

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 25, 2025 — 9:15 a.m. to 12:15 p.m., only

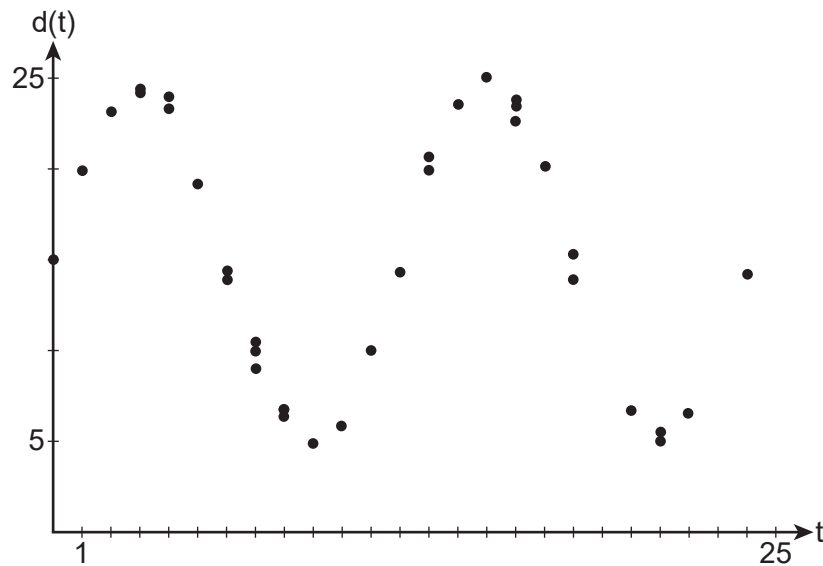
MODEL RESPONSE SET

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Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

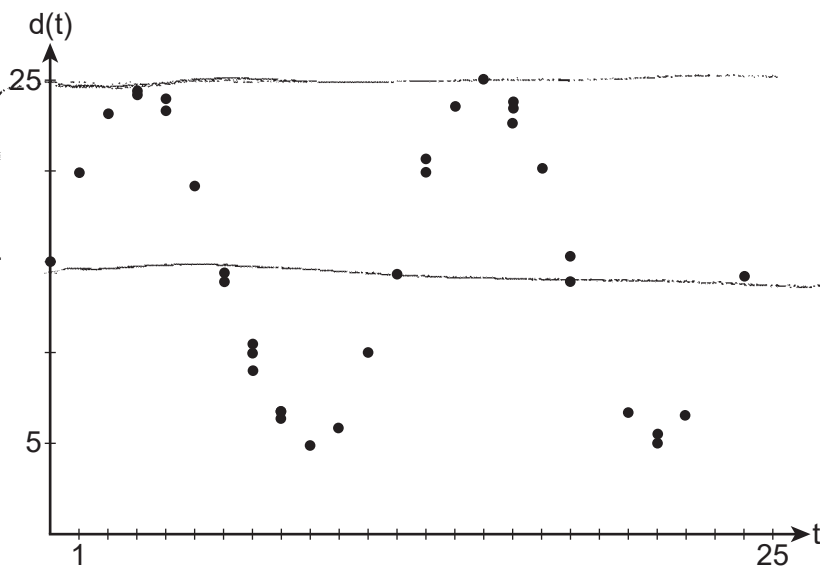
$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the *nearest integer*. Justify your answer.

$A=10$ because C is the midline which is at 10 and from 10 to 25 there is 10 spaces away.

Score 2: The student gave a complete and correct response.

Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

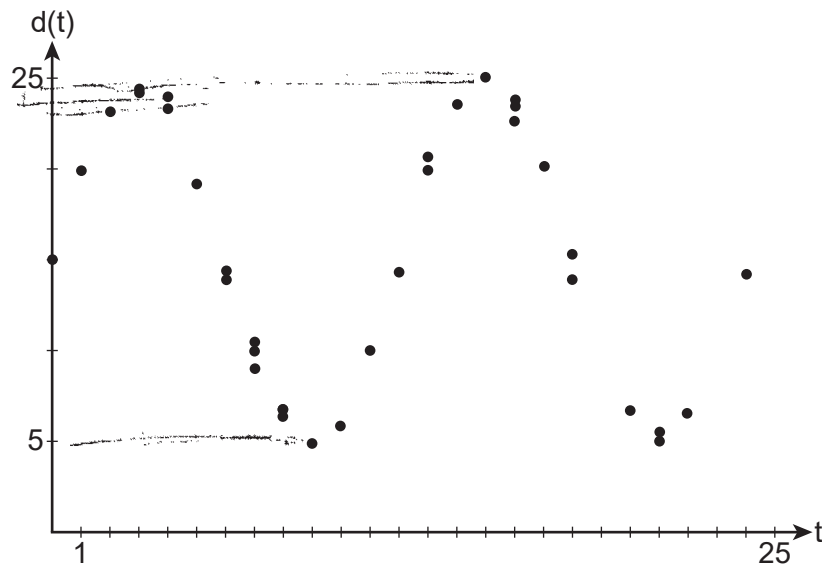
$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the *nearest integer*. Justify your answer.

10

Score 2: The student gave a complete and correct response.

Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

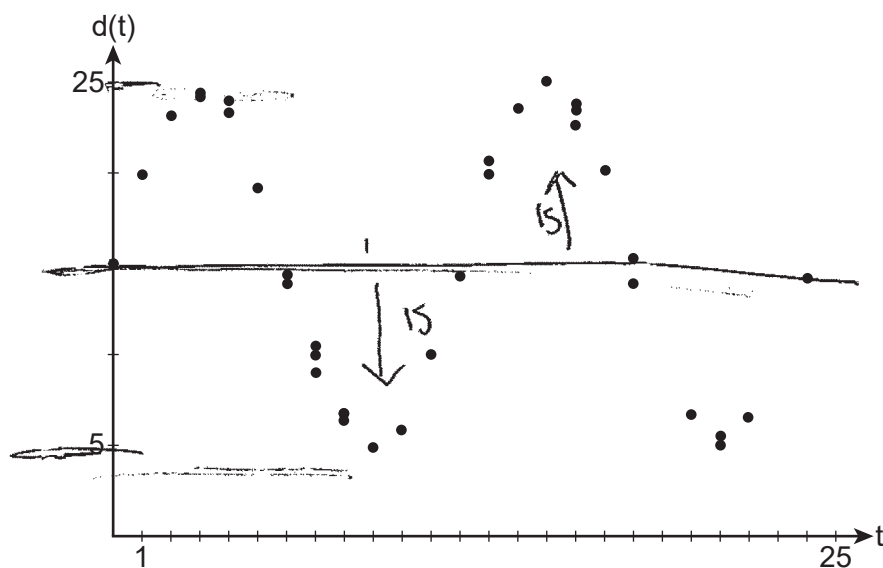
$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the nearest integer. Justify your answer.

$A = 20$ because A is the amplitude of the function and looking at the graph, the amplitude can be seen within the minimum value of 5 and maximum value of 25; $25 - 5 = 20$.

Score 1: The student did not divide by two when calculating the amplitude.

Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

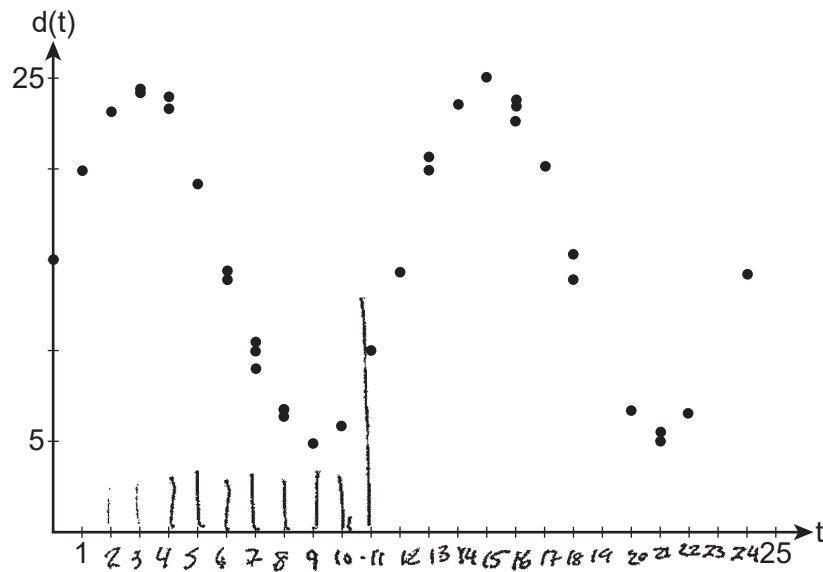
$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the *nearest integer*. Justify your answer.

$A = 15$ $a = \text{amplitude}$

Score 1: The student made a computational error.

Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

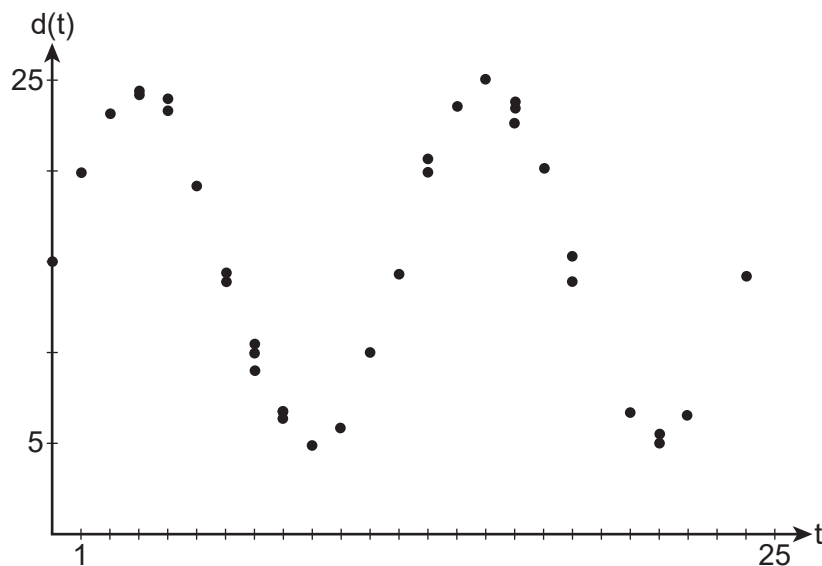
$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the nearest integer. Justify your answer.

$$\begin{aligned}
 & d(t) = A \sin\left(\frac{\pi}{6}t\right) + C \\
 & 25 = 11 \sin\left(\frac{\pi}{6}24\right) + C \\
 & \quad \downarrow \\
 & 25 = 11 \times -25 - 13 + C \\
 & 25 = -25 - 13 + C \\
 & 25 = -38 + C \\
 & C = 63
 \end{aligned}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 25

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.



The depth of the water can be modeled with a trigonometric function of the form

$d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A , to the *nearest integer*. Justify your answer.

$A = 25$ because that
is the max the line
will go

Score 0: The student did not show enough correct work to receive any credit.

Question 26

26 Algebraically determine the solution to the equation below.

$$\sqrt{x-2} + x = 4$$

$$\sqrt{x-2} = (-x+4)$$

$$x-2 = (-x+4)(-x+4)$$

$$x-2 = x^2 - 8x + 16$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$$\cancel{x=6} \quad x=3$$

$$\sqrt{6-2} + 6 = 4$$

$$2+6 \neq 4$$

$$\sqrt{3-2} + 3 = 4$$

$$1+3 = 4 \checkmark$$

Score 2: The student gave a complete and correct response.

Question 26

26 Algebraically determine the solution to the equation below.

$$\sqrt{x-2} + x = 4$$

$$(\sqrt{x-2})^2 = (4-x)^2$$

$$x-2 = (4-x)(4-x)$$

$$x-2 = 16 - 4x - 4x + x^2$$

$$x-2 = x^2 - 8x + 16$$

$$x^2 - 9x + 18$$

$$(x-3)(x-6)$$

$$\boxed{x=3} \quad x=6$$

Score 2: The student gave a complete and correct response.

Question 26

26 Algebraically determine the solution to the equation below.

$$\sqrt{x-2} + x = 4$$

$$(\sqrt{x-2})^2 = (4-x)^2$$

$$x-2 = 16-8x+x^2$$

$$x^2 - 9x + 18 = 0$$

$$(x-3)(x-6)$$

$$x = \{3, 6\}$$

Score 1: The student did not reject the extraneous root.

Question 26

26 Algebraically determine the solution to the equation below.

$$\sqrt{x-2} + x = 4$$

$$\sqrt{x-2} = (4-x)$$

$$x-2 = (4-x)(4-x)$$

$$16 - 4x - 4x + x^2$$

$$x-2 = 16 - 8x + x^2$$

$$x = 18 - 8x + x^2$$

$$2(1)$$

$$0 = 18 - 9x + x^2$$

$$\frac{8 \pm \sqrt{8}}{2}$$

$$\frac{8 \pm \sqrt{4} \sqrt{-2}}{2}$$

$$\frac{8 \pm 2\sqrt{-2}}{2}$$

$$4 \pm \sqrt{-2}$$

$$4 \pm \sqrt{2} i$$

Score 1: The student made a substitution error.

Question 26

26 Algebraically determine the solution to the equation below.

$$\begin{aligned} \sqrt{x-2} + x &= 4 \\ \sqrt{x-2} &= 4-x \\ (\sqrt{x-2})^2 &= (4-x)^2 \\ x-2 &= x^2 - 8x + 16 \\ -x^2 + 8x - 16 &= -x^2 + 8x - 16 \\ -x^2 + 9x - 18 &= 0 \\ \text{No Solution} \end{aligned}$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 26

26 Algebraically determine the solution to the equation below.

$$\sqrt{x-2} + x = 4$$

$$\begin{array}{r} \sqrt{x-2} + x = 4 \\ -x \quad -x \\ \hline \end{array}$$

$$x-2 = x^2 - 4x - 4x + 16$$

$$\begin{array}{r} x+2 = x^2 - 16x + 16 \\ -x-2 \quad -x-2 \\ \hline 0 = x^2 - 17x + 14 \end{array}$$

answer: $\frac{17 + \sqrt{233}}{2}$

Score 0: The student made multiple errors.

