MATHEMATICS B

Thursday, August 13, 2009 — 8:30 to 11:30 a.m., only

Print Your Name: ____________________________

Print Your School’s Name: ____________________________

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 19. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.  

1 Which equation is represented by the accompanying graph?

\[ \begin{array}{ll}
(1) & y = 2^x \\
(2) & y = -2^x \\
(3) & y = 2^{-x} \\
(4) & y = x^2 - 2 \\
\end{array} \]

2 What are the coordinates of the turning point of the parabola whose equation is \( y = -x^2 + 4x + 1 \)?

\[ \begin{array}{ll}
(1) & (-2,-11) \\
(2) & (-2,-3) \\
(3) & (2,5) \\
(4) & (2,13) \\
\end{array} \]

3 The graph of the equation \( y = |\sin x| \) will contain no points in Quadrants

\[ \begin{array}{ll}
(1) & I \text{ and } II \\
(2) & II \text{ and } III \\
(3) & III \text{ and } IV \\
(4) & I \text{ and } IV \\
\end{array} \]
4 What is the value of \[ \sum_{k=0}^{2} 3(2)^k \]?

(1) 15  
(2) 19  
(3) 21  
(4) 43

5 Expressed in simplest form, \( \frac{\sqrt{-20}}{\sqrt{5}} \) is equivalent to

(1) \(-2i\)  
(2) \(2i\)  
(3) \(\sqrt{2}i\)  
(4) \(\frac{2i}{\sqrt{5}}\)

6 On a graph, if point A represents \(2 - 3i\) and point B represents \(-2 - 5i\), which quadrant contains \(3A - 2B\)?

(1) I  
(2) II  
(3) III  
(4) IV

7 In the accompanying diagram of triangles \(BAT\) and \(FLU\), \(\angle B \cong \angle F\) and \(BA \cong FL\). Which statement is needed to prove \(\triangle BAT \cong \triangle FLU\)?

(1) \(\angle A \cong \angle L\)  
(2) \(\overline{AT} \cong \overline{LU}\)  
(3) \(\angle A \cong \angle U\)  
(4) \(\overline{BA} \parallel \overline{FL}\)
8 Which type of transformation is \((x, y) \rightarrow (x + 2, y - 2)\)?

(1) dilation  (3) rotation
(2) reflection  (4) translation

9 Which functions are positive for angles terminating in Quadrant II?

(1) sine and cosine  (3) sine and tangent
(2) sine and secant  (4) sine and cosecant

10 What is \(\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}}\) expressed in simplest form?

(1) 1  (3) \(\frac{\sqrt{3}}{6}\)
(2) 0  (4) \(2\sqrt{3}\)

11 Banks use the formula \(A = P(1 + r)^x\) when they compound interest annually. If \(P\) represents the amount of money invested and \(r\) represents the rate of interest, which expression represents \(\log A\), where \(A\) represents the amount of money in the account after \(x\) years?

(1) \(x \log P + \log (1 + r)\)
(2) \(\log P + x \log (1 + r)\)
(3) \(\log P + x \log 1 + r\)
(4) \(\log P + \log x + \log (1 + r)\)

12 If the equation of the axis of symmetry of a parabola is \(x = 2\), at which pair of points could the parabola intersect the \(x\)-axis?

(1) (3,0) and (5,0)  (3) (3,0) and (1,0)
(2) (3,0) and (2,0)  (4) (−3,0) and (−1,0)
13 Jack is driving from New York to Florida. The number of hours that he drives and the speed at which he drives are inversely proportional. Which graph could be used to describe this situation if one axis represents speed and the other represents hours?

Use this space for computations.

14 What is the length of the altitude of an equilateral triangle whose side has a length of 8?

(1) 32
(2) $4\sqrt{3}$
(3) 4
(4) 4

15 What is the third term in the expansion of $(3x - 2)^5$?

(1) $1,080x^2$
(2) $270x^3$
(3) $540x^3$
(4) $1,080x^3$
16 If the dilation $D_k$ is an isometry, what must be the value of $k$?

(1) 1  (3) −2  
(2) 2  (4) 0

17 If $f(x) = x^2$ and $g(x) = 2x + 1$, which expression is equivalent to $(f \circ g)(x)$?

(1) $2x^2 + 1$  (3) $4x^2 + 1$  
(2) $2(x + 1)^2$  (4) $4x^2 + 4x + 1$

18 What is the inverse of the function $y = 2x - 3$?

(1) $y = \frac{x + 3}{2}$  (3) $y = -2x + 3$  
(2) $y = \frac{x}{2} + 3$  (4) $y = \frac{1}{2x - 3}$

19 If $a > 0$, which function represents the reflection of $y = a^x$ in the $y$-axis?

(1) $y = -a^x$  (3) $y = \left(\frac{1}{a}\right)^{-x}$  
(2) $y = \left(\frac{1}{a}\right)^x$  (4) $x = a^y$

