

**The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION**

**ALGEBRA 2/
TRIGONOMETRY**

Thursday, January 29, 2015 — 9:15 a.m. – 12:15 p.m.

SAMPLE RESPONSE SET

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Question 28

28 If p and q vary inversely and p is 25 when q is 6, determine q when p is equal to 30.

$$(p)(q) \quad (25)(6) = 30q$$
$$\frac{150}{30} = \frac{30q}{30}$$

$$5 = q$$

Score 2: The student has a complete and correct response.

Question 28

28 If p and q vary inversely and p is 25 when q is 6, determine q when p is equal to 30.

$$p = \frac{k}{q}$$

$$p = \frac{k}{q}$$

$$6 \cdot 25 = \frac{k}{6} \cdot 6$$

$$q \cdot 30 = \frac{150}{q} \cdot q$$

$$150 = k$$

$$\frac{30q}{30} = \frac{150}{30}$$

$$q = 5$$

Score 2: The student has a complete and correct response.

Question 28

28 If p and q vary inversely and p is 25 when q is 6, determine q when p is equal to 30.

$$\frac{25}{6} = \frac{30}{q}$$

$$\frac{180}{25} = \frac{25q}{25}$$

$$q = 7.2$$

Score 1: The student applied direct variation.

Question 28

28 If p and q vary inversely and p is 25 when q is 6, determine q when p is equal to 30.

$$p = 25 \quad q = 6$$

$$\frac{p}{25} = \frac{q}{6}$$

$$30 : ?$$

$$25 : 6 \times$$

$$4.1666666667 \rightarrow 4.17$$

$$30 = p$$

$$\text{when } q = 7 \checkmark$$

$$\hookrightarrow \times 4.17 = 29.169$$

↓ rounded

$$\boxed{30}$$

Score 0: The student applied direct variation and made multiple rounding errors.

Question 29

29 Express in simplest form:

$$\frac{\frac{36 - x^2}{(x + 6)^2}}{x - 3}$$
$$\frac{x^2 + 3x - 18}{x^2 + 3x - 18}$$

$$\begin{array}{c} \frac{36 - x^2}{(6-x)(6+x)} \\ \frac{(x+6)(x+6)}{x^2 + 3x - 18} \\ \hline \end{array}$$
$$\begin{array}{c} \frac{(6-x)(6+x)}{(x+6)(x+6)} \\ \frac{(x-3)}{(x-3)(x+6)} \\ \hline \end{array}$$
$$\begin{array}{c} \boxed{1} \\ \boxed{(6-x)} \\ \hline \end{array}$$

Score 2: The student has a complete and correct response.

Question 29

29 Express in simplest form:

$$\frac{36 - x^2}{(x + 6)^2}$$

$\cancel{(6-x)(6+x)} \quad \cancel{(x+6)(x+6)}$

$$\frac{x-3}{x^2 + 6x - 18}$$
$$\frac{(x-3)(x+6)}{1}$$
$$\frac{-18}{(6)(3) | 3}$$
$$\frac{1}{x+6}$$

$$\frac{\frac{6+x}{x+6} \left(\frac{(x+6)}{1}\right)}{\frac{1}{x+6}}$$
$$\boxed{6+x}$$

Score 1: The student factored all the expressions correctly, but made one simplification error.

Question 29

29 Express in simplest form:

$$\frac{\frac{36 - x^2}{(x + 6)^2}}{\frac{x - 3}{x^2 + 3x - 18}}$$
$$\frac{(x-6)(x+6)}{(x+6)(x+6)}$$
$$\frac{x-3}{(x+6)(x+6)}$$

$$\frac{x-6}{x+6} \cdot \frac{x+6}{1}$$

$$\boxed{x-6}$$

Score 1: The student made one factoring error when factoring $36 - x^2$.

Question 29

29 Express in simplest form:

$$\frac{36 - x^2}{(x + 6)^2}$$

$$\frac{x - 3}{x^2 + 3x - 18}$$

$$(x-6)(x+3)$$

Score 0: The student has a correct response that was obtained by an obviously incorrect procedure.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.

$$\begin{aligned} \log_e 12 &= 4x \\ 2.4849 &= 4x \\ 4 \\ 0.62 &= x \end{aligned}$$

Score 2: The student has a complete and correct response.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.

$$\log_e 12 = 4x$$

$$\frac{\log e}{\log 12} = 4x$$

$$\frac{4.6024796044}{4} = 1.1506074011$$

$$x = 1.1506074011$$

$$x \approx 0.15$$

Score 1: The student applied the change of base rule for logarithms incorrectly.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.

