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25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = (\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}
\]

\[
x = 4 \pm \sqrt{-36}
\]

\[
x = 4 \pm 6i
\]

Yes, the equation has imaginary solutions because there is a negative number under the radical.

**Score 2:** The student gave a complete and correct response.
25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

Yes the equation does have imaginary solutions because when you graph it, it doesn’t pass through the x axis. Meaning it has no real roots or solutions.

Score 2: The student gave a complete and correct response.
Question 25

25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

\[ \sqrt{b^2-4ac} \]

\[ (-4)^2 + 4(1)(13) \]

\[ \sqrt{68} \]

No, the # under the $\sqrt{}$ is positive, which means it is not imaginary.

Score 1: The student used the wrong formula for the discriminant.
Question 25

Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

\[
\begin{align*}
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} \\
& = \frac{4 \pm \sqrt{16 - 52}}{2} \\
& = \frac{4 \pm \sqrt{-36}}{2} \\
& = \frac{4 \pm 6i}{2} \\
& = 2 \pm 3i
\end{align*}
\]

Yes, it does have imaginary solutions.

Score 1: The student simplified incorrectly.
Question 25

25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

\[ x^2 - 4x + 13 = 0 \]
\[ -13 = -13 \]
\[ x - 4x = -13 + 4 \]
\[ x - 4x + 2x = \bar{9} \]
\[ (x - 2)(x + 2) = 3 \]

No, the equation $x^2 - 4x + 13 = 0$ doesn’t have imaginary solution, it has only one solution which is 3.

Score 0: The student gave a completely incorrect response.
Question 25

25 Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

\[
\begin{align*}
x^2 - 4x + 13 &= 0 \\
x &= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} \\
x &= \frac{4 \pm \sqrt{-36}}{2} \\
x &= 2 \pm \sqrt{-36} \\
x &= 2 \pm 6i
\end{align*}
\]

Score 0: The student made a simplification error and did not indicate the solutions are imaginary.
26 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

\[ S_n = \frac{a_1 - a_1 r^n}{1 - r} \]

\[ S_5 = \frac{6 - 6(0.80)^5}{1 - 0.80} \]

\[ a_1 = 6 \]
\[ r = 0.80 \]

\[ S_5 = 20.17 \]

A child travels 20.17 feet in total in the first five swings.

**Score 2:** The student gave a complete and correct response.
The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

\[
\begin{align*}
1\text{st swing} &= 6 \text{ feet} \\
2\text{nd swing} &= 4.8 \text{ feet} \\
3\text{rd swing} &= 3.84 \text{ feet} \\
4\text{th swing} &= 3.072 \text{ ft} \\
5\text{th swing} &= 2.4576 \text{ ft} \\
\text{Total distance} &= 20.1696 \text{ ft}
\end{align*}
\]

Score 2: The student gave a complete and correct response.
26 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

\[ A = a_n \cdot 0.8 \]
\[ = 6 \cdot 0.8 = 4.8 \]
\[ 4.8 \cdot 0.8 = 3.84 \]
\[ 3.072 \]
\[ 2.4576 \]
\[ 1.9600685 \]

Total = 16.14 feet

Score 1: The student calculated the sum of swings 2 through 6.
Question 26

26 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

Swing 1: 6 ft

6 \times 0.8 = 4.8 ft

Swing 2: 4.8 ft

4.8 \times 0.8 = 3.84 ft

Swing 3: 3.84 ft

3.84 \times 0.8 = 3.072 ft

Swing 4: 3.072 ft

3.072 \times 0.8 = 2.4576 ft

Swing 5: 2.4576 ft

6 + 4.8 + 3.84 + 3.072 + 2.4576 = 20.7 ft

Score 1: The student wrote 20.7 instead of 20.17.
The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

Score 0: The student did not show enough correct work to receive any credit.
Question 27

27 Solve algebraically for $n$: \[ \frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}. \]

\[ \frac{2}{n^2} + \frac{3}{n} \cdot \frac{n}{n} = \frac{4}{n^2} \]
\[ \frac{2}{n^2} + \frac{3n}{n^2} = \frac{4}{n^2} \]
\[ 2 + 3n = 4 \]
\[ 3n = 2 \]
\[ n = \frac{2}{3} \]

Score 2: The student gave a complete and correct response.
Question 27

27 Solve algebraically for \( n \):
\[
\frac{2}{x^2} + \frac{3}{x} = \frac{4}{x^2}
\]

\[
2 + 3n = 4
\]

\[
3n = 2
\]

\[
n = \frac{2}{3}
\]

Score 2: The student gave a complete and correct response.
27 Solve algebraically for \( n \): \( \frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2} \).

\[
\begin{align*}
\frac{n^3}{n^2} + \frac{2n^2}{n^2} + \frac{3n}{n^2} &= \frac{4}{n^2} \\
2n + 3n^2 &= 4n \\
-4n &= -4n \\
-2n + 3n^2 &= 0 \\
(-2 + 3n) &= 0 \\
\boxed{n = 0} & \\
-2 + 3n &= 0 \\
+2 & \\
3n &= \frac{2}{3} \\
\boxed{n = \frac{2}{3}}
\end{align*}
\]

