ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 23, 2020 — 1:15 to 4:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The expression $\sqrt[4]{81x^8y^6}$ is equivalent to

(1) $3x^2y^{\frac{3}{2}}$

(2) $3x^4y^2$

(3) $9x^2y^{\frac{3}{2}}$

(4) $9x^4y^2$

2 Chet has \$1200 invested in a bank account modeled by the function $P(n) = 1200(1.002)^n$, where $P(n)$ is the value of his account, in dollars, after n months. Chet's debt is modeled by the function $Q(n) = 100n$, where $Q(n)$ is the value of debt, in dollars, after n months.

After n months, which function represents Chet's net worth, $R(n)$?

(1) $R(n) = 1200(1.002)^n + 100n$

(2) $R(n) = 1200(1.002)^{12n} + 100n$

(3) $R(n) = 1200(1.002)^n - 100n$

(4) $R(n) = 1200(1.002)^{12n} - 100n$

3 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1: $25x^2 + (5x^2 - 5)^2$

Step 2: $25x^2 + 25x^2 + 25$

Step 3: $50x^2 + 25$

Step 4: $75x^2$

What statement is true regarding Emmeline's work?

(1) Emmeline's work is entirely correct.

(2) There is a mistake in step 2, only.

(3) There are mistakes in step 2 and step 4.

(4) There is a mistake in step 4, only.

4 Susan won \$2,000 and invested it into an account with an annual interest rate of 3.2%. If her investment were compounded monthly, which expression best represents the value of her investment after t years?

Use this space for computations.

(1) $2000(1.003)^{12t}$

(2) $2000(1.032)^{\frac{t}{12}}$

(3) $2064^{\frac{t}{12}}$

(4) $\dfrac{2000(1.032)^t}{12}$

5 Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

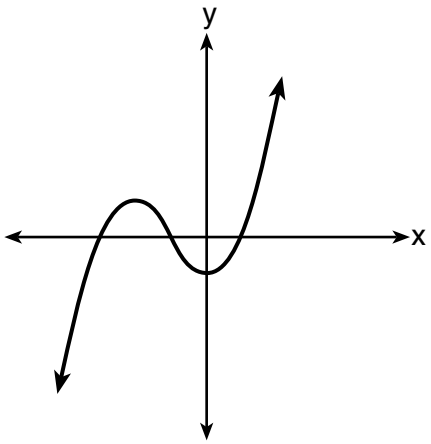
Which function satisfies the given conditions?

$f(x) = x^4 + 2x^2 + 1$

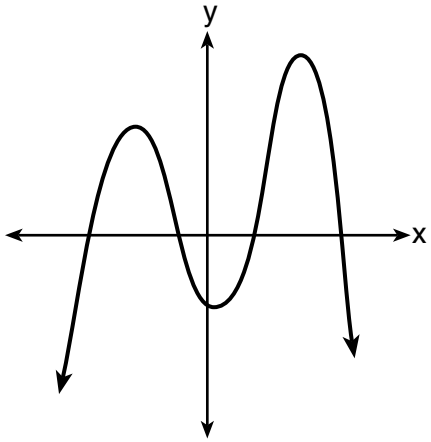
(1)

$f(x) = -x^3 + 2x - 6$

(3)



(2)



(4)

6 The expression $(x + a)^2 + 5(x + a) + 4$ is equivalent to

(1) $(a + 1)(a + 4)$

(2) $(x + 1)(x + 4)$

(3) $(x + a + 1)(x + a + 4)$

(4) $x^2 + a^2 + 5x + 5a + 4$

Use this space for computations.