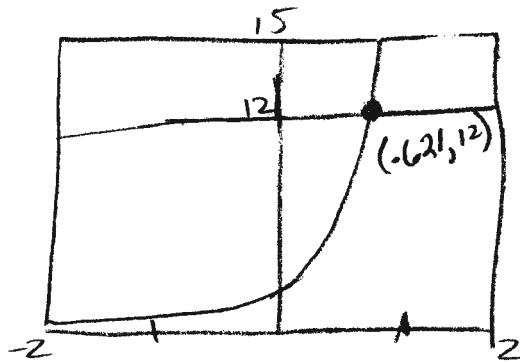
$$\begin{aligned} e^{4x} &= 12 \\ \cancel{4} \cancel{x \log e} &= \frac{12}{\cancel{4}} \\ x \log e &= 3 \\ \frac{x \log e}{\log e} &= \frac{3}{\log e} \\ x &= \frac{3}{\log e} \\ x &= 3 / \log(e^{\ln(1)}) = 6.907755279 \end{aligned}$$

$x = 6.91$

Score 1: The student did not take the log of both sides of the equation.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.



$$x = -.62$$

Score 1: The student solved the problem graphically instead of algebraically.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.

$$\begin{aligned} e^{4x} &= 12 \\ 4x &= \log 12 \\ x &= .27 \end{aligned}$$

Score 1: The student used log base 10 instead of using log base e . The student then solved for x correctly.

Question 30

30 Solve $e^{4x} = 12$ algebraically for x , rounded to the *nearest hundredth*.

$$e^{4x} = 12$$

$$\ln^{12} = 4x$$

$$\frac{102754.79}{4} = \cancel{4x}$$

$$x = 40688.10$$

Score 0: The student has a completely incorrect response.

Question 31

31 Determine, to the *nearest minute*, the degree measure of an angle of $\frac{5}{11}\pi$ radians.

$$\frac{5(180)}{11} = 81.8$$

$$81^\circ 49'$$

Score 2: The student has a complete and correct response.

Question 31

31 Determine, to the *nearest minute*, the degree measure of an angle of $\frac{5}{11}\pi$ radians.

$$\frac{5\pi}{11} \cdot \frac{180}{\pi} = \frac{(5)(180)}{11} = 81.8$$

∴

$$\frac{\theta}{10} = \frac{x}{60}$$

$$\frac{(8)(60)}{10} = 48$$

$$81^\circ 48'$$



Answer

Score 1: The student rounded prematurely.

Question 31

31 Determine, to the *nearest minute*, the degree measure of an angle of $\frac{5}{11}\pi$ radians.

$$\frac{\frac{5\pi}{11}}{\frac{\pi}{180}} = .0249$$

$$0^\circ 1' 29.724''$$

130 min

Score 0: The student made multiple errors.

Question 32

- 32 The probability of Ashley being the catcher in a softball game is $\frac{2}{5}$. Calculate the exact probability that she will be the catcher in *exactly* five of the next six games.

$${}_6C_5 \cdot \left(\frac{2}{5}\right)^5 \cdot \left(\frac{3}{5}\right)^1 = .036864$$

Score 2: The student has a complete and correct response.

Question 32

32 The probability of Ashley being the catcher in a softball game is $\frac{2}{5}$. Calculate the exact probability that she will be the catcher in *exactly* five of the next six games.

$$\begin{aligned} & \frac{2}{5} \quad nCr \quad (p_{\text{success}})^r \quad (p_{\text{fails}})^{n-r} \\ & \text{6} \quad \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^1 = .0368\dots \\ & \text{total } 6 \\ & \text{success } 5 \\ & 3.7\% \end{aligned}$$

Score 1: The student did not state the exact probability.

Question 32

- 32 The probability of Ashley being the catcher in a softball game is $\frac{2}{5}$. Calculate the exact probability that she will be the catcher in *exactly* five of the next six games.

$$6C5 (.4)^5 (.6) = \boxed{.037}$$

Score 1: The student did not state the exact probability.

Question 32

32 The probability of Ashley being the catcher in a softball game is $\frac{2}{5}$. Calculate the exact probability that she will be the catcher in *exactly* five of the next six games.

$$P =$$

$$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$$

,01024

1.024% probability

Score 0: The student has a completely incorrect response.

