Question 28

28 Solve algebraically for $x$:

$$\sqrt{2x + 1} + 4 = 8$$

\[
\sqrt{2x + 1} = 4 \\
2x + 1 = 16 \\
x = 7.5
\]

**Score 2:** The student has a complete and correct response.
Question 28

28 Solve algebraically for $x$:

\[
\sqrt{2x + 1} + 4 = 8 \\
-4 - 4
\]

\[
(\sqrt{2x + 1})^2 = (4.1)^2
\]

\[
2x + 1 = 16
\]

\[
2x + 1 = 16 \\
-1 - 1
\]

\[
\frac{2x}{2} = 15 \\
\frac{2}{2}
\]

\[
x = 7.5
\]

\[
\sqrt{2x + 1} + 4 = 8 \\
-4 - 4
\]

\[
(\sqrt{2x + 1})^2 = (4.1)^2
\]

\[
2x + 1 = -16 \\
-1 - 1
\]

\[
\frac{2x}{2} = -17 \\
\frac{2}{2}
\]

\[
x = -8.5 \text{ Reject}
\]

Score 1: The student made an error by treating the square root as an absolute value.
Question 28

28 Solve algebraically for $x$:

$$\sqrt{2x + 1} + 4 = 8$$

$$\sqrt{2x + 1} = 4$$

$$2x + 1 = 64$$

$$2x = 63$$

$$x = 31.5$$

Score 1: The student made an error when squaring both sides of the equation.
Question 28

28 Solve algebraically for $x$:

$$\sqrt{2x + 1} + 4 = 8$$

\[ \begin{align*}
\sqrt{2x + 1} &= 4 \\
2x + 1 &= 4 \\
2x &= 3 \\
x &= \frac{3}{2}
\end{align*} \]

Score 1: The student made a conceptual error by not squaring both sides of the equation.
28 Solve algebraically for $x$:

$$\sqrt{2x + 1} + 4 = 8$$

\[
\begin{align*}
\sqrt{2x+1} + 4 &= 8 \\
\sqrt{2x+1} &= 4 \\
2x + 1 &= 16 \\
x &= 7 \\
\sqrt{x} &= \sqrt{3.75} \\
x &= 1.936491673
\end{align*}
\]

Score 0: The student made one error when squaring the radical and a second error by not stating $\pm\sqrt{3.75}$. 
Question 29

29 Factor completely:

\[ x^3 + 3x^2 + 2x + 6 \]

\[ x(x^2 + 2) + 2(x + 3) \]

\[ (x^2 + 2)(x + 3) \]

Score 2: The student has a complete and correct response.
29 Factor completely:

\[ x^3 + 3x^2 + 2x + 6 \]

\[ x^2(x+3) + 2(x+3) \]

\[ (x^2+2)(x+3) \]

\[ (x-2)(x-1)(x+3) \]

**Score 1:** The student incorrectly factored \((x^2 + 2)\).
Question 29

29 Factor completely:

\[ x^3 + 3x^2 + 2x + 6 \]

\[
\begin{align*}
  &x^3 + 3x^2 \\
  &x^2(x + 3) \\
  &2x + 6 \\
  &2(x + 3) \\
  \hline
  &2x^2(x + 3)(x + 3)
\end{align*}
\]

Score 1: The student made an error in factoring by grouping.
Question 29

29 Factor completely:

\[ x^3 + 3x^2 + 2x + 6 \]

\[ x^3 + 3x^2 + 2x + 6 \]
\[ = x^2(x + 3) + 2(x + 3) \]
\[ = (x^2 + 2)(x + 3) \]
\[ x^2 + 2 = 0 \quad x + 3 = 0 \]
\[ x = -3 \quad x = -\sqrt{2} \]

Score 1: The student made an error by treating the expression as an equation.
29 Factor completely:

\[ x^3 + 3x^2 + 2x + 6 \]

\[ x(x^2 + 3x + 2) + 6 \]

\[ (x+6)(x^2 + 3x + 2) = 0 \]

\[ (x+6)(x+2)(x+1) = 0 \]

\[ x+6 = 0 \quad x+2 = 0 \quad x+1 = 0 \]

\[ x = -6 \quad x = -2 \quad x = -1 \]

