

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Thursday, August 16, 2018 — 12:30 to 3:30 p.m., only

## MODEL RESPONSE SET

### Table of Contents

Question 25.....	2
Question 26.....	7
Question 27.....	12
Question 28.....	16
Question 29.....	21
Question 30.....	26
Question 31.....	30
Question 32.....	35
Question 33.....	41
Question 34.....	46
Question 35.....	54
Question 36.....	60
Question 37.....	67

Question 25

25 Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

Grouping  
method

$$\begin{aligned} & \overbrace{x^4 - 4x^2 - 12} \\ & \quad -12x^4 \\ & \quad \quad \quad \swarrow \quad \searrow \\ & \quad \quad -6x^2 \quad 2x^2 \\ & (x^4 - 6x^2) + (2x^2 - 12) \\ & x^2(x^2 - 6) + 2(x^2 - 6) \\ & \boxed{(x^2 + 2)(x^2 - 6)} \\ & \quad \downarrow \\ & \text{answer} \end{aligned}$$

**Score 2:** The student gave a complete and correct response.

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**Question 25**

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**25** Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

$$x^4 - 4x^2 - 12$$
$$(x^2 - 6)(x^2 + 2)$$

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**Score 2:** The student gave a complete and correct response.

Question 25

25 Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

$$\begin{array}{cc} (x^2-6)(x^2+2) & \\ \downarrow & \downarrow \\ x^2=6 & x^2=-2 \\ x=\pm\sqrt{6} & x=\pm i\sqrt{2} \end{array}$$

$$\begin{array}{l} (x^2-6)(x^2+2) \\ x^4-6x^2+2x^2-12 \\ x^4-4x^2-12 \end{array}$$

$$\left\{ -\sqrt{6}, \pm i\sqrt{2} \right\}$$

**Score 1:** The student factored correctly, but then went on to solve an equation.

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**Question 25**

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25 Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

~~\*~~

$$\frac{(x^2 - 6)(x^2 + 2)}{\boxed{(x - 3)(x + 2)(x + 2)(x + 1)}}$$

**Score 1:** The student initially factored correctly, but showed incorrect work beyond the correct answer.

Question 25

25 Over the set of integers, factor the expression  $x^4 - 4x^2 - 12$ .

$$(x^2 - 1)(x + 6)(x^2 + 3x + 2)$$
$$(x - 3)(x + 2)(x + 2)(x + 1)$$
$$x = 3, x = -2, x = -2, x = -1$$

~~$$(x^2 - 2x - 3)(x^2 + 2x + 4)$$
$$(x - 3)(x + 1)(x + 2)$$~~



**Score 0:** The student did not do enough correct work to receive any credit.

Question 26

26 Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$$
$$\frac{2\sqrt[2]{x^3}}{4\sqrt[4]{16x^4}}$$
$$\frac{2x\sqrt{x}}{2x}$$
$$\sqrt{x}$$

Score 2 The student gave a complete and correct response.

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**Question 26**

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26 Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$16^{\frac{1}{4}} = 2 \quad \frac{2x^{\frac{3}{2}}}{2x} = \frac{x^{\frac{3}{2}}}{x} = \boxed{\sqrt{x}}$$

**Score 2:** The student gave a complete and correct response.

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**Question 26**

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**26** Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$\frac{2(\sqrt{x})^3}{\sqrt[4]{16(x)}} \\ \frac{\cancel{2}(\sqrt{x})^3}{\cancel{2}x} \\ \boxed{\frac{(\sqrt{x})^3}{x}}$$

**Score 1:** The student did not simplify completely.

Question 26

26 Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$\frac{(2\sqrt{2x})^3}{\sqrt[4]{16x^4}}$$

$$\frac{2^{\frac{3}{2}}}{2^{\frac{1}{2}}}$$

$$\frac{\sqrt{2x}^3}{2x}$$

$$\frac{\cancel{2x}\sqrt{2x}}{\cancel{2x}}$$

$$\sqrt{2x}$$

**Score 1:** The student applied the exponent to  $2x$  instead of  $x$ .

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**Question 26**

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**26** Express the fraction  $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$  in simplest radical form.

$$\frac{\sqrt[3]{2x^3}}{2x}$$
$$\frac{\sqrt{x^2}}{x}$$

**Score 0:** The student made multiple errors applying exponent rules.

**Question 27**

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where  $t$  is time in years after 1950 and  $p(t)$  is the population in millions. Determine the average rate of change of  $p(t)$  in millions of people per year, from  $4 \leq t \leq 8$ . Round your answer to the nearest hundredth.

$$p(4) = 2560e^{0.017185(4)}$$

$$p(4) = 2742.16364$$

$$p(8) = 2560e^{0.017185(8)}$$

$$p(8) = 2937.289621$$

$$8 - 4 = 4$$

$$2937.289621 - 2742.16364$$

$$= 195.125981$$

$$\frac{195.125981}{4} = 48.78149525$$

$$\approx 48.78$$



**Score 2:** The student gave a complete and correct response.

Question 27

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where  $t$  is time in years after 1950 and  $p(t)$  is the population in millions. Determine the average rate of change of  $p(t)$  in millions of people per year, from  $4 \leq t \leq 8$ . Round your answer to the nearest hundredth.

$1950 - 2560$   
 $1960 - 3040$

$p(t) = 2560e^{0.017185t}$   
 $p(4) = 2560e^{0.017185(4)}$   
 $2560e^{.06874}$   
 $p(4) = 2742.1636$

$p(8) = 2560e^{0.017185(8)}$   
 $13748$   
 $p(8) = 2937.28962$

$\frac{y_2 - y_1}{x_2 - x_1}$   
 $\frac{8 - 4}{2937.3 - 2742.2}$   
 $\frac{4}{195.1} = .02$   
 rate of change

**Score 1:** The student made a substitution error when finding the average rate of change.

**Question 27**

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where  $t$  is time in years after 1950 and  $p(t)$  is the population in millions. Determine the average rate of change of  $p(t)$  in *millions of people per year*, from  $4 \leq t \leq 8$ . Round your answer to the *nearest hundredth*.

$$p(4) = 2560e^{0.017185(4)}$$
$$= 2742.1636$$

$$p(8) = 2560e^{0.017185(8)}$$
$$= 2937.2896$$

$$\begin{array}{r} 2937.2896 \\ - 2742.1636 \\ \hline 195.126 \end{array}$$

The population changed by  
195.13.

**Score 1:** The student failed to divide by four before rounding.

**Question 27**

27 The world population was 2560 million people in 1950 and 3040 million in 1960 and can be modeled by the function  $p(t) = 2560e^{0.017185t}$ , where  $t$  is time in years after 1950 and  $p(t)$  is the population in millions. Determine the average rate of change of  $p(t)$  in millions of people per year, from  $4 \leq t \leq 8$ . Round your answer to the nearest hundredth.

$$p(4) = 2560e^{0.017185 \cdot 4} = 2742.2$$

$$p(8) = 2560e^{0.017185 \cdot 8} = 2988.2$$

$$\frac{2988.2 - 2742.2}{8 - 4} = \frac{246}{4} = 61.5$$

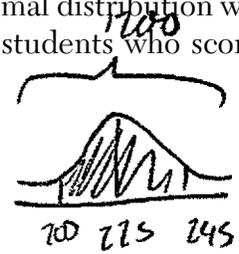
Average rate of change is 62 million  
People every year

**Score 0:** The student made an error evaluating  $p(8)$  and rounded incorrectly.

Question 28

28 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

$$\text{normalcdf}(200, 245, 225, 18) = .7843$$



$$.7843 \times 1200 = 941$$

941 students

**Score 2:** The student gave a complete and correct response.

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**Question 28**

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**28** The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

$$1200 (.784) = \boxed{941 \text{ students}}$$

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**Score 2:** The student gave a complete and correct response.

**Question 28**

**28** The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

1200 students

mean - 225

S.D - 18

btw: 200 - 245

$$\text{Normalcdf}(200, 245, 225, 18) = .7843063697$$

Z-score

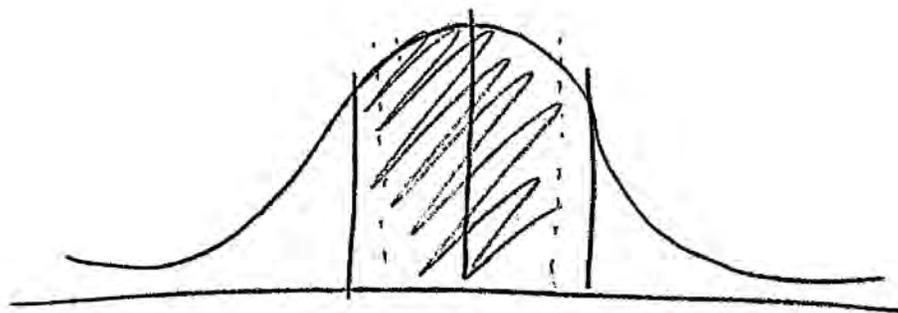
$$\frac{200 - 245}{18} = 180.\bar{3}$$

78.4% is  
expected to  
score btw  
200 and  
245

**Score 1:** The student failed to determine the number of students.

**Question 28**

**28** The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.



941 students

**Score 1:** The student did not show enough work to determine the number of students.

**Question 28**

28 The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

1200 students mean = 225  $\sigma = 18$   
between 200 and 245

$$t\text{-score} = \frac{x - \bar{x}}{s} = \frac{200 - 225}{18} = -1.25 = -1.25$$

$$z\text{-score} = \frac{x - \bar{x}}{s} = \frac{245 - 225}{18} = 1.11 = 1.11$$

$$.089566162 \approx .0896 \approx 9.0\%$$

$$\frac{x}{1200} = \frac{9}{100}$$

$$100x = 10800$$

$$x = 108 \text{ students}$$

**Score 0:** The student did not show enough correct work to receive any credit.

Question 29

29 Algebraically solve for x:

$$\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}$$

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$$\frac{-3}{x+3} + 1 = \frac{x}{6}$$
$$\frac{-3}{x+3} + \frac{1(x+3)(6)}{1} = \frac{x}{6} \frac{(x+3)(6)}{6}$$
$$-18 + (x+3)(6) = x(x+3)$$
$$-18 + 6x + 18 = x^2 + 3x$$
$$6x = x^2 + 3x$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$

$x=0, x=3$

Score 2: The student gave a complete and correct response.

**Question 29**

29 Algebraically solve for x:

$$\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}$$

$$\text{LCD} = 6(x+3)$$

$$\cancel{6(x+3)} \left( \frac{-3}{\cancel{x+3}} + \frac{1}{2} \right) = \frac{x}{6} - \frac{1}{2}$$

$$-18 + 3x + 9 = x^2 + 3x - 3x - 9$$

$$3x - 9 = x^2 - 9$$

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$$\boxed{\begin{array}{l} x=0 \\ x=3 \end{array}}$$

**Score 2:** The student gave a complete and correct response.

**Question 29**

29 Algebraically solve for x:

$$\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}$$

$$\frac{-3}{x+3} + \frac{3}{6} = \frac{x}{6} - \frac{3}{6}$$

$$-3(6) + 3(x+3) = x(x+3) - 3(x+3)$$

$$-18 + 3x + 3 = x^2 + 3x - 3x - 9$$

$$3x - 15 = x^2 - 9$$

$$x^2 - 3x + 6 = 0$$

$$\frac{3 \pm \sqrt{9 - 4(1)(6)}}{2(1)} = \frac{3 \pm \sqrt{-15}}{2}$$

$$x = \frac{3 \pm i\sqrt{15}}{2}$$

**Score 1:** The student failed to properly distribute the three.

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**Question 29**

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29 Algebraically solve for  $x$ :

$$\frac{-3}{x+3} + \frac{1}{2} = \frac{x}{6} - \frac{1}{2}$$

$$\frac{-18}{6x+18} + \frac{3x+9}{6x+18} = \frac{x^2+3x}{6x+18} - \frac{3x+9}{6x+18}$$

$$-18 + 3x + 9 = x^2 + 3x - 3x - 9$$

$$-18 + 6x + 18 = x^2 + 3x$$

$$\frac{3x}{x} = \frac{x^2}{x}$$

$$\boxed{3 = x}$$

**Score 1:** The student lost a solution by dividing by  $x$ .

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**Question 29**

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29 Algebraically solve for x:

$$\left(\frac{2}{2}\right) \frac{-3}{x+3} \left(\frac{x+1}{x+3}\right) = \frac{x}{6} - \frac{1}{2} \left(\frac{3}{2}\right)$$

$$\frac{-6}{2x+6} \cdot \frac{x+3}{2x+6} = \frac{x}{6} - \frac{3}{6}$$

$$\begin{array}{r} -6 + x + 3 = x - 3 \\ \quad \quad \quad +3 \quad \quad +3 \end{array}$$