**Score 1:** The student did not reject the extraneous solution.
27 Solve algebraically for $n$: \( \frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2} \).

\[
2n^{-2} + 3n^{-1} = 4n^{-2} \\
\frac{3n^{-1}}{n^{-2}} = \frac{2n^{-2}}{n} \\
3 = \frac{2n^{-2}}{n} \\
\frac{3}{2} = \frac{2n}{n} \\
\boxed{n = \frac{3}{2}}
\]

**Score 1:** The student incorrectly simplified the right side of the equation.
27 Solve algebraically for \( n \): \[ \frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}. \]

Score 0: The student made multiple errors.
28 Factor completely over the set of integers:

\[-2x^4 + x^3 + 18x^2 - 9x\]

\[
-2x^4 + x^3 + 18x^2 - 9x
\]

\[
x^3 (-2x + 1) - 9x (-2x + 1)
\]

\[
(x^3 - 9x)(-2x + 1)
\]

\[
x(x^2 - 9)(-2x + 1)
\]

\[
x(x + 3)(x - 3)(-2x + 1)
\]

**Score 2:** The student gave a complete and correct response.
28 Factor completely over the set of integers:

\[
-2x^4 + x^3 + 18x^2 - 9x
\]

\[
= -x^2(2x^2-x) + 9(2x^2-x)
\]

\[
= -(x^2 - 9)(2x^2 - x)
\]

\[
= -(x-3)(x+3)(2x-1)
\]

Score 2: The student gave a complete and correct response.
28 Factor completely over the set of integers:

\[-2x^4 + x^3 + 18x^2 - 9x\]

\[-x^3(2x-1) + 9x(2x-1)\]

\[(-x^3 + 9x)(2x-1)\]

\[x(-x^2 + 9)(2x-1)\]

**Score 1:** The student did not factor completely.
28 Factor completely over the set of integers:

\[-2x^4 + x^3 + 18x^2 - 9x\]

\[-x^3(2x-1) + 9x(2x-1)\]

\[(-x^3 + 9x)(2x-1)\]

\[-x(\frac{3}{2} + 9)(2x-1)\]

**Score 1:** The student made one factoring error.
28 Factor completely over the set of integers:

\[-2x^4 + x^3 + 18x^2 - 9x\]

\[x(-2x^3 + x^2 + 18x - 9)\]

**Score 0:** The student did not show enough correct work to receive any credit.
The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Wear Glasses</th>
<th>Don’t Wear Glasses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue Eyes</td>
<td>0.14</td>
<td>0.26</td>
</tr>
<tr>
<td>Brown Eyes</td>
<td>0.11</td>
<td>0.24</td>
</tr>
<tr>
<td>Green Eyes</td>
<td>0.10</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

\[
P(LIS) = \frac{14}{35} = 0.4 \quad P(LIS) \neq P(L)\]

\[P(L) = 0.4\]

\[0.4 = 0.4\]

They are independent because \(P(LIS) = P(L)\)

**Score 2:** The student gave a complete and correct response.
The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

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<td>0.10</td>
</tr>
</tbody>
</table>

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

\[
(0.14)(0.38) = 0.14 \\
0.14 = 0.14 \checkmark
\]

Yes, they are independent because the values are equal.

Score 2: The student gave a complete and correct response.
The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

<table>
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<tr>
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</tr>
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</table>

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

\[
P(A) = P(A|B)
\]

\[
0.4 = \frac{0.4}{0.35}
\]

\[
0.4 \neq 1.11428...
\]

No, they are not equal, making them not independent from each other.

Score 1: The student made one error in determining \(P(A|B)\).
29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

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Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

\[
\begin{align*}
P(A) &= \frac{4}{1} = 0.4 \\
P(B) &= \frac{35}{1} = 0.35 \\

P(A \text{ and } B) &= P(A)P(B) \\
0.14 &= 0.14
\end{align*}
\]

not independent  \text{ b/c prob } =

**Score 1:** The student made an incorrect conclusion based on appropriate work.
The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

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Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

No, because
\[ P(A \cap B) = P(A) \times P(B) \]
\[ P(A \cap B) = 0.14 \times 0.35 \]
Since they do not equal having blue eyes and wearing glasses, the events are not independent.

