

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Wednesday, June 22, 2022 — 9:15 a.m. to 12:15 p.m., only

## MODEL RESPONSE SET

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**Question 25**

25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

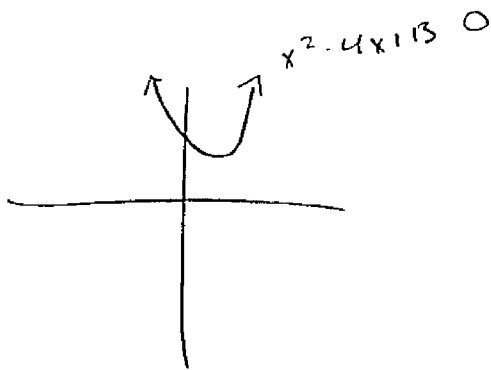
$$x = \frac{4 \pm \sqrt{-36}}{2}$$

Yes the equation has imaginary solution because there is a negative number under the radical.

**Score 2:** The student gave a complete and correct response.

Question 25

25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.



Yes the equation does have imaginary solutions because when you graph it, it doesn't pass through the x axis. Meaning it has no real roots or solutions

**Score 2:** The student gave a complete and correct response.

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**Question 25**

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25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

$$\sqrt{b^2 + 4ac}$$
$$(-4)^2 + 4(1)(13)$$
$$\sqrt{68}$$

no, the # under the  $\sqrt{\quad}$  is positive, which means it is not imaginary.

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**Score 1:** The student used the wrong formula for the discriminant.

**Question 25**

25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

$$x^2 - 4x + 13 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$\frac{4 \pm \sqrt{16 - 4(13)}}{2}$$

$$\frac{4 \pm \sqrt{-36}}{2}$$

$$\frac{4 \pm 6i}{2}$$

$$\frac{2 \pm 3i}{1}$$

Yes, it does have  
imaginary solutions.

**Score 1:** The student simplified incorrectly.

**Question 25**

25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

$$x^2 - 4x + 13 = 0$$
$$-13 = -13$$

$$x - 4x = -13 + 4$$

$$x - 4x + 2x$$

$$x - 4x + 2x = \sqrt{9}$$

$$(x - 2)(x + 2) = 3$$

No, the equation  $x^2 - 4x + 13 = 0$  doesn't have imaginary solution, it has only one solution which is 3.

**Score 0:** The student gave a completely incorrect response.

**Question 25**

25 Does the equation  $x^2 - 4x + 13 = 0$  have imaginary solutions? Justify your answer.

$$\begin{aligned}x^2 - 4x + 13 &= 0 \\x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\x &= \frac{-(-4) \pm \sqrt{4^2 - 4(1)(13)}}{2(1)} \\x &= \frac{4 \pm \sqrt{-36}}{\cancel{2}2} \\x &= 2 \pm \sqrt{-36}\end{aligned}$$

**Score 0:** The student made a simplification error and did not indicate the solutions are imaginary.

**Question 26**

**26** The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

$$S_5 = \frac{6 - 6(0.80)^5}{1 - 0.80}$$

$$a_1 = 6$$
$$r = 0.80$$

$$S_5 = 20.17$$

A child travels 20.17 feet in total  
in the first five swings

**Score 2:** The student gave a complete and correct response.



**Question 26**

**26** The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

$$\begin{aligned} 1\text{st swing} &= 6 \text{ feet} \\ 2\text{nd swing} &= 4.8 \text{ feet} \\ 3\text{rd swing} &= 3.84 \text{ feet} \\ 4\text{th swing} &= 3.072 \text{ ft} \\ 5\text{th swing} &= 2.4576 \text{ ft} \\ \hline &20.1696 \text{ ft} \end{aligned}$$

Total distance: 20.17 feet

**Score 2:** The student gave a complete and correct response.

Question 26

26 The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings

$$\begin{aligned} A &= A_{n-1}(-.80) \\ &= 6(-.80) = 4.8 \quad 1) \\ &4.8(-.80) = 3.84 \quad 2) \\ &3.072 \quad 3) \\ &2.4576 \quad 4) \\ &1.96608 \quad 5) \end{aligned}$$

$= 16.13568$

↓  
16.14  
feet

**Score 1:** The student calculated the sum of swings 2 through 6.

**Question 26**

**26** The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

$$\text{Swing 1: } 6 \text{ ft}$$

$$6 \cdot 0.8 = 4.8$$

$$\text{Swing 2: } 4.8 \text{ ft}$$

$$4.8 \cdot 0.8 = 3.84$$

$$\text{Swing 3: } 3.84 \text{ ft}$$

$$3.84 \cdot 0.8 = 3.072$$

$$\text{Swing 4: } 3.072 \text{ ft}$$

$$3.072 \cdot 0.8 = 2.4576$$

$$\text{Swing 5: } 2.4576$$

$$6 + 4.8 + 3.84 + 3.072 + 2.4576 =$$

$$20.7 \text{ ft}$$

**Score 1:** The student wrote 20.7 instead of 20.17.

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**Question 26**

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**26** The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the nearest hundredth of a foot, a child travels in the first five swings.

$$\begin{aligned}6 \text{ feet} & \quad 80\% \text{ of } 6 = 4.8 \\ & \quad 80\% \text{ of } 4.8 = 3.84 \\ & \quad 80\% \text{ of } 3.84 = 3.072 \\ & \quad 80\% \text{ of } 3.072 = 2.4576 \\ & \quad 80\% \text{ of } 2.4576 = 1.96608 \\ & \quad 161.36 \text{ ft}\end{aligned}$$

**Score 0:** The student did not show enough correct work to receive any credit.

Question 27

27 Solve algebraically for  $n$ :  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$ .

$$\frac{2}{n^2} + \frac{3 \cdot n}{n} = \frac{4}{n^2}$$

$$\frac{2}{n^2} + \frac{3n}{n^2} = \frac{4}{n^2}$$

$$\begin{array}{r} 2 + 3n = 4 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\frac{3n}{3} = \frac{2}{3}$$

$$\boxed{n = \frac{2}{3}}$$

**Score 2:** The student gave a complete and correct response.

Question 27

27 Solve algebraically for  $n$ :  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$ .

$$2 + 3n = 4$$

$$3n = 2$$

$$n = \frac{2}{3}$$

**Score 2:** The student gave a complete and correct response.

Question 27

27 Solve algebraically for  $n$ :  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$ .

$$n^3 \cdot \frac{2}{n^2} + \frac{n^3 \cdot 3}{n} = \frac{n^3 \cdot 4}{n^2}$$

$$2n + 3n^2 = 4n$$
$$-4n$$

$$-2n + 3n^2 = 0$$

$$n(-2 + 3n) = 0$$

$$n = 0$$

$$-2 + 3n = 0$$

$$+2 \qquad +2$$
$$\frac{3n}{3} = \frac{2}{3}$$

$$n = \frac{2}{3}$$

**Score 1:** The student did not reject the extraneous solution.

**Question 27**

27 Solve algebraically for  $n$ :  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$ .

$$2n^{-2} + 3n^{-1} = 4n^{-2}$$

$$\frac{3n^{-1}}{n^{-1}} = \frac{2n^{-2}}{n^{-1}}$$

$$3 = \frac{2n^{-2}}{n^{-1}}$$

$$\frac{3}{2} = \frac{2n}{2}$$

$$\boxed{n = \frac{3}{2}}$$

**Score 1:** The student incorrectly simplified the right side of the equation.



Question 27

27 Solve algebraically for  $n$ :  $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$ .

$$\begin{array}{l} \frac{2+3n}{n^2} - \frac{4}{n^2} \\ \hline 2+3n = -4 \\ -2 \quad -2 \\ 3n = -6 \\ \frac{3}{3} \quad \frac{3}{3} \\ n = 2 \end{array}$$

~~Handwritten scribbles and crossed-out work are present to the left of the main work.~~

**Score 0:** The student made multiple errors.

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**Question 28**

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28 Factor completely over the set of integers:

$$-2x^4 + x^3 + 18x^2 - 9x$$

$$-2x^4 + x^3 + 18x^2 - 9x$$

$$x^3(-2x+1) - 9x(-2x+1)$$

$$(x^3 - 9x)(-2x+1)$$

$$x(x^2 - 9)(-2x+1)$$

$$x(x+3)(x-3)(-2x+1)$$

---

**Score 2:** The student gave a complete and correct response.

Question 28

28 Factor completely over the set of integers:

$$(-2x^4 + x^3) + 18x^2 - 9x$$

$$-x^2(2x^2 - x) + 9(2x^2 - x)$$

$$\begin{array}{l} (-x^2 + 9) (2x^2 - x) \\ -1(x^2 - 9) \end{array}$$

$$-1(x-3)(x+3) \quad x(2x-1)$$

**Score 2:** The student gave a complete and correct response.

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**Question 28**

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28 Factor completely over the set of integers:

$$\underline{-2x^4 + x^3} + \underline{18x^2 - 9x}$$

$$-x^3(2x-1) + 9x(2x-1)$$

$$(-x^3 + 9x)(2x-1)$$

$$x(-x^2 + 9)(2x-1)$$

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**Score 1:** The student did not factor completely.

Question 28

28 Factor completely over the set of integers:

$$\underbrace{-2x^4 + x^3} + \underbrace{18x^2 - 9x}$$

$$-x^3(2x-1) + 9x(2x-1)$$

$$(-x^3 + 9x)(2x-1)$$

$$-x(x^2 + 9)(2x-1)$$

**Score 1:** The student made one factoring error.

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**Question 28**

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**28** Factor completely over the set of integers:

$$-2x^4 + x^3 + 18x^2 - 9x$$

$$x(-2x^3 + x^2 + 18x - 9)$$

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**Score 0:** The student did not show enough correct work to receive any credit.