Question 33

33 If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$\begin{aligned} & 2xi(i - 4(-1)) \\ & 2xi(i + 4) \\ & 2xi^2 + 8xi \\ & \underline{2x(-1) + 8xi} \\ & -2x + 8xi \\ & x(-2 + 8i) \end{aligned}$$

Score 2: The student has a complete and correct response, followed by a correct factorization.

Question 33

33 If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$\begin{aligned} & 2xi(i+4) \\ & 2xi^2 + 8xi \\ & \underline{-2x + 8xi} \\ & \quad x \\ & \boxed{-2 + 8i} \end{aligned}$$

Score 1: The student made an error by dividing by x .

Question 33

33 If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$2xi(i - 4i^2)$$

$$2xi^2 - 8xi^3$$

$$-8xi^3 + 2xi^2$$

$$-8x(-i) + 2x(-1)$$

$$8xi - 2x$$

Score 1: The student did not express the answer in $a + bi$ form.

Question 33

33 If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$2xi(i - 4i^2)$$

$$2xi^2 - 8xi^3$$

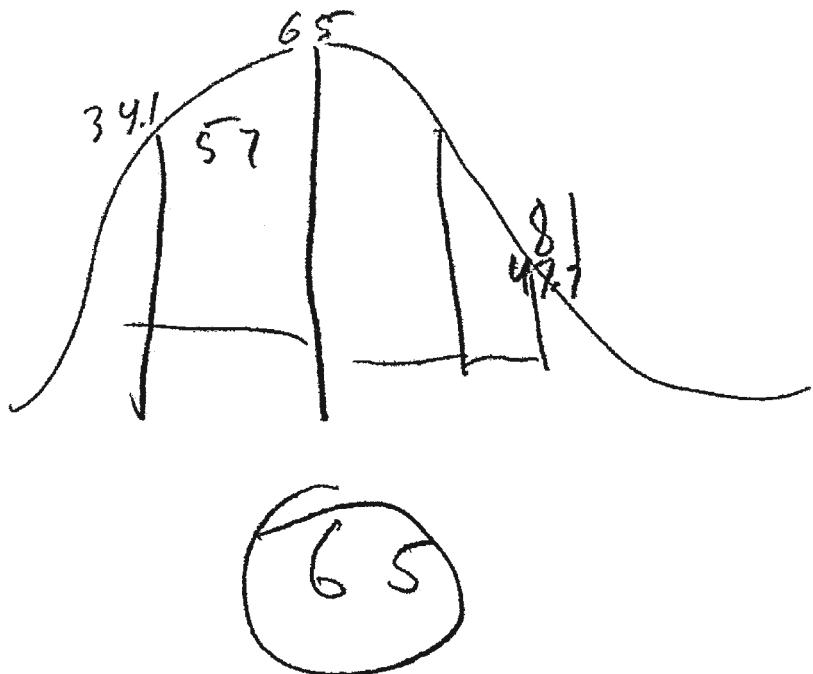
$$-2x - 8x$$

$$-6x$$

Score 0: The student made two errors, simplifying i^3 and combining the resulting like terms.

Question 34

- 34 On a test that has a normal distribution of scores, a score of 57 falls one standard deviation below the mean, and a score of 81 is two standard deviations above the mean. Determine the mean score of this test.



Score 2: The student has a complete and correct response. The student has enough work to justify the solution of 65.

Question 34

- 34 On a test that has a normal distribution of scores, a score of 57 falls one standard deviation below the mean, and a score of 81 is two standard deviations above the mean. Determine the mean score of this test.

$$\begin{array}{ccccccc} -1 & 0 & \text{MEAN} & 2 \\ 57 & +8 & 63 & +8 & 73 & +8 & 81 \end{array}$$

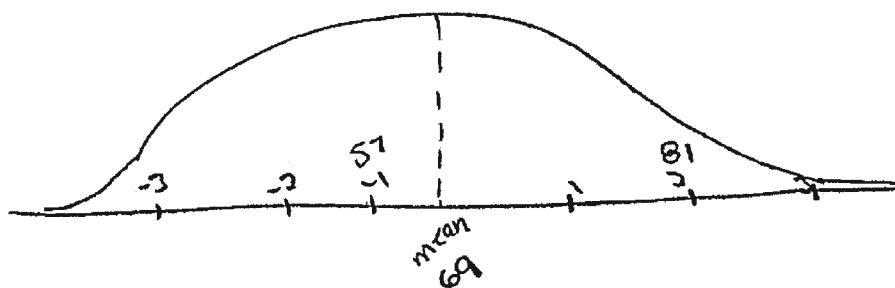
$$\begin{aligned} S &= 8 \\ r & \quad \boxed{\bar{x} = 63} \end{aligned}$$

Score 1: The student made a computational error when adding 8 to 57.