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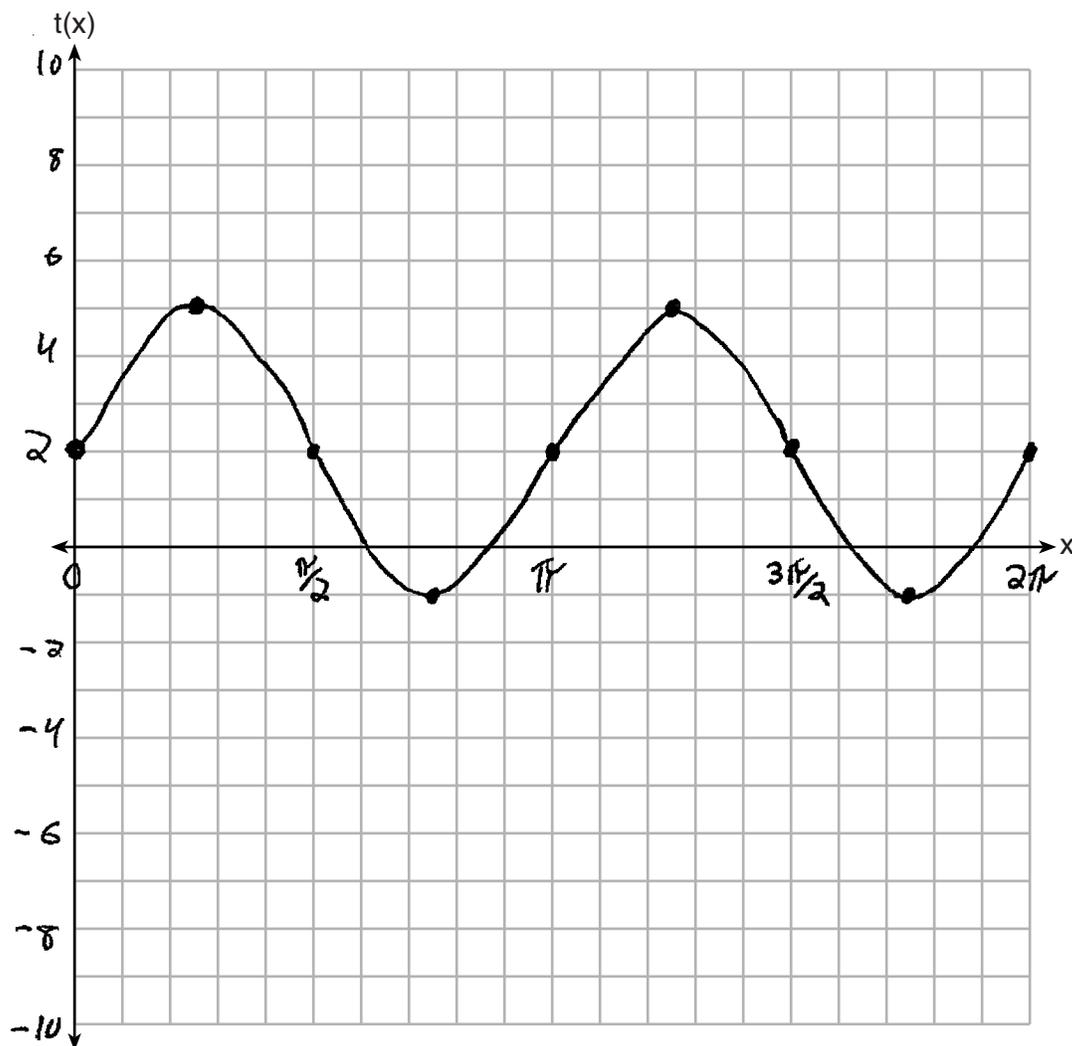
$$-6 + x + 6 = x$$

$$x = 0$$

**Score 0:** The student did not show enough correct work to receive any credit.

Question 30

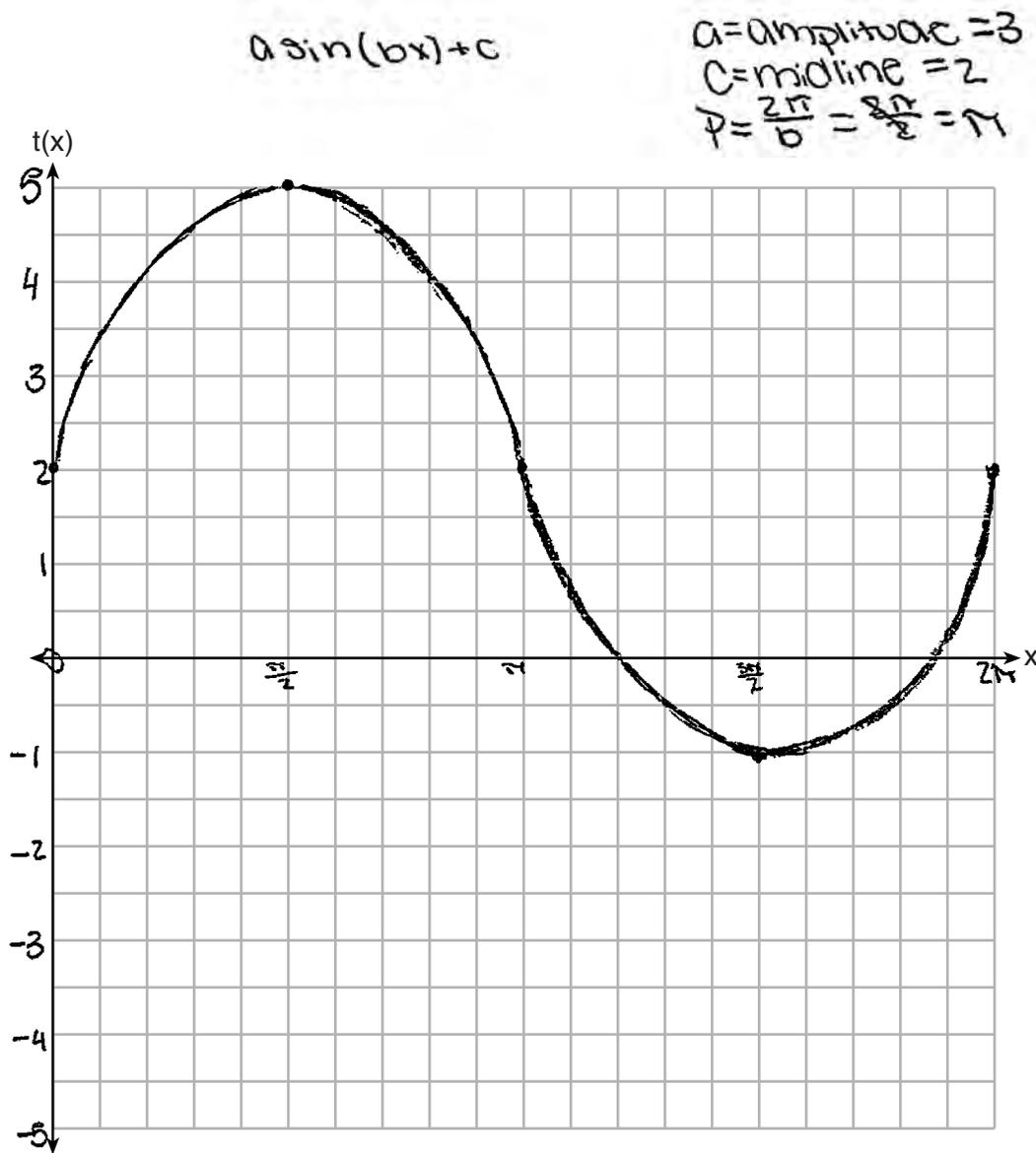
30 Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



Score 2: The student gave a complete and correct response.

Question 30

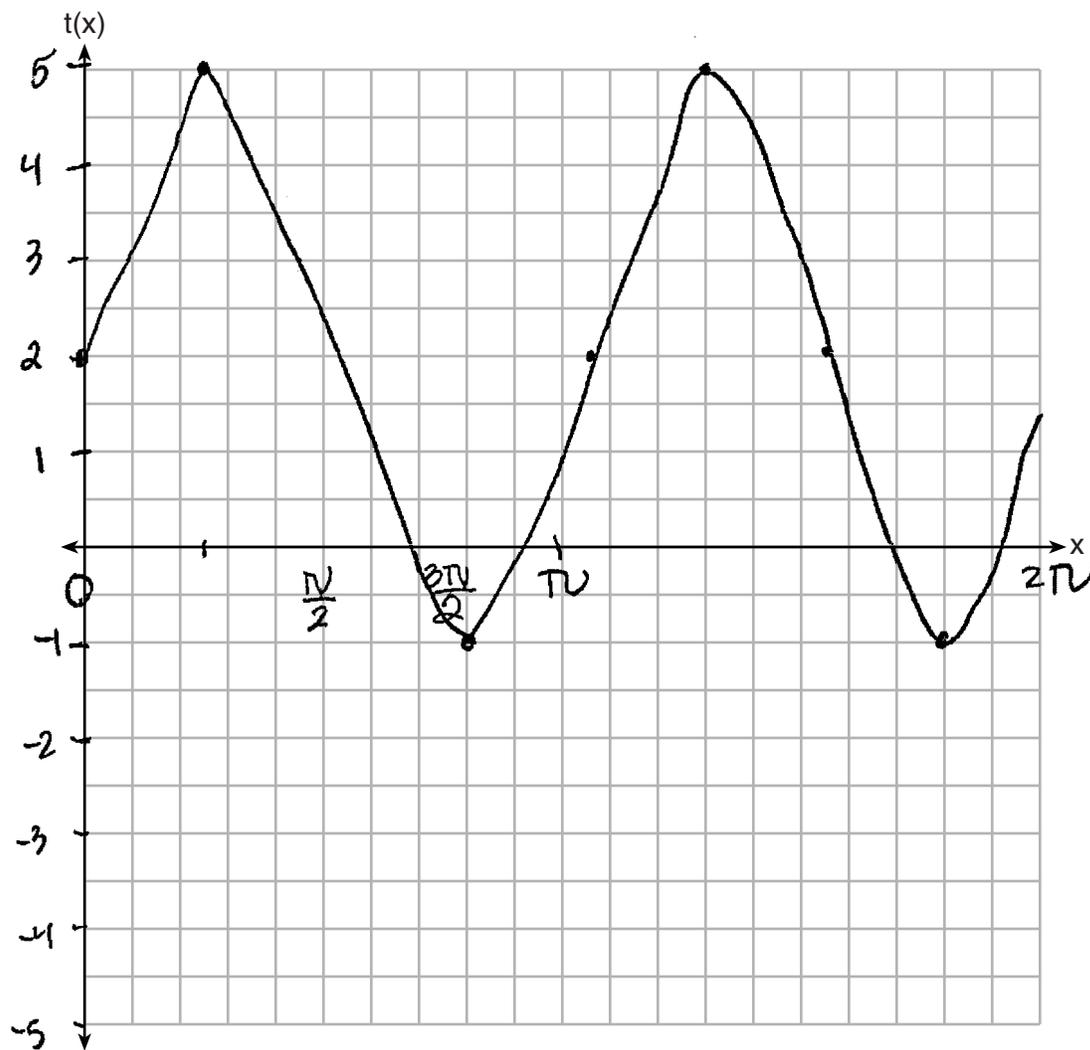
30 Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