**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

		S	N
		Wear Glasses	Don't Wear Glasses
L	Blue Eyes	0.14	0.26
B	Brown Eyes	0.11	0.24
G	Green Eyes	0.10	0.15

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

$$P(L|S) = \frac{.14}{.35} = .4 \quad P(L|S) \stackrel{?}{=} P(L)$$

$$P(L) = .4$$

$$.4 = .4$$

They are independent  
because  $P(L|S) = P(L)$

**Score 2:** The student gave a complete and correct response.

**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	Wear Glasses	Don't Wear Glasses
Blue Eyes	0.14	0.26
Brown Eyes	0.11	0.24
Green Eyes	0.10	0.15

A

.35

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

$$(.4)(.35) = .14$$

$$.14 = .14 \checkmark$$

Yes, they are independent because the values are equal.

**Score 2:** The student gave a complete and correct response.



**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	<b>Wear Glasses</b>	<b>Don't Wear Glasses</b>
<b>Blue Eyes</b>	0.14	0.26
<b>Brown Eyes</b>	0.11	0.24
<b>Green Eyes</b>	0.10	0.15

$$= .4$$

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

$$P(A) = P(A|B)$$

$$.4 = \frac{.4}{.35}$$

$$.4 \neq 1.1428\dots$$

No, They are not equal making them not independent from each other

**Score 1:** The student made one error in determining  $P(A|B)$ .

**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	Wear Glasses	Don't Wear Glasses
Blue Eyes	0.14	0.26
Brown Eyes	0.11	0.24
Green Eyes	0.10	0.15

.35

.4

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

$$P(A) = \frac{.4}{1} = .4$$

$$P(B) = \frac{.35}{1} = .35$$

$$P(A \text{ and } B) = P(A)P(B)$$
$$.14 = .14$$

not independent b/c prob =

**Score 1:** The student made an incorrect conclusion based on appropriate work.

**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	Wear Glasses	Don't Wear Glasses
Blue Eyes	0.14	0.26
Brown Eyes	0.11	0.24
Green Eyes	0.10	0.15

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

No, be  $P(A \cap B) = P(B)$   
 $P(.14) = 1.35$   
 $A = \text{Blue}$   
 $B = \text{Glasses}$   
since they do not equal having blue eyes and wearing glasses is not independent and is dependent

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 29**

29 The relative frequency table shows the proportion of a population who have a given eye color and the proportion of the same population who wear glasses.

	Wear Glasses	Don't Wear Glasses	
Blue Eyes	0.14	0.26	0.30
Brown Eyes	0.11	0.24	
Green Eyes	0.10	0.15	
	0.35		

Given the data, are the events of having blue eyes and wearing glasses independent? Justify your answer.

$$\frac{0.14}{0.30} = 0.466\ldots = 47\%$$

$$\frac{0.14}{0.35} = 0.4 = 40\%$$

$$40\% \neq 47\%$$

$\therefore$  is dependent

**Score 0:** The student made a calculation error and did not complete a test for independence.

**Question 30**

30 For  $x \neq 0$  and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of  $a$ .

$$\sqrt[3]{81x^{15}y^9} = (3^a x^5 y^3)^3$$

$$81 \cancel{x^{15}} \cancel{y^9} = 3^{3a} \cancel{x^{15}} \cancel{y^9}$$

$$81 = 3^{3a}$$

$$3^4 = 3^{3a}$$

$$\frac{4}{3} = \frac{3a}{3}$$

$$a = \frac{4}{3}$$

**Score 2:** The student gave a complete and correct response.

**Question 30**

30 For  $x \neq 0$  and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of  $a$ .

$$\left(\sqrt[3]{81x^{15}y^9}\right)^3 = \left(3^a x^5 y^3\right)^3$$

$$\frac{81x^{15}y^9}{x^{15}y^9} = \frac{3^{3a}x^{15}y^9}{x^{15}y^9}$$

$$\log(81) = \log(3^{3a})$$

$$\frac{\log(81)}{\log(3)} = \frac{3a \log(3)}{\log(3)}$$

$$4 = \frac{3a}{3}$$

$$a = \boxed{\frac{4}{3}}$$

**Score 2:** The student gave a complete and correct response.

**Question 30**

30 For  $x \neq 0$  and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of  $a$ .

$$\begin{aligned} \sqrt[3]{81} \sqrt[3]{x^{15}} \sqrt[3]{y^9} &= 3^a x^5 y^3 \\ \sqrt[3]{27} \sqrt[3]{3} \downarrow \downarrow \downarrow & \quad \downarrow \downarrow \downarrow \\ 3 \cdot \sqrt[3]{3} \downarrow \downarrow \downarrow & \quad \downarrow \downarrow \downarrow \\ 3 \cdot 3^{\frac{1}{3}} \cdot x^5 y^3 &= 3^a x^5 y^3 \\ 3^{\frac{1}{3}} x^5 y^3 &= 3^a x^5 y^3 \\ 3^{\frac{1}{3}} &= 3^a \\ a &= \frac{1}{3} \end{aligned}$$

**Score 1:** The student multiplies exponents instead of adding.

Question 30

30 For  $x \neq 0$  and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of  $a$ .

$$\sqrt[3]{81x^{15}y^9} = 3^a = x^5 y^3$$

$$81^{1/3} x^{15/3} y^{9/3} = 3^a x^5 y^3$$

$$\frac{27x^5 y^3}{3^a} = \frac{3^a x^5 y^3}{3^a}$$

$$3^a x^5 y^3 = x^5 y^3$$

$$a = 3$$

**Score 0:** The student did not show enough correct work to receive any credit.



**Question 30**

30 For  $x \neq 0$  and  $y \neq 0$ ,  $\sqrt[3]{81x^{15}y^9} = 3^a x^5 y^3$ . Determine the value of  $a$ .

$$\sqrt[3]{81x^{15}y^9}^2 = (3^a x^5 y^3)^2$$

$$\frac{81x^{15}y^9}{-y^6} = \frac{3^{2a} x^{10} y^6}{-y^6}$$

$$\frac{81x^{15}y^3}{-x^{10}} = \frac{3^{2a} x^{10}}{-x^{10}}$$
$$\frac{81x^5y^3}{3} = \frac{3^{2a}}{3}$$

$$\frac{81x^5y^3}{3} = 2a$$

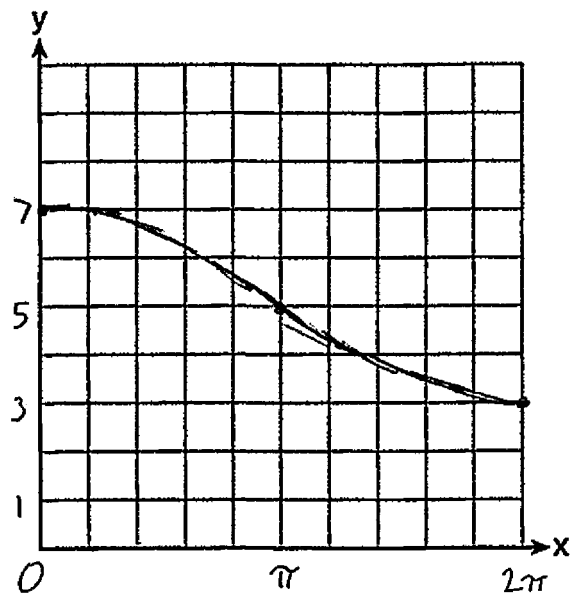
$$\frac{27x^5y^3}{2} = \frac{2a}{2}$$

$$13.5x^5y^3 = a$$

**Score 0:** The student gave a completely incorrect response.

**Question 31**

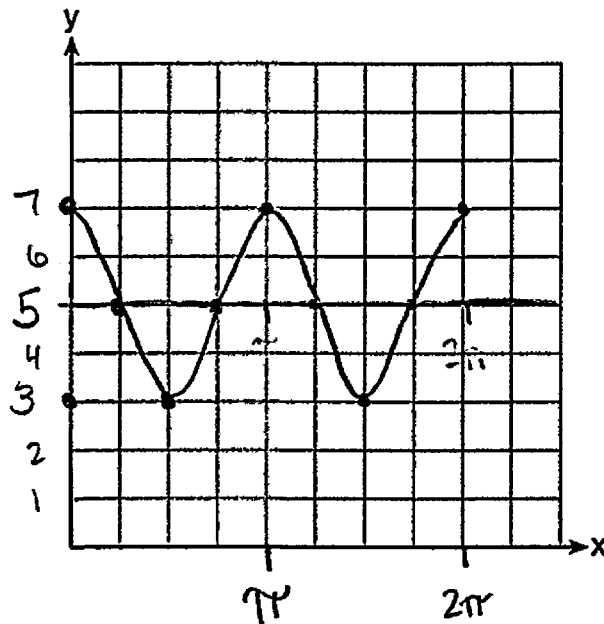
**31** Graph  $y = 2\cos\left(\frac{1}{2}x\right) + 5$  on the interval  $[0, 2\pi]$ , using the axes below.



**Score 2:** The student gave a complete and correct response.

Question 31

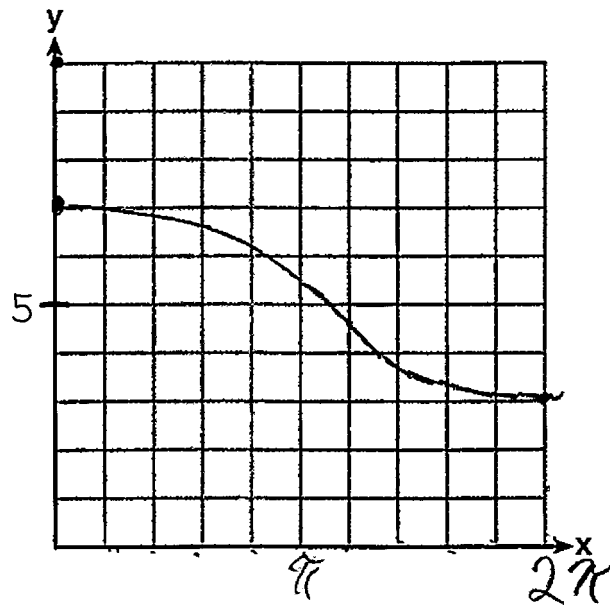
31 Graph  $y = 2\cos\left(\frac{1}{2}x\right) + 5$  on the interval  $[0, 2\pi]$ , using the axes below.



