

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, August 16, 2017 — 12:30 to 3:30 p.m.

MODEL RESPONSE SET

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Question 25

25 Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

rewrite $\frac{4}{3}$ as $\frac{1}{3} \times \frac{4}{1}$, using the power to a power rule

Score 2: The student gave a complete and correct response.

Question 25

25 Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

-8 can be cube rooted, followed by being raised to the 4th power, making the answer 16.

Score 2: The student gave a complete and correct response.

Question 25

25 Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

$$(-8)^{\frac{4}{3}} = (\sqrt[3]{-8})^4 = (-2)^4 = 16$$

Score 1: The student gave a correct justification, not an explanation.

Question 25

25 Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

$\sqrt[3]{(-8)^4}$

set up $(-8)^{\frac{4}{3}}$ as $\frac{\text{power}}{\text{root}}$, $\sqrt[3]{(-8)^4}$

$(-8)^4 = -4096$?

$\sqrt[3]{-4096} = 16i$

Score 0: The student made multiple errors and did not provide an explanation.

Question 26

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

$$\begin{array}{l} P(W \cap D) = .4 \\ P(D) = .5 \end{array} \qquad \frac{.4}{.5} = \boxed{.8} \qquad P(W|D)$$

Score 2: The student gave a complete and correct response.

Question 26

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

50% - drug

50% - sugar pill

receiving drug + getting well = 40%.

$$\frac{40}{50} = .8 = 80\%$$

Score 2: The student gave a complete and correct response.

Question 26

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

	Drug	Sugar	
Well	.4	.4	.8
Not Well	.1	.1	.2
	.50	.50	1

Handwritten annotations: A bracket on the left side of the table groups the 'Well' and 'Not Well' rows. An arrow points from the top-left corner of this bracket to the value .4 in the 'Well' row, 'Drug' column. Another arrow points from the bottom-left corner of the bracket to the value .50 in the 'Drug' column, 'Total' row.

$.8$

Score 1: The student gave a correct answer based on the drug column in the table, even though there is no evidence to support the data in the sugar column.

Question 26

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

$$\begin{aligned} &P(W) \cdot P(D) \\ &(.4)(.5) \\ &\boxed{.20} \end{aligned}$$

Score 0: The student made an error confusing independence with conditional probability, and substituted incorrectly for $P(W)$, which is actually unknown.

Question 27

27 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$\begin{aligned}(x^2 + y^2)^2 &= (x^2 - y^2)(x^2 - y^2) + (2xy)(2xy) \\(x^2 + y^2)^2 &= (x^4 - 2x^2y^2 + y^4) + 4x^2y^2 \\(x^2 + y^2)^2 &= x^4 + 2x^2y^2 + y^4 \\(x^2 + y^2)^2 &= (x^2 + y^2)(x^2 + y^2) \\(x^2 + y^2)^2 &= (x^2 + y^2)^2\end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 27

27 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$\begin{array}{c} x^2 \quad y^2 \\ \begin{array}{|c|c|} \hline x^4 & x^2y^2 \\ \hline x^2y^2 & y^4 \\ \hline \end{array} \end{array}$$

$$x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 \quad \checkmark$$

Score 2: The student gave a complete and correct response.

Question 27

27 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

The image shows a handwritten algebraic proof of the identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$. The student expands both sides of the equation. On the left side, $(x^2 + y^2)^2$ is expanded to $x^4 + 2x^2y^2 + y^4$. On the right side, $(x^2 - y^2)^2 + (2xy)^2$ is expanded to $x^4 - 2x^2y^2 + y^4 + 4x^2y^2$. The student then shows that the $2x^2y^2$ term on the left is equal to the $4x^2y^2$ term on the right, with a checkmark indicating the equality.

$$\begin{aligned} \cancel{x^4} + 2x^2y^2 + \cancel{y^4} &= \cancel{x^4} - 2x^2y^2 + \cancel{y^4} + 4x^2y^2 \\ 2x^2y^2 &= 4x^2y^2 \quad \checkmark \end{aligned}$$

Score 2: The student gave a complete and correct response given there are no domain restrictions for addition and subtraction.

Question 27

27 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$\begin{array}{r} \cancel{x^4} + 2x^2y^2 + \cancel{y^4} = \cancel{x^4} - 2x^2y^2 + \cancel{y^4} + 2x^2y^2 \\ \hline -\cancel{x^4} + 2x^2y^2 - \cancel{y^4} \quad -\cancel{x^4} + 2x^2y^2 - \cancel{y^4} \\ \hline 4x^2y^2 \neq 2x^2y^2 \end{array}$$

Score 1: The student made an error squaring $2xy$.

Question 27

27 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$\text{IF } x=2$$

$$\text{IF } y=4$$

$$(2^2 + 4^2)^2 = (2^2 - 4^2)^2 + (2(2)(4))^2$$

$$(4 + 16)^2 = (4 - 16)^2 + (16)^2$$

$$400 = 144 + 256$$

$$400 = 400$$

Score 0: The student did not verify the identity for all values of x and y .

Question 28

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said “What are the odds I got all of that kind?” Mrs. Jones replied, “simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual.”

Explain how this simulation could be used to solve the problem.

The student could choose a number to represent black jellybeans, then see how many times that number would have been rolled 4 times in a row after 250 simulations of rolling a die.

Score 2: The student gave a complete and correct response.

Question 28

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said “What are the odds I got all of that kind?” Mrs. Jones replied, “simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual.”

Explain how this simulation could be used to solve the problem.

On a die, there are six numbers (1-6). If her student rolled the die 250 times, the student could calculate the probability of getting each of the numbers. If the student identified each flavor of jelly bean with a number (ex. black is six), the student could see how likely the chance would be of getting black/six.

Score 1: The student gave an incomplete explanation.

Question 28

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said “What are the odds I got all of that kind?” Mrs. Jones replied, “simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual.”

Explain how this simulation could be used to solve the problem.

This simulation would work because the dice takes the place of the 6 different flavors of jelly beans and the numbers on the dice represent each of the 6 flavors. Each 4 times the dice is rolled, the numbers would represent the 4 jelly beans that were picked out of the bag.

Score 1: The student gave an incomplete explanation.

Question 28

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said “What are the odds I got all of that kind?” Mrs. Jones replied, “simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual.”

Explain how this simulation could be used to solve the problem.

Rolling the dice 250 times is testing probability because the chances of rolling a dice 250 times and getting the same number more than 4 times might be high or low but you would be able to test that theory out of 250 times.

Score 0: The student did not explain the simulation.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$a_1 = 4$$

$$a_n = 2a_{n-1} + 1$$

5 10 20 40

$$9 = 2 \cdot 4 + 1$$

$$19 = 2 \cdot 9 + 1$$

$$39 = 2 \cdot 19 + 1$$

Determine the eighth term in Candy's sequence.

$$a_6 = 2(79) + 1 = 159$$

$$a_7 = 2(159) + 1 = 319$$

$$a_8 = 2(319) + 1 = 639$$

Score 2: The student gave a complete and correct response.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$2x + 1$$

Determine the eighth term in Candy's sequence.

$$6 \quad 2(79) + 1 = 159$$

$$7 \quad 2(159) + 1 = 319$$

$$8 \quad 2(319) + 1 = 639$$

Score 1: The student showed appropriate work to find 639.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$a_n = (a_{n-1} \cdot 2) + 1$$

Determine the eighth term in Candy's sequence.

$$\begin{array}{cccccccc} & +5 & +10 & +20 & +40 & +80 & +160 & +320 \\ 4, & 9, & 19, & 39, & 79, & 159, & 319, & 639 \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{array}$$

$$a_8 = 639$$

Score 1: The student did not identify $a_1 = 4$.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$\begin{aligned} a_1 &= 4 \\ a_n &= 5(2^{n-1}) - 1 \end{aligned}$$

Determine the eighth term in Candy's sequence.

$$639$$

Score 1: The student did not write a recursive formula.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$\frac{9}{4} = 2.25$$

$$n(2.25)^x$$

Determine the eighth term in Candy's sequence.

$$a_8 = (4)(2.25)^7 \quad \cancel{a_n = a_1 + (n-1)d}$$

$$a_8 = 1167.717041$$

Score 0: The student provided no correct work.

Question 29

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79,

Write a recursive formula for Candy's sequence.

$$n = 2n + 1$$

Determine the eighth term in Candy's sequence.

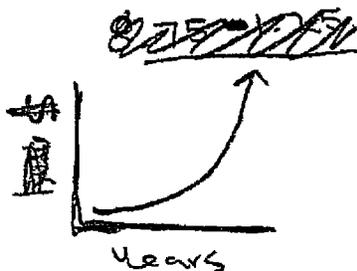
4, 9, 19, 39, 79, 159, 317, 633

633

Score 0: The student did not provide a recursive formula and made a computational error.

Question 30

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was \$1.25 an hour and in 2015, it was \$8.75. Algebraically determine the rate of growth to the nearest percent.



$$8.75 = 1.25(x)^{2015-1966}$$

$$8.75 = 1.25(x)^{49}$$

$$\sqrt[49]{7} = \sqrt[49]{x}$$

$$1.0405 \approx \boxed{4\% \text{ growth}}$$

Score 2: The student gave a complete and correct response.