7 Given $x \neq -2$, the expression $\frac{2x^2+5x+8}{x+2}$ is equivalent to

(1) $2x^2 + \frac{9}{x+2}$

(2) $2x + \frac{7}{x+2}$

(3) $2x + 1 + \frac{6}{x+2}$

(4) $2x + 9 - \frac{10}{x+2}$

8 Which situation best describes conditional probability?

(1) finding the probability of an event occurring two or more times

(2) finding the probability of an event occurring only once

(3) finding the probability of two independent events occurring at the same time

(4) finding the probability of an event occurring given another event had already occurred

9 Which expression is *not* a solution to the equation $2^t = \sqrt{10}$?

(1) $\frac{1}{2}\log_2 10$

(2) $\log_2\sqrt{10}$

(3) $\log_4 10$

(4) $\log_{10} 4$

10 What is the solution set of $x = \sqrt{3x+40}$?

(1) $\{-5,8\}$

(2) $\{8\}$

(3) $\{-4,10\}$

(4) $\{\ \}$

Use this space for computations.

11 Consider the data in the table below.

	Right Handed	Left Handed
Male	87	13
Female	89	11

What is the probability that a randomly selected person is male given the person is left handed?

(1) $\frac{13}{200}$

(2) $\frac{13}{100}$

(3) $\frac{13}{50}$

(4) $\frac{13}{24}$

12 The function $N(x) = 90(0.86)^x + 69$ can be used to predict the temperature of a cup of hot chocolate in degrees Fahrenheit after x minutes. What is the approximate average rate of change of the temperature of the hot chocolate, in degrees per minute, over the interval [0, 6]?

(1) -8.93

(2) -0.11

(3) 0.11

(4) 8.93

13 A recursive formula for the sequence 40, 30, 22.5, … is

(1) $g_n = 40\left(\frac{3}{4}\right)^n$

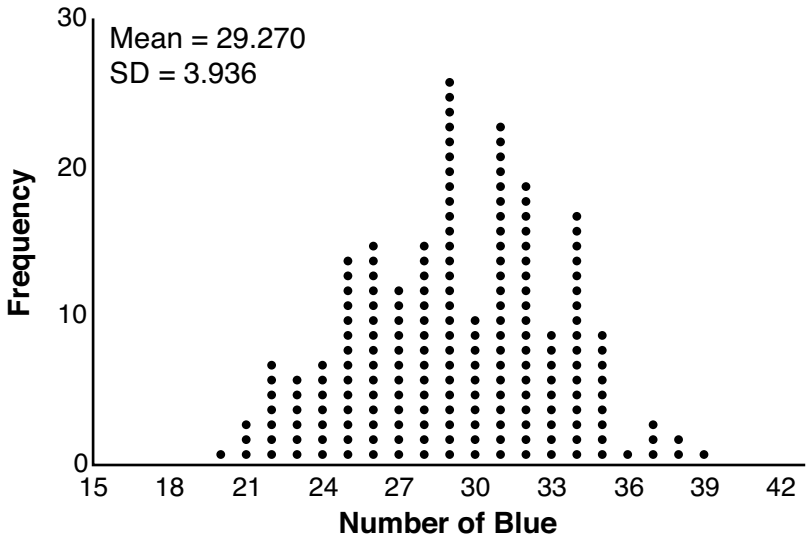
(2) $g_1 = 40$
$g_n = g_{n-1} - 10$

(3) $g_n = 40\left(\frac{3}{4}\right)^{n-1}$

(4) $g_1 = 40$
$g_n = \frac{3}{4}g_{n-1}$

Use this space for computations.

14 The J & B candy company claims that 45% of the candies it produces are blue, 30% are brown, and 25% are yellow. Each bag holds 65 candies. A simulation was run 200 times, each of sample size 65, based on the premise that 45% of the candies are blue. The results of the simulation are shown below.



Bonnie purchased a bag of J & B's candy and counted 24 blue candies. What inference can be made regarding a bag of J & B's with only 24 blue candies?

(1) The company is not meeting their production standard.

(2) Bonnie's bag was a rarity and the company should not be concerned.

(3) The company should change their claim to 37% blue candies are produced.

(4) Bonnie's bag is within the middle 95% of the simulated data supporting the company's claim.

15 Which investigation technique is most often used to determine if a single variable has an impact on a given population?

(1) observational study

(2) random survey

(3) controlled experiment

(4) formal interview

16 As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will

(1) decrease from 1 to 0

(2) decrease from 0 to -1

(3) increase from -1 to 0

(4) increase from 0 to 1

Use this space for computations.