**Score 1:** The student used an incorrect period.

Question 31

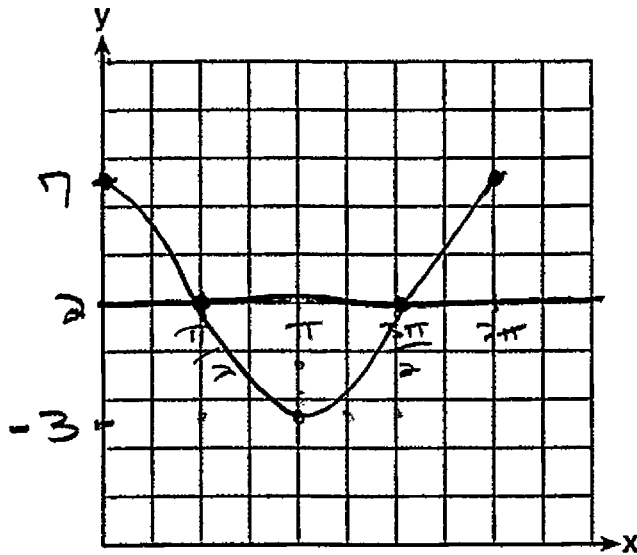
31 Graph  $y = 2\cos\left(\frac{1}{2}x\right) + 5$  on the interval  $[0, 2\pi]$ , using the axes below.



**Score 1:** The student made one graphing error at  $x = \pi$ .

Question 31

31 Graph  $y = 2\cos\left(\frac{1}{2}x\right) + 5$  on the interval  $[0, 2\pi]$ , using the axes below.



$$y = 2 \cos \frac{1}{2} x + 5$$

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 32**

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**32** A cup of coffee is left out on a countertop to cool. The table below represents the temperature,  $F(t)$ , in degrees Fahrenheit, of the coffee after it is left out for  $t$  minutes.

$t$	0	5	10	15	20	25
$F(t)$	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation,  $F(t)$ , to model the temperature of the coffee. Round all values to the nearest thousandth.

$$F(t) = 169.136(0.971)^t$$

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**Score 2:** The student gave a complete and correct response.

**Question 32**

**32** A cup of coffee is left out on a countertop to cool. The table below represents the temperature,  $F(t)$ , in degrees Fahrenheit, of the coffee after it is left out for  $t$  minutes.

<b>t</b>	0	5	10	15	20	25
<b>F(t)</b>	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation,  $F(t)$ , to model the temperature of the coffee. Round all values to the *nearest thousandth*.

$$y = a \times b^x$$
$$a = 169.136$$
$$b = 0.971$$

**Score 1:** The student made a notation error by not using  $F(t)$  and  $t$ .

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**Question 32**

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**32** A cup of coffee is left out on a countertop to cool. The table below represents the temperature,  $F(t)$ , in degrees Fahrenheit, of the coffee after it is left out for  $t$  minutes.

<b>t</b>	0	5	10	15	20	25
<b>F(t)</b>	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation,  $F(t)$ , to model the temperature of the coffee. Round all values to the *nearest thousandth*.

$$F(t) = 171.426(.970)^x$$

**Score 0:** The student made a notation error and wrote an exponential function with incorrect values.



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**Question 32**

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**32** A cup of coffee is left out on a countertop to cool. The table below represents the temperature,  $F(t)$ , in degrees Fahrenheit, of the coffee after it is left out for  $t$  minutes.

<b>t</b>	0	5	10	15	20	25
<b>F(t)</b>	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation,  $F(t)$ , to model the temperature of the coffee. Round all values to the *nearest thousandth*.

$$F(t) = .141x^2 - 7.171x + 178.525$$

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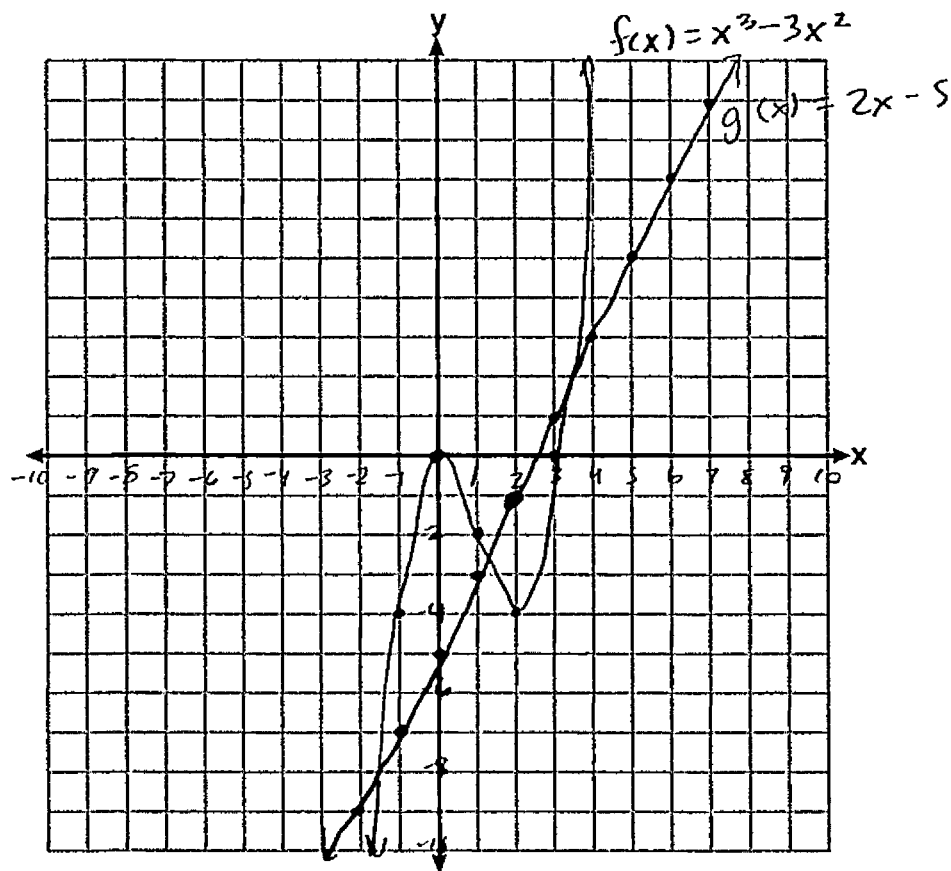
**Score 0:** The student made a notation error and used a quadratic regression.

**Question 33**

33 On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

$$\begin{array}{r} x^3 - 3x^2 = 2x - 5 \\ \underline{-2x + 5} \end{array}$$

$$x^3 - 3x^2 - 2x + 5 = 0$$

3 solutions

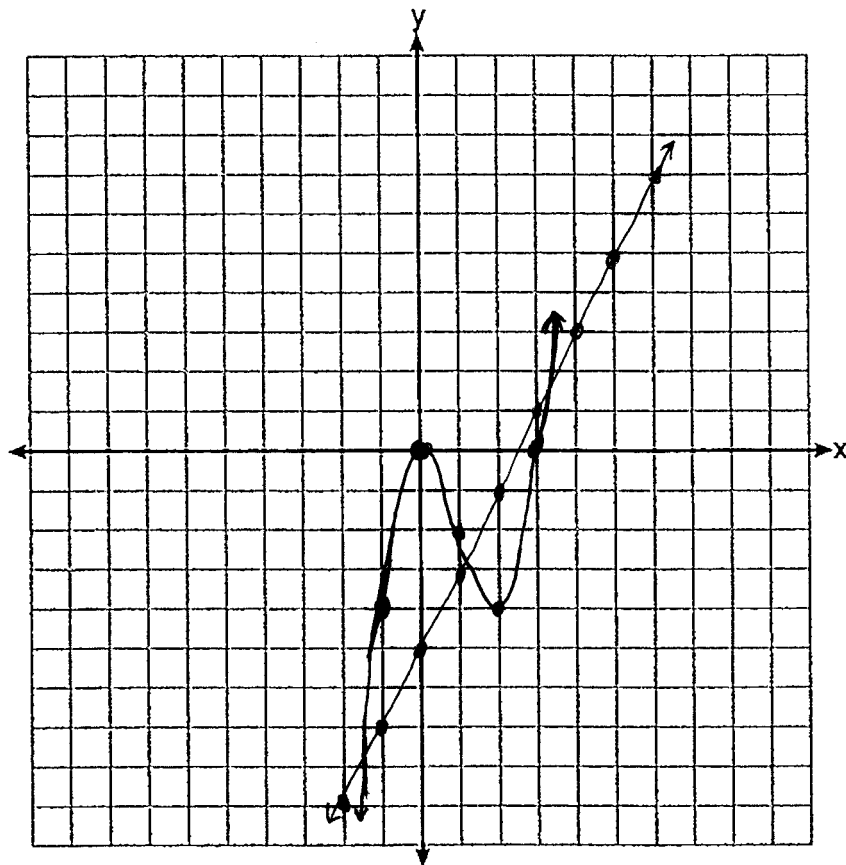
**Score 4:** The student gave a complete and correct response.

