The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Friday, June 21, 2019 — 1:15 to 4:15 p.m., only

MODEL RESPONSE SET

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25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

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Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

Score 2: The student gave a complete and correct response.
25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

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Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

\[
\frac{\Delta y}{\Delta x} = \frac{13.9 - 9.4}{4 - 1} = 1.5
\]

Interpret what this means in the context of the problem.

On average, the number of hours of daylight increased 1.5 hours per month from January - April.

Score 2: The student gave a complete and correct response.
Question 25

The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

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Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

\[
\frac{13.9 - 9.4}{4 - 1} = \frac{4.5}{3} = 1.5
\]

Interpret what this means in the context of the problem.

On average, the temperature increased by 1.5 degrees every month.

Score 1: The student gave an incorrect interpretation.
25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

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Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

\[
\frac{\Delta y}{\Delta x} = \frac{4.5}{3} = 1.5 \text{ hrs/month}
\]

Interpret what this means in the context of the problem.

Every month from January to April, there are 1.5 more hours of daylight.

Score 1: The student gave an incomplete interpretation.
The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

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Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

\[
\text{Jan} \rightarrow \text{Apr.} \quad \frac{9.4 - 13.9}{3} = \frac{4.5}{3}
\]

Interpret what this means in the context of the problem.

A means from January to April, the number of daylight hours increases by 1.5.

**Score 0:** The student found an incorrect average rate of change and wrote an incomplete interpretation.
26 Algebraically solve for $x$:

\[
\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}
\]

\[
\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}
\]

\[
\frac{(7)(x+1)}{(2x)(x+1)} - \frac{4x}{(2x)(x+1)} = \frac{1}{4}
\]

\[
\frac{3x + 7}{(2x)(x+1)} < \frac{1}{4}
\]

\[
(2x)(x+1) = (4)(3x+7)
\]

\[
2x^2 + 2x = 12x + 28
\]

\[
-12x - 2x
\]

\[
2x^2 - 10x = 28
\]

\[
-28 - 28
\]

\[
2x^2 - 10x - 28 = 0
\]

\[
2(x^2 - 5x - 14) = 0
\]

\[
2(x - 2)(x + 7) = 0
\]

$x = -2$, $x = 7$

Score 2: The student gave a complete and correct response.
26 Algebraically solve for $x$:

\[
\begin{align*}
28(x+1) - 14x &= 2x(x+1) \\
12x + 28 - 10x &= 2x^2 + 2x \\
0 &= 2x^2 - 12x + 2x - 28 \\
0 &= 2x^2 - 10x - 28 \\
(2x + 4)(x - 7) &= 0 \\
2x + 4 &= 0 \quad \Rightarrow \quad x - 7 = 0 \\
2x &= -4 \quad \Rightarrow \quad x = 7 \\
2 &= 2 \\
\therefore x &= 7 \\
\end{align*}
\]

**Score 2:** The student gave a complete and correct response.
Algebraically solve for $x$:

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\frac{7}{-y} = \frac{2}{-1} = \frac{1}{y}$$

$$\frac{7 - \frac{2}{x+1}}{2x + 1} = \frac{1}{y}$$

$$\frac{7x + 1}{2x^2 + 2x} - \frac{4x}{2x^2 + 2x} = \frac{1}{y}$$

$$\frac{3x + 7}{2x^2 + 2x} \times \frac{1}{y}$$

$$4(3x + 7) = 2x^2 + 2x$$

$$12x + 28 = ?x^2 + 12x$$

$$10x + 28 = 2x^2$$

$$2x^2 - 10x - 28 = 0$$

$$2x^2 + 4x - 10x - 20 = 0$$

$$2x(x + 2) - 10(x + 2) = 0$$

$$2x - 10 = 0, \quad x + 2 = 0$$

$$x = 5, \quad x = -2$$

$-2$ is extraneous root.