Question 34

- 34 On a test that has a normal distribution of scores, a score of 57 falls one standard deviation below the mean, and a score of 81 is two standard deviations above the mean. Determine the mean score of this test.

$$81.81^{\circ}$$



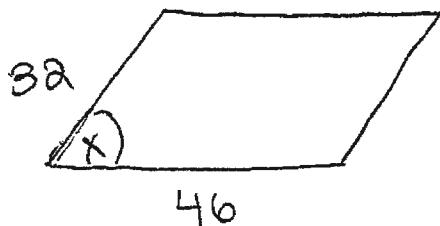
$$\frac{81 - 57}{13.8} = 69$$

mean = 69

Score 0: The student has a completely incorrect response.

Question 35

- 35 The area of a parallelogram is 594, and the lengths of its sides are 32 and 46. Determine, to the nearest tenth of a degree, the measure of the acute angle of the parallelogram.



$$\text{area} = 594$$

$$594 = (32)(46)(\sin x)$$

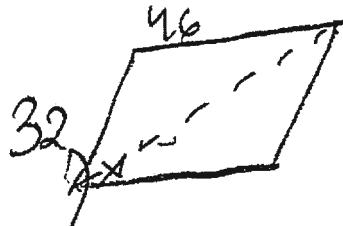
$$\frac{594}{1472} = \frac{1472(\sin x)}{1472}$$

$$\cdot 4035326087 = \sin x$$
$$\sin^{-1} (.4035326087) = 23.8^\circ = x$$

Score 2: The student has a complete and correct response.

Question 35

- 35 The area of a parallelogram is 594, and the lengths of its sides are 32 and 46. Determine, to the nearest tenth of a degree, the measure of the acute angle of the parallelogram.



$$594/2 = 297$$

$$K = \frac{1}{2} ab \sin C$$

$$297 = \frac{1}{2} (32)(46) \sin C$$

$$\frac{297}{736} = \frac{736 \sin C}{736}$$

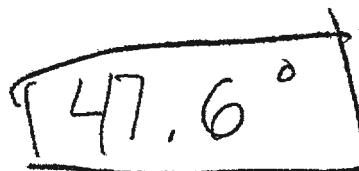
$$.4035326087 = \sin C$$

$$\sin^{-1}(.4035326087)$$

$$\sin = 23.79920549$$

$$= 23.8$$

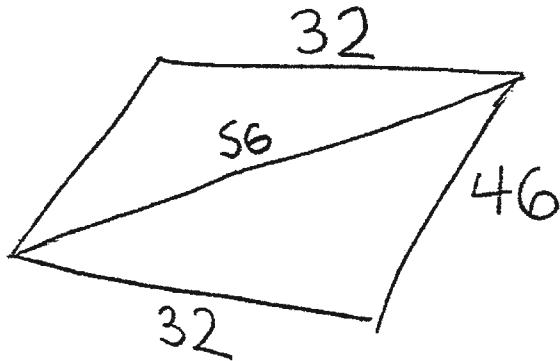
$$\begin{array}{r} 23.8 \\ \times 2 \\ \hline 47.6 \end{array}$$



Score 1: The student made an error by doubling 23.8.

Question 35

35 The area of a parallelogram is 594, and the lengths of its sides are 32 and 46. Determine, to the nearest tenth of a degree, the measure of the acute angle of the parallelogram.



$$32^2 + 46^2 = x^2$$

$$32^2 = 56^2 + 46^2 - 2(56)(46)\cos A$$

$$\begin{aligned} 1024 &= 3136 + 2116 - (5152)\cos A \\ -5252 &- 582\cancel{52} \end{aligned}$$

$$\frac{-4288}{-5152} = -\frac{5152\cos A}{-5152}$$

$$(0.8206521739)^{-1} (\cos A)^{-1}$$

$$\cos A = 34.84986775$$

34.8°

Score 0: The student made multiple errors. The student used the Pythagorean Theorem and then found the measure of an acute angle of the triangle that was not an acute angle of the parallelogram.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$y = 733.646(0.786)^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$y = 733.646(0.786)^{12}$$
$$y = 40.7 \boxed{41}$$