**Question 33**

**33** On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

3

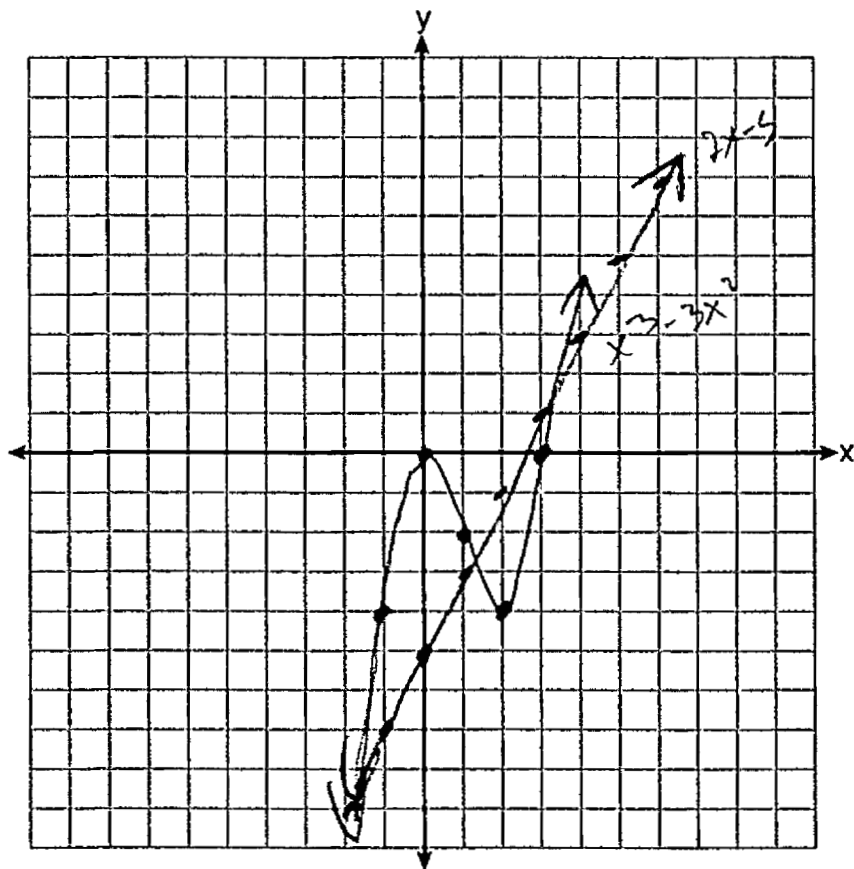
**Score 4:** The student gave a complete and correct response.

**Question 33**

**33** On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

there are no solutions.

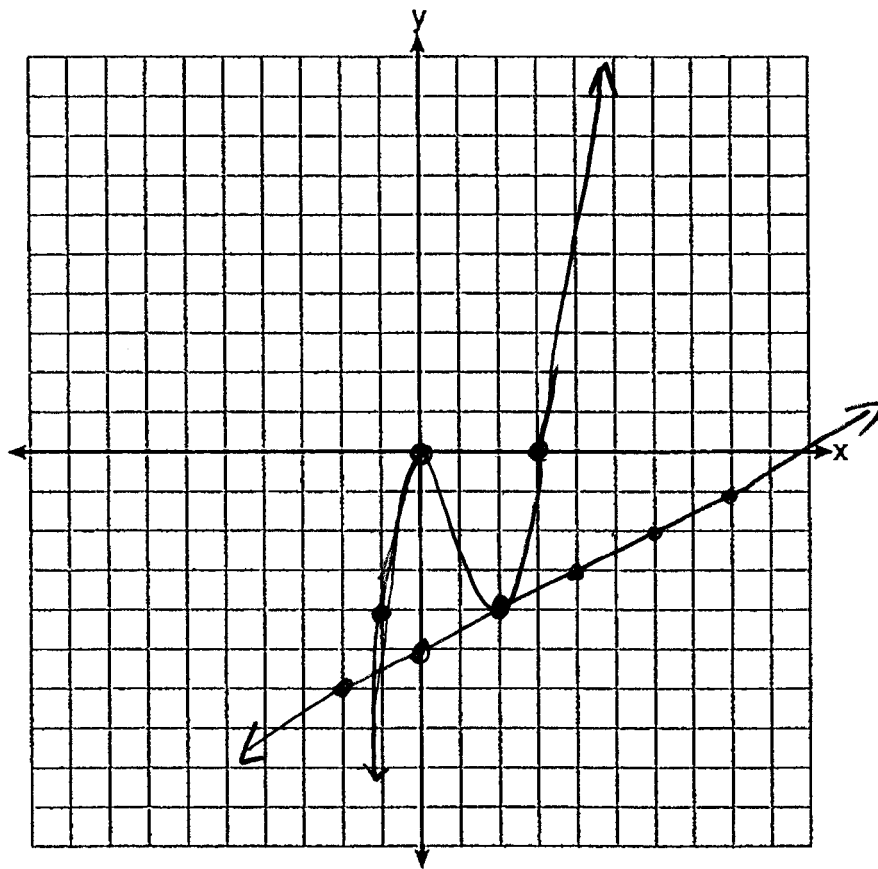
**Score 3:** The student incorrectly stated the number of solutions.

**Question 33**

**33** On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

2

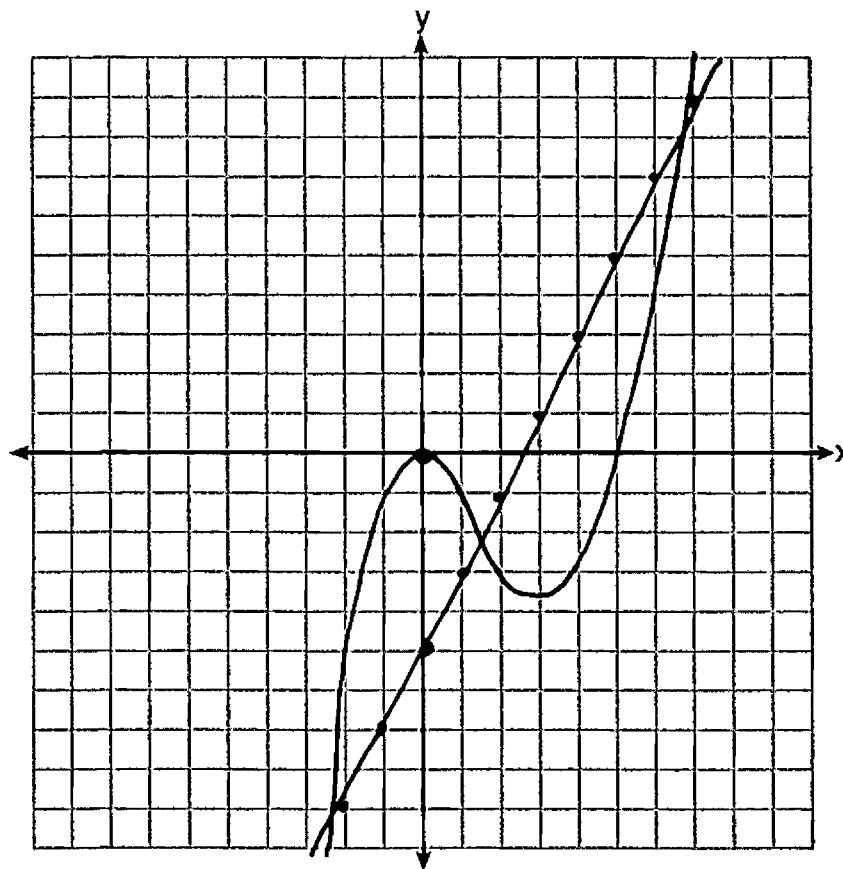
**Score 3:** The student graphed  $y = g(x)$  incorrectly.

Question 33

33 On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$
$$g(x) = 2x - 5$$

$x^3 - 3x^2$   
 $2x - 5$



State the number of solutions to the equation  $f(x) = g(x)$ .

1/3 ~~zero~~ solutions

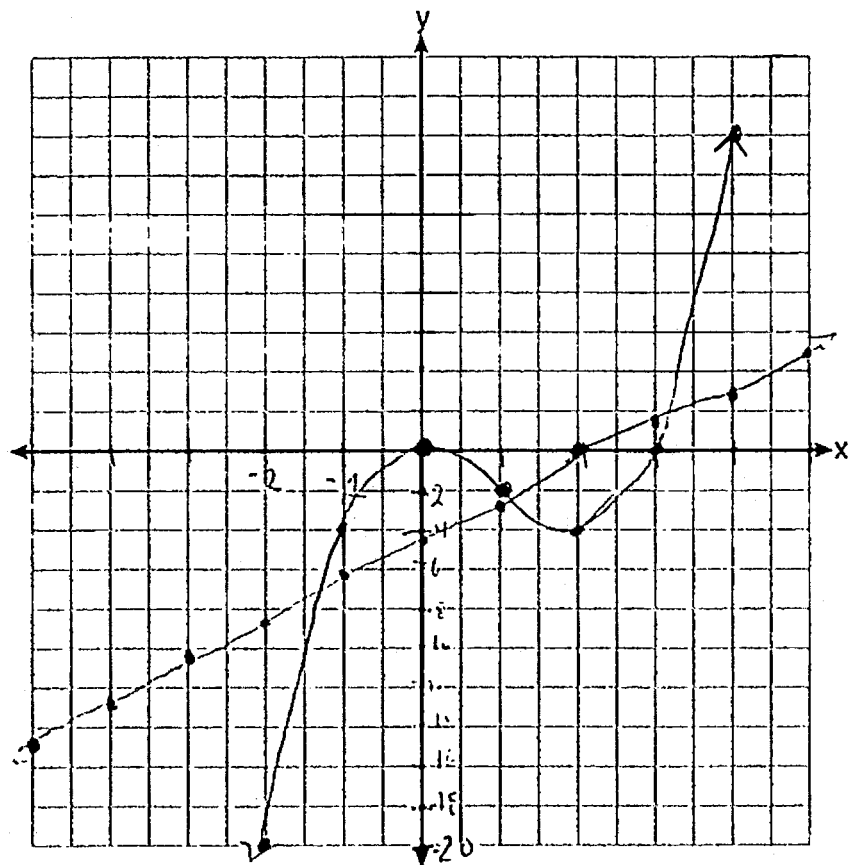
**Score 2:** The student graphed  $y = f(x)$  incorrectly.