**Score 0:** The student did not show enough correct work to receive any credit.
29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

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<td>0.15</td>
</tr>
</tbody>
</table>

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

\[ \frac{0.14}{0.30} = 0.466... = 47\% \]

\[ \frac{0.14}{0.35} = 0.4 = 40\% \]

\[ 40\% \neq 47\% \]

\[ \therefore \text{is dependent} \]

**Score 0:** The student made a calculation error and did not complete a test for independence.
For $x \neq 0$ and $y \neq 0$, $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$. Determine the value of $a$.

\[
\sqrt[3]{81x^{15}y^9} = (3^a x^5 y^3)^3
\]

\[
81x^{15}y^9 = 3^{3a}x^{15}y^9
\]

\[
\frac{81}{3} = 3^{3a}
\]
\[
4 = 3^{3a}
\]
\[
\frac{4}{3} = \frac{3a}{3}
\]
\[
a = \frac{4}{3}
\]

**Score 2:** The student gave a complete and correct response.
30 For \( x \neq 0 \) and \( y \neq 0 \), \( \sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3 \). Determine the value of \( a \).

\[
\left( \sqrt[3]{81x^{15}y^9} \right) = \left( 3^a x^5 y^3 \right)^3 \\
\frac{81x^{15}y^9}{x^{15}y^9} = \frac{3^{3a} x^{15} y^9}{x^{15} y^9} \\
\log(81) = \log(3^{3a}) \\
\frac{10 \log(81)}{10 \log(3)} = \frac{2a}{10 \log(3)} \\
\frac{4}{3} = \frac{2a}{10 \log(3)} \\
a = \frac{3}{2} \\
q = \sqrt[3]{\frac{4}{3}}
\]

Score 2: The student gave a complete and correct response.
Question 30

30 For \( x \neq 0 \) and \( y \neq 0 \), \( \sqrt[3]{81x^{15}y^9} = 3^{a}x^5y^3 \). Determine the value of \( a \).

\[
\sqrt[3]{81x^{15}y^9} = \sqrt[3]{3^{4}x^{15}y^{9}} = \sqrt[3]{3^{4}} \cdot \sqrt[3]{x^{15}} \cdot \sqrt[3]{y^{9}} = 3^{\frac{4}{3}} \cdot x^{5} \cdot y^{3} = 3^{a}x^5y^3
\]

\[
3^{\frac{4}{3}} = 3^{a}
\]

\[
a = \frac{4}{3}
\]

Score 1: The student multiplies exponents instead of adding.
30 For $x \neq 0$ and $y \neq 0$, $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$. Determine the value of $a$.

\[
\sqrt[3]{81x^{15}y^9} = 3^a = x^5 y^3
\]

\[
\frac{8/3 \cdot x^{15/3} \cdot y^{9/3}}{\frac{27x^5y^3}{3^a}} = \frac{3^a x^5 y^3}{3^a}
\]

\[
a = 3
\]
30 For $x \neq 0$ and $y \neq 0$, $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$. Determine the value of $a$.

\[ \sqrt[3]{81x^{15}y^9} = (3^a x^5 y^3)^2 \]

\[ \frac{81x^{15}y^9}{3} = 3^{2a} x^{10} y^{6} \]

\[ \frac{81x^{15}y^9}{3} = 3^{2a} x^{10} y^{6} \]

\[ \frac{81x^{5}y^3}{3} = 3^{2a} \]

\[ \frac{27x^{5}y^3}{a} = \frac{3^{2a}}{a} \]

\[ 13.5x^{5}y^3 = a \]

**Score 0:** The student gave a completely incorrect response.
31 Graph \( y = 2\cos\left(\frac{1}{2}x\right) + 5 \) on the interval \([0, 2\pi]\), using the axes below.

*Score 2:* The student gave a complete and correct response.
31 Graph \( y = 2\cos\left(\frac{1}{2}x\right) + 5 \) on the interval \([0, 2\pi]\), using the axes below.

Score 1: The student used an incorrect period.
31 Graph \( y = 2\cos\left(\frac{1}{2}x\right) + 5 \) on the interval \([0, 2\pi]\), using the axes below.

Score 1: The student made one graphing error at \( x = \pi \).
Question 31

31 Graph \( y = 2\cos\left(\frac{1}{2}x\right) + 5 \) on the interval \([0, 2\pi]\), using the axes below.

Score 0:  The student did not show enough correct work to receive any credit.
32 A cup of coffee is left out on a countertop to cool. The table below represents the temperature, \( F(t) \), in degrees Fahrenheit, of the coffee after it is left out for \( t \) minutes.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 t & 0 & 5 & 10 & 15 & 20 & 25 \\
\hline
 F(t) & 180 & 144 & 120 & 104 & 93.3 & 86.2 \\
\hline
\end{array}
\]

Based on these data, write an exponential regression equation, \( F(t) \), to model the temperature of the coffee. Round all values to the nearest thousandth.

\[
F(t) = 169.136 (0.971)^t
\]

**Score 2:** The student gave a complete and correct response.
32 A cup of coffee is left out on a countertop to cool. The table below represents the temperature, \( F(t) \), in degrees Fahrenheit, of the coffee after it is left out for \( t \) minutes.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( 0 )</th>
<th>( 5 )</th>
<th>( 10 )</th>
<th>( 15 )</th>
<th>( 20 )</th>
<th>( 25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(t) )</td>
<td>180</td>
<td>144</td>
<td>120</td>
<td>104</td>
<td>93.3</td>
<td>86.2</td>
</tr>
</tbody>
</table>

Based on these data, write an exponential regression equation, \( F(t) \), to model the temperature of the coffee. Round all values to the nearest thousandth.