Question 30

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was \$1.25 an hour and in 2015, it was \$8.75. Algebraically determine the rate of growth to the nearest percent.

$$8.75 = 1.25 (x)^{49}$$

$$7 = x^{49}$$

$$\sqrt[49]{7} = x$$

$$\boxed{x = 1.04}$$

Score 1: The student did not determine the rate of growth.

Question 30

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was \$1.25 an hour and in 2015, it was \$8.75. Algebraically determine the rate of growth to the nearest percent.

$$A = e^{rt}$$

$$7.5 = e^{r \cdot 49}$$

$$\frac{\ln 7.5}{49} = \frac{r \cdot 49}{49}$$

$$r = .0411 \dots$$

$$\boxed{r = 4\%}$$

Score 1: The student made an error by subtracting $8.75 - 1.25$.

Question 30

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was \$1.25 an hour and in 2015, it was \$8.75. Algebraically determine the rate of growth to the nearest percent.

$$A = 1.25 e^{k(2015-1966)} + 8.75$$
$$- 8.75 = 3.397 \dots$$

$$\ln 3.397 \dots = 49k$$

$$1.7733 = 49k$$

$$0.036 = k$$

$$\boxed{4\%}$$

Score 0: The student obtained a correct answer, but made multiple errors.

Question 31

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.

$$j(x) = x^4 - 3x^2 - 4$$

$$j(-x) = (-x)^4 - 3(-x)^2 - 4$$

$$j(-x) = x^4 - 3x^2 - 4$$

Even
same

$$\boxed{j(-x) = j(x)}$$

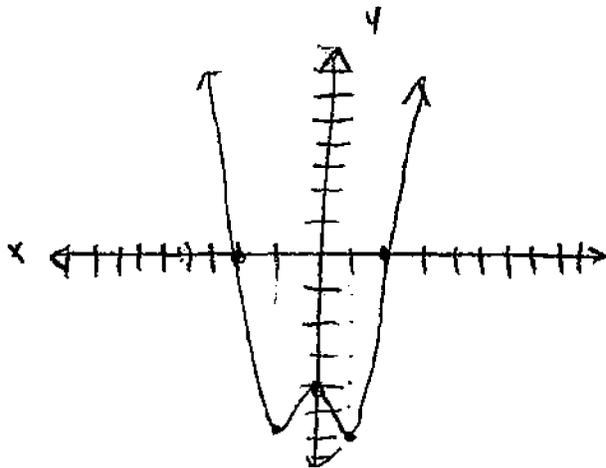
function as
as $j(x)$

$j(-x)$ is the

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.



$j(x)$ is even
b/c it is symmetric
across the
y-axis

Score 1: The student used a method other than algebraic.

Question 31

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.

$$j(-1) = -6$$

$$j(1) = -6$$

$j(x)$ is even

Score 1: The student did not verify for all values of $-x$.

Question 31

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.

even because all exponents are even.

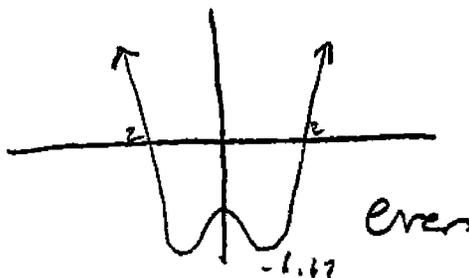
Score 1: The student used a method other than algebraic.

Question 31

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.

$$x^4 - 3x^2 - 4$$

$$\cancel{x^2} / \cancel{x^2} - 3$$



Even has two
arrows going same
way up

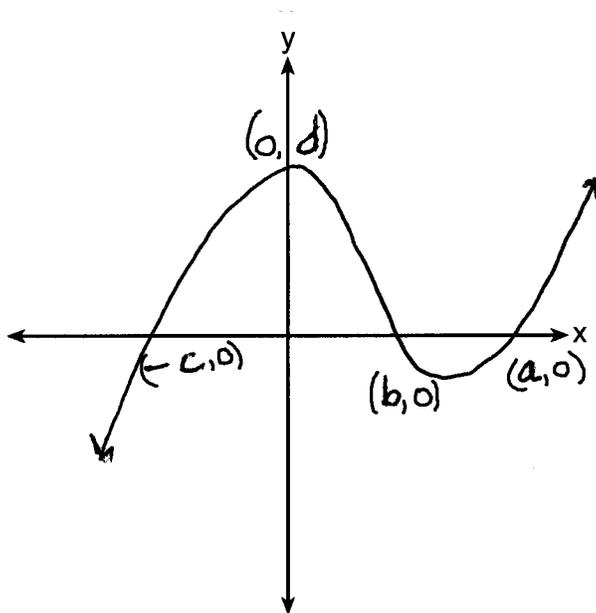
it's not odd

because
it will have

Score 0: The student incorrectly justified an even function and used a method other than algebraic.

Question 32

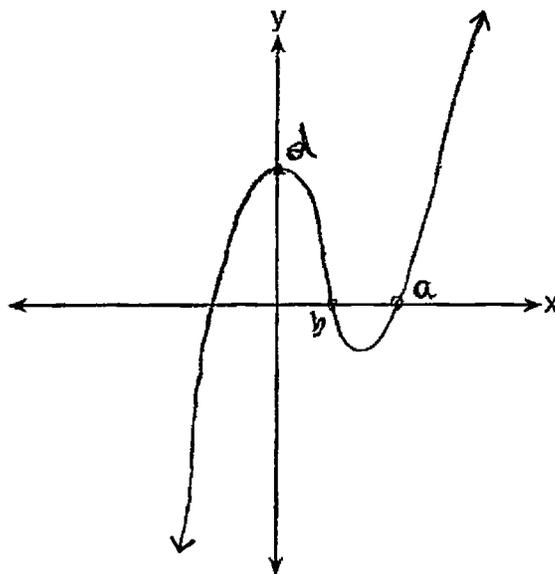
32 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where a, b , and c are positive, $a > b$, and $p(x)$ has a positive y -intercept of d . Label all intercepts.



Score 2: The student gave a complete and correct response.

Question 32

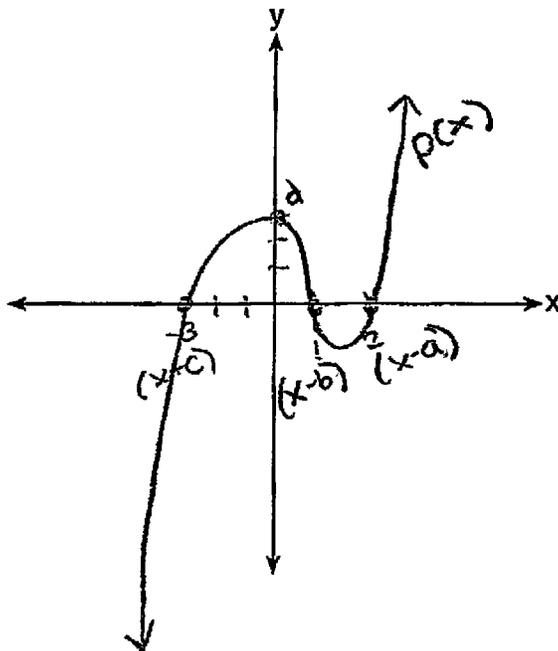
32 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where a, b , and c are positive, $a > b$, and $p(x)$ has a positive y -intercept of d . Label all intercepts.



Score 1: The student did not label the intercept at $-c$.

Question 32

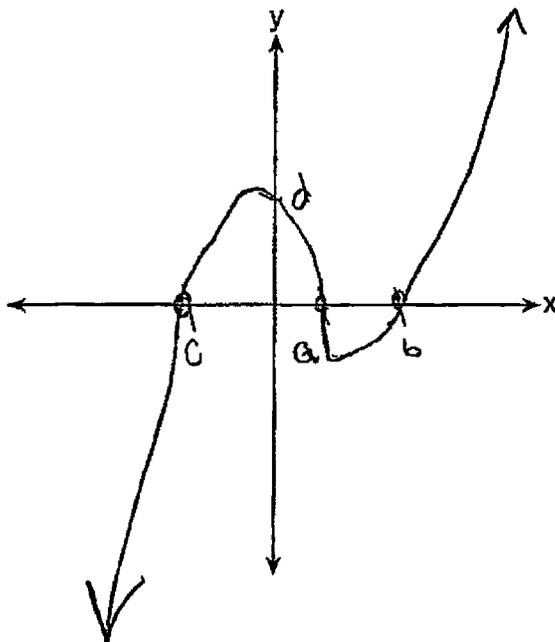
32 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where $a, b,$ and c are positive, $a > b,$ and $p(x)$ has a positive y -intercept of $d.$ Label all intercepts.



Score 1: The student did not label the x -intercepts correctly.

Question 32

32 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where a, b , and c are positive, $a > b$, and $p(x)$ has a positive y -intercept of d . Label all intercepts.



Score 0: The student made multiple labeling errors.