Question 27

27 Factor the expression completely.

$$(x - 1)^2 + 5(x - 1) - 6$$

$$= x^2 - 2x + 1 + 5x - 5 - 6$$

$$= x^2 + 3x - 10$$

$$= (x - 2)(x + 5)$$

Score 2: The student gave a complete and correct response.

Question 27

27 Factor the expression completely.

$$(x - 1)^2 + 5(x - 1) - 6$$

$$\begin{aligned} & (x-1)(x-1) + 5x - 5 - 6 \\ & a^2 + 5a - 6 \\ & (a+6)(a-1) \\ & (x-1+6)(x-2) \\ & (x+5)(x-2) \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 27

27 Factor the expression completely.

$$(x - 1)^2 + 5(x - 1) - 6$$

$$x^2 - 2x + 1 + 5x - 5 - 6$$

$$= x^2 - 2x + 5x - 11 + 1$$

$$= x^2 + 3x - 10$$

$$= (x + 5)(x - 2)$$

$$\boxed{x = -5 \quad x = 2}$$

Score 1: The student made an error by solving for x .

Question 27

27 Factor the expression completely.

$$\begin{array}{r} x-1 \\ x \quad \begin{array}{|c|c|} \hline x^2 & -x \\ \hline -x & 1 \\ \hline \end{array} \\ -1 \end{array}$$

$$(x-1)^2 + 5(x-1) - 6$$

$$(x-1)(x-1) + 5(x-1) - 6$$

$$\cancel{x^2(x-1)(x-1)}$$

$$\cancel{x^2} (2x+1) + 5x - 5 - 6$$

$$x^2 + 3x - 10$$

Score 1: The student found a correct quadratic expression but did not factor.

Question 27

27 Factor the expression completely.

$$(x-1)(x-1)$$

$$x^2 - x - 1 + 1$$

$$x^2 - 2x + 1 + 5x - 11$$

$$x^2 - 3x + 10$$

$$(x-5)(x+2)$$

$$\begin{array}{c} (x-5)(x+2) \\ \hline x=5 \quad x=2 \end{array}$$

$$(x-1)^2 + 5(x-1) - 6$$

$$(x-1)^2 + 5x - 5 - 6$$

Score 0: The student made multiple errors.

Question 27

27 Factor the expression completely.

$$(x-1)(x-1) + 5(x) - 5(1) - 6$$

$$x^2 - 1x - 1x + 1 + 5x - 5 - 6$$

$$x^2 - 2x - 10$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 28

- 28 The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise	
Female	87	94	21	202
Male	68	110	18	196
	155	204	39	398

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$P(M|F) = \frac{\frac{94}{398}}{\frac{202}{398}} = \frac{94}{398} \cdot \frac{398}{202} = \frac{94}{202} = \frac{47}{101}$$

Score 2: The student gave a complete and correct response.

Question 28

- 28 The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise	
Female	87	94	21	202
Male	68	110	18	196

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{202}$$

Score 2: The student gave a complete and correct response.

Question 28

- 28 The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise
Female	87	94	21
Male	68	110	18

202

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{202} = .46535$$

Score 1: The student did not express the final answer as an exact probability.

Question 28

- 28** The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise
Female	87	94	21
Male	68	110	18

204

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{204}$$

Score 1: The student calculated the incorrect conditional probability.

Question 28

- 28 The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise	
Female	87	94	21	202
Male	68	110	18	196
	155	204	39	

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{204} = 46.1\%$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 28

- 28** The results of a survey of the students at the local high school regarding the topic “What I Do to Relax” are displayed in the table below.

	Read	Listen to Music	Exercise
Female	87	94	21
Male	68	110	18

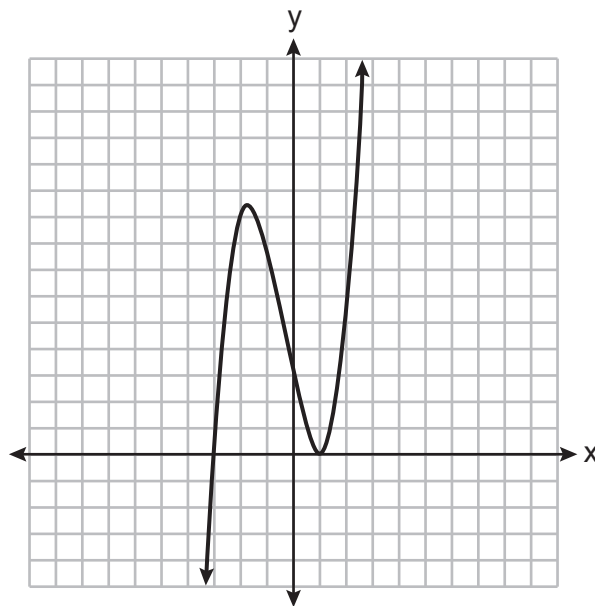
If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{204} = \frac{47}{102}$$

Score 0: The student made multiple errors.

Question 29

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.



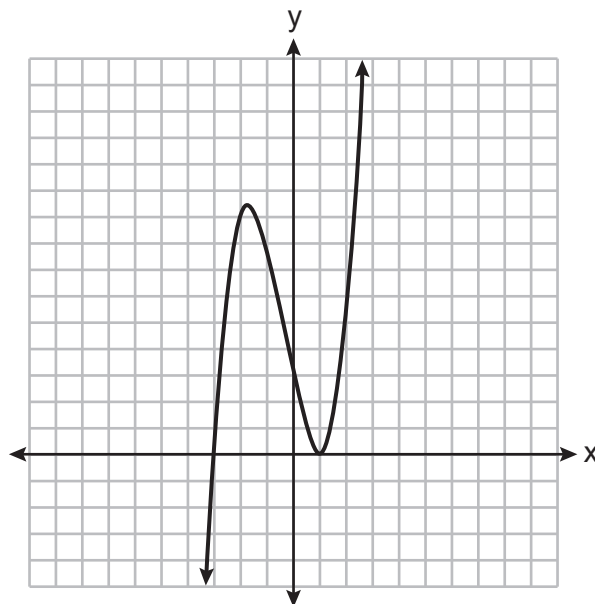
Write an equation for $f(x)$.

$$f(x) = (x+3)(x-1)^2$$

Score 2: The student gave a complete and correct response.

Question 29

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.



Write an equation for $f(x)$.

Zeros : -3 , 1 , 1

$$(x+3)(x-1)(x-1)$$

$$y = 1(x+3)(x-1)(x-1)$$

$$y = 1(x^2 - x + 3x - 3)(x-1)$$

$$y = 1(x^2 + 2x - 3)(x-1)$$

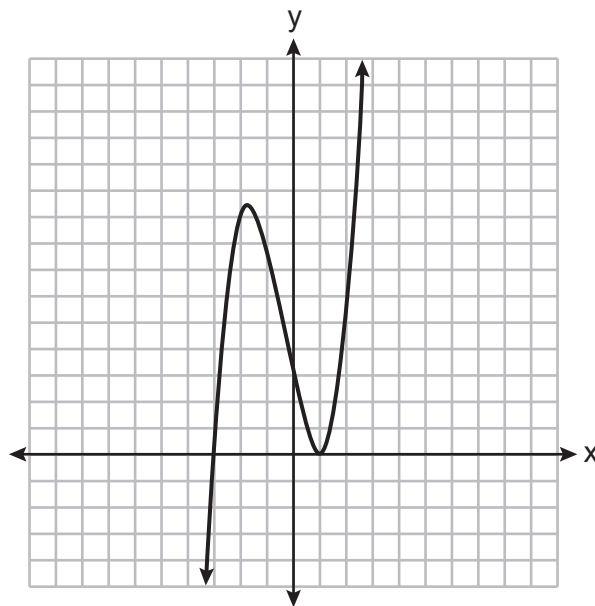
$$y = 1(x^3 - x^2 + 2x^2 - 2x - 3x + 3)$$

$$y = x^3 + x^2 - 5x + 3$$

Score 2: The student gave a complete and correct response.

Question 29

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.



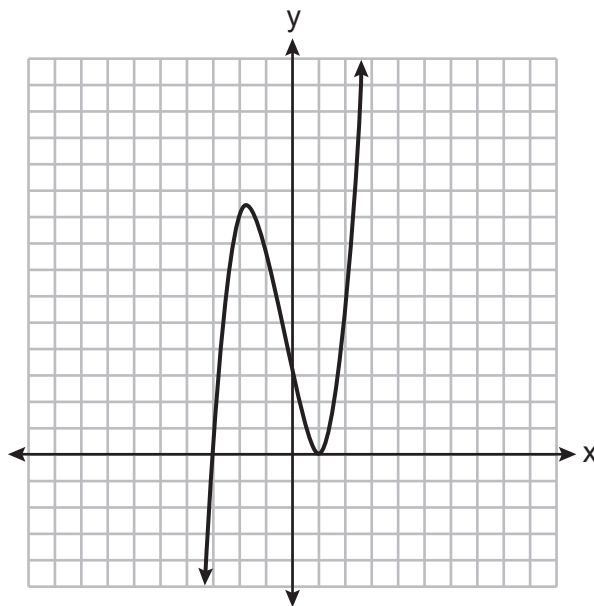
Write an equation for $f(x)$.

$$y = (x-1)(x+3)$$

Score 1: The student did not recognize the repeated root.

Question 29

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.



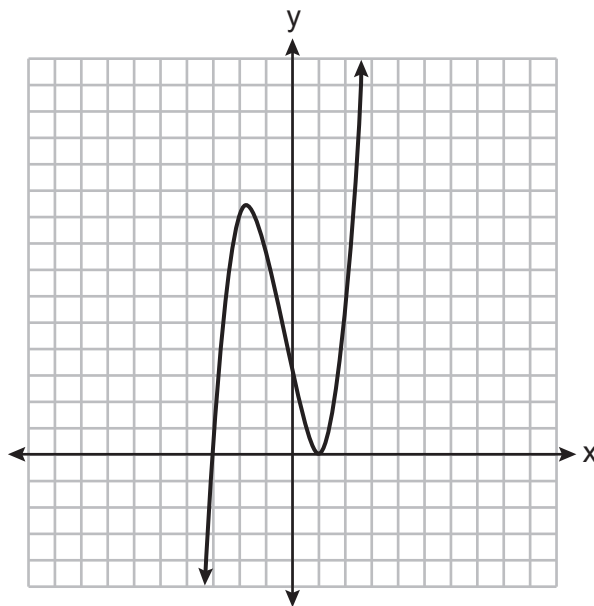
Write an equation for $f(x)$.

$$f(x) = (x^3 - 1)$$

Score 0: The student did not show enough correct work to receive any credit.

Question 29

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.



Write an equation for $f(x)$.

$$f(x) = x^3(x-3)(x+1) + 6$$

Score 0: The student did not show enough correct work to receive any credit.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$$x = \frac{2}{3}y + 6$$

$$\frac{3}{2}(x-6) = \frac{2}{3}y \cdot \frac{3}{2}$$

$$\frac{3}{2}(x-6) = f^{-1}(x)$$

$$f^{-1}(x) = \frac{3}{2}x - 9$$

Score 2: The student gave a complete and correct response.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$\hat{y}$ $\nwarrow$ $\nearrow$

$$x = \frac{2}{3}y + 6$$

$$\frac{3}{2} \cdot \frac{2}{3}y = \frac{(x-6)3}{2}$$

$$y = \frac{3x-18}{2}$$

$$\downarrow$$
$$f^{-1}(x) = \frac{3x-18}{2}$$

Score 2: The student gave a complete and correct response.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$$y = \frac{2}{3}x + 6$$

$$x = \frac{2}{3}y + 6$$

$$\frac{3}{2}(x-6) = \left(\frac{2}{3}\right)y \quad \frac{3}{2}$$

$$\frac{3}{2}(x-6) = y$$

$$\frac{3}{2}x - 9 = y$$

$$f^{-1}(x) = 3x - 9$$

Score 1: The student made a transcription error.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$$y = \frac{2}{3}x + 6$$

$$x = \frac{3}{2}y + \underline{6}$$

$$\frac{3}{2} \left(\frac{2}{3}y \right) = (x - 6) \left(\frac{3}{2} \right)$$

$$y = \frac{3}{2}x - 9$$

Score 1: The student did not use $f^{-1}(x)$ when writing the equation.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$$f(x) = \frac{2}{3}x + 6$$

$$f^{-1}(x) = -\frac{2}{3}x - 6$$

Score 0: The student incorrectly found the inverse.

