The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/ TRIGONOMETRY

Friday, June 14, 2013 — 1:15 – 4:15 p.m.

SAMPLE RESPONSE SET

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28 Determine the sum and the product of the roots of the equation $12x^2 + x - 6 = 0$.

$$\frac{12x^2 + x - 6 = 0}{12}$$

$$x^2 + \frac{x}{12} - \frac{1}{2} = 0$$

$$\boxed{\begin{array}{l} \text{sum} = \frac{-1}{12} \\ \text{product} = \frac{-1}{2} \end{array}}$$

Score 2: The student has a complete and correct response.

28 Determine the sum and the product of the roots of the equation $12x^2 + x - 6 = 0$.

$12x^2 + x - 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-1 \pm \sqrt{1^2 - 4(12)(-6)}}{24}$

$\frac{-1 \pm 17}{24}$ $S = \frac{16}{24} + \frac{-18}{24}$

$S = \frac{-2}{24} = \boxed{\frac{-1}{12}}$

$P = \frac{16}{24} \cdot \frac{-18}{24}$

$P = \frac{288}{576} = \boxed{\frac{1}{2}}$

Score 1: The student made a computational error by omitting a negative sign.

28 Determine the sum and the product of the roots of the equation $12x^2 + x - 6 = 0$.

$12x^2 + x - 6 = 0$

$\frac{-(6)}{2(12)}$ $\boxed{\frac{-1}{24} = \text{sum}}$

$\frac{-6}{12}$ $\boxed{-.5 = \text{product}}$

Score 1: The student made a conceptual error by using the expression $-\frac{b}{2a}$ to find the sum of the roots.

28 Determine the sum and the product of the roots of the equation $12x^2 + x - 6 = 0$.

$r = \frac{-b}{a}$ $r = \frac{-c}{12} = +\frac{1}{2}$

$r = \frac{a}{c}$ $r = \frac{12}{-6} = \frac{2}{-1} = -2$

Sum of roots $= \boxed{\frac{1}{2}}$

product of roots $= \boxed{-2}$

Score 0: The student made multiple errors. For the sum of the roots, the student used *c* rather than *b* in the formula and for the product of the roots, the student used an incorrect formula.

29 Solve algebraically for x:

$$\log_{27}(2x - 1) = \frac{4}{3}$$

$$27^{4/3} = 2x-1$$

$$81 = 2x-1$$

$$\frac{82}{2} = \frac{2x}{2}$$

$$\boxed{41 = x}$$

Score 2: The student has a complete and correct response.

29 Solve algebraically for x:

$$\log_{27}(2x - 1) = \frac{4}{3}$$

$\boxed{x = 41}$

81

27

81

80 . 9

$81 = 2x \cancel{-1}$
$+1 \qquad\quad +1$

$\frac{82}{2} = \frac{2x}{2}$

Score 2: The student computed $27^{\frac{4}{3}} = 81$ (without showing that calculation), but completed the solution appropriately.

29 Solve algebraically for x:

$$\log_{27}(2x - 1) = \frac{4}{3}$$

$$\log_{27}(2x-1) = \frac{4}{3}$$

$$(2x-1)^{\frac{4}{3}} = 27$$

$$\left(\sqrt[3]{(2x-1)^4}\right)^3 = (27)^3$$

$$\cancel{\sqrt[4]{(2x-1)^4}} = \sqrt[4]{19683}$$

$$2x-1 = 11.8446617$$

$$+1 \quad +1$$

$$2x = 12.8446617$$

$$\div 2 \quad \div 2$$

$$\boxed{x = 6.42233058}$$

Score 1: The student made a conceptual error in rewriting the log equation as an exponential equation, but solved the resulting equation appropriately.

29 Solve algebraically for x:

$$\log_{27}(2x - 1) = \frac{4}{3}$$

$$\frac{\log(2x-1)}{\log(27)} = \frac{4}{3}$$

$$\frac{2x-1}{27} = \frac{4}{3}$$

$$6x - 3 = 108$$
$$+3 \quad +3$$

$$\frac{6x}{6} = \frac{111}{6}$$

$$x = 1.83$$

$$x = 1.83$$

Score 0: The student began with the change of base formula for the logarithm. The student then made a conceptual error by "dividing out" the "log" and made a computational error in solving for x.