Question 33

33 Solve for all values of p : $\frac{3p}{p+3} - \frac{2}{p-5} = \frac{p}{p+3} - \frac{p-5}{p-5}$

$$\frac{3p^2 + 9p}{(p+3)(p-5)} - \frac{2p-10}{(p+3)(p-5)} = \frac{p^2 - 5p}{(p+3)(p-5)}$$

$$\frac{3p^2 + 7p + 10}{(p+3)(p-5)} = \frac{p^2 - 5p}{(p+3)(p-5)}$$

$$\begin{array}{r} 3p^2 + 7p + 10 = p^2 - 5p \\ -p^2 + 5p \end{array}$$

$$\begin{array}{r} 2p^2 + 12p + 10 = 0 \\ 10p + 12p \\ (2p+2)(p+5) = 0 \end{array}$$

$$\begin{array}{r} 2p+2=0 \\ -2 \quad -2 \\ \hline 2p=-2 \\ \frac{2}{2} \quad \frac{-2}{2} \\ \hline p=-1 \end{array}$$

$$\begin{array}{r} p+5=0 \\ -5 \quad -5 \\ \hline p=-5 \end{array}$$

Check

$$\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$$

$$\frac{3(-1)}{-1-5} - \frac{2}{-1+3} = \frac{-1}{-1+3}$$

$$\frac{-3}{-6} - \frac{2}{2} = \frac{-1}{2}$$

$$\frac{3}{6} - \frac{6}{6} = \frac{-1}{2}$$

$$-\frac{3}{6} = -\frac{1}{2}$$

$$-\frac{1}{2} = -\frac{1}{2}$$

Check

$$\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$$

$$\frac{3(5)}{-5-5} - \frac{2}{-5+3} = \frac{-5}{-5+3}$$

$$\frac{-15}{-10} - \frac{2}{-2} = \frac{-5}{-2}$$

$$\frac{-15}{-10} + 1 = \frac{3}{2}$$

$$\frac{3}{2} + \frac{2}{2} = \frac{5}{2}$$

$$\frac{5}{2} = \frac{5}{2}$$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve for all values of p : $\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$

$$3p(p+3) + -2(p-5) = p(p-5)$$

$$3p^2 + 7p + 10 = p^2 - 5p$$

$$-(p^2 - 5p) = -(p^2 - 5p)$$

$$2p^2 + 12p + 10 = 0$$

$$2(p+1)(p+5) = 0$$

$$p+1=0 \text{ or } p+5=0$$

$$-1 \quad \cancel{5} \text{ Reject}$$

Score 3: The student incorrectly rejected one of the solutions.

Question 33

33 Solve for all values of p : $\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3} \cdot \frac{(p-5)(p+3)}$

$$3p(p+3) + 2(p-5) = p(p-5)$$

$$3p^2 + 9p + 2p - 10 = p^2 - 5p$$

$$\begin{array}{r} -p^2 + 5p \\ \hline \end{array}$$

$$2p^2 + 14p - 10 = 0$$

$$\frac{2(p^2 + 7p - 5) = 0}{2} \quad \frac{0}{2}$$

$$p^2 + 7p - 5 = 0$$

$$a=1 \quad b=7 \quad c=-5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{49 + 20}}{2}$$

$$= \frac{-7 \pm \sqrt{69}}{2}$$

$$= \frac{-7 \pm 2\sqrt{21}}{2}$$

$$= \frac{2(-4 \pm \sqrt{21})}{2}$$

$$= -4 \pm \sqrt{21}$$

$$\{-4 + \sqrt{21}, -4 - \sqrt{21}\}$$

Score 2: The student made a transcription error in the first line of the solution and did not check for extraneous solutions.

Question 33

33 Solve for all values of p : $\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$

$$\frac{3p}{p-5} = \frac{p}{p+3} + \frac{2}{p+3}$$

$$\frac{3p}{p-5} = \frac{2+p}{p+3} \cdot \frac{p+3}{2+p}$$

$$\frac{p+3}{2+p} \cdot \frac{3p}{p-5}$$

$$\frac{3p}{p-5} = \frac{2+p}{p+3}$$

$$3p(p+3) = \frac{p+3}{(2+p)(p-5)}$$

$$3p^2 + 9p = 2p + p^2 - 10 - 5p$$

$$2p^2 + 12p = -10$$

$$2p^2 + 12p + 10 = 0$$

$$(\cancel{p^2 + 10})(\cancel{p^2 + 12})$$

Score 2: The student wrote a correct quadratic equation in standard form.

Question 33

33 Solve for all values of p : $\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$

$$+ \frac{2}{p+3} \quad + \frac{2}{p+3}$$

$$\frac{3p}{p-5} = \frac{p}{p+3} + \frac{2}{p+3}$$

$$\frac{3p}{p-5} = \frac{2+p}{p+3}$$

$$3p^2 + 9p = p^2 - 3p - 10$$

$$2p^2 + 9p = 3p - 10$$

$$2p^2 + 12p = -10$$

$$2p^2 + 12p + 10$$

Score 1: The student wrote a correct quadratic expression.

Question 33

33 Solve for all values of p : $\frac{3p}{p-5} - \frac{2}{p+3} = \frac{p}{p+3}$

$$\begin{array}{r} p-5 \quad p \\ p-5 \quad 3 \end{array}$$

$$p^2 - 2p - 15$$

$$\frac{-3p}{p^2 - 2p + 15} - \frac{2}{p^2 - 2p + 15} = \frac{p}{p^2 - 2p + 15}$$

$$\begin{array}{r} 3p - 2 = p \\ -5p \quad -3p \end{array}$$

$$\frac{-2}{-2} = \frac{-2p}{-2}$$

$$p = 1$$

Score 0: The student did not show enough correct work to receive any credit.

Question 34

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

per day, + .25

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

161. < 5 days 2.25
21 days 6.25 4.00 diff. card = \$1.

$$a_n = a_1 + (n-1)d$$

$$a_n = 1.25 + (n-1) \cdot .25$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

\$16.00 60 -> 25

$$a_1 = 1.25 + (60-1) \cdot .25$$
$$1.25 + 14.75$$

Score 4: The student gave a complete and correct response.

Question 34

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$d = \frac{4}{16} \rightarrow .25$$
$$a_n = a_5 + (n-5)d$$
$$a_n = 2.25 + (n-5)(.25)$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$a_{60} = 2.25 + (55)(.25)$$
$$a_{60} = 16$$

Score 4: The student gave a complete and correct response.

Question 34

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

5 days \rightarrow \$2.25
1 card

21 days \rightarrow 6.25
1 card

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$a_5 = 2.25$$

$$a_{21} = 6.25$$

$$6.25 = a_1 + d(20)$$

$$2.25 = a_1 + d(4)$$

$$2.25 = a_1 + 1(4)(.25)$$

$$a_1 = 1.25$$

$$4 = 16d \quad d = .25$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$a_n = 1.25 + .25(n-1)$$

$$a_n = .25n + 1$$

$$a_{60} = 1.25 + .25(60-1)$$

$$\boxed{\$16}$$

Score 4: The student gave a complete and correct response.

Question 34

- 34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$\begin{aligned} a &= \text{amount due per day late} \\ n &= \text{Number of days late} \\ .25n + 1.00 &\rightarrow \text{Amount needed to replace card} \end{aligned}$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$.25 \cdot 60 = 15$$

$$15 + 1 = 16$$

\$16 is needed to pay for overdue book and replace card.

Score 3: The student wrote an expression for a_n .

Question 34

- 34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

$$\frac{2.25}{a_5} \quad \frac{6.25}{a_{21}} \quad .25/d$$

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$a_n = 2.25 + (n-1).25$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$a_n = 2.25 + (n-1).25$$
$$a_{60} = 2.25 + (59).25$$

$$a_{60} = \$17$$

Score 2: The student made a conceptual error writing the formula for a_n by not adjusting the number of common differences, but found an appropriate amount based on that error.

Question 34

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

5 21
2.25 6.25

4 days = 1 dollar
25¢ = 1 day

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$\frac{5}{2.25}$ $\frac{21}{6.25}$ $\frac{25}{7.25}$ $\frac{29}{8.25}$ $\frac{33}{9.25}$ $\frac{37}{10.25}$ $\frac{41}{11.25}$ $\frac{45}{12.25}$ $\frac{49}{13.25}$ $\frac{53}{14.25}$ $\frac{57}{15.25}$ $\frac{60}{16.00}$

\$16.00

Score 1: The student correctly determined \$16 without using a formula.

Question 34

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$a_n = \sum_{k=1}^n (2.25)^{k-1}$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

\$16.00

Score 1: The student found the correct amount of money, but did not show any work.