**Score 0:** The student factored by grouping incorrectly and treated the expression as an equation.
30 Solve algebraically for the *exact* value of $x$:

\[ \log_8 16 = x + 1 \]

\[ 8^{x+1} = 16 \]
\[ 2^{3x+3} = 2^4 \]
\[ 3x + 3 = \frac{4}{3} \]
\[ 3x = \frac{1}{3} \]
\[ x = \frac{1}{3} \]

**Score 2:** The student has a complete and correct response.
30 Solve algebraically for the exact value of $x$:

$$\log_8 16 = x + 1$$

$\frac{\log 16}{\log 8} = x + 1$

$1.3 = x + 1$

$0.3 = x$

**Score 2:** The student has a complete and correct response.
30 Solve algebraically for the exact value of x:

\[
\log_8 16 = x + 1
\]

\[
2 = x + 1
\]

\[
x = 1
\]

**Score 2:** The student has a complete and correct response.
Question 30

30 Solve algebraically for the exact value of $x$:

$$\log_8 16 = x + 1$$

Score 1: The student made an error by not raising 8 to the power of $(x + 1)$. 
30 Solve algebraically for the exact value of $x$:

$$\log_8 16 = x + 1$$

Score 1: The student made an error by calculating $\log 16$. 
Question 30

30 Solve algebraically for the exact value of $x$:

\[
\log_8 16 = x + 1
\]

\[
\log_8 16 = \log_8 (2^{x+1})
\]

\[
x + 1 = 2^{x+1}
\]

\[
4 = 3x - 3
\]

\[
+3 + 3
\]

\[
\frac{4}{3} = 3x
\]

\[
\frac{4}{3} = x
\]

Score 1: The student made a transcription error by writing $(x - 1)$ instead of $(x + 1)$. 
30 Solve algebraically for the exact value of $x$:

$$\log_8 16 = x + 1$$

$$16(x + 1) = 8$$

$$x + 1 = \frac{1}{2}$$

$$x = -\frac{1}{2}$$

**Score 0:** The student wrote a completely incorrect response.
31 Determine how many eleven-letter arrangements can be formed from the word “CATTARAUGUS.”

\[
\frac{11!}{3! \cdot 2! \cdot 2!} = 1663200
\]

Score 2: The student has a complete and correct response.
Question 31

31 Determine how many eleven-letter arrangements can be formed from the word “CATTARAUGUS.”

\[
\frac{11!}{3! \cdot 2! \cdot 2!} = 1,663,200
\]

Score 2: The student has a complete and correct response.
31 Determine how many eleven-letter arrangements can be formed from the word “CATTARAUGUS.”

\[
\frac{11!}{3! \cdot 2! \cdot 2!} = 3,326,400
\]

**Score 1:** The student divided by an incorrect denominator.
31 Determine how many eleven-letter arrangements can be formed from the word “CATTARAUGUS.”

\[
\frac{11!}{3! + 2! + 2!} = \frac{11!}{10} = 3,991,680
\]

**Score 1:** The student added in the denominator instead of multiplying.
31 Determine how many eleven-letter arrangements can be formed from the word “CATTARAUGUS.”

\[ 11! = 39,916,800 \]

Score 0: The student only evaluated 11!. 
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

\[
\frac{-13\pi}{18^\circ} = \frac{-13\pi}{18^\circ} = -2.27
\]

Score 2: The student has a complete and correct response.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

Score 2: The student has a complete and correct response.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

Score 1:  The student did not include $\pi$ in the formula.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

Score 1: The student did not express the answer to the nearest hundredth.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

\[
130 \times \left( \frac{\pi}{180} \right) = \frac{13\pi}{18}
\]

\[
\frac{-13\pi}{18}
\]