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

3 "s"

3 "t"

2 "i"

1 "a"

1 "c"

$$\frac{10!}{3!\,3!\,2!} = 50400$$

50400 ways

Score 2: The student has a complete and correct response.

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

$$\frac{10!}{3! \cdot 3! \cdot 2!} = \frac{3628800}{72}$$

$$= \boxed{50{,}400 \text{ ways}}$$

Score 2: The student has a complete and correct response.

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

$$\frac{10!}{3!3!2!} = \boxed{7257600}$$

Score 1: The student wrote the correct expression, but made a calculator entry error by computing $\frac{10!}{3!3!2!}$, excluding the parentheses around the denominator.

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

$$\frac{10!}{3!\,3!} = \text{[illegible]}$$

$$\frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = \frac{3628800}{36}$$

$$\boxed{100800}$$

Score 1: The student made a conceptual error by omitting 2! in the denominator.

30 Find the number of possible different 10-letter arrangements using the letters of the word "STATISTICS."

Statistics

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} =$

3628800

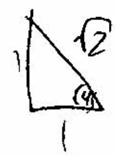
Score 0: The student response is equivalent to 10!.

31 Express the product of cos 30° and sin 45° in simplest radical form.

$\cos 30 = \frac{\sqrt{3}}{2}$ $\quad$ $\sin 45 = \frac{1}{\sqrt{2}}$

$\frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{6}}{4}$



Score 2: The student has a complete and correct response.

31 Express the product of cos 30° and sin 45° in simplest radical form.

$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 45 = \frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

Score 1: The student did not complete the multiplication.

31 Express the product of cos 30° and sin 45° in simplest radical form.

cos30 · sin45 = .6123724357

Score 0: The student failed to express both trigonometric expressions in radical form and only showed the answer as a decimal.

32 Find, algebraically, the measure of the obtuse angle, to the *nearest degree*, that satisfies the equation $5\csc\theta = 8$.

$\csc\Theta = \frac{8}{5} \longrightarrow \sin\Theta = \frac{5}{8}$ $\quad \sin^{-1}(5/8) \doteq 39°$

$180° - 39° = \boxed{141°}$

Score 2: The student has a complete and correct response.

32 Find, algebraically, the measure of the obtuse angle, to the *nearest degree*, that satisfies the equation $5 \csc \theta = 8$.

$$\frac{5\csc\theta}{5} = \frac{8}{5}$$

$$\csc\theta = \frac{8}{5}$$

$$\sin\theta = \frac{5}{8}$$

$$= 38.68218745$$

$$+\ 90$$

$$128.6821875$$

Answer: 129°

Score 1: The student found the correct reference angle, but made a conceptual error in finding the obtuse angle by adding 90° to the reference angle rather than subtracting it from 180°.

32 Find, algebraically, the measure of the obtuse angle, to the *nearest degree*, that satisfies the equation $5 \csc \theta = 8$.

$$\frac{5\csc\theta}{5} = \frac{8}{5}$$

$$\csc\theta = \frac{8}{5}$$

$$\frac{1}{\text{sine}\left(\frac{5}{8}\right)} = 91.67506653$$

$$\boxed{92^\circ}$$

Score 0: The student demonstrated a lack of understanding of the reciprocal function.

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express g(10) in simplest form.

$g(x) = (10a\sqrt{1-10})^2$

$(10a\sqrt{-9})(10a\sqrt{-9})$

$100a^2 \cdot -9$

$-900a^2$

Score 2: The student has a complete and correct response.

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express g(10) in simplest form.