\[
y = a \cdot b^x
\]

\[
a = 169.136
\]

\[
b = 0.971
\]

**Score 1:** The student made a notation error by not using \( F(t) \) and \( t \).
A cup of coffee is left out on a countertop to cool. The table below represents the temperature, $F(t)$, in degrees Fahrenheit, of the coffee after it is left out for $t$ minutes.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
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</tr>
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<tbody>
<tr>
<td>$F(t)$</td>
<td>180</td>
<td>144</td>
<td>120</td>
<td>104</td>
<td>93.3</td>
<td>86.2</td>
</tr>
</tbody>
</table>

Based on these data, write an exponential regression equation, $F(t)$, to model the temperature of the coffee. Round all values to the nearest thousandth.

$$F(t) = 171.426(0.970)^t$$

**Score 0:** The student made a notation error and wrote an exponential function with incorrect values.
A cup of coffee is left out on a countertop to cool. The table below represents the temperature, $F(t)$, in degrees Fahrenheit, of the coffee after it is left out for $t$ minutes.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
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</tr>
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<td>180</td>
<td>144</td>
<td>120</td>
<td>104</td>
<td>93.3</td>
<td>86.2</td>
</tr>
</tbody>
</table>

Based on these data, write an exponential regression equation, $F(t)$, to model the temperature of the coffee. Round all values to the nearest thousandth.

$$F(t) = 0.141e^{-7.171t} + 78.525$$

**Score 0:** The student made a notation error and used a quadratic regression.
33 On the set of axes below, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

State the number of solutions to the equation $f(x) = g(x)$.

$$x^3 - 3x^2 = 2x - 5$$

$$\begin{array}{c|c}
-5 & -2x + 5 \\
\hline
+1 & 0
\end{array}$$

$$x^3 - 3x^2 - 2x + 5 = 0$$

Score 4: The student gave a complete and correct response.
33 On the set of axes below, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

State the number of solutions to the equation $f(x) = g(x)$.

3

**Score 4:** The student gave a complete and correct response.
33 On the set of axes below, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

State the number of solutions to the equation $f(x) = g(x)$.

[Graph of $f(x)$ and $g(x)$]

There are no solutions.

**Score 3:** The student incorrectly stated the number of solutions.
33 On the set of axes below, graph \( y = f(x) \) and \( y = g(x) \) for the given functions.

\[
\begin{align*}
f(x) &= x^3 - 3x^2 \\
g(x) &= 2x - 5
\end{align*}
\]

State the number of solutions to the equation \( f(x) = g(x) \).

\( 2 \)

**Score 3:** The student graphed \( y = g(x) \) incorrectly.
Question 33

33 On the set of axes below, graph \( y = f(x) \) and \( y = g(x) \) for the given functions.

\[
\begin{align*}
  f(x) &= x^3 - 3x^2 \\
  g(x) &= 2x - 5
\end{align*}
\]

State the number of solutions to the equation \( f(x) = g(x) \).

\[
\begin{array}{c}
\text{3 Solutions}
\end{array}
\]

Score 2: The student graphed \( y = f(x) \) incorrectly.
Question 33

33 On the set of axes below, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

State the number of solutions to the equation $f(x) = g(x)$.

There are no solutions to the equation $f(x) = g(x)$.

Score 2: The student only correctly graphed $y = f(x)$. 
33 On the set of axes below, graph \( y = f(x) \) and \( y = g(x) \) for the given functions.

\[
f(x) = x^3 - 3x^2 \\
g(x) = 2x - 5
\]

State the number of solutions to the equation \( f(x) = g(x) \).

There are zero solutions.

**Score 1:** The student made a domain error graphing \( y = f(x) \) and showed no further correct work.
33 On the set of axes below, graph \( y = f(x) \) and \( y = g(x) \) for the given functions.

\[
\begin{align*}
f(x) &= x^3 - 3x^2 \\
g(x) &= 2x - 5
\end{align*}
\]

State the number of solutions to the equation \( f(x) = g(x) \).

\[
x^3 - 3x^2 - 2x + 5 = 0
\]

\[
(x - 3) \left( x^2 - 2 - \frac{1}{x - 3} \right)
\]

**Score 1:** The student only correctly graphed \( y = g(x) \).
33 On the set of axes below, graph \( y = f(x) \) and \( y = g(x) \) for the given functions.

\[
\begin{align*}
f(x) &= x^3 - 3x^2 \\
g(x) &= 2x - 5
\end{align*}
\]

State the number of solutions to the equation \( f(x) = g(x) \).

\[0 \text{ solutions}\]

**Score 0:** The student gave a completely incorrect response.
A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, $t$, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi \sqrt{\frac{L}{g}}$ where $L$ is the length of the pendulum in meters and $g$ is a constant of $9.81 \text{ m/s}^2$.