Score 1: The student did not express the answer to the nearest hundredth.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

\[
\frac{-130 \pi}{180} = \frac{-130\pi}{180} = \frac{-13\pi}{18} = -2.27\pi
\]

Score 1: The student incorrectly included $\pi$ in the final answer.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

\[ -130^\circ \cdot \frac{\pi}{180^\circ} = -\frac{130\pi}{180} = -\frac{13\pi}{18} \approx 0.72\pi \]

**Score 0:** The student made an error when dividing $-13$ by $18$ and did not express the answer to the nearest hundredth.
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

\[
\begin{align*}
-130^\circ & \cdot \frac{180}{\pi} \\
& = -\frac{23400}{\pi} \\
& \approx -7448.451337
\end{align*}
\]

Score 0: The student used the wrong conversion and did not round to the nearest hundredth.
33 Determine the area, to the nearest integer, of $\triangle SRO$ shown below.

\[ K = \frac{1}{2} \times 15 \times 31.6 \times \sin 128 \]
\[ K = 194.1390345 \]
\[ K = 194 \]

Score 2: The student has a complete and correct response.
33 Determine the area, to the nearest integer, of \( \triangle SRO \) shown below.

\[
\begin{align*}
\quad & \quad \\
S & \quad R \\
15 \quad & \quad 31.6 \\
38^\circ & \quad 17^\circ \\
S & \quad O
\end{align*}
\]

\[
180 - (38 + 17) = 125
\]

\[
A = 15 \cdot 31.6 \cdot \sin 125
\]

\[
388.278069
\]

\[
A \approx 388
\]

**Score 1:** The student did not divide by 2.
33 Determine the area, to the nearest integer, of \( \triangle SRO \) shown below.

\[
\frac{1}{2} ab \sin \left( \frac{\pi}{25} \right)
\]

\[
\frac{1}{2} (15)(31.6) \sin (125)
\]

\[K \approx 1941.13\]

**Score 1:** The student did not round to the nearest integer.
33 Determine the area, to the nearest integer, of \( \triangle SRO \) shown below.

\[ A = \frac{1}{2} \cdot 15 \cdot 31.6 \cdot \sin(125) \]

\[ A = 7.5 \cdot 32 \cdot \frac{\sqrt{2}}{2} \]

\[ A = 7.5 \cdot 16 \cdot \sqrt{2} \]

\[ A = 7.5 \cdot 16 \cdot 1 \]

\[ A = 120 \]

\[ A \approx 120 \]

Score 1: The student substituted correctly into the area formula.
Question 33

Determine the area, to the nearest integer, of \( \triangle SRO \) shown below.

\[
\begin{align*}
A &= \frac{1}{2}bh \\
&= \frac{1}{2} \times 15 \times 31.6 \\
&= 237
\end{align*}
\]

Score 0: The student used the incorrect formula.
Determine the area, to the nearest integer, of \( \triangle SRO \) shown below.

\[
\frac{x}{\sin 125^\circ} = \frac{31.6}{\sin 38^\circ}
\]

\[
x = 25.885
\]

\[
42.0445
\]

**Score 0:** The student wrote irrelevant work.
Question 34

34 Prove that the equation shown below is an identity for all values for which the functions are defined:

\[
csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta
\]

\[
\frac{1}{\sin \theta} \cdot \sin^2 \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta
\]

\[
cosec = \cosec
\]

Score 2: The student has a complete and correct response.
34 Prove that the equation shown below is an identity for all values for which the functions are defined:

\[ \csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta \]

Score 1: The student did not prove the equation works for all values of \( \theta \).
34 Prove that the equation shown below is an identity for all values for which the functions are defined:

\[ \csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta \]

\[ \frac{1}{\sin \theta} \cdot (1 - \cos^2 \theta) \cdot \frac{\cos \theta}{\sin \theta} \]

**Score 1:** The student wrote all the trigonometric functions in terms of \( \sin \theta \) and \( \cos \theta \), but showed no further correct work.
34 Prove that the equation shown below is an identity for all values for which the functions are defined:

\[
csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta
\]

\[
\left( \frac{1}{\sin \theta} \right) \sin^2 \theta \cdot \frac{\sin \theta}{\cos \theta} = \cos \theta
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