$g(10) = (a10\sqrt{1-10})^2$

$g(10) = (a10\sqrt{-9})^2$

$g(10) = (a10 \cdot 3i)^2$

$g(10) = (10a^2 \cdot 3i)^2$

$g(10) = (100a2 \cdot -9)$

$\boxed{g(10) = (-900a2)}$

$\sqrt{-9}$

$\sqrt{9} \quad \sqrt{-1}$

$3 \quad i$

Score 2: The student has a complete and correct response.

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express g(10) in simplest form.

$g(10) = (a10\sqrt{1-10})^2$

$g(10) = (a10 \cdot 3)^2$

$g(10) = (30a)^2$

$\boxed{g(10) = 900a^2}$

Score 1: The student made a conceptual error in simplifying $\sqrt{1-10}$.

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express g(10) in simplest form.

$$X = \left(10\sqrt{1-10}\right)^2$$

$$-9$$

$$3$$

$$(30i)^2$$

$$X = 900i$$

Score 0: The student dropped the *a* from the problem and made a computational error when squaring $30i$.

33 If $g(x) = \left(ax\sqrt{1-x}\right)^2$, express g(10) in simplest form.

$$g(10) = (a(10)\sqrt{1-10})^2$$

$$g(10) = (10a\sqrt{-9})^2$$

$$g(10) = (10a \cdot 3i)^2$$

$$g(10) = 100a - 9$$

Score 0: The student made two distinct errors on the last step. The student failed to square the *a* and failed to multiply by –9.

34 Express $\dfrac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

$$\frac{\cot x \sin x}{\sec x} = \cos^2 x$$

$$\frac{\dfrac{\cos x}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{1}}{\dfrac{1}{\cos x}}$$

$$\frac{\cos x}{1} \cdot \frac{\cos x}{1}$$

$$\cos^2 x$$

Score 2: The student has a complete and correct response.

34 Express $\dfrac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

$$\cot x = \frac{1}{\tan x} \quad \text{OR} \quad \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1}$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{\cancel{\cos x}}{\cancel{1}} \cdot \frac{\cancel{1}}{\cancel{\cos x}} = \boxed{1}$$

Score 1: The student simplified the numerator correctly, but made a conceptual error by multiplying by $\dfrac{1}{\cos x}$.

34 Express $\dfrac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

$$\frac{\dfrac{1}{\tan(x)} \cdot \dfrac{\sin(x)}{1}}{\dfrac{1}{\cos(x)}}$$

$$\frac{\sin(x)}{\text{tax}(x)} \cdot \frac{\cos(x)}{1}$$

Score 0: The student made two simplification errors. The numerator of the expression $\dfrac{\sin x}{\tan x}$ was not simplified and the final product was not simplified.

34 Express $\dfrac{\cot x \sin x}{\sec x}$ as a single trigonometric function, in simplest form, for all values of x for which it is defined.

$$\frac{\frac{\sin}{\cos}x \ \sin x}{\frac{1}{\cos x}} \qquad \frac{\cos x}{\cos x} = x$$

Score 0: The student made an incorrect substitution for $\cot x$ and made a conceptual error simplifying the expression.

35 On a multiple-choice test, Abby randomly guesses on all seven questions. Each question has four choices. Find the probability, to the *nearest thousandth*, that Abby gets *exactly* three questions correct.

$n=7$
$r=3$
$p=1/4$
$q=3/4$

$\underbrace{{}_7C_3}_{35}(1/4)^3(3/4)^4$

$35(1/64)(81/256)$

$\frac{2835}{16384}$

$.173034668$

$\boxed{.173}$

Score 2: The student has a complete and correct response.

35 On a multiple-choice test, Abby randomly guesses on all seven questions. Each question has four choices. Find the probability, to the *nearest thousandth*, that Abby gets *exactly* three questions correct.

$n = 7$

$r = 3$

$p = 3/7$

$q = 4/7$

${}_7C_3(3/7)^3(4/7)^4$

$.293755153$

$.294$

Score 1: The student made a conceptual error in finding the probability of success on a single trial, but then found an appropriate probability.