Score 1: The student incorrectly identified $-2$ as an extraneous root.
Algebraically solve for $x$:

\[
\frac{7x+2}{2x(x+1)} = \frac{1}{4}
\]

\[
\frac{7x+2 - 4x}{2x(x+1)} = \frac{1}{4}
\]

\[
2x(x+1) = 28x + 28 - 12x
\]

\[
2x^2 + 2x = 12x + 28
\]

\[
-12x - 28
\]

\[
2x^2 - 10x - 28 = 0
\]

\[
2(x^2 - 5x - 14) = 0
\]

\[
2(x + 2)(x - 7) = 0
\]

\[
2x + 2 = 0 \quad \text{or} \quad 2x - 14 = 0
\]

\[
x = -2 \quad \text{or} \quad x = 7
\]

\[
\text{or}
\]

\[
2x = -2 \quad \text{or} \quad 2x = 2
\]

\[
x = -1
\]

\[
\text{undefined}
\]

**Score 1:** The student made a computational error by not distributing the 2 correctly.
26 Algebraically solve for \( x \):

\[
\frac{7}{2x} - \frac{2}{x + 1} = \frac{1}{4}
\]

\[
\frac{7}{2x} \cdot \frac{x + 1}{x + 1} - \frac{2}{x + 1} \cdot \frac{2x}{2x} = \frac{1}{4}
\]

\[
\frac{7x + 7 - 4x}{2x(x + 1)} = \frac{1}{4}
\]

\[
\frac{3x + 7}{2x} = \frac{1}{4}
\]

\[
\frac{3x + 7}{2} = \frac{1}{4}
\]

\[
\frac{3x}{2} = \frac{3}{4}
\]

\[
3x = 0.75
\]

\[
\frac{3}{3} = \frac{0.75}{3}
\]

\[
x = \frac{9}{4}
\]

\[
x = 2.25
\]

\[
x = \frac{9}{4}
\]

**Score 0:** The student made a conceptual error and a computational error.
27 Graph \( f(x) = \log_2(x + 6) \) on the set of axes below.

**Score 2:** The student gave a complete and correct response.
27 Graph $f(x) = \log_2(x + 6)$ on the set of axes below.

Score 2: The student gave a complete and correct response.
27 Graph \( f(x) = \log_2(x + 6) \) on the set of axes below.

**Score 1:** The student made an error graphing the end behavior as \( x \to -6 \).
27 Graph \( f(x) = \log_2(x + 6) \) on the set of axes below.

**Score 0:** The student made multiple graphing errors.
28 Given \( \tan \theta = \frac{7}{24} \), and \( \theta \) terminates in Quadrant III, determine the value of \( \cos \theta \).

**Score 2:** The student gave a complete and correct response.
28 Given \( \tan \theta = \frac{7}{24} \), and \( \theta \) terminates in Quadrant III, determine the value of \( \cos \theta \).

\[
\begin{align*}
7^2 + 24^2 &= x^2 \\
49 + 576 &= x^2 \\
\sqrt{625} &= x \\
x &= 25
\end{align*}
\]

Score 2: The student gave a complete and correct response.
28 Given \( \tan \theta = \frac{7}{24} \), and \( \theta \) terminates in Quadrant III, determine the value of \( \cos \theta \).

\[
\cos Q = \frac{A}{H} = \frac{24}{25}
\]

Score 1: The student did not consider the quadrant.
28 Given \( \tan \theta = \frac{7}{24} \), and \( \theta \) terminates in Quadrant III, determine the value of \( \cos \theta \).

Score 0:  The student did not show enough correct work to receive any credit.
Question 29

29 Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[2]{x^2}\right)\left(\sqrt[3]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

\[
\left(\sqrt[2]{x^2}\right)\left(\sqrt[3]{x^3}\right) = \sqrt[2]{x^2} \cdot \sqrt[3]{x^3} = \sqrt[6]{x^6} = \sqrt[35]{x^6}
\]

She is not correct because when you convert the expression into radical form and multiply, add the exponents, the answer should be $\sqrt[35]{x^6}$.

Score 2: The student gave a complete and correct response.
29 Kenzie believes that for $x \geq 0$, the expression $\left( \frac{\sqrt[n]{x^2}}{\sqrt[n]{x^3}} \right)$ is equivalent to $\sqrt[n]{x^6}$. Is she correct? Justify your response algebraically.