Question 34

- 34** Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed \$2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed \$6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n , the n th term of this sequence.

$$a_n = .45 + (n - 1)d$$

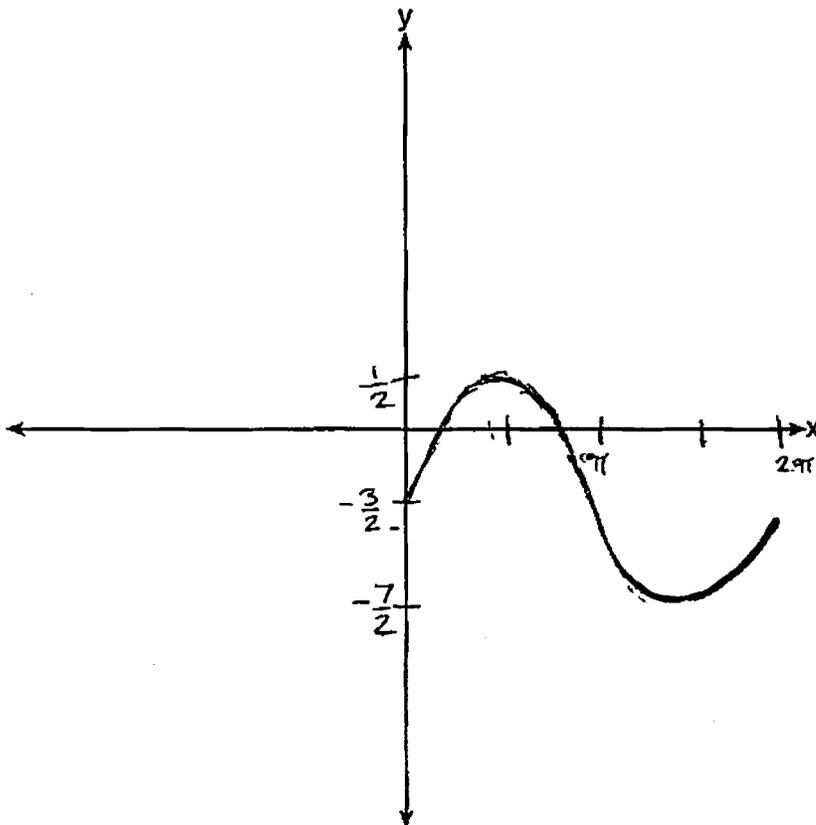
Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$a_{60} = .45 + (60 - 1)d$$
$$\$15.93$$

Score 0: The student did not show any correct work.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .



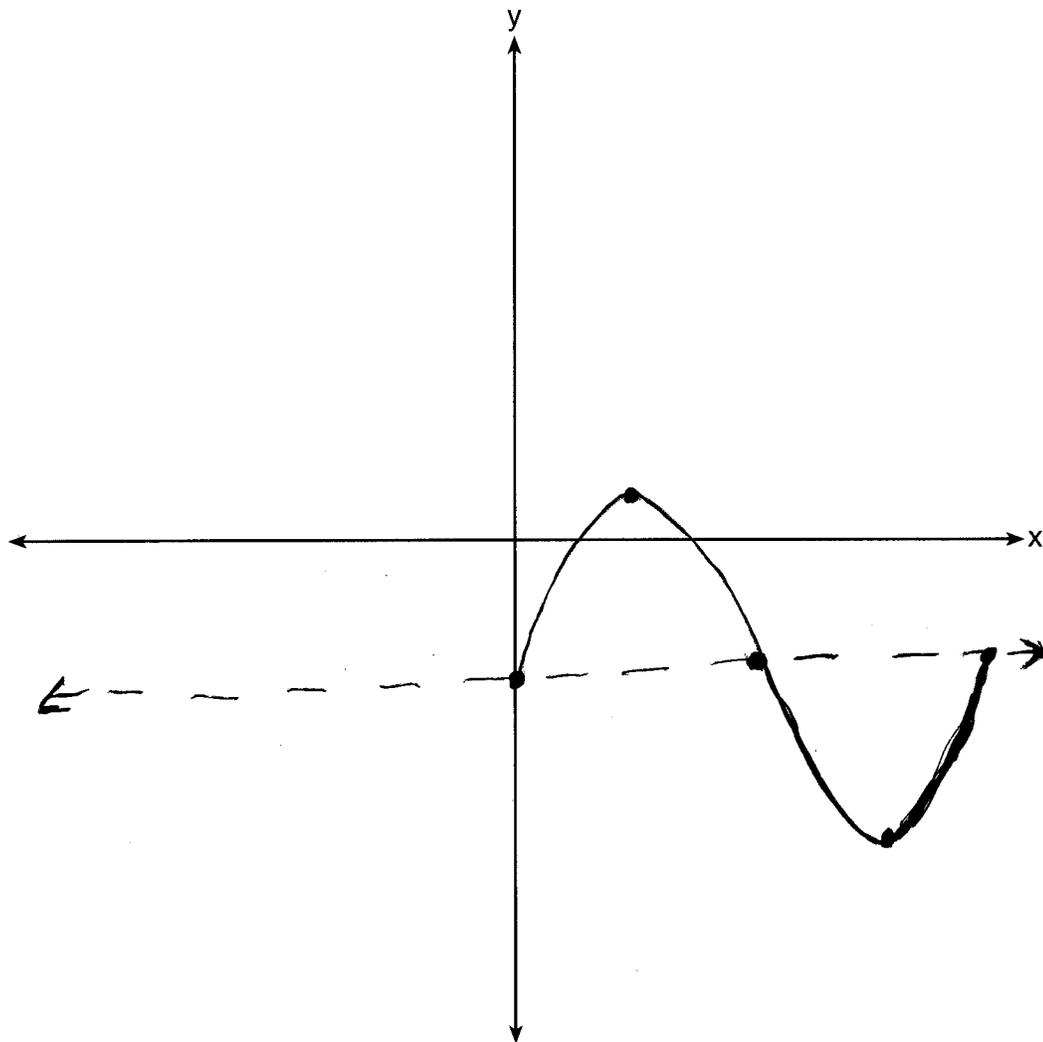
- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

It is moved $\frac{\pi}{3}$ units to the right (on the x-axis).

Score 4: The student gave a complete and correct response.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .



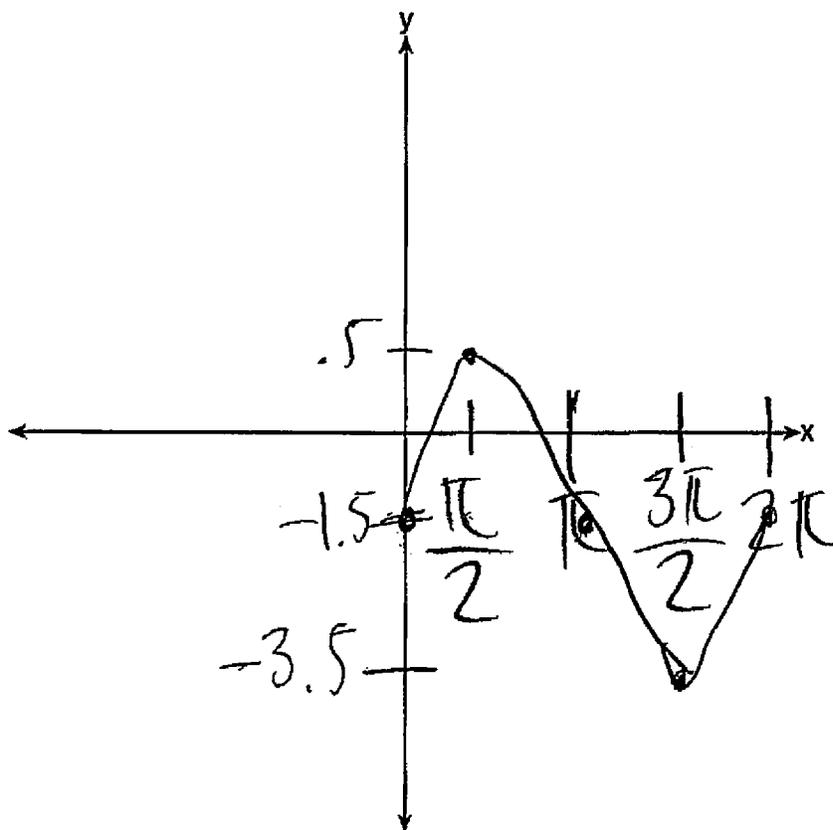
- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

shifted to the right $\frac{\pi}{3}$

Score 3: The student did not label the sketch.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .



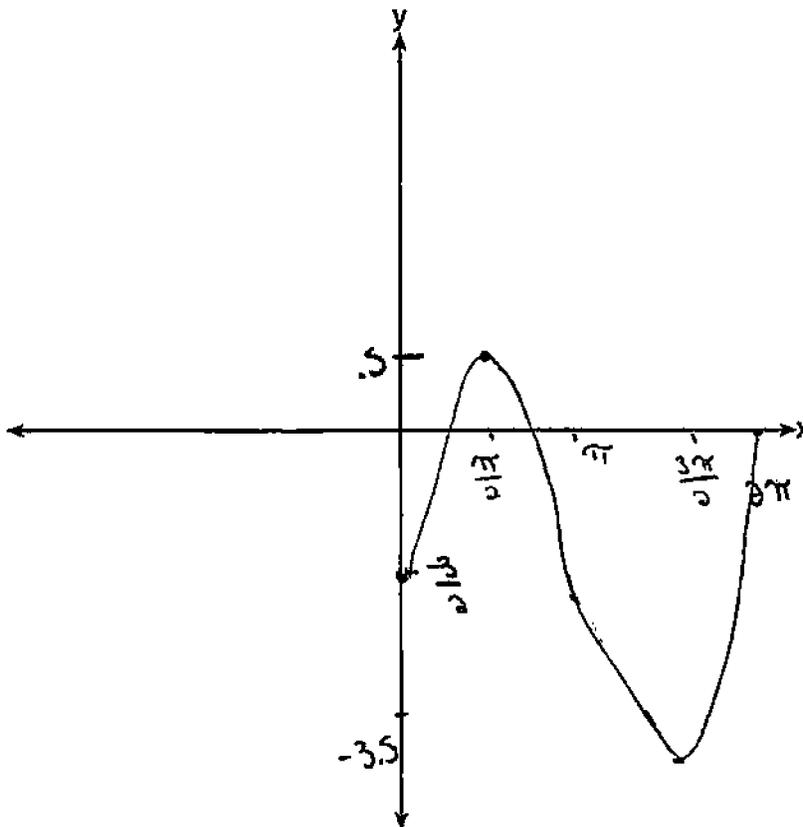
- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

The values would be doubled

Score 3: The student only received credit for the sketch.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .

