DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of \( \pi \) or in radical form. \[ \text{[60]} \]

1 In the accompanying diagram of \( \triangle ABC \), \( 
\overline{CB} \cong \overline{CA} \). \( 
\overline{BA} \) is extended to \( D \), and \( \angle B = 42 \). Find \( \angle CAD \).

2 In the accompanying diagram, line \( a \) is parallel to line \( b \), and line \( t \) is a transversal. If \( \angle 1 = 97 \) and \( \angle 2 = 44 \), find \( \angle 3 \).

3 In equilateral triangle \( ABC \), \( AB = 14 \). Find the perimeter of the triangle that joins the midpoints of the sides of triangle \( ABC \).

4 For the binary operation \( a \ast b = 3a - 2b^2 \), find the value of \( 4 \ast 5 \).

5 Solve for \( x \):
\[
\frac{x}{3} + \frac{x}{18} = \frac{7}{6}
\]

6 A committee of four teachers is to be chosen from ten teachers. What is the total number of different committees that can be formed?

7 What is the distance between points \( (-3,2) \) and \( (5,-6) \), expressed to the nearest tenth?

8 In a plane, what is the total number of points 3 units from the line whose equation is \( x = 4 \) and 3 units from point \( (4,-2) \)?

9 The midpoint of \( AB \) has coordinates \( (-3,2) \), and the coordinates of point \( A \) are \( (1,4) \). What are the coordinates of point \( B \)?

10 Factor completely: \( 3x^2y - 12xy \)

11 The area of a square is 9. If this square and an equilateral triangle have equal perimeters, find the length of one side of the triangle.

Directions (12–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

12 What is the value of \( (C \ast O) \ast (L \ast D) \) in the system defined below?

\[
\begin{array}{c|cccc}
\ast & C & O & L & D \\
\hline
C & C & D & O & L \\
O & D & C & O & L \\
L & O & C & L & D \\
D & O & O & C & L \\
\end{array}
\]

(1) \( C \)  (3) \( L \)

(2) \( O \)  (4) \( D \)
13. What is the total number of different seven-letter arrangements that can be formed using the letters in the word “COCONUT”?

1. \(7P_7\)  
2. \(\frac{7!}{4!}\)  
3. 7!  
4. \(\frac{7!}{2!2!}\)

14. Which transformation moves point \((x, y)\) to point \((4x, 4y)\)?

1. dilation  
2. reflection  
3. rotation  
4. translation

15. In \(\triangle NAP\), \(m\angle N = 45\) and \(m\angle A = 70\). Which statement best describes \(\triangle NAP\)?

1. \(\overline{NA}\) is the longest side.  
2. \(\overline{NP}\) is the longest side.  
3. \(\overline{AP}\) is the longest side.  
4. \(\overline{AP}\) and \(\overline{NP}\) are the longest sides and are congruent.

16. Expressed in simplest form, \(\frac{x^2 - 64}{x^2 - 16x + 64}, x \neq 8\), is equivalent to

1. 1  
2. -1  
3. \(\frac{x - 8}{x + 8}\)  
4. \(\frac{x + 8}{x - 8}\)

17. Which statement is true?

1. A quadrilateral is always a parallelogram.  
2. A square is always a parallelogram.  
3. A parallelogram is never a rhombus.  
4. A trapezoid never has two congruent sides.

18. Which set of numbers cannot be the measures of the three sides of a triangle?

1. \([2,3,4]\)  
2. \([3,4,6]\)  
3. \([5,7,12]\)  
4. \([5,8,12]\)

19. In the accompanying diagram of right triangle \(\triangle CAR\), \(m\angle A = 90\), \(m\angle C = 59\), and \(CR = 15\). If \(AR\) is represented by \(c\), which equation can be used to find \(c\)?

1. \(\sin 59^\circ = \frac{c}{15}\)  
2. \(\cos 59^\circ = \frac{c}{15}\)  
3. \(\tan 59^\circ = \frac{c}{15}\)  
4. \(\sin 31^\circ = \frac{c}{15}\)

20. What is the inverse of the statement “If two parallel lines are cut by a transversal, the corresponding angles are equal”?

1. If the corresponding angles of two lines cut by a transversal are equal, the lines are parallel.  
2. If two nonparallel lines are cut by a transversal, the corresponding angles are not equal.  
3. If the corresponding angles of two lines cut by a transversal are not equal, the lines are not parallel.  
4. Corresponding angles of parallel lines are equal.