Score 2: The student gave a complete and correct response. It is indicated that Kenzie is incorrect.
29 Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[2]{x^2}\right)\left(\sqrt[3]{x^3}\right)$ is equivalent to $\sqrt[35]{x^{6}}$. Is she correct? Justify your response algebraically.

\[
\left( x^{\frac{2}{3}} \right) \cdot \left( x^{\frac{3}{5}} \right) = x^{\frac{2}{3} + \frac{3}{5}} = x^{\frac{10 + 9}{15}} = x^{\frac{19}{15}}
\]

\[
\left( x^{\frac{2}{3}} \right) = \sqrt[3]{x^{2}}
\]

\[
\left( x^{\frac{3}{5}} \right) = \sqrt[5]{x^{3}}
\]

Yes, she is correct.

Score 1: The student applied exponent properties incorrectly.
29 Kenzie believes that for $x \geq 0$, the expression $\left( \sqrt[2]{x^2} \right) \left( \sqrt[3]{x^3} \right)$ is equivalent to $\sqrt[6]{35x^6}$. Is she correct? Justify your response algebraically.

$$\left( \sqrt[2]{x^2} \right) \left( \sqrt[3]{x^3} \right) = 1.84767919$$

$$\sqrt[6]{(2)^6} = 1.126178081$$

No, when plugging in a tester they are not the same.

Score 1: The student used a method other than algebraic by showing a contradiction.
29 Kenzie believes that for \( x \geq 0 \), the expression \( \frac{\sqrt[7]{x^2}}{\sqrt[5]{x^3}} \) is equivalent to \( \sqrt[35]{x^6} \). Is she correct? Justify your response algebraically.

\[
\frac{\sqrt[7]{x^2}}{\sqrt[5]{x^3}} = (x^2)^\frac{1}{7} \quad \frac{1}{5} \\
(x^2)^\frac{1}{7} \cdot (x^3)^\frac{1}{5} = x^\frac{\frac{2}{7} + \frac{3}{5}}{1} = x^\frac{35}{35} = (\sqrt[35]{x^6})^1
\]

**Score 0:** The student made multiple errors.
Question 30

30 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

\[
\frac{P(x)}{x-1} = x^2 + 7 + \frac{5}{x-1} \\
\begin{align*}
\times x(x-1) &+ 7(x-1) + \left(\frac{5}{x-1}\right)(x-1) \\
\times^3 - x^2 + 7x - 7 + 5 \\
P(x) = x^3 - x^2 + 7x - 2
\end{align*}
\]

Score 2: The student gave a complete and correct response.
30 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

Score 2: The student gave a complete and correct response.
30 When the function \( p(x) \) is divided by \( x - 1 \) the quotient is \( x^2 + 7 + \frac{5}{x - 1} \). State \( p(x) \) in standard form.

\[
\frac{p(x)}{x - 1} = x^2 + 7 + \frac{5}{x - 1} \\
\frac{(x-1)}{(x-1)} \left(x^2 + 7\right) + \frac{5}{x - 1} \\
x^3 - x^2 + 7x - 7 + 5 \\
x - 1 \\
x^3 - x^2 + 7x - 2 \\
x - 1 \\
p(x) = x^3 - x^2 + 7x - 2
\]

**Score 2:** The student gave a complete and correct response.
# Question 30

30 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

\[
\begin{align*}
&x^3 - x^2 + 7x - 7 + \frac{5}{x - 1} \\
&x^3 - x^2 + 12x - 12
\end{align*}
\]

**Score 1:** The student incorrectly distributed the $x - 1$ to the rational term.
30 When the function \( p(x) \) is divided by \( x - 1 \) the quotient is \( x^2 + 7 + \frac{5}{x - 1} \). State \( p(x) \) in standard form.

\[
x - 1 (x^2) = \boxed{x^3 - x^2} \]
\[
x - 1 (7) = \boxed{7x - 7} \]
\[
x - 1 \left( \frac{x^2}{x - 1} + 7 \right) \]
\[
x - 1 \sqrt{x^3 - x^2 + 7x - 7}
\]
\[
- x^3 + x^2
\]
\[
- 7x - 7
\]

**Score 1:** The student excluded the remainder.
30 When the function $p(x)$ is divided by $x-1$ the quotient is $x^2 + 7 + \frac{5}{x-1}$. State $p(x)$ in standard form.