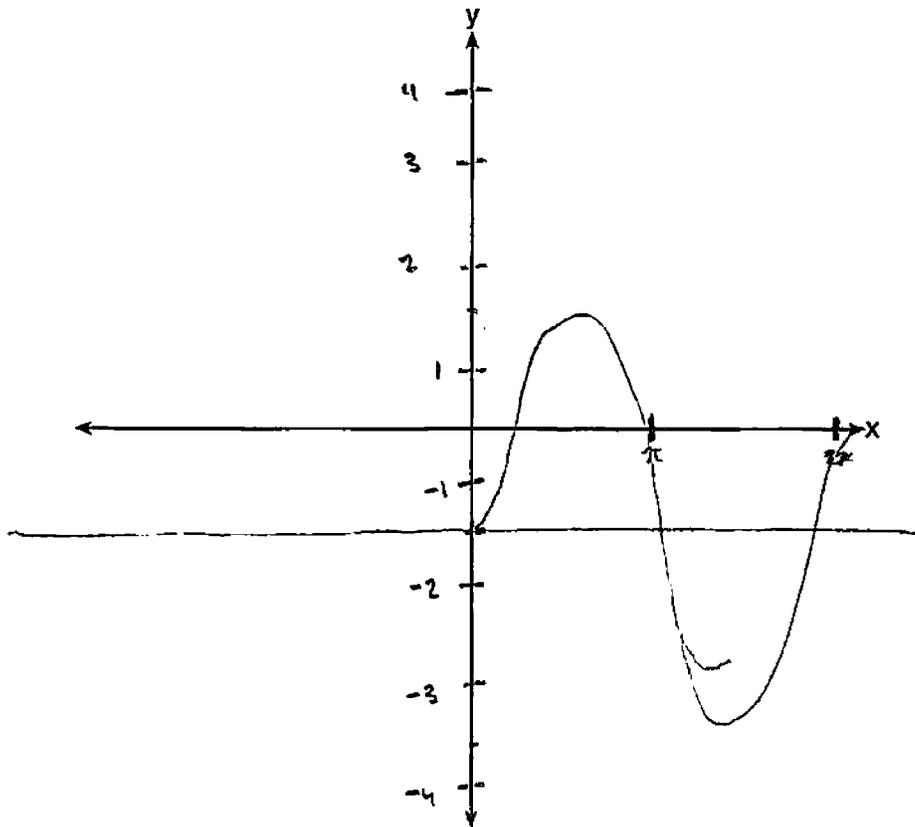


- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

Score 2: The student made one graphing error with no explanation.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .

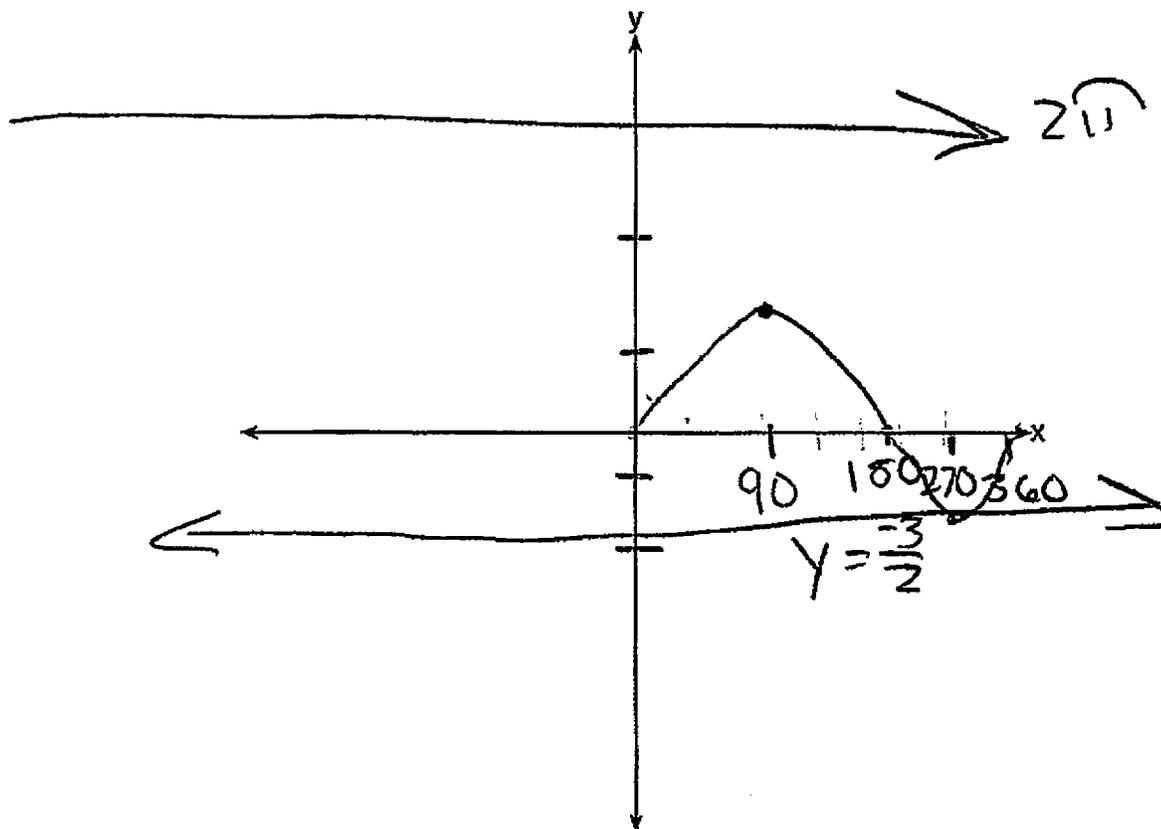


- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part *a*.

Score 1: The student made two graphing errors with no explanation.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .



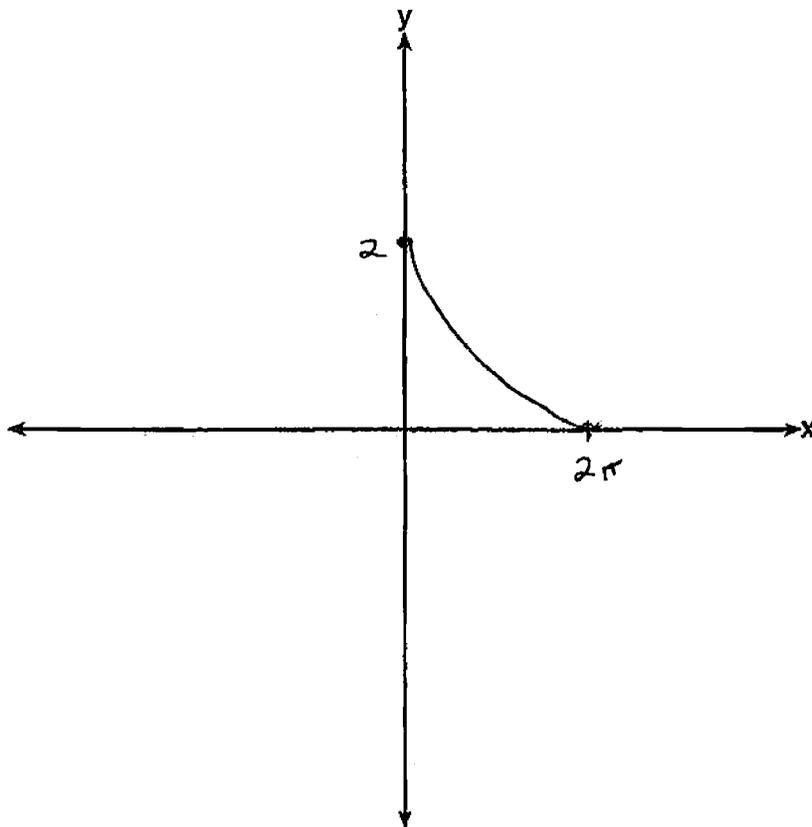
- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

$y = 2 \sin \dots$ increases
 sketch from a did not

Score 0: The student made several errors, and wrote an incorrect explanation.

Question 35

- 35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π .