**Score 0:** The student did not substitute for \( \cot \theta \) correctly and showed no further correct work.
35 Find the difference when \( \frac{4}{3} x^3 - \frac{5}{8} x^2 + \frac{7}{9} x \) is subtracted from \( 2x^3 + \frac{3}{4} x^2 - \frac{2}{9} \).

\[
\left(2x^3 + \frac{3}{4} x^2 - \frac{2}{9}\right) - \left(\frac{4}{3} x^3 - \frac{5}{8} x^2 + \frac{7}{9} x\right)
\]

\[
\frac{2}{3} x^3 + \frac{11}{8} x^2 - \frac{7}{9} x - \frac{2}{9}
\]

**Score 2:** The student has a complete and correct response.
35 Find the difference when \( \frac{4}{3} x^3 - \frac{5}{8} x^2 + \frac{7}{9} x \) is subtracted from \( 2x^3 + \frac{3}{4} x^2 - \frac{2}{9} \).

\[
2x^3 + \frac{3}{4} x^2 - \frac{2}{9} - \frac{4}{3} x^3 + \frac{5}{8} x^2 - \frac{7}{9} x
\]

\[
\frac{2}{3} x^3 + \frac{11}{8} x^2 - \frac{7}{9} x - \frac{2}{9}
\]

**Score 2:** The student has a complete and correct response.
35 Find the difference when \( \frac{4}{3} x^3 - \frac{5}{8} x^2 + \frac{7}{9} x \) is subtracted from \( 2x^3 + \frac{3}{4} x^2 - \frac{2}{9} \).

\[
(2x^3 + \frac{3}{4} x^2 - \frac{2}{9}) - (\frac{4}{3} x^3 - \frac{5}{8} x^2 + \frac{7}{9} x)
\]

\[
2x^3 + \frac{3}{4} x^2 - \frac{2}{9} - \frac{4}{3} x^3 + \frac{5}{8} x^2 - \frac{7}{9} x
\]

\[
\frac{2}{3} x^3 + \frac{11}{8} x^2 - 1
\]

**Score 1:** The student made a transcription error by not writing \(-\frac{7}{9} x\).
35 Find the difference when \( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \) is subtracted from \( 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \).

\[
\left( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \right) - \left( 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \right) \\
\left( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \right) - 2x^3 - \frac{3}{4}x^2 + \frac{2}{9} \\
- \frac{5}{8}x^2 - \frac{3}{4}x^2 = \frac{11}{8}x^2 \\
- \frac{2}{3}x^3 - \frac{11}{8}x^2 + \frac{7}{9}x - \frac{2}{9}
\]

**Score 1:** The student subtracted in the wrong order.
Question 35

35 Find the difference when \( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \) is subtracted from \( 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \).

Score 1: The student did not distribute the negative.
35 Find the difference when \( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \) is subtracted from \( 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \).

\[
\begin{array}{c}
\frac{2}{3}x^3 - \frac{1}{8}x^2 + \frac{5}{9}x \\
\end{array}
\]

**Score 0:** The student did not distribute the negative and combined unlike terms.
36 Find the exact roots of \( x^2 + 10x - 8 = 0 \) by completing the square.

\[
(x^2 + 10x + 25) - 33 = 0
\]

\[
(x + 5)^2 = 33
\]

\[
x + 5 = \pm \sqrt{33}
\]

\[
x = -5 \pm \sqrt{33}
\]

**Score 4:** The student has a complete and correct response.
36 Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

\[
\begin{align*}
x^2 + 10x &= 8 \\
x^2 + 10x + 25 &= 8 + 25 \\
(x + 5)^2 &= 33 \\
x + 5 &= \pm \sqrt{33} \\
x &= -5 \pm \sqrt{33}
\end{align*}
\]