21. Which statement is not valid for proving that two triangles are congruent?

1. \(SAS \equiv SAS\)  
2. \(SSA \equiv SSA\)  
3. \(ASA \equiv ASA\)  
4. \(AAS \equiv AAS\)

22. The coordinates of the endpoints of the base of an isosceles triangle are \((4,3)\) and \((10,3)\). The coordinates of the vertex of this triangle could be

1. \((3,7)\)  
2. \((4,7)\)  
3. \((4,-4)\)  
4. \((7,-4)\)
23 Which equation represents a circle whose center is \((-2,1)\) and whose radius is 5?

\[
\begin{align*}
(1) & \quad (x-2)^2 + (y+1)^2 = 25 \\
(2) & \quad (x+2)^2 + (y-1)^2 = 10 \\
(3) & \quad (x+2)^2 + (y-1)^2 = 25 \\
(4) & \quad (x-2)^2 + (y+1)^2 = 10
\end{align*}
\]

24 Which biconditional statement demonstrates DeMorgan’s Law?

\[
\begin{align*}
(1) & \quad \sim(p \land q) \iff (\sim p \lor \sim q) \\
(2) & \quad (\sim p \land \sim q) \iff (p \lor q) \\
(3) & \quad (p \lor q) \iff (\sim p \land \sim q) \\
(4) & \quad (p \land \sim q) \iff (\sim p \lor q)
\end{align*}
\]

25 What is the positive value of \(n\) if the slope of the line joining \((6,n)\) and \((7,n^2)\) is 20?

\[
\begin{align*}
(1) & \quad 49 \\
(2) & \quad 13 \\
(3) & \quad 5 \\
(4) & \quad 4
\end{align*}
\]

26 If two angles of one triangle are congruent respectively, to two angles of another triangle, then these triangles must be

\[
\begin{align*}
(1) & \quad \text{isosceles} \\
(2) & \quad \text{similar} \\
(3) & \quad \text{congruent} \\
(4) & \quad \text{equilateral}
\end{align*}
\]

27 What is the \(y\)-intercept of the graph of the parabola whose equation is \(y = x^2 - 2x - 8\)?

\[
\begin{align*}
(1) & \quad (4,0) \\
(2) & \quad (0,4) \\
(3) & \quad (-8,0) \\
(4) & \quad (0,-8)
\end{align*}
\]

28 What are the coordinates of the turning point of the parabola whose equation is \(y = x^2 - 4x + 3\)?

\[
\begin{align*}
(1) & \quad (2,-1) \\
(2) & \quad (2,3) \\
(3) & \quad (-2,14) \\
(4) & \quad (-2,-9)
\end{align*}
\]

29 The length of the hypotenuse of a right triangle is \(\sqrt{15}\) and the length of one leg is 3. What is the length of the other leg?

\[
\begin{align*}
(1) & \quad 6 \\
(2) & \quad 9 \\
(3) & \quad \sqrt{6} \\
(4) & \quad 3\sqrt{2}
\end{align*}
\]

30 What is the image of \((1,-3)\) when reflected in the \(y\)-axis?

\[
\begin{align*}
(1) & \quad (-1,3) \\
(2) & \quad (3,-1) \\
(3) & \quad (-3,-1) \\
(4) & \quad (-1,-3)
\end{align*}
\]

31 Given the true statement: “If you want to help preserve the environment, you conserve fuel.” Which statement must also be true?

\[
\begin{align*}
(1) & \quad \text{If you conserve fuel, you want to help preserve the environment.} \\
(2) & \quad \text{If you do not conserve fuel, you do not want to help preserve the environment.} \\
(3) & \quad \text{If you want to help preserve the environment, you do not conserve fuel.} \\
(4) & \quad \text{If you do not want to help preserve the environment, you do not conserve fuel.}
\end{align*}
\]

32 The graph of the equation \(2y = 3x + 6\) is perpendicular to the graph of the line represented by the equation

\[
\begin{align*}
(1) & \quad y = -\frac{2}{3}x - 2 \\
(2) & \quad y = \frac{2}{3}x - 2 \\
(3) & \quad y = \frac{1}{3}x - 2 \\
(4) & \quad y = -\frac{1}{3}x - 2
\end{align*}
\]