Score 0: The student made an error distributing the $x^2$ and did not state $p(x)$ in standard form.
31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

\[ a_1 = 6 \]
\[ a_n = a_{n-1} \cdot 1.5 \]

Check:
\[ a_2 = a_1 \cdot 1.5 \]
\[ a_2 = 6 \cdot 1.5 \]
\[ a_2 = 9 \]
\[ a_2 = \sqrt{ } \checkmark \]

**Score 2:** The student gave a complete and correct response.
31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

\[ a_1 = 6 \\
\[ a_n = a_{n-1} \cdot \frac{3}{2} \]

Score 2: The student gave a complete and correct response.
31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

\[ a_1 = 6 \]
\[ a_n = a_1 \left( \frac{3}{2} \right)^{n-1} \]

**Score 1:** The student received credit for writing \( a_1 = 6 \).
31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

$$a_n = 1.5a_{n-1}$$

**Score 1:** The student did not write the initial term.
31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, \ldots

\[
\begin{align*}
\alpha_n &= 60 \\
\alpha_n &= 9, 1.5
\end{align*}
\]

Score 0: The student did not show enough correct work to receive any credit.
Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

\[
\text{Robin's coin } = \frac{43}{100} = .43
\]

\[
.499 \pm 2(.049) \rightarrow (.401, .597)
\]

Since .43 is within the interval of (.401, .597) her coin is likely not unfair.

**Score 2:** The student gave a complete and correct response.
Question 32

32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

\[ \frac{43}{100} = 0.43 \]

No because .43 falls inside the 95% student deviation.
32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

Score 1: The student gave a correct explanation based on an inappropriate interval.
32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

No, 43 is within 2 standard deviations from the mean.

Score 1: The student gave an explanation, but provided no statistical evidence.
32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Do the results of the simulation provide strong evidence that Robin’s coin is unfair? Explain your answer.

Yes, her coin is more than 1 standard deviation away. Although it isn’t more than 1.5 deviations, it is still much less than the mean.

Score 0: The student did not show enough correct statistical evidence to receive any credit.
33 Factor completely over the set of integers: $16x^4 - 81$

\[
16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3)
\]

Sara graphed the polynomial \( y = 16x^4 - 81 \) and stated “All the roots of \( y = 16x^4 - 81 \) are real.” Is Sara correct? Explain your reasoning.

\[
a = 16 \quad b = 0 \quad c = -81
\]

\[
4x^2 + 9 = 0 \quad 2x + 3 = 0 \quad 2x - 3 = 0
\]

\[
\sqrt{4x^2} = \pm 3 \quad \sqrt{9} = 3
\]

NO, when you make mini equations, \( 4x^2 + 9 = 0 \) can be solved for \( x \), but your answer is an imaginary number, meaning not all roots of \( y = 16x^4 - 81 = 0 \) are real.

**Score 4:** The student gave a complete and correct response.
Factor completely over the set of integers: $16x^4 - 81$

$$\frac{4x^2 - 9}{16} \cdot \frac{4x^2 + 9}{81} = \frac{(2x - 3)(2x + 3)(4x^2 + 9)}{16 \cdot 81}$$

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

Sara is incorrect. Although 2 of the roots are real, we know at least one root is nonreal because, when using the quadratic formula to determine the roots for the factor $4x^2 + 9$, the discriminant is negative.

Score 4: The student gave a complete and correct response.
33 Factor completely over the set of integers: $16x^4 - 81$

\[(4x^2 - 9)(4x^2 + 9)(2x-3)(2x+3)(4x^2 + 9)\]

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

No because the graph only crosses the x-axis two times meaning only 2 real roots not 4.