**Score 1:** The student made a graphing error.

Question 30

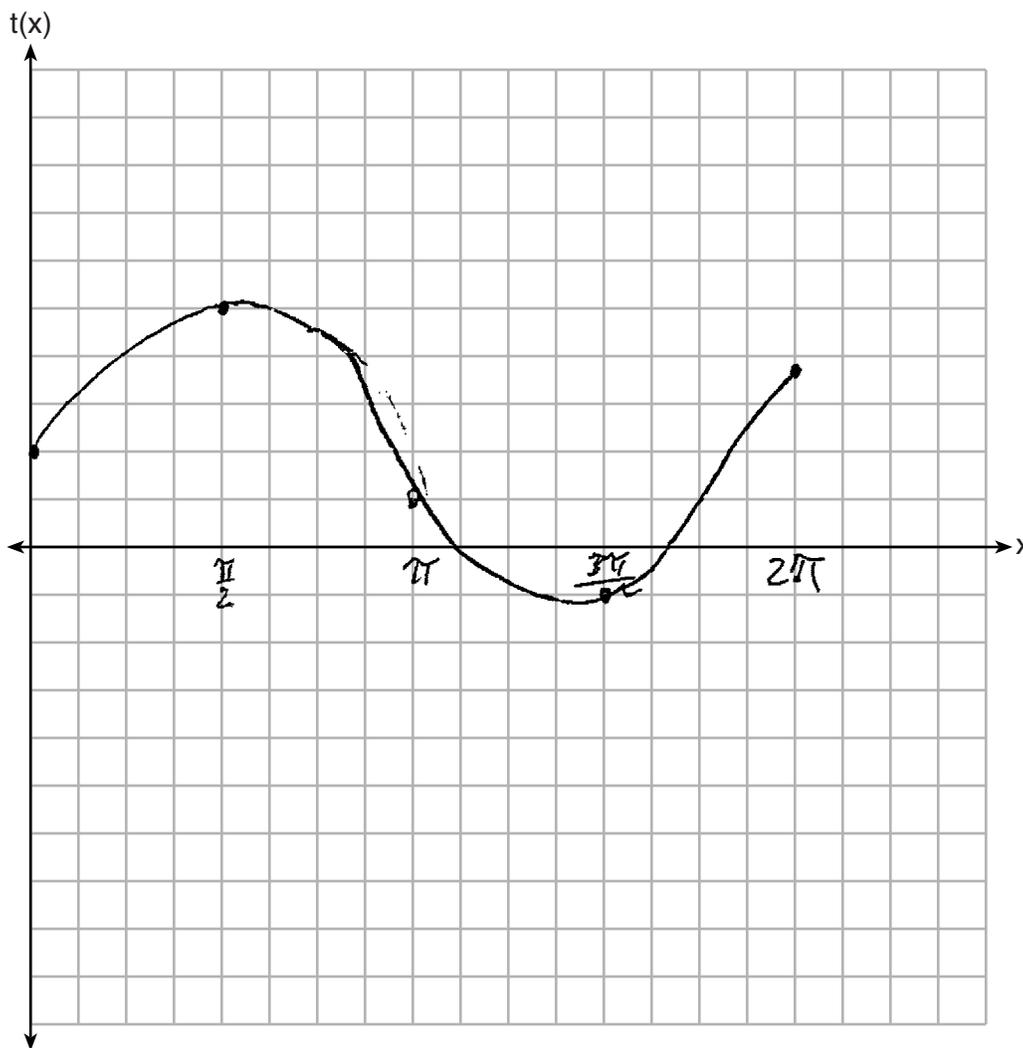
30 Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



**Score 0:** The student made multiple graphing errors.

Question 30

30 Graph  $t(x) = 3\sin(2x) + 2$  over the domain  $[0, 2\pi]$  on the set of axes below.



**Score 0:** The student made multiple graphing errors.

**Question 31**

31 Solve the following system of equations algebraically.

$$x^2 + y^2 = 400$$

$$y = x - 28$$

$$\begin{aligned} x^2 + (x-28)^2 &= 400 \\ x^2 + x^2 - 56x + 784 &= 400 \\ \underline{-400 \quad -400} \end{aligned}$$

$$\begin{aligned} 2x^2 - 56x + 384 &= 0 \\ \cancel{2}(x^2 - 28x + 192) &= \cancel{0} \end{aligned}$$

$$x^2 - 28x + 192 = 0$$

$$(x-16)(x-12) = 0$$

$$\begin{array}{l|l} x-16=0 & x-12=0 \\ \hline x=16 & x=12 \end{array}$$

$$\begin{array}{ll} y=16-28 & y=12-28 \\ y=-12 & y=-16 \end{array}$$

$x=16, y=-12$
$x=12, y=-16$

**Score 2:** The student gave a complete and correct response.

Question 31

31 Solve the following system of equations algebraically.

$$x^2 + y^2 = 400$$

$$y = x - 28$$

$$x^2 + (x-28)^2 = 400$$

$$x^2 + -x(x-28) - 28(x-28) = 400$$

$$x^2 + x^2 - 28x - 28x + 784 = 400$$

$$2x^2 - 56x + 784 = 400$$

$$-400 \quad -400$$

$$2x^2 - 56x + 384 = 0$$

$$2(x^2 - 28x + 192) = 0$$

$$2(x^2 - 12x - 16x + 192) = 0$$

$$2(x(x-12) - 16(x-12)) = 0$$

$$2(x-16)(x-12) = 0$$

$$\begin{array}{l} \overline{x=16} \\ \underline{x=12} \end{array}$$

$$(16, -12)$$

$$(12, -16)$$

$$y = 16 - 28$$

$$\boxed{y = -12}$$

$$y = 12 - 28$$

$$\boxed{y = -16}$$

$$\begin{array}{r} 192 \\ 1 \times 192 \\ 2 \times 96 \\ 3 \times 64 \\ 4 \times 48 \\ 5 \times 32 \\ 6 \times 24 \\ 7 \times 16 \end{array}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Solve the following system of equations algebraically.

	$x$	$-28$
$x$	$x^2$	$-28x$
$-28$	$-28x$	$784$

$$x^2 + y^2 = 400$$

$$y = x - 28$$

$$x^2 + x^2 - 56x + 784 = 400$$

$$2x^2 - 56x + 384$$

$$A = 2$$

$$B = -56$$

$$C = 384$$

$$x = \frac{56 \pm \sqrt{3136 - 4(2)(384)}}{4}$$

$$x = \frac{56 \pm \sqrt{64}}{4}$$

$$x = \frac{56 \pm 8}{4}$$

$$\begin{aligned} x &= 16 \\ x &= 12 \end{aligned}$$

**Score 1:** The student failed to find the corresponding  $y$ -values.

**Question 31**

**31** Solve the following system of equations algebraically.

$$x^2 + y^2 = 400$$

$$y = x - 28$$

$$\begin{aligned}
 & x^2 + (x-28)^2 = 400 \\
 & x^2 + (x-28)(x-28) = 400 \\
 & x^2 + x^2 - 28x - 28x + 784 = 400 \\
 & 2x^2 - 56x + 784 = 400 \\
 & \quad \quad \quad -400 \quad -400 \\
 & 2x^2 - 56x + 384 = 0 \\
 & x = \frac{56 \pm \sqrt{(-56)^2 - 4(2)(384)}}{2(2)} \\
 & x = \frac{56 \pm \sqrt{6208}}{4} \\
 & x = \frac{56 + 78}{4} \\
 & x = \frac{134}{4} \\
 & x = 34
 \end{aligned}$$

$y = 34 - 28$   
 $y = 6$   
  
 $x = 34$   
 $y = 6$

**Score 1:** The student correctly determined  $2x^2 - 56x + 384 = 0$ .

Question 31

31 Solve the following system of equations algebraically.

$$x^2 + y^2 = 400$$

$$y = x - 28$$
$$x^2 + (x - 28)^2 = 400$$

$$x + x - 28 = 20$$

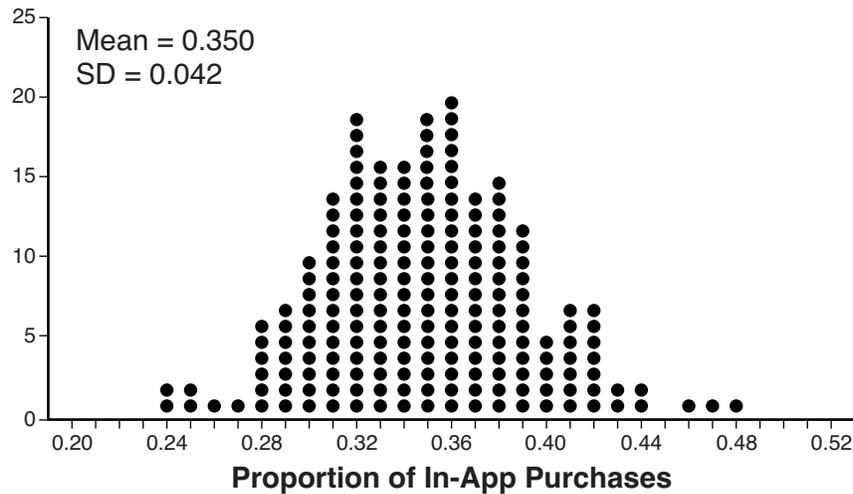
$$x + x \quad \begin{array}{r} + 28 \\ \hline = 48 \\ \hline 2 \end{array}$$

$$x = 24$$

**Score 0:** The student made a conceptual error and only found a value for  $x$ .

**Question 32**

**32** Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

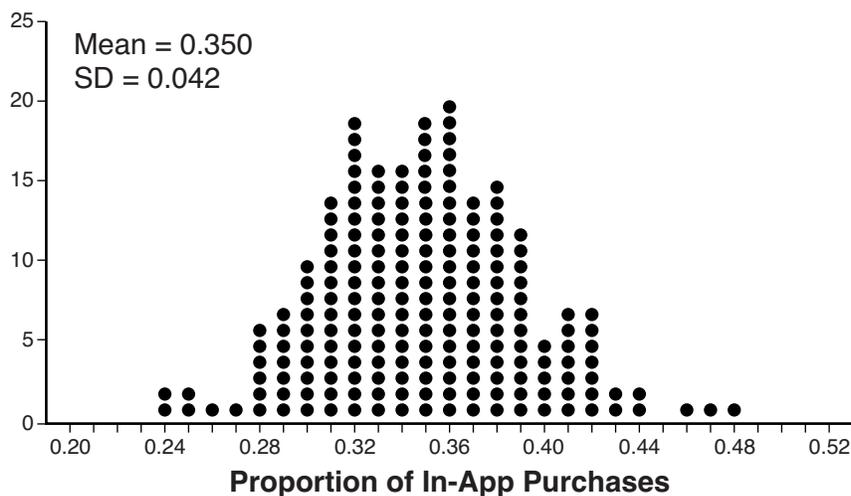
$ME = 2SD = 2(.042) = .084 = .08$  are expected to  
 A ME of .08 means that 27 - 43% of users ~~will make~~  
 make in-app purchases.

$$\begin{array}{r} .35 \\ + .08 \\ \hline .43 \end{array} \quad \begin{array}{r} .35 \\ - .08 \\ \hline .27 \end{array}$$

**Score 2:** The student gave a complete and correct response.

### Question 32

**32** Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

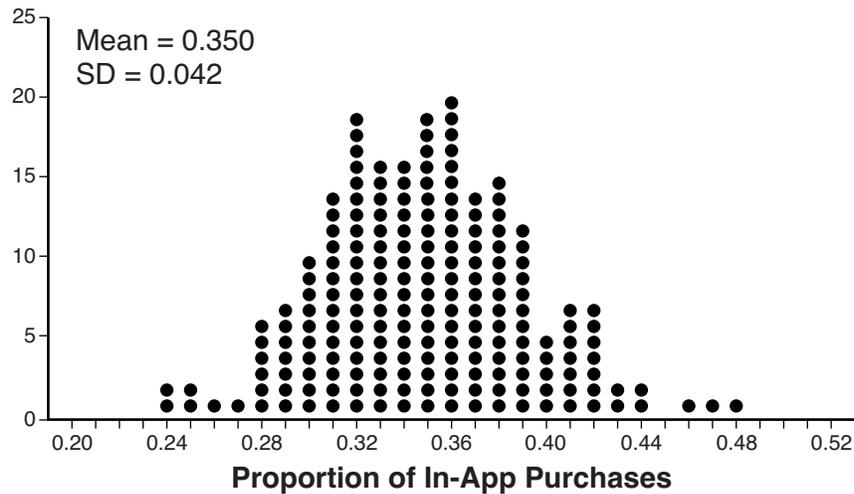
$$2SD = .08$$

This means that 95% falls between  
 $.35 \pm .08$ .