Score 4: The student has a complete and correct response.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$y = ab^x$$
$$a = 733.646$$
$$b = 0.786$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$733.646 (0.786)^{12}$$

$$\textcircled{41}$$

Score 4: The student has a complete and correct response.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$y = 743.477(0.784)^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$y = 743.477(-.784)^{12}$$

$$y = 40.092271$$

40

Score 3: The student did not clear the frequency list (on the TI-84 Plus 2.55 operating system) that was left after doing question #25. All work after that is complete and correct.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

	1990	1992	1995	1999	2004	2007	2009
Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$733.646 \cdot 0.786^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$733.646 \cdot 0.786^{12} = 40.7903$$

$$\approx 41$$

in the year 2002

Score 3: The student wrote an expression instead of an equation.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$y = 672.704 (0.787)^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$2002 - 1990 = 12$$

$$y = 672.704 (0.787)^{12}$$

$$37.977$$

38

Score 2: The student solved an incorrect regression equation appropriately.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$a = 733.6458141$$

$$a = 733.646$$

$$b = .7864904337$$

$$b = .786$$

$$y = (733.646)(.786^x)$$

$$\cdot 0103045344$$

$$y = 7.33646$$

$$\cdot 010$$

$$y = 7.34$$

$$y = (733.646)(.786^{12})$$

$$y = 453.2435642$$

year 2002

$$y = 453.244$$

Score 2: The student wrote the correct equation and found a and b to the nearest thousandth. No further correct work was shown.

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$y = a(b)^x$$

$$y = 733.645 (.786)^x$$

$$y = 733.645 (.786)^{2002}$$

$$y = \textcircled{0}$$

Score 1: The student made a rounding error when writing the equation and made a conceptual error by substituting in 2002 for x .

Question 36

- 36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$733.646 \times 0.78649^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$\begin{aligned}y &= 733.646 \times 0.78649^{2002} \\&= 1.1047\end{aligned}$$

Score 0: The student wrote an expression instead of an equation, made multiple rounding errors, and used 2002 for x .

Question 37

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.

$$a_1 = 2$$

$$a_n = 3(a_{n-1})^{-2}$$

$$a_2 = 3(a_1)^{-2} \rightarrow a_2 = 3(2)^{-2}$$

$$a_2 = \frac{3}{4} = .75$$

$$a_3 = \frac{16}{3} = 5.\overline{333}$$

$$a_4 = \frac{27}{256} = .1046875$$

Score 4: The student has a complete and correct response. The work beyond the correct solution is also correct.

Question 37

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.

$$a_1 = 2$$

$$a_n = 3(a_{n-1})^{-2}$$

$$\begin{aligned}a_2 &= 3(2)^{-2} & a_3 &= 3(0.75)^{-2} & a_4 &= 3\left(\frac{16}{3}\right)^{-2} \\&= .75 & &= \frac{16}{3} & &= \frac{27}{256}\end{aligned}$$

$$2, .75, \frac{16}{3}, \frac{27}{256}$$

Score 3: The student did not express a_2 as a fraction.

Question 37

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.

$$a_1 = 2$$

$$a_n = 3(a_{n-1})^{-2}$$

$$a_2 = 3(2-1)^{-2} = 3$$

$$a_3 = 3(3-1)^{-2} = \frac{3}{4}$$

$$a_4 = 3(.75-1)^{-2} = 48$$

Score 2: The student made a conceptual error by interpreting the subscript index as an operation of subtracting 1.

Question 37

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.

$$a_1 = 2$$

$$a_n = 3(a_{n-1})^{-2}$$

$$a_2 = 3(2-1)^{-2} = 3$$

$$a_3 = 3(3-1)^{-2} = .75$$

$$a_4 = 3(.75-1)^{-2} = 4.8$$

Score 1: The student made a conceptual error by interpreting the subscript index as an operation of subtracting 1 and the student did not express a_2 as a fraction.

Question 37

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.

$$a_1 = 2$$

$$a_n = 3(a_{n-1})^{-2}$$

$$\begin{aligned}a_2 &= \\a_2 &= 3(a_1 - 1)^{-2} \\a_2 &= 3(2 - 1)^{-2} \\a_2 &= \frac{1}{81}\end{aligned}$$