- b) Explain any differences between a sketch of $y = 2 \sin \left(x - \frac{\pi}{3} \right) - \frac{3}{2}$ and the sketch from part a.

when its sin it would start
from the bottom in part a
started from the top

Score 0: The student gave a completely incorrect response.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$y = a(b^x) \quad \begin{matrix} a = 4.168 \\ b = 3.981 \end{matrix}$$

$$y = 4.168(3.981)^x$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$y = 4.168(3.981)^x$$

$$\frac{100}{4.168} = \frac{4.168(3.981)^x}{4.168}$$

$$23.412 = (3.981)^x$$

$$\log 23.412 = \log 3.981^x$$

$$\log 23.412 = x \log 3.981$$

$$\frac{\log 23.412}{\log 3.981} = \frac{x \log 3.981}{\log 3.981}$$

$$2.300 = x$$

$$x = 2.25 \text{ hours or } 2 \text{ hours} + 15 \text{ minutes}$$

Score 4: The student gave a complete and correct response.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$y = (4.168) \cdot (3.981)^x$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$100 = (4.168)(3.981)^x$$

x	y
2.25	93.306
2.5	131.8

2.25 hours

Score 4: The student gave a complete and correct response.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

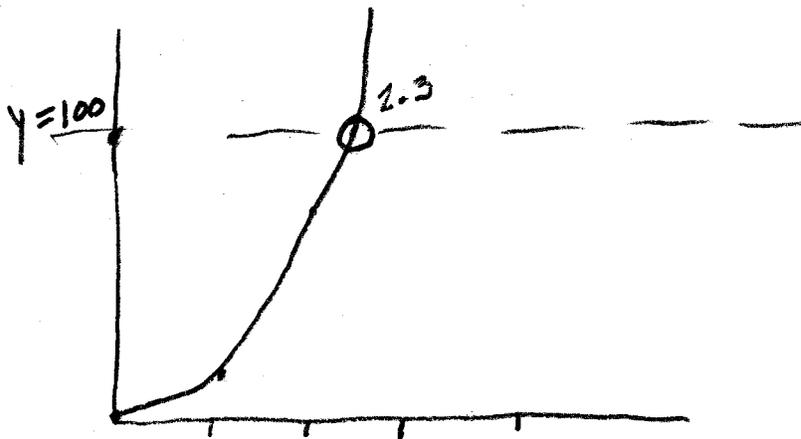
Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$y = ab^x$$

$$a = 4.168$$

$$b = 3.981$$



The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$2.3 \approx 2.25 \text{ hrs}$$

Score 4: The student gave a complete and correct response.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$y = (a)(b^x)$$
$$y = 4.168(3.981)^x$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$100 \geq 4.168(3.981)^x$$
$$\frac{100}{4.168} \geq (3.981)^x$$
$$\frac{\log(100 \div 4.168)}{\log(3.981)} \geq x$$
$$x \leq 2.3$$

Score 3: The student did not round to the nearest quarter hour.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

STAT 1 Edit...
 STAT CALC 0 ExpReg
 $y = ab^x$
 $y = 4.16798(3.98061)^x$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$y = 4.16798(3.98061)^x$
 $100 = 4.16798(3.98061)^x$
 $23.99243 = 3.98061^x$

Score 2: The student made an error rounding the coefficients and did not finish solving for x .

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$a \cdot b^x$$

$$a = 4.674377358$$

$$b = 3.898204241$$

$$y = 4.674 \cdot 3.898^x$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$x = 2.25 \text{ hrs}$$

Score 2: The student made a computational error finding the regression equation and wrote 2.25 (based on their equation) without showing work.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$Y = ab^x \quad a = 4.167983971 \quad b = 3.9869454$$

$$Y = 4.168(3.981^x)$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$100 = 4.168(3.981^x)$$

$$\frac{100}{4.168} = 16.592808 \times$$

$$\frac{16.592808}{16.592808} = 16.592808$$

$$6.026707 = x$$

$$6.02 \text{ hours}$$

Score 1: The student received credit for finding and correctly rounding the regression coefficients.

Question 36

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$2252.567x - 2758.336$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

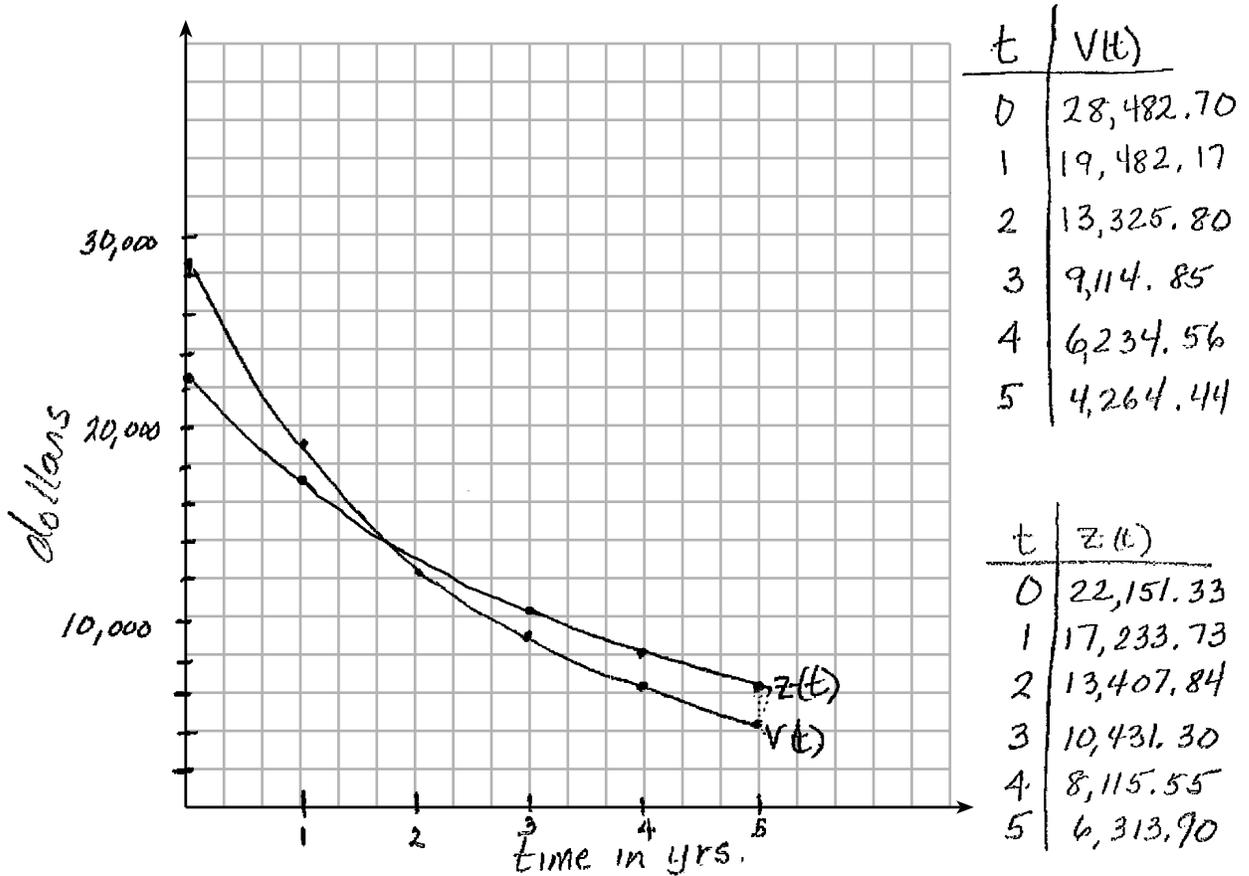
$$\begin{aligned} 2252.567(100) - 2758.336 \\ 2252.567(-2658.336) \\ - 5988079.949 \end{aligned}$$

Score 0: The student showed no correct work.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 6: The student gave a complete and correct response.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

$$V(t) = Z(t) \text{ when } t \approx 1.9523346$$

at $t = 1.95$ yrs, the value of the car is exactly equal to the unpaid amount of Zach's loan.

