The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, June 14, 2018 — 1:15 to 4:15 p.m., only

MODEL RESPONSE SET

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A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Favorite Type of Program</th>
<th>Sports</th>
<th>Reality Show</th>
<th>Comedy Series</th>
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<tr>
<td>Senior</td>
<td>83</td>
<td>110</td>
<td>67</td>
</tr>
<tr>
<td>Freshman</td>
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<td>103</td>
<td>54</td>
</tr>
</tbody>
</table>

A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

\[
\frac{103}{213}
\]

Score 2: The student gave a complete and correct response.
25 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

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A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

\[
p = \frac{103}{536}
\]

Score 1: The student used the total number of students instead of the number of students who prefer to watch reality shows.
A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

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A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

\[
\frac{110}{213} \times 100 = 51.63\%
\]

Score 1:  The student gave a non-exact probability.
25 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

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A student response is selected at random from the results. State the exact probability the student response is from a freshman, given the student prefers to watch reality shows on television.

\[
\frac{103}{276} \times 100 = 37.3\%
\]

Score 0: The student calculated the incorrect conditional probability and stated a non-exact probability.
26 On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$.

Score 2: The student gave a complete and correct response.
26 On the grid below, graph the function \( f(x) = x^3 - 6x^2 + 9x + 6 \) on the domain \(-1 \leq x \leq 4\).

Score 1: The student did not draw a smooth curve on the interval \(1 \leq x \leq 4\).
26 On the grid below, graph the function \( f(x) = x^3 - 6x^2 + 9x + 6 \) on the domain \(-1 \leq x \leq 4\).

Score 0: The student plotted the \( y \)-intercept incorrectly and made a domain error, as indicated by the arrows.
26 On the grid below, graph the function \( f(x) = x^3 - 6x^2 + 9x + 6 \) on the domain \(-1 \leq x \leq 4\).

**Score 0:** The student made multiple errors.
27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

$$2x^2 + 5x + 8 = 0$$

$a = 2$
$b = 5$
$c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(8)}}{4}$$
$$x = \frac{-5 \pm \sqrt{49}}{4}$$
$$x = \frac{-5 \pm 7}{4}$$
$$x = \frac{2}{4}, \frac{-6}{4}$$
$$x = \frac{1}{2}, -\frac{3}{2}$$

Score 2: The student gave a complete and correct response.
**Question 27**

27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

\[
\begin{align*}
\frac{2x^2 + 5x + 8}{2} &= 0 \\
x^2 + \frac{5}{2}x + 4 &= 0 \\
x^2 + \frac{5}{2}x &= -4 \\
x^2 + \frac{5}{2}x + \frac{25}{16} &= -4 + \frac{25}{16} \\
\sqrt{(x + \frac{5}{4})^2} &= \sqrt{\frac{39}{16}} \\
x + \frac{5}{4} &= \pm \frac{\sqrt{39}}{4} \\
x &= -\frac{5}{4} \pm \frac{\sqrt{39}}{4}
\end{align*}
\]

**Score 2:** The student gave a complete and correct response.
Question 27

27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ a = 2 \]
\[ b = 5 \]
\[ c = 8 \]

\[ x = \frac{-5 \pm \sqrt{25 - 4(2)(8)}}{2(2)} \]
\[ x = \frac{-5 \pm \sqrt{-39}}{4} \]
\[ x = \frac{-5 \pm i\sqrt{39}}{4} \]

Score 1: The student did not express the answer in $a + bi$ form.
27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

\[
-\frac{5 \pm \sqrt{5^2 - 4(2)(8)}}{2} \\
-\frac{5 \pm \sqrt{-39}}{2} \\
-\frac{5}{2} \pm \frac{i\sqrt{39}}{2}
\]

Score 1: The student used an incorrect denominator.
Question 27

27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-5 \pm \sqrt{25 - 64}}{4} \]

\[ x = \frac{-5 \pm \sqrt{-39}}{4} \]

\[ x = \frac{-5 \pm 13i}{4} \]

Score 0: The student incorrectly simplified the radical and did not express the answer in $a + bi$ form.
28 Chuck’s Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

![How's My Driving? Call 1-555-DRIVING](image)

The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

They are performing a survey, and a possible bias is that only those with time to call will call, thus making the employee of the month the one who drives by the most people with a lot of free time.