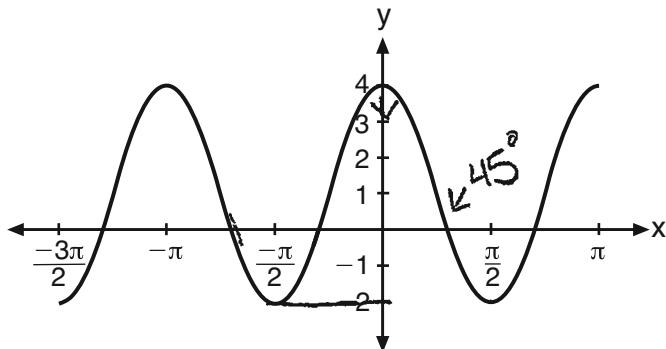
$$\left. \begin{aligned}a_3 &= 6 \\a_3 &= 3(a_2 - 1)^{-2} \\a_3 &= 3(6 - 1)^{-2} \\a_3 &= \frac{1}{225}\end{aligned} \right\}$$

$$\left. \begin{aligned}a_4 &= 8 \\a_4 &= 3(a_3 - 1)^{-2} \\a_4 &= 3(8 - 1)^{-2} \\a_4 &= \frac{1}{441}\end{aligned} \right\}$$

Score 0: The student has a completely incorrect response.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$$a = 3$$

$$b = 2$$

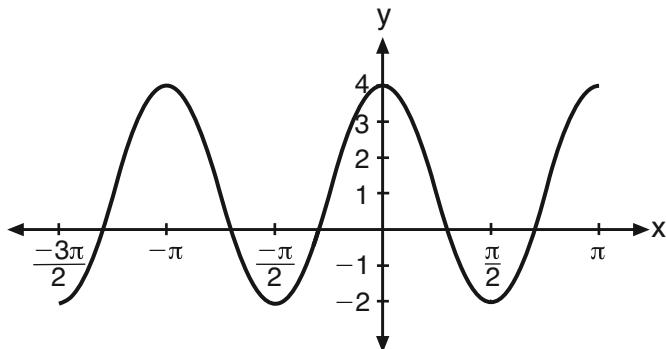
$$c = 1$$

$$y = 3 \cos 2x + 1$$

Score 4: The student has a complete and correct response.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$$a = 3$$

$$b = 2$$

$$c = 1$$

b is period

$$a = \text{amp} = 3 \cdot \frac{4 - (-2)}{2} = 3$$

$$c = \text{mid} = \frac{4 + -2}{2} = 1$$

$$\frac{2\pi}{\text{Freq}}$$

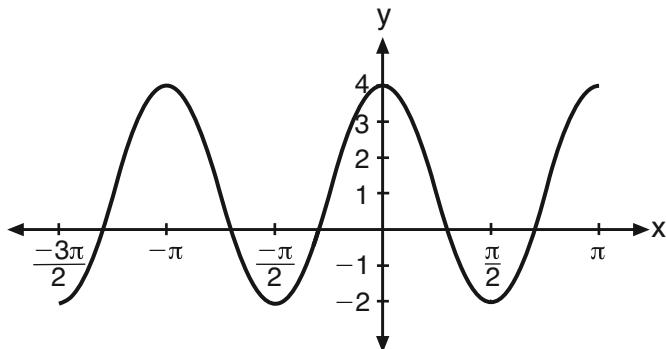
$$\text{Amp} = \frac{\text{high} - \text{low}}{2}$$

$$\text{MS} = \frac{\text{high} + \text{low}}{2}$$

Score 3: The student stated the correct values of a , b , and c , but did not write the equation for the graph.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

a = amplitude

$$a = \frac{|4 - -2|}{2}$$

$$a = 3$$

b = frequency

$$b = \frac{2\pi}{\text{period}}$$

$$b = \frac{2\pi}{\pi}$$

$$b = 2$$

c = vertical translation

$$c = \frac{4 + -2}{2}$$

$$c = \frac{2}{2}$$

$$c = 1$$

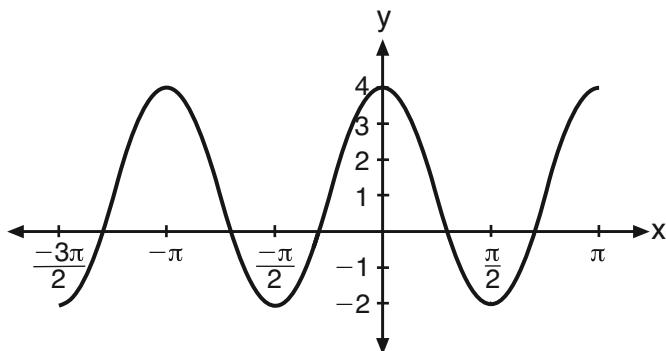
$$a \cos bx + c$$

$$3 \cos 2x + 1$$

Score 3: The student stated the correct values of a , b , and c , but wrote an expression instead of an equation for the graph.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$a = 3$ amplitude