35 On a multiple-choice test, Abby randomly guesses on all seven questions. Each question has four choices. Find the probability, to the *nearest thousandth*, that Abby gets *exactly* three questions correct.

$n = 7$

$r = 3$

$p = 12/28$

$q = 16/28$

$${}_7C_3\left(12/28\right)^{7-3}\left(16/28\right)^3$$

$= .220$

Score 0: The student made a conceptual error in finding the value of p and made a second conceptual error by reversing the exponents.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$-x^2 + 10x - 13 = 0$$

$$x^2 - 10x + 13 = 0$$

$$\frac{10 \pm \sqrt{100 - 4(1)(13)}}{2(1)}$$

$$\frac{10 \pm \sqrt{100 - 52}}{2}$$

$$\frac{10 \pm \sqrt{48}}{2}$$

$$\frac{10 \pm \sqrt{16 \cdot 3}}{2}$$

$$\frac{\cancel{10}^{5} \pm \cancel{4}^{2}\sqrt{3}}{\cancel{2}}$$

$$5 \pm 2\sqrt{3}$$

Score 4: The student has a complete and correct response.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$13 = 10x - x^2$$

$$-x^2 + 10x = 13$$

$$\cancel{-x^2 + 10x + 25 = 38}$$

$$x^2 - 10x = -13$$

$$x^2 - 10x + 25 = 12$$

$$(x-5)(x-5) = 12$$

$$(x-5)^2 = 12$$

$$x - 5 = \pm\sqrt{12}$$

$$x = \pm\sqrt{12} + 5$$

Score 3: The student did not simplify $\sqrt{12}$.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$\frac{13}{x} = \frac{10-x}{1}\left(\frac{x}{x}\right)$$

$$\cancel{\frac{13}{x} = 10x - x^2}$$

$$\frac{13x}{x} = \frac{10-x}{1}$$

$$13 = 10x - x^2$$
$$-13 \qquad -13$$

$$-x^2 + 10x - 13$$

$$x^2 - 10x + 13$$

$a = 1$
$b = -10$
$c = 13$

$$\frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

$$\frac{10 \pm \sqrt{(-10)^2 - 4(1)(13)}}{2(1)}$$

$$\frac{10 \pm \sqrt{48}}{2}$$

$$\frac{10 \pm \sqrt{48}}{2}$$

$5 \pm$

$$\frac{10 \pm \sqrt{48}}{2}$$

Score 3: The student wrote and solved a correct quadratic equation, but the radical was not simplified.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$13 = 10 - x^2$

$x^2 = -3$

$x = \pm\sqrt{-3}$

$x = \pm i\sqrt{3}$

Score 2: The student made a conceptual error when simplifying the proportion.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$13 = 10x - x^2$$

$$-13 = -10x + x^2$$

$$x^2 - 10x + 25 = -13 + 25$$

$$(x-5)^2 = -12$$

$$\sqrt{(x-5)^2} = \sqrt{-12}$$

$$x - 5 = \pm\sqrt{-12}$$

$$x = 5 \pm \sqrt{-12}$$

Score 2: The student made a computational error when adding -13 and 25 and did not simplify the radical.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$13 = 10x - x^2$

$-10x \quad -10x$

$3x = x^2$

$0 = x^2 - 3x$

$0 = (x-3)x$

$x = 3 \quad x = 0$

Score 1: The student made a conceptual error combining 13 and $-10x$. The student also dropped the negative sign in front of x^2, but an appropriate solution was found.

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$\frac{13}{x} = \frac{10-x}{1}$$

$$10x - x^2 = 13$$
$$-13$$

$$10x - x^2 - 13$$

$$x^2 - 10x + 13 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{10 \pm \sqrt{10 + 52}}{2} \qquad \frac{\overset{5}{\cancel{10}} \pm \sqrt{62}}{\cancel{2}}$$

$$x = 5 \pm \sqrt{62}$$

Score 0: The student made two computational errors (did not square 10 and added 52) and one conceptual error (did not divide the entire numerator by 2).