Score 2: The student gave a complete and correct response.
28 Chuck’s Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

The driver who receives the highest number of positive comments will win the recognition. Explain one statistical bias in this data collection method.

A possible bias could occur because there may be one car driving behind this truck for a long time, so they may not get as many calls as another truck.

Score 2: The student gave a complete and correct response.
Chuck’s Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

The driver who receives the highest number of positive comments will win the recognition. Explain one statistical bias in this data collection method.

One possible bias is that drivers can persuade people to call.

Score 1: The student gave an incomplete explanation.
28 Chuck’s Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

The driver who receives the highest number of positive comments will win the recognition. Explain one statistical bias in this data collection method.

Depending on the age of the driver their driving abilities may be impaired. The biased is that it is unfair calling about someone driving if their age is a large factor.

Score 0: The student’s response is irrelevant.
Chuck’s Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.

![How's My Driving? Call 1-555-DRIVING](image)

The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

**Randomly choose 5 from each 10 comments.**

**Score 0:** The student gave a completely incorrect response.
29 Determine the quotient and remainder when \((6a^3 + 11a^2 - 4a - 9)\) is divided by \((3a - 2)\).

Express your answer in the form \(q(a) + \frac{r(a)}{d(a)}\).

\[
\begin{array}{c|ccccc}
\multicolumn{2}{c|}{2a^2 + 5a + 2} \\
\cline{2-6}
3a - 2 & 6a^3 + 11a^2 - 4a - 9 \\
\multicolumn{2}{c|}{6a^3 - 4a^2} \\
\cline{2-6}
\multicolumn{2}{c|}{15a^2 - 4a - 9} \\
\multicolumn{2}{c|}{15a^2 - 10a} \\
\cline{2-6}
\multicolumn{2}{c|}{6a - 9} \\
\multicolumn{2}{c|}{6a - 4} \\
\multicolumn{2}{c|}{-5} \\
\end{array}
\]

\[
\begin{array}{c}
2a^2 + 5a + 2 - \frac{5}{3a - 2} = \text{remainder}
\end{array}
\]

Score 2: The student gave a complete and correct response.
Question 29

29 Determine the quotient and remainder when \((6a^3 + 11a^2 - 4a - 9)\) is divided by \((3a - 2)\).

Express your answer in the form \(q(a) + \frac{r(a)}{d(a)}\).

\[
\begin{array}{cccc}
2 & 3 & 6 & 11 & -4 & -9 \\
\downarrow & 4 & 10 & 4 \\
6 & 15 & 6 & -5 \\
\end{array}
\]

\[\Rightarrow 2 \ 5 \ 2 \ -\frac{5}{3}\]

\[2a^2 + 5a + 2 \ - \frac{5}{3(a-\frac{2}{3})}\]

\[2a^2 + 5a + 2 \ - \frac{5}{3a-2}\]

Score 2: The student gave a complete and correct response.
29 Determine the quotient and remainder when \((6a^3 + 11a^2 - 4a - 9)\) is divided by \((3a - 2)\).

Express your answer in the form \(q(a) + \frac{r(a)}{d(a)}\).

Score 1: The student made one computational error.
29 Determine the quotient and remainder when \((6a^3 + 11a^2 - 4a - 9)\) is divided by \((3a - 2)\).

Express your answer in the form \(q(a) + \frac{r(a)}{d(a)}\).

\[
\begin{array}{c}
3a-2 \overline{6a^3+11a^2-4a-9} \\
- \quad 6a^3-4a^2 \\
\hline
\quad 15a^2-4a \\
- \quad 15a^2-10a \\
\hline
\quad 6a-9 \\
- \quad 6a-4 \\
\hline
\quad -5
\end{array}
\]

\[
\text{Quotient} = 2a^2 + 5a + 2 \\
\text{Remainder} = \frac{9}{5}
\]

**Score 0:** The student made an error expressing the remainder and did not express the answer in the required form.
29 Determine the quotient and remainder when \((6a^3 + 11a^2 - 4a - 9)\) is divided by \((3a - 2)\).

Express your answer in the form \(q(a) + \frac{r(a)}{d(a)}\).