$\boxed{x = -5 \pm 5.744562647}$

**Score 3:** The student did not give the exact values of $x$ as the final answer.
36 Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

\[
\begin{align*}
x^2 + 10x - 8 &= 0 \\
+8 &+8 \\
x^2 + 10x &= 8 \\
 x^2 + 10x + 25 &= 8 + 25 \\
(x + 5)^2 &= 33 \\
\sqrt{(x + 5)^2} &= \sqrt{33} \\
x + 5 &= \sqrt{33} \\
x &= \sqrt{33} - 5
\end{align*}
\]

**Score 3:** The student did not write $\pm \sqrt{33}$.
Question 36

Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

\[ x^2 + 10x = 8 \]

\[ x^2 + 10x + 25 = 8 + 25 \]

\[ (x + 5)^2 = 33 \]

\[ x + 5 = \pm \sqrt{33} \]

\[ x = -5 \pm i \sqrt{33} \]

Score 2: The student made a conceptual error by adding 25 to the left and subtracting 25 from the right.
36 Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

\[ x^2 + 10x - 8 = 0 \]

\[ a = 1 \quad b = 10 \quad c = -8 \]

\[ x = \frac{-10 \pm \sqrt{100 - (-32)}}{2} \]

\[ x = \frac{-10 \pm \sqrt{132}}{2} \]

\[ x = \frac{-10 \pm 2\sqrt{33}}{2} \]

\[ x = -5 \pm \sqrt{33} \]

**Score 2:** The student used the quadratic formula to solve for $x$. 
36 Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

\[
x^2 + 10x - 8 = 0
\]

\[
x^2 + 10x + 100 = 8 + 100
\]

\[
(x + 10)^2 = 108
\]

\[
x + 10 = \pm \sqrt{108}
\]

\[
x = -10 \pm \sqrt{108}
\]

\[
x = -10 \pm 6\sqrt{3}
\]

**Score 1:** The student made a conceptual error in completing the square by adding 100 to both sides and not writing $\pm \sqrt{108}$.
Find the exact roots of $x^2 + 10x - 8 = 0$ by completing the square.

**Score 1:** The student used a method other than completing the square and did not give exact values of $x$. 
36 Find the exact roots of \(x^2 + 10x - 8 = 0\) by completing the square.

\[a = 1\quad b = 10\quad c = -8\]

\[x = \frac{-10 \pm \sqrt{100 - 4(1)(-8)}}{2(1)}\]

\[x = \frac{-10 \pm \sqrt{132}}{2}\]

\[x = -10.7445626465\]

\[x = 10.7445626465\]

**Score 1:** The student used the quadratic formula and did not give the exact value of \(x\).
36 Find the exact roots of \( x^2 + 10x - 8 = 0 \) by completing the square.

\[
\begin{align*}
    x^2 + 10x &= 8 \\
    x^2 + 10x + 25 &= 8 \\
    (x+5)(x+5) &= 8 \\
    (x+5)^2 &= 8^2 \\
    (x+5)^2 &= 64 \\
    (x+5)^2 - 64 &= 0
\end{align*}
\]

**Score 0:** The student made a conceptual error by not adding 25 to both sides of the equation and another conceptual error by squaring the 8. The student also did not solve for \( x \).
The table below gives the relationship between \( x \) and \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.2</td>
<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for \( y \) as a function of \( x \), rounding all values to the nearest hundredth.

\[
y = 2.19 \cdot (3.23)^x
\]

Using this equation, predict the value of \( x \) if \( y \) is 426.21, rounding to the nearest tenth.

[Only an algebraic solution can receive full credit.]

\[
426.21 = 2.19 \cdot (3.23)^x
\]

\[
194.62 = 3.23^x
\]

\[
\log_{3.23} 194.62 = x
\]

\[
\therefore x = 4.5
\]

**Score 4:** The student has a complete and correct response.
37 The table below gives the relationship between \( x \) and \( y \).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</tr>
</tbody>
</table>

Use exponential regression to find an equation for \( y \) as a function of \( x \), rounding all values to the nearest hundredth.