36 Solve the equation below algebraically, and express the result in simplest radical form:

$$\frac{13}{x} = 10 - x$$

$$\frac{13}{x} = \frac{10-x}{1}$$

$$10x - x^2 = 13$$

$$-x^2 + 10x - 13$$

$$-x^2 + 10x = 13$$

$$-x^2 + 10x + 25 = 13$$

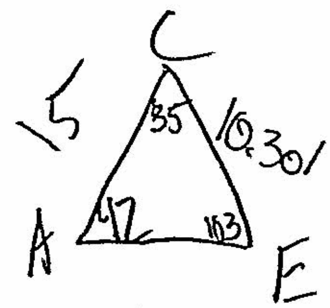
$$(-x + 5)(-x + 5)$$

$$-x = -5 \quad -x = 5$$

$$x = 5 \quad x = 5 \quad x = -13$$

Score 0: The student made two conceptual errors when attempting to complete the square and when solving the resulting equation.

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.



$$\frac{15}{\sin 103} = \frac{x}{\sin 42}$$

$$\frac{x\sin 103}{\sin 103} = \frac{10.0370}{\sin 103}$$

$$x = 10.3000$$

$$K = \frac{1}{2}ab\sin C$$

$$K = \frac{1}{2}(15)(10.30)(\sin 35)$$

$$K = \frac{1}{2}(154.515)(.5736)$$

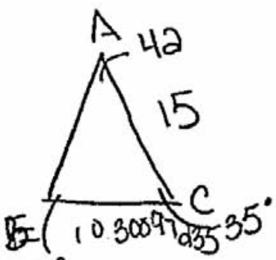
$$K = 77.2575(.5736)$$

$$K = 44.3149$$

$$K = 44$$

Score 4: The student has a complete and correct response.

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.



$$\begin{array}{r} 103 \\ +\ 42 \\ \hline 145 \end{array} \qquad \begin{array}{r} 180 \\ -\ 145 \\ \hline 35^\circ \end{array} \qquad 163$$

$$\frac{a}{\sin A} = \frac{e}{\sin E}$$

$$\frac{a}{\sin 42} = \frac{15}{\sin 103}$$

$$\frac{a\sin 103}{\sin 103} = \frac{15\sin 42}{\sin 103}$$

$$a = 10.30097235\ldots..$$

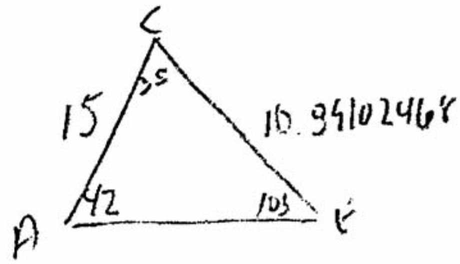
$$A = \tfrac{1}{2}ab\sin C$$

$$A = \tfrac{1}{2}(10.30097235)(15)\sin 35$$

$$A = 44^{2}ft$$

Score 3: The student labeled the answer with the wrong units.

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.



$$\frac{a}{\sin A} = \frac{e}{\sin E}$$

$$\frac{x}{\sin 42} = \frac{15}{\sin 105}$$

$$x(\sin 105) = 15(\sin 42)$$

$$x = 10.39102468$$

$$K = \frac{1}{2} cb \sin C$$

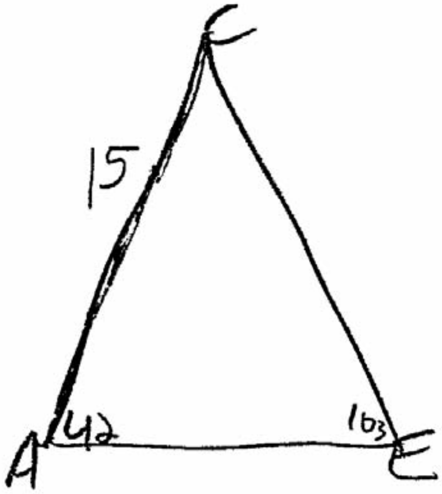
$$K = \frac{1}{2}(15)(10.39102468) \sin 35$$

$$K = 77.93268511(\sin 35)$$

$$K = 44.70035179$$

Score 2: The student wrote the wrong measure for the obtuse angle and the student did not round the final answer to the nearest square mile.