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

$$t = 2\pi \sqrt{\frac{67}{9.81}}$$

$$t = 2\pi \sqrt{6.829765345}$$

$$t = 2\pi \left(2.6013382013\right)$$

$$t = 11.0 \text{ s}$$

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

$$\sin \left(\frac{9.81}{2\pi}\right) = \sqrt{\frac{L}{9.81}}$$

$$\sin \left(\frac{9.81}{2\pi}\right)^2 = \frac{L}{9.81} \cdot 9.81$$

$$\left(\frac{9.81}{2\pi}\right)^2 \cdot 9.81 = 1$$

$$2.29 = L$$

**Score 4:** The student gave a complete and correct response.
A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, $t$, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi \sqrt{L/g}$, where $L$ is the length of the pendulum in meters and $g$ is a constant of 9.81 m/s$^2$.

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

**Score 3:** The student rounded incorrectly.
Question 34

34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, \( t \), in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation

\[ t = \frac{2\pi \sqrt{\frac{L}{g}}}{2\pi} \]

where \( L \) is the length of the pendulum in meters and \( g \) is a constant of \( 9.81 \text{ m/s}^2 \).

The first Foucault pendulum was constructed in 1851 and has a pendulum length of \( 67 \text{ m} \). Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes \( 9.6 \) seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

\[
\frac{9.6}{2\pi} = \frac{2\pi}{2\pi} \sqrt{\frac{L}{\text{9.81 m/s}^2}}
\]

\[
\left(\frac{9.6}{2\pi}\right)^2 = \left(\frac{\sqrt{L}}{\text{9.81 m/s}^2}\right)^2
\]

\[
\left(\frac{9.6}{2\pi}\right)^2 = \frac{L}{\text{9.8 m/s}^2}
\]

\[
L = 22.9
\]

Score 3: The student did not find the period.
34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, \( t \), in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation

\[
t = 2\pi \sqrt{\frac{L}{g}}
\]

where \( L \) is the length of the pendulum in meters and \( g \) is a constant of 9.81 m/s\(^2\).

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

\[
\frac{9.6}{2\pi} = \frac{8.8 \sqrt{\frac{L}{9.81}}}{2\pi} = \left(15.07964474\right)^2 \left(\sqrt{\frac{L}{9.81}}\right)^2
\]

\[
(9.81) \times 2.37 = 39.56854 = \frac{L}{9.81(9.81)}
\]

\[
L = 2.230.8
\]

Score 2: The student incorrectly calculated the period and made a computational error.
34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, \( t \), in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation \( t = 2\pi \sqrt{\frac{L}{g}} \) where \( L \) is the length of the pendulum in meters and \( g \) is a constant of 9.81 m/s\(^2\).

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

\[
9.6 = 2\pi \sqrt{\frac{L}{9.81}}
\]

\[
15.06076447 = \sqrt{\frac{L}{9.81}}
\]

\[
227.3956854 = \frac{L}{9.81}
\]

\[
23.129988 = L
\]

\[
23.2
\]

**Score 1:** The student did not find the period and made two errors when determining the length of the pendulum.
34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, $t$, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi \sqrt{\frac{L}{g}}$ where $L$ is the length of the pendulum in meters and $g$ is a constant of 9.81 m/s².

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

Score 1: The student only determined the period correctly.
34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, \( t \), in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation

\[
t = 2\pi \sqrt{\frac{L}{g}}
\]

where \( L \) is the length of the pendulum in meters and \( g \) is a constant of 9.81 m/s\(^2\).

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

\[
t = \pi \sqrt{\frac{67 \text{ m}}{9.81 \text{ m/s}^2}}
\]

\[
t = 2.613382013
\]

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

**Score 0:** The student did not show enough correct work to receive any credit.
35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Mean = 0.651  
SD = 0.034

Proportion of residents who drive to work

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

\[
\frac{0.651 - 0.062}{2} = 0.619
\]

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[
\frac{122}{200} = 0.61
\]

No, the campaign wasn’t effective because the proportion of people who drive to work is within the 95% interval. Yes, there was a decrease, but it wasn’t statistically significant.

Score 4: The student gave a complete and correct response.
35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

![Histogram showing simulation results](image)

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

\[ \bar{x} - 2\sigma = 0.54 \]
\[ \bar{x} + 2\sigma = 0.72 \]

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[ \frac{122}{200} = 0.61 \]

No, because \( \frac{122}{200} \) is included in the interval \([0.54, 0.72]\).

Score 4: The student gave a complete and correct response.
In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

\[
\text{Mean } = 0.651, \quad \text{SD } = 0.034
\]

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[
\frac{122}{200} = 0.61
\]

No because the proportion of people still driving to work is within the previous 95% confidence interval.

Score 3: The student made a rounding error.
Question 35

35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[
\text{It was effective b/c } \frac{122}{200} = 0.61 \\
\text{lies between the } 95\% \text{ interval.}
\]

Score 3: The student did not state a correct conclusion.
In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

\[0.550 \quad \underline{0.570} \quad 0.590 \quad 0.610 \quad 0.630 \quad 0.650 \quad 0.670 \quad 0.690 \quad 0.710 \quad 0.730\]

Proportion of residents who drive to work

\[\text{Mean} = 0.651\]
\[\text{SD} = 0.034\]

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[\frac{112}{200} = 0.56\]

No, it was not effective. The amount who drive was not within the confidence interval.