Question 30

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

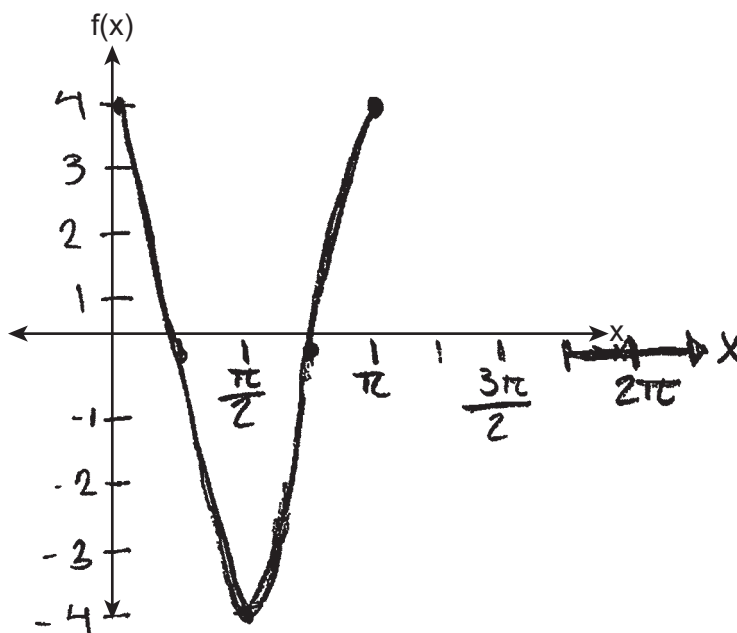
$$f^{-1}(x) = \frac{2}{3}(-1) + 6$$

$$\boxed{f^{-1}(x) = \frac{16}{3}}$$

Score 0: The student evaluated $f(-1)$.

Question 31

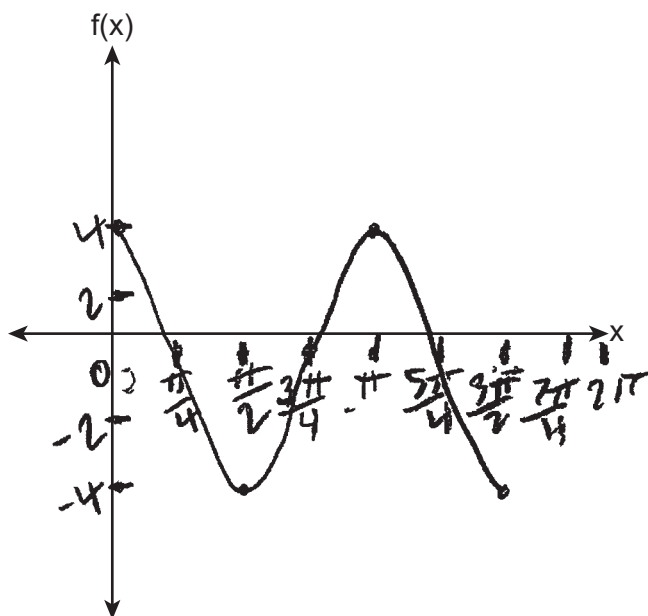
- 31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.



Score 2: The student gave a complete and correct response.

Question 31

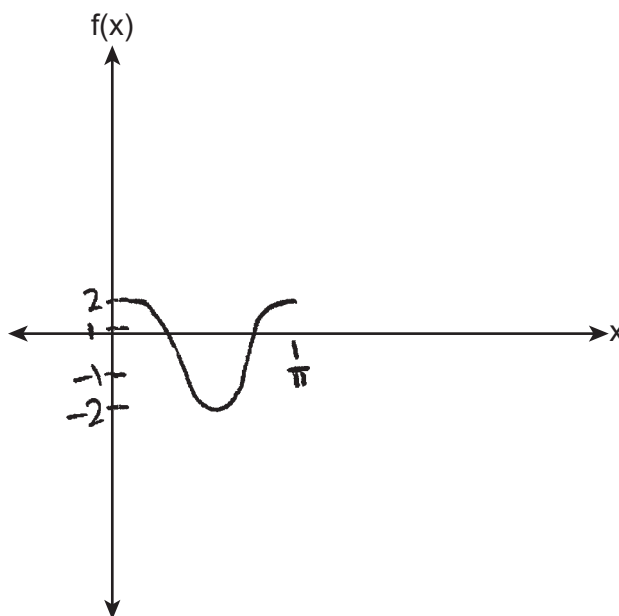
- 31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.



Score 2: The student gave a complete and correct response.

Question 31

- 31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.

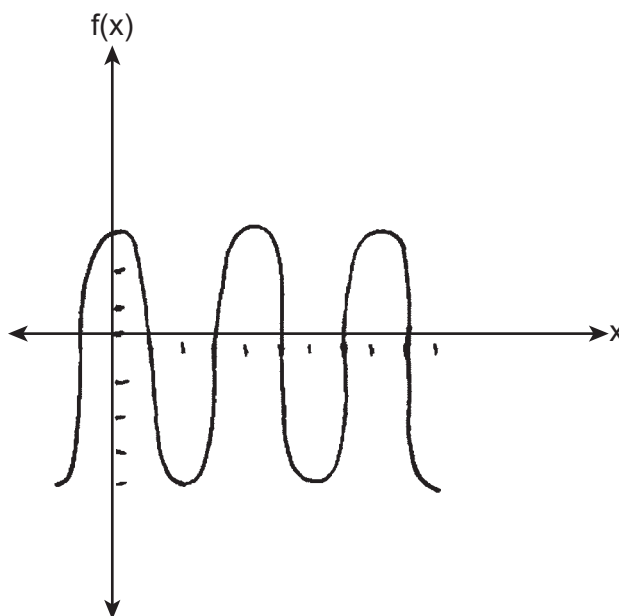


$$\frac{2\pi}{1} \times \frac{1}{2} = \pi$$

Score 1: The student graphed the amplitude incorrectly.

Question 31

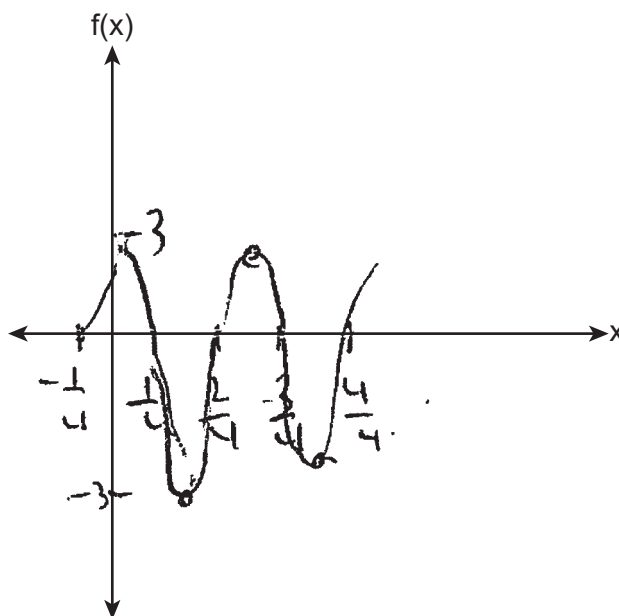
- 31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.



Score 1: The student did not indicate a scale on the x -axis.

Question 31

- 31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.



Score 0: The student made multiple graphing errors.

Question 32

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

$$(.615, .795) \rightarrow (61.5\%, 79.5\%)$$

No, 67% is within the
range between 61.5% and 79.5%

Score 2: The student gave a complete and correct response.

Question 32

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

-7 The contest
does not provide
evidence to challenge
the claim
7.05
 ± 0.045 .67 is within
INT .66, .75

Score 2: The student gave a complete and correct response.

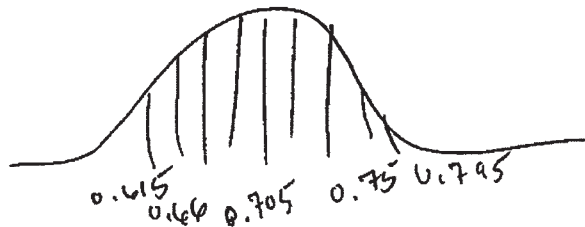
Question 32

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

mean : 0.705

SD : 0.045



No, the contest result doesn't provide evidence to contradict the designer's claim.

Score 1: The student wrote an incomplete justification.

Question 32

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

$$MOE = 2\sigma = 2(0.045) = .09$$

$$(.615, .795)$$
$$(61.5, 79.5)$$

The results show the designer's claim being true because ~~70%~~ 70% fits in the ~~it~~ confidence interval.

Score 1: The student used 70% in their justification.

Question 32

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

No Cause 67% is close to
the designers claim of 70%.

Score 0: The student wrote an incorrect justification.

Question 33

33 Solve algebraically for x : $\frac{x(x-4)}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$2(x-4) = x(2x+3)$$

$$\begin{array}{r} 2x - 8 = 2x^2 + 3x \\ -2x \quad +8 \end{array}$$

$$2x^2 + x + 8 = 0$$

$a=2 \quad b=1 \quad c=8$

$$\frac{-1 \pm \sqrt{1-4(2)(8)}}{2(2)}$$

$$\frac{-1 \pm \sqrt{-63}}{4}$$

$63 = 9 \cdot 7$

$$\frac{-1 \pm 3i\sqrt{7}}{4}$$

$$\boxed{\frac{-1}{4} \pm \frac{3i\sqrt{7}}{4}}$$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\begin{array}{r} 2x-8 = 2x^2+3x \\ -2x+8 \quad \quad -2x \end{array}$$

$$\begin{array}{l} a=2 \\ b=1 \\ c=8 \end{array} \quad \begin{array}{r} 2x^2+x+8 \\ -1 \pm \sqrt{1-64} \\ \hline 4 \end{array}$$

$$\begin{array}{r} \sqrt{63} \\ -1 \pm 3\sqrt{7} \\ \hline 4 \end{array}$$

$$\boxed{\begin{array}{l} x = \frac{-1}{4} + \frac{3i\sqrt{7}}{4} \\ x = \frac{-1}{4} - \frac{3i\sqrt{7}}{4} \end{array}}$$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\begin{array}{r} 2x - 8 = 2x^2 + 3x \\ -2x + 8 \end{array}$$

$$0 = 2x^2 + x + 8$$

$$(2x \quad)(x \quad)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 - 64}}{4}$$

$$x = \frac{-1 \pm i\sqrt{63}}{4}$$

$$x = \frac{-1}{4} \pm \frac{i\sqrt{63}}{4}$$

Score 3: The student did not simplify the radical.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\frac{\cancel{(x-4)}^{\cancel{(x-4)}(x)}}{x} = \frac{\cancel{(x-4)}^{\cancel{(x-4)}(x)}(2x+3)}{\cancel{x-4}}$$

LCD: $(x-4)(x)$

$$\begin{array}{r} 2x-8 \\ +8 \end{array} = \begin{array}{r} 2x^2+3 \\ +8 \end{array}$$

$$\begin{array}{ccc} 2x^2 & -2x & +11 \\ a & b & c \end{array} = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(2)(11)}}{2(2)}$$

$$\frac{2 \pm \sqrt{-84}}{4}$$

$$\sqrt{21} \sqrt{4}$$

$$\frac{\frac{1}{2} \pm \frac{1}{2}i\sqrt{21}}{2}$$

$$\boxed{\frac{1}{2} \pm \frac{i\sqrt{21}}{2}}$$

$$\boxed{\frac{1}{2} - \frac{i\sqrt{21}}{2}}$$

Score 3: The student made a computational error.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\frac{2}{x} = \frac{2x+3}{x-4}$$

$$2x-8 = 2x^2+3x$$

$$2x^2+x+8=0$$

$$D = b^2 - 4ac = 1 - 4(2)(8) = 1 - 8(8) = 1 - 64 = -63$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{-63}}{4} = \frac{-1 \pm i\sqrt{63}}{4}$$

$$\boxed{\begin{array}{l} x_1 = \frac{-1 + i\sqrt{63}}{4} \\ x_2 = \frac{-1 - i\sqrt{63}}{4} \end{array}}$$

Score 2: The student did not express in $a + bi$ form and did not simplify the radical.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\frac{2}{x} = \frac{2x+3}{x-4}$$

$$x(2x+3) = 2(x-4)$$

$$\begin{array}{r} 2x^2 + 3x = 2x - 4 \\ -2x + 4 \quad -2x + 4 \\ \hline \end{array}$$

$$2x^2 + x + 4 = 0$$

$$a=2 \quad b=1 \quad c=4$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{-28}}{4} \quad \begin{matrix} 14 \\ 2 \end{matrix}$$

$$x = \frac{-2}{4} \pm \frac{2i\sqrt{7}}{2}$$

$$\boxed{x = -\frac{1}{2} + i\sqrt{7} \quad x = -\frac{1}{2} - i\sqrt{7}}$$

Score 1: The student made three errors.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$2(x-4) = x(2x+3)$$

$$\begin{array}{l} (2x-8) = 2x^2+3x \\ -2x \quad +8 \end{array}$$

$$0 = 2x^2 + 1x + 8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(8)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{-63}}{4}$$

$$x = -1 \pm \frac{63i}{4}$$

Score 1: The student showed work to find a correct quadratic equation in standard form.

