

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 22, 2020 — 9:15 a.m. to 12:15 p.m.

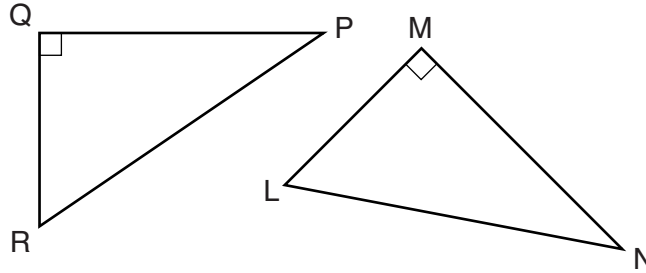
MODEL RESPONSE SET

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Question 25

25 In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML .



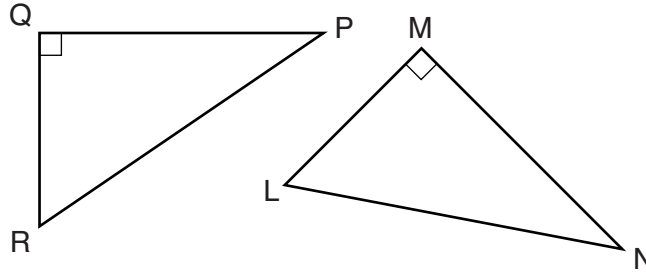
Write a set of three congruency statements that would show ASA congruency for these triangles.

$$\begin{aligned}\angle R &\cong \angle M \\ \angle P &\cong \angle N \\ \overline{QP} &\cong \overline{MN}\end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 25

25 In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML .



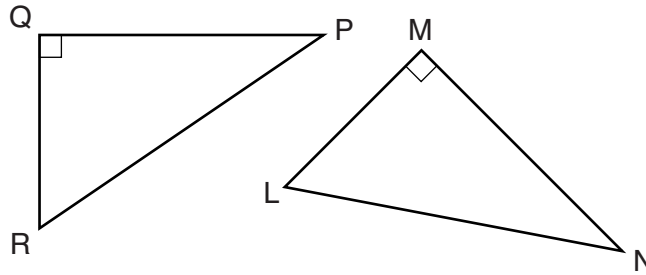
Write a set of three congruency statements that would show ASA congruency for these triangles.

$$\begin{aligned}\angle Q &\cong \angle M \\ \overline{QR} &\cong \overline{ML} \\ \angle R &\cong \angle L\end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 25

25 In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML .



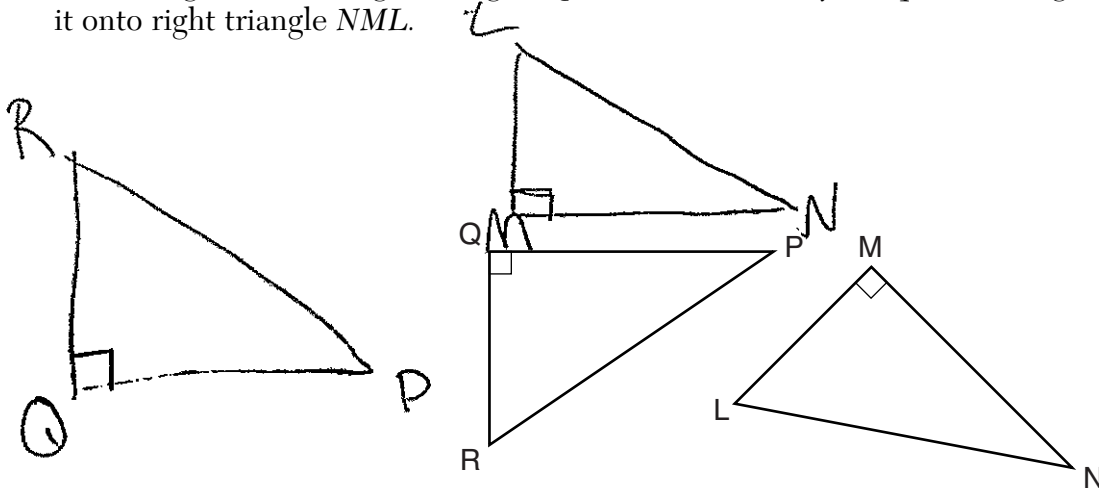
Write a set of three congruency statements that would show ASA congruency for these triangles.

- ① $\angle Q \cong \angle M$
- ② $\overline{PR} \cong \overline{NL}$
- ③ $\angle P \cong \angle N$

Score 1: The student wrote congruency statements for AAS.

Question 25

25 In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML .



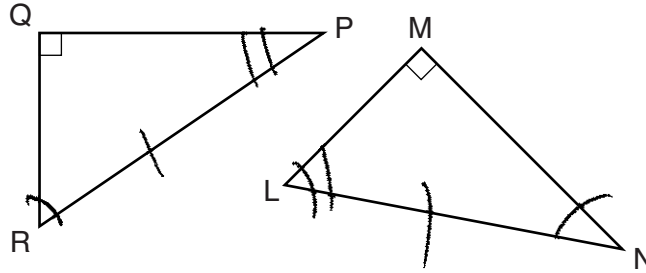
Write a set of three congruency statements that would show ASA congruency for these triangles.

- ① $\angle Q \cong \angle M$ because they are both right angles and all right angles are congruent (Angle)
- ② Since $\triangle PQR$ is formed by a rigid motion of $\triangle NML$ and rigid motions preserve shape and size then $\overline{QP} \cong \overline{MN}$, $\overline{QR} \cong \overline{ML}$, $\overline{RP} \cong \overline{LN}$, $\angle R \cong \angle L$, $\angle P \cong \angle N$
- ③ By ASA $\triangle PQR \cong \triangle NML$

Score 1: The student wrote all corresponding congruency statements, but did not specify which congruencies were for ASA.

Question 25

25 In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML .



ASA

Write a set of three congruency statements that would show ASA congruency for these triangles.

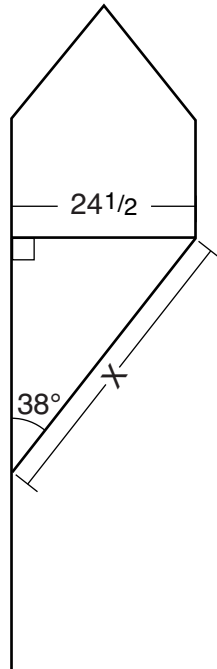
$$\begin{aligned} \angle R &\cong \angle N \\ \angle L &\cong \angle P \\ \overline{RP} &\cong \overline{LN} \end{aligned}$$

Score 0: The student stated only one correct corresponding congruency statement, $\overline{RP} \cong \overline{LN}$.

Question 26

26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of 38° with the vertical post. Determine and state the approximate length of the support beam, x , to the nearest inch.

SOH CAH - TOA



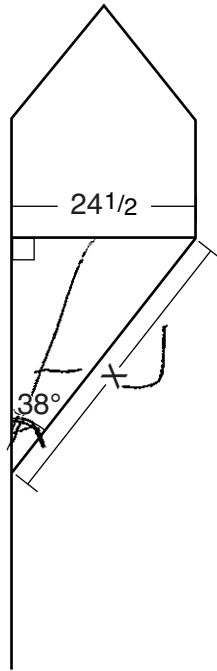
$$\sin 38^\circ = \frac{24\frac{1}{2}}{x}$$
$$x = 39.79459651$$

$x \approx 40$ inches

Score 2: The student gave a complete and correct response.

Question 26

26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of 38° with the vertical post. Determine and state the approximate length of the support beam, x , to the nearest inch.



$$\sin 38 = \frac{24.5}{x}$$

~~to~~

$$24.5 = \frac{\sin 38 x}{\sin 38}$$

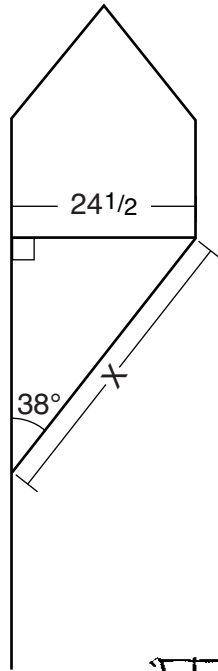
$$x \approx 82.667$$

83 in

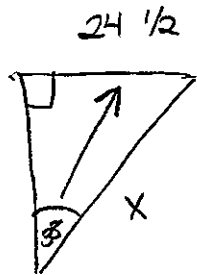
Score 1: The student made an error by using 38° as a radian measure.

Question 26

26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of 38° with the vertical post. Determine and state the approximate length of the support beam, x , to the nearest inch.



SOH
CAH
TOA



$$\frac{\tan(38)}{\tan(38)} = \frac{24.5}{\tan(38)}$$

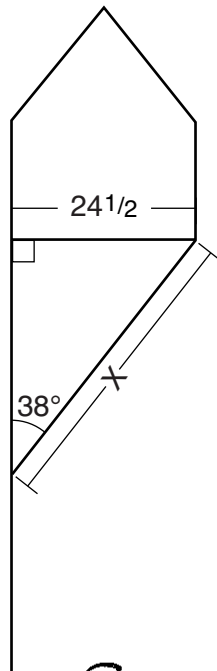
$$\frac{\tan(38) \cdot x}{\tan(38)} = \frac{24.5}{\tan(38)}$$

$$x = 31.35 \quad x \approx 31$$

Score 1: The student used an incorrect trigonometric equation, but solved it correctly.

Question 26

- 26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of 38° with the vertical post. Determine and state the approximate length of the support beam, x , to the nearest inch.



$$\begin{aligned}A^2 + B^2 &= C^2 \\38^2 + 24^2 &= C^2 \\1444 + 576 &= C^2 \\\sqrt{2020} &= C^2 \\c &= 44.94\end{aligned}$$

Score 0: The student gave a completely incorrect response.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

$$43 \text{ lb} / \text{ft}^3$$

$$V = lwh$$

$$8 \cdot 3 \cdot \frac{1}{12} = 2$$

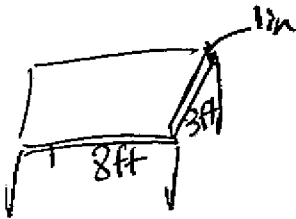
$$43 \cdot 2 =$$

$$86 \text{ pounds}$$

Score 2: The student gave a complete and correct response.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

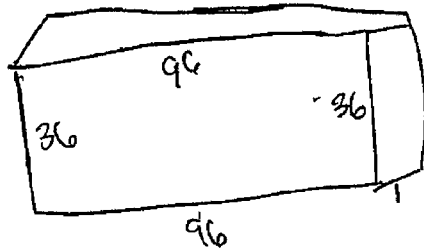
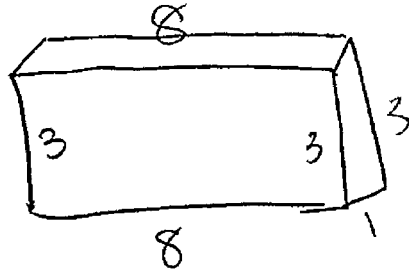


$$\begin{aligned} 12\text{in} &= 1\text{ft} \\ \frac{1\text{ft}}{12\text{in}} &= 0.083333 \\ 8 \cdot 3 \cdot 0.083333 &= 1.999992 \\ 43 \cdot 1.999992 &= 85.99967 \\ &\downarrow \\ &86 \\ &\text{weight of table} \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.