**Score 1:** The student did not refer to the given context.

### Question 32

32 Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the nearest hundredth, for the simulated results. In the given context, explain what this value represents.

$$\text{Margin of Error} = 2(\text{S.D.})$$

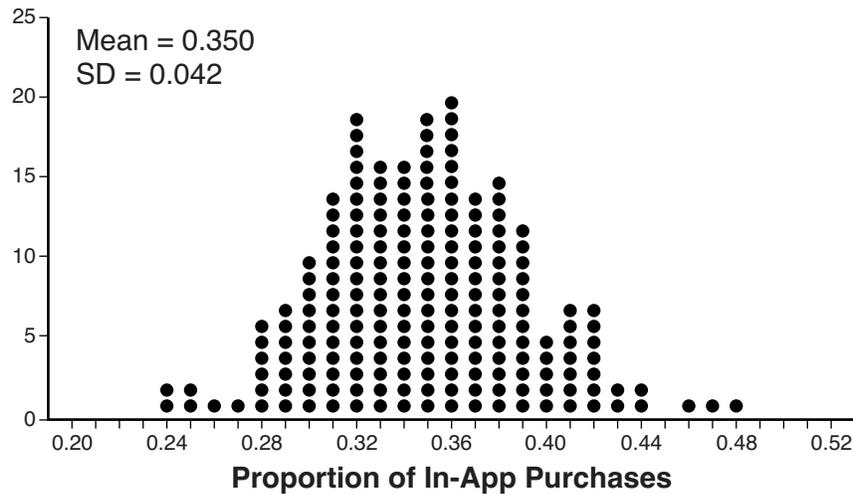
$$M_oE = 2(0.042)$$

$$M_oE = .08$$

**Score 1:** The student did not provide an explanation.

### Question 32

**32** Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

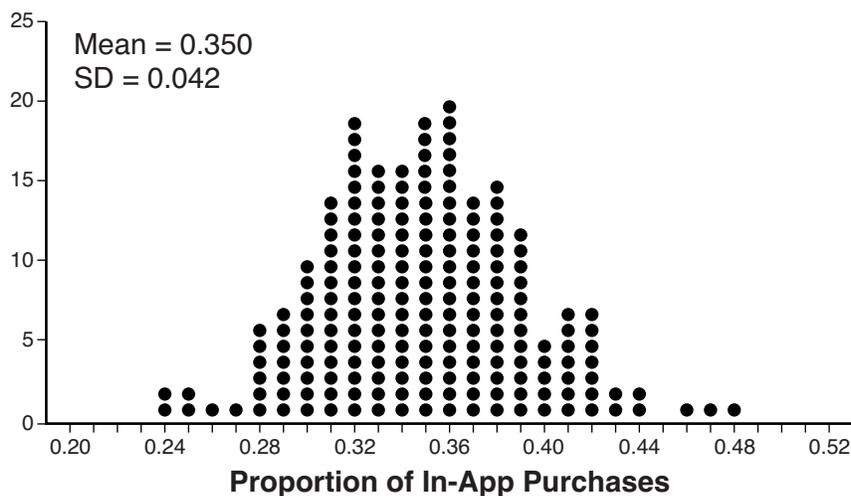
$$ME = .04$$

You can expect the percent of people making in-app purchases to be within 4% of 35%.

**Score 1:** The student stated an incorrect margin of error, but provided an appropriate explanation.

### Question 32

**32** Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

$$\frac{(35)(1+35)}{2}$$

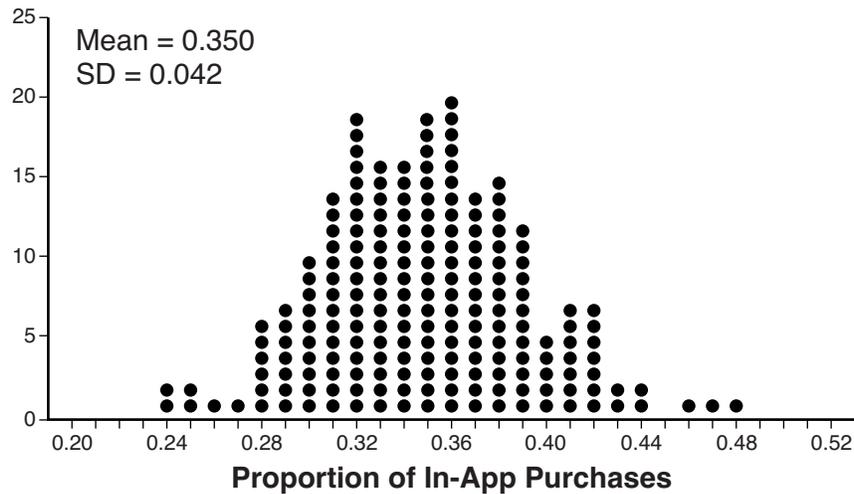
$$z(0.042) = \boxed{0.84}$$

This represents the max and the min values of where the data falls and how many make in-app purchases.

**Score 0:** The student made a rounding error stating the margin of error and gave an incorrect explanation.

### Question 32

32 Some smart-phone applications contain “in-app” purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below.



Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results. In the given context, explain what this value represents.

$$.35 \pm 2(.042)$$

$$.266 - .434$$

$$.168$$

this means 95%  
of the data  
falls within this  
range.

**Score 0:** The student did not show enough correct work to get any credit.

**Question 33**

**33** Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$2x + 3y - 4z = -1$$

$$x - 2y + 5z = 3$$

$$-4x + y + z = 16$$

$$\begin{array}{r} 1) \quad -2(x - 2y + 5z = 3) \\ \quad 2x + 3y - 4z = -1 \\ \quad \underline{-2x + 4y - 10z = -6} \end{array}$$

$$7y - 14z = -7$$

$$\begin{array}{r} 2) \quad (-4x + y + z = 16) \\ \quad 4(x - 2y + 5z = 3) \\ \quad \underline{-4x + 1y + z = 16} \\ \quad 4x - 8y + 20z = 12 \\ \quad \underline{-7y + 21z = 28} \end{array}$$

$$\begin{array}{r} 3) \quad 7y - 14z = -7 \\ \quad \underline{-7y + 21z = 28} \end{array}$$

$$\frac{7z = 21}{7} = \frac{21}{7}$$

$$\boxed{z = 3}$$

$$\begin{array}{r} 4) \quad -7y + 21(3) = 28 \\ \quad -7y + 63 = 28 \\ \quad \quad \underline{-63 \quad -63} \\ \quad \underline{-7y = -35} \\ \quad \quad \underline{-7 \quad -7} \end{array}$$

$$\boxed{y = 5}$$

$$\begin{array}{r} 5) \quad 2(x) + 3(5) - 4(3) = -1 \\ \quad 2(x) + 15 - 12 = -1 \end{array}$$

$$\begin{array}{r} \quad 2(x) + 3 = -1 \\ \quad \quad \underline{-3 \quad -3} \end{array}$$

$$\frac{2(x) = -4}{2} = \frac{-4}{2}$$

$$\boxed{x = -2}$$

**Score 4:** The student gave a complete and correct response.

**Question 33**

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$2x + 3y - 4z = -1$$

$$x - 2y + 5z = 3$$

$$-4x + y + z = 16$$

$$2(2x + 3y - 4z = -1)$$

$$4x + 6y - 8z = -2$$

$$-4x + y + z = 16$$

$$7y - 7z = 14$$

$$y = 2 + z$$

$$\boxed{y = 2 + 3 = 5}$$

$$4(x - 2y + 5z = 3)$$

$$4x - 8y + 20z = 12$$

$$-4x + y + z = 16$$

$$-7y + 21z = 28$$

$$-7(2 + z) + 21z = 28$$

$$-14 - 7z + 21z = 28$$

$$-14 + 14z = 28$$

$$14z = 42$$

$$\boxed{z = 3}$$

$$x - 2(5) + 5(3) = 3$$

$$x - 10 + 15 = 3$$

$$x + 5 = 3$$

$$x = 2$$

**Score 3:** The student made one computational error solving for  $x$ .

Question 33

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$\begin{aligned} & \cancel{2}x + 3y - 4z = -1 \\ & -2\cancel{x} + 4y + 5z = 3 \\ & -4x + y + z = 16 \end{aligned}$$

$$\begin{aligned} & 2x + 3y - 4z = -1 \\ & -2x + 4y - 10z = -3 \end{aligned}$$

$$\begin{aligned} & 7y - 14z = -4 \\ & -7y + 7z = -4 \end{aligned}$$

$$\begin{aligned} -7z &= -8 \\ \overline{-7} & \quad \overline{-7} \end{aligned}$$

$$z = \frac{8}{7}$$

$$x - 2\left(\frac{-4}{7}\right) + 5\left(\frac{8}{7}\right) = 3$$

$$x = \frac{-27}{7}$$

$$\begin{aligned} & -4x + y + z = 6 \\ & 4\cancel{x} + 6y - 8z = -2 \end{aligned}$$

$$-7y - 7z = 4$$

$$7y - 7\left(\frac{2}{7}\right) = 4$$

$$\frac{7y}{7} = \frac{-4}{7}$$

$$y = \frac{-4}{7}$$

Score 2: The student made two or more computational errors.

**Question 33**

**33** Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$2x + 3y - 4z = -1$$

$$x - 2y + 5z = 3$$

$$-4x + y + z = 16$$

$$\begin{array}{r} 2x + 3y - 4z = -1 \\ + (-2x + 4y - 10z) = -6 \\ \hline 7y - 14z = -7 \end{array}$$

$$\begin{array}{r} 4x + 6y - 8z = -2 \\ -4x + y + z = 16 \\ \hline 7y - 7z = 14 \end{array}$$

**Score 1:** The student only made two equations eliminating the same variable.

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**Question 33**

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**33** Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ .

$$2x + 3y - 4z = -1$$

$$x - 2y + 5z = 3$$

$$-4x + y + z = 16$$

$$1(2x + 3y - 4z = -1)$$

$$2(x - 2y + 5z = 3)$$

$$\begin{array}{r} 2x + 3y - 4z = -1 \\ -(2x - 4y + 10z = 6) \\ \hline 7y - 14z = -7 \end{array}$$

**Score 0:** The student did not do enough correct work to receive a credit.

Question 34

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$\begin{aligned} 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 &= \\ 2(1) - (-1) - 35(1) + 16(-1) + 48 &= \\ 2 + 1 - 35 - 16 + 48 &= 0 \end{aligned}$$

That  $x+1$  is a factor  
of  $2x^4 - x^3 - 35x^2 + 16x + 48$

Algebraically find the remaining zeros of  $j(x)$ .

$$2x^4 - x^3 - 35x^2 + 16x + 48$$

$$\begin{array}{r|rrrrrr} -1 & 2 & -1 & -35 & 16 & 48 \\ & \downarrow & -2 & 3 & 32 & -48 \\ \hline & 2 & -3 & -32 & 48 & 0 \end{array}$$

$$2x^3 - 3x^2 - 32x + 48 = 0$$

$$x^2(2x-3) - 16(2x-3) = 0$$

$$(x^2-16)(2x-3) = 0$$

$$(x-4)(x+4)(2x-3) = 0$$

$$x = 4, -4, \frac{3}{2}$$

**Score 4:** The student gave a complete and correct response.

**Question 34**

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 = 0$$

It shows us that  $x + 1$   
is a factor of  $j(x)$

Algebraically find the remaining zeros of  $j(x)$ .

$$2x^4 - x^3 - 35x^2 + 16x + 48 = 0$$

$$2(-4)^4 - (-4)^3 - 35(-4)^2 + 16(-4) + 48 = 0$$

$$2(4)^4 - (4)^3 - 35(4)^2 + 16(4) + 48 = 0$$

$$2(1.5)^2 - 4(1.5)^3 - 35(1.5)^2 + 16(1.5) + 48 = 0$$

$$x = 1.5, x = -1, x = -4, x = 4$$

**Score 3:** The student did not find the remaining zeros algebraically.

Question 34

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$x+1$  is a divisible factor of  $j(x)$ , because there would not be a remainder since  $j(-1) = 0$ . because it intercepts the  $x$ -axis; it acts as a root/zero.

$$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48$$

$$j(-1) = 2 + 1 - 35 - 16 + 48$$

$$j(-1) = 0$$

$$\begin{array}{r|rrrrr} -1 & 2 & -1 & -35 & 16 & 48 \\ & & -2 & 3 & 32 & -48 \\ \hline & 2 & -3 & -32 & 48 & 0 \end{array}$$

Algebraically find the remaining zeros of  $j(x)$ .

$$(2x^3 - 3x^2 - 32x + 48)(x + 1)$$

$$2x^3 - 3x^2 = 32x + 48$$

$$x^2(2x - 3) - 16(2x - 3)$$

$$(x^2 - 16)(2x - 3)$$

$$x = 4$$

$$x = \frac{3}{2}$$

$$x = -1$$

**Score 3:** The student omitted one of the zeros.

**Question 34**

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$\begin{array}{r}
 2x^3 - 3x^2 - 32x + 48 \\
 x+1 \overline{) 2x^4 - x^3 - 35x^2 + 16x + 48} \\
 \underline{2x^4 + 2x^3} \phantom{+ 48} \\
 -3x^3 - 35x^2 \phantom{+ 16x + 48} \\
 \underline{-3x^3 - 3x^2} \phantom{+ 48} \\
 -32x^2 + 16x + 48 \\
 \underline{-32x^2 - 32x} \phantom{+ 48} \\
 48x + 48 \\
 \underline{48x + 48} \\
 0
 \end{array}$$

$x+1$  is a factor because when you divide it into  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$  the remainder is zero.