$b = \pi$ period

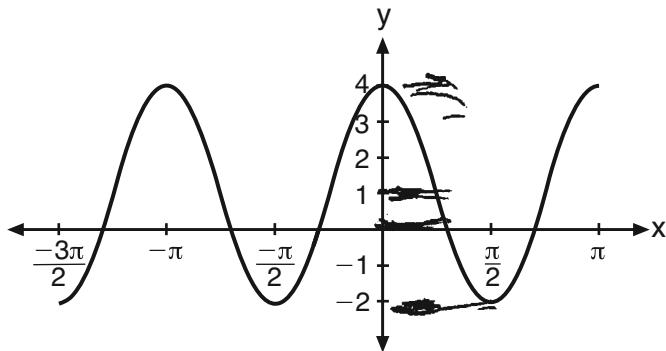
$c = 1$ vertical shift

$$y = 3 \cos \pi x + 1$$

Score 3: The student only stated the correct values for a and c , but an appropriate equation was written.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$$a = 3$$

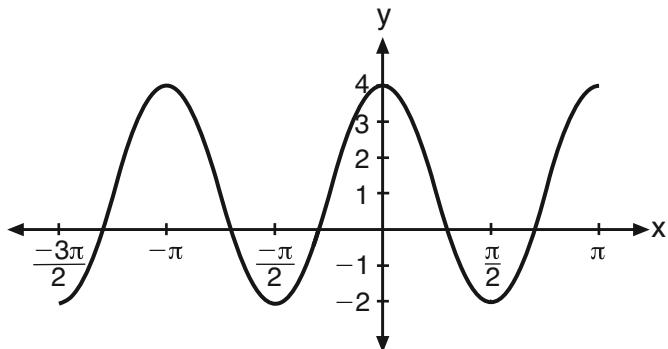
$$b = 1$$

$$c = 1$$

Score 2: The student only stated the correct values for a and c . No further correct work was shown.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$$a = \text{amp.}$$

$$b = \text{period}$$

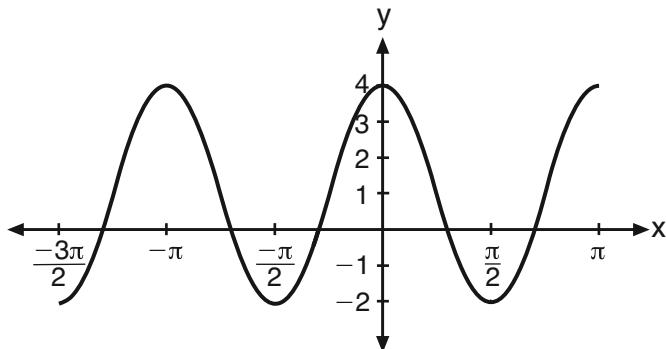
$$c = \text{freq.}$$

$$4 \cos 2x + 2$$

Score 1: The student only wrote the correct value for b and wrote an expression instead of an equation for the graph.

Question 38

- 38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where a , b , and c are real numbers.



State the values of a , b , and c , and write an equation for the graph.

$$\begin{aligned}a &= -2 \\b &= \frac{\pi}{2} \\c &= +4\end{aligned}$$

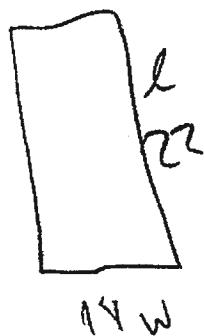
$$y = -2 \cos \frac{\pi}{2}x + 4$$

Score 0: The student has a completely incorrect response.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]



$$\begin{aligned} (l+x) \cdot (w+x) &\leq 800 \\ (22+x)(14+x) &\leq 800 \\ 308 + 36x + x^2 &\leq 800 \\ x^2 + 36x - 492 &= 0 \\ x^2 + 36x + 324 &= 992 + 324 \\ (x+18)^2 &= \sqrt{816} \\ x+18 &= 28.57 \\ x &= 10.57 \\ x &\approx 10 \text{ ft} \quad \text{must round down} \end{aligned}$$

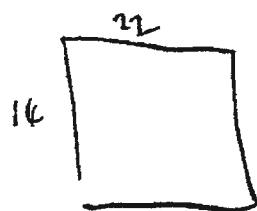
(can be expanded
10 feet)

