The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 21, 2000 — 1:15 to 4:15 p.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
1. The set \{a, b, c, d\} and the operation ① are shown in the accompanying table. What is the identity element for the operation?

<table>
<thead>
<tr>
<th>①</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<tbody>
<tr>
<td>a</td>
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</table>

2. In the accompanying diagram of isosceles triangle SUM, \(\overline{SM} \cong \overline{UM}\) and \(\angle MUD\) is an exterior angle formed by extending \(\overline{SU}\) to D. If \(m\angle MUD = 124\), find \(m\angle M\).

3. In \(\triangle BGL\), \(m\angle B = 53\) and \(m\angle G = 66\). Which is the longest side of this triangle?

4. Solve for the positive value of \(x\):

\[
\frac{x + 4}{2} = \frac{4}{x - 3}
\]

5. If a translation maps A(-1,5) to A'(2,9), what are the coordinates of B', the image of B(2,-2) under the same translation?

6. If one of the roots of the equation \(x^2 + kx = 28\) is 4, find the value of \(k\).

7. What is the total number of different four-digit numerals that can be formed using the digits 1, 9, 9, and 9?

8. In the accompanying diagram, \(\overline{ACB}\) is a straight line, \(m\angle DCA = 44\), and \(\overline{CE}\) bisects \(\angle DCB\). Find \(m\angle ECB\).

9. In the accompanying diagram, parallel lines \(\ell\) and \(m\) are cut by transversal \(r\), \(m\angle 1 = 3x + 40\), and \(m\angle 2 = 5x - 20\). Find \(m\angle 1\).

10. In the accompanying diagram of \(\triangle SRT\), \(\overline{LM} \parallel \overline{RT}\). If \(SL = 4\), \(LR = 3\), and \(RT = 21\), find \(LM\).

11. Lake High School has nine mathematics teachers. How many different four-teacher committees can be formed from these nine mathematics teachers?

12. Find the distance between points (14, -4) and (2,1).
13 In the accompanying diagram of right triangle $ABC$, a right angle is at $C$, $AB = 26$, and $m\angle A = 27$. Find the length of $BC$ to the nearest tenth.

Directions (14–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

14 The coordinates of three of the vertices of rectangle $RECT$ are $R(-1,1)$, $E(3,1)$, and $C(3,5)$. What are the coordinates of vertex $T$?

(1) $(-5,3)$ (3) $(1,3)$
(2) $(-1,5)$ (4) $(3,-5)$

15 What is the negation of the statement $m \wedge \sim q$?

(1) $\sim m \lor q$ (3) $m \lor \sim q$
(2) $\sim m \land q$ (4) $m \land q$

16 If the statements $a \rightarrow \sim b$, $\sim b \rightarrow c$, and $a$ are true, which statement must also be true?

(1) $b$ (3) $c$
(2) $\sim a$ (4) $\sim c$

17 If a conditional statement is true, what must also be true?

(1) the negation of the statement
(2) the converse of the statement
(3) the inverse of the statement
(4) the contrapositive of the statement

18 Which statement is true about the lines formed by the graphs of the equations $y = x - 3$ and $x = y - 3$?

(1) They are identical.
(2) They intersect but are not perpendicular.
(3) They are parallel.
(4) They are perpendicular.