17 Consider the following patterns:

I. 16, −12, 9, −6.75, …
II. 1, 4, 9, 16, …
III. 6, 18, 30, 42, …
IV. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

Which pattern is geometric?

(1) I
(2) II
(3) III
(4) IV

18 Consider the system below.

$$x + y + z = 9$$
$$x - y - z = -1$$
$$x - y + z = 21$$

Which value is *not* in the solution, (x,y,z), of the system?

(1) −8
(2) −6
(3) 11
(4) 4

19 Which statement regarding polynomials and their zeros is true?

(1) $f(x) = (x^2 - 1)(x + a)$ has zeros of 1 and $-a$, only.
(2) $f(x) = x^3 - ax^2 + 16x - 16a$ has zeros of 4 and a, only.
(3) $f(x) = (x^2 + 25)(x + a)$ has zeros of ± 5 and $-a$.
(4) $f(x) = x^3 - ax^2 - 9x + 9a$ has zeros of ± 3 and a.

20 If a solution of $2(2x - 1) = 5x^2$ is expressed in simplest $a + bi$ form, the value of b is

(1) $\frac{\sqrt{6}}{5}i$
(2) $\frac{\sqrt{6}}{5}$
(3) $\frac{1}{5}i$
(4) $\frac{1}{5}$

Use this space for computations.

21 Which value, to the *nearest tenth*, is the *smallest* solution of $f(x) = g(x)$ if $f(x) = 3\sin\left(\frac{1}{2}x\right) - 1$ and $g(x) = x^3 - 2x + 1$?

(1) -3.6

(2) -2.1

(3) -1.8

(4) 1.4

22 Expressed in simplest $a + bi$ form, $(7 - 3i) + (x - 2i)^2 - (4i + 2x^2)$ is

(1) $(3 - x^2) - (4x + 7)i$

(2) $(3 + 3x^2) - (4x + 7)i$

(3) $(3 - x^2) - 7i$

(4) $(3 + 3x^2) - 7i$

23 Written in simplest form, the fraction $\frac{x^3 - 9x}{9 - x^2}$, where $x \neq \pm 3$, is equivalent to

(1) $-x$

(2) x

(3) $\frac{-x(x+3)}{(3+x)}$

(4) $\frac{x(x-3)}{(3-x)}$

24 According to a study, 45% of Americans have type O blood. If a random number generator produces three-digit values from 000 to 999, which values would represent those having type O blood?