Question 33

33 On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

There are no solutions to the equations  $f(x) = g(x)$

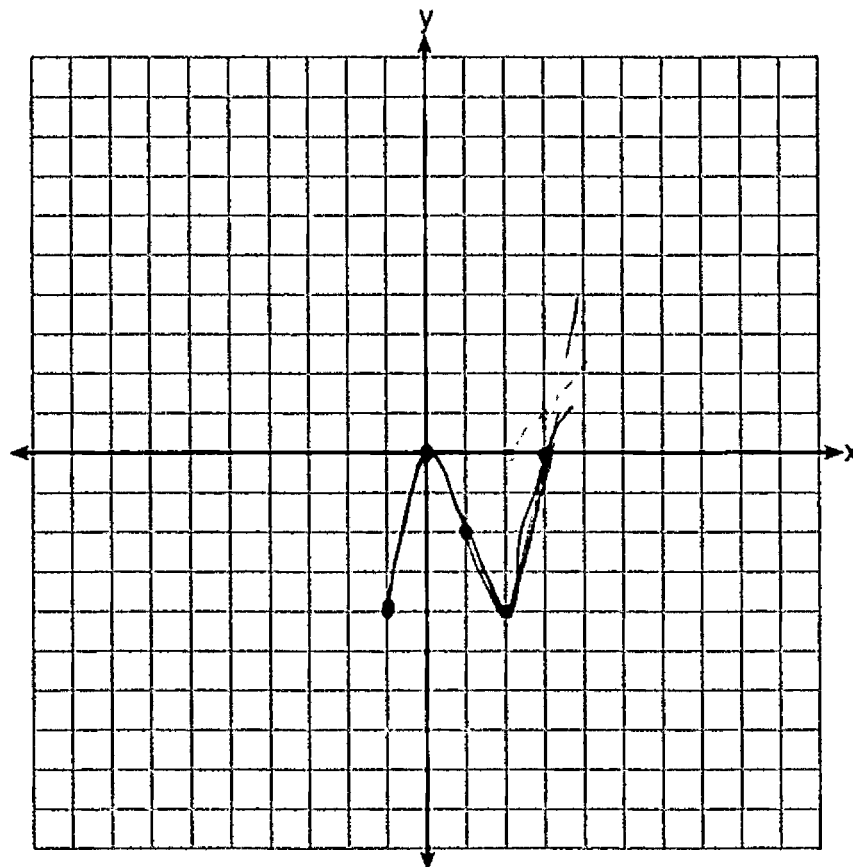
**Score 2:** The student only correctly graphed  $y = f(x)$ .

**Question 33**

**33** On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

There are zero solutions.

**Score 1:** The student made a domain error graphing  $y = f(x)$  and showed no further correct work.

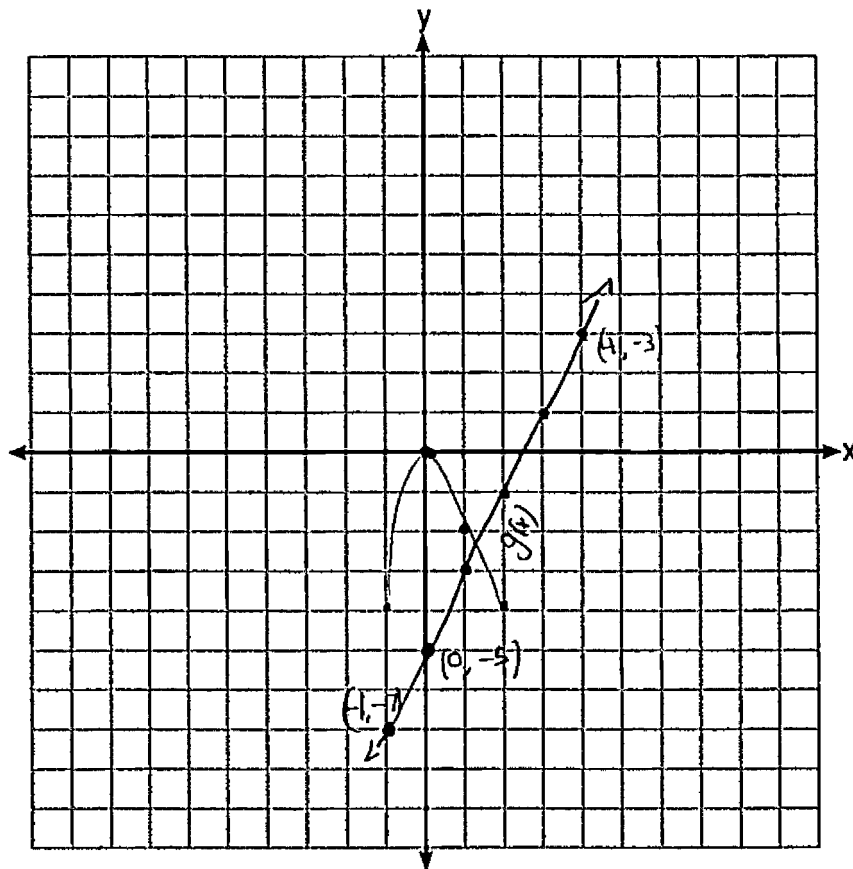


**Question 33**

33 On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

$$x^3 - 3x^2 - 2x + 5 = 0$$

$$(x - 3) \left( x^2 - 2 - \frac{1}{x-3} \right)$$

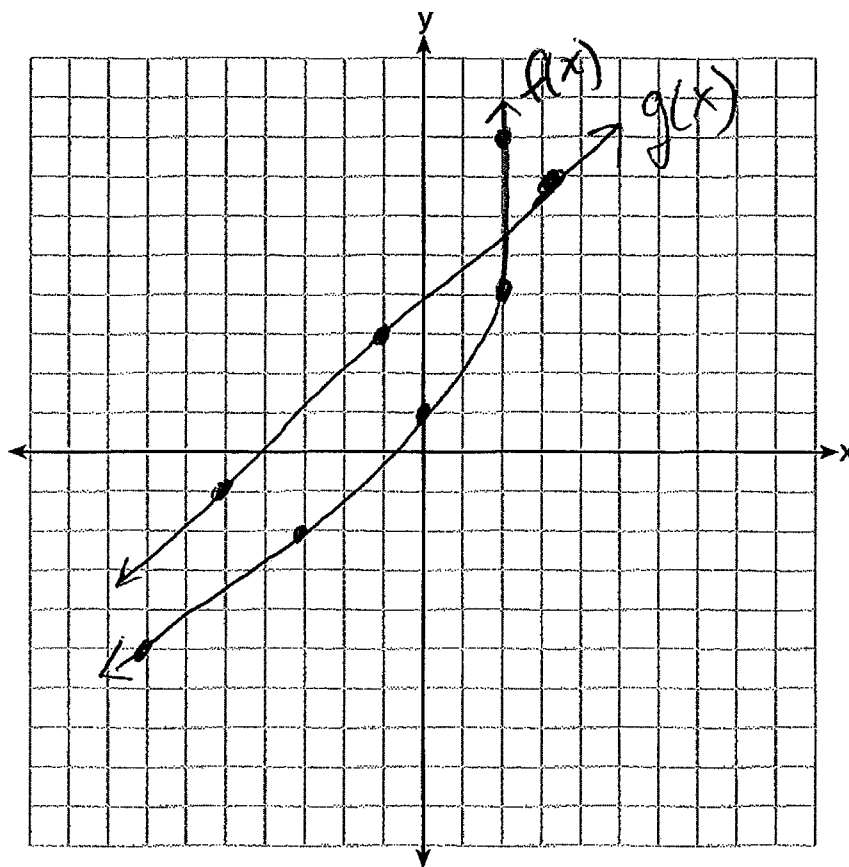
**Score 1:** The student only correctly graphed  $y = g(x)$ .

**Question 33**

**33** On the set of axes below, graph  $y = f(x)$  and  $y = g(x)$  for the given functions.

$$f(x) = x^3 - 3x^2$$

$$g(x) = 2x - 5$$



State the number of solutions to the equation  $f(x) = g(x)$ .