Score 0: The student made multiple errors.
30 The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]
\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence.

\[
\begin{align*}
    a_2 &= 1 + 2(3) = 7 \\
    a_3 &= 1 + 2(7) = 15 \\
    a_4 &= 1 + 2(15) = 31
\end{align*}
\]

Answer: 3, 7, 15, 31

Can this sequence be represented using an explicit geometric formula? Justify your answer.

No, an explicit geometric formula cannot be used as there is no common ratio between the numbers.

Score 2: The student gave a complete and correct response.
30 The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]
\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence.

\[
\begin{align*}
q_1 &= 3 \\
q_2 &= 7 \\
q_3 &= 15 \\
q_4 &= 31
\end{align*}
\]

Can this sequence be represented using an explicit geometric formula? Justify your answer.

**Score 1:** The student correctly determined the first four terms only.
30 The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]
\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence.

\[
\begin{align*}
    a_1 &= 1 + 6 = 7 \\
    a_2 &= 1 + 14 = 15 \\
    a_3 &= 1 + 28 = 29 \\
    a_4 &= 1 + 56 = 57
\end{align*}
\]

Can this sequence be represented using an explicit geometric formula? Justify your answer.

No. Since there is no common ratio that would give us these terms exactly.

Score 1: The student incorrectly stated the first four terms, but gave a correct justification based on those terms.
30 The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]
\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence.

\[ a_2 = 1 + 2a_1 = 7 \]
\[ a_3 = 1 + 2a_2 = 15 \]
\[ a_4 = 1 + 2a_3 = 30 \]

Can this sequence be represented using an explicit geometric formula? Justify your answer.

No because this is an arithmetic sequence, you have to multiply to get the next term. In a geometric sequence you add.

Score 0: The student made a computational error and gave an incorrect justification.
The recursive formula to describe a sequence is shown below.

\[ a_1 = 3 \]
\[ a_n = 1 + 2a_{n-1} \]

State the first four terms of this sequence.

\[ a_2 = 1 + 2a_1 = 1 + 2(3) = 7 \]
\[ a_3 = 1 + 2a_2 = 1 + 2(7) = 15 \]
\[ a_4 = 1 + 2a_3 = 1 + 2(15) = 31 \]
\[ a_5 = 1 + 2a_4 = 1 + 2(31) = 63 \]

Can this sequence be represented using an explicit geometric formula? Justify your answer.

Yes, because if you find the difference once you can then multiply those differences and get 2.
The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

\[
M = \frac{P\left(\frac{r}{12}\right)(1 + \frac{r}{12})^n}{\left(1 + \frac{r}{12}\right)^n - 1}
\]

- \(M\) = monthly payment
- \(P\) = amount borrowed
- \(r\) = annual interest rate
- \(n\) = total number of monthly payments

\[
M = \frac{137,250\left(\frac{.036}{12}\right)(1 + \frac{.036}{12})^{360}}{\left(1 + \frac{.036}{12}\right)^{360} - 1}
\]

\[
M = 137,250\left(\frac{.036}{12}\right)(1 + \frac{.036}{12})^{360}
\]

\[
\left(1 + \frac{.036}{12}\right)^{360} - 1
\]

\[
6241 \text{ dollars}
\]

**Score 2:** The student gave a complete and correct response.
The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

\[
M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}
\]

- \(M\) = monthly payment
- \(P\) = amount borrowed
- \(r\) = annual interest rate
- \(n\) = total number of monthly payments

\[
M = \frac{137250 \left( \frac{0.036}{12} \right) \left( 1 + \frac{0.036}{12} \right)^{360}}{\left( 1 + \frac{0.036}{12} \right)^{360} - 1}
\]

\[
M = \frac{137250 \left( 0.003 \right) \left( 1.003 \right)^{360}}{\left( 1.003 \right)^{360} - 1}
\]

\[
M = \frac{137250 \left( 0.003 \right) \left( 1.094027 \right)}{\left( 1.094027 \right) - 1}
\]

\[
M = \frac{4790.811333}{0.094027}
\]

\[
M = 4790.811333
\]

**Score 1:** The student used 30 instead of 360.
The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

\[ M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1} \]

- \( M \) = monthly payment
- \( P \) = amount borrowed
- \( r \) = annual interest rate
- \( n \) = total number of monthly payments

\[ M = \frac{152500 \left( \frac{0.036}{12} \right) \left( 1 + \frac{0.036}{12} \right)^{300}}{\left( 1 + \frac{0.036}{12} \right)^{300} - 1} \]

The monthly payment would be about $40.