Question 33

33 Solve algebraically for x : ~~$\frac{2}{x} = \frac{2x+3}{x-4}$~~ . Express your answers in simplest $a + bi$ form.

$$2x - 8 = 2x^2 + 3x$$

$$2x^2 - x + 8 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{1 \pm \sqrt{-63}}{2(2)}$$

$$\frac{1 \pm \sqrt{-63}}{4}$$

$$\begin{array}{l} b^2 - 4ac \\ 1 - 63 \\ -63 \end{array}$$

Score 1: The student made a computational error and did not express the answer in simplest $a + bi$ form.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\frac{2}{x} = \frac{2x+3}{x-4}$$

$$x(2x+3) = 2(x-4)$$

$$2x^2 + 3x = 2x - 8$$
$$\begin{array}{r} -2x^2 \quad -2x \quad +8 \\ \hline \end{array}$$

$$2x^2 - x + 8 = 0$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 33

33 Solve algebraically for x : $\frac{2}{x} = \frac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$\begin{aligned} \frac{(x-4) \cancel{2} \cdot \cancel{(x-4)} \cdot \frac{2x+3}{(x-4)(x-4)}}{(x-4) \cancel{(x-4)}} &\rightarrow \frac{2(x-4) + 2x^2 + 3x}{x(x-4)} \\ &\rightarrow \frac{2x-8+2x^2+3x}{x^2-4x} + \frac{\overbrace{2x^2+5x-8}^{x^2+5x-16}}{x^2-4x} \end{aligned}$$

$$\boxed{\frac{2x^2+5x-8}{x^2-4x}}$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

$$95\% \approx 2 \text{ standard deviations}$$

$$750 \pm 2(20) = 710, 790$$

$$\begin{array}{l} 95\% \text{ of students score} \\ \text{within the interval of} \\ 710 \leq p \leq 790 \end{array}$$

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$\text{normalcdf}(10^{-99}, 760, 750, 20) = .6914624678$$

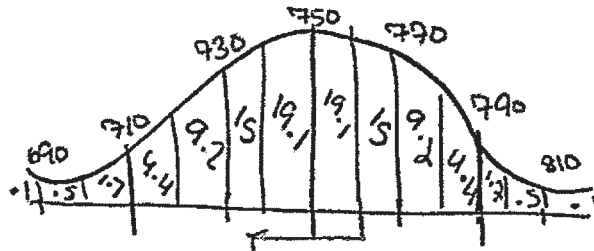
$$\begin{array}{l} 69\% \text{ of accepted students} \\ \text{scored a 760 or less} \end{array}$$

Score 4: The student gave a complete and correct response.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.



$$710 - 790$$

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$19.1 + 19.1 + 18 + 9.2 + 4.4 + 1.7 + .6 =$$

$$69\%$$

Score 4: The student gave a complete and correct response.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

95% is 2 deviations away from the mean.

$20(2) = 40$
40 is 2 standard deviations away

$$750 \pm 40$$
$$790$$

$$750 - 40$$
$$710$$

$$710 - 750$$

To the nearest whole percent, determine the percentage of accepted students who scored a 760 or less.

$$\text{normal cdf}(10^{-99}, 760, 750, 20)$$

$$.691$$

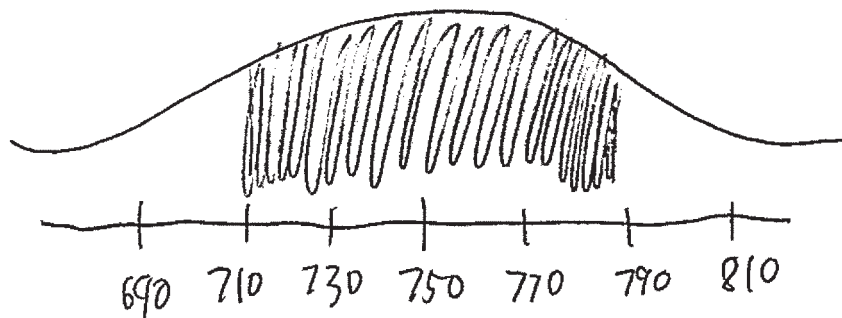
$$69\%$$

Score 3: The student made a transcription error.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.



$$710 \leq X \leq 790$$

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

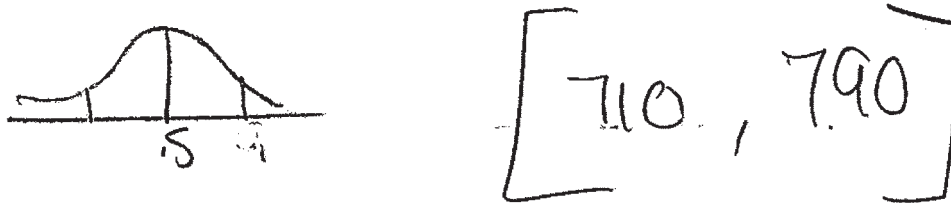
69%

Score 3: The student did not show work to find the percent.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.



To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$\frac{760 - 750}{20} = .5$$
$$(-999, .5, 0, 1)$$
$$69.14$$

70%

Score 2: The student did not show enough work to determine the interval and incorrectly rounded the percent.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

$$\begin{array}{ccc} 750 + 2(20) & \text{and} & 750 - 2(20) \\ \downarrow & & \swarrow \\ & 710 & 790 \end{array}$$

A score between 710 and 790

To the nearest whole percent, determine the percentage of accepted students who scored a 760 or less.

$$\begin{array}{r} 710 \neq 760 \\ 760 \\ - 710 \\ \hline 50 \end{array} \quad \begin{array}{r} 790 \\ - 710 \\ \hline 80 \end{array}$$

63%

$$\frac{50}{80} = .625$$

Score 2: The student correctly determined the interval.

Question 34

- 34** A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

710 - 790

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

Score 1: The student did not show work to determine the interval.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

normal cdf (0, 760, 750, 20)

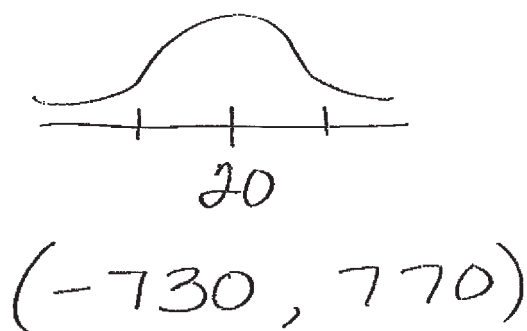
$$\hookrightarrow \approx .7 = 70\%$$

Score 1: The student incorrectly rounded the percent.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.



To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$760 - 20 = 740\%$$

Score 0: The student did not show enough correct work to receive any credit.

Question 34

- 34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

$$(690, 812.5)$$

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$69.1\%$$

Score 0: The student did not show enough correct work to receive any credit.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned}
 & (3x^2 - 4x + 7)(x - 2) - (x - 2)^3 \\
 & 3x^3 - 4x^2 + 7x - 6x^2 + 8x - 14 \quad \begin{matrix} (x-2)(x-2) \\ (x^2 - 4x + 4)(x-2) \end{matrix} \\
 & (3x^3 - 10x^2 + 15x - 14) - (x^3 - 4x^2 + 4x - 2x^2 + 8x - 8) \\
 & \cancel{(3x^3 - 10x^2 + 15x - 14)} - \cancel{(x^3 - 4x^2 + 4x - 2x^2 + 8x - 8)} \\
 & \boxed{2x^3 - 4x^2 + 3x - 6}
 \end{aligned}$$

Score 4: The student gave a complete and correct response.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned} & (3x^2 - 4x + 7)(x - 2) - (x - 2)^3 \\ &= (x - 2)[(3x^2 - 4x + 7) - (x - 2)^2] \\ &= (x - 2)[3x^2 - 4x + 7 - (x^2 + 4 - 4x)] \\ &= (x - 2)(2x^2 + 3) \\ &= 2x^3 + 3x - 4x^2 - 6 \\ &= 2x^3 - 4x^2 + 3x - 6 \end{aligned}$$

Score 4: The student gave a complete and correct response.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned}
 & c(x) \cdot d(x) - [d(x)]^3 \\
 & 3x^2 - 4x + 7 \cdot x - 2 - [x - 2]^3 \\
 & \underline{(3x^3 - 10x^2 + 15x - 14)} - \underline{x^3 - 6x^2 + 12x - 8} \\
 & \boxed{2x^3 - 16x^2 + 27x - 22}
 \end{aligned}$$

$$\begin{array}{r}
 \begin{array}{r|rr}
 & 3x^2 & -4x & 7 \\
 \times & 3x^2 & -4x^2 & 7x \\
 \hline
 -2 & -6x^2 & 8x & -14
 \end{array} \\
 3x^3 - 4x^2 + 7x - 6x^2 + 8 \\
 3x^3 - 10x^2 + 15x - 14 \\
 \hline
 (x-2)^3 \\
 \hline
 (x-2)(x-2)(x-2) \\
 x^2 - 2x - 2x + 4 \\
 (x^2 - 4x + 4)(x-2) \\
 x^3 - 2x^2 - 4x^2 + 8x + 4x \\
 x^3 - 6x^2 + 12x - 8
 \end{array}$$

Score 3: The student made a computational error distributing the negative.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned}
 & (3x^2 - 4x + 7)(x - 2) - [x - 2]^3 \\
 & 3x^3 - 4x^2 + 7x - 6x^2 + 8x - 14 \\
 & [x - 2][x - 2][x - 2] \\
 & x^2 - 2x - 2x + 4 \\
 & [x^2 - 4x + 4][x - 2] \\
 & x^3 - 4x^2 - 8 - 2x^2 + 8x - 8 \\
 & x^3 - 6x^2 + 8x - 16 \\
 & 3x^3 - 10x^2 + 15x - 14 - x^3 - 6x^2 + 8x - 16 \\
 & 2x^3 - 16x^2 + 23x - 30
 \end{aligned}$$

Score 2: The student made two computational errors.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned} & (3x^2 - 4x + 7) \cdot (x - 2) & (x-2)(x-2)(x-2) \\ & & x^2 + x^2 - 2x + 2x \\ & & -8 + 4 \\ & 3x^3 - 4x^2 - 14x + 14 & \\ & 3x^3 - 10x^2 + 15x - 14 \end{aligned}$$

Score 1: The student correctly determined $c(x) \cdot d(x)$.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned} & (3x^2 - 4x + 7)(x - 2) - (x - 2)^3 \\ & 3x^3 - 6x^2 - 4x^2 + 8x + 7x - 14 - (\cancel{x^3} + 4x^2 + 8x - 8) \\ & 2x^3 - 14x^2 + 7x + 6 \end{aligned}$$

Score 1: The student incorrectly determined $[d(x)]^3$ and then made a computational error.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$\begin{aligned}
 & (3x^2 - 4x + 7)(x - 2) - [x - 2]^3 \\
 & (3x^2 - 6x^2 - 4x^2 + 8x + 7x - 14) \\
 & -7x^2 + 15x - 14 - [x - 2]^3 \\
 & \quad \downarrow \\
 & \quad x + 2^3 = 18 \\
 & -7x^2 + 15x - 14 + 18 \\
 & -7x^2 + 15x + 4 \text{ - Answer}
 \end{aligned}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 35

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$3x^2 - 4x + 7 - (x-2)^3$$

$$3x^2 - 4x + 7 - (x-2)(x-2)(x-2)$$

$$3x^2 - 4x + 7 - (x-2)(x^2 - 4x + 4)$$

$$3x^2 - 4x + 7 - x^3 - 4x^2 + 4x$$

Score 0: The student did not satisfy the criteria for one or more credits.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$C_n = \frac{a - ar^n}{1-r}$$

$$C_n = \frac{85,000 - 85,000(1.025)^n}{1-(1.025)}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$C_{10} = \frac{85,000 - 85,000(1.025)^{10}}{1-(1.025)} \approx \$952,300$$

Score 4: The student gave a complete and correct response.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$C_n = \frac{a_1(1-r^n)}{1-r} \rightarrow C_n = \frac{85000(1-1.025^n)}{1-1.025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$C_{10} = \frac{85000(1-1.025^{10})}{1-1.025}$$

$$C_{10} = 952,300$$

Score 4: The student gave a complete and correct response.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$
$$S_n = \frac{85,000 - 85,000(1.025)^n}{1 - 1.025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$S_{10} = \frac{85,000 - 85,000(1.025)^{10}}{1 - 1.025}$$
$$S_{10} = 952,287.45$$

\$952,300

Score 3: The student made a notation error.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$C_n = \frac{85000 - 85000(0.025)^n}{1 - 0.025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$C_{10} = \frac{85000 - 85000(0.025)^{10}}{1 - 0.025}$$

$$C_{10} = \frac{85000}{0.975}$$

$$C_{10} = 87200$$

Score 3: The student used the incorrect “ r ” value.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

0.025

$$S_n = \frac{(85,000 - 85,000(1.025^n))}{(1 - 1.025)}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$S_{10} = \frac{(85,000 - 85,000(1.025^{10}))}{(1 - 1.025)}$$

$$S_{10} = \frac{-23807.18626}{-0.025}$$

$$S_{10} = 952287.4504$$

$$S_{10} = 952,300.45$$

Score 2: The student made a notation error and a rounding error.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$S_n = \frac{85000 - 85000(.025)^n}{1 - .025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$\begin{aligned} S_n &= \frac{85000 - 85000(.025)^{10}}{1 - .025} \\ &= \$87179.48718 \\ &\quad \boxed{\$87200} \end{aligned}$$

Score 2: The student made a notation error and used the incorrect " r " value.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$S_n = \frac{85000 - 85000(1.025)^n}{1 - 1.025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$= \frac{85000 - 85000(1.025)^{10}}{1 - 1.025}$$

$$= 846134.0978$$

$$\approx 846134.10$$

Score 1: The student made a notational error writing the series formula, incorrectly substituted for " n ," and made a rounding error.

Question 36

- 36** Christopher works for a defense contractor and earned \$85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n , for Christopher's total earnings over n years.

$$85,000 + 2.5n$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$\begin{aligned} 85,000 + 2.5(10) \\ = 85,025. \end{aligned}$$

Score 0: The student did not show enough relevant, correct course-level work to earn any credit.