0 solutions

**Score 0:** The student gave a completely incorrect response.

**Question 34**

34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

$$\begin{aligned}t &= 2\pi\sqrt{\frac{67}{9.81}} \\t &= 2\pi\sqrt{6.829765545} \\t &= 2\pi(2.613382013) \\t &= 16.4\end{aligned}$$

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

$$\begin{aligned}\frac{9.6}{2\pi} &= \frac{2\pi\sqrt{\frac{L}{9.81}}}{2\pi} \\ \left(\frac{9.6}{2\pi}\right)^2 &= \sqrt{\frac{L}{9.81}}^2 \\ 9.81 \cdot \left(\frac{9.6}{2\pi}\right)^2 &= \frac{L}{9.81} \cdot 9.81 \\ \left(\frac{9.6}{2\pi}\right)^2 \cdot 9.81 &= L \\ 22.9 &= L\end{aligned}$$

**Score 4:** The student gave a complete and correct response.

**Question 34**

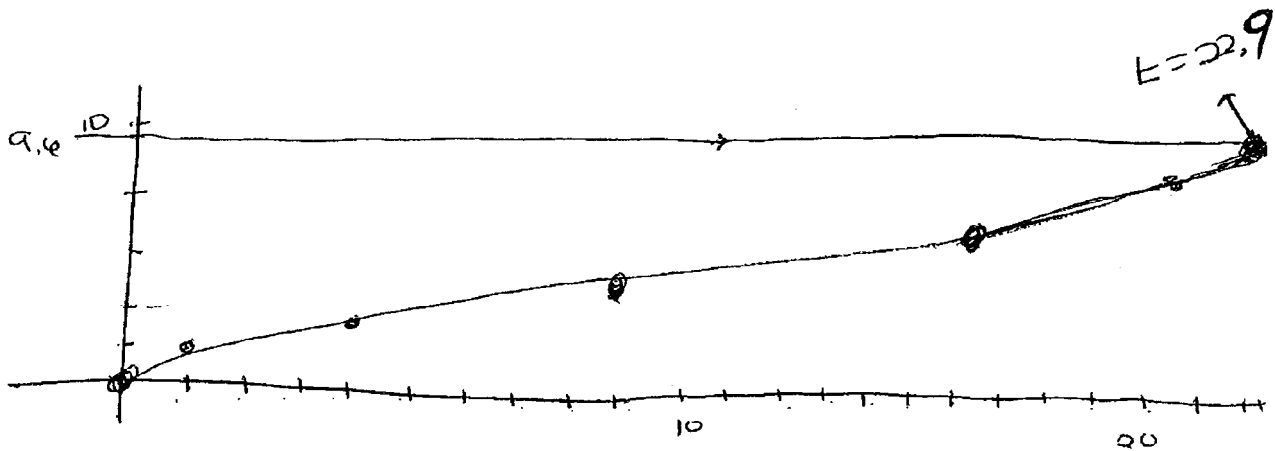
**34** A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

$$(67, 16.42)$$

$$16.42$$

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.



**Score 3:** The student rounded incorrectly.

**Question 34**

34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

$$\frac{9.6}{2\pi} = \frac{2\pi\sqrt{\frac{L}{9.81 \text{ m/s}^2}}}{2\pi}$$

$$\left(\frac{9.6}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{9.81 \text{ m/s}^2}}\right)^2$$

$$\left(\frac{9.6}{2\pi}\right)^2 = \frac{L}{9.8 \text{ m/s}^2}$$

$$\boxed{L = 22.9}$$

**Score 3:** The student did not find the period.

**Question 34**

**34** A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

$$2\pi \rightarrow 6.3$$

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

$$\begin{aligned} \frac{9.6}{2\pi} &= \frac{2\pi}{2\pi} \sqrt{\frac{L}{9.81}} && L = \text{length} \\ (15.07964474)^2 &= \left(\sqrt{\frac{L}{9.81}}\right)^2 \\ (9.81) 227.3956854 &= \frac{L}{9.81} \quad (\cancel{9.81}) \\ L &= 2230.8 \end{aligned}$$

**Score 2:** The student incorrectly calculated the period and made a computational error.

**Question 34**

**34** A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

$$9.6 = 2\pi \sqrt{\frac{L}{9.81}}$$

$$15.07964474 = \sqrt{\frac{L}{9.81}}$$

$$227.3956854 = \frac{L}{9.81}$$

$$23.179988 = L$$

$$23.2$$

**Score 1:** The student did not find the period and made two errors when determining the length of the pendulum.

### Question 34

34 A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

16.4 seconds

~~16.4~~ (67, 16.42) if graph it.

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

23m

**Score 1:** The student only determined the period correctly.



**Question 34**

**34** A Foucault pendulum can be used to demonstrate that the Earth rotates. The time,  $t$ , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation  $t = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in meters and  $g$  is a constant of  $9.81 \text{ m/s}^2$ .

The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

$$t = 2\pi\sqrt{\frac{L}{g}}$$

$$t = 2\pi\sqrt{\frac{67\text{m}}{9.81\text{m/s}^2}}$$

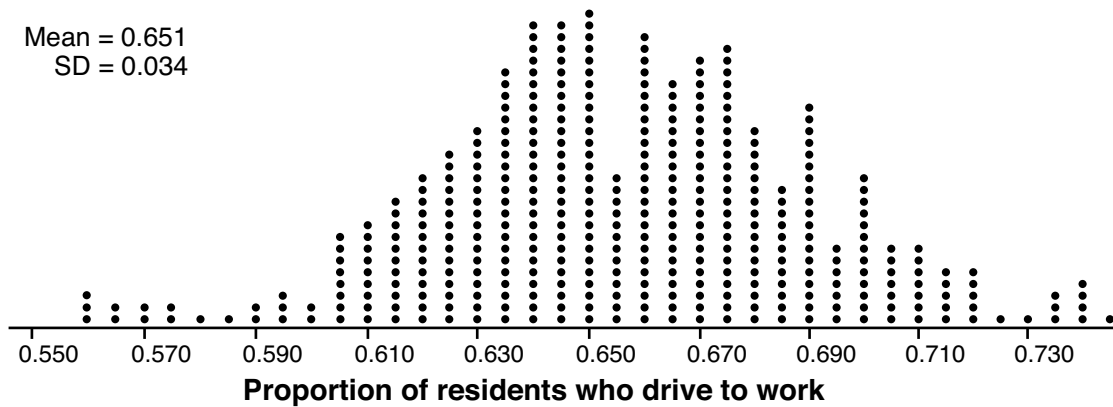
$$t = 2\pi(2.613382013)$$

Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the nearest hundredth.

$$0.034(2) = 0.068$$

$\begin{array}{r} 0.651 \\ + .068 \\ \hline .719 \\ \downarrow \\ .72 \end{array}$	$\begin{array}{r} 0.651 \\ - .068 \\ \hline .583 \\ \downarrow \\ .58 \end{array}$	$.58 \text{ to } .72$
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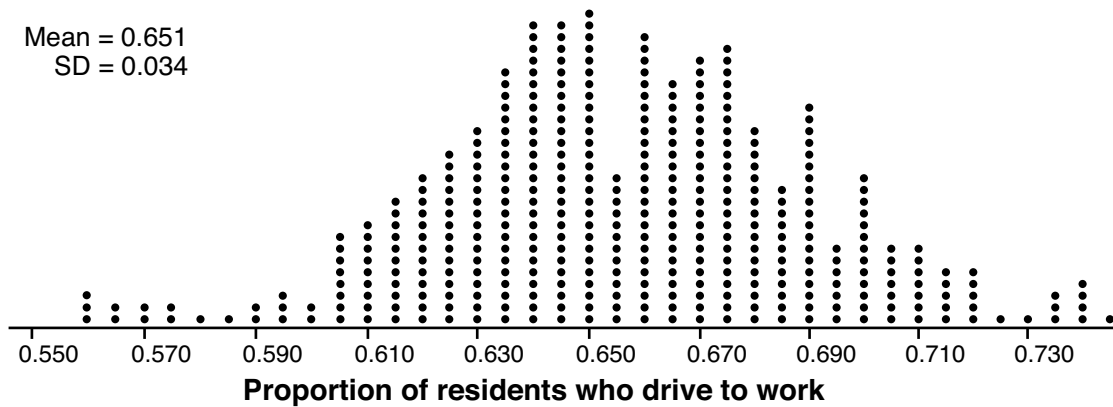
One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

$\frac{122}{200} = 0.61$	<p>No, the campaign wasn't effective because the proportion of people who drive to work is within the 95% interval. Yes, there was a decrease, but it wasn't statistically significant.</p>
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**Score 4:** The student gave a complete and correct response.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

$$\bar{x} - 2\sigma = .58$$
$$\bar{x} + 2\sigma = .72$$

$$[.58, .72]$$

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

$$\frac{122}{200} = .61$$

No, because  $\frac{122}{200}$  is included in the interval  $[.58, .72]$ .

**Score 4:** The student gave a complete and correct response.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

$$.651 - (2) .034$$

$$.651 + (2) .034$$

$$.583 - .719$$

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

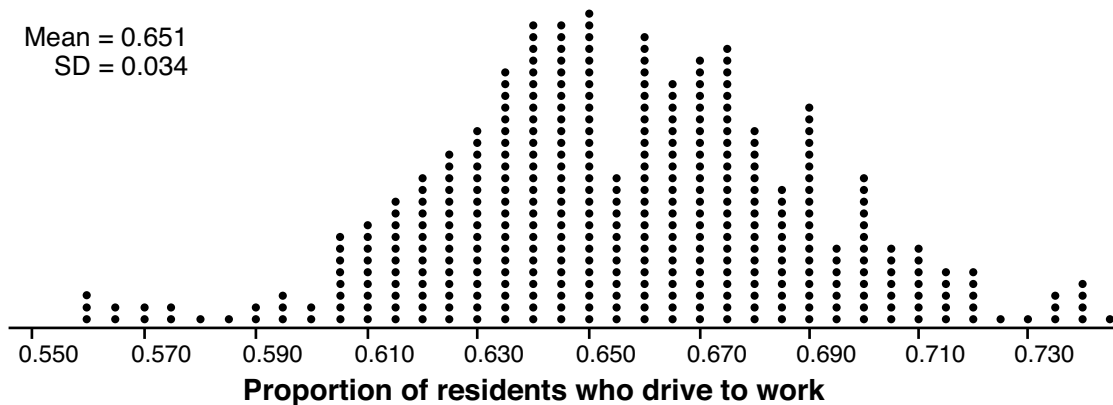
$$\frac{122}{200} = .61$$

No because the proportion of people still driving to work is within the previous 95% confidence interval.

