25 Express \((1 - i)^3\) in \(a + bi\) form.

\[
\begin{align*}
(1 - i)(1 - i)(1 - i) \\
(1 - i - i + i^2)(1 - i) \\
(1 - 2i + 1)(1 - i) \\
(-2i)(1 - i) \\
-2i + 2i^2 \\
-2i - 2 \\
-2 - 2i
\end{align*}
\]

Score 2: The student gave a complete and correct response.
25 Express \((1 - i)^3\) in \(a + bi\) form.

\[
\begin{align*}
i^1 &= -i \\
i^2 &= -1 \\
i^3 &= -i \\
i^4 &= 1
\end{align*}
\]

\(-2 - 2i\)

Score 2: The student gave a complete and correct response.
25 Express \((1 - i)^3\) in \(a + bi\) form.

\[
(1 - i) (1 - i)^2
\]

\[
(1 - i) (1 - 2i + i^2)
\]

\[
(1 - i)(-2i)
\]

\[
-2i - 2
\]

**Score 2:** The student gave a complete and correct response.
25 Express \((1 - i)^3\) in \(a + bi\) form.

\[
(1 - i)^3 = (1 - i)(1 - i)(1 - i)
\]

\[
= (1 - i - i + i^2)(1 - i)
\]

\[
= (1 - 2i - i^2)(1 - i)
\]

\[
= (1 + 2i + 1)(1 - i)
\]

\[
= 2(1 - i)
\]

\[
= 2 - 2i
\]

Score 1: The student did not simplify powers of \(i\).
25 Express $(1 - i)^3$ in $a + bi$ form.

\[
\begin{align*}
3^0 \cdot (1)^3 \cdot (-i)^0 &= 1 + \\
3^1 \cdot (1)^2 \cdot (-i)^1 &= -3i + \\
3^2 \cdot (1)^1 \cdot (-i)^2 &= 3 + \\
3^3 \cdot (1)^0 \cdot (-i)^3 &= i
\end{align*}
\]

\[\sqrt{4} = 2i \pm i\]

**Score 1:** The student made one computational error.
25 Express \((1 - i)^3\) in \(a + bi\) form.

Score 0: The student gave a completely incorrect response.
25 Express \((1 - i)^3\) in \(a + bi\) form.

\[
(1 - i)^2 (1 - i) \\
(1 + i^2) (1 - i) \\
1 - i + i^2 - i^3 \\
-2i
\]

**Score 0:** The student made multiple errors.
26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

The population is the truckload of oranges. 

The sample are the oranges in the pails.

State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

It's pretty likely that most of the oranges are satisfactory.

Score 2: The student gave a complete and correct response.
26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

Sample = 3 pails of 50 oranges
Population = truckload

State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

Some of the population may also be unsatisfactory.

Score 2: The student gave a complete and correct response.
An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

Sample: 50 oranges  
Population: 3 pails

State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

It is likely that approximately 5% of all the oranges are unsatisfactory.

Score 1: The student only stated a correct conclusion.
An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

The population is 50 oranges and the sample is 3 pails.

State one conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

95% of the population is satisfactory.

Score 0: The student only identified the sample correctly.
27 Using the unit circle below, explain why $\csc \theta = \frac{1}{y}$.

$csc \theta$ is equal to $\frac{1}{\sin \theta}$, and $\sin \theta$ on a unit circle (with radius = 1) is equal to the y value of the point. So, if $\sin \theta = y$ and $\csc \theta = \frac{1}{\sin \theta}$ it is also true to say $\csc \theta = \frac{1}{y}$.

**Score 2:** The student gave a complete and correct response.
27 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).

\[
\csc \theta = \frac{h}{o}
\]

\[
csc \theta = \frac{1}{y} \text{ because on the unit circle the hypotenuse is always 1 and } y \text{ is the opposite leg.}
\]

\[
csc \theta = \frac{\text{hyp}}{\text{opp}}.
\]

**Score 2:** The student gave a complete and correct response.
27 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).

\[ \csc = \frac{1}{\sin} \]

\[ \sin \theta = \frac{y}{1} = y \]

\[ \csc = \frac{1}{\sin} = \frac{1}{y} \]
27 Using the unit circle below, explain why \( \csc \theta = \frac{1}{y} \).