**Score 1:** The student substituted correct values into the formula, but showed no further correct work.
The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the nearest dollar.

\[
M = \frac{P \left( \frac{r}{12} \right) \left( 1 + \frac{r}{12} \right)^n}{\left( 1 + \frac{r}{12} \right)^n - 1}
\]

where:
- \( M \) = monthly payment
- \( P \) = amount borrowed
- \( r \) = annual interest rate
- \( n \) = total number of monthly payments

\[
M = \frac{152,500 \left( \frac{0.0036}{12} \right) \left( 1 + \frac{0.0036}{12} \right)^{360}}{\left( 1 + \frac{0.0036}{12} \right)^{360} - 1}
\]

\[M = 449.60 \approx 45\]

Score 0: The student made multiple errors.
Question 32

32 An angle, $\theta$, is in standard position and its terminal side passes through the point $(2, -1)$. Find the exact value of $\sin \theta$.

Score 2: The student gave a complete and correct response.
Question 32

32 An angle, θ, is in standard position and its terminal side passes through the point (2, -1). Find the exact value of sin θ.

\[
\begin{align*}
\text{By Pythagoras, } c^2 &= a^2 + b^2 \\
c^2 &= (2)^2 + (-1)^2 \\
c^2 &= 5 \\
\frac{c}{\sqrt{5}} &= \frac{-1 \cdot \sqrt{5}}{\sqrt{5}} = \frac{-\sqrt{5}}{5}
\end{align*}
\]

Score 2: The student gave a complete and correct response.
32 An angle, $\theta$, is in standard position and its terminal side passes through the point $(2, -1)$. Find the exact value of $\sin \theta$.

Score 1: The student incorrectly represented the answer.
32 An angle, \( \theta \), is in standard position and its terminal side passes through the point \((2, -1)\). Find the exact value of \( \sin \theta \).

Score 1: The student omitted a negative sign.
32 An angle, $\theta$, is in standard position and its terminal side passes through the point $(2, -1)$. Find the exact value of $\sin \theta$.

Score 0: The student gave a completely incorrect response.
33 Solve algebraically for all values of $x$:

\[
\sqrt{6 - 2x} + x = 2(x + 15) - 9
\]

\[
\begin{align*}
\sqrt{6 - 2x} &= 2x + 30 - 9 - x \\
6 - 2x &= (x + 21)^2 \\
6 - 2x &= x^2 + 42x + 441 \\
-6 + 2x &= x^2 + 42x + 441 \\
0 &= x^2 + 42x + 435 \\
0 &= (x + 29)(x + 15) \\
x + 15 &= 0 \quad x + 15 = 0 \\
x &= -29 \quad x = -15
\end{align*}
\]

**Check**

\[
\begin{align*}
\sqrt{6 - 2(-29)} + (-29) &= 2((-29) + 15) - 9 \\
8 + -29 &= -28 - 9 \\
-21 &= -37
\end{align*}
\]

\[
\begin{align*}
\sqrt{6 - 2(-15)} + (-15) &= 2((-15) + 15) - 9 \\
6 + -15 &= 0 - 9 \\
-9 &= -9
\end{align*}
\]

Score 4: The student gave a complete and correct response.
33 Solve algebraically for all values of $x$:

$$\sqrt{6-2x} + x = 2(x + 15) - 9$$

\[
\begin{align*}
\sqrt{6-2x} + x &= 2x + 21 \\
(\sqrt{6-2x})^2 &= (x + 21)^2 \\
6 - 2x &= x^2 + 42x + 441 \\
-6 &= x^2 + 42x - 441 \\
x^2 + 42x + 435 &= 0
\end{align*}
\]

\[
(x + 69)(x + 15) = 0
\]

\[
x = -69, x = 15
\]

\[
\begin{align*}
x &= -69 \\
x &= 15
\end{align*}
\]