Using this equation, predict the value of \( x \) if \( y \) is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

\[
y = a(b^x)
\]

\( a = 2.19 \)
\( b = 3.23 \)

\[
426.21 = 2.19(3.23^x)
\]

\[
194.6164384 = 3.23^x
\]

\[
\log 194.6164384 = x \log 3.23
\]

\[
x = 4.5
\]

Score 4: The student has a complete and correct response.
37 The table below gives the relationship between x and y.

<table>
<thead>
<tr>
<th>x</th>
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<td>y</td>
<td>4.2</td>
<td>33.5</td>
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<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for y as a function of x, rounding all values to the nearest hundredth.

Using this equation, predict the value of x if y is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

\[
y = a \cdot b^x
\]

\[
a = 8.26
\]

\[
b = 2.32
\]

\[
\frac{426.21}{8.26} = 51.599 = 2.32^x
\]

\[
\log 51.599 = x \cdot \log 2.32
\]

\[
\frac{\log 51.599}{\log 2.32} = x = 4.6859
\]

\[
x = 4.7
\]

**Score 3:** The student solved an incorrect exponential regression equation appropriately.
Question 37

37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
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<th>4</th>
<th>5</th>
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<td>33.5</td>
<td>113.1</td>
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<td>523.6</td>
</tr>
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</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

$\text{\begin{align*} \quad y &= 2.19 (3.23)^x \end{align*}}$

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth.

[Only an algebraic solution can receive full credit.]

$\text{\begin{align*} \quad 426.21 &= 2.19 (3.23)^x \end{align*}}$

Score 2: The student did not solve for $x$. 

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Question 37

37 The table below gives the relationship between $x$ and $y$.

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<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

\[
y = ab^x \]

\[
a \approx 2.19, \quad b \approx 3.23
\]

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

\[
\log 426.21 = \log 2.19(3.23)^x
\]

\[
\log 426.21 = \log 2.19(3.23)^x - \log 2.19(3.23)
\]

\[
\frac{\log 426.21}{\log 2.19(3.23)} = x
\]

Score 2: The student wrote a correct exponential regression equation, but made a conceptual error by not applying the product rule.
Question 37

37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4.2</td>
<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

Score 2: The student wrote a correct exponential regression equation, but made a conceptual error by subtracting 2.19 instead of dividing.
The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

$$y = a \cdot b^x, \quad y = 0.71(4.05)^x$$

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth.

[Only an algebraic solution can receive full credit.]

$$\frac{426.21}{0.71} = (4.05)^x$$

$$\ln \left( \frac{426.21}{0.71} \right) = x \ln 4.05$$

$$x = \frac{\ln \left( \frac{426.21}{0.71} \right)}{\ln 4.05}$$

$$x = 4.57$$

**Score 2:** The student solved an incorrect exponential regression equation appropriately, but did not round 4.57 to the nearest tenth.
37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

$$y = 8.24 \cdot (2.32)^x$$

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

Score 1: The student wrote an incorrect exponential regression equation.
37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tr>
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<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth.

[Only an algebraic solution can receive full credit.]

$$y = a \cdot b^x$$

$$\frac{426.21}{8.26} = \frac{8.26 \cdot 2.32^x}{8.26}$$

$$a = 8.26$$

$$b = 2.32$$

$$\frac{523.6}{2.32} = \frac{2.32^x}{2.32}$$

$$22.2$$

**Score 1:** The student solved an incorrect exponential equation, but made a conceptual error by dividing by 2.32.
37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth.

[Only an algebraic solution can receive full credit.]