Score 4: The student gave a complete and correct response.
33 Factor completely over the set of integers: $16x^4 - 81$

\[
\begin{align*}
16x^4 - 81 &= (4x^2 - 9)(4x^2 + 9) \\
&= (2x + 3)(2x - 3)(2x + 3)(2x - 3) \\
&= (2x + 3)^2(2x - 3)
\end{align*}
\]

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

No. Because it has 4 possible zeros, but only crosses the x-axis twice. (Graphing calculator)

Score 3: The student made one factoring error.
Question 33

33 Factor completely over the set of integers: $16x^4 - 81$

$$
\begin{align*}
(4x^2 - 9)(4x^2 + 9) \\
\quad \quad \quad 4x^2 - 9 = 0 \\
\quad \quad \quad 4x^2 = 9 \\
\quad \quad \quad x^2 = \frac{9}{4} \\
\quad \quad \quad x = \pm \frac{3}{2}
\end{align*}
$$

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

Not all of the roots of $16x^4 - 81$ are real, because if it would be real it wouldn’t had an imaginary number.
33 Factor completely over the set of integers: $16x^4 - 81$

\[
(4x^2 - 9)(4x^2 + 9)
\]

\[
(2x + 3)(2x - 3)(2x + 3i)(2x - 3i)
\]

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

Roots: $\frac{-3}{2}, \frac{3}{2}, \frac{3i}{2}, \frac{-3i}{2}$

No, $b/c \frac{3i}{2}$ and $\frac{-3i}{2}$ are imaginary #’s

Score 3: The student did not factor over the set of integers.
33 Factor completely over the set of integers: $16x^4 - 81$

$$\frac{16x^4 - 81}{(4x^2+9)(4x^2-9)}$$

$$= \frac{(2x+3)(2x-3)}{2x+3} \frac{4x^2-9}{2x-3}$$

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

Sara is incorrect because if you plug into $y = \ldots$ and go to 2nd graph to the table and scroll up you can see there are some unreal roots.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17</td>
<td>1.34e6</td>
</tr>
<tr>
<td>-16</td>
<td>1.05e6</td>
</tr>
<tr>
<td>-15</td>
<td>809,919</td>
</tr>
<tr>
<td>-14</td>
<td>6,145,75</td>
</tr>
</tbody>
</table>

Score 2: The student only received credit for factoring completely.
33 Factor completely over the set of integers: $16x^4 - 81$

$$16x^4 - 81 = (4x^2 - 9)(4x^2 + 9) = (2x - 3)(2x + 3)(4x^2 + 9)$$

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.” Is Sara correct? Explain your reasoning.

$$2x - 3 = 0 \quad 2x + 3 = 0$$
$$2x = 3 \quad 2x = -3$$
$$x = \frac{3}{2} \quad x = -\frac{3}{2}$$

No, because 1.5 is a real number, but -1.5 is not a real number because it is negative.

Score 1: The student made one factoring error and gave an incorrect explanation.
33 Factor completely over the set of integers: $16x^4 - 81$

\[ 16x^4 - 81 \]

Sara graphed the polynomial $y = 16x^4 - 81$ and stated “All the roots of $y = 16x^4 - 81$ are real.”

Is Sara correct? Explain your reasoning.

Sarah is incorrect because some of the roots are imaginary.

Score 0: The student’s explanation was not sufficient to receive any credit.
The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years.

\[
s(t) = 200 \left( \frac{1}{2} \right)^{\frac{t}{15}}
\]

Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

\[
\frac{20}{200} = 200 \left( \frac{1}{2} \right)^{\frac{t}{15}}
\]

\[
\frac{1}{10} = \left( \frac{1}{2} \right)^{\frac{t}{15}}
\]

\[
\log(\frac{1}{10}) = \frac{t}{15} \log(\frac{1}{2})
\]

\[
3.32193 = \frac{t}{15}
\]

\[
49.8289 = t
\]

50. years

**Score 4:** The student gave a complete and correct response.
34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years.

\[
s(t) = 200 \left( \frac{1}{2} \right)^{t/15}
\]

Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

\[
\frac{1}{10} = 200 \left( \frac{1}{2} \right)^{t/15}
\]

\[
\frac{1}{200} = \left( \frac{1}{2} \right)^{t/15}
\]

\[
100 \left( \frac{1}{200} \right) = \frac{t}{15} \log \left( \frac{1}{2} \right)
\]

\[
t = 1041 \text{ years}
\]

**Score 3:** The student made an error assuming that \( \frac{1}{10} \) of a gram of the substance remained.
Question 34

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after $t$ years.