(I tried)

Score 0: The student showed no appropriate work and did not write an explanation.
Question 28

28 The function $M(t)$ represents the mass of radium over time, $t$, in years.

$$M(t) = 100e^{\frac{\ln(\frac{1}{2})}{1590}}$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

Score 2: The student gave a complete and correct response.
28 The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{(\ln 2)t}{1590}}
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

Decay

\[ \ln 0.5 \approx -0.6931 \]

When \( t \geq 0 \), \( \frac{(\ln 0.5)t}{1590} \) will be negative.

Therefore, the exponent will be negative, representing exponential decay

Score 2: The student gave a complete and correct response.
28 The function $M(t)$ represents the mass of radium over time, $t$, in years.

$$M(t) = 100e^{\frac{(\ln 0.5)t}{1590}}$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

Decay because the $\frac{1}{2}$ signifies that it is decay, not growth.

**Score 1:** The student gave an incomplete explanation.
28 The function \( M(t) \) represents the mass of radium over time, \( t \), in years.

\[
M(t) = 100e^{\frac{ln\frac{1}{2}}{1590}} \leq t \leq 1590
\]

Determine if the function \( M(t) \) represents growth or decay. Explain your reasoning.

\[
100e^{\frac{ln\frac{1}{2}}{1590}} = 99.91
\]

\[
100e^{\frac{ln\frac{1}{2}}{1540}} = 94.738
\]

\text{decay}

**Score 1:** The student showed appropriate work, but did not write an explanation.
28 The function $M(t)$ represents the mass of radium over time, $t$, in years.

$$M(t) = 100e^{\frac{t-1590}{1590}}$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

Decay!

Score 0: The student did not write an explanation.
28 The function $M(t)$ represents the mass of radium over time, $t$, in years.

$$M(t) = 100e^{\frac{(\ln 2)t}{1590}}$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

$M(t) = 100e^{\left(\frac{\ln 2}{1590}\right)t}$, decay of .05

Score 0: The student showed no appropriate work and did not write an explanation.
Question 29

29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and $-\frac{2}{3}$.

Score 2: The student gave a correct sketch.
Question 29

29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and $-2$.

Score 2: The student gave a correct sketch.
29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and −2.

Score 1: The student made one graphing error.
29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and −2.

Score 1: The student produced an insufficient sketch.
On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

Score 0: The student did not provide a sketch.
30 Given the equal terms $\sqrt[3]{x^5}$ and $y^\frac{5}{6}$, determine and state $y$, in terms of $x$.

\[3 \sqrt[3]{x^5} = y^{\frac{5}{6}}\]
\[(x^{\frac{5}{3}})^{\frac{6}{5}} = (y^{\frac{5}{6}})^{\frac{6}{5}}\]
\[x^{\frac{30}{15}} = y\]
\[x^2 = y\]

Score 2: The student gave a complete and correct response.
30 Given the equal terms $\sqrt[3]{x^5}$ and $y^{\frac{5}{6}}$, determine and state $y$, in terms of $x$.

$$
\sqrt[3]{x^5} = y^{\frac{5}{6}} \\
\Rightarrow x^{\frac{5}{3}} = y^{\frac{5}{6}} \\
\Rightarrow y = x^{\frac{15}{5}} = x^3
$$

Score 1: The student made a transcription error.
30 Given the equal terms $\sqrt[3]{x^5}$ and $y^{\frac{5}{6}}$, determine and state $y$, in terms of $x$.

\[3\sqrt[3]{x^5} = y^{\frac{5}{6}}\]

\[x = y^{\frac{5}{6}}\]

\[y = (x^{\frac{3}{5}})^{\frac{6}{5}}\]

**Score 1:** The student interchanged the root and power.
30 Given the equal terms $\sqrt[3]{x^5}$ and $y^\frac{5}{6}$, determine and state $y$, in terms of $x$.

\[ x' \leftarrow \text{only goes into } 5 \text{val} \]

\[ x\sqrt{x^2} \rightarrow x \cdot x \rightarrow x^2 \]

**Score 0:** The student used an incorrect procedure to get $x^2$. 
31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