Weight = 43 (288)
 $W = 12,384 \text{ lbs}$

$$V = Bh$$

$$V = 96(1)$$

$$V = 96(36)$$

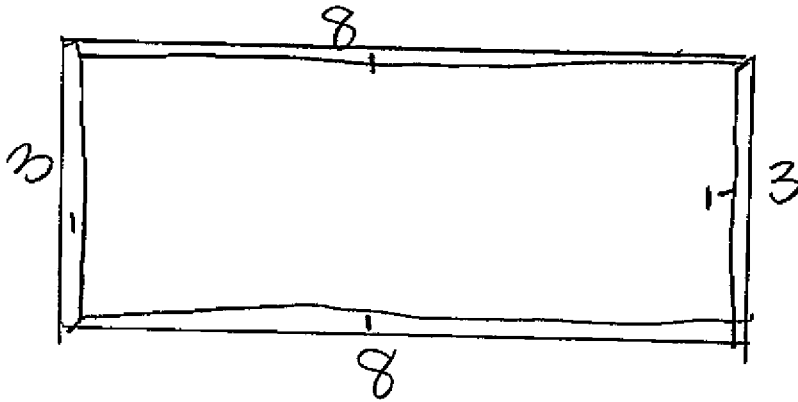
$$V = 3456 \text{ in}^3$$

$$V = 288 \text{ ft}^3$$

Score 1: The student made an error when converting 3456 cubic inches to 288 cubic feet.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.



$$V = l \cdot w \cdot h$$

$$V = 8 \cdot 1 \cdot 3$$

$$24 \text{ feet}^3 \cdot 43 \text{ pounds}$$

1032 pounds

Score 1: The student did not convert the 1-inch thickness to feet.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

$$1 \text{ inch} = \frac{1}{12} \text{ foot} = 0.083$$

$$8 \times 3 \times 0.083 = 1.992$$

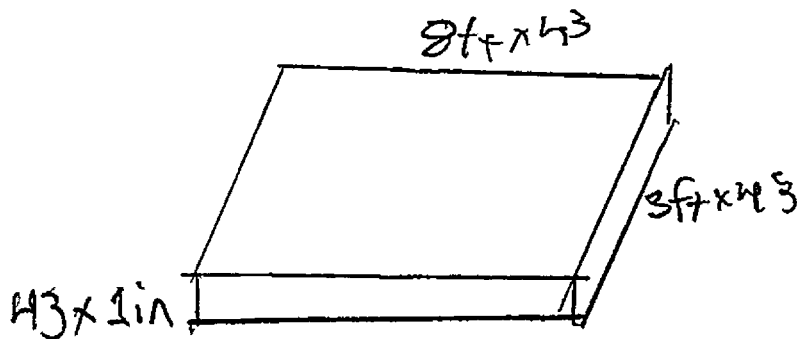
$$\begin{array}{r} \times 43 \\ \hline \end{array}$$

$$85.656$$

Score 1: The student made a rounding error when stating the weight of the table.

Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.



$$L = 344 \text{ cf}$$

$$W_{\text{ft}} = 129 \text{ cf}$$

$$\text{Thick} = 3 \text{ in}$$

$$V = L \times W \times h$$

$$V = h \times W \times T$$

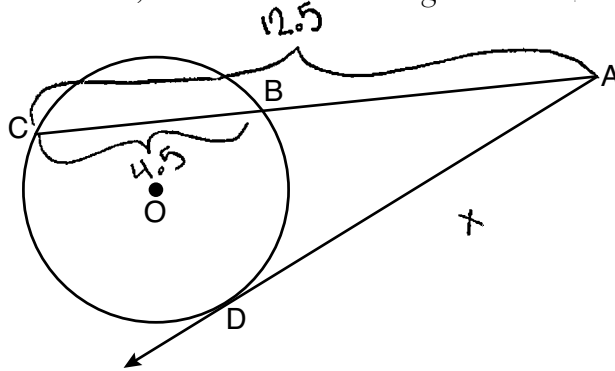
$$V = 344 \times 129 \times \frac{3}{12}$$

$$W_{\text{T}} = 1609 \text{ pounds}$$

Score 0: The student gave a completely incorrect response.

Question 28

28 In the diagram below of circle O , secant \overline{ABC} and tangent \overline{AD} are drawn.



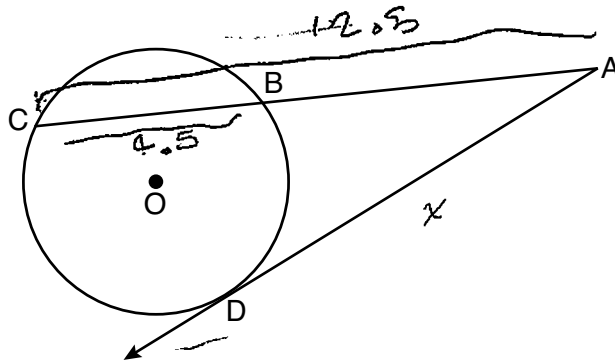
If $CA = 12.5$ and $CB = 4.5$, determine and state the length of \overline{DA} .

$$\begin{aligned} & \text{(outside)(whole)} = \text{(outside)(outside)} \\ & (AB)(AC) = (AD)(AD) \\ & \begin{array}{l} 12.5 - 4.5 \\ \vee \\ \overline{AB} = 8 \end{array} \quad \nearrow \\ & (8)(12.5) = x(x) \\ & \sqrt{100} = \sqrt{x^2} \\ & x = 10 \\ & \boxed{DA = 10} \end{aligned}$$

Score 2: The student gave a complete and correct response.

Question 28

28 In the diagram below of circle O , secant \overline{ABC} and tangent \overline{AD} are drawn.



If $CA = 12.5$ and $CB = 4.5$, determine and state the length of \overline{DA} .

$$\overline{DA} = 7.5$$

$$x^2 = 12.5(4.5)$$

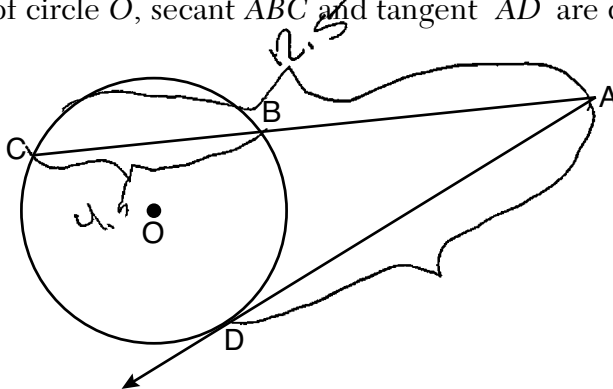
$$\sqrt{x^2} = \sqrt{56.25}$$

$$x = 7.5$$

Score 1: The student made an error by using 4.5 instead of 8.

Question 28

28 In the diagram below of circle O , secant \overline{ABC} and tangent \overline{AD} are drawn.



If $CA = 12.5$ and $CB = 4.5$, determine and state the length of \overline{DA} .

$$17(8) = (x)(x)$$

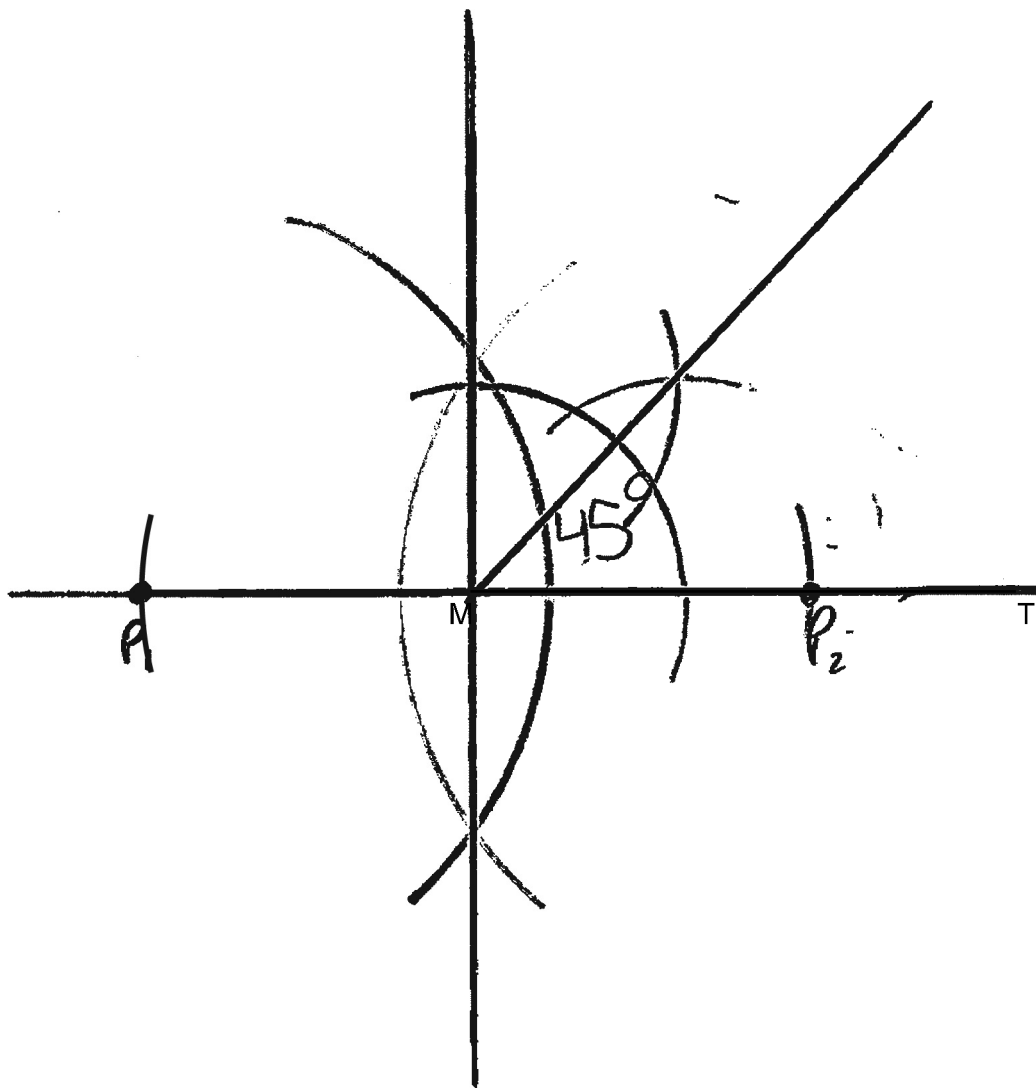
$$136 = x^2$$

$$\overline{DA} = 11.6$$

Score 0: The student made an error with $AC = 12.5 + 4.5$ and made a rounding error to find DA .