$$s(t) = 200 (0.5)^t$$

Determine algebraically, to the nearest year, how long it will take for $\frac{1}{10}$ of this substance to remain.

$$\frac{1}{10} \cdot 200 = 20$$

$$20 = 200 (0.5)^t$$

$$0.1 = (0.5)^t$$

$$\log 0.1 = \log (0.5)^t$$

$$\log 0.1 = t \log 0.5$$

$$t = \frac{\log 0.1}{\log 0.5}$$

$$t = 3.32 \text{ years}$$

Score 3: The student made an error writing the equation for $s(t)$, assuming $t$ was the number of half-lives.
The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years.

\[
S(t) = 200 \left( \frac{1}{2} \right)^t
\]

Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

\[
\frac{1}{10} = \frac{200}{200} \left( \frac{1}{2} \right)^t
\]

\[
\log \left( \frac{1}{200} \right) = \log \left( \frac{1}{2} \right)^t
\]

\[
\log \left( \frac{1}{200} \right) = t \log \frac{1}{2}
\]

\[
t = \frac{\log \frac{1}{200}}{\log \frac{1}{2}}
\]

\[
t = 10.96578428
\]

\[
11 \text{ years}
\]

**Score 2:** The student wrote an incorrect equation and made an error assuming \( \frac{1}{10} \) of a gram of the substance remained.
The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years.

\[
\frac{200}{100} = 200(1 - r)^{\frac{15}{t}}
\]

Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

\[
\frac{20}{200} = 200(1 - 0.045)^x
\]

\[
x = \frac{\log(0.1955)}{\log(0.956)}
\]

It will take 50 years to only have \( \frac{1}{10} \) of the substance to remain.

Score 1: The student received no credit for the first part and showed incomplete algebraic work on the second part.
The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, \( s(t) \), of 200 grams of this substance that remains after \( t \) years.

\[
s(t) = 200 \left( \frac{1}{2} \right)^{\frac{t}{15}}
\]

Determine algebraically, to the nearest year, how long it will take for \( \frac{1}{10} \) of this substance to remain.

\[
s(t) = 200 \left( \frac{1}{2} \right)^{\frac{t}{15} \cdot \frac{5}{6}} \rightarrow 71 \text{ years}
\]

Score 1: The student received 1 credit for the equation.
34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after $t$ years.

$$s(t) = 200(1 - 0.15)^t$$

Determine algebraically, to the nearest year, how long it will take for $\frac{1}{10}$ of this substance to remain.

\[
\begin{align*}
20g &= 200(0.85)^t \\
\frac{20}{200} &= \frac{200}{200} (0.85)^t \\
0.1 &= 0.85^t \\
\log_{0.85} 0.1 &= t \log_{0.85} 0.85 \\
13 &\text{ years}
\end{align*}
\]

Score 0: The student did not show enough correct work to receive any credit.
35 Determine an equation for the parabola with focus $(4, -1)$ and directrix $y = -5$. (Use of the grid below is optional.)

\[ \sqrt{(x-4)^2 + (y+1)^2} = (y+5)^2 \]

\[ x^2 - 8x + 16 + y^2 + 2y + 1 = (y+5)^2 \]

\[ x^2 - 8x + 16 - 24 = 0 \]

\[ \frac{x^2 - 8x - 8}{8} = \frac{2y}{8} \]

\[ y = \frac{1}{8}x^2 - x - 1 \]

**Score 4:** The student gave a complete and correct response.
35 Determine an equation for the parabola with focus \((4, -1)\) and directrix \(y = -5\).
(Use of the grid below is optional.)

\[
\gamma = \frac{1}{2(-1-(-5))} (x-4)^2 + \frac{-1+(-5)}{2}
\]

\[
\gamma = \frac{1}{8} (x-4)^2 - 3
\]

**Score 4:** The student gave a complete and correct response.
Question 35

Determine an equation for the parabola with focus $(4, -1)$ and directrix $y = -5$. 
(Use of the grid below is optional.)

\[ y = \frac{1}{4p} (x-h)^2 - k \]
\[ y = \frac{-1}{8} (x-4)^2 - 3 \]

\[ y = -3 \quad x = 4 \]
\[ y = \frac{-5+1}{2} = \frac{-6}{2} = -3 \]

Score 3: The student used an incorrect value for $p$. 

