point \(E\)

10 In the diagram of right triangle \(ABC\), \(CD\) intersects hypotenuse \(AB\) at \(D\).

If \(AD = 4\) and \(DB = 6\), which length of \(AC\) makes \(CD \perp AB\)?

1. \(2\sqrt{6}\)  
2. \(2\sqrt{10}\)  
3. \(2\sqrt{15}\)  
4. \(4\sqrt{2}\)
11 Segment \( CD \) is the perpendicular bisector of \( AB \) at \( E \). Which pair of segments does not have to be congruent?

(1) \( AD, BD \)  
(2) \( AC, BC \)  
(3) \( \overline{AE}, \overline{BE} \)  
(4) \( \overline{DE}, \overline{CE} \)

12 In triangle \( CHR \), \( O \) is on \( \overline{HR} \), and \( D \) is on \( \overline{CR} \) so that \( \angle H \cong \angle RDO \).

If \( RD = 4 \), \( RO = 6 \), and \( OH = 4 \), what is the length of \( CD \)?

(1) \( 2\frac{2}{3} \)  
(2) \( 6\frac{2}{3} \)  
(3) 11  
(4) 15

13 The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

(1) circle  
(2) square  
(3) triangle  
(4) rectangle

14 The diagonals of rhombus \( TEAM \) intersect at \( P(2,1) \). If the equation of the line that contains diagonal \( \overline{TA} \) is \( y = -x + 3 \), what is the equation of a line that contains diagonal \( \overline{EM} \)?

(1) \( y = x - 1 \)  
(2) \( y = x - 3 \)  
(3) \( y = -x - 1 \)  
(4) \( y = -x - 3 \)
15 The coordinates of vertices $A$ and $B$ of $\triangle ABC$ are $A(3,4)$ and $B(3,12)$. If the area of $\triangle ABC$ is 24 square units, what could be the coordinates of point $C$?

(a) (3,6)  
(b) (8,−3)  
(c) (−3,8)  
(d) (6,3)

16 What are the coordinates of the center and the length of the radius of the circle represented by the equation

$$x^2 + y^2 −4x + 8y + 11 = 0$$

(a) center $(2,−4)$ and radius 3  
(b) center $(-2,4)$ and radius 3  
(c) center $(2,−4)$ and radius 9  
(d) center $(-2,4)$ and radius 9

17 The density of the American white oak tree is 752 kilograms per cubic meter. If the trunk of an American white oak tree has a circumference of 4.5 meters and the height of the trunk is 8 meters, what is the approximate number of kilograms of the trunk?

(a) 13  
(b) 9694  
(c) 13,536  
(d) 30,456
18 Point $P$ is on the directed line segment from point $X(-6,-2)$ to point $Y(6,7)$ and divides the segment in the ratio 1:5. What are the coordinates of point $P$?

(1) $(4, \frac{1}{2})$  
(2) $(-\frac{1}{2}, -4)$  
(3) $(-4 \frac{1}{2}, 0)$  
(4) $(-4, -\frac{1}{2})$

19 In circle $O$, diameter $AB$, chord $BC$, and radius $OC$ are drawn, and the measure of arc $BC$ is $108^\circ$.

Some students wrote these formulas to find the area of sector $COB$:

Amy $\frac{3}{10} \cdot \pi \cdot (BC)^2$

Beth $\frac{108}{360} \cdot \pi \cdot (OC)^2$

Carl $\frac{3}{10} \cdot \pi \cdot (\frac{1}{2} AB)^2$

Dex $\frac{108}{360} \cdot \pi \cdot \frac{1}{2} (AB)^2$

Which students wrote correct formulas?

(1) Amy and Dex  
(2) Beth and Carl  
(3) Carl and Amy  
(4) Dex and Beth
20 Tennis balls are sold in cylindrical cans with the balls stacked one on top of the other. A tennis ball has a diameter of 6.7 cm. To the nearest cubic centimeter, what is the minimum volume of the can that holds a stack of 4 tennis balls?

(1) 236  (2) 282  (3) 564  (4) 945

21 Line segment $A'B'$, whose endpoints are $(4,-2)$ and $(16,14)$, is the image of $AB$ after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of $AB$?

(1) 5  (2) 10  (3) 20  (4) 40

22 Given: $\triangle ABE$ and $\triangle CBD$ shown in the diagram below with $DB \cong BE$

![Diagram of triangles ABE and CBD with corresponding parts labeled]

Which statement is needed to prove $\triangle ABE \cong \triangle CBD$ using only SAS $\cong$ SAS?

(1) $\angle CDB \cong \angle AEB$  (2) $\angle AFD \cong \angle EFC$

(3) $\overline{AD} \cong \overline{CE}$  (4) $\overline{AE} \cong \overline{CD}$

Use this space for computations.
23 In the diagram below, \( BC \) is the diameter of circle \( A \).

Point \( D \), which is unique from points \( B \) and \( C \), is plotted on circle \( A \). Which statement must always be true?

(1) \( \triangle BCD \) is a right triangle.
(2) \( \triangle BCD \) is an isosceles triangle.
(3) \( \triangle BAD \) and \( \triangle CBD \) are similar triangles.
(4) \( \triangle BAD \) and \( \triangle CAD \) are congruent triangles.