<table>
<thead>
<tr>
<th>Comedy Series</th>
<th>Drama Series</th>
<th>Reality Series</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td>95</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>Females</td>
<td>80</td>
<td>70</td>
<td>110</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>135</td>
<td>180</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

Two events are independent if

\[
P(E \text{ and } M) = P(E) \cdot P(M)
\]

\[
\frac{70}{490} = \frac{180}{490} \cdot \frac{230}{490}
\]

\[
\frac{70}{490} = \frac{1400}{240100}
\]

\[
\frac{1}{7} \neq \frac{414}{2401}
\]

No, because

\[
P(E \text{ and } M) \neq P(E) \cdot P(M)
\]

Score 2: The student gave a complete and correct response.
The results of a survey of the student body at Central High School about television viewing preferences are shown below.

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</tr>
</tbody>
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Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

\[
P(m|R) \neq P(m) \]

\[
\frac{70}{180} \neq \frac{230}{490}
\]

\[
\approx 0.3888... \neq 0.4693...
\]

The events are not independent because \( P(m|R) \neq P(m) \)

Score 2: The student gave a complete and correct response.
31. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

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<td>180</td>
<td>490</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

\[
\frac{230}{490} = .47 \\
\frac{70}{230} = .30
\]

No because they have different probabilities.

Score 1: The student found one of the probabilities incorrectly in comparison.
The results of a survey of the student body at Central High School about television viewing preferences are shown below.

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</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

\[
\begin{align*}
\frac{P(m) \cdot P(R)}{P(m \text{ and } R)} &= \frac{\frac{230}{490} \cdot \frac{180}{490}}{\frac{70}{490}} \\
&= \frac{411600}{240100} \neq \frac{70}{490}
\end{align*}
\]

**Score 1:** The student gave a correct justification, but did not state ‘no’.
31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

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<td>135</td>
<td>490</td>
</tr>
</tbody>
</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

\[
\frac{230}{490} = 0.46, \quad \frac{180}{490} = 0.36
\]

different probabilities.

**Score 0:** The student found one of the probabilities incorrectly in comparison and did not state “no”.
31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

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</tr>
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</table>

Are the events “student is a male” and “student prefers reality series” independent of each other? Justify your answer.

No, because the amount of male students who prefer reality shows is dependent on the amount of total males there are.

**Score 0:** The student compared incorrect probabilities and gave an incorrect justification.
32 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

\[
\begin{array}{c|cc}
\multicolumn{1}{r}{x-2} & 3x^2 + 7x - 20 \\
\hline
& -3x^2 + 6x \\
& \\
& 13x - 20 \\
& -13x + 26 \\
& 6
\end{array}
\]

\( (3x+13) + \frac{6}{x-2} \)

Score 2: The student gave a complete and correct response.
32 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

\[
\begin{array}{c|cccc}
2 & 3 & 7 & -20 \\
   &   & 6 & 24 \\
\hline
   & 3 & 13 & 6
\end{array}
\]

Answer:

\[
3x + 13 + \frac{6}{x-2}
\]

Score 2: The student gave a complete and correct response.
Question 32

32 Given \(f(x) = 3x^2 + 7x - 20\) and \(g(x) = x - 2\), state the quotient and remainder of \(\frac{f(x)}{g(x)}\), in the form \(q(x) + \frac{r(x)}{g(x)}\).

\[
\begin{array}{c}
\underline{3x^2+7x-20} \div (x-2) \\
\underline{3x^2-6x} \) \\
\underline{-} \\
\underline{7x-22} \\
\underline{22} \\
\end{array}
\]

\[
(3x+1) \div \frac{-22}{x-2}
\]

Score 1: The student used \(-2\) instead of 2.
32 Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

Score 1: The student did not give the answer in the required form.
Question 32

32 Given \( f(x) = 3x^2 + 7x - 20 \) and \( g(x) = x - 2 \), state the quotient and remainder of \( \frac{f(x)}{g(x)} \), in the form \( q(x) + \frac{r(x)}{g(x)} \).