Score 2: The student only received credit for the interval.
35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

![Simulation Results]

Mean = 0.651
SD = 0.034

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

\[0.58 \leq \hat{p} \leq 0.72\]

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[\frac{122}{200} = 0.61\]

Yes, it is in the 95% range and close to the mean.

Score 2: The student did not show work for the interval and stated an incorrect conclusion.
35 In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

$M \pm 2\sigma$

$0.651 \pm 2(0.034)$

$0.583 \leq p \leq 0.719$

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

Score 1: The student made a rounding error and showed no further correct work.
In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city’s Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.

![Histogram of simulation results]

Mean = 0.651
SD = 0.034

Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

0.63 - 0.69

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city’s campaign was effective? Use statistical evidence from the simulation to explain your answer.

\[
\frac{122}{200} = 0.61
\]

Yes, the percentage of people who drove to work in the sample population decreased.

Score 0: The student wrote an incorrect interval and received no credit for the explanation.
Question 36

36 Solve the system of equations algebraically:

\[
\begin{align*}
    x^2 + y^2 &= 25 \\
    y + 5 &= 2x
\end{align*}
\]

\[
\begin{align*}
    y + \frac{5}{2} &= 2x \\
    y &= 2x - 5
\end{align*}
\]

\[
\begin{align*}
    x^2 + (2x - 5)^2 &= 25 \\
    x^2 + 4x^2 - 20x + 25 &= 25 \\
    5x^2 - 20x &= 0 \\
    5x(x - 4) &= 0
\end{align*}
\]

\[
\begin{align*}
    x &= 0 \\
    y &= \frac{5}{5} = 1 \\
    x &= 4 \\
    y &= \frac{20}{5} = 4
\end{align*}
\]

Solutions

(0, -5)

(4, 3)

Check

\[
\begin{align*}
    (4)^2 + (3)^4 &= 25 \\
    16 + 81 &= 25 \checkmark \\
    (0)^2 + (5)^4 &= 25 \\
    0 + 625 &= 25 \checkmark
\end{align*}
\]

Score 4: The student gave a complete and correct response.
36 Solve the system of equations algebraically:

\[
\begin{align*}
  x^2 + y^2 &= 25 \\
  y + 5 &= 2x
\end{align*}
\]

\[
y = 2x - 5
\]

\[
x^2 + (2x-5)^2 = 25
\]

\[
x^2 + (2x-5)(2x-5) = 25
\]

\[
x^2 + 4x^2 - 20x + 25 = 25
\]

\[
5x^2 - 20x = 0
\]

\[
5x(x-4) = 0
\]

\[
x = 0 \quad x = 4
\]

\[
y + 5 = 2(0) \quad y + 5 = 2(4)
\]

\[
y = -5 \quad y = 3
\]

**Score 4:** The student gave a complete and correct response.
Question 36

Solve the system of equations algebraically:

\[ x^2 + y^2 = 25 \]
\[ y + 5 = 2x \]

\[ y = 2x - 5 \]

\[ x^2 + (2x - 5)^2 = 25 \]
\[ x^2 + 4x^2 - 20x + 25 = 25 \]
\[ 5x^2 - 20x = 0 \]
\[ 5x(x - 4) = 0 \]
\[ x = 0 \quad | \quad x = 4 \]

\[ y = 2x - 5 \]
\[ y = 3 \]

\[ y = 5 \]

\[ y = 8 \]

Score 3: The student stated only one solution.
36 Solve the system of equations algebraically:

\[
\begin{align*}
x^2 + y^2 &= 25 \\
y + 5 &= 2x
\end{align*}
\]

\[
\begin{align*}
x^2 + (2x-5)^2 &= 25 \\
x^2 + (2x-5)(2x-5) &= 25 \\
x^2 + 4x^2 - 10x - 10x + 25 &= 25 \\
-9x + 25 &= 0 \\
5x - 20 &= 0 \\
x &= 4
\end{align*}
\]

**Score 2:** The student received credit for writing a quadratic equation in standard form.
Question 36

36 Solve the system of equations algebraically:

\[
\begin{align*}
 x^2 + y^2 &= 25 \\
 y + 5 &= 2x
\end{align*}
\]

\[
\begin{align*}
 y &= 2x - 5 \\
 \sqrt{x^2 - (2x - 5)^2} &= \sqrt{25} \\
 3x + 5 &= \frac{5}{-5} \\
 x &= \frac{0}{3} \\
 x &= 0 \\
 y &= -5
\end{align*}
\]