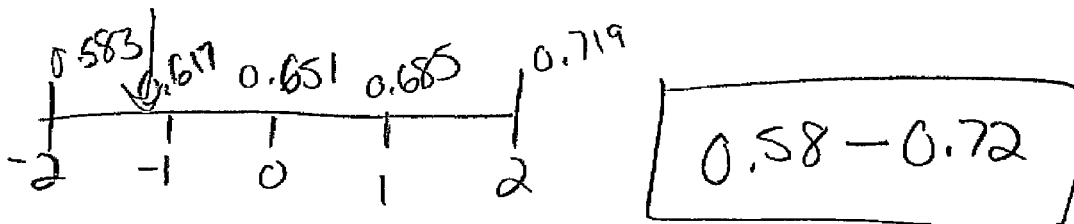
**Score 3:** The student made a rounding error.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.



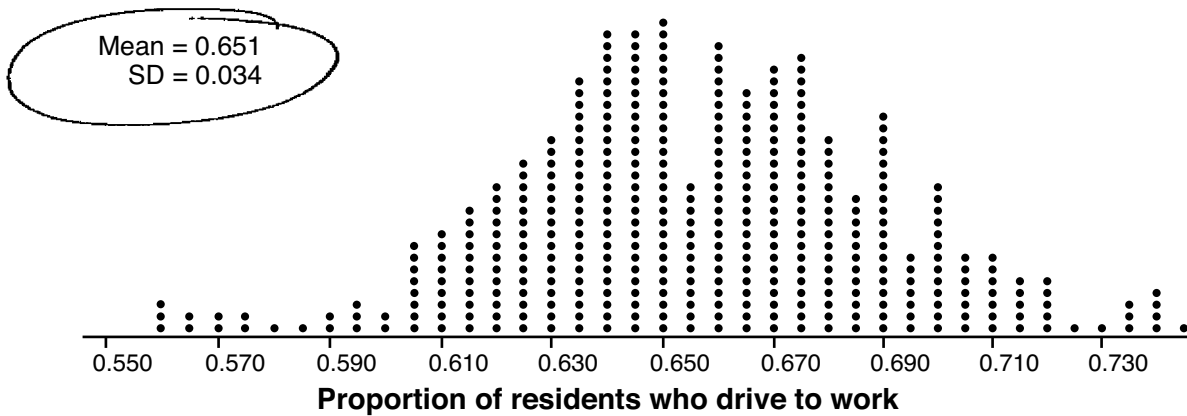
One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

It was effective b/c  $\frac{122}{200} = 0.61$   
the survey's result lies between the 95% interval

**Score 3:** The student did not state a correct conclusion.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, ~~run 400 times~~ based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

$$[.58, .72] \qquad \begin{array}{l} 0.651 - 2(0.034) \\ 0.651 + 2(0.034) \end{array}$$

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

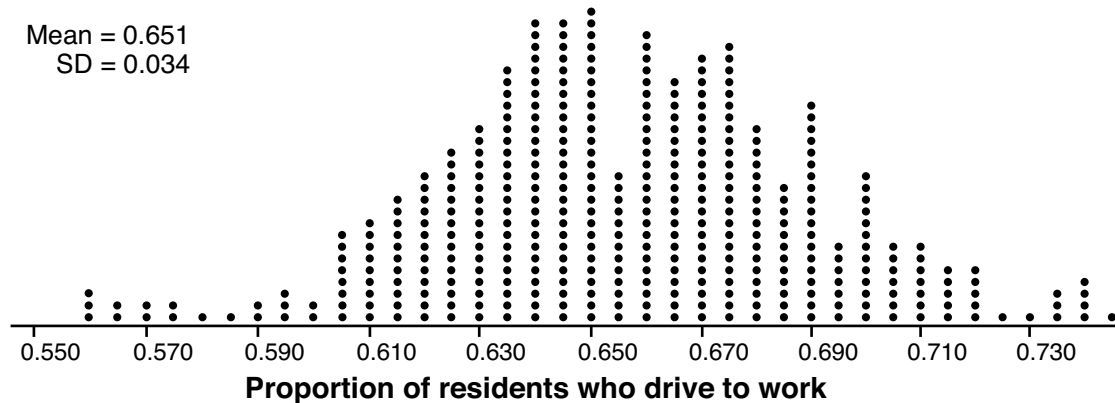
$$\frac{112}{200} = 0.56$$

No, it was not effective.  
The amount who drive was not within the confidence interval.

**Score 2:** The student only received credit for the interval.

### Question 35

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

*.58 - .72*

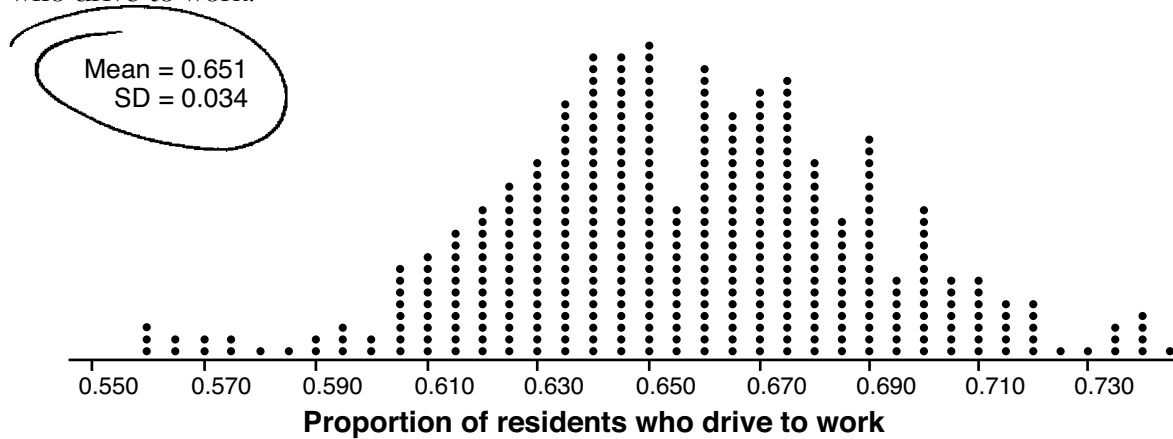
One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

$\frac{122}{200} = .61$  Yes it is in  
the 95% range  
and close to the  
mean

**Score 2:** The student did not show work for the interval and stated an incorrect conclusion.

**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

$$M \pm 2\sigma$$

$$= 0.651 \pm 2(0.034)$$

$$= 0.583 + 0.719$$

One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

988

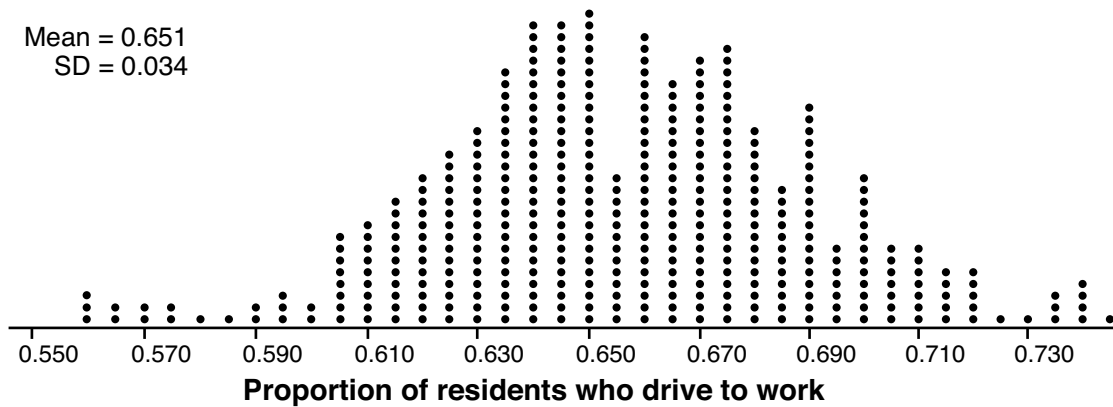
It was 99% effective

**Score 1:** The student made a rounding error and showed no further correct work.



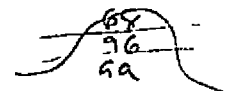
**Question 35**

**35** In order to decrease the percentage of its residents who drive to work, a large city launches a campaign to encourage people to use public transportation instead. Before starting the campaign, the city's Department of Transportation uses census data to estimate that 65% of its residents drive to work. The Department of Transportation conducts a simulation, shown below, run 400 times based on this estimate. Each dot represents the proportion of 200 randomly selected residents who drive to work.



Use the simulation results to construct a plausible interval containing the middle 95% of the data. Round your answer to the *nearest hundredth*.

.63 - .69



One year after launching the campaign, the Department of Transportation conducts a survey of 200 randomly selected city residents and finds that 122 of them drive to work. Should the department conclude that the city's campaign was effective? Use statistical evidence from the simulation to explain your answer.

$\frac{122}{200} = 61\%$   
Yes b/c the percentage of people who drove to work in the sample population decreased.

**Score 0:** The student wrote an incorrect interval and received no credit for the explanation.

Question 36

36 Solve the system of equations algebraically:

$$\begin{aligned} x^2 + y^2 &= 25 \\ y + 5 &= 2x \end{aligned}$$

	$2x$	$-5$
$2x$	$4x^2$	$-10x$
$-5$	$-10x$	$25$

$$\begin{aligned} y + 5 &= 2x \\ -5 & \\ \hline y &= 2x - 5 \end{aligned}$$

$$\begin{aligned} x^2 + (2x - 5)^2 &= 25 \\ x^2 + 4x^2 - 20x + 25 &= 25 \\ 5x^2 - 20x &= 0 \end{aligned}$$

$$5x^2 - 20x = 0$$

$$x(5x - 20) = 0$$

$x=0$	$\frac{5x}{5} = \frac{20}{5}$
	$x=4$

Solutions  
(0, -5)  
(4, 3)

$$\begin{aligned} y + 5 &= 2x \\ y + 5 &= 2(0) \\ y + 5 &= 0 \\ -5 & \\ \hline y &= -5 \end{aligned}$$

$$\begin{aligned} y + 5 &= 2(4) \\ y + 5 &= 8 \\ -5 & \\ \hline y &= 3 \end{aligned}$$

Check

$$(4)^2 + (3)^2 = 25$$

$$\begin{aligned} 16 + 9 &= 25 \\ 25 &= 25 \checkmark \end{aligned}$$

$$\begin{aligned} (0)^2 + (-5)^2 &= 25 \\ 0 + 25 &= 25 \\ 25 &= 25 \checkmark \end{aligned}$$

Score 4: The student gave a complete and correct response.