Question 29

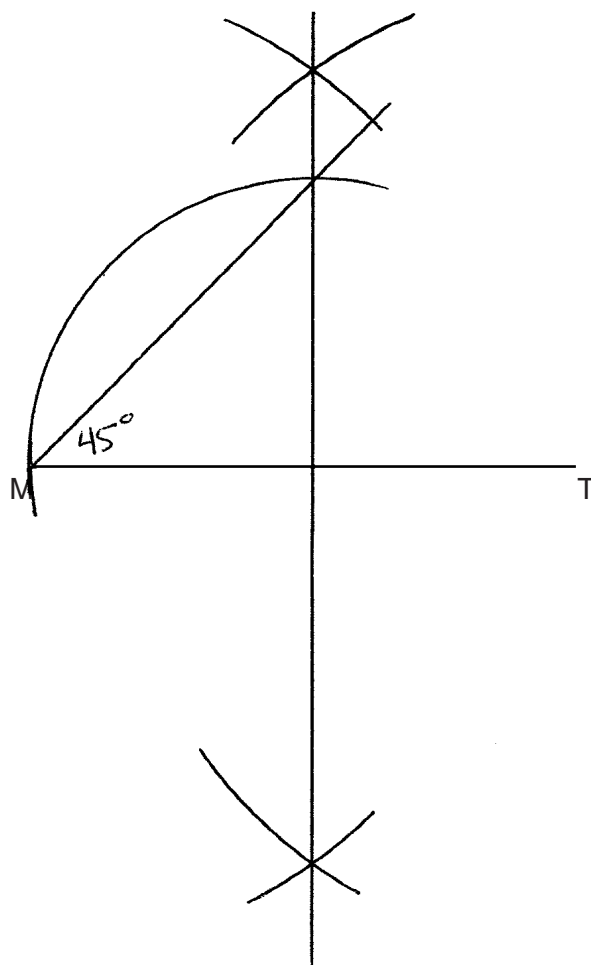
29 Given \overline{MT} below, use a compass and straightedge to construct a 45° angle whose vertex is at point M .
[Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 29

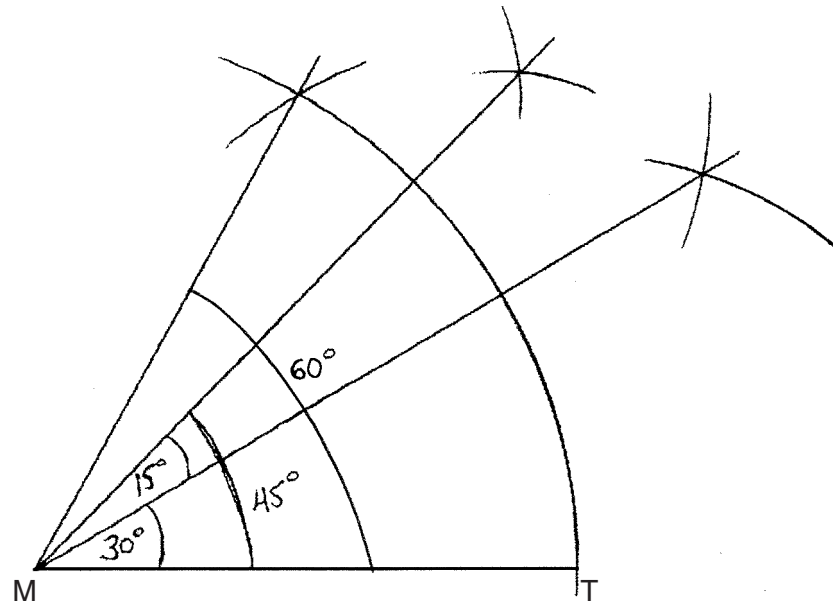
29 Given \overline{MT} below, use a compass and straightedge to construct a 45° angle whose vertex is at point M .
[Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 29

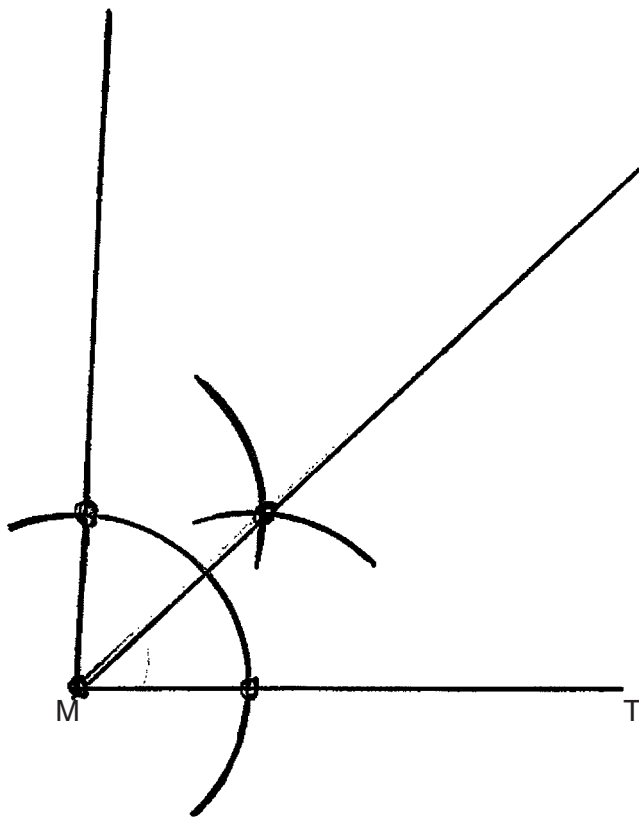
29 Given \overline{MT} below, use a compass and straightedge to construct a 45° angle whose vertex is at point M .
[Leave all construction marks.]



Score 2: The student gave a complete and correct response. The student constructed a 60° angle using an equilateral triangle and then bisected that 60° angle to get a 30° angle. Lastly, the student bisected a 30° angle to combine the other 30° angle with the 15° angle to get a 45° angle.

Question 29

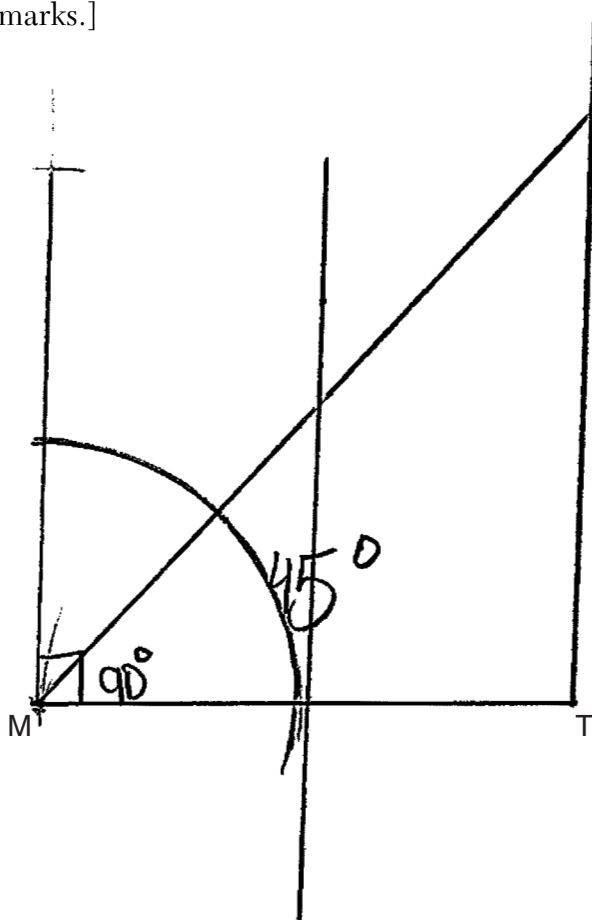
29 Given \overline{MT} below, use a compass and straightedge to construct a 45° angle whose vertex is at point M .
[Leave all construction marks.]



Score 1: The student did not construct the line perpendicular to \overline{MT} through M , but correctly bisected the angle.

Question 29

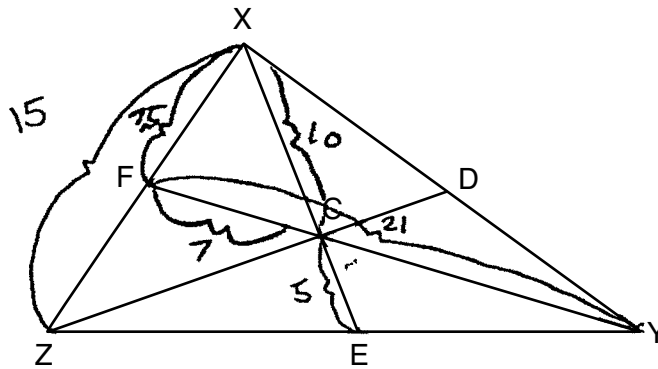
29 Given \overline{MT} below, use a compass and straightedge to construct a 45° angle whose vertex is at point M .
[Leave all construction marks.]



Score 0: The student gave a completely incorrect response.

Question 30

30 In $\triangle XYZ$ shown below, medians \overline{XE} , \overline{YF} , and \overline{ZD} intersect at C .



If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle CFX .

$$7 + 7.5 + 10 = 24.5$$

$$21/3 = 7$$

$$\triangle CFX \text{ perimeter} = 24.5$$

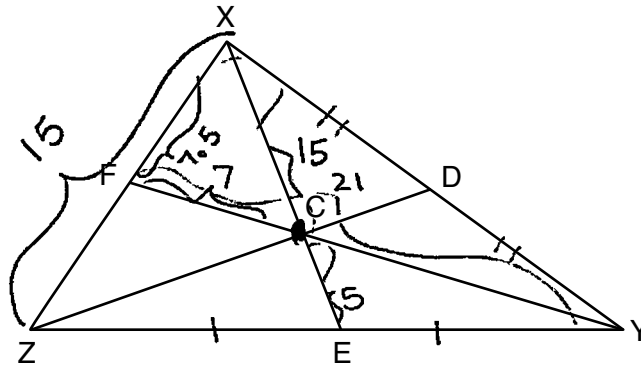
$$5 \cdot 2 = 10$$

$$15/2 = 7.5$$

Score 2: The student gave a complete and correct response.