Score 0: The student wrote completely incorrect work.
38 Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0^\circ \leq x < 360^\circ \).

\[
\begin{align*}
L & = e^{2x} - 1 + e^{2x} \\
G & = e^{2x} - \cos x - 1 = 0 \\
A & = 2 \\
b & = -1 \\
c & = -1
\end{align*}
\]

\[
\begin{align*}
C & = \frac{1 \pm \sqrt{1 - (-8)}}{4} = \frac{1 \pm \sqrt{9}}{4} \\
n_1 & = \frac{1 + 3}{4} = 1 \\
n_2 & = \frac{1 - 3}{4} = \frac{1}{2}
\end{align*}
\]

\( \gamma = \arccos(1) \)

\( \chi = 0 \)

\( \gamma = \arccos(1/2) \)

\( \chi = 120, 240 \)

Score 4: The student has a complete and correct response.
Question 38

38 Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0^\circ \leq x < 360^\circ \).

\[
\begin{align*}
\cos 2x &= \cos x \\
2 \cos^2 x - 1 &= \cos x \\
2 \cos^2 x - \cos x - 1 &= 0 \\
(2 \cos x + 1)(\cos x - 1) &= 0 \\
\cos x &= -\frac{1}{2} \quad \cos x = 1 \\
x &= 120^\circ \quad 0^\circ \quad 240^\circ
\end{align*}
\]

**Score 4:** The student has a complete and correct response.
38 Solve the equation $\cos 2x = \cos x$ algebraically for all values of $x$ in the interval $0^\circ \leq x < 360^\circ$.

\[
\begin{align*}
\cos 2x &= \cos x \\
2\cos^2 x - 1 &= \cos x \\
2\cos^2 x - \cos x - 1 &= 0 \\
(2\cos x - 1)(\cos x + 1) &= 0 \\
2\cos x - 1 &= 0 \quad \text{or} \quad \cos x + 1 = 0 \\
\cos x &= \frac{1}{2} \quad \text{or} \quad \cos x = -1 \\
x &= 60^\circ \text{ and } 300^\circ \quad \text{or} \quad x = 180^\circ \\
\end{align*}
\]

Score 3: The student made one factoring error.
Question 38

38 Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0^\circ \leq x < 360^\circ \).

\[
\cos 2x = \cos x \\
2 \cos^2 x - 1 = \cos x \\
2 \cos^2 x - \cos x - 1 = 0 \\
(2\cos x + 1)(\cos x - 1) = 0 \\
2\cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0 \\
\cos x = -\frac{1}{2} \quad \text{or} \quad \cos x = 1 \\
\]

\( x = 120^\circ \text{ or } 240^\circ \quad x = 0^\circ \text{ or } 360^\circ \)

**Score 3:** The student stated a value that is not included in the domain.
Question 38

38 Solve the equation $\cos 2x = \cos x$ algebraically for all values of $x$ in the interval $0^\circ \leq x < 360^\circ$.

\[ y = \cos x - \cos 2x \]

\[ x = 0, 120, 240 \]

Score 2: The student used a method other than algebraic.
Question 38

Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0^\circ \leq x < 360^\circ \).

\[
\begin{align*}
\cos 2x &= \cos x \\
-\cos x &= -\cos x \\
\cos x &= 0 \\
x &= \cos^{-1}(0) \\
&= 90^\circ, 270^\circ
\end{align*}
\]

Score 2: The student made a conceptual error by subtracting \( \cos x \).
38 Solve the equation $\cos 2x = \cos x$ algebraically for all values of $x$ in the interval $0^\circ \leq x < 360^\circ$.

\[
\begin{align*}
\sin 2x \cos x - \sin x &= 0 \\
\cos x (2\sin x - 1) &= 0 \\
\cos x &= 0 \\
\sin x &= \frac{1}{2} \\
\theta &= 30^\circ, 150^\circ, 210^\circ, 330^\circ
\end{align*}
\]

Score 1: The student made a conceptual error by using the formula for $\sin 2x$ and did not find all values of $x$. 
38 Solve the equation $\cos 2x = \cos x$ algebraically for all values of $x$ in the interval $0^\circ \leq x < 360^\circ$.

\[
\begin{align*}
1 - 2\sin^2 x &= \cos x \\
\cos^2 x &= \cos x \\
\cos^2 x - \cos x &= 0 \\
\cos x (\cos x - 1) &= 0 \\
\cos x &= 0, \quad \cos x - 1 &= 0 \\
x &= 90^\circ, \quad \cos x = 1 \\
x &= 0^\circ
\end{align*}
\]

\[x = 0^\circ, 90^\circ\]