**Score 1:** The student found a correct quadratic equation in one variable, but showed no further correct work.
36 Solve the system of equations algebraically:

\[
\begin{align*}
\begin{cases}
\frac{x^2}{100} + \frac{y^2}{25} &= 25 \\
\frac{x^2}{5} - \frac{y^2}{100} &= 2x \cdot y \\
\frac{-5y}{y} &= -2 \cdot \frac{y}{y}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\sqrt{x^2 - 25} &= 5 \\
\frac{x + y}{2} &= 5 \\
\frac{x^2 + (2x - 5)^2}{2} &= 25 \\
(2x - 5)(2x - 5) &= 25 \\
x^2 - 16x + 25 &= 0
\end{align*}
\]

\[
\begin{align*}
5x^2 - 20x + 25 &= 25 \\
-2x^2 - 2 &= 0
\end{align*}
\]

\[
\begin{align*}
2x - 5 &= 2x - 5 \\
25 &= 3x - 5 \\
\frac{3x}{2} &= 3 \\
\frac{30}{2} &= x
\end{align*}
\]

\[
\begin{align*}
y + 5 &= 2 \cdot 10 \\
\frac{y - 5}{5} &= \frac{20}{5} \\
y &= 15
\end{align*}
\]

\[
\begin{align*}
x &= 10
\end{align*}
\]

**Score 1:** The student found a correct quadratic equation in one variable but earned no credit for \(x = 4\) and \(y = 3\) because no work was shown to solve the quadratic equation.
36 Solve the system of equations algebraically:

\[
\begin{align*}
    x^2 + y^2 &= 25 \\
    y + 5 &= 2x
\end{align*}
\]

\[
\begin{align*}
    y &= 2x - 5 \\
    y^2 &= -x^2 + 25 \\
    y &= \sqrt{-x^2 + 25} \\
    \sqrt{-x^2 + 25} &= 2x - 5 \\
    \sqrt{x^2 + 30} &= 2x
\end{align*}
\]

\[
\begin{align*}
    x + 30 &= 2x \\
    x &= 30
\end{align*}
\]

\[
\begin{align*}
    y + 5 &= 2(30) \\
    y &= 55
\end{align*}
\]

**Score 0:** The student did not show enough correct work to receive any credit.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where \( a_0 \) represents the population in 1910 and \( n \) represents the number of years since 1910.

\[
\begin{align*}
  a_0 &= 92.2 \\
  a_n &= 1.015a_{n-1}
\end{align*}
\]

Identify the percentage of the annual rate of growth from the equation \( a_n = 1.015a_{n-1} \).

\[
1.015 - 1 = 0.015 \times 100 = 1.5\
\]

\[
\boxed{1.5\%}
\]

Write an exponential function, \( P \), where \( P(t) \) represents the United States population in millions of people, and \( t \) is the number of years since 1910.

\[
P(t) = 92.2 \times 1.015^t
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\begin{align*}
300 &= 92.2 \times 1.015^t \\
\frac{300}{92.2} &= (1.015)^t \\
\log\left(\frac{300}{92.2}\right) &= t \log(1.015) \\
\frac{\log(300)}{\log(1.015)} &= t \\
\approx 79.24 &= t \\
&\approx 79
\end{align*}
\]

Score 6: The student gave a complete and correct response.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where \(a_0\) represents the population in 1910 and \(n\) represents the number of years since 1910.

\[
a_0 = 92.2 \\
a_n = 1.015a_{n-1}
\]

Identify the percentage of the annual rate of growth from the equation \(a_n = 1.015a_{n-1}\).

\[
1.5\%
\]

Write an exponential function, \(P\), where \(P(t)\) represents the United States population in millions of people, and \(t\) is the number of years since 1910.

\[
P(t) = 92.2e^{0.015t}
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\frac{300}{92.2} = \frac{92.2e^{0.015t}}{92.2}
\]

\[
\ln(\frac{300}{92.2}) = \frac{0.015t \ln e}{0.015} = \frac{0.015t}{0.015}
\]

\[
t = 78.65 \\
t = 79
\]

**Score 6:** The student gave a complete and correct response.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where $a_0$ represents the population in 1910 and $n$ represents the number of years since 1910.

\[
\begin{align*}
a_0 &= 92.2 \\
a_n &= 1.015a_{n-1}
\end{align*}
\]

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

\[
\text{\begin{circle}0.015\end{circle}}
\]

Write an exponential function, $P$, where $P(t)$ represents the United States population in millions of people, and $t$ is the number of years since 1910.

\[
P(t) = 92.2 \cdot 1.015^t
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\begin{align*}
300 &= 92.2 \cdot 1.015^t \\
\log 1.015 &= \log 3.253796095 \\
\log 1.015 &= \log 3.253796095 \\
\frac{\log 1.015}{\log 1.015} &= \frac{\log 3.253796095}{\log 1.015} \\
\therefore t &= 79
\end{align*}
\]

\textbf{Score 5:} The student wrote the percent incorrectly.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where \( a_0 \) represents the population in 1910 and \( n \) represents the number of years since 1910.

\[
\begin{align*}
    a_0 &= 92.2 \\
    a_n &= 1.015a_{n-1}
\end{align*}
\]

Identify the percentage of the annual rate of growth from the equation \( a_n = 1.015a_{n-1} \).

It will grow 1.5 percent per year.