Question 30

30 In $\triangle XYZ$ shown below, medians \overline{XE} , \overline{YF} , and \overline{ZD} intersect at C .



If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle CFX .

$$15 \div 2 = 7.5$$

$$21 \times \frac{1}{3} = 7$$

$$5 \div \frac{1}{3} = 15$$

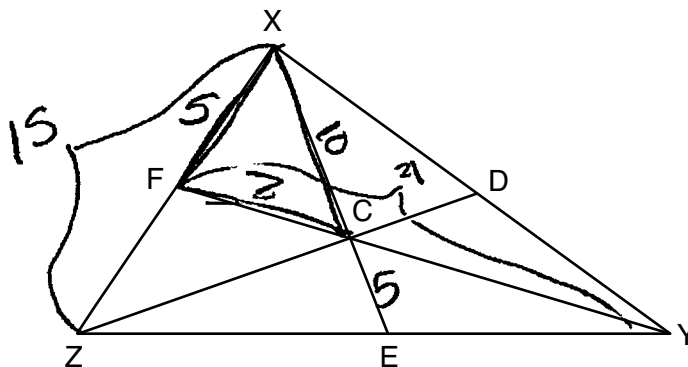
$$15 + 7.5 + 7 = 29.5$$

perimeter = 29.5 units

Score 1: The student made an error in determining XC .

Question 30

30 In $\triangle XYZ$ shown below, medians \overline{XE} , \overline{YF} , and \overline{ZD} intersect at C .



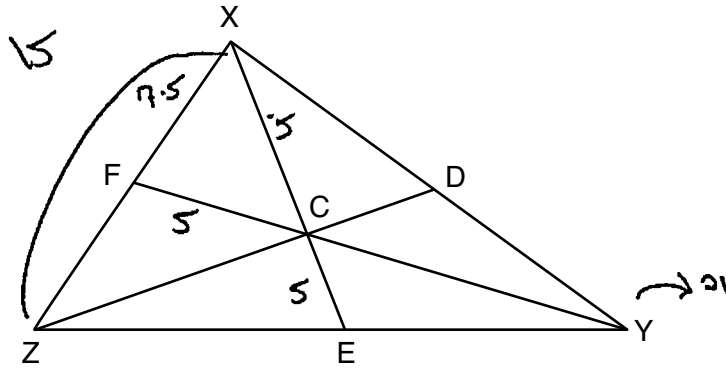
If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle CFX .

Perimeter = 22

Score 1: The student found correct lengths for \overline{CF} and \overline{CX} .

Question 30

30 In $\triangle XYZ$ shown below, medians \overline{XE} , \overline{YF} , and \overline{ZD} intersect at C .



If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle CFX .

$$\frac{15}{7.5} = \frac{x+5}{5}$$

$$7.5x + 37.5 = 7.5$$

$$\begin{array}{r} 7.5x + 37.5 = 7.5 \\ -37.5 \quad -37.5 \\ \hline 7.5x = -37.5 \\ \frac{7.5x}{7.5} = \frac{-37.5}{7.5} \\ x = -5 \end{array}$$

$$7.5 + 5 + 5 = 17.5$$

Perimeter of $\triangle CFX = 17.5$

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$5x - 4y = 10$$
$$+4y \quad +4y$$

$$4y + 10 = 5x$$
$$-10 \quad -10$$

$$\frac{4y = 5x - 10}{4}$$

$$y = \frac{5}{4}x - \frac{5}{2}$$

$$m = \frac{5}{4}$$

$$m_{\perp} = -\frac{4}{5}$$

$$(y - 12) = -\frac{4}{5}(x - 5)$$

Score 2: The student gave a complete and correct response.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$5x - 4y = 10$$

$$\frac{-4y}{-4} = \frac{-5x + 10}{-4}$$

$$y = \frac{5}{4}x + \frac{-5}{2}$$

$$m_{\perp} = -\frac{4}{5}$$

$$y = mx + b$$

$$12 = \left(-\frac{4}{5}\right)(5) + b$$

$$12 = -4 + b$$

$$12 + 4 = b$$

$$16 = b$$

$$y = -\frac{4}{5}x + 16$$

Score 2: The student gave a complete and correct response.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$5x - 4y = 10$$

$$\frac{5x}{4} - \frac{10}{4} = \frac{4y}{4}$$

$$\left(\frac{5}{4}\right)x - 2.5 = y$$

$$m = \frac{5}{4}$$

$$y - y_1 = m(x - x_1)$$

$$\begin{array}{r} y - 12 = \frac{5}{4}(x - 5) \\ \underline{+12} \qquad \qquad \qquad \underline{+12} \end{array}$$

$$y = \frac{5}{4}(x - 5) + 12$$

Score 1: The student wrote an equation of the line parallel and passing through $(5,12)$.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$\frac{4y}{4} = \frac{5x - 10}{4}$$

$$y = -\frac{4}{5}x + 8$$

$$y = \frac{5}{4}x - \frac{5}{2}$$

$$8 = b$$

$$y = -\frac{4}{5}x + b$$

$$12 = -\frac{4}{5}(5) + b$$

$$12 = 4 + b$$

Score 1: The student made one computational error in determining the y -intercept.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$5x + 4y = 10$$

$$\frac{4y = 5x - 10}{4} \quad \frac{4}{4} \quad \frac{5x}{4} \quad \frac{-10}{4}$$

$$(y - y_1) = m(x - x_1)$$

$$(y - 12) = m(x - 5)$$

$$y = \frac{-5}{4}x - 2.5$$

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 31

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

$$5x - 4y = 10$$

$$5x - 4y = 10$$

$$-4y = \frac{10}{1} - 5x$$

$$-4y = \frac{1}{10} - 5x$$

$$y = 12$$

$$y = m \cdot x + b$$

Score 0: The student gave a completely incorrect response.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral *NATS* is a rhombus.

[The use of the set of axes below is optional.]

$$AT = \sqrt{(8-1)^2 + (1-2)^2}$$

$$TS = \sqrt{(8-3)^2 + (1+4)^2}$$

$$NS = \sqrt{(3+4)^2 + (-4+3)^2}$$

$$NA = \sqrt{(1+4)^2 + (2+3)^2}$$

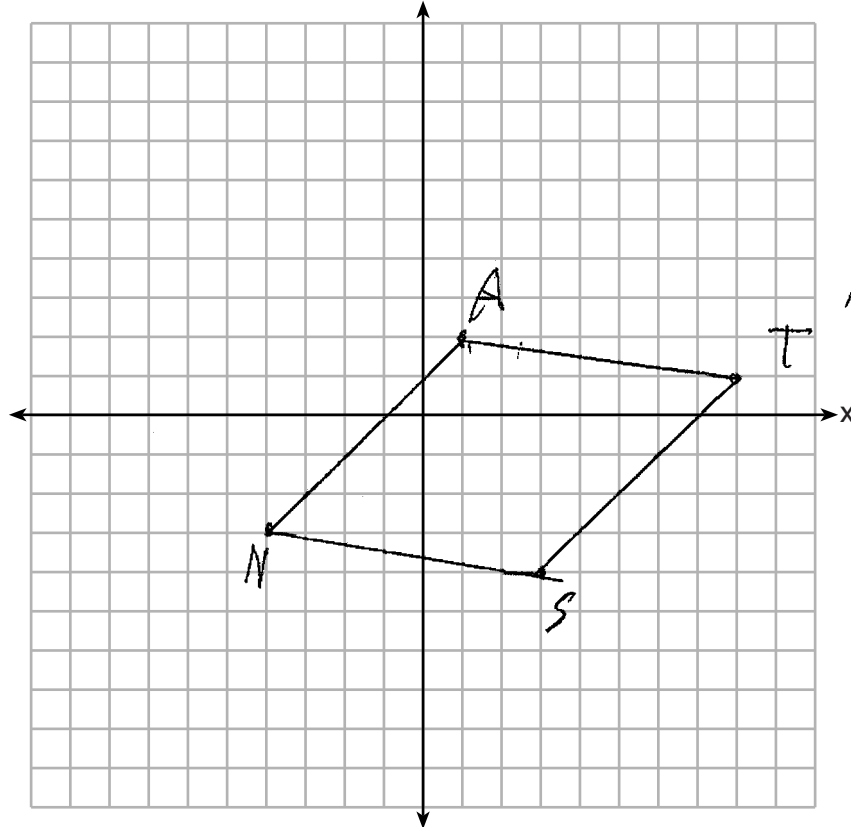
$$\sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} = TS$$

$$\sqrt{7^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} = AT$$

$$\sqrt{49+1} = \sqrt{50} = 5\sqrt{2} = NS$$

$$\sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2} = NA$$

$5\sqrt{2}$
 $5\sqrt{2} = AT$
 $5\sqrt{2}$
 $5\sqrt{2}$
 $NA = 5\sqrt{2}$



All sides
 are equal
 in the quadrilateral
 NATS
 it is a Rhombus

Score 4: The student gave a complete and correct response.

Question 32

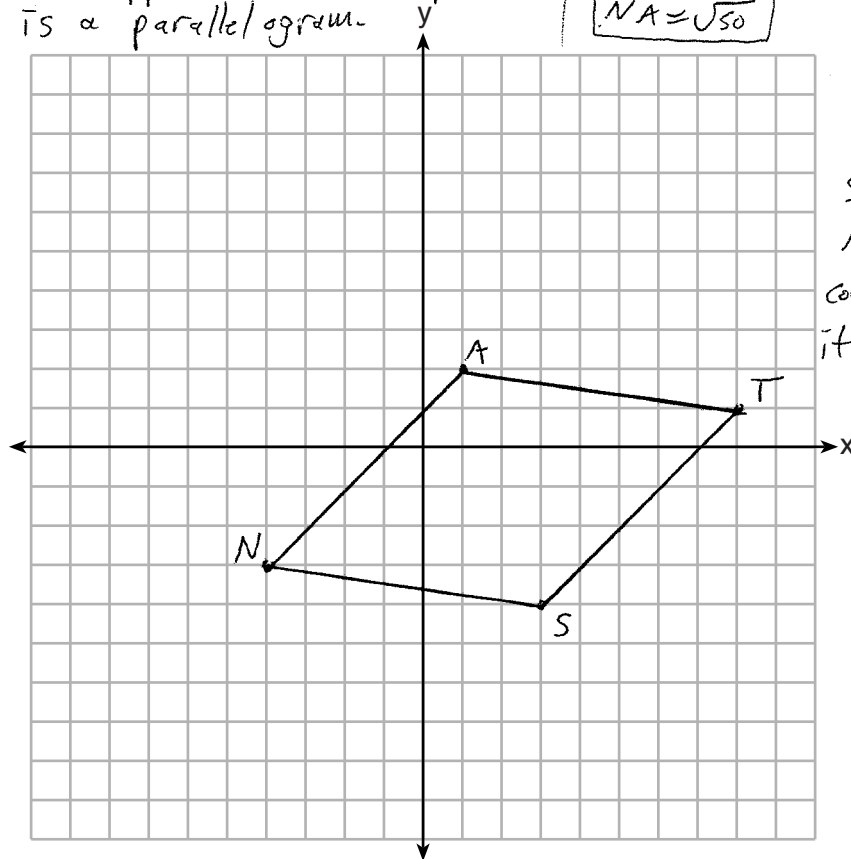
32 Quadrilateral $NATS$ has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral $NATS$ is a rhombus.
 [The use of the set of axes below is optional.]