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned}
 A(t) &= A_0 e^{kt} \\
 125 &= 500 e^{k(60.34)} \\
 \frac{125}{500} &= \frac{500 e^{k(60.34)}}{500} \\
 .25 &= e^{60.34k} \\
 \ln .25 &= 60.34k \\
 &\rightarrow \frac{-1.386294361}{60.34} = \frac{60.34k}{60.34} \\
 k &= -.022974716 \\
 \boxed{k = -.023}
 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = 500e^{-.023t}$$

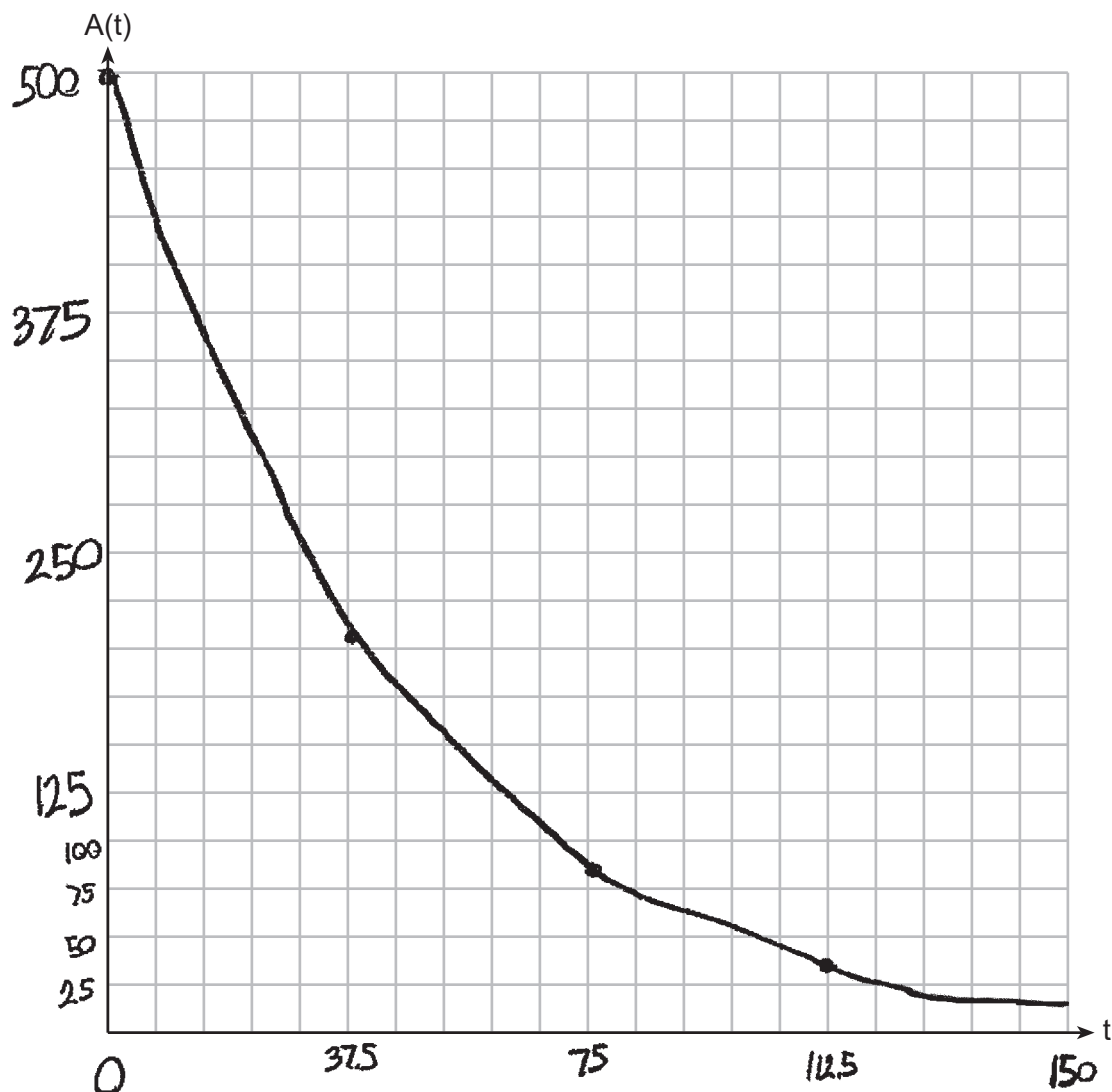
Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\frac{\Delta y}{\Delta x} = \frac{125 - 500}{60 - 0}$$

$$-6.2 \text{ g/yr}$$

Explain what this value means in the given context.

The mass of the element decreases about 6.2 grams every year

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the *nearest thousandth*.

$$\begin{aligned}\frac{125}{500} &= \frac{500 \cdot e^{k(60.34)}}{500} \\ 0.25 &= e^{k(60.34)} \\ \ln 0.25 &= k(60.34) \\ -1.386294361 &= k(60.34)\end{aligned}$$

$k = -0.023$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = 500e^{(-0.023)t}$$

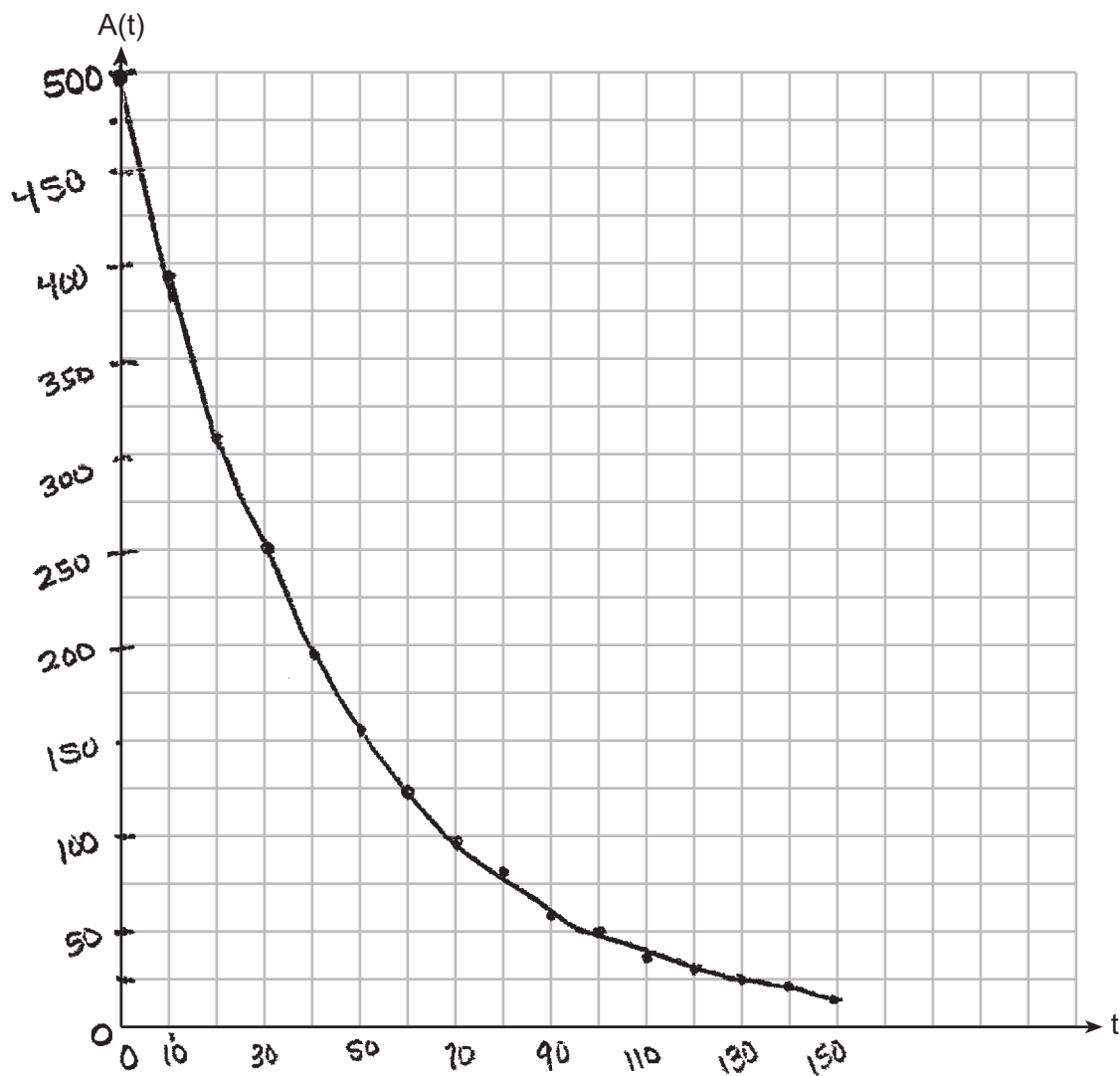
Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\begin{array}{c} (0, 500) \\ (60, 125.79) \end{array} \quad \frac{\Delta y}{\Delta x} = \frac{500 - 125.79}{0 - 60} = \frac{374.21}{-60} = \boxed{-6.2}$$

Explain what this value means in the given context.

-6.2 represents that the mass of Cesium-137 decays at an average rate of 6.2 grams per year.

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned} A(t) &= A_0 e^{kt} \\ \frac{125}{500} &= \frac{500}{500} e^{k(60.34)} \\ .25 &= e^{k(60.34)} \\ \frac{\ln(.25)}{60.34} &= \left(k \frac{60.34}{60.34} \right) \ln e \quad k \approx -.023 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = 500 e^{(-.023)(t)}$$

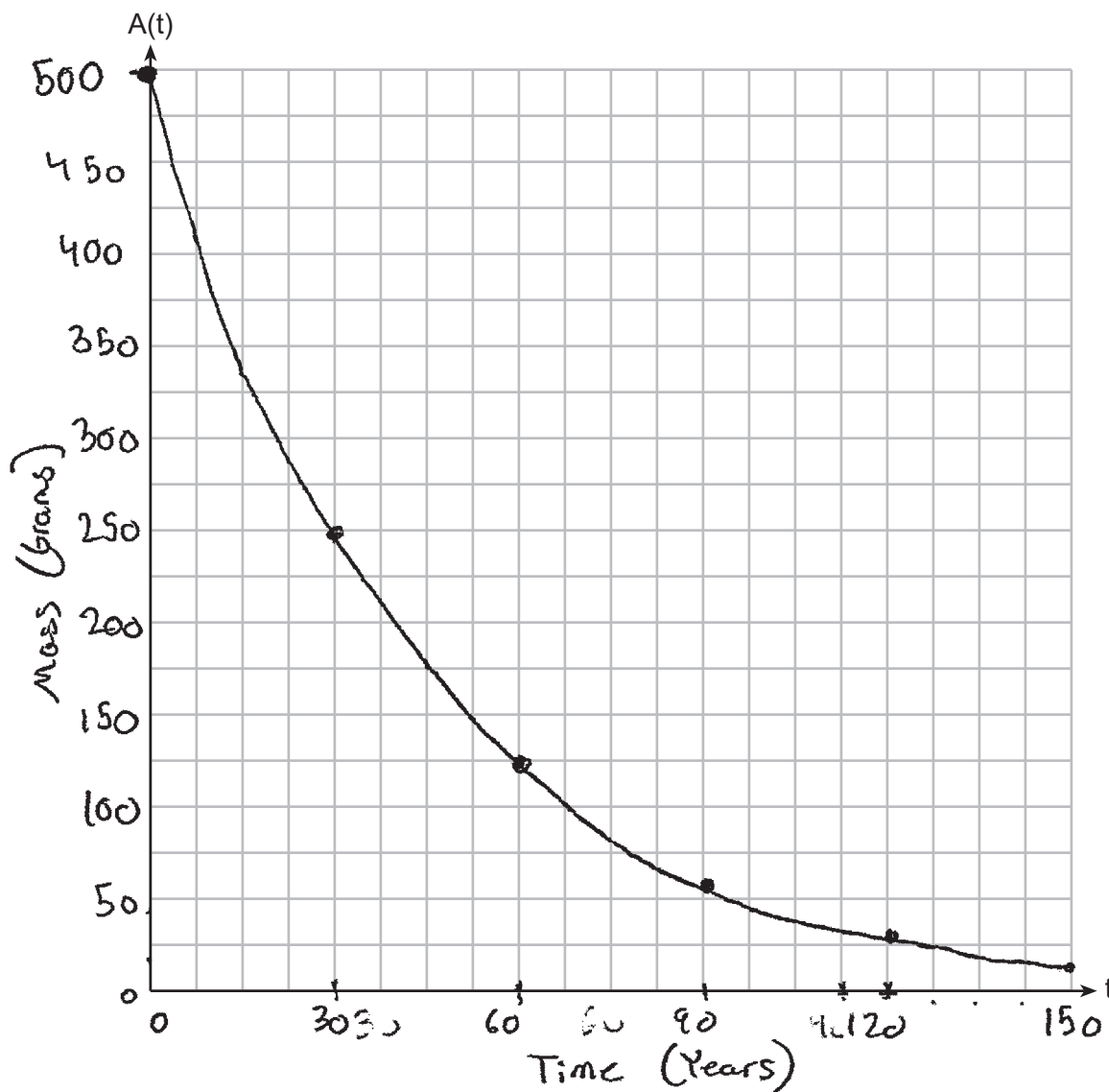
Question 37 is continued on the next page.

Score 5: The student made a rounding error calculating the average rate of change.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\frac{\Delta Y}{\Delta x} = \frac{500 - 125}{0 - 60} = -6.3$$

Explain what this value means in the given context.