(1) between 000 and 045, inclusive

(2) between 000 and 444, inclusive

(3) between 000 and 449, inclusive

(4) between 000 and 450, inclusive

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

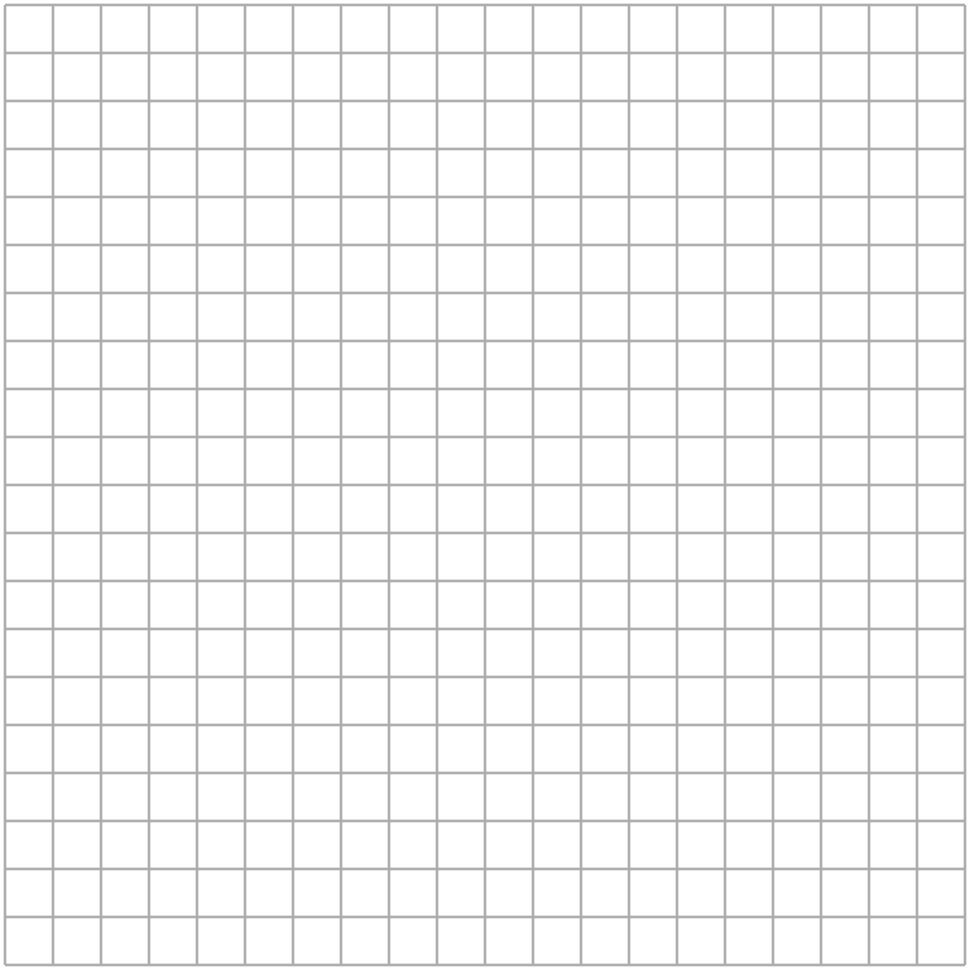
25 For n and $p > 0$, is the expression $\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4}$ equivalent to $p^{18} n^6 \sqrt{p}$? Justify your answer.

26 Show why $x - 3$ is a factor of $m(x) = x^3 - x^2 - 5x - 3$. Justify your answer.

27 Describe the transformation applied to the graph of $p(x) = 2^x$ that forms the new function $q(x) = 2^{x-3} + 4$.

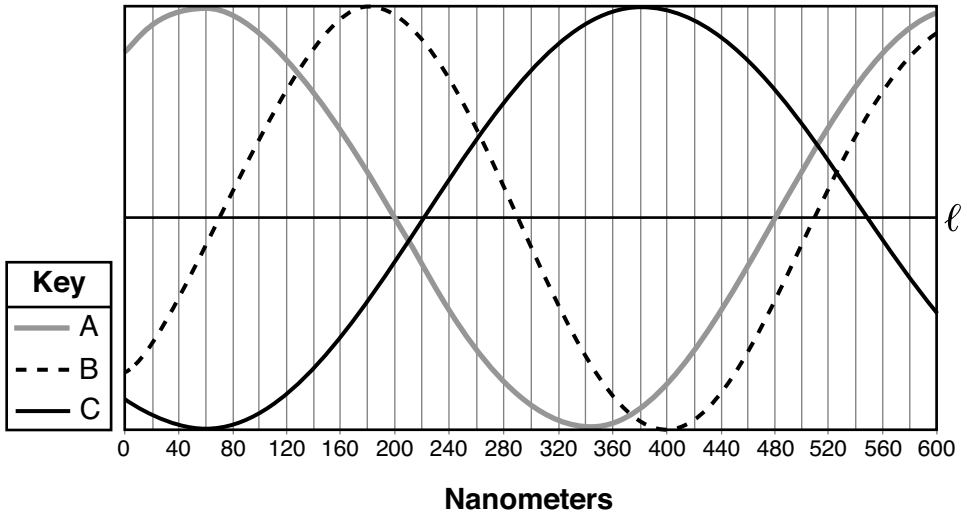
28 The parabola $y = -\frac{1}{20}(x - 3)^2 + 6$ has its focus at (3,1). Determine and state the equation of the directrix.

(The use of the grid below is optional.)



29 Given the geometric series $300 + 360 + 432 + 518.4 + \ldots$, write a geometric series formula, S_n, for the sum of the first n terms. Use the formula to find the sum of the first 10 terms, to the *nearest tenth*.