$$\begin{array}{l} \text{slope } \overline{NA} = \frac{2 - (-3)}{1 - (-4)} \\ \quad = \frac{5}{5} \\ \boxed{m_{\overline{NA}} = 1} \end{array} \quad \left| \quad \begin{array}{l} \text{slope } \overline{AT} = \frac{1 - 2}{8 - 1} \\ \quad = \frac{-1}{7} \\ \boxed{m_{\overline{AT}} = -\frac{1}{7}} \end{array} \quad \left| \quad \begin{array}{l} \text{slope } \overline{TS} = \frac{-4 - 1}{3 - 8} \\ \quad = \frac{-5}{-5} \\ \boxed{m_{\overline{TS}} = 1} \end{array} \quad \left| \quad \begin{array}{l} \text{slope } \overline{NS} = \frac{-4 - (-3)}{3 - (-4)} \\ \quad = \frac{-1}{7} \\ \boxed{m_{\overline{NS}} = -\frac{1}{7}} \end{array}$$

Since \overline{NA} + \overline{TS} have the same slope, $\overline{NA} \parallel \overline{TS}$
 Since \overline{AT} + \overline{NS} have the same slope, $\overline{AT} \parallel \overline{NS}$
 Since both pairs of opposite sides are parallel,
 quad $NATS$ is a parallelogram.

$$\begin{array}{l} \overline{NA} = \sqrt{(1 - (-4))^2 + (2 - (-3))^2} \\ \quad = \sqrt{5^2 + 5^2} \\ \quad = \sqrt{25 + 25} \\ \boxed{\overline{NA} = \sqrt{50}} \end{array} \quad \left| \quad \begin{array}{l} \overline{AT} = \sqrt{(8 - 1)^2 + (1 - 2)^2} \\ \quad = \sqrt{7^2 + (-1)^2} \\ \quad = \sqrt{49 + 1} \\ \boxed{\overline{AT} = \sqrt{50}} \end{array}$$



Since parallelogram $NATS$ has 2 \cong consecutive sides, it is a rhombus.

Score 4: The student gave a complete and correct response.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral *NATS* is a rhombus.
 [The use of the set of axes below is optional.]

$$m = \frac{\text{rise}}{\text{run}} \quad m_{\overline{NA}} = \frac{1}{1} > \overline{NA} \parallel \overline{ST}$$

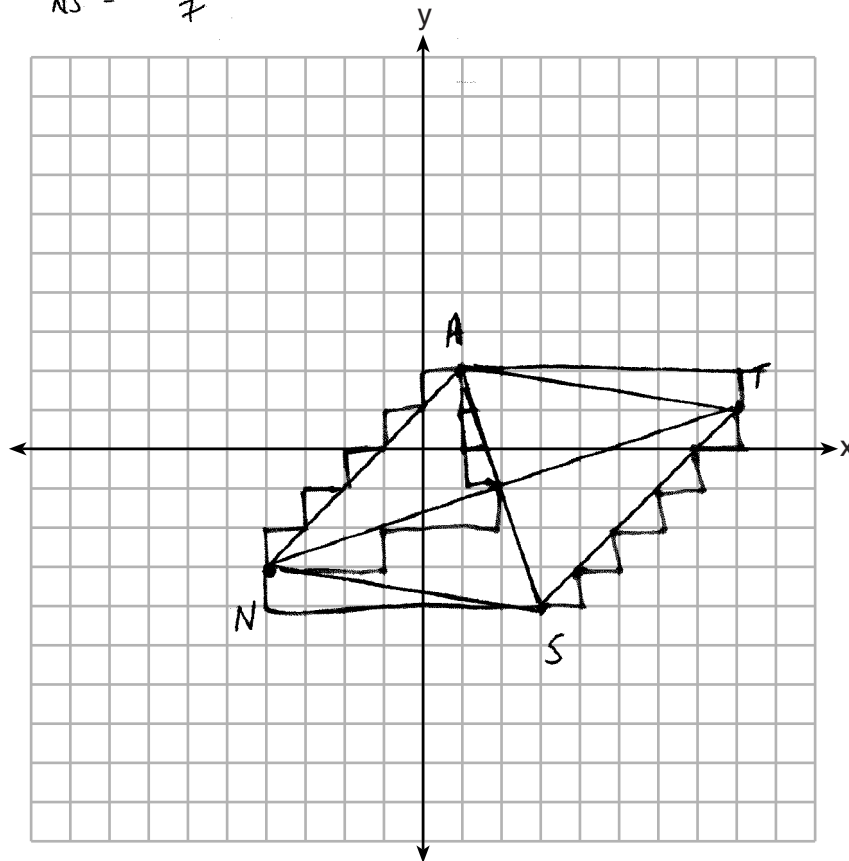
$$m_{\overline{ST}} = \frac{1}{1}$$

$$m_{\overline{AS}} = -\frac{3}{1} \quad m_{\overline{AT}} = -\frac{1}{7} > \overline{AT} \parallel \overline{NS}$$

$$m_{\overline{NT}} = \frac{1}{3} \quad m_{\overline{NS}} = -\frac{1}{7}$$

} \perp

Quadrilateral *NATS* is a ~~rhombus~~ ^{parallelogram} because it has 2 pairs of opposite sides \parallel . It is a rhombus because it has diagonals that are \perp .



Score 4: The student gave a complete and correct response.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral *NATS* is a rhombus.

[The use of the set of axes below is optional.]

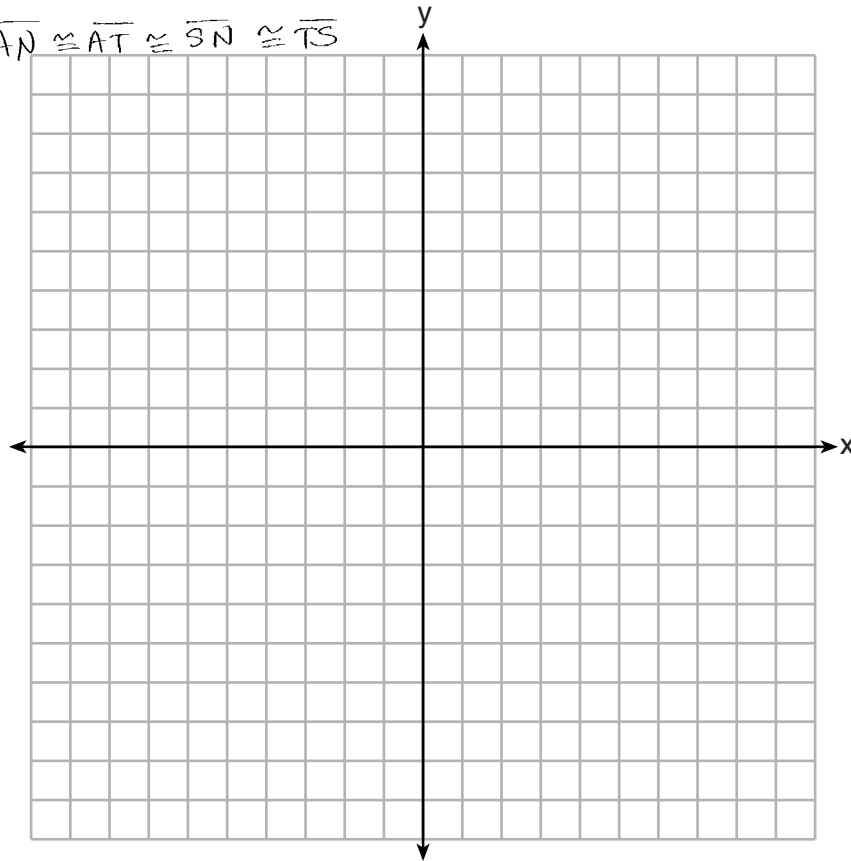
$$\overline{AN} = \sqrt{(1 - -4)^2 + (2 - -3)^2} = \sqrt{25 + 25} = \sqrt{50}$$

$$\overline{AT} = \sqrt{(8 - 1)^2 + (1 - 2)^2} = \sqrt{49 + 1} = \sqrt{50}$$

$$\overline{SN} = \sqrt{(3 - -4)^2 + (-4 - -3)^2} = \sqrt{49 + 1} = \sqrt{50}$$

$$\overline{TS} = \sqrt{(8 - 3)^2 + (1 - -4)^2} = \sqrt{25 + 25} = \sqrt{50}$$

$$\overline{AN} \cong \overline{AT} \cong \overline{SN} \cong \overline{TS}$$



Score 3: The student did not write a concluding statement.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$

Prove quadrilateral *NATS* is a rhombus.
 [The use of the set of axes below is optional.]

$$\begin{aligned} \overline{NA} &\cong \overline{ST} \\ \overline{AT} &\cong \overline{NS} \\ \overline{NA} &\cong \overline{AT} \end{aligned}$$

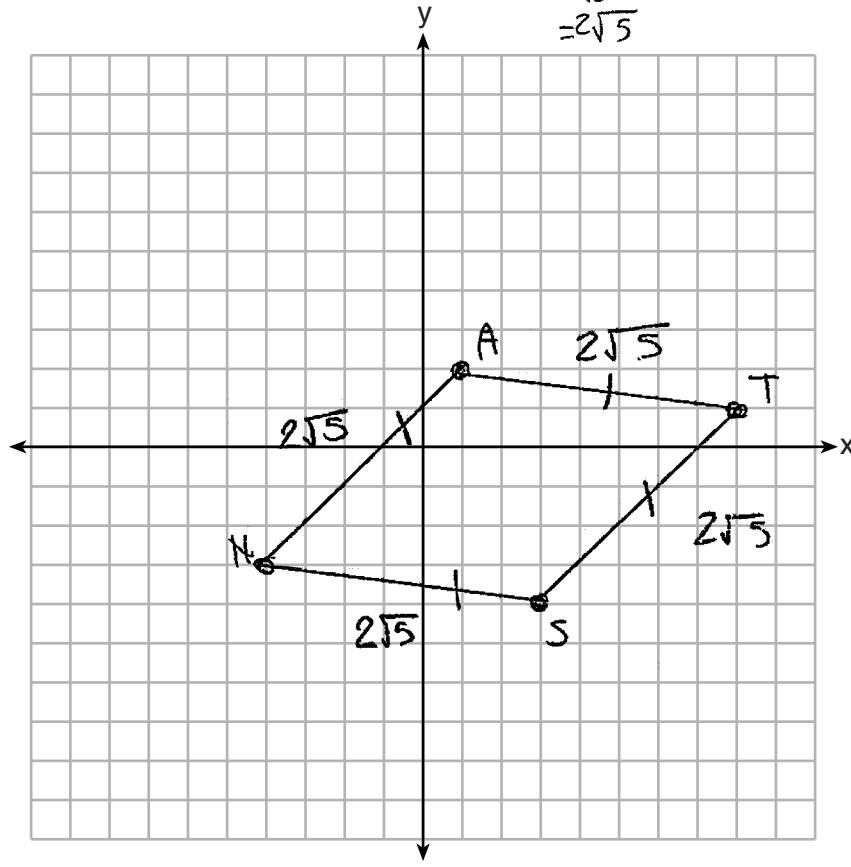
$$\begin{aligned} \overline{NS} &= \sqrt{(-4-1)^2 + (-3-8)^2} \\ &= \sqrt{-5^2 + -8^2} \\ &= \sqrt{25+25} \\ &= \sqrt{50} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{array}{r} \textcircled{5} \overline{)50} \\ \underline{50} \\ 00 \\ \underline{00} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

$$\begin{aligned} \overline{NA} &= \sqrt{(2+3)^2 + (1+4)^2} \\ &= \sqrt{5^2 + 5^2} \\ &= \sqrt{25+25} \\ &= \sqrt{50} \end{aligned}$$

$$\begin{aligned} \overline{AT} &= \sqrt{(1-2)^2 + (8-1)^2} \\ &= \sqrt{-1^2 + 7^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \overline{NS} &= \sqrt{(-4+3)^2 + (-3+4)^2} \\ &= \sqrt{1^2 + 1^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2} \\ &= \sqrt{2} \end{aligned}$$



Score 2: The student made a simplification error and did not write a concluding statement.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4,-3)$, $A(1,2)$, $T(8,1)$, and $S(3,-4)$.