33 What are the values of \(x\) in the equation \(x^2 + 4x - 1 = 0\)?

\[
\begin{align*}
(1) & \quad -4 \pm \sqrt{5} \\
(2) & \quad -4 \pm \sqrt{3} \\
(3) & \quad -2 \pm \sqrt{5} \\
(4) & \quad -2 \pm \sqrt{3}
\end{align*}
\]

34 Which statement describes the graphs of the equations \(x = -1\) and \(x^2 + y^2 = 4\)?

\[
\begin{align*}
(1) & \quad \text{They do not intersect.} \\
(2) & \quad \text{They intersect in the second quadrant, only.} \\
(3) & \quad \text{They intersect in the third quadrant, only.} \\
(4) & \quad \text{They intersect in the second and in the third quadrants.}
\end{align*}
\]

35 The sides of a triangle have lengths 3, 4, and 5. What is the length of the shortest side of a similar triangle that has a perimeter of 36?

\[
\begin{align*}
(1) & \quad 12 \\
(2) & \quad 9 \\
(3) & \quad 3 \\
(4) & \quad 15
\end{align*}
\]
36 Solve the following system of equations graphically or algebraically and check:

\[ y = -2x^2 - 4x + 2 \quad [8, 2] \]
\[ 2x = y - 2 \]

37 a Perform the indicated operation and express the result as a single fraction in simplest form:

\[ \frac{a}{a-b} - \frac{b^2}{a^2-ab} \quad [4] \]

b Express the quotient in simplest form:

\[ \frac{x^2-12x+36}{x-6} + \frac{x^2-36}{6} \quad [2] \]

c Solve for x:

\[ \frac{x^2-2}{2x-1} = \frac{2x+2}{5} \quad [4] \]

38 Triangle ABC has vertices A(-2,1), B(5,5), and C(-1,-7).

a Show that \( \triangle ABC \) is isosceles. \([4]\)

b Find the length of the altitude drawn from A to BC. \([4]\)

c Using the results obtained in parts a and b, find, to the nearest degree, the measure of a base angle of this triangle. \([2]\)

39 The length of one base of a trapezoid is 3 centimeters longer than the length of the other base. The altitude of the trapezoid is twice the length of the shorter base. If the area of the trapezoid is 40 square centimeters, find the length of the longer base to the nearest tenth of a centimeter. \([10]\)

40 Theresa has five mysteries, four adventure stories, and three historical novels. She can choose eight books for her summer reading.

a What is the total number of different eight-book selections that Theresa can choose? \([2]\)

b What is the probability that an eight-book selection will contain three mysteries, two adventure stories, and three historical novels? \([4]\)

c What is the probability that an eight-book selection will contain no adventure stories? \([2]\)

d What is the probability that an eight-book selection will contain no mysteries? \([2]\)
Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: If I do not rollerblade, then I go to the gym.
   If I ride my bike and I go to the gym, then I am tired.
   I am not tired.
   I ride my bike.

   Let \( R \) represent: “I rollerblade.”
   Let \( G \) represent: “I go to the gym.”
   Let \( B \) represent: “I ride my bike.”
   Let \( T \) represent: “I am tired.”

   Prove: I rollerblade. [10]

42 Given: \( \triangle BFD, \triangle AGE, \overline{ABCDE} \),
   \( \angle FBC \equiv \angle FDC, \overline{CGF} \) bisects \( \angle BFD \),
   \( \overline{AB} \equiv \overline{ED} \), and \( \angle CGA \equiv \angle CGE \).

   Prove: \( \triangle ACG \equiv \triangle ECG \) [10]
Your answers to Part I should be recorded on this answer sheet.

**Part I**

Answer 30 questions from this part.

1 .................. 11 .................. 21 .................. 31 .................. 
2 .................. 12 .................. 22 .................. 32 .................. 
3 .................. 13 .................. 23 .................. 33 .................. 
4 .................. 14 .................. 24 .................. 34 .................. 
5 .................. 15 .................. 25 .................. 35 .................. 
6 .................. 16 .................. 26 .................. 
7 .................. 17 .................. 27 .................. 
8 .................. 18 .................. 28 .................. 
9 .................. 19 .................. 29 .................. 
10 .................. 20 .................. 30 .................. 

Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

__________________________
Signature