Score 6: The student has a complete and correct response.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]



$$(14+x) \cdot 22+x = 800$$

$$(14+x)(22+x)$$

$$\begin{array}{r} 308 + 36x + x^2 = 800 \\ -800 \\ \hline -492 \end{array}$$

$$x^2 + 36x - 492 = 0$$

$$\frac{-36 \pm \sqrt{36^2 - 4(-492)}}{2}$$

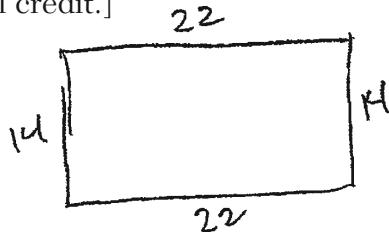
+ 10

Score 6: The student has a complete and correct response.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]



$$(14+x)(22+x) \leq 800$$

$$308 + 36x + x^2 \leq 800$$

$$x^2 + 36x + 308 \leq 800$$

$$x^2 + 36x - 492 \leq 0$$

$$\frac{-36 \pm \sqrt{1296 + 1968}}{2}$$

$$\frac{-36 \pm \sqrt{3264}}{2}$$

rejecting -36

$$\frac{-36 + 57.13142743}{2}$$

$$\frac{21.13142743}{2}$$

10.5657 feet
 ≈ 11 feet

Score 5: The student showed appropriate work to find x , but an incorrect solution was stated.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]



$$A \leq 800$$

$$(x+22)(x+14) \leq 800$$

$$x^2 + 14x + 22x + 308 = 800$$

$$\begin{array}{r} 22 \\ \times 14 \\ \hline 88 \\ 220 \\ \hline 308 \end{array}$$

$$x^2 + 36x - 492 \leq 0$$

$$\begin{array}{r} 78120 \\ -308 \\ \hline 492 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{r} 246 \\ 2 \sqrt{492} \\ \hline \end{array}$$

$$x = \frac{-36 \pm \sqrt{1296 - 4(1)(-492)}}{2}$$

$$\begin{array}{r} 41 \\ 12 \sqrt{492} \\ \hline 48 \\ 3 \\ \hline 16 \sqrt{492} \\ \hline 48 \\ 12 \end{array}$$

$$x = \frac{-36 \pm \sqrt{2264}}{2}$$

$$x = 10 \text{ ft}$$

$$x = \frac{-36 \pm \sqrt{566}}{2}$$

$$x = -13 \pm \sqrt{566}$$

$$x = 10.791$$

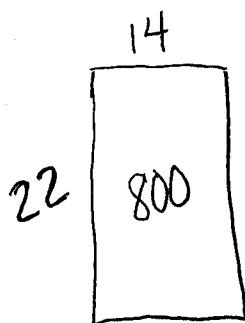
$$\begin{array}{r} 3441 \\ \hline 72 \\ 9 \end{array}$$

Score 4: The student made two computational errors. A computational error was made in simplifying under the radical and a computational error was made in dividing by 2.

Question 39

39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]



$$\text{add 12} \quad (26)(34) = 884$$

$$\text{add 11} \quad (25)(33) = 825$$

$$\text{add 10} \quad (24)(32) = 768$$

A hand-drawn box containing the text "10 Feet".

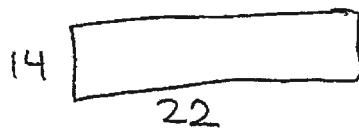
Score 3: The student used a method other than algebraic to solve the problem.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]

$$(14+x)(22+x) = 800$$



$$308 + 14x + 22x + x^2 = 800$$

$$\begin{array}{r} x^2 + 36x + 308 = 800 \\ \underline{-800} \end{array}$$

$$x^2 + 36x - 492 = 0$$

Score 2: The student wrote a correct quadratic equation in standard form, but no further correct work was shown.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]

$$(22+x)(14+x) \leq 800$$

~~18.888888888888887~~

10 feet

Score 1: The student did not show algebraic work to support a correct answer.

Question 39

- 39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]

$$w = 14 \quad l = 22$$
$$\text{area} = 800 \text{ ft}^2$$

$$a = l \cdot w$$

$$(w+3) \cdot (l+3) = \text{area}$$

$$17 \cdot 25 = 425$$

$$20 \cdot 28 = 560$$

$$23 \cdot 31 = 713$$

$$24 \cdot 32 = 768$$

$$26 \cdot 34 = 884$$

$$\text{width} = 24.5 \text{ ft}$$

$$\text{length} = 32.5 \text{ ft}$$

$$\text{area} = 796.25 \text{ ft}^2$$

Score 0: The student used a method other than algebraic, but did not obtain a correct solution.