**Score 4:** The student gave a complete and correct response.
33 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

$$\frac{-x}{\sqrt{6 - 2x}} = 2x + 30 - 9$$

$$-x$$

$$\sqrt{(6 - 2x)^2} = (x + 21)^2$$

$$6 - 2x = (x + 21)(x + 21)$$

$$6 - 2x = x^2 + 42x + 441$$

$$-6 + 2x$$

$$0 = x^2 + 42x + 435$$

$$0 = (x + 15)(x + 29)$$

$$x + 15 = 0$$

$$x = -15$$

$$x + 29 = 0$$

$$x = -29$$

$$\{ -29, -15 \}$$

**Score 3:** The student failed to reject $-29$. 

29.
33 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

\[
\begin{align*}
\sqrt{6 - 2x} + x &= 2x + 30 - 9 \\
\sqrt{6 - 2x} &= x + 21 \\
6 - 2x &= (x + 21)^2 \\
6 - 2x &= x^2 + 42x + 441 \\
0 &= x^2 + 44x + 435 \\
0 &= (x + 15)(x + 29) \\
x &= -15, x = -29
\end{align*}
\]

**Score 3:** The student found the correct values, but then made a rejection error.
Question 33

33 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

$$\begin{align*}
\sqrt{6 - 2x} + x &= 2x + 21 - 9 \\
\sqrt{6 - 2x} &= x + 12 \\
6 - 2x &= (x + 12)(x + 12) \\
6 - 2x &= x^2 + 42x + 144 \\
2x &= x^2 + 44x + 148 \\
0 &= x^2 + 44x + 435
\end{align*}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-44 \pm \sqrt{44^2 - 4(1)(435)}}{2(1)}$$

$$x = \frac{-44 \pm \sqrt{1936 - 1740}}{2}$$

$$x = \frac{-44 \pm \sqrt{196}}{2}$$

$$x = -22 \pm 7$$

$$x = -19 \quad \text{or} \quad x = -29$$

Score 2: The student made a computational error and failed to reject.
33 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

$$\left(\sqrt{6 - 2x}\right)^2 = (x + 21)^2$$

$$6 - 2x = (x + 21)(x + 21)$$

$$0 = x^2 + 44x + 435$$

$$(x + 15)(x + 29)$$

$x = 15$  $x = 29$

**Score 2:** The student wrote a correct quadratic equation set equal to zero.
33 Solve algebraically for all values of $x$:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

Score 1: The student did not do enough work to receive a second point.
33 Solve algebraically for all values of \( x \):

\[ \sqrt{6 - 2x} + x = 2(x + 15) - 9 \]

**Score 0:** The student did not show enough correct work to receive any credit.
33 Solve algebraically for all values of $x$:

\[
\sqrt{6 - 2x} + x = 2(x + 15) - 9
\]

\[
\frac{6 - 2x + x}{-x} = \frac{2x + 30 - 9}{-x}
\]

\[
(\sqrt{6 - 2x})^2 = (x - 21)^2
\]

\[
6 - 2x = x^2 - 21
\]

\[
6 + 2x = 6 + 2x
\]

\[
0 = x^2 + 2x - 27
\]

\[
0 = x^2 + 9x + 3x - 21
\]

\[
0 = x(x + 9) + 3(x - 9)
\]

\[
0 = (x + 3)(x + 9)
\]

\[
\begin{array}{cc}
0 &= x + 3 \\
-3 &= -3
\end{array}
\]

\[
\begin{array}{cc}
0 &= x + q \\
-q &= -q
\end{array}
\]

\[
\begin{array}{c}
x = -3 \\
x = -9
\end{array}
\]

**Score 0:** The student made multiple conceptual and computational errors, and failed to reject.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

\[
23 - 18 = 5
\]

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

**Score 4:** The student gave a complete and correct response.
Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

Yes - 5 is greater than 3.13, so it doesn’t fall in the 95%, meaning it is statistically significant.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

$$23 - 18 = 5$$

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

**Score 4:** The student gave a complete and correct response.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

\[0.030 + 2(1.548) = 3.126\]
\[0.030 - 2(1.548) = -3.07\]

\[3.07 \text{ to } 3.13\]

Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

Yes, a difference of 5 or greater occurred only 3 times out of 1000 which is statistically significant.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

\[ 23 - 18 = 5 \]

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 3: The student gave an incomplete response regarding the statistical significance of the results.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