19 Which graph represents the equation $y = x^2 - 3$?

(1) $x^2 - 3$ (3) $x^2 - 3$
(2) $x^2 - 3$ (4) $x^2 - 3$

20 If $x \# y = xy + yx$, what is the value of $2 \# 5$?

(1) 20 (3) 42
(2) 35 (4) 57

21 The image of $P(6,-9)$ after a dilation with respect to the origin is $(4,-6)$. What is the constant of dilation?

(1) $\frac{1}{3}$ (3) $\frac{3}{2}$
(2) $\frac{2}{3}$ (4) $-2$

22 In the accompanying diagram, $ABC \equiv EDC$, $AD$ and $BE$ are drawn, and $\angle 1 \equiv \angle 2$.

Triangle $ADC$ can be proved congruent to triangle $EBC$ by

(1) $HL \equiv HL$ (3) $ASA \equiv ASA$
(2) $SAS \equiv SAS$ (4) $AAA \equiv AAA$
23 The sum of \( \frac{x - 6}{3} + \frac{x + 2}{x} \) is
   (1) \( x \)  
   (2) \( \frac{x - 4}{3} \)  
   (3) \( \frac{2x - 4}{3x} \)  
   (4) \( \frac{x^2 - 3x + 6}{3x} \)

24 Which equation represents the axis of symmetry of the graph of the equation \( y = x^2 - 4x + 5 \)?
   (1) \( x = 2 \)  
   (2) \( x = -2 \)  
   (3) \( y = 2 \)  
   (4) \( y = -2 \)

25 If each exterior angle of a regular polygon measures 40°, what is the total number of sides in the polygon?
   (1) 5  
   (2) 6  
   (3) 8  
   (4) 9

26 What are the coordinates of \( A' \), the image of point \( A(4, -2) \) after a reflection in the origin?
   (1) \( (-4, 2) \)  
   (2) \( (4, 2) \)  
   (3) \( (-4, -2) \)  
   (4) \( (-2, 4) \)

27 Points \( A, B, C, \) and \( D \) are midpoints of the sides of square \( \text{JETS} \).

If the area of \( \text{JETS} \) is 36, the area of \( \text{ABCD} \) is
   (1) \( 9\sqrt{2} \)  
   (2) \( 18\sqrt{2} \)  
   (3) 9  
   (4) 18

28 If a tree casts a 90-foot shadow at the same time that a 3-foot pole held perpendicular to the ground casts a 5-foot shadow, what is the height of the tree, expressed in feet?
   (1) 18  
   (2) 54  
   (3) 72  
   (4) 150

29 In the accompanying diagram, \( \triangle RST \) is a right triangle, \( SU \) is the altitude to hypotenuse \( RT \), \( RT = 16 \), and \( RU = 7 \).

What is the length of \( ST \)?
   (1) \( 3\sqrt{7} \)  
   (2) \( 4\sqrt{7} \)  
   (3) \( 9 \)  
   (4) \( 12 \)

30 The midpoint of \( \overline{AB} \) is \( M \), the coordinates of \( A \) are \((a, b)\), and the coordinates of \( B \) are \((a + 4, 5b)\). What are the coordinates of \( M \)?
   (1) \( (2, 2b) \)  
   (2) \( (a + 2, 3b) \)  
   (3) \( (2a + 4, 6b) \)  
   (4) \( \left( \frac{a + 4}{2}, \frac{5b}{2} \right) \)

31 If the graphs of the equations \( x^2 + y^2 = 16 \) and \( y = 4 \) are drawn on the same set of axes, what is the total number of points common to both graphs?
   (1) 1  
   (2) 2  
   (3) 3  
   (4) 0

32 The roots of the equation \( x^2 - 6x - 2 = 0 \) are
   (1) \( 3\pm \sqrt{11} \)  
   (2) \( -3\pm \sqrt{11} \)  
   (3) \( 3\pm \sqrt{7} \)  
   (4) \( -3\pm \sqrt{7} \)

33 What is the slope of a line that is perpendicular to the line whose equation is \( y - 2x = 5 \)?
   (1) \( \frac{1}{2} \)  
   (2) \( 2 \)  
   (3) \( -\frac{1}{2} \)  
   (4) \( -2 \)

34 What is an equation of the straight line whose slope is 3 and that passes through point \((-2, 0)\)?
   (1) \( y = 3x - 2 \)  
   (2) \( y = 3x + 6 \)  
   (3) \( x = 3y - 2 \)  
   (4) \( -2x = 3y + 1 \)

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct the angle bisector of \( \angle ABC \).
36 Solve the following system of equations graphically or algebraically and check.