Algebraically find the remaining zeros of  $j(x)$ .

$$\begin{aligned}
 &2x^3 - 3x^2 - 32x + 48 \\
 &x^2(2x - 3) - 16(2x - 3) \\
 &(x^2 - 16)(2x - 3) \\
 &\left. \begin{array}{l} x^2 - 16 = 0 \\ x^2 = 16 \\ x = 4 \\ x = -4 \end{array} \right\} \begin{array}{l} 2x - 3 = 0 \\ 2x = 3 \\ x = \frac{3}{2} \end{array}
 \end{aligned}$$

$$x = \left\{ -4, \frac{3}{2}, 4 \right\}$$

**Score 2:** The student did not evaluate  $j(-1)$  and made a transcription error writing the answers.

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**Question 34**

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34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$j(-1) = 0$$

$x+1$  is a factor of  $j(x)$

Algebraically find the remaining zeros of  $j(x)$ .

---

**Score 2:** The student did not find the remaining zeros.

### Question 34

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$\begin{aligned} & 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 \\ & -2 + 1 - 35 - 16 + 48 \\ & -4 \end{aligned}$$

$\therefore x+1$  is not a factor of  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$  because there is a remainder.

Algebraically find the remaining zeros of  $j(x)$ .

**Score 1:** The student received one credit for an explanation based on a calculation error.

### Question 34

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.

$$j(-1) = 2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48$$

$$j(-1) = 0$$

This tells me that  $(x+1)$  is a zero of function  $j(x)$ .

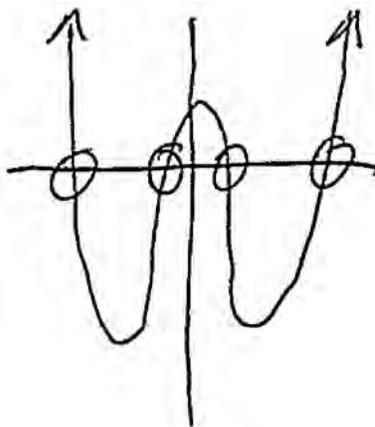
Algebraically find the remaining zeros of  $j(x)$ .

Remaining zeros:  $(-4), (4), (1)$

**Score 1:** The student only received credit for evaluating  $j(-1)$  correctly.

### Question 34

34 Evaluate  $j(-1)$  given  $j(x) = 2x^4 - x^3 - 35x^2 + 16x + 48$ . Explain what your answer tells you about  $x + 1$  as a factor.



Algebraically find the remaining zeros of  $j(x)$ .

$$\begin{aligned} &(4, 0)(-4, 0) \\ &(1, 0)(-1, 0) \end{aligned}$$

$$\begin{aligned} &2(-1)^4 - (-1)^3 - 35(-1)^2 + 16(-1) + 48 \\ &+512 + 84 + 560 - 64 + 48 \end{aligned}$$

**Score 0:** The student used a graphical method and did not find the correct zeros.

Question 35

35 Determine, to the nearest tenth of a year, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

$$A = Pe^{rt} \quad \text{Solve for } t$$

$$\text{let } P = \$500$$

$$\text{let } A = (500)(2)$$

$$r = 3\frac{3}{4}\% = 3.75\% = .0375$$

It would take approximately 18.5 years for the investment to double.

$$(500)(2) = 500e^{.0375t}$$

$$\frac{1000}{500} = \frac{500e^{.0375t}}{500}$$

$$2 = e^{.0375t}$$

apply ln to both sides  
of e. apply on both sides  
ln e = 1

$$\ln 2 = \ln e^{.0375t}$$

$$\ln 2 = .0375t \ln e$$

$$\ln 2 = .0375t(1)$$

$$\frac{\ln 2}{.0375} = \frac{.0375t}{.0375}$$

$$t \approx 18.5$$

Score 4: The student gave a complete and correct response.

**Question 35**

35 Determine, to the nearest tenth of a year, how long it would take an investment to double at a  $3\frac{3}{4}\%$  <sup>3.75%</sup> interest rate, compounded continuously.

$$A = Pe^{rt}$$
$$1000 = 500e^{.0375t}$$
$$2 = e^{.0375t}$$
$$\log 2 = \log e^{.0375t}$$
$$\frac{\log 2}{\log e} = \frac{.0375t \log e}{\log e}$$
$$\frac{.6931471806}{.0375} = \frac{.0375t}{.0375}$$
$$t \approx 18.5 \text{ years}$$

**Score 4:** The student gave a complete and correct response.

**Question 35**

**35** Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

$$\frac{2000}{1000} = \frac{1000e^{.0375t}}{1000}$$

$$2 = e^{.0375t}$$

$$\log 2 = \log e^{.0375t}$$

$$\frac{\log 2}{\log 1.038211997} = \frac{\log(1.038211997)t}{\log 1.038211997}$$

$$t = 18.48392485$$

$$t = 18 \text{ years}$$

**Score 3:** The student made one rounding error.

**Question 35**

**35** Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

$$\frac{500}{500} (1 + 0.0375)^x = \frac{1000}{500}$$

$$\log (1.0375)^x = \log 2$$

$$x \frac{\log 1.0375}{\log 1.0375} = \frac{\log 2}{\log 1.0375}$$

$$x = \boxed{18.8 \text{ years}}$$

**Score 2:** The student wrote an incorrect equation, but showed appropriate work.

Question 35

35 Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

Lets use \$100

$$200 = 100(1 + 0.0375)^x$$

~~0.0375~~

~~0.9625~~

3.75%

18.8

**Score 1:** The student wrote an incorrect equation and provided insufficient work to determine 18.8.

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**Question 35**

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**35** Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a  $3\frac{3}{4}\%$  interest rate, compounded continuously.

$$100 \cdot 1.0375$$

$$\frac{6.007}{7.275} = \frac{x}{100}$$

18.8 years

---

**Score 0:** The student did not do enough correct work to receive any credit.

### Question 36

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

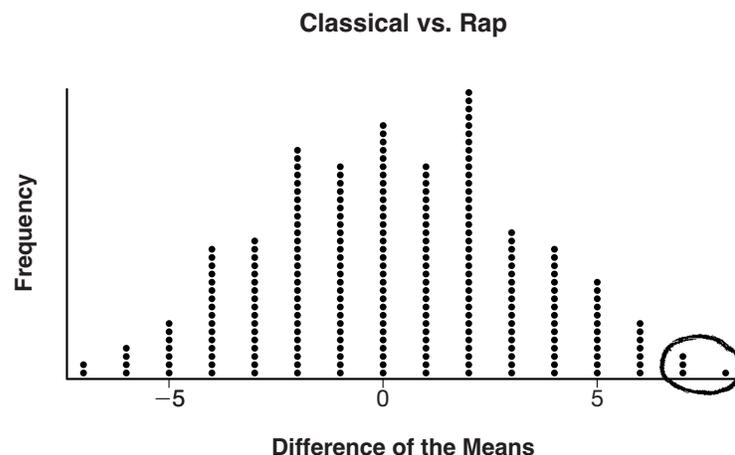
Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John obtained this value by calculating the mean score for each group and then subtracting one from the other. This value represents that the classical group's mean score was 7; higher than that of the rap group.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes, because there is less than 5% chance of this difference occurring due to random chance, so it is likely that the difference was due to the different types of music and was therefore significant.

**Score 4:** The student gave a complete and correct response.

### Question 36

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

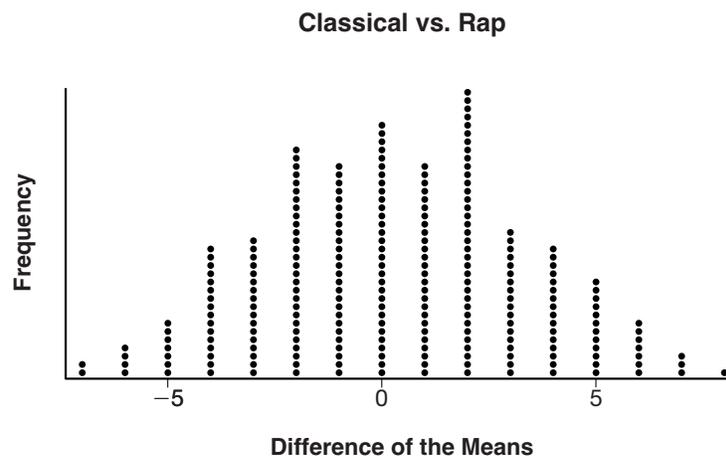
Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John added up all the test scores for rap and averaged them, averaged all the test scores for classical and averaged them, and then subtracted one from the other to get the mean difference. In this context, this value represents that on average, someone listening to classical music during the test scored 7 points higher than those listening to rap.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes, because if it were closer to the mean difference of zero, we wouldn't think anything of it because it's so common, but since it was so rare it shows that there may be a significant difference in quiz scores.

**Score 3:** The student provided insufficient evidence for a significant difference.

**Question 36**

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

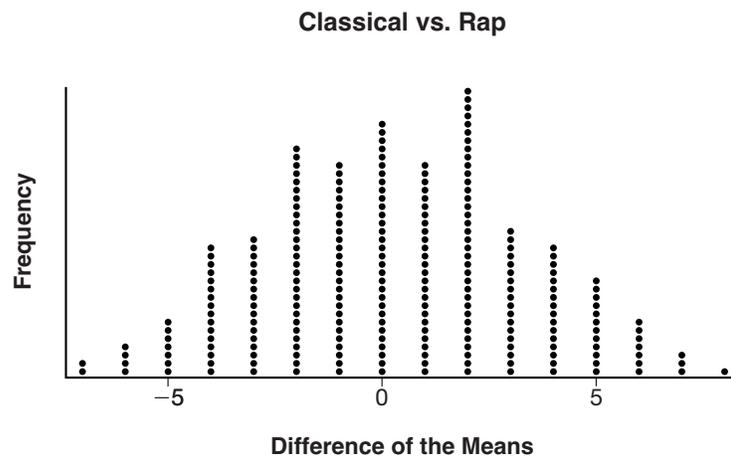
Classical: 74, 83, 77, 77, 84, 82, 90, 89 **82.5**

Rap: 77, 80, 78, 74, 69, 72, 78, 69 **74 ish**

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John added the values of each group together and divided each by 8. Then he subtracted the two values. This shows that, on average, classical gives scores 7 higher than rap.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

No. The simulation shows that the original was an outlier, and the experiment is a standard distributed graph centered around 0.

**Score 2:** The student only received credit for the first part.

### Question 36

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

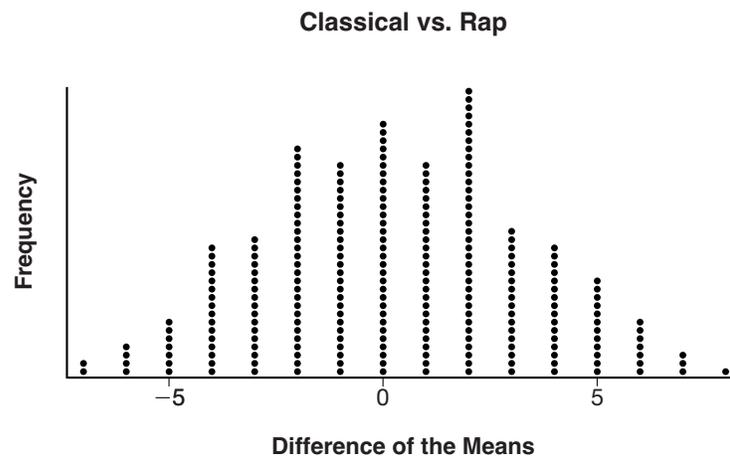
Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

It means that the mean grades of the classical music group are approximately 7 points higher than the mean grade of the rap music group.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes there may be a significant difference b/c 7 does not fall in the center of distribution.