Prove quadrilateral *NATS* is a rhombus.
 [The use of the set of axes below is optional.]

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d_{AS} = \sqrt{(2+4)^2 + (1-3)^2} \quad d_{NT} = \sqrt{(1+3)^2 + (8+4)^2} \quad m_{AS} = \frac{2+4}{1-3} = \frac{6}{-2} = -3$$

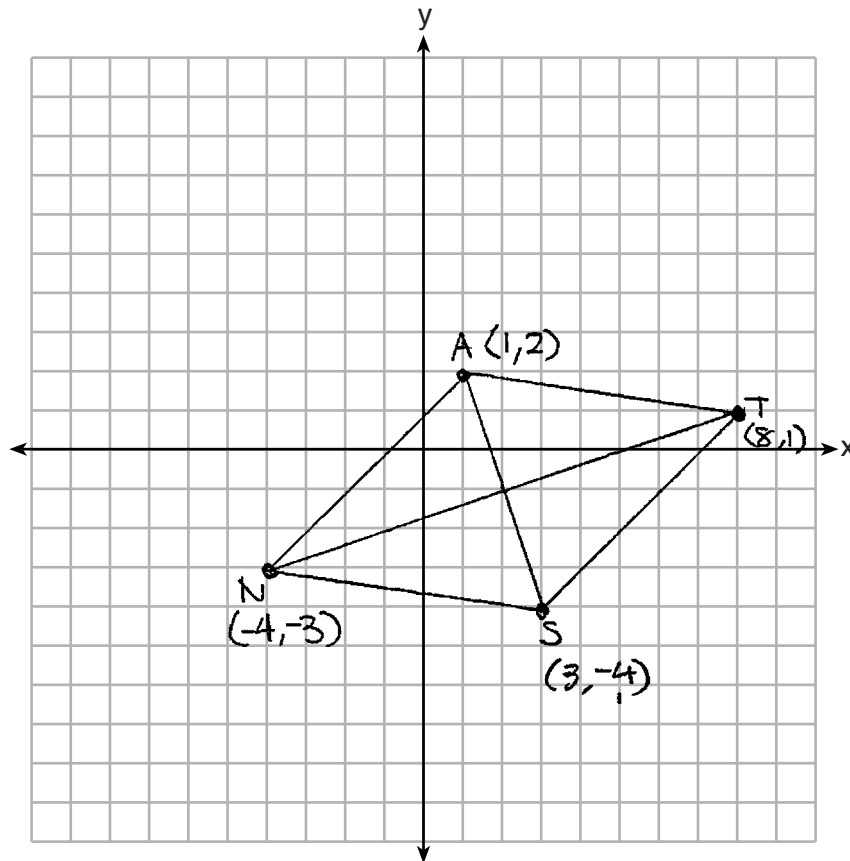
$$d = \sqrt{6^2 + (-2)^2} \quad d = \sqrt{4^2 + 12^2} \quad m_{NT} = \frac{1+3}{8+4} = \frac{4}{12} = \frac{1}{3}$$

$$d = \sqrt{36+4} \quad d = \sqrt{16+144}$$

$$d = \sqrt{40} \quad d = \sqrt{160}$$

$$\left. \begin{array}{l} m_{AS} = -3 \\ m_{NT} = \frac{1}{3} \end{array} \right\} AS \perp NT$$

Quadrilateral *NATS* is a rhombus because the diagonals are perpendicular to each other



Score 2: The student made a conceptual error by concluding that a quadrilateral with perpendicular diagonals is a rhombus.

Question 32

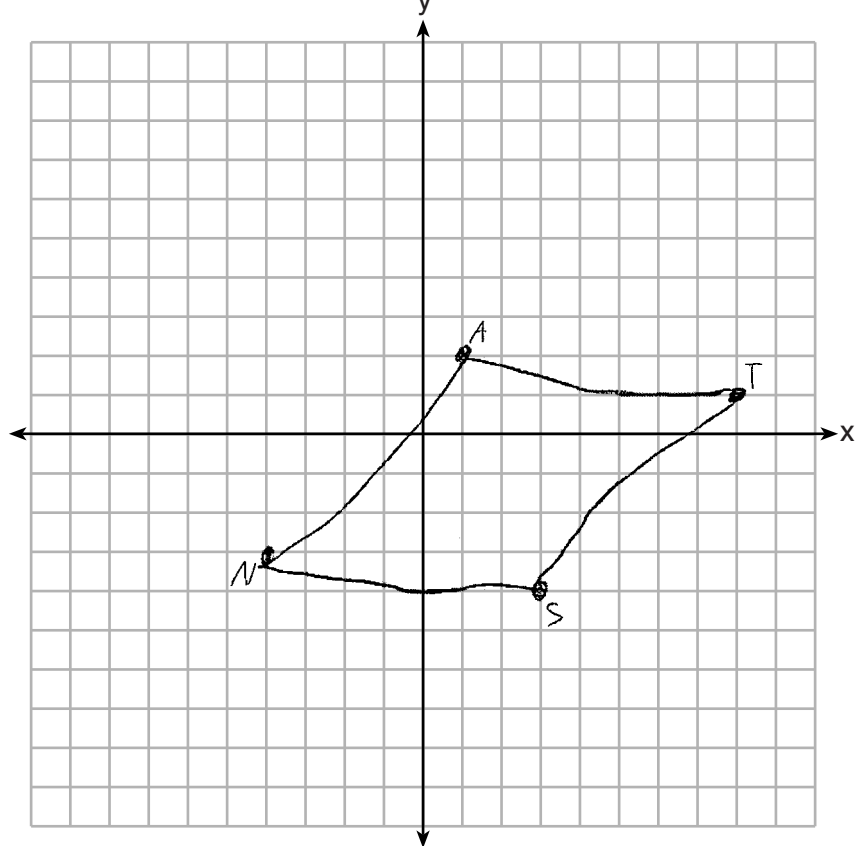
32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral *NATS* is a rhombus.
 [The use of the set of axes below is optional.]

$$\begin{aligned} \text{Slope } NA &= \frac{2 - (-3)}{1 - (-4)} = 1 \\ \text{Slope } TS &= \frac{-4 - 1}{3 - 8} = 1 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Slope } NA \\ \text{Slope } TS \end{aligned}} \right\} \overline{NA} \parallel \overline{TS}$$

yes this is a rhombus because both sides are congruent

$$\begin{aligned} \text{Distance } NA &= \sqrt{(1 - (-4))^2 + (2 - (-3))^2} = \sqrt{50} = 5\sqrt{2} \\ \text{Distance } TS &= \sqrt{(3 - 8)^2 + (-4 - 1)^2} = \sqrt{50} = 5\sqrt{2} \end{aligned}$$



Score 1: The student had correct work to prove *NATS* is a parallelogram, but the concluding statement was incorrect.

Question 32

32 Quadrilateral *NATS* has coordinates $N(-4, -3)$, $A(1, 2)$, $T(8, 1)$, and $S(3, -4)$.

Prove quadrilateral *NATS* is a rhombus.

[The use of the set of axes below is optional.]

step 2: $m = \frac{y_2 - y_1}{x_2 - x_1}$

step 3: $A(1, 2)$ $T(8, 1)$ $m = \frac{1 - 2}{8 - 1} = \frac{-1}{7}$
 $T(8, 1)$ $S(3, -4)$ $m = \frac{-4 - 1}{3 - 8} = \frac{-5}{-5} = 1$
 $S(3, -4)$ $N(-4, 3)$ $m = \frac{3 - (-4)}{-4 - 3} = \frac{7}{-7} = -1$