**Question 36**

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$

$$y + 5 = 2x$$

$$y = 2x - 5$$

$$x^2 + (2x - 5)^2 = 25$$

$$x^2 + (2x - 5)(2x - 5) = 25$$

$$x^2 + 4x^2 - 20x + 25 = 25$$

$$5x^2 - 20x = 0$$

$$5x(x - 4) = 0$$

$$x = 0 \quad x = 4$$

$$y + 5 = 2(0) \quad y + 5 = 2(4)$$

$$y = -5$$

$$y = 3$$

**Score 4:** The student gave a complete and correct response.

Question 36

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$

$$y + 5 = 2x$$

$$y = 2x - 5$$

$$\begin{aligned} &(2x-5)(2x-5) \\ &4x^2 - 10x - 10x + 25 \\ &4x^2 - 20x + 25 \end{aligned}$$

$$x^2 - (2x-5)^2 = 25$$

$$x^2 - 4x^2 + 20x + 25 = 25$$

$$5x^2 - 20x + 25 = 25$$

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$$5x = 0 \quad \boxed{x=4}$$

$$y = 0$$

(reject)

$$\begin{aligned} y+5 &= 2x \\ y+5 &= 2(4) \\ y+5 &= 8 \\ \boxed{y=3} \end{aligned}$$

**Score 3:** The student stated only one solution.

Question 36

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$

$$y + 5 = 2x \quad y = 2x - 5$$

$$x^2 + (2x - 5)^2 = 25$$

$$x^2 + (2x - 5)(2x - 5) = 25$$

$$x^2 + 4x^2 - 10x - 10x + 25 = 25$$

$$(5x^2 - 20x) = 0$$

$$x(5x - 20) = 0$$

$$5x = 20$$

$$x = 4$$

$$y + 5 = 2(8)$$

$$y + 5 = 16$$

$$-5 \quad -5$$

$$y = 11$$

**Score 2:** The student received credit for writing a quadratic equation in standard form.

Question 36

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$

$$y + 5 = 2x$$

$$\begin{array}{r} -5 \quad -5 \\ y = 2x - 5 \end{array}$$

$$\sqrt{x^2 + (2x - 5)^2} = \sqrt{25}$$

$$\begin{array}{r} * \quad 3x + 5 = 5 \\ \quad \quad -5 \quad -5 \end{array}$$

$$\begin{array}{r} 3x = 0 \\ \quad \quad 3 \quad 3 \end{array}$$

$$\begin{array}{l} x = 0 \\ y = -5 \end{array}$$

**Score 1:** The student found a correct quadratic equation in one variable, but showed no further correct work.

Question 36

36 Solve the system of equations algebraically:

$$\begin{cases} x=4 \\ y=3 \end{cases}$$

$$x^2 + y^2 = 25$$

$$x^2 - 5y = -2xy + 5$$

$$\frac{-5y}{y} = \frac{-2xy}{y}$$

$$x^2 = 25$$

$$\sqrt{x^2} \sqrt{y^2} = 25$$

$$x + y = 25 - x$$

$$y = 2x - 5$$

$$\begin{aligned} y &= 25 - x \\ y &= 2x - 5 \end{aligned}$$

$$y = 2x - 5$$

$$y^2 = 25 - x^2$$

$$x^2 + (2x - 5)^2 = 25$$

$$(2x - 5)(2x - 5)$$

$$4x^2 - 10x + 25$$

$$5x^2 - 20x + 25 = 25$$

$$-20x - 25$$

$$\begin{array}{r} 25 - x = 2x - 5 \\ \hline 1x \quad +x \end{array}$$

$$\begin{array}{r} 25 = 3x - 5 \\ \hline +5 \end{array}$$

$$\begin{array}{r} 30 = 3x \\ \hline /3 \end{array}$$

$$x = 10$$

$$y + 5 = 2 \cdot 10$$

$$\begin{array}{r} y + 5 = 20 \\ \hline -5 \end{array}$$

$$y = 15$$

**Score 1:** The student found a correct quadratic equation in one variable but earned no credit for  $x = 4$  and  $y = 3$  because no work was shown to solve the quadratic equation.

Question 36

36 Solve the system of equations algebraically:

$$x^2 + y^2 = 25$$

$$y + 5 = 2x$$

$$y = 2x - 5$$

$$y^2 = -x^2 + 25$$

$$y = \sqrt{-x^2 + 25}$$

$$\sqrt{-x^2 + 25} = 2x - 5$$

$$\sqrt{-x^2 + 30} = 2x$$

$$x + 30 = 2x$$

$$x = 30$$

$$y + 5 = 2x$$

$$y + 5 = 2(30)$$

$$y + 5 = 60$$

$$y = 55$$

$$y = 55$$

**Score 0:** The student did not show enough correct work to receive any credit.



**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$

$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

$$1.015 - 1 = 0.015 \times 100 = 1.5$$

$$\boxed{1.5\%}$$

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = 92.2(1.015)^t$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$300 = 92.2(1.015)^t$$

$$\frac{300}{92.2} = (1.015)^t$$

$$\frac{\log\left(\frac{300}{92.2}\right)}{\log(1.015)} = \frac{t \log(1.015)}{\log(1.015)}$$

$$79.24 = t$$

$$t \approx 79$$

**Score 6:** The student gave a complete and correct response.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

1.5%

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = 92.2 e^{.015t}$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$\frac{300}{92.2} = \frac{92.2 e^{.015t}}{92.2}$$
$$\frac{\ln\left(\frac{300}{92.2}\right)}{.015} = \frac{.015t \ln e}{.015 \cdot 1}$$
$$t = 78.65$$
$$t = 79$$

**Score 6:** The student gave a complete and correct response.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

.015

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = 92.2(1.015)^t$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$300 = 92.2(1.015)^t$$
$$\frac{300}{92.2} = 92.2$$
$$3.253796095 = (1.015)^t$$
$$t \log 1.015 = \log 3.253796095$$
$$\frac{\log 1.015}{\log 1.015} = \frac{\log 3.253796095}{\log 1.015}$$
$$t = 79$$

**Score 5:** The student wrote the percent incorrectly.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

it will grow 1.5 percent  
Per year

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = 92.2(1.015)^t$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$\frac{300,000,000}{92.2} = \frac{92.2(1.015)^t}{92.2}$$
$$\frac{\log 3253796.095}{\log 1.015} = t \frac{\log 1.015}{\log 1.015}$$
$$1007.16 = t$$
$$t = 1007$$

**Score 5:** The student made an incorrect substitution for the population.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

$$1.015(92.2) = 93.8875$$

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$y = 92.2(1.015)^x$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$\frac{300}{92.2} = \frac{92.2(1.015)^x}{92.2}$$

$$\frac{300}{92.2} = 1.015^x$$

$$\frac{\log\left(\frac{300}{92.2}\right)}{\log(1.015)} = x \frac{\log(1.015)}{\log(1.015)}$$

$$x = 79$$

**Score 4:** The student did not identify the annual growth rate and made a notation error in writing the exponential function.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

$$1.015 - 1 = .015 \times 100 = 1.5\%$$

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = 92.2^{1.5t}$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$P(t) = 92.2^{1.5t}$$
$$\log(300 = 92.2^{1.5t})$$
$$\log 300 = 1.5t \log 92.2$$
$$\frac{\log 300}{\log 92.2} = 1.5t$$
$$\frac{\log 300}{1.5 \log 92.2} = t \quad .84 = t$$

**Score 3:** The student wrote an incorrect exponential function, then made a rounding error.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

$$a_1 = 1.015(92.2) = 93.6$$

94%

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P = 92.2(1.015)^x$$

$$\begin{aligned} 93.6 &= 1 \\ 95.0 &= 2 \\ 96.4 &= 3 \\ 97.8 &= 4 \\ 99.3 &= 5 \end{aligned}$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$300 = 92.2(1.015)^x$$
$$x = 79$$

**Score 2:** The student did not find the correct percentage, made a notation error, and showed no work for  $x = 79$ .

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$\begin{aligned} a_0 &= 92.2 \\ a_n &= 1.015a_{n-1} \end{aligned}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$92.2(1.015)^t$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

**Score 1:** The student wrote a correct expression.



**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

1.5%

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

~~$P(t) = 92.2$~~   $(1.015)^t$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

~~$a_1 = 93$~~

~~$a_2 =$~~

~~$a_3 =$~~

~~$a_4 =$~~

$$(1.015)^{300} = 87.06$$

87 years

**Score 1:** The student correctly found the percent of annual growth, but showed no further correct work.

**Question 37**

37 The population, in millions of people, of the United States can be represented by the recursive formula below, where  $a_0$  represents the population in 1910 and  $n$  represents the number of years since 1910.

$$a_0 = 92.2$$

$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation  $a_n = 1.015a_{n-1}$ .

93.583  ~~$A = A_0 r$~~   $(1.015)(92.2)$  10% growth

Write an exponential function,  $P$ , where  $P(t)$  represents the United States population in millions of people, and  $t$  is the number of years since 1910.

$$P(t) = A_0 (n-1)^t$$

$$P(t) = 'A_0' (n-1)^t$$

According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

$$P(t) = 300 \times 10^8 (n-1)^t$$

**Score 0:** The student did not show enough correct work to earn any credit.