Algebra II – June ’19 [59]
35 Determine an equation for the parabola with focus $(4, -1)$ and directrix $y = -5$.
(Use of the grid below is optional.)

$y = \left( x - 4 \right)^2 - 3$

Score 2: The student correctly found the vertex and received 1 credit for the equation.
35 Determine an equation for the parabola with focus $(4, -1)$ and directrix $y = -5$.

(Use of the grid below is optional.)

\[ y = \frac{1}{8} (x - 4) - 3 \]

Score 2: The student correctly found the vertex and received 1 credit for the equation.
35 Determine an equation for the parabola with focus (4, −1) and directrix \( y = −5 \).
(Use of the grid below is optional.)

\[ f(x) = (x^2 - 4) - 3 \]

**Score 1:** The student correctly found the vertex, but made multiple errors writing the equation.
35 Determine an equation for the parabola with focus \((4, -1)\) and directrix \(y = -5\).

(Use of the grid below is optional.)

\[
y = \frac{1}{8}(x - 4) - 5
\]

\[
\frac{1}{4p} = \frac{1}{8}
\]

\[
p = 2
\]

Score 0: The student did not show enough correct work to receive any credit.
36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

<table>
<thead>
<tr>
<th>Practice Time</th>
<th>Juan Wins</th>
<th>Filipe Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Long</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[ P(F|L) = \frac{12}{27} \]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[ P(F|L) \neq \frac{P(F)}{P(L)} \]

\[ \frac{12}{27} \neq \frac{22}{45} \]

\[ 0.44 \neq 0.488 \quad \text{not independent} \]

Score 4: The student gave a complete and correct response.
Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

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<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[
P(F \mid L) = \frac{12}{27}
\]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[
P(F \text{ and } L) = P(F) \cdot P(L)
\]

\[
\frac{12}{45} = \frac{22}{45} \cdot \frac{27}{45}
\]

\[
0.2667 \neq 0.2933
\]

No, the two events are not independent.

**Score 4:** The student gave a complete and correct response.
Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

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<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[
P(F|S^c) = \frac{P(F \cap S^c)}{P(S^c)} = \frac{12/45}{27/45} = 0.160 \quad \text{(or 16%)}
\]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[
P(F \cap S^c)^2 = P(F) \cdot P(S^c)
\]

\[
\frac{12}{45} \neq \left( \frac{22}{45} \right) \left( \frac{27}{45} \right)
\]

0.267 \neq 0.293

Not independent

**Score 3:** The student made a computational error finding \(p(f|s^c)\).
36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

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<th></th>
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</tr>
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<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[ P(F|L) = \frac{12}{27} = 0.44 \]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[ P(A) = P(A|B) \]
\[ P(F)^2 = P(F|B) \]
\[ \frac{22}{45} = 0.44 \]
\[ \frac{12}{27} \neq 0.44 \]
not independent

**Score 3:** The student made an error rounding to 0.48.
Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

<table>
<thead>
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<th>Filipe Wins</th>
</tr>
</thead>
<tbody>
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<td>10</td>
</tr>
<tr>
<td>Long Practice Time</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[
\frac{12}{15+12} = \frac{12}{27}
\]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

The events of “Filipe wins” and “long practice” are dependent on one another because as “long practices” are done, the less times the event of “Filipe wins”,

**Score 2:** The student only received credit for the first part.
Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

<table>
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<tr>
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<th>Filipe Wins</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>Long Practice Time</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

\[
\frac{12}{27}
\]

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[
\frac{2a}{45} = \frac{18}{27}
\]

No, because they are different %.

Score 1: The student received one credit for \(\frac{12}{27}\).
36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

<table>
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</tr>
</thead>
<tbody>
<tr>
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<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

Probability that Filipe wins the next match is \( \frac{13}{28} \)

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

\[ \frac{12}{27} = 0.444 \approx 44\% \]

They're independent to each other because they don't have the same proportion, it's only a 44% between them.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13\cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

\[
\begin{align*}
B P & : 2\pi \\
B & : 3\pi \\
(0) & : 3\pi \\
P & : 2.5
\end{align*}
\]

Interpret what the period represents in this context.