Cesium-137 decays an average 6.3 grams per year between years zero and sixty,

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned} 125 &= 500 e^{k(60.34)} \\ \frac{1}{4} &= e^{k(60.34)} \\ \frac{\log e \frac{1}{4}}{60.34} &= \frac{k(60.34)}{60.34} \\ k &= -0.023 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = 500 e^{-0.023t}$$

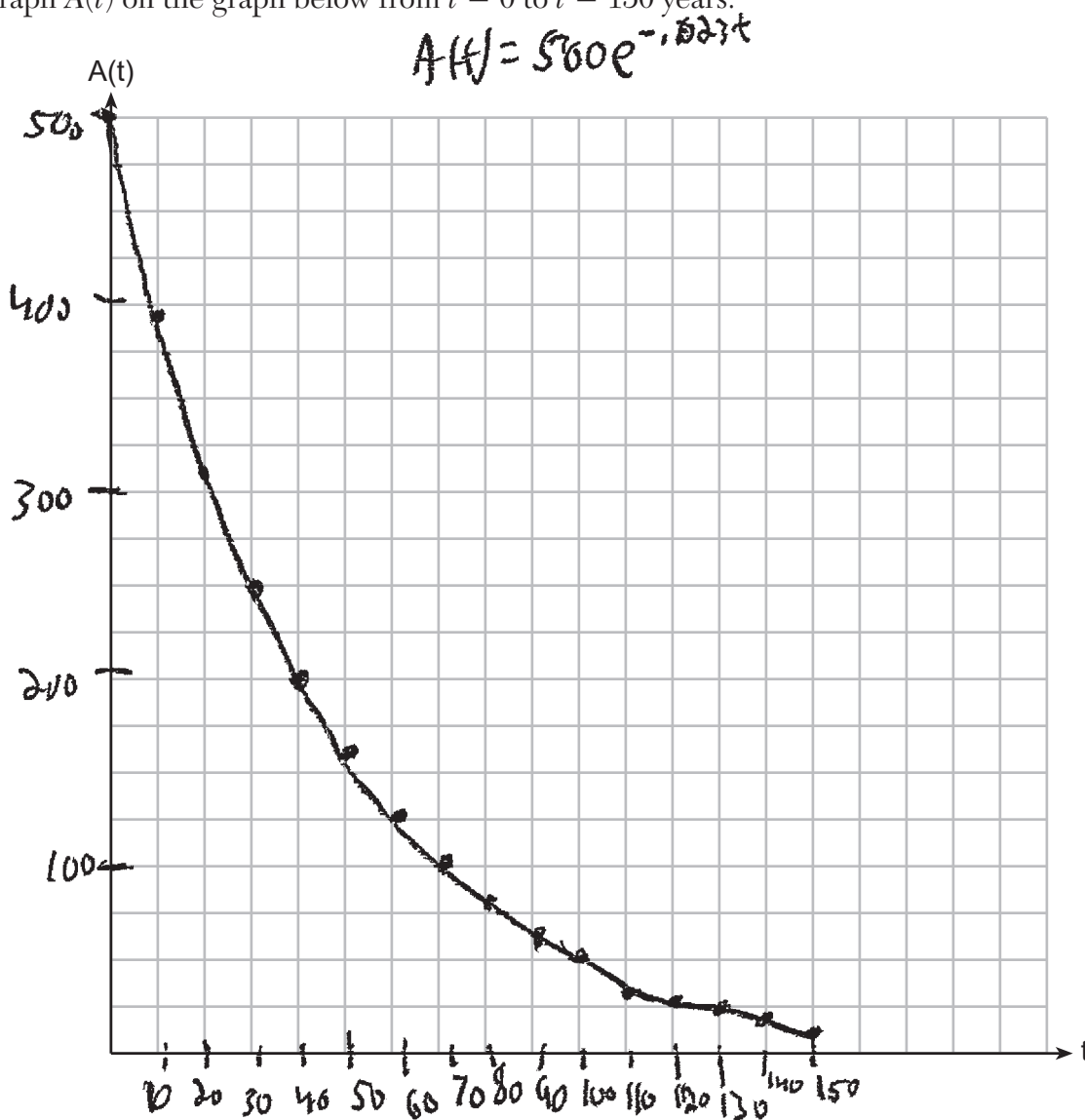
Question 37 is continued on the next page.

Score 5: The student incorrectly calculated the average rate of change.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$A(60) = 500e^{-0.023(60)}$$

$$A(60) = 125.78927653$$

$$\frac{125.78927653}{60} = 2.09648794216$$

Explain what this value means in the given context.

between years 0 and 60, the average change per year is 2.1 grams.

Question 37

37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\frac{125}{500} = e^{k(60.34)}$$

$$\ln\left(\frac{125}{500}\right) = \ln e^{k(60.34)}$$

$$\ln\left(\frac{125}{500}\right) = 60.34k \cdot \cancel{\ln e}^1$$

$$\frac{\ln\left(\frac{125}{500}\right)}{60.34} = \frac{60.34k}{60.34}$$

$$\frac{125}{500} = \frac{500 e^{k(60.34)}}{500}$$

$$k = -0.022974715962$$

$$\boxed{k = -0.023}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$\boxed{A(t) = 500 e^{-0.023t}}$$

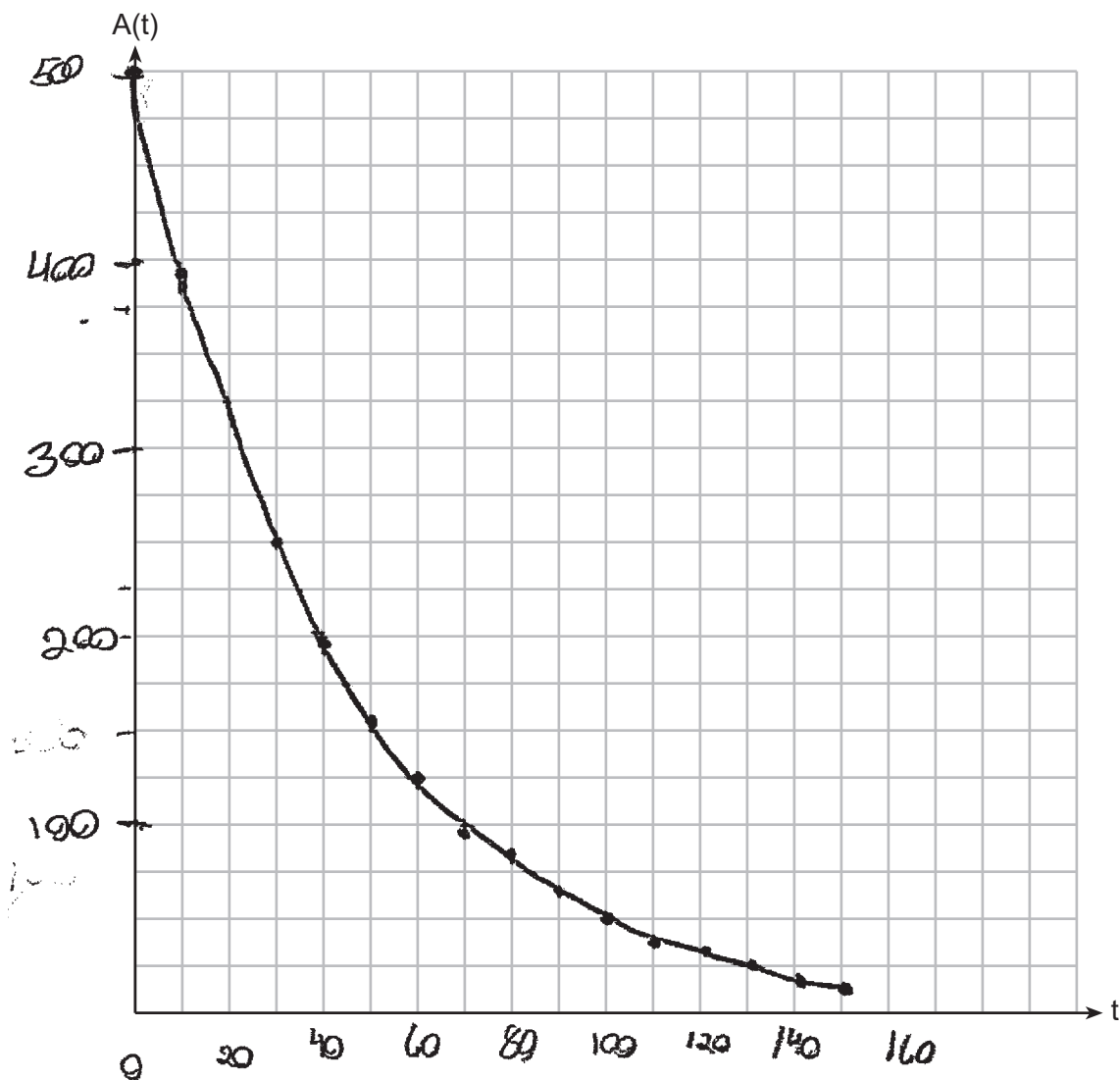
Question 37 is continued on the next page.

Score 4: The student incorrectly calculated the average rate of change, and the explanation was incomplete.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\frac{175 - 500}{60 - 0} = -6.25$$

Explain what this value means in the given context.

The value is the rate by which the line decreases

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned}
 A_0 &= 500 \\
 t &= 60.34 \\
 A &= 125 \\
 A &= A_0 e^{kt} \\
 125 &= 500 e^{60.34k} \\
 0.25 &= e^{60.34k} \\
 e^k &\approx 0.977 \\
 e^k &= 0.977 \\
 \ln 0.977 &= k \\
 k &= -0.023
 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$\begin{aligned}
 A(t) &= 500 e^{-0.023t} \\
 A(t) &= \frac{500}{e^{0.023t}}
 \end{aligned}$$

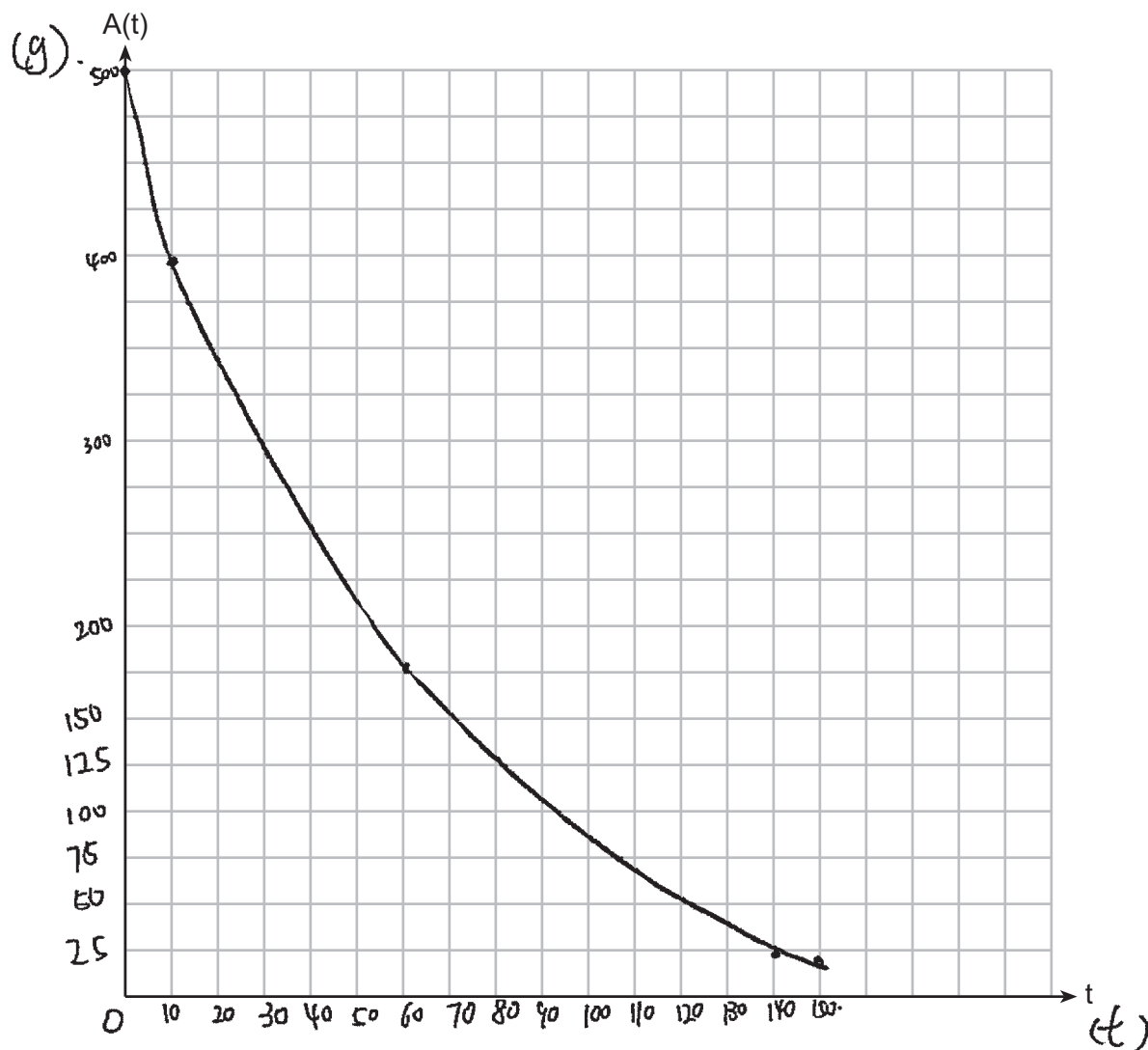
Question 37 is continued on the next page.

Score 3: The student received credit for finding “ k ” and writing a correct function.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\begin{aligned} A(0) &= 500 \times e^{-0.023 \times 0} \\ &= 500 \times 1 \\ &= 500 \end{aligned}$$

$$\begin{aligned} A(60) &= 500 \times e^{-1.38} \\ &= 500 \times 0.25 \\ &\approx 125.79 \end{aligned}$$

Explain what this value means in the given context.