\[
\begin{array}{c|c|c}
  & x-2 & \\
\hline
5 & 3 & 5 \\
3 & 15 & 17 \\
2 & 30 & \underline{30} \\
1 & 50 & \underline{50} \\
\hline
& 3x^2+7x-20 & (3x-5)(x+4) \\
\end{array}
\]

\[
\frac{3x^2+7x-20}{x-2} = (3x-5)(x+4)
\]

\[
3x(x+4) - 5(x+4)
\]

Score 0: The student did not use a correct procedure to find a quotient that has a remainder.
33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

$$= 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$$

$$= 2x^3 + hx^2 - 8x^2 + 3x - 4hx + 12 + k$$

$$-8x^2 + hx^2 = -10x^2$$

$$x(-8h) = x^2(-10)$$

$$-8h = -10$$

$$h = 2$$

$$3x - 4hx = 11x$$

$$x(3 - 4h) = x(11)$$

$$3 - 4h = 11$$

$$-4h = 8$$

$$h = -2$$

$$-12 + k = -7$$

$$k = 5$$

Score 4: The student gave a complete and correct response.
Question 33

33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

\[
\begin{align*}
2x^3 - 10x^2 + 11x - 7 &= (x - 4)(2x^2 + hx + 3) + k \\
2x^3 - 10x^2 + 11x - 7 &= \underline{2x^3 - 8x^2} - 3x + 12 \\
2x^3 - 10x^2 + 11x - 7 &= \underline{h}x^2 - 4hx + k \\
-2x^2 + 8x + 5 &= h - 4h + k
\end{align*}
\]

\[
\begin{align*}
h &= -3 \\
k &= 5
\end{align*}
\]

Score 4: The student gave a complete and correct response.
Question 33

33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

\[
2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k
\]

\[
\begin{align*}
2x^3 & - 4x^2 + 11x - 7 = 2x^3 + hx^2 + 3x + 6x^2 + 3hx + 12 + k \\
2x^3 & - 4x^2 + 11x - 7 = 2x^3 + hx^2 + 3x + 6x^2 + 3hx + 12 + k \\
-2x^2 & + 8x + 10 = hx^2 - 4hx + k \\
-2(x^2 - 4x - 5) & = hx^2 - 4hx + k
\end{align*}
\]

\[
\begin{align*}
h & = -2 \\
k & = 10
\end{align*}
\]

Score 3: The student made one computational error.
33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

\[
\begin{align*}
2x^3 - 10x^2 &+ 11x - 7 \\
(x - 5) &\quad (x - 4) \\
(x - 4)(2x^2 + hx + 3) &+ h \\
2x^3 &+ h x^2 + 3x - 4hx - 12 + h \\
2x^3 &+ (h - 8)x^2 - hx - 12 + h \\
\hline
& h = 18 \\
& h = 5
\end{align*}
\]

Score 2: The student only found the correct value for $k$. 
33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

\[
2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \\
= 2x^3 + hx^2 - 8x^2 + 3x - 4hx - 12 + k
\]

**Score 1:** The student distributed correctly.
33 Algebraically determine the values of $h$ and $k$ to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx + 12 + k$$

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 - 8hx^2 + hx + 12 + k$$

**Score 0:** The student did not show enough correct work to receive any credit.
Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

**Score 4:** The student gave a complete and correct response.
Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian’s plan follows an arithmetic pattern because from the graph it is visible that one mile extra is added each week.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

\[ a_1 = 10 \]
\[ a_{n-1} + 1 = a_n \]

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

\[ a_n = 13 + (n-1) \cdot 1 \]

Score 3: The student did not express the explicit formula in simplest form.
34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian’s, because one mile is added each week

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

\[ t_1 = 10 \]
\[ t_n = t_{n-1} + 1 \]

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

\[ t_n = 13 + n - 1 \]

Score 3:  The student gave an incorrect recursive definition.
34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian because she increased the distance run by 1 mile each week.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

\[ a_1 = 10 \]
\[ a_n = a_{n-1} + 1 \]

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.
Question 34

34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

Score 1: The student gave an incorrect explanation and recursive definition, and did not simplify the explicit definition.
Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian’s plan follows an arithmetic pattern because the common difference is 2.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

\[ a_n = a_{n-1} + 1 \]

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

\[ a_n = \]

Score 1: The student did not state a recursive or explicit definition correctly.
34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian’s 12-week plan and Josh’s 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Josh’s plan has a pattern of dropping one mile after going up n+1 each time.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

Jillian’s plan: \( y = x + a \)

Jillian’s plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in simplest form, to represent the number of miles run each week for the full-marathon training plan.

\[ y = x + 12 \]

Score 0: The student made multiple errors.
35 The guidance department has reported that of the senior class, 2.3% are members of key club, \( K \), 8.6% are enrolled in AP Physics, \( P \), and 1.9% are in both.