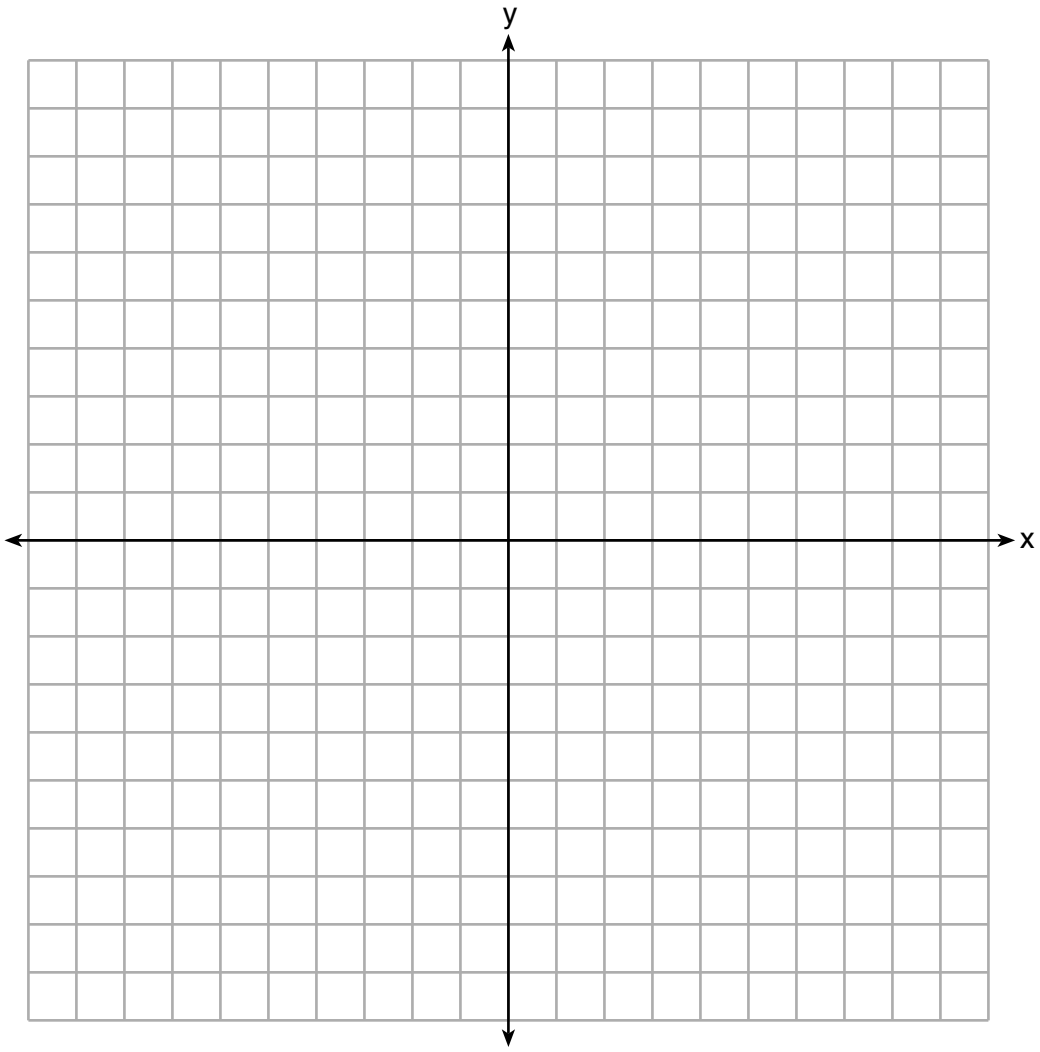
30 Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled ℓ.



Based on the graph, which light wave has the longest period? Justify your answer.

31 Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria, B, in terms of the number of hours, t, since the experiment began.