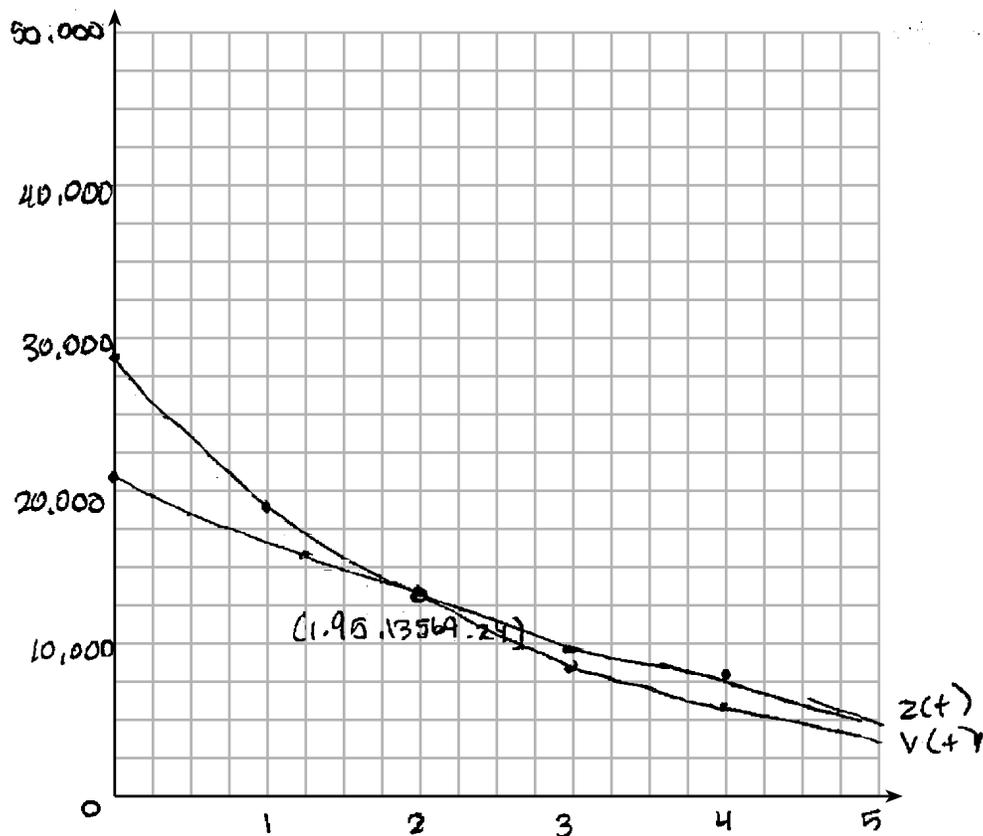
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

6 yrs because at that point, the value of the car is less than \$3000.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 5: The student wrote the answer to the second part as a coordinate pair.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

Solve intersect at $(1.95, 13569.24)$. This means that the value of the car at that time was the same as the unpaid amount of Zach's at that time.

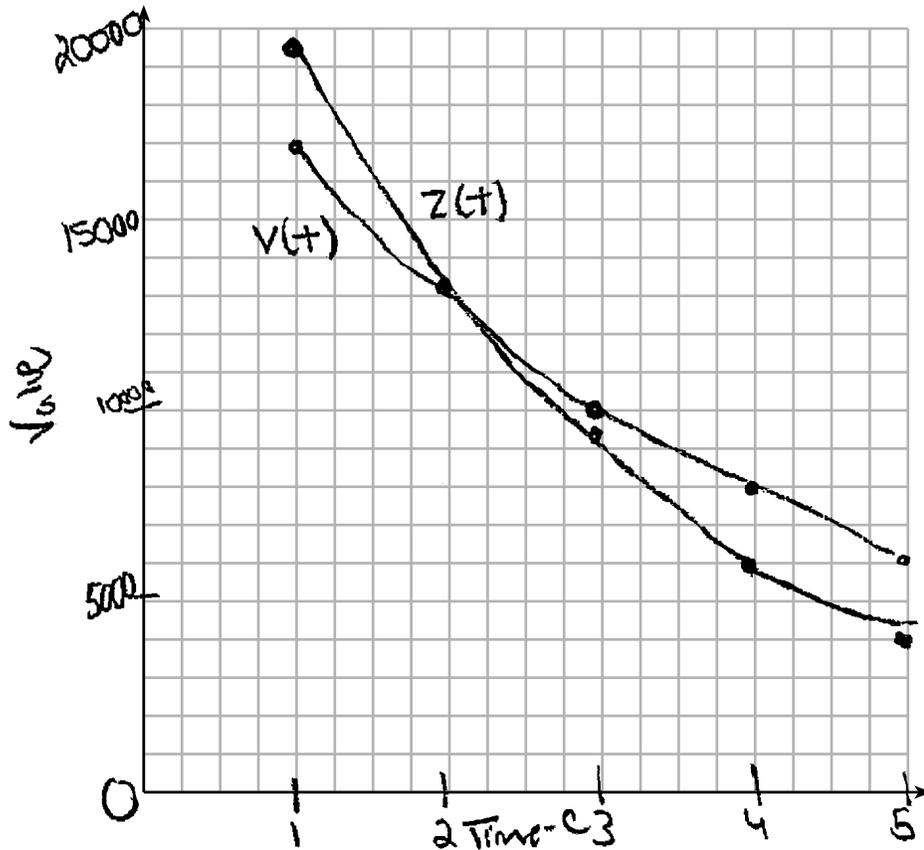
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

Around the 6th year would be reasonable because the value of the car is \approx \$3000.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 4: The student made one graphing error, then did not state where the graphs intersect.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

That Zach has a loan equal to the exact value of his purchased model.

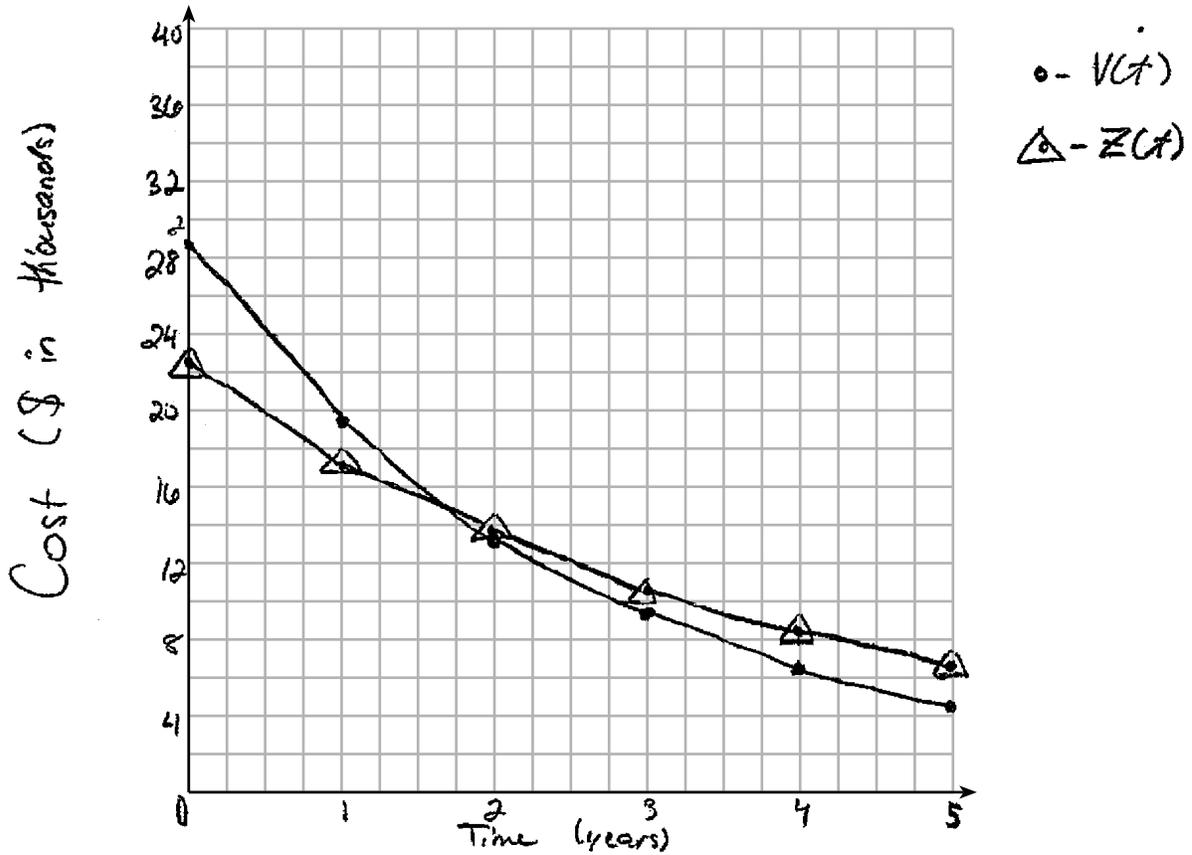
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

By six years his collision policy will equal the value of the model approximately so it is useless.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 3: The student received credit for the graph and the contextual interpretation on the second part.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

$V(t) = Z(t)$ after about 1.5 years. This means that the value of the car is equal to the unpaid amount of Zach's loan.