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned} \frac{125}{500} &= \frac{500 e^{k(60.34)}}{500} \\ \ln .25 &= \ln e^{k(60.34)} \\ -1.386 &= k(60.34) \\ -0.0229 &= k \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$\begin{aligned} A(t) &= 500 e^{-.0229t} \\ t &= \text{time in years} \end{aligned}$$

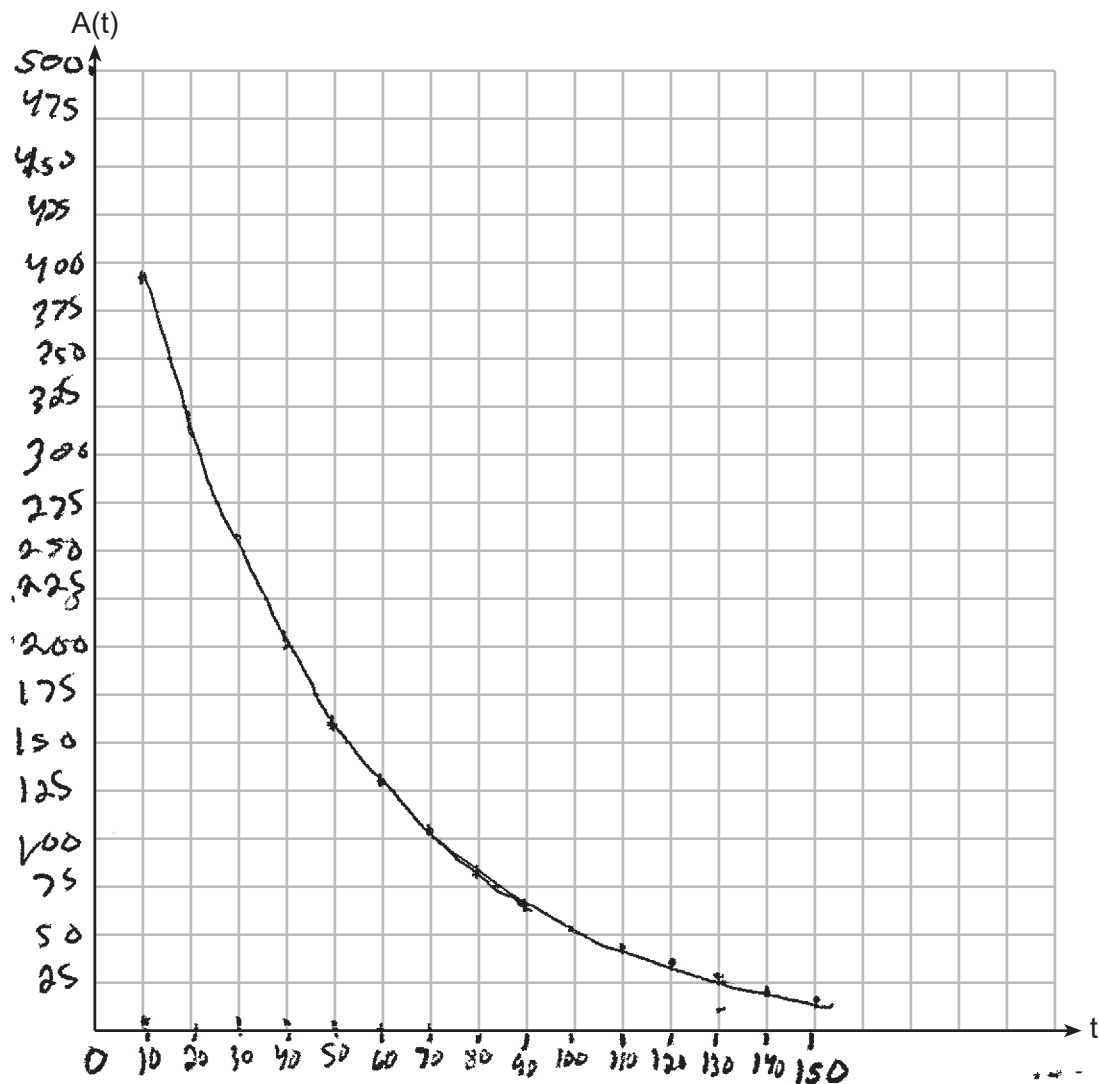
Question 37 is continued on the next page.

Score 3: The student made a rounding error, wrote a correct equation based on their “ k ” value, and found the correct average rate of change.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$-6.2$$

Explain what this value means in the given context.

how many grams it decreases

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned}
 A(t) &= A_0 e^{kt} \\
 \frac{125}{500} &= \frac{500}{500} e^{k(60.34)} \\
 0.25 &= e^{k(60.34)} \\
 \ln 0.25 &= k(60.34) \quad \cancel{\ln e} \\
 \frac{\ln 0.25}{60.34} &= \frac{k(60.34)}{60.34} \\
 k &= -1.386
 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$\begin{aligned}
 A(t) &= A_0 e^{kt} \\
 A(t) &= 500e^{-1.386t}
 \end{aligned}$$

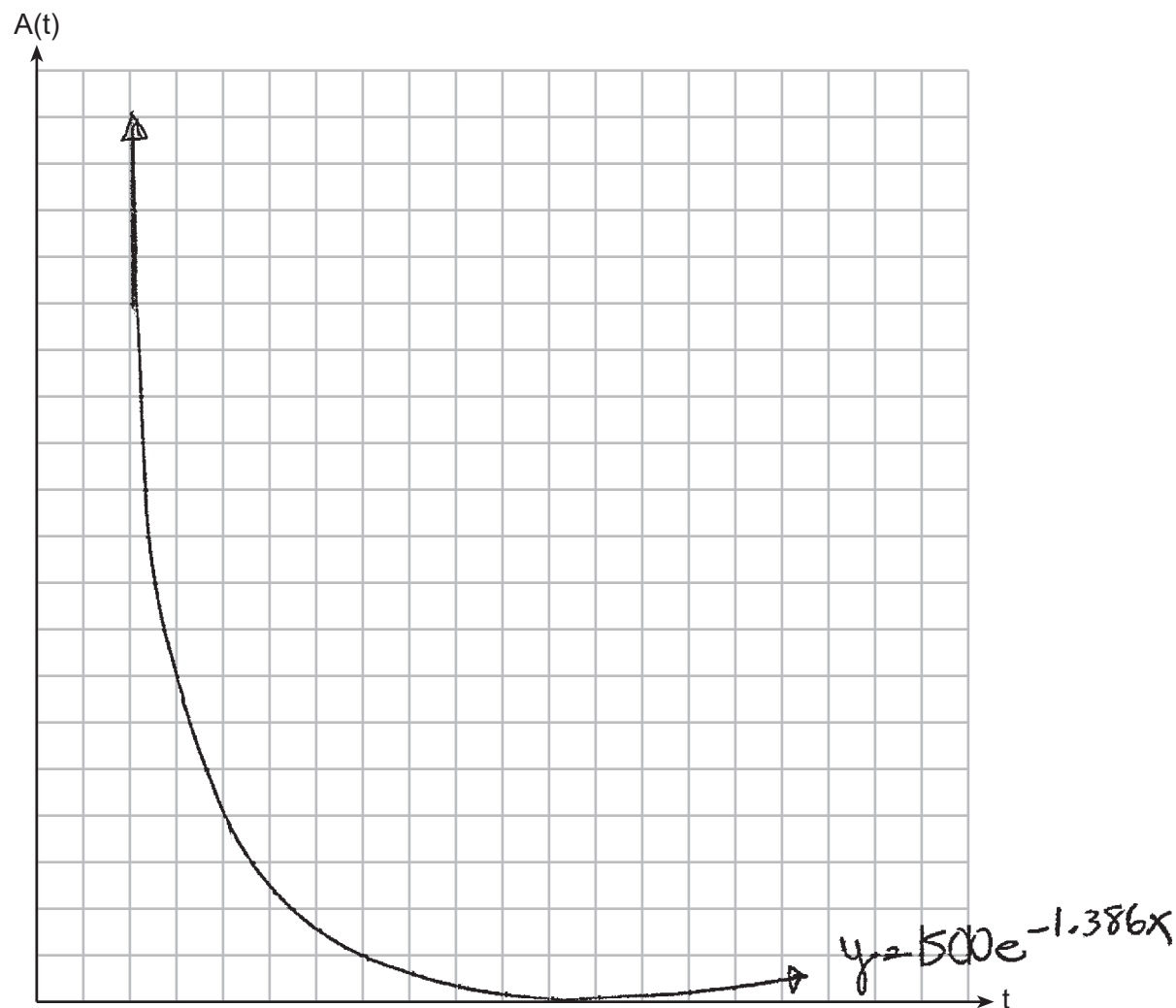
Question 37 is continued on the next page.

Score 2: The student made a calculation error finding “ k ” but wrote a correct equation based on their “ k ” value.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the *nearest tenth*.

Explain what this value means in the given context.

The sample loses an average
of grams.

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned} \frac{125}{500} &= \frac{500}{500} e^{60.34K} \\ .25 &= e^{60.34K} \\ \frac{\ln .25}{60.34} &= \frac{60.34K}{60.34} \\ K &= -0.0230 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$\begin{aligned} \frac{A(t)}{A_0} &= \frac{A_0 e^{kt}}{A_0} \\ \frac{A(t)}{A_0} &= e^{kt} \\ \frac{\ln\left(\frac{A(t)}{A_0}\right)}{k} &= t \end{aligned}$$

$$t = \frac{\ln\left(\frac{A(t)}{A_0}\right)}{k}$$

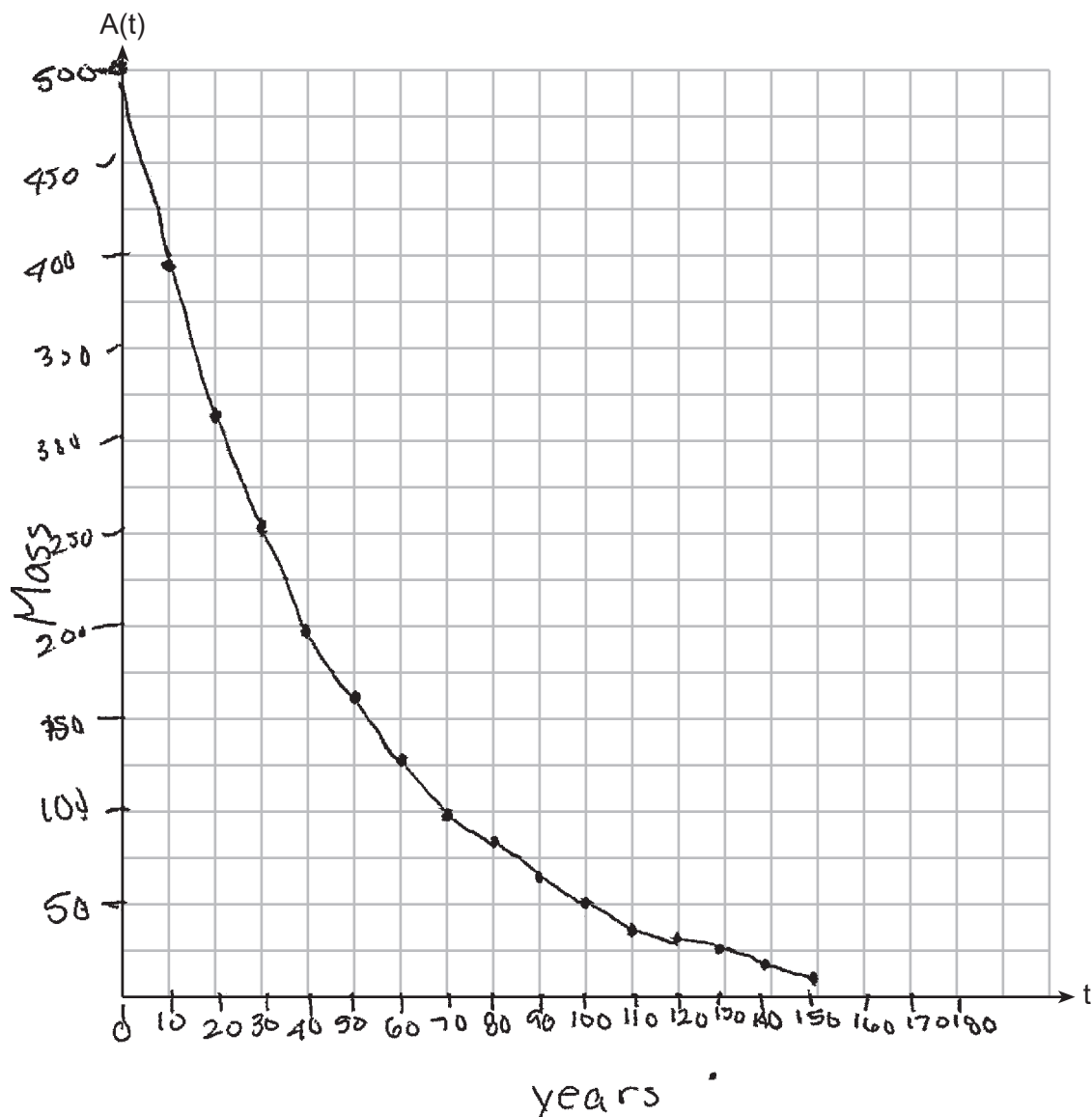
Question 37 is continued on the next page.

Score 2: The student made a rounding error solving for “ k ” and drew a correct graph.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

Explain what this value means in the given context.