No it is not because 5 doesn’t fall within 2 standard deviations.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

\[
23 - 18 = 5
\]

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 3: The student made a rounding error.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

\[
0.30 \pm 2(1.848) \\
-3.066 - 3.126
\]

Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

The difference of the means is statistically significant as the value lies well outside the interval.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 2: The student received one point for each of the first two parts.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

\[-3.07 \text{ to } 3.13\]

Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

Yes, since the difference between Joseph’s mean was 5 while the simulation was 0.2.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

\[23 - 18 = 5\]

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 2: The student made a rounding error and gave an incorrect explanation.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

\[ 0.030 \pm 3.096 \]
\[ (-3.066, 3.126) \]

Is the difference in means in Joseph’s experiment statistically significant based on the simulation? Explain.

Yes, since there is a clear difference that students given scented paper did better than with unscented.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 1: The student received only one credit for the interval.
Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.
Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

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Calculate the difference in means in the experimental test grades (scented – unscented).

\[-5\]

A simulation was conducted in which the subjects’ scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

Score 0: The student gave a completely incorrect response.
Question 34 continued.

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth.

\[-3.07 \quad 3.126\]

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.
35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account $t$ years after the account is opened, given that no more money is deposited into or withdrawn from the account.

$$C(t) = 63,000 \left(1 + \frac{0.0255}{12}\right)^{12t}$$

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

$$\frac{100,000}{63,000} = \left(1 + \frac{0.0255}{12}\right)^{12t}$$

$$1.5873 = \left(1 + \frac{0.0255}{12}\right)^{12t}$$

$$\log(1.5873) = 12t \log\left(1 + \frac{0.0255}{12}\right)$$

$$t = \frac{\log(1.5873)}{12 \log\left(1 + \frac{0.0255}{12}\right)} = 18.14$$

Score 4: The student gave a complete and correct response.
Question 35

35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = 63000 \left( 1.002125 \right)^{12t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
100000 = 63000 \left( 1.002125 \right)^{12t}
\]

\[
1.567301587 = \left( 1.002125 \right)^{12t}
\]

\[
\log_{1.002125} (1.567301587) = 12t
\]

\[
217.6803875 = 12t
\]

\[
t = 18.1382829
\]

\[
18.14 \text{ years}
\]

Score 4: The student gave a complete and correct response.
35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = Pe^{rt} = 63,000e^{0.0255t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
100,000 = 63,000e^{0.0255t} \Rightarrow \ln \left( \frac{100,000}{63,000} \right) = 0.0255t \Rightarrow \frac{0.4620354896 - \ln 63000}{0.0255} = t \Rightarrow 18.12 = t
\]

**Score 3:** The student used the wrong formula, but evaluated it correctly.
Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = 63000 \left(1 + \frac{0.0255}{12}\right)^{12t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
\frac{100,000}{63000} = \left(1 + \frac{0.0255}{12}\right)^{12t}
\]

\[
1.587 = \left(1 + \frac{0.0255}{12}\right)^{12t}
\]

\[
1.587 = \left(1.002125\right)^{12t}
\]

\[
\log_{1.002125}1.587 = 12t
\]

\[
\frac{109.10228}{12} = 12t
\]

\[
18.13 \text{ years}
\]

**Score 3:** The student made an error by rounding too early.
Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = 63,000 \left( 1 + \frac{0.0255}{12} \right)^{12t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
\frac{100,000}{63,000} = \left( 1 + \frac{0.0255}{12} \right)^{12t}
\]

\[
\ln \frac{\frac{100}{63}}{\ln \left( 1 + \frac{0.0255}{12} \right)} = 12t \ln \left( 1 + \frac{0.0255}{12} \right)
\]

\[
12t = \ln \left( \frac{100}{63} \right) - \ln \left( 1 + \frac{0.0255}{12} \right)
\]

Score 2: The student failed to solve the equation for \( t \).
Question 35

35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account $t$ years after the account is opened, given that no more money is deposited into or withdrawn from the account.