\[ y = x^2 + 2x - 3 \]
\[ 2x + y = -3 \]  \[\text{[8.2]}\]

37 The coordinates of the vertices of \( \triangle ABC \) are \( A(-4,1), B(4,9), \) and \( C(9,-2) \). Point \( M(1,6) \) lies on \( AB \).

a Show by means of coordinate geometry that \( CM \perp AB \).  \[\text{[4]}\]

b Find, to the nearest degree, the measure of angle \( A \).  \[\text{[6]}\]

38 a On graph paper, draw and label the graph of circle \( A \), which is represented by the equation \( x^2 + y^2 = 9 \).  \[\text{[2]}\]

b On the same set of axes, draw the image of circle \( A \) after the translation \((x,y) \rightarrow (x + 5,y - 3)\) and label it \( B \).  \[\text{[3]}\]

c On the same set of axes, draw the image of circle \( B \) after a reflection in the \( x \)-axis and label it \( C \).  \[\text{[3]}\]

d What is the area of the triangle formed by connecting the centers of the circles drawn in parts a, b, and c?  \[\text{[2]}\]

39 Answer both a and b.

a For all values of \( y \) for which the expressions are defined, express the quotient in simplest form:

\[ \frac{2y^2 - 6y}{2y^2 - 7y - 4} : \frac{y^2 + y - 12}{y^2 - 16} \]  \[\text{[5]}\]

b Lorraine won a contest and can select six compact discs (CD's) from a list of 12 CD's. The list contains 5 rock, 3 jazz, and 4 classical CD's. What is the probability that a random selection includes 3 rock, 1 jazz, and 2 classical CD's?  \[\text{[5]}\]

40 In the accompanying diagram, \( \triangle ABC \sim \triangle A'B'C' \) and \( A'B' = 4 \).

a If \( AC \) is 2 more than \( AB \), and \( A'C' \) is 6 more than \( AB \), find \( AB \). [Only an algebraic solution will be accepted.]  \[\text{[8]}\]

b Using the results from part a, determine the smallest possible integral value of \( BC \). Justify your answer.  \[\text{[1.1]}\]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: If Mike is the catcher, then Robin plays first base.
If Uk-Hae is not in the lineup, Mike is the catcher.
If Edgardo plays second base and Robin plays first base, then Luis is the centerfielder.
Luis is not the centerfielder.
Edgardo plays second base.

Let E represent: “Edgardo plays second base.”
Let L represent: “Luis is the centerfielder.”
Let M represent: “Mike is the catcher.”
Let R represent: “Robin plays first base.”
Let U represent: “Uk-Hae is in the lineup.”

Prove: Uk-Hae is in the lineup. [10]

42 Given: E is the midpoint of $\overline{AD}$, $\overline{BA} \perp \overline{AD}$, $\overline{CD} \perp \overline{AD}$, $\overline{BC}$, $\overline{BE}$, and $\overline{CE}$ are drawn, and $\angle 1 \cong \angle 2$.

Prove: $\overline{AB} \cong \overline{DC}$ [10]
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SEQUENTIAL MATH – COURSE II

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ANSWER SHEET

Part I Score .................
Part II Score .................
Part III Score .................
Total Score .................
Rater’s Initials: .................

Pupil ........................................... Sex: □ Male □ Female  Grade ....................
Teacher ........................................... School ..................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

1 .........................  11 .........................  21 .........................  31 .........................
2 .........................  12 .........................  22 .........................  32 .........................
3 .........................  13 .........................  23 .........................  33 .........................
4 .........................  14 .........................  24 .........................  34 .........................
5 .........................  15 .........................  25 .........................  35 Answer question 35 on the other side of this sheet.
6 .........................  16 .........................  26 .........................
7 .........................  17 .........................  27 .........................
8 .........................  18 .........................  28 .........................
9 .........................  19 .........................  29 .........................
10 .........................  20 .........................  30 .........................
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

________________________________________
Signature