$m = \frac{3 - (-4)}{-4 - 3} = \frac{7}{-7} = -1$

$N(-4, 3)$ $A(1, 2)$ $m = \frac{2 - 3}{1 - (-4)} = \frac{-1}{5} = -\frac{1}{5}$

Step one

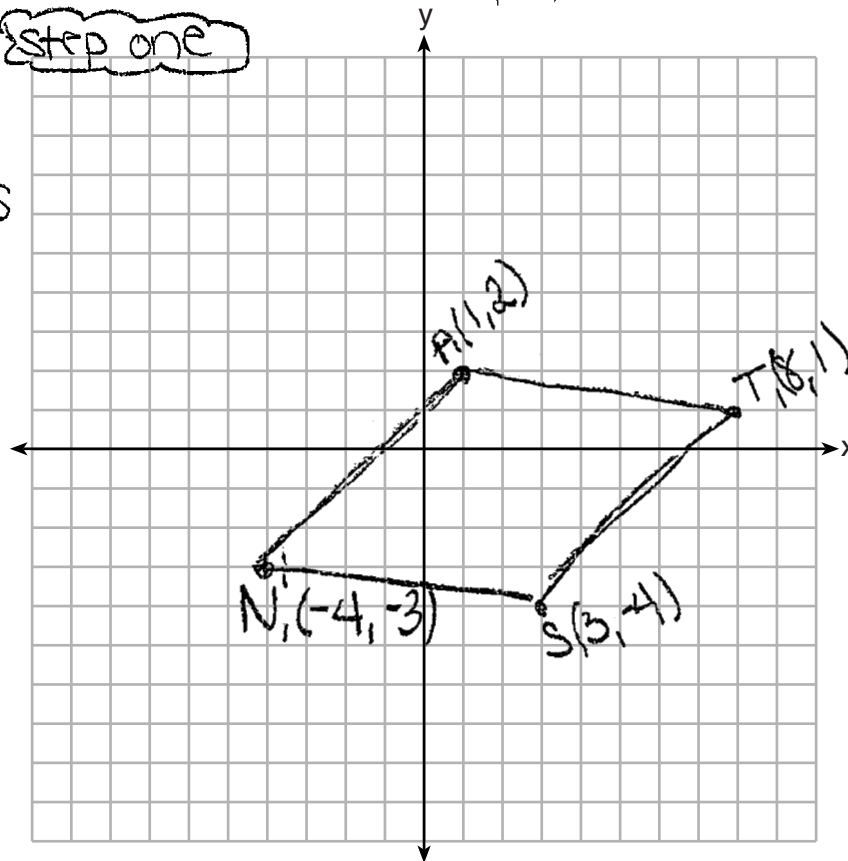
Step 4:
quad *NATS*
is not a
rhombus

$A \not\cong T$

$T \cong S$

$S \cong N$

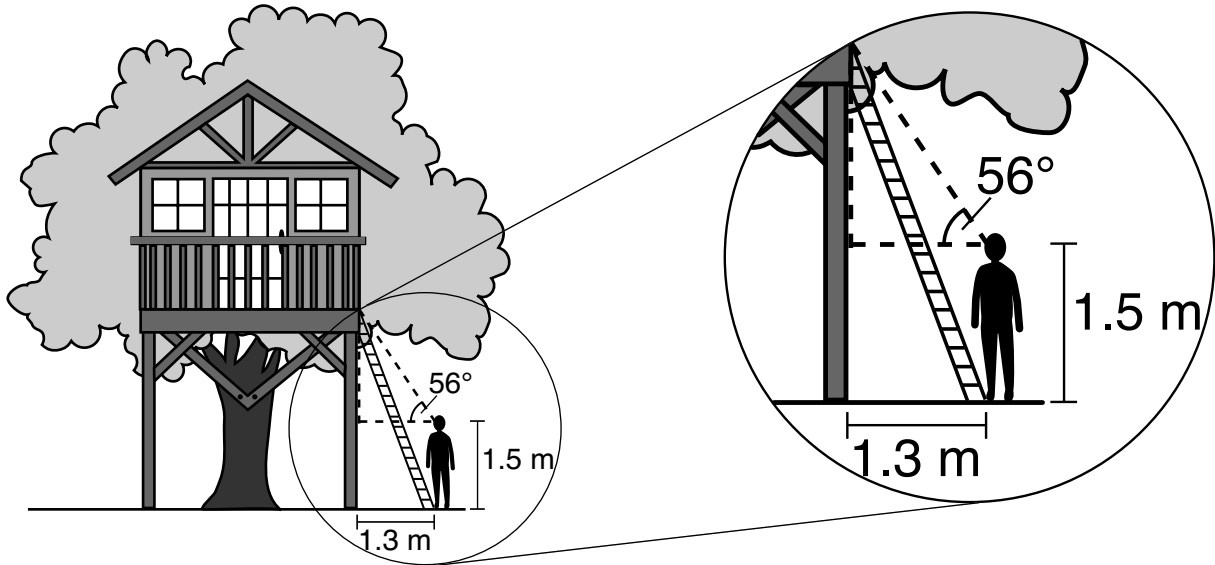
$N \cong A$



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

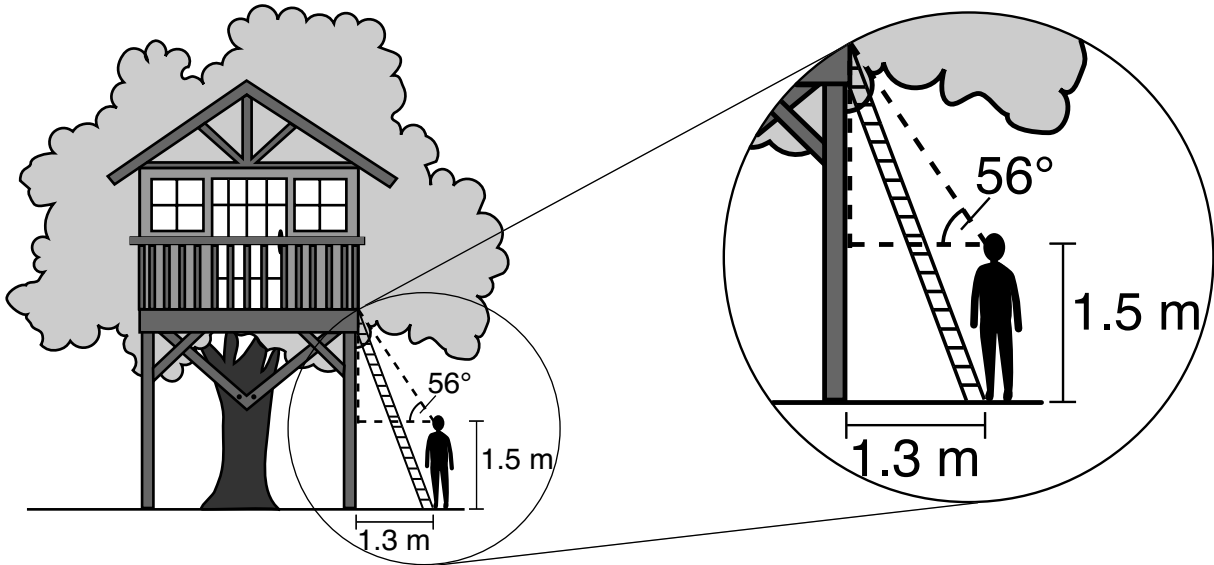
$$\begin{array}{l}
 \begin{array}{c} x \\ \triangle \\ 1.3 \end{array} \quad \begin{array}{l} 1.3 \\ \tan 56 = \frac{x}{1.3} \\ x = 1.3 \tan 56 \\ x = 1.927329 \\ \hline + 1.5 \\ \text{Stilt} = 3.427329 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \begin{array}{c} y \\ \triangle \\ 1.3 \end{array} \quad \begin{array}{l} \tan \theta = \frac{3.427329}{1.3} \\ \theta = \tan^{-1}(3.427329/1.3) \\ \theta = 69.228395 \\ \cos \theta = \frac{1.3}{y} \\ y = 3.665 \\ \text{3.7} \end{array}
 \end{array}$$

Score 4: The student gave a complete and correct response.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

$\tan 56 = \frac{x}{1.3}$
 $x = 1.3(\tan 56)$
 $x = 1.927329259$

$$(3.427329259)^2 + (1.3)^2 = c^2$$

$$11.74658585 + 1.69 = c^2$$

$$13.43658585 = c^2$$

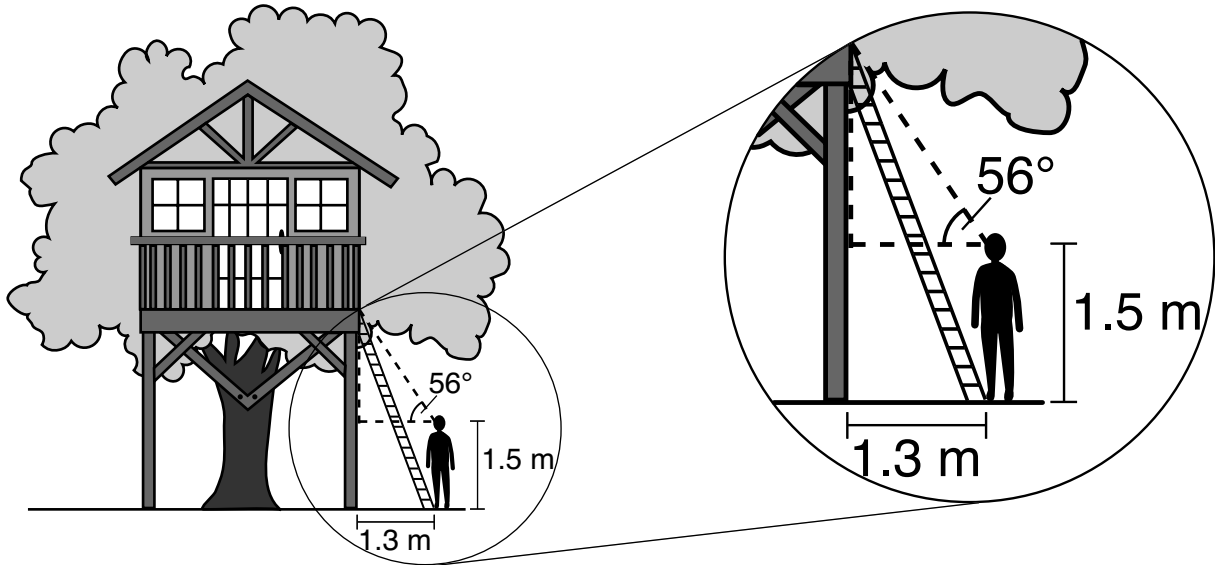
$$3.66594884 = c$$

3.7 meters

Score 4: The student gave a complete and correct response.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

$\tan 56 = \frac{x}{1.3}$
 $1.3 \tan 56 = x$
 $1.927329259 = x$
 ≈ 1.93

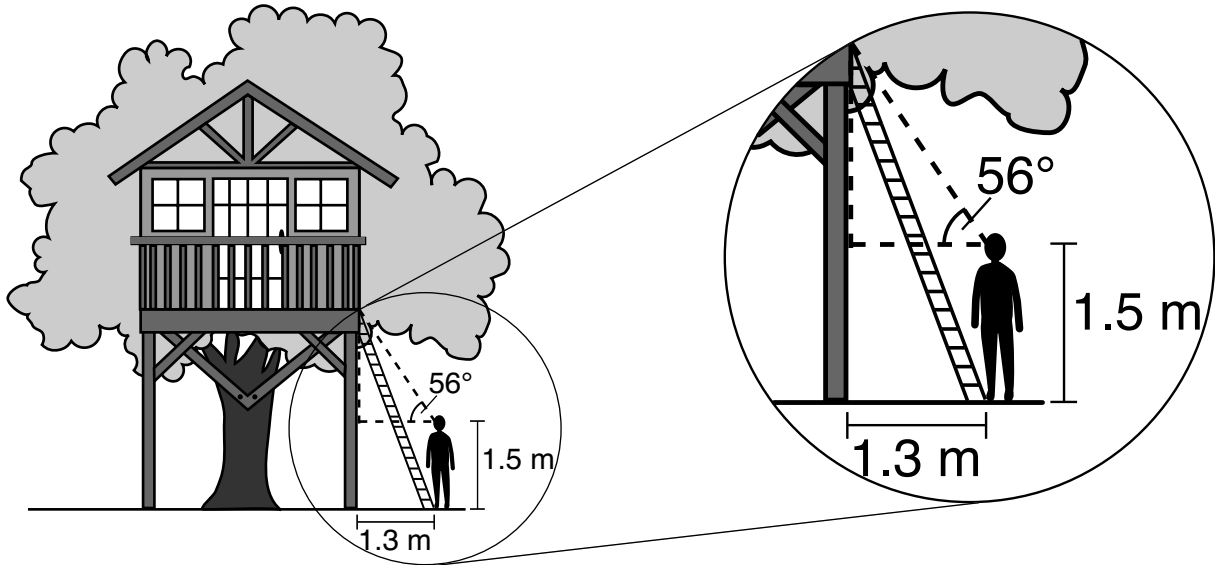
$+1.93$
 1.5
3.43
 round up

$3.43^2 + 1.3^2 = 12$
 $11.7649 + 1.69 = \sqrt{13.4549}$
 $3.668092147 = \sqrt{13.4549}$
3.5 m