$$C(t) = 63000 \cdot e^{0.0255t}$$

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

$$100000 = 63000 \cdot e^{0.0255t}$$

$$\ln\left(\frac{100000}{63000}\right) = \ln(e^{0.0255t})$$

$$0.415515444 = 0.0255t$$

$$t \approx 16.29$$

Score 2: The student used the wrong formula then made a computational error.
35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = \$63,000 \left(1 + \frac{0.0255}{12}\right)^{12t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
C(t) = 63,000 \left(1 + \frac{0.0255}{12}\right)^{12t}
\]

\[
C(t) = 63,000 \left(1 + \frac{0.0255}{12}\right)^{20 \times 12}
\]

20 years

Score 1: The student wrote the correct function, but did no further correct work.
Question 35

35 Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = 63,000 \left( \frac{1.0255}{12} \right)^{12t}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
\frac{100,000}{63,000} = \left( \frac{1.0255}{12} \right)^{12t}
\]

\[
\log \frac{1.5873615867}{1} = \log \left( \frac{1.0255}{12} \right)^{12t}
\]

\[
\log 1.5873615867 = \log (0.0255) = 12t
\]

\[
12 \cdot \log(0.0255) = 12t \cdot \log(0.0255)
\]

\[-0.6072565906 = t\]

\[t = 0.006 \text{ years}\]

Score 1: The student gave the wrong equation and made a transcription error and a rounding error.
Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account $t$ years after the account is opened, given that no more money is deposited into or withdrawn from the account.

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

Score 0:  The student wrote an expression, but showed no further correct work.
Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, \( C(t) \), that represents the amount of money in the account \( t \) years after the account is opened, given that no more money is deposited into or withdrawn from the account.

\[
C(t) = 63,000 \cdot e^{0.0255(t)}
\]

Calculate algebraically the number of years it will take for the account to reach $100,000, to the nearest hundredth of a year.

\[
C(t) = 63,000 \cdot e^{0.0255(t)}
\]

\[
C(t) = 63,000 \cdot 1.0179
\]

\[
C(t) = 63,913.3453
\]

\[
C(t) = 63,000 \cdot e^{0.0155(t)}
\]

\[
C(t) = 63,351.3891
\]

\[
C(t) = 63,000 \cdot e^{0.0383(t)}
\]

\[
C(t) = 67,186.78315
\]

\[
C(t) = 63,000 \cdot e^{0.178(t)}
\]

\[
C(t) = 99,479
\]

\[
C(t) = 63,000 \cdot e^{0.255(t)}
\]

\[
C(t) = 179,143.01101
\]

Score 0: The student gave the wrong equation and was unsuccessful using trial-and-error.
The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3} t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval \( 1 \leq t \leq 2 \).

\[
\frac{14 - 7}{1 - 2} = \frac{11}{-1} = \boxed{-11}
\]

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

Score 4: The student gave a complete and correct response.
The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3} t\right) + 8$, where $t$ represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston’s height on the interval $1 \leq t \leq 2$.

At what value(s) of $t$, to the nearest tenth of a second, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

Score 4: The student gave a complete and correct response.
36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval \( 1 \leq t \leq 2 \).

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

\[
\begin{array}{c|c|c}
\hline
\text{t} & h(t) & \text{Avg. Rate of Change} \\
\hline
1 & 14 & \frac{14 - 2}{1 - 2} = \frac{12}{-1} = -12 \\
2 & 2 & \\
\hline
\end{array}
\]

\[
0 = 12\cos\left(\frac{\pi}{3}t\right) + 8 \\
-8 = 12\cos\left(\frac{\pi}{3}t\right) \\
\frac{-8}{12} = \cos\left(\frac{\pi}{3}t\right) \\
\cos^{-1}\left(-\frac{2}{3}\right) = \frac{\pi}{3}t \\
\text{Ref \# = 0.841069} \\
A \approx 2.301 \quad \text{and} \quad 3.983 = \frac{\pi}{3}t \\
B \approx 3.983 \quad \text{and} \quad 3.983 = \frac{\pi}{3}t
\]

\boxed{3.2 = t \quad \text{and} \quad 3.8 = t}

\textbf{Score 4:} \quad \text{The student gave a complete and correct response.}
36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3} t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston’s height on the interval \( 1 \leq t \leq 2 \).

\[
\frac{\Delta y}{\Delta x} = \frac{12\cos\left(\frac{\pi}{3} t\right) + 8}{t - 2}
\]