Determine the probability of \( P \) given \( K \), to the nearest tenth of a percent.

\[
\Pr(P|K) = \frac{\Pr(P \cap K)}{\Pr(K)} = \frac{1.9}{2.3} \approx 82.6\%
\]

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

If we choose a student who is a member of key club, they have an 82.6% probability of being in AP Physics.

Score 4: The student gave a complete and correct response.
35 The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both.

Determine the probability of $P$ given $K$, to the nearest tenth of a percent.

\[
\frac{1.9}{2.3} \approx 82.6\% 
\]

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

Score 3: The student did not provide a statement.
The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both.

Determine the probability of $P$ given $K$, to the nearest tenth of a percent.

$$P(K | P) = \frac{P(K \cap P)}{P(P)} = \frac{0.019}{0.086} = 0.2209$$

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

The probability that a student is a member of the key club given that the student is enrolled in AP Physics.

**Score 2:** The student found 22.1% and wrote an appropriate statement.
The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both.

Determine the probability of $P$ given $K$, to the nearest tenth of a percent.

\[
\frac{0.023 \times 100}{0.086} = 0.19
\]

\[
X = 22.1\%
\]

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

If a student is enrolled in key club, then there is a 19% that he will also be enrolled in AP physics.

**Score 1:** The student made a conceptual error and did not base the statement on the calculation.
The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both.

Determine the probability of $P$ given $K$, to the nearest tenth of a percent.

$\left(\frac{P}{K}\right) = \left(\frac{2.3}{8.6}\right) = 0.278$,

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

Of the students who are taking AP physics and are members of the key club, 19.7% of them will be enrolled in both.

Score 0: The student made multiple conceptual errors.
Question 35

35 The guidance department has reported that of the senior class, 2.3% are members of key club, $K$, 8.6% are enrolled in AP Physics, $P$, and 1.9% are in both.

Determine the probability of $P$ given $K$, to the nearest tenth of a percent.

\[
\begin{align*}
2.3\% &= K \\
8.6\% &= P \\
1.9\% &= B \\
\frac{12.8}{128} &= \frac{2.3}{100} \approx 18.9\%
\end{align*}
\]

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

18.9% of the seniors are members of key club.

Score 0: The student made multiple errors.
**Question 36**

36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

\[
20000 = PMT \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right)
\]

\[
PMT = 400.7589719
\]

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

\[
21,000 - x = 300 \left[ \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right]
\]

\[
-x = -6028.407545
\]

\[
x = 6028
\]

**Score 4:** The student gave a complete and correct response.
Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = \frac{PMT \left(1 - (1 + i)^{-n}\right)}{i} \]

- \( P_n \): present amount borrowed
- \( n \): number of monthly pay periods
- \( PMT \): monthly payment
- \( i \): interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

Score 3: The student did not show work to find $6028.
Question 36

36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

\[ \begin{align*}
20000 &= PMT \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \\
20000 &= PMT \left( 49.90530818 \right) \\
\text{PMT} &= 400.76
\end{align*} \]

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

Score 2: The student found $400.76 correctly.
Question 36

36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

\[ 20000 = x \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \]

\[ x = \frac{20000}{49.9053} \]

\[ x = 400.8 \]

\[ i = 0.00625 \]
\[ PMT = x \]
\[ n = 60 \text{ months} \]
\[ P_n = 20000 \]

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

\[ n = 60 \]
\[ PMT = 300 \]
\[ i = 0.00625 \]

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

\[ P_n = 300 \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right) \]

\[ P_n \approx 149.72 \]

Score 2: The student made a rounding error and did not subtract from $21,000.
Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right) \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

\[ x = \frac{300 \left( 1 - (1 + 0.00625)^{-60} \right)}{0.00625} \]