Write an exponential function, \( P \), where \( P(t) \) represents the United States population in millions of people, and \( t \) is the number of years since 1910.

\[
P(t) = 92.2 \times 1.015^t
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\begin{align*}
    \frac{300,000,000}{92.2} &= 325379.6095 \\
    \log 325379.6095 &= t \log 1.015 \\
    \frac{\log 325379.6095}{\log 1.015} &= t \\
    1007.16 &= t \\
    t &= 1007
\end{align*}
\]

**Score 5:** The student made an incorrect substitution for the population.
Question 37

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where \( a_0 \) represents the population in 1910 and \( n \) represents the number of years since 1910.

\[
a_0 = 92.2 \\
\frac{a_n}{a_{n-1}} = 1.015
\]

Identify the percentage of the annual rate of growth from the equation \( \frac{a_n}{a_{n-1}} = 1.015 \).

\[
1.015(92.2) = 93.8876
\]

Write an exponential function, \( P \), where \( P(t) \) represents the United States population in millions of people, and \( t \) is the number of years since 1910.

\[
y = 92.2 \cdot (1.015)^x
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\frac{300}{92.2} = \frac{92.2 \cdot (1.015)^x}{92.2} \\
\frac{300}{92.2} = (1.015)^x \\
\log\left(\frac{300}{92.2}\right) = x \log(1.015) \\
\frac{\log(1.015)}{\log(1.015)} = x \\
x = \frac{\log(300)}{\log(1.015)}
\]

\[
\frac{\log(300)}{\log(1.015)} = 79
\]

Score 4: The student did not identify the annual growth rate and made a notation error in writing the exponential function.
Question 37

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where $a_0$ represents the population in 1910 and $n$ represents the number of years since 1910.

\[
\begin{align*}
    a_0 &= 92.2 \\
    a_n &= 1.015a_{n-1}
\end{align*}
\]

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

\[
1.015 - 1 = 0.015 \times 100 = 1.5\%
\]

Write an exponential function, $P$, where $P(t)$ represents the United States population in millions of people, and $t$ is the number of years since 1910.

\[
P(t) = 92.2^{1.5t}
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
\begin{align*}
    P(t) &= 92.2^{1.5t} \\
    \log(300) &= 92.2^{1.5t} \\
    \log(300) &= 1.5t \log 92.2 \\
    \frac{\log 300}{\log 92.2} &= 1.5t \\
    \frac{\log 300}{1.5} &= t \\
    9.4 &= t
\end{align*}
\]

Score 3: The student wrote an incorrect exponential function, then made a rounding error.
37 The population, in millions of people, of the United States can be represented by the recursive
down formula below, where $a_0$ represents the population in 1910 and $n$ represents the number of years
since 1910.

\[
a_0 = 92.2
\]

\[
a_n = 1.015a_{n-1}
\]

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

\[
\sigma = 1.015(92.2) = 93.10
\]

Write an exponential function, $P$, where $P(t)$ represents the United States population in millions
of people, and $t$ is the number of years since 1910.

\[
P(t) = 92.2(1.015)^t
\]

According to this model, determine algebraically the number of years it takes for the population of
the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
300 = 92.2(1.015)^x
\]

\[
x = 79
\]

**Score 2:** The student did not find the correct percentage, made a notation error, and showed
no work for $x = 79$. 
The population, in millions of people, of the United States can be represented by the recursive formula below, where $a_0$ represents the population in 1910 and $n$ represents the number of years since 1910.

$$a_0 = 92.2$$

$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

Write an exponential function, $P$, where $P(t)$ represents the United States population in millions of people, and $t$ is the number of years since 1910.

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

**Score 1:** The student wrote a correct expression.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where \( a_0 \) represents the population in 1910 and \( n \) represents the number of years since 1910.

\[
a_0 = 92.2 \\
a_n = 1.015a_{n-1}
\]

Identify the percentage of the annual rate of growth from the equation \( a_n = 1.015a_{n-1} \).

1.5 %

Write an exponential function, \( P \), where \( P(t) \) represents the United States population in millions of people, and \( t \) is the number of years since 1910.

\[
P(t) = a_0 \cdot (1.015)^t
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
(1.015)^{300} = 87.060
\]

87 years

Score 1: The student correctly found the percent of annual growth, but showed no further correct work.
37 The population, in millions of people, of the United States can be represented by the recursive formula below, where $a_0$ represents the population in 1910 and $n$ represents the number of years since 1910.

\[
\begin{align*}
  a_0 &= 92.2 \\
  a_n &= 1.015a_{n-1}
\end{align*}
\]

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$.

Write an exponential function, $P$, where $P(t)$ represents the United States population in millions of people, and $t$ is the number of years since 1910.

\[
\begin{align*}
P(t) &= A_0 (n-1)^t \\
P(t) &= 300 \times 10^8 (n-1)^t
\end{align*}
\]

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the nearest year.

\[
P(t) = 300 \times 10^8 (n-1)^t
\]

Score 0: The student did not show enough correct work to earn any credit.