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

\[
0 = 12\cos\left(\frac{\pi}{3} t\right) + 8
\]

\[
-8 = 12\cos\left(\frac{\pi}{3} t\right)
\]

\[
-\frac{8}{12} = \cos\left(\frac{\pi}{3} t\right)
\]

\[
-\frac{2}{3} = \cos\left(\frac{\pi}{3} t\right)
\]

\[
t = 2.2, 3.8
\]

Score 3: The student made a graphing error in the justification.
36 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$, where $t$ represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston’s height on the interval $1 \leq t \leq 2$.

\[
\frac{\Delta h}{\Delta t} = \frac{2\cdot 14}{2 - 1} = -\frac{12}{1} = -12
\]

At what value(s) of $t$, to the nearest tenth of a second, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

\[
\begin{align*}
X &= 2.2 \\
X &= 3.8
\end{align*}
\]

**Score 3:** The student did not provide a justification.
36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval \( 1 \leq t \leq 2 \).

Score 2: The student correctly computed the average rate of change.

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

Answer: Average rate of change is \(-12\).
36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3} t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston’s height on the interval \( 1 \leq t \leq 2 \).

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

2.2 and 3.8

Score 1: The student incorrectly computed the average rate of change and did not provide a justification.
36 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$, where $t$ represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \leq t \leq 2$.

\[
\text{AROC} = -12
\]

At what value(s) of $t$, to the nearest tenth of a second, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

\[
0 = 12\cos\left(\frac{\pi}{3}t\right) + 8
\]

\[
-8 = 12\cos\left(\frac{\pi}{3}t\right)
\]

\[
-\frac{8}{12} = \cos\left(\frac{\pi}{3}t\right)
\]

Use a calculator.

\[
2.3 \text{ seconds}
\]

Score 1: The student provided the correct average rate of change, but showed no work.
36 The height, \( h(t) \) in cm, of a piston, is given by the equation \( h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8 \), where \( t \) represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston’s height on the interval \( 1 \leq t \leq 2 \).

At what value(s) of \( t \), to the nearest tenth of a second, does \( h(t) = 0 \) in the interval \( 1 \leq t \leq 5 \)? Justify your answer.

**Score 0:** The student did not show enough correct work to receive any credit.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

\[
P(16) = \log(16 - 4) \\
= \log(12) \\
= 1.083 \\
= 1.1 \\
\]

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

**Score 6:** The student gave a complete and correct response.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

\[
1.6742 \rightarrow 1.7
\]

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2} x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

\[
14,000 \text{ weekly visits}
\]

Score 6: The student gave a complete and correct response.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

\[
1.1
\]

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

\[
14
\]

**Score 5:** The student gave 14 instead of 14,000.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Score 5: The student made an error graphing the \( x \)-intercept of \( y = P(x) \).
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Score 4: The student made a graphing error and stated 14 instead of 14,000.
37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the \textit{nearest tenth}?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Score 3: The student made a graphing error and did not attempt the third part.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

\[
\log(16000 - 4) = 4.2
\]

Graph \( y = P(x) \) on the axes below.

![Graph of \( y = P(x) \)](image)

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

**Score 2:** The student incorrectly substituted into \( P(x) \), but evaluated it correctly. The student only received one credit for graphing \( y = P(x) \) due to a lack of precision.
37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

\[
\frac{1}{2}x - 6 = 4 - 2
\]

\[
\frac{1}{2}x = 10.2
\]

Score 1: The student incorrectly substituted into \( P(x) \), but evaluated it correctly, and showed no further correct work.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2}x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Score 1: The student made a rounding error and multiple graphing errors, but received credit for reading the graph correctly in the third part.
Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is \( P(x) = \log(x - 4) \), where \( x \) is the number of visits per week in thousands and \( P(x) \) is the website’s popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph \( y = P(x) \) on the axes below.

An alternative rating model is represented by \( R(x) = \frac{1}{2} x - 6 \), where \( x \) is the number of visits per week in thousands. Graph \( R(x) \) on the same set of axes. For what number of weekly visits will the two models provide the same rating?

**Score 0:** The student incorrectly graphed the linear function and did nothing else.
37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where $x$ is the number of visits per week in thousands and $P(x)$ is the website’s popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

Graph $y = P(x)$ on the axes below.

An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where $x$ is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Score 0: The student did not do enough correct work to receive any credit.