Score 1: The student did not take off the original down payment and showed no further correct work.
Question 36

36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the nearest cent.

\[ P_n = \frac{PMT \left( \frac{1 - (1 + i)^{-n}}{i} \right)}{100} \]

- \( P_n \) = present amount borrowed
- \( n \) = number of monthly pay periods
- \( PMT \) = monthly payment
- \( i \) = interest rate per month

\[
21,000 = PMT \left( \frac{1 - (1 + 0.00625)^{-60}}{0.00625} \right)
\]

\[
21,000 = PMT \times \frac{0.16}{0.016}
\]

\[
PMT = \$1,312.5
\]

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

\[
21,000 = 300 \left( \frac{1 - (1 + 0.025)^{-60}}{0.025} \right)
\]

\[
21,000 = 300 \times 0.16
\]

\[
21000 = 4.8 - 4.8
\]

\[
\$1,995.2
\]

Score 0: The student made multiple errors.
Question 37

37 The speed of a tidal wave, $s$, in hundreds of miles per hour, can be modeled by the equation

$$s = \sqrt{t} - 2t + 6,$$

where $t$ represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

$$0 = \sqrt{t} - 2t + 6$$

$$\left(\sqrt{t} - 6\right)^2 = \left(\sqrt{t}\right)^2$$

$$4t^2 - 24t + 36 = t$$

$$4t^2 - 25t + 36 = 0$$

$$(4t - 9)(t - 4) = 0$$

$t = \frac{9}{4}, t = 4$

rejected

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

$$s = \sqrt{1} - 2(1) + 6 = 5$$

$$s = \sqrt{3} - 2(3) + 6 = 1.732050808$$

$$\text{score 6: The student gave a complete and correct response.}$$
The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation \( s = \sqrt{t} - 2t + 6 \), where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

\[
\begin{align*}
0 &= \sqrt{t} - 2t + 6 \\
(\sqrt{t} - 6)^2 &= (\sqrt{t})^2 \\
4t^2 - 24t + 36 &= t \\
3t^2 - 25t + 36 &= 0 \\
x &= \frac{25 + \sqrt{625 - 144}}{6} \\
&= \frac{25 + 17}{6} \\
x &= 6.67 \\
\text{or} \quad x &= 2.25
\end{align*}
\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

When \( t = 1 \): 
\[
s = \sqrt{1} - 2(1) + 6 = 5 \text{ mph}
\]
When \( t = 3 \): 
\[
s = \sqrt{3} - 2(3) + 6 = \sqrt{3} - 6 + 6 = \sqrt{3}
\]

\[
5 - \sqrt{3} = 4.4926 \approx 4.5 \text{ miles}
\]

**Score 5:** The student did not convert \( \sqrt{3} \) to miles.
The speed of a tidal wave, $s$, in hundreds of miles per hour, can be modeled by the equation 

$$s = \sqrt{t} - 2t + 6,$$

where $t$ represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

\[
\begin{align*}
\sqrt{t} - 2t + 6 &= 0 \\
-6 &-6 \\
\sqrt{t} &= -2t + 6 \\
(\sqrt{t}) &= (2t - 6)^2 \\
\sqrt{t} &= 2t - 6 \\
4t^2 - 12t + 36 &= 0 \\
4t^2 - 24t + 36 &= 0
\end{align*}
\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
\begin{align*}
S &= \sqrt{1} - 2(1) + 6 \\
S &= 5 \quad \text{500 mph} \\
S &= \sqrt{3} - 2(3) + 6 \\
S &= \sqrt{3} \approx 1.73 \quad \text{173 mph}
\end{align*}
\]

**Score 4:** The student found a correct quadratic equation in standard form and 327.
The speed of a tidal wave, $s$, in hundreds of miles per hour, can be modeled by the equation 

$$s = \sqrt{t} - 2t + 6,$$

where $t$ represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

$$\begin{align*}
\sqrt{t} - 2t + 6 &= 0 \\
(\sqrt{t})^2 &= (2t - 6)^2 \\
4t^2 - 24t + 36 &= 0 \\
4t^2 - 24t + 36 &= 0 \\
4t^2 - 24t + 36 &= 0 \\
4t^2 - 24t + 36 &= 0 \\
4t^2 - 24t + 36 &= 0.
\end{align*}$$

2x faster b/c of the coefficient 2 being used.