24 In the diagram below, \( ABCD \) is a parallelogram, \( AB \) is extended through \( B \) to \( E \), and \( CE \) is drawn.

If \( CE \equiv BE \) and \( \angle D = 112^\circ \), what is \( \angle E \)?

(1) 44°  (3) 68°
(2) 56°  (4) 112°
25 Lines $AE$ and $BD$ are tangent to circles $O$ and $P$ at $A$, $E$, $B$, and $D$, as shown in the diagram below.

If $AC:CE = 5:3$, and $BD = 56$, determine and state the length of $CD$. 


![Diagram of two circles with tangents and segments labeled A, B, C, D, E, O, and P.]
In the diagram below, \( \triangle ABC \) has coordinates \( A(1,1), B(4,1), \) and \( C(4,5) \). Graph and label \( \triangle A'B'C' \), the image of \( \triangle ABC \) after the translation five units to the right and two units up followed by the reflection over the line \( y = 0 \).
27 A regular hexagon is rotated in a counterclockwise direction about its center. Determine and state the minimum number of degrees in the rotation such that the hexagon will coincide with itself.
28 In the diagram of \( \triangle ABC \) shown below, use a compass and straightedge to construct the median to \( AB \). [Leave all construction marks.]
Triangle $MNP$ is the image of triangle $JKL$ after a $120^\circ$ counterclockwise rotation about point $Q$. If the measure of angle $L$ is $47^\circ$ and the measure of angle $N$ is $57^\circ$, determine the measure of angle $M$. Explain how you arrived at your answer.
A circle has a center at (1,–2) and radius of 4. Does the point (3.4,1.2) lie on the circle? Justify your answer.
31 In the diagram below, a window of a house is 15 feet above the ground. A ladder is placed against the house with its base at an angle of 75° with the ground. Determine and state the length of the ladder to the nearest tenth of a foot.
32 Using a compass and straightedge, construct and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation with a scale factor of 2 and centered at $B$. [Leave all construction marks.]

Describe the relationship between the lengths of $AC$ and $A'C'$. 

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]
33 The grid below shows $\triangle ABC$ and $\triangle DEF$.

Let $\triangle A'B'C'$ be the image of $\triangle ABC$ after a rotation about point $A$. Determine and state the location of $B'$ if the location of point $C'$ is $(8,-3)$. Explain your answer.

Is $\triangle DEF$ congruent to $\triangle A'B'C'$? Explain your answer.
As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.

Determine and state, to the nearest tenth of a degree, the measure of \( \theta \), the projection angle.
Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.  \[12\]

35 Given: Circle \(O\), chords \(AB\) and \(CD\) intersect at \(E\)

![Diagram of a circle with chords AB and CD intersecting at E](image)

Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Prove this theorem by proving \(AE \cdot EB = CE \cdot ED\).
A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.

The desired density of the shaved ice is 0.697 g/cm³, and the cost, per kilogram, of ice is $3.83. Determine and state the cost of the ice needed to make 50 snow cones.
### High School Math Reference Sheet

1 inch = 2.54 centimeters  
1 meter = 39.37 inches  
1 mile = 5280 feet  
1 mile = 1760 yards  
1 mile = 1.609 kilometers  
1 kilometer = 0.62 mile  
1 pound = 16 ounces  
1 pound = 0.454 kilogram  
1 kilogram = 2.2 pounds  
1 cup = 8 fluid ounces  
1 pint = 2 cups  
1 quart = 2 pints  
1 gallon = 4 quarts  
1 gallon = 3.785 liters  
1 liter = 0.264 gallon  
1 liter = 1000 cubic centimeters

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Pythagorean Theorem</th>
<th>Quadratic Formula</th>
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<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
<td>$a^2 + b^2 = c^2$</td>
<td>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</td>
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<td>Parallelogram</td>
<td>$A = bh$</td>
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<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
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<tr>
<td>Circle</td>
<td>$C = \pi d$ or $C = 2\pi r$</td>
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<td>General Prisms</td>
<td>$V = Bh$</td>
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<td>$V = \pi r^2h$</td>
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<tr>
<td>Sphere</td>
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<tr>
<td>Cone</td>
<td>$V = \frac{1}{3} \pi r^2h$</td>
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<td>Pyramid</td>
<td>$V = \frac{1}{3} Bh$</td>
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<th>Geometric Sequence</th>
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<tr>
<td>Circle</td>
<td>$A_n = a_1 + (n - 1)d$</td>
<td>$a_n = a_1r^{n-1}$</td>
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<tr>
<td>Geometric Series</td>
<td>$S_n = \frac{a_1 - a_1r^n}{1 - r}$ when $r \neq 1$</td>
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<td>1 degree</td>
<td>$\frac{\pi}{180}$</td>
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</tbody>
</table>

| Exponential Growth/Decay | $A = A_0e^{k(t - t_0)} + B_0$ | |

Geometry (Common Core) – Aug. ’16
Scrap Graph Paper — This sheet will *not* be scored.
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