32 Graph $y = x^3 - 4x^2 + 2x + 7$ on the set of axes below.



Part III

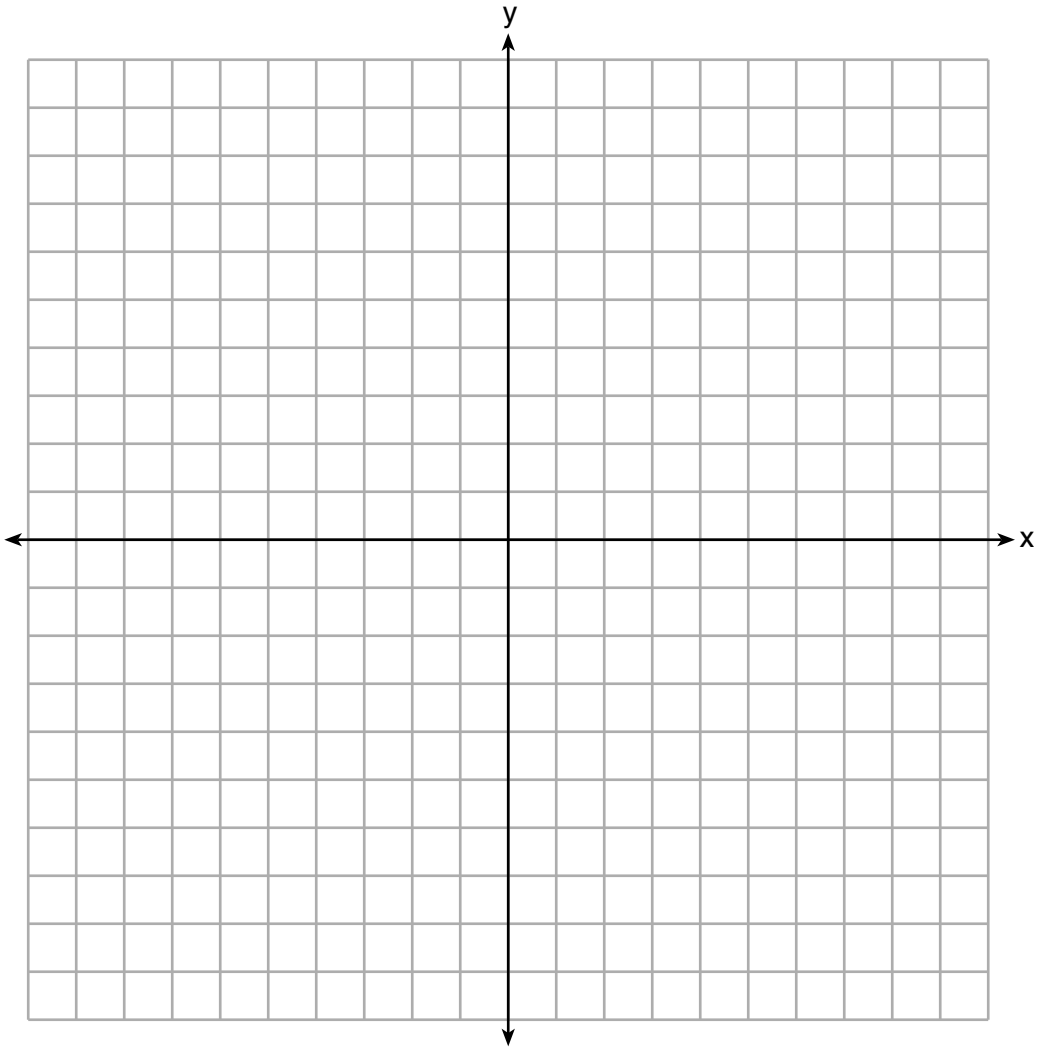
Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Sonja is cutting wire to construct a mobile. She cuts 100 inches for the first piece, 80 inches for the second piece, and 64 inches for the third piece. Assuming this pattern continues, write an explicit equation for a_n, the length in inches of the nth piece.

Sonja only has 40 feet of wire to use for the project and wants to cut 20 pieces total for the mobile using her pattern. Will she have enough wire? Justify your answer.

34 Graph the following function on the axes below.

$$f(x) = \log_3(2 - x)$$



State the domain of f.

State the equation of the asymptote.

35 Algebraically solve the following system of equations.

$$(x - 2)^2 + (y - 3)^2 = 16$$
$$x + y - 1 = 0$$

36 The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	**Altitude** (km)	0	1	2	3	4	5
y	**Air Pressure** (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*.