The period of \( f(t) \) represents the amount of time it would take the tire to spin one full rotation.

Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.
On the grid below, graph \textit{at least one} cycle of \( f(t) \) that includes the \( y \)-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

\textit{No} because at its max height, the tire can only reach 26 feet, as proven through adding the absolute value of \( a \) (13) to \( d \) (the midpoint, 13).
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13\cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

\[
\text{period} = \frac{2\pi}{0.8\pi} = 2.5 \text{ seconds}
\]

Interpret what the period represents in this context.

**Score 6:** The student gave a complete and correct response.
On the grid below, graph at least one cycle of \( f(t) \) that includes the \( y \)-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the maximum of the slousoideal curve is 26.
Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where $t$ represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of $f(t)$.

\[ \text{period is } 2.5, \]

Interpret what the period represents in this context.

\[ \text{It takes } 2.5 \text{ seconds for the nail to do a full revolution on the tire.} \]

**Question 37 is continued on the next page.**

**Score 5:** The student made one graphing error.
Question 37 continued.

On the grid below, graph at least one cycle of $f(t)$ that includes the $y$-intercept of the function.

![Graph](image)

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, it peaks at 28 inches.
Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where $t$ represents the time (in seconds) since the nail first became caught in the tire. Determine the period of $f(t)$.

Interpret what the period represents in this context.

Score 4: The student did not interpret the period and gave an incomplete justification in the last part.
On the grid below, graph at least one cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, it does not because the cosine graph's amplitude is 13 and the midline is 13.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where $t$ represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of $f(t)$.

$$\frac{\pi}{0.8\pi} = 2.5 \text{ seconds},$$

Interpret what the period represents in this context.

It takes 2.5 seconds for the nail to go from highpoint back to highpoint.

**Score 3:** The student made a labeling error on the graph and did not answer the last part.
On the grid below, graph at least one cycle of \( f(t) \) that includes the \( y \)-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13\cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

The period is \( 0.8 \) second, this represents how many times it takes for the nail to reach the original height it came stuck at.

Interpret what the period represents in this context.
Question 37 continued.

On the grid below, graph *at least one* cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where $t$ represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of $f(t)$.

\[
\begin{align*}
\frac{\pi}{0.8\pi} &= \frac{1}{0.8} \\
p &= 2.5
\end{align*}
\]

Interpret what the period represents in this context.

If it takes $2.5$ seconds for the nail to complete $\frac{5}{9}$ of a full rotation, then the period is the time it takes to complete one full rotation.

Question 37 is continued on the next page.

Score 2: The student received credit for the period and the interpretation.
On the grid below, graph *at least one* cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where $t$ represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of $f(t)$.

Interpret what the period represents in this context.

Score 2: The student drew a correct graph.
On the grid below, graph at least one cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the nail would not be short enough to go in the fire. It would just tip over.
Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13\cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

\[
\text{Period} = \frac{2\pi}{\text{Frequency}} = \frac{2\pi}{0.8} = 2.5
\]

Interpret what the period represents in this context.

Score 1: The student received credit for correctly finding the period.
On the grid below, graph at least one cycle of \( f(t) \) that includes the \( y \)-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

NO
Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function \( f(t) = -13\cos(0.8\pi t) + 13 \), where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

Interpret what the period represents in this context.

\[ f(t) \text{ represents one cycle of the wheel.} \]

\[ f(t) \text{ represents how high the nail is from the ground.} \]

Score 1: The student received one credit for the graph.
On the grid below, graph *at least one* cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.
37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function 

\[ f(t) = -13\cos(0.8\pi t) + 13, \]

where \( t \) represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of \( f(t) \).

Interpret what the period represents in this context.

\( f(t) \) represents one cycle of the wheel, and how high the nail is from the ground.

Question 37 is continued on the next page.

Score 0:  The student did not show enough correct work to receive any credit.
On the grid below, graph *at least one* cycle of $f(t)$ that includes the $y$-intercept of the function.

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the maximum height is 23.5 inches.