**Score 2:** The student received partial credit for each part.

### Question 36

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

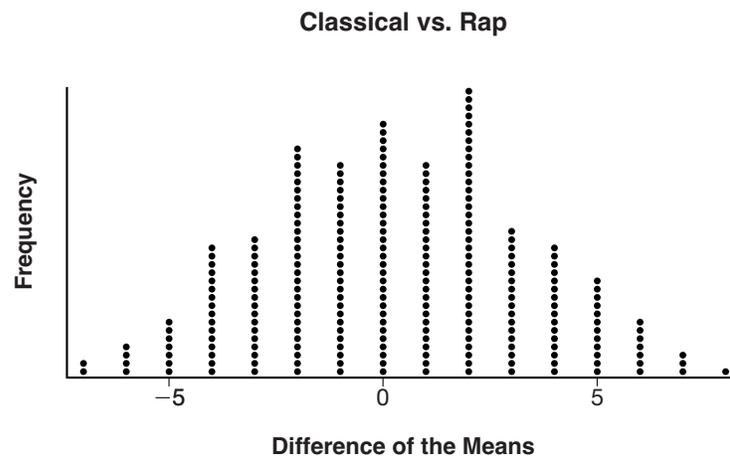
Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

He found the mean of each by adding them together and dividing by 8, and then subtracted the means of each.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

yes, because there is a large variation in the chart.

**Score 1:** The student received partial credit for the first part.

**Question 36**

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

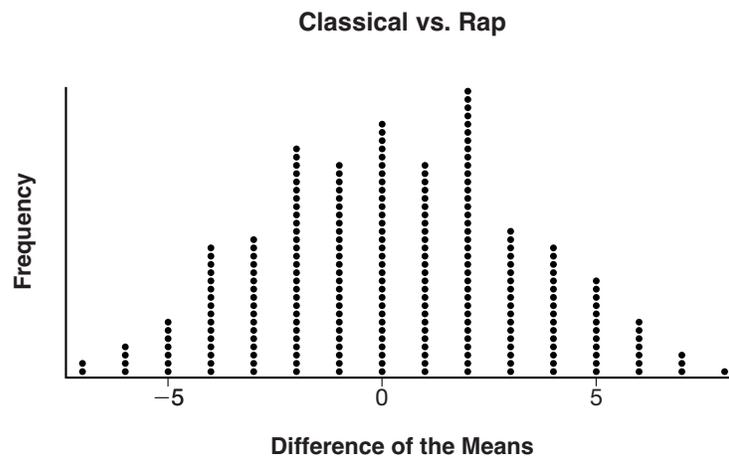
Classical: 74, 83, 77, 77, 84, 82, 90, 89

Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John obtained the value by finding the means of both groups then subtracted them to get the difference.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

yes, the theory that there may be a significant difference in quiz scores is proven through the stimulation.

**Score 1:** The student received partial credit for the first part.

**Question 36**

**36** To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

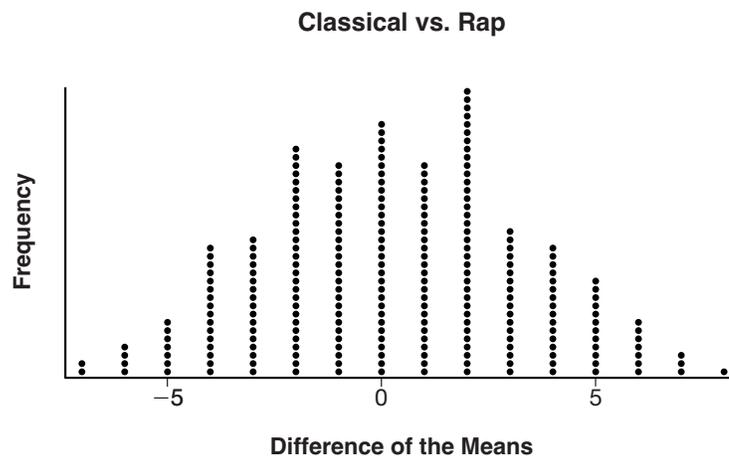
Classical: 74, 83, 77, 77, 84, 82, 90, 89 82  
 Rap: 77, 80, 78, 74, 69, 72, 78, 69 84

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer.

John obtained his values by subtracting  $Q_1 = 77$  from  $Q_3 = 84$ . This value represents the mean difference of the two experimental groups.

To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.

$\bar{X} = 81.91467286$   
 $\Sigma x = 48903$   
 $\Sigma x^2 = 4023321$   
 $S_x = 5.411373062$   
 $\sigma_x = 5.406839024$   
 $dn = 597$   
 $\min x = 74$   
 $Q_1 = 77$   
 $Med = 82$   
 $Q_3 = 84$   
 $\max x = 90$



Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

This simulation does support the theory that there may be a significant difference in the quiz scores because the values range from a low frequency to a very high frequency as depicted in the simulation.

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 37**

**37** A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$P(x) = R(x) - C(x)$$

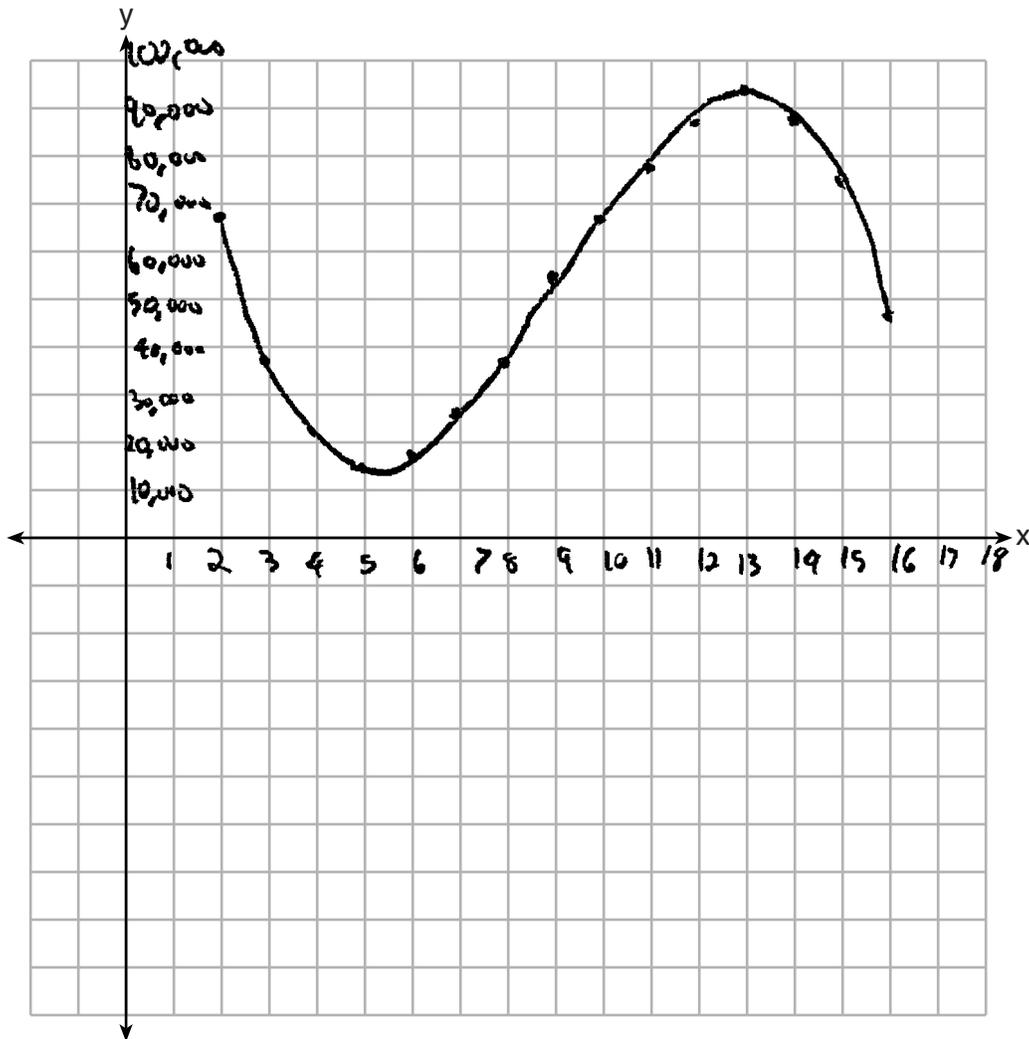
$$550x^3 - 12000x^2 + 83000x + 7000 - (880x^3 - 21000x^2 + 150000x - 160000)$$

$$P(x) = -330x^3 + 9000x^2 - 67000x + 167000$$

**Score 6:** The student gave a complete and correct response.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

The company was least profitable in the fifth year and most profitable in the 13<sup>th</sup> year because it made \$15,000 in year 5 which was the lowest in the interval and \$95,000 in year 13 which was the highest in the interval.

**Question 37**

**37** A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

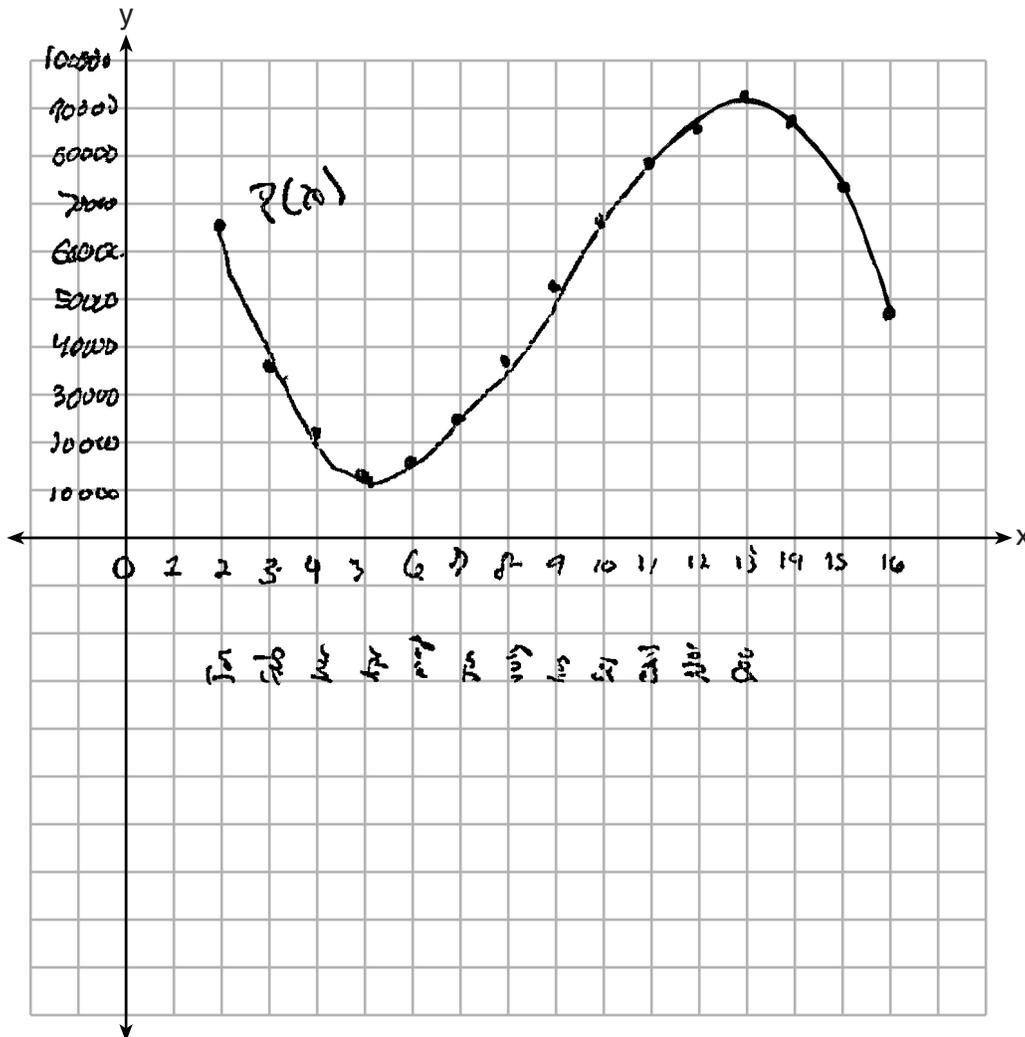
The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$\begin{array}{r} 550x^3 - 12,000x^2 + 83,000x + 7,000 \\ - 880x^3 - 21,000x^2 + 150,000x - 160,000 \\ \hline P(x) = -330x^3 + 9,000x^2 - 67,000x + 167,000 \end{array}$$

**Score 5:** The student misunderstood the meaning of the independent variable.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

least in april b/c thats when the graph is the lowest and  
 December b/c thats when its the highest

**Question 37**

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$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