Use this equation to algebraically determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is 29 kPa.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2+8t+15}$$

$$a(t) = \frac{9}{t+3}$$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer.

Question 37 is continued on the next page.

Question 37 is continued

Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

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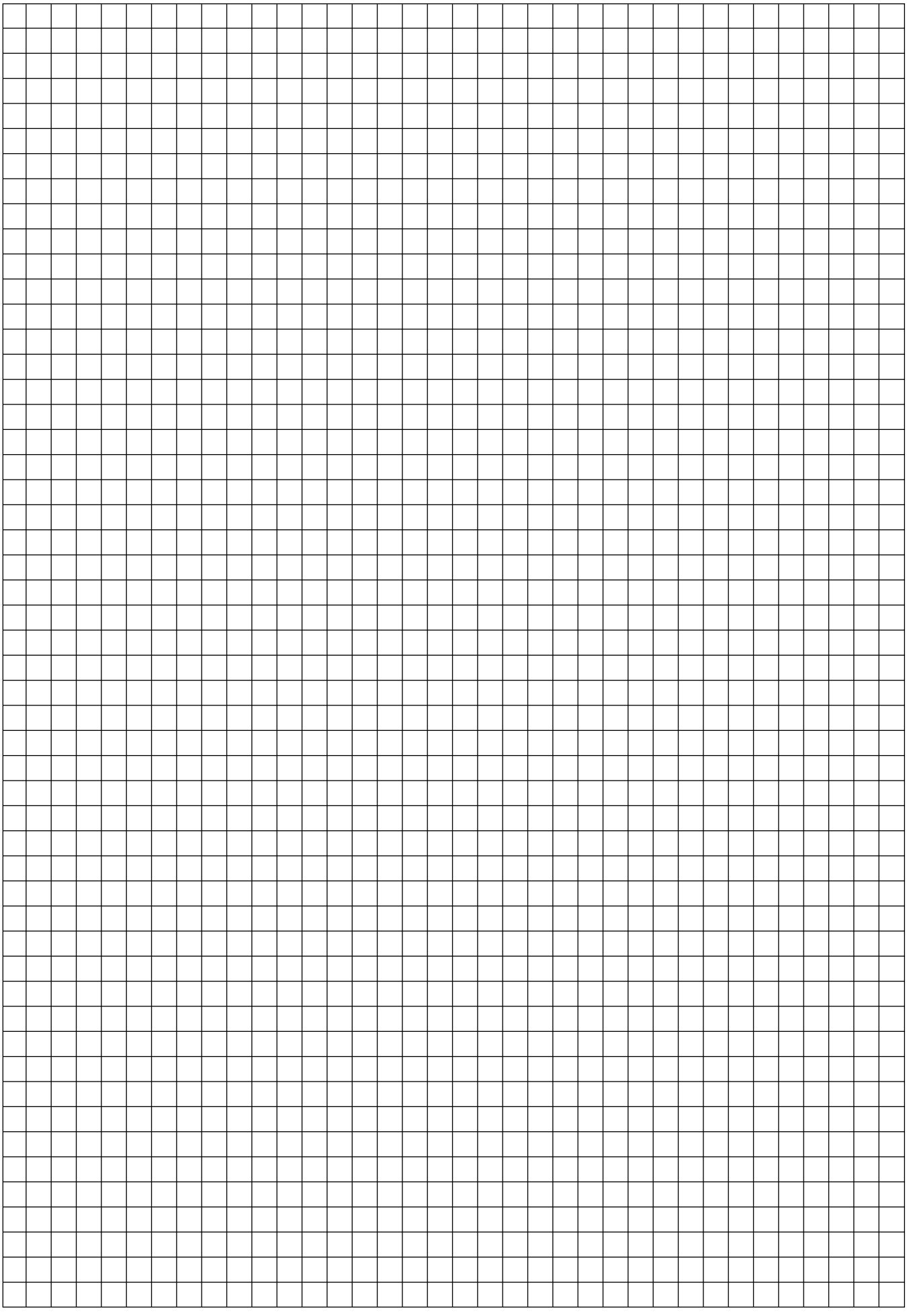