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned}\frac{125}{500} &= \frac{500e^{k(60.34)}}{500} \\ 0.25 &= e^{k(60.34)} \\ \ln 0.25 &= k(60.34) \\ -1.38629 &= k(60.34) \\ k &= -0.022975\end{aligned}$$

$$\boxed{k = -0.022}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = A_0 e^{-0.022t}$$

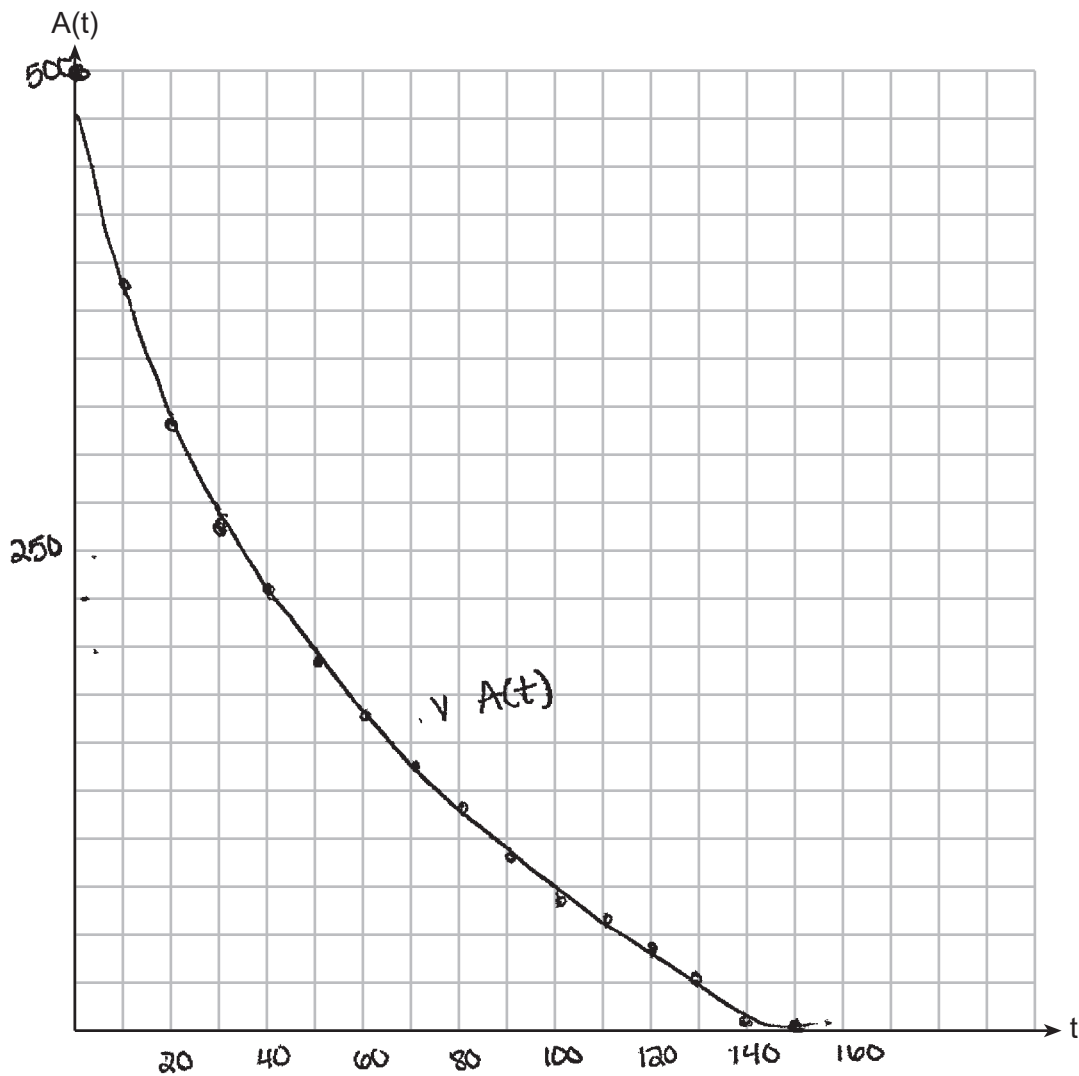
Question 37 is continued on the next page.

Score 1: The student earned one point for showing correct work to find “ k ” but made a rounding error stating the value.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\left| \frac{133.548 - 500}{60 - 0} \right| = |-6.1072|$$

Explain what this value means in the given context.

it's decreasing

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

Handwritten work:

$$A(t) = A_0 e^{kt}$$

$$\frac{125}{500} = \frac{500 e^{k \cdot 125}}{500}$$

$$\log \frac{1}{4} = 125k$$

$$\frac{-0.60206}{125} = \frac{125k}{125}$$

$$k = -0.006$$

~~$\log \frac{1}{4} = \log 125k$~~

$$\frac{1}{4} = e^{k \cdot 125}$$

$$125 = 500(e)^{-0.006 \cdot 60.34}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$A(t) = 500e^{-0.006t}$$

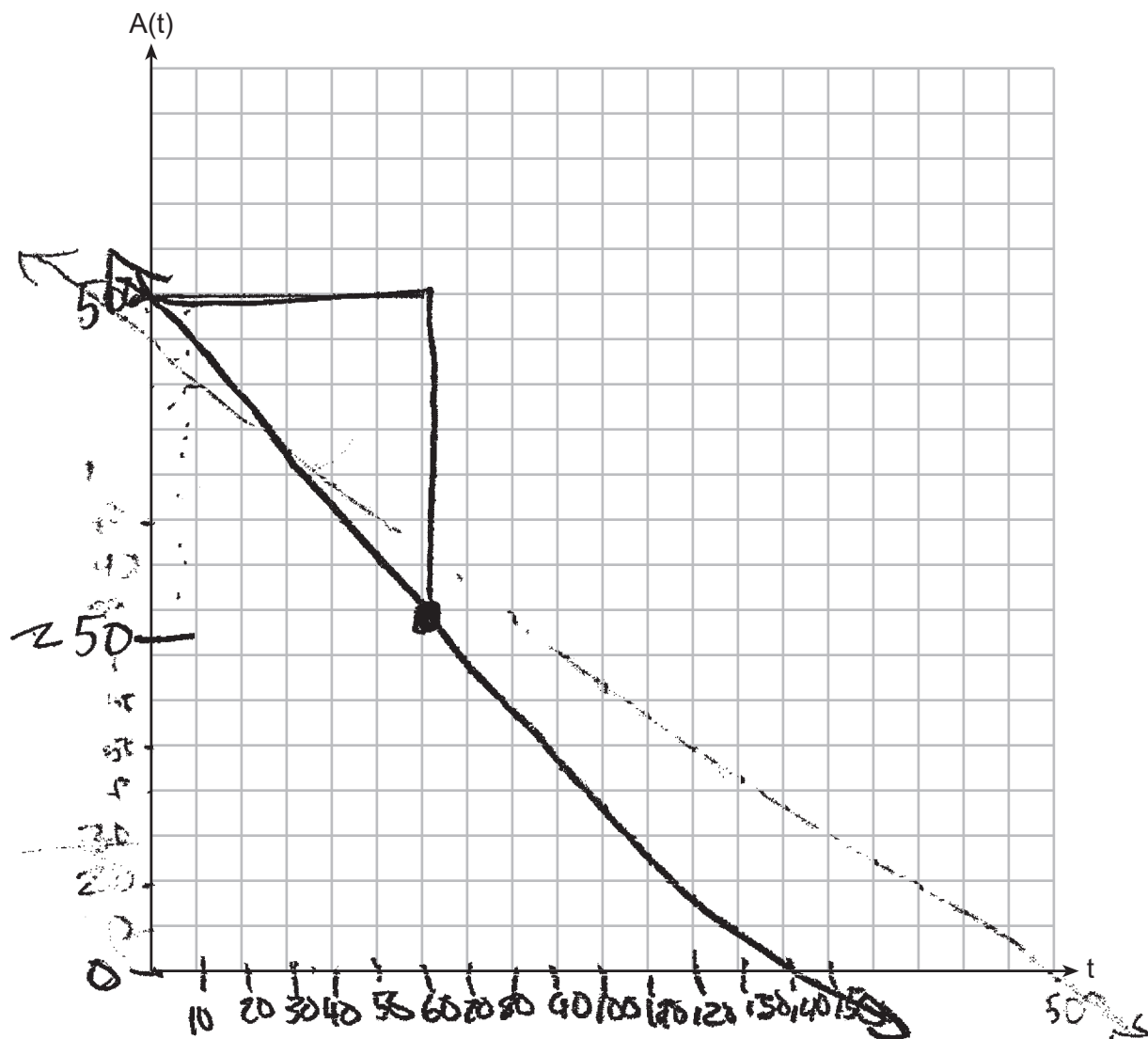
Question 37 is continued on the next page.

Score 1: The student earned credit for the function written using the incorrect value of " k ."

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$500 \cdot e^{-0.006 \cdot 60}$$

$$\frac{8}{500}$$

$$(60, 348.838)$$

$$500 \cdot e^{-0.006 \cdot 60}$$

$$-151.2$$

$$(0, 500)$$

$$\frac{60 - 0}{348.838 - 500} = \frac{60}{-151.162}$$

Explain what this value means in the given context.

This means that over the course of those 60 years the mass decreases

Question 37

- 37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k , to the nearest thousandth.

$$\begin{aligned} 125 &= 500 e^{k(60.34)} \\ \frac{125}{500} &= \frac{500}{500} e^{k(60.34)} \\ 0.25 &= e^{k(60.34)} \\ \frac{\ln 0.25}{60.34 \ln} &= \frac{k(60.34 \ln)}{60.34 \ln} \quad k = -0.0134 \end{aligned}$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t , in years.

$$500 e^{-0.0134(t)}$$

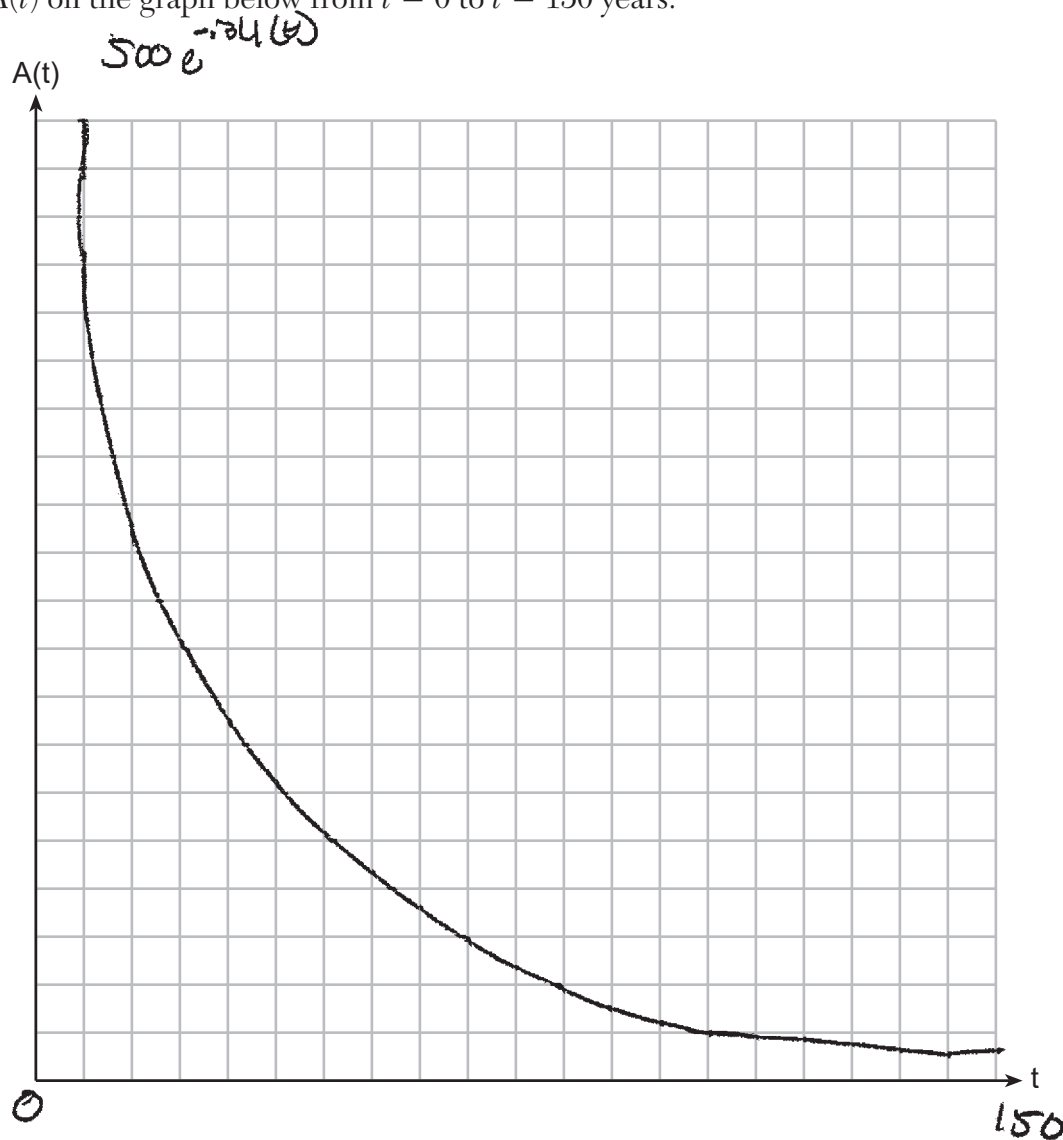
Question 37 is continued on the next page.

Score 0: The student did not satisfy the criteria for one or more credits.

Question 37

Question 37 continued

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.



Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the nearest tenth.

$$\begin{aligned}
 t(0) &= 500 \\
 t(60) &= 6.4 \times 10^{-7} \\
 \frac{6.4 \times 10^{-7}}{500} &= 1.28 \times 10^{-9} \\
 &= 1.4 \times 10^{-4}
 \end{aligned}$$

Explain what this value means in the given context.

This represents how much the cesium-137 is decaying