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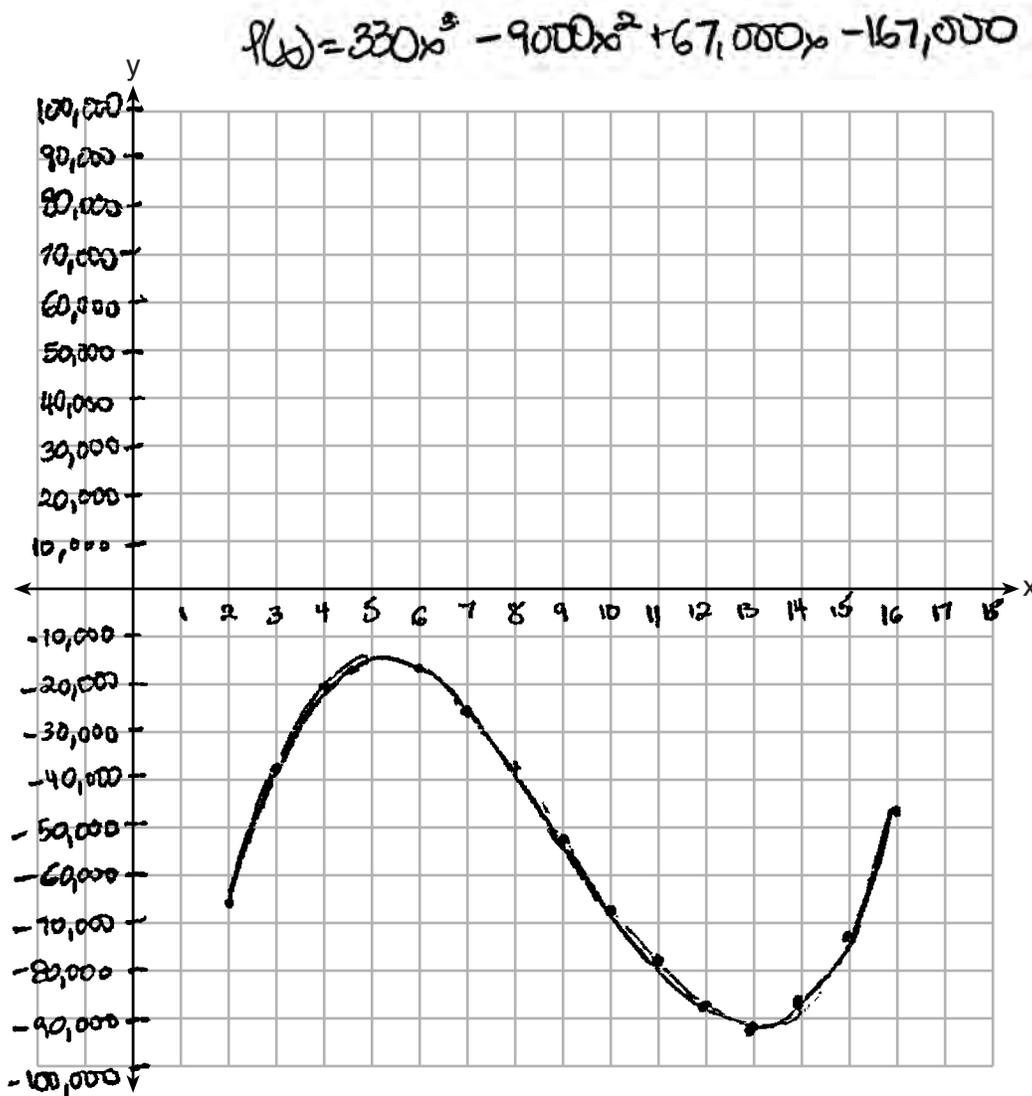
The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$\begin{aligned} C(x) - R(x) &= P(x) \\ 880x^3 - 21,000x^2 + 150,000x - 160,000 &- (550x^3 - 12,000x^2 + 83,000x + 7000) \\ \boxed{330x^3 - 9,000x^2 + 67,000x - 167,000} \end{aligned}$$

**Score 5:** The student made an error finding  $P(x)$ .

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

The company is least profitable at 13 years, and most profitable at 5 years. I determined my answer by observing the minimum and maximum values of the range.

**Question 37**

**37** A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

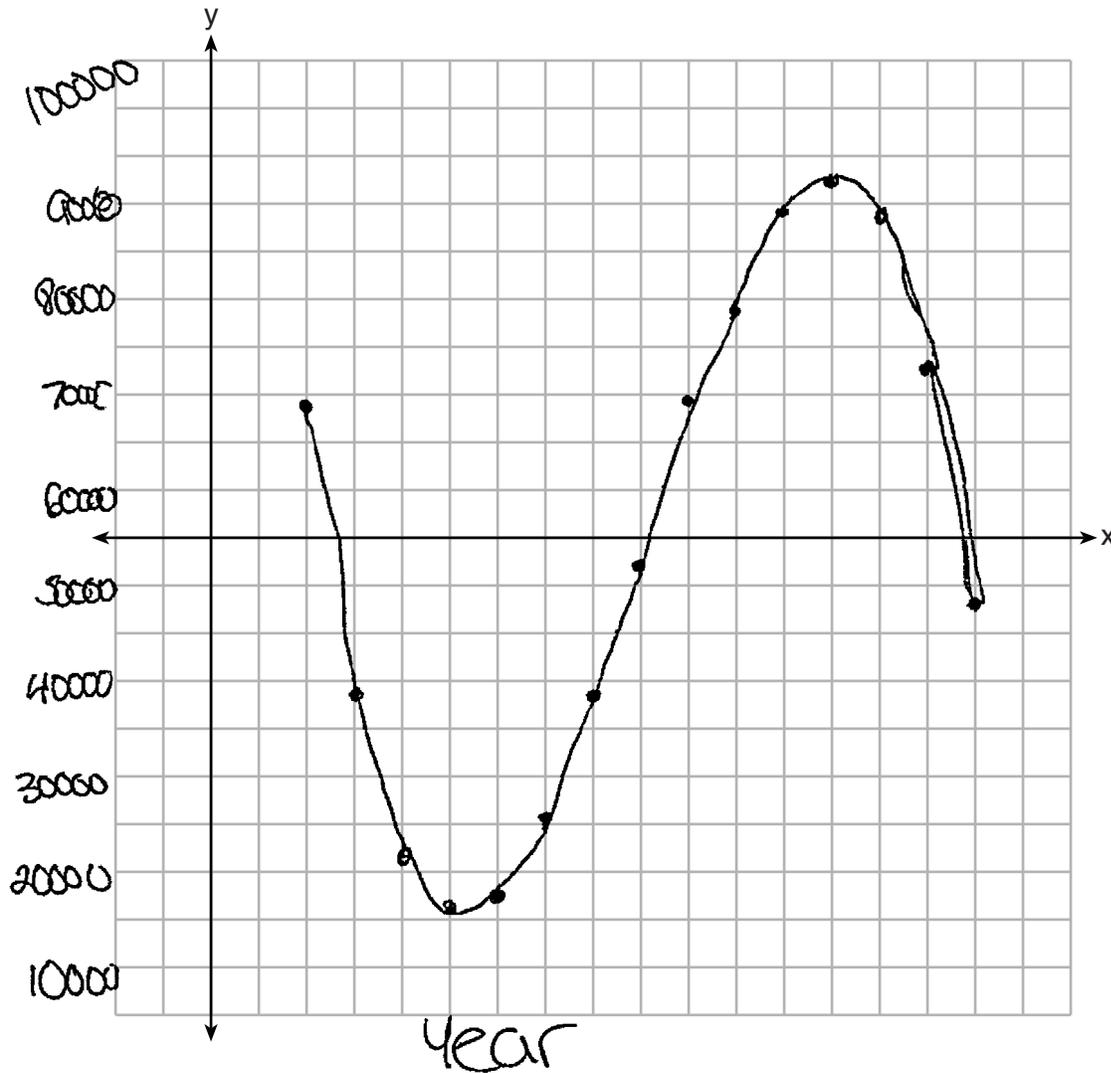
$$\begin{array}{r} 550x^3 - 12000x^2 + 83000x + 7000 \\ 880x^3 - 21000x^2 + 150000x - 160000 \end{array}$$

$$P(x) = -330x^3 + 9000x^2 - 67000x + 167000$$

**Score 4:** The student received one credit each for  $P(x)$ , the graph, 5, and 13.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

13

The company was the least profitable during year 5 and the most profitable during year 13.

**Question 37**

37 A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$P(x) = R(x) - C(x)$

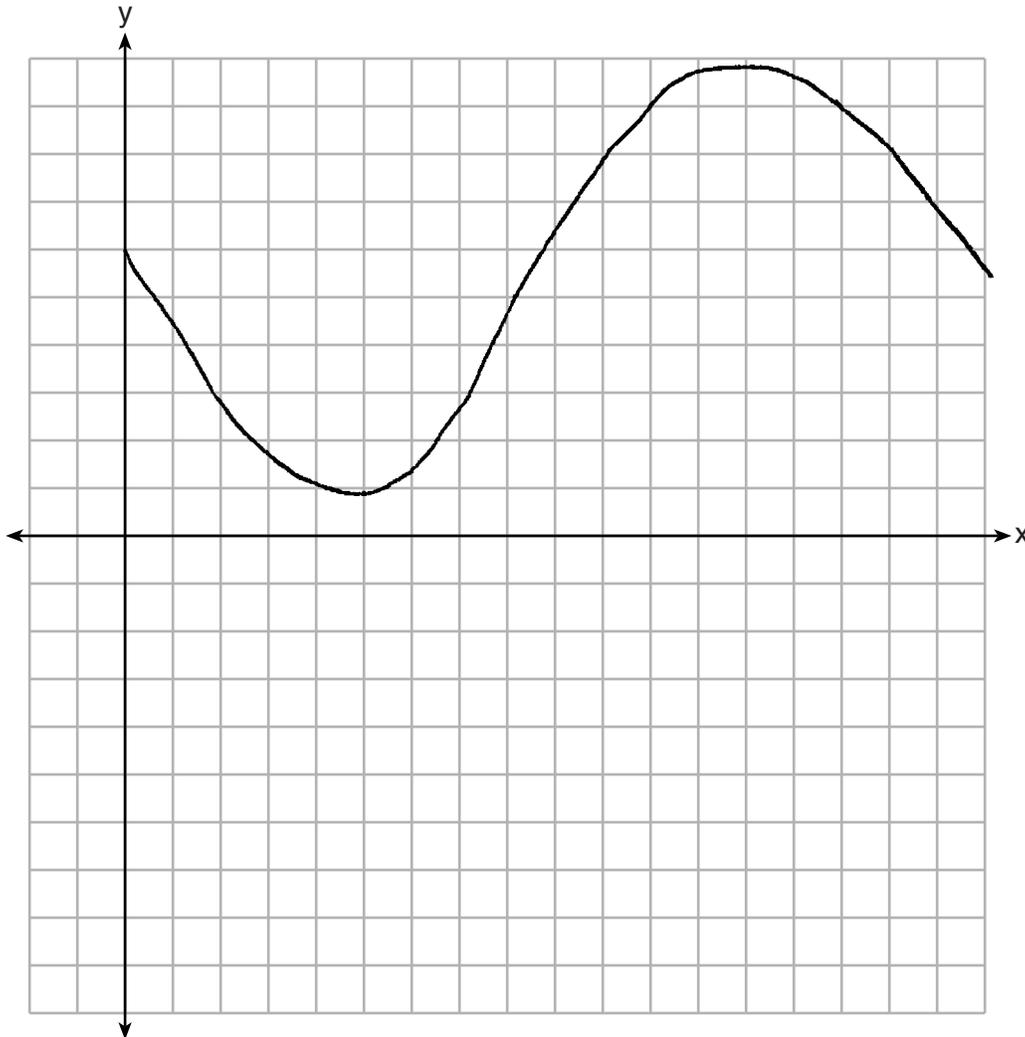
$P(x) = (550x^3 - 12,000x^2 + 83,000x + 7000) - (880x^3 - 21,000x^2 + 150,000x - 160,000)$

$P(x) = -330x^3 + 9000x^2 + 67000x + 167000$

**Score 3:** The student received credit for finding  $P(x)$ , 5, and 13.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

The company was the least profitable at around the 5<sup>th</sup> year. It was the most profitable during about the 13<sup>th</sup> year.