**Score 1:** The student made a conceptual error when replacing $1 - 2 \sin^2 x$ with $\cos^2 x$ and did not find all the values of $x$. 
38 Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0^\circ \leq x < 360^\circ \).

\[
2 \sin x \cos x = \cos x
\]

\[
\frac{2 \sin x}{\cos x} = 1
\]

\[
\sin x = \frac{1}{2}
\]

\[
x = 30
\]

**Score 0:** The student made conceptual errors by using an incorrect substitution for \( \cos 2x \) and then dividing both sides by \( \cos x \). The student did not find all values of \( x \).
39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 6: The student has a complete and correct response.
39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \equiv ED$, and $GF \equiv FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 6: The student has a complete and correct response.
39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,  
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,  
$BE \cong ED$, and $GF \cong FB$  

Find $m\angle A$ to the nearest tenth.  

Find $BC$ to the nearest tenth.  

$$\frac{10}{\sin 50^\circ} = \frac{12}{\sin C} = 64.8^\circ$$  
$$180 - 64.8 - 50 = 63.2^\circ = m\angle D$$  
$$\frac{x}{\sin 63.2^\circ} = \frac{10}{\sin 50^\circ}$$  
$$x = 11.7$$  
$$BC = 11.7$$  

Score 5: The student made one rounding error in $m\angle A$.  

Question 39
Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,

$m \angle ABG = 40$, $m \angle GBD = 90$, $m \angle C < 90$,

$BE \cong ED$, and $GF \cong FB$

Find $m \angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 5: The student showed appropriate work to find 11.7 and found $BG$ to be 16.
Question 39

39 Given: $DC = 10, AG = 15, BE = 6, FE = 10,$
$m\angle ABG = 40, m\angle GBD = 90, m\angle C < 90,$
$BE \equiv ED, \text{ and } GF \equiv FB$

Find m\angle A to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 4: The student made one conceptual error by assuming $GF$ and $FB$ are congruent to $BE$.
Question 39

39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 4: The student found $\angle C$ correctly, and $BG = 16$ but made a computational error when evaluating $\sin A$. 

Algebra 2/Trigonometry – Jan. ’16  

[81]
Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 3: The student correctly found $\angle A$ and the student found $BG$. 
Question 39

39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,

$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,

$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 3: The student found $m\angle C$ and $BG$, but showed no further work.
Question 39

39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m \angle ABG = 40$, $m \angle GBD = 90$, $m \angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m \angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 3: The student showed appropriate work to find $BC$, but did not round properly.
Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 2: The student found $m\angle C$, but showed no further work.
39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$, $m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

Score 1: The student found $\overline{BG}$, but showed no further work.
Question 39

39 Given: $DC = 10$, $AG = 15$, $BE = 6$, $FE = 10$,
$m\angle ABG = 40$, $m\angle GBD = 90$, $m\angle C < 90$,
$BE \cong ED$, and $GF \cong FB$

Find $m\angle A$ to the nearest tenth.

Find $BC$ to the nearest tenth.

$$x^2 = 12^2 + 10^2 - 2(12)(10)\cos 50$$
$$x^2 = 144 + 100 - 154.249$$
$$x^2 = 89.731$$
$$x = 9.47$$
$$x = 9.5$$

Score 0: The student wrote a completely incorrect response.
39 Given: $DC = 10, AG = 15, BE = 6, FE = 10,$
\[m\angle ABG = 40, m\angle GBD = 90, m\angle C < 90,\]
\[BE \equiv ED, \text{ and } GF \equiv FB\]

\[a^2 = b^2 + c^2 - 2bc \cos A\]
\[a = 10^2 + 12^2 - 2(10)(12) \cos 50\]
\[a = 89.730976\]
\[a = 89.7\]

Find $m\angle A$ to the nearest tenth. $= 89.7$

Find $BC$ to the nearest tenth.

**Score 0:** The student wrote a completely incorrect response.