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.



$$\frac{15}{\sin 103} = \frac{X}{\sin 42}$$

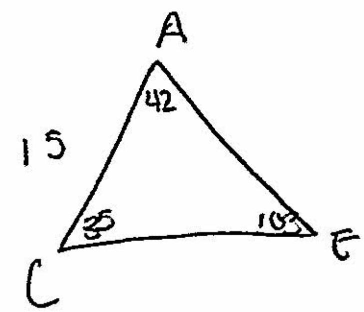
$$\frac{15(\sin 42)}{\sin 103} = \frac{(\sin 103)x}{\sin 103}$$

$X = 10.30097235^{2}$ mi.

$X \approx 10 \text{ miles}^2$

Score 1: The student made a correct substitution into the Law of Sines, but made an error in labeling the length of $\overline{CE}$ as miles^2.

37 A ranch in the Australian Outback is shaped like triangle ACE, with $m\angle A = 42$, $m\angle E = 103$, and $AC = 15$ miles. Find the area of the ranch, to the *nearest square mile*.



$K = \frac{1}{2}ab \sin C$

$K = \frac{1}{2}(42)(103)\sin(35)$

1240.65 sq mi.

$$\begin{array}{r} 42 \\ 103 \\ \hline 145 \end{array}$$

$$\begin{array}{r} 180 \\ -145 \\ \hline \end{array}$$

Score 0: The student incorrectly used the area of a triangle formula by substituting the measures of the angles for *a* and *b* instead of the lengths of the sides.

38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

32.32 38.2 44.08

$\bar{X}$

−1 +1

5.88 SD

6 scores

For these data, what is the interquartile range?

$41 - 37 = 4$

$(Q_3) - (Q_1) = IQR$

Score 4: The student has a complete and correct response.

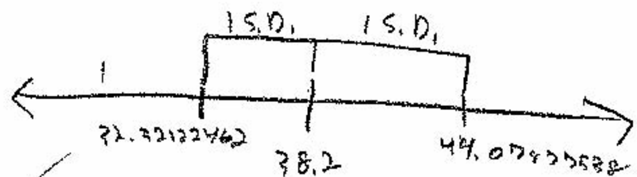
38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

1-Var stats L_1

$\bar{x} = 38.2$

$\sigma x = 5.878775383$



$\cancel{29}, \cancel{28}, 39, 37, \cancel{45}, 40, 41, 38, 37, \cancel{48}$

$\boxed{6 \text{ scores}}$

For these data, what is the interquartile range?

$Q_3 = 41$

$Q_1 = 37$

$$\begin{array}{r} 41 \\ +37 \\ \hline 78 \end{array} \qquad 78 \div 2 = 39$$

Interquartile range: $\boxed{39}$

Score 3: The student found Q_1 and Q_3, but instead of subtracting those values, they were averaged.

38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

$\bar{X} = 38.2$

$\sigma x = 5.8788$

$+\ 5.8788$

11.76

15

$+19.1$

$34.1 + 34.1 = 68.2$

11

For these data, what is the interquartile range?

$Q1 = 37 \quad Q3 = 41$

Score 2: The student correctly found the mean, population standard deviation, Q_1, and Q_3, but did not answer either part of the question.

38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

$\bar{x} = 38.2$

$\sigma_x = 5.9$

For these data, what is the interquartile range?

$Q_3 - Q_1$

Score 1: The student correctly found the mean and population standard deviation, but did not find the number of scores or interquartile range.

38 Ten teams competed in a cheerleading competition at a local high school. Their scores were 29, 28, 39, 37, 45, 40, 41, 38, 37, and 48.

How many scores are within one population standard deviation from the mean?

6 scores

For these data, what is the interquartile range?