**Question 37**

**37** A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$\begin{array}{l} \text{Revenue} \\ \text{Costs} \end{array} \quad \begin{array}{l} R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000 \\ C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000 \end{array}$$

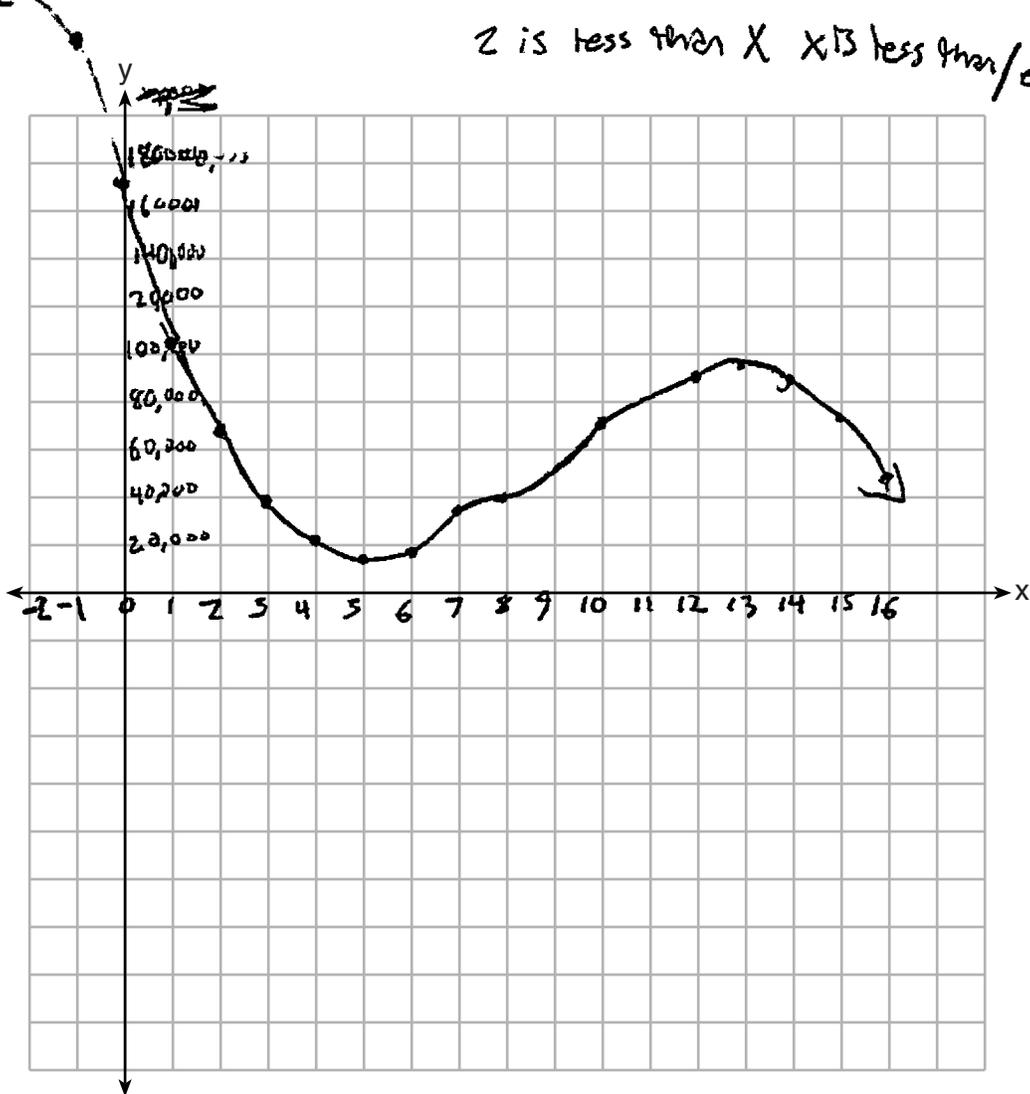
The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$P(x) = -330x^3 + 9000x^2 - 67,000x + 167,000$$

**Score 2:** The student received one credit for  $P(x)$  and only one credit for graphing  $P(x)$  outside the domain.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

Least profitable was in 75 years (5.15)  
 Most profitable was in 15 years (1.15)  
 -2-0 don't count for you can't make money with no cars. so you find highest and lowest points, and you multiply the x value by 15 for it says  $P(x)/L(x)$  is in fifteen years.

**Question 37**

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$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

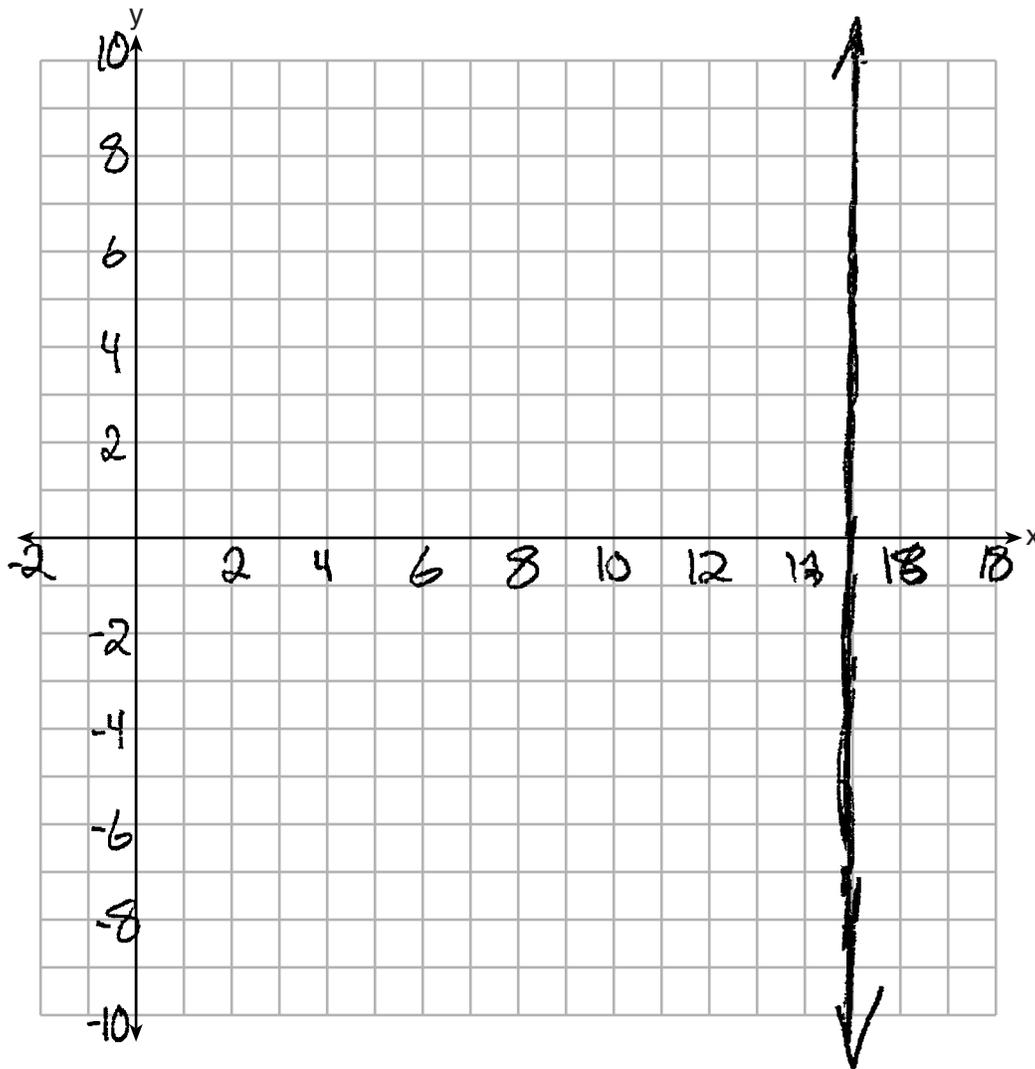
$$R(x) - C(x) = P(x)$$

$$P(x) = -330x^3 + 9000x^2 - 67000x + 167000$$

**Score 1:** The student only found  $P(x)$  correctly.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the *nearest* year. Explain how you determined your answer.

most profitable in 2017  
least profitable in 1972

**Question 37**

37 A major car company analyzes its revenue,  $R(x)$ , and costs  $C(x)$ , in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years,  $x$ , using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7,000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function,  $P(x)$ , as a polynomial in standard form.

$$P(x) = R(x) - C(x)$$

$$P(x) = 550x^3 - 12000x^2 + 83000x + 7000 - (880x^3 - 21000x^2 + 150000x - 160000)$$

$$550x^3 - 12000x^2 + 83000x + 7000 - 880x^3 + 21000x^2 - 150000x + 160000$$

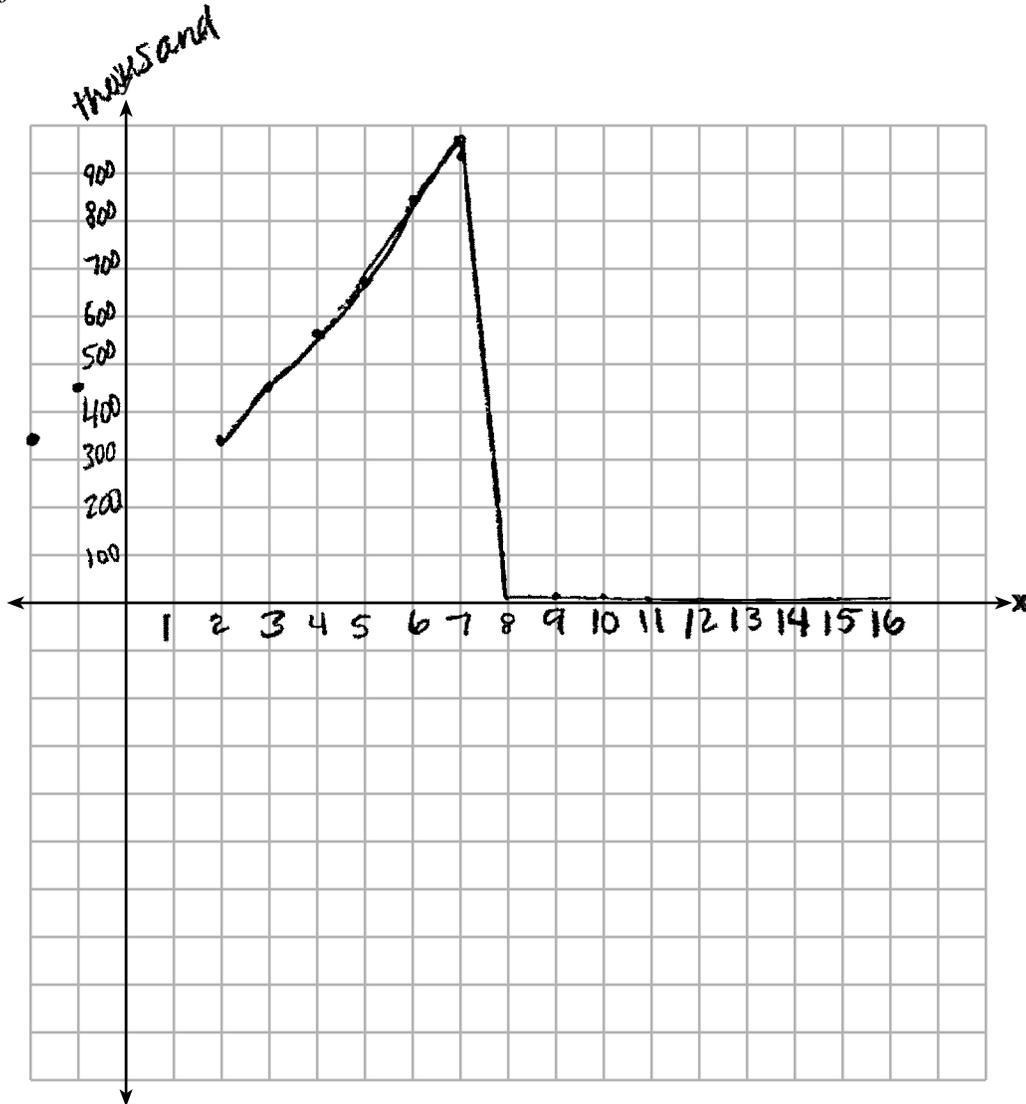
$$-330x^3 + 9000x^2 + 68000x + 167000$$

$$P(x) = -330x^3 + 9000x^2 + 68000x + 167000$$

**Score 0:** The student made a computational error finding  $P(x)$  and showed no further correct work.

Question 37 continued.

Graph  $y = P(x)$  on the set of axes below over the domain  $2 \leq x \leq 16$ .



Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.

The company was least profitable between years 8 to 16 because that's after there was a drop in profit. The company was most profitable between years 2 to 7, where their profit kept increasing.