Score 4: The student found 4 correctly.
37 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation \( s = \sqrt{t} - 2t + 6 \), where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

\[
\begin{align*}
0 &= \left(\sqrt{t}\right)^2 - 2t + 6 \\
&= \left(\sqrt{t} - 2t + 6\right)^2 \\
&= 4t^2 - 24t + 36 \\
0 &= 4t^2 - 24t + 36
\end{align*}
\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
\begin{align*}
\sqrt{1} - 2(1) + 6 &= 4 - 2 + 6 \\
1 - 2 + 6 &= 5 \\
\sqrt{3} - 2(3) + 6 &= -1 - 2 + 6 \\
\sqrt{3} &= -3 + 6
\end{align*}
\]

\( s - \sqrt{3} \) is rounded to 3 miles/hour.

**Score 3:** The student found a correct quadratic equation, but did not convert to miles.
The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation 
\[ s = \sqrt{t} - 2t + 6, \]
where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

\[
0 = \sqrt{t} - 2t + 6
\]

\[
(2t - 6)(2t + 6) = (\sqrt{t})^2
\]

\[
4t^2 - 12t - 12t + 36 = 0
\]

\[
4t^2 - 24t + 36 = 0
\]

\[
4t^2 - 25t + 36 = 0
\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
s = \sqrt{1} - 2(1) + 6 \quad s = \sqrt{3} - 2(3) + 6
\]

\[
s = 5 \quad s = 1.7
\]

3.3 hundreds of miles per hour

Score 3: The student made more than two mechanical errors.
Question 37

The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation 
\[ s = \sqrt{t} - 2t + 6 \]
where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

\[
\begin{align*}
0 &= \sqrt{t} - 2t + 6 \\
(\sqrt{t} - 2t + 6)^2 &= 0 \\
-4t^2 + 24t + 36 &= 0 \\
t &= \frac{-24 \pm \sqrt{(-24)^2 - 4 \cdot 4 \cdot 36}}{2 \cdot 4} \\
t &= \frac{-24 \pm 144}{8} \\
t &= \frac{120}{8} \quad \text{or} \quad t = 0
\end{align*}
\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
\begin{align*}
\text{Difference} &= s(1) - s(3) \\
&= (\sqrt{1} - 2(1) + 6) - (\sqrt{3} - 2(3) + 6) \\
&= \sqrt{1} - 2\sqrt{3} + 14
\end{align*}
\]

Score 3: The student didn’t reject \( \frac{9}{4} \).
37 The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation

\[
 s = \sqrt{t} - 2t + 6
\]

where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

\[\begin{align*}
0 &= \sqrt{t} - 2t + 6 \\
-6 &= \sqrt{t} - 2t \\
-36 &= t - 2t \\
36 &= t
\end{align*}\]

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
\begin{align*}
5 &= \sqrt{1} - 2(1) + 6 \\
5 &= 1 - 2 + 6 \\
5 &= 500 \text{ mph} \\
5 &= \sqrt{3} - 2(3) + 6 \\
5 &= \sqrt{3} - 6 + 6 \\
5 &= 173 \text{ mph}
\end{align*}
\]

\[500 - 173 = 327 \text{ mph}\]

Score 2:  The student found 327.
Question 37

37 The speed of a tidal wave, $s$, in hundreds of miles per hour, can be modeled by the equation 
$$s = \sqrt{t} - 2t + 6,$$
where $t$ represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

Score 1: The student did not convert to miles.
The speed of a tidal wave, \( s \), in hundreds of miles per hour, can be modeled by the equation 
\[ s = \sqrt{t} - 2t + 6 \]
where \( t \) represents the time from its origin in hours. Algebraically determine the time when \( s = 0 \).

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the nearest mile per hour? Justify your answer.

\[
\begin{align*}
3 &= \sqrt{1} - 2(1) + 6 \\
+ &= 2.5 \\
3 &= \sqrt{2.5} - 2(2.5) + 6 \\
\end{align*}
\]

\( |1| = 1 \) mile

**Score 0:** The student did not show enough correct work to receive any credit.