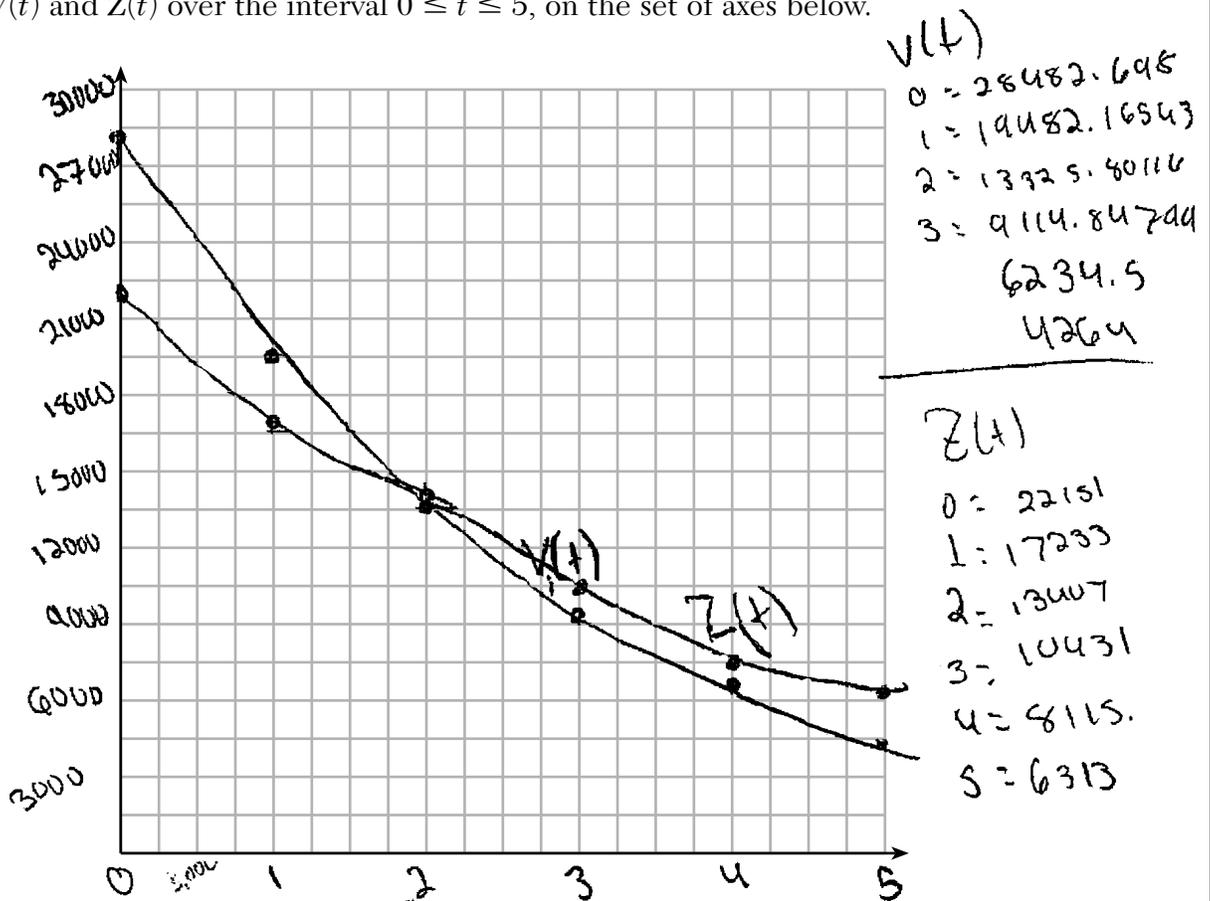
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

It would be reasonable to cancel the policy after 5 years because the value of the car is lowest.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 2: The student received credit for correctly drawing each graph.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

between 2 & 2.5 years

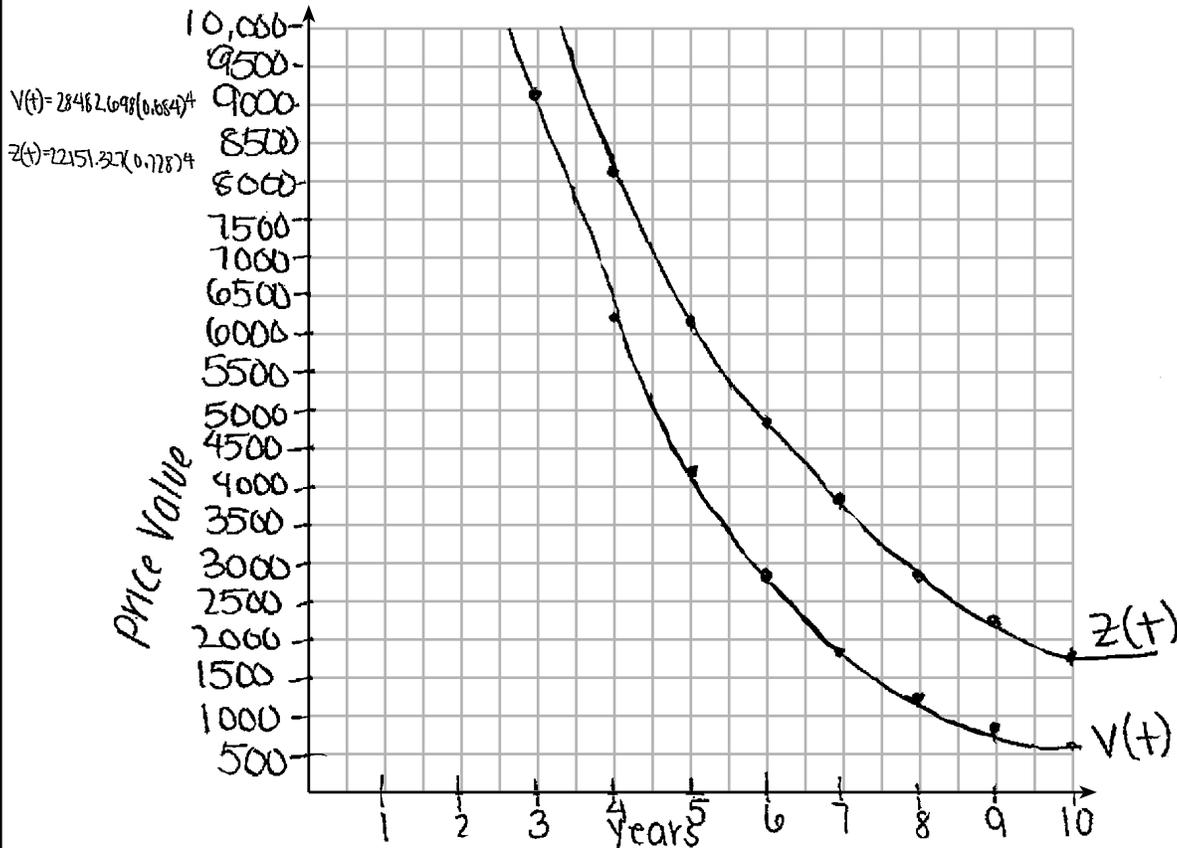
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

2

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 1: The student earned one point for the graph.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

$(1.9523346, 13564.237)$
 \therefore At almost 2 years, the car will be worth
\$13,564.24.

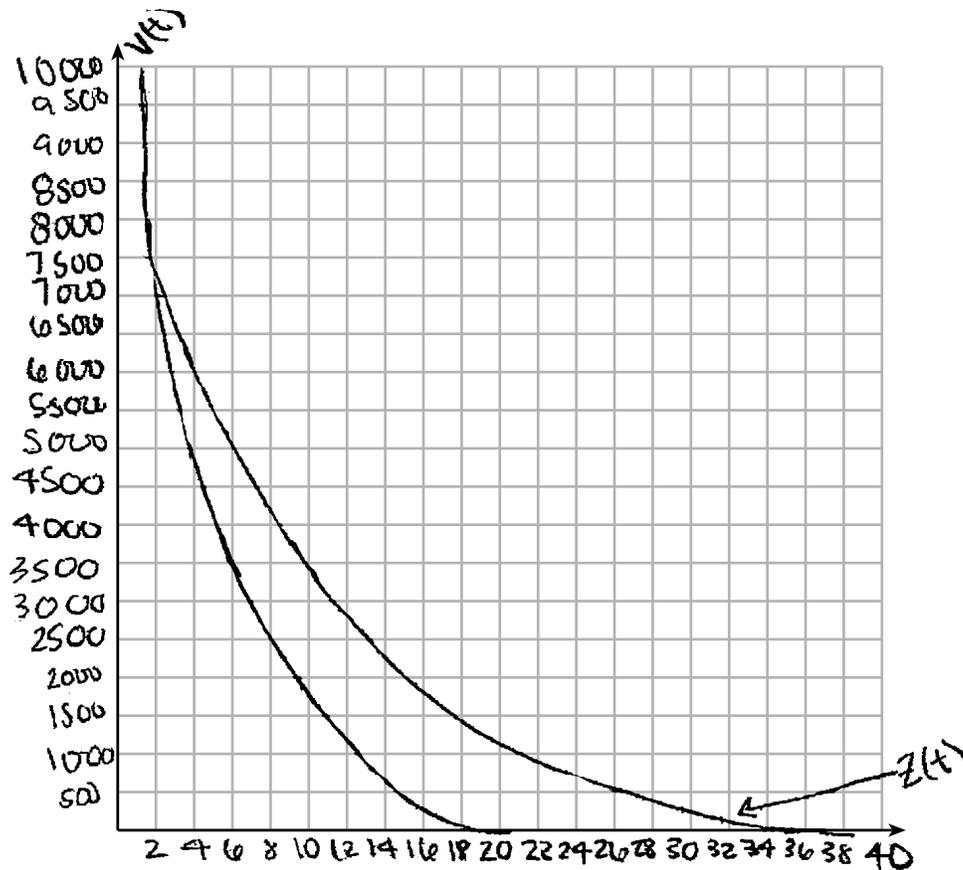
Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

8 years
 \therefore the car will only cost \$12,973.30
which is less than the insurance.

Question 37

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.



Score 0: The student made several graphing errors and showed no other correct work.

Question 37

State where $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

$$(1.95, 13569.24)$$

Where the two costs of the cars meet

Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

After 30 because he would almost be done paying