20 The graph of the equation $2x^2 - 3y^2 = 4$ forms

(1) a circle  (3) a hyperbola  
(2) an ellipse  (4) a parabola
Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

21 Evaluate the expression \((x + 3)^{\frac{1}{3}} + (x - 3)^0 + (x + 2)^{\frac{-2}{3}}\) when \(x = 6\).

22 Solve algebraically for \(x\): \(27^x = 9^{x+2}\)
23 Solve for the negative value of $x$: $|2x + 5| + 1 = 13$

24 In physics class, Esther learned that force due to gravity can be determined by using the formula $F = \frac{Gm_1m_2}{r^2}$. Solve for $r$ in terms of $F$, $G$, $m_1$, and $m_2$. 
25 In the accompanying diagram of circle $O$, $\overline{PC}$ is a tangent, $\overline{PBA}$ is a secant, $\angle mAB = 132$, and $\angle mCB = 46$. Find $\angle mP$.

26 The accompanying graph shows a trigonometric function. State an equation of this function.
27 Kathy swims laps at the local fitness club. As she times her laps, she finds that each succeeding lap takes a little longer as she gets tired. If the first lap takes her 33 seconds, the second lap takes 38 seconds, the third takes 42 seconds, the fifth takes 50 seconds, and the seventh lap takes 54 seconds, state the power regression equation for this set of data, rounding all coefficients to the nearest hundredth.

Using your written regression equation, estimate the number of seconds that it would take Kathy to complete her tenth lap, to the nearest tenth of a second.
Dave is the manager of a construction supply warehouse and notes that 60% of the items purchased are heating items, 25% are electrical items, and 15% are plumbing items. Find the probability that at least three out of the next five items purchased are heating items.
The heights of a sample of female students at Oriskany High School are normally distributed with a mean height of 65 inches and a standard deviation of 0.6 inch.

What percent of this sample is between 63.8 inches and 66.2 inches?

Above what height, in inches, would the top 2.3% of this sample population be found?
30 Express in simplest form: \[
\frac{5}{a + b} - \frac{5}{a - b} - \frac{10}{a^2 - b^2}
\]
31 Solve the equation $3x^2 + 5 = 4x$ and express the roots in simplest $a + bi$ form.
The drawing for a right triangular roof truss, represented by \( \triangle ABC \), is shown in the accompanying diagram. If \( \angle ABC \) is a right angle, altitude \( BD = 4 \) meters, and \( DC \) is 6 meters longer than \( AD \), find the length of base \( AC \), in meters.
33 Given: \( T(-1,1), R(3,4), A(7,2), \) and \( P(-1,-4) \)

Prove: \( TRAP \) is a trapezoid.

\( TRAP \) is not an isosceles trapezoid.

[The use of the grid on the next page is optional.]
Question 33 continued
34 Firefighters dug three trenches in the shape of a triangle to prevent a fire from completely destroying a forest. The lengths of the trenches were 250 feet, 312 feet, and 490 feet.

Find, to the nearest degree, the smallest angle formed by the trenches.

Find the area of the plot of land within the trenches, to the nearest square foot.
Formulas

Area of Triangle

\[ K = \frac{1}{2}ab \sin C \]

Functions of the Sum of Two Angles

\[
\begin{align*}
\sin (A + B) &= \sin A \cos B + \cos A \sin B \\
\cos (A + B) &= \cos A \cos B - \sin A \sin B 
\end{align*}
\]

Functions of the Difference of Two Angles

\[
\begin{align*}
\sin (A - B) &= \sin A \cos B - \cos A \sin B \\
\cos (A - B) &= \cos A \cos B + \sin A \sin B 
\end{align*}
\]

Law of Sines

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Law of Cosines

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Functions of the Double Angle

\[
\begin{align*}
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
\cos 2A &= 1 - 2 \sin^2 A
\end{align*}
\]

Functions of the Half Angle

\[
\begin{align*}
\sin \frac{1}{2} A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\
\cos \frac{1}{2} A &= \pm \sqrt{\frac{1 + \cos A}{2}}
\end{align*}
\]

Normal Curve

Standard Deviation

[19]
Scrap Graph Paper — This sheet will not be scored.
The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, August 13, 2009 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Student ............................................. Sex: ☐ Male ☐ Female Grade ............

Teacher ............................................. School ....................................

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 ................. 6 ................. 11 ................. 16 .................

2 ................. 7 ................. 12 ................. 17 .................

3 ................. 8 ................. 13 ................. 18 .................

4 ................. 9 ................. 14 ................. 19 .................

5 ................. 10 ................. 15 ................. 20 .................

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

[23]
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**Total Raw Score**

| Maximum Total | 88 |

Checked by

**Scaled Score**
(from conversion chart)

**Rater’s/Scorer’s Name**
(minimum of three)