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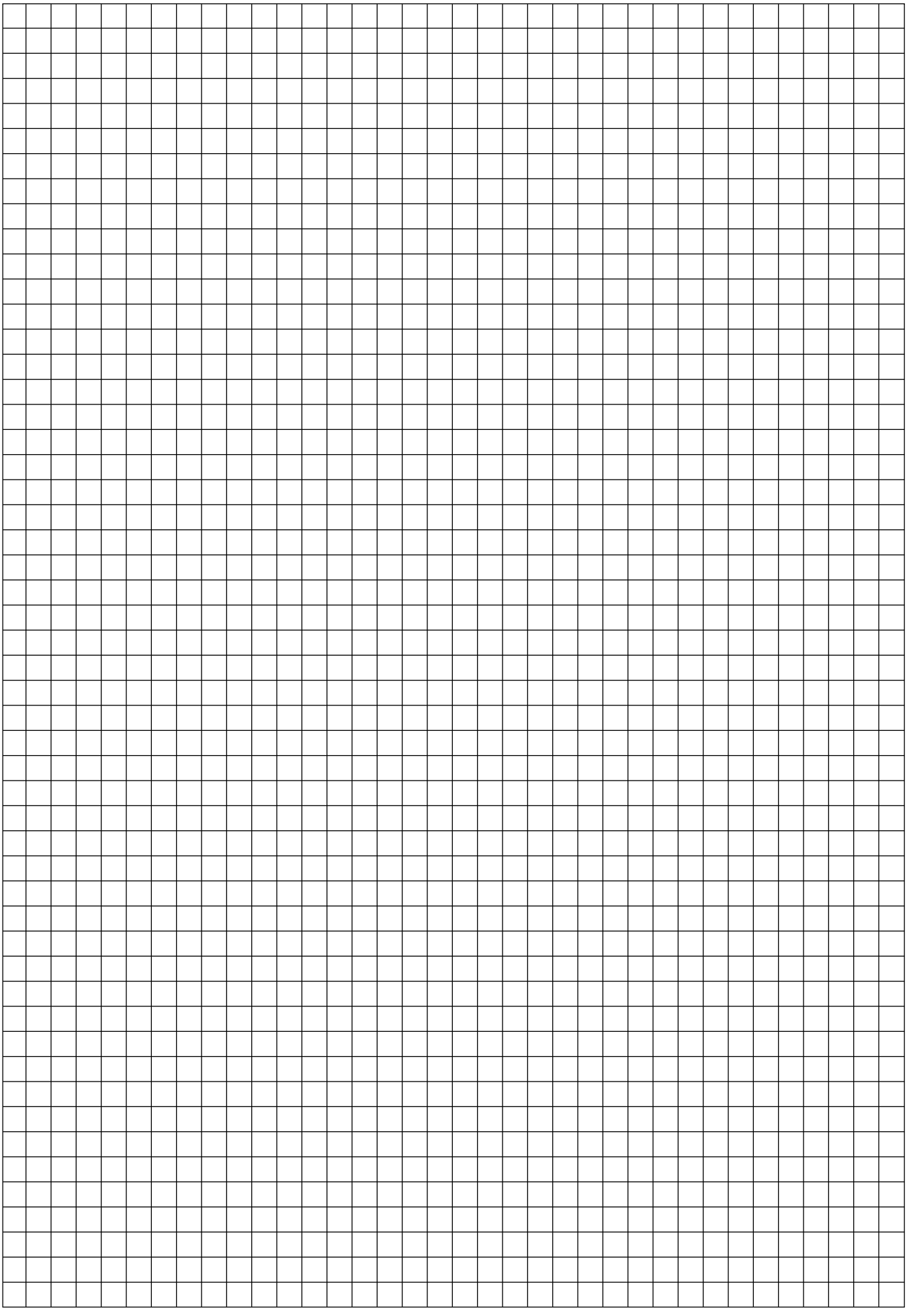

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High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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