37-41

Score 0: The student did not show work for the first part of the question and did not find the interquartile range.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$x^4 + 4x^3 + 4x^2 + 16x = 0$

$x(x^3 + 4x^2 + 4x + 16) = 0$

$x(x^2(x+4) + 4(x+4))$

$x(x^2+4)(x+4)$

$x = 0 \quad x^2 = -4 \qquad x = -4$

$x = \sqrt{-4}$

$x = \pm 2i$

$x = 0$

$x = 2i$

$x = -2i$

$x = -4$

Score 6: The student has a complete and correct response.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$(x^4 + 4x^3) + (4x^2 + 16x) = 0$$

$$x^3(x+4) + 4x(x+4) = 0$$

$$(x^3 + 4x)(x+4) = 0$$

$$x(x^2 - 4)\ (x+4) = 0$$

$x = 0$ $\qquad$ $x = -4$

$(x+2)(x-2)$

$x = -2$ $\quad$ $x = 2$

Score 5: The student factored $x^3 + 4x$ incorrectly, but appropriate solutions were found.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$(x^4 + 4x^3) + (4x^2 + 16x) = 0$$

$$x^3(x+4) + 4x(x+4) = 0$$

$$(x^3 + 4x)(x+4) = 0$$

$x^3 + 4x = 0$ or $x + 4 = 0$

$x(x^2+4) = 0$ $\quad$ $x = -4$

$x = 0$ or $x^2 + 4 = 0$

$x^2 = -4$

no solution

$$\boxed{x = 0 \text{ or } x = -4}$$

Score 4: The student found two correct solutions, but did not solve $x^2 + 4 = 0$ correctly.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$+16x \quad +16x$

$(x^4 + 4x^3)(+4x^2 + 16x) = 0$

$x^3(x + 4) + 4x(x + 4) = 0$

$(x^3 + 4x)(x + 4) = 0$

$x^3 + 4x = 0$

$-4x \quad -4x$

$\frac{x^3}{x} = \frac{-4x}{x}$

$\sqrt{x^2} = \sqrt{-4}$

$x =$

reject

$x + 4 = 0$

$-4 \quad -4$

$x = -4$

Score 3: The student found $(x^3 + 4x)(x + 4) = 0$. The student did not find two solutions.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$x^4 + 4x^3 + 4x^2 + 16x = 0$

$x^3(x+4) + 4x(x+4)$

$\boxed{(x^3+4x)(x+4) = 0}$

Score 3: The student found $(x^3 + 4x)(x + 4) = 0$, but no further work was shown.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$\frac{x^4+4x^3+4x^2+16x=0}{x}$$

$$x(\underbrace{x^3+4x^2}+\underbrace{4x+16})=0$$

$$x[x^2(x+4)+4(x+4)]=0$$

Score 2: The student factored out a greatest common factor of x correctly. The student grouped correctly and factored out a greatest common factor from each binomial.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$x(x^3 + 4x^2 + 4x + 16) = 0$$

Score 1: The student factored out the common factor of x, but did not do any other work.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16\cancel{x}$$

$$+16x \quad \cancel{+16x}$$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

Score 0: The student set the equation equal to zero, but no further work is shown.

39 Solve algebraically for all values of x:

$$x^4 + 4x^3 + 4x^2 = -16x$$

$x^4+4x^3+4x^2+16x=0$

$x^2(x^2+4x+4+16)=0$

$x^2(x^2+4x+20)=0$

~~$x^2(x \quad)(x \quad)=0$~~

$x=\frac{-4\pm\sqrt{(4)^2-4(1)(20)}}{2}$

$\frac{-4\pm\sqrt{16-80}}{2}$

$\frac{-4\pm\sqrt{-64}}{2}$

$\frac{-4\pm 8}{2}$

$x=\frac{-4+8}{2}$

$x=2$

$x=\frac{-4-8}{2}$

$x=-6$

Score 0: The student set the equation equal to zero, but made a conceptual error in factoring out a common factor. The student made a second conceptual error in solving $x^2 + 4x + 20 = 0$.