Score 3: The student made one rounding error when finding the length of the ladder.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

Handwritten student work:

$$\begin{array}{r} 1.927329259 \\ + 1.5 \\ \hline 3.427329259 \end{array}$$

$$1.3 \cdot \tan 56 = \frac{x}{1.3} \cdot 1.3$$

$$1.3 \tan 56 = x$$

$$x = 1.927329259 \text{ m} + 1.5 \text{ m}$$

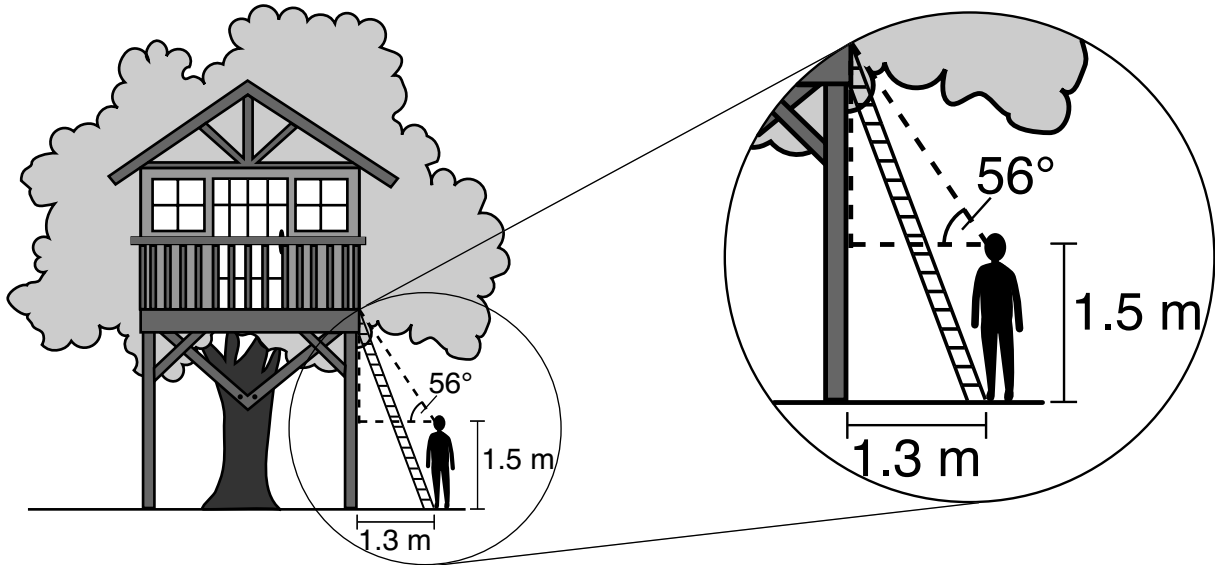
$$=$$

he would need to buy at least a 3.4 meter tall ladder to reach his treehouse.

Score 3: The student found the length of the stilt, but no further correct work was shown.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

$$\tan 56 = \frac{x}{1.3}$$

$$x = 1.3 \cdot \tan 56$$

$$x = 1.9273295907$$

Minimum length =
1.9 meters

$$\tan 56 = \frac{0.42732925907}{1}$$

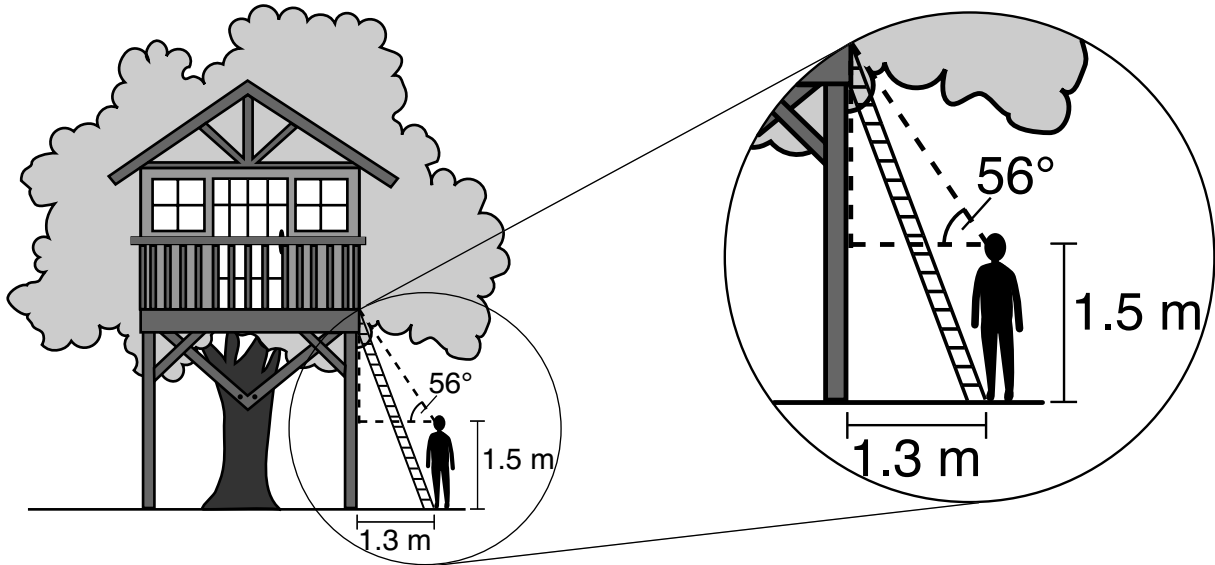
$$0.427... = y \cdot \tan 56$$

$$y = 0.288257224736$$

Score 2: The student found the altitude from the sight line to the top of the ladder, but no further correct work was shown.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.



Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

$$\frac{\tan(56)}{1} = \frac{x}{1.3}$$

$$x = \tan(56)$$

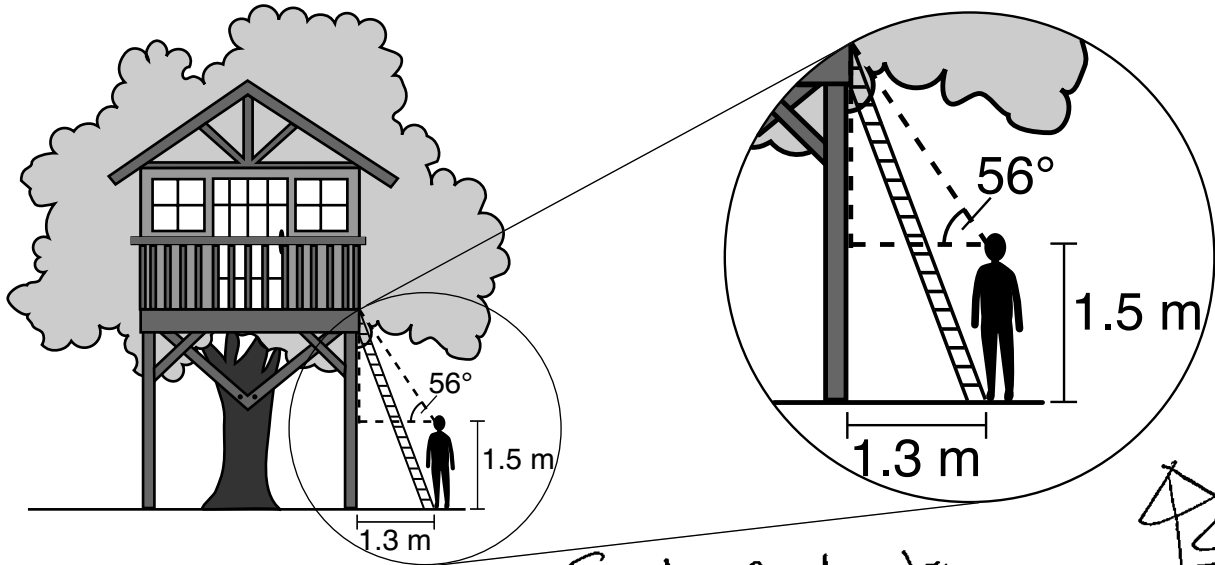
$$x \approx 3.2$$

The minimum length of ladder David will need to buy is 3.2 meters of ladder.

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work was shown.

Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.

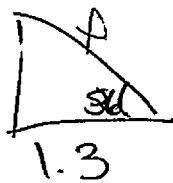


Soh cah toa



Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

Add 1.5
at END!!



$$\cos 56 = \frac{x}{1.3} \cdot 1.3$$

$$x = .72695$$

$$+ 1.5$$

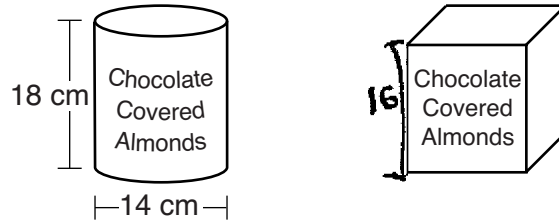
$$2.22695$$

the minimum length
of a ladder david will
have to buy is
2.2 meters

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

$$\pi(49)(18) = 16x^2$$

$$882\pi = 16x^2$$

$$55.125\pi = x^2$$

$$13.1597 = x$$

$$13.2 = x$$

cm

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

~~$30(60) = 41800 \text{ cm}^2$~~

$$13.2 \cdot 4 = 52.8$$

41 per shelf
6 shelves

60 ||||
80

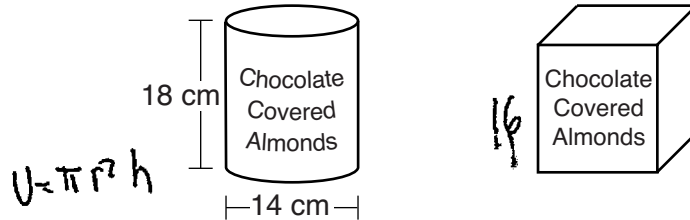
$$\frac{80}{13.2} \approx 6$$

24 New Containers

Score 4: The student gave a complete and correct response.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

Handwritten work for the volume calculation:

Cylinder
 $V = \pi (7)^2 (18)$
 $V = 2770.88472$

Rectangular Prism
 $V = Bh$
 $2770.88472 = (16)x^2$
 $173.2 = x^2$

The final answer is boxed as 13.1.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Handwritten work for the shelf fitting problem:

$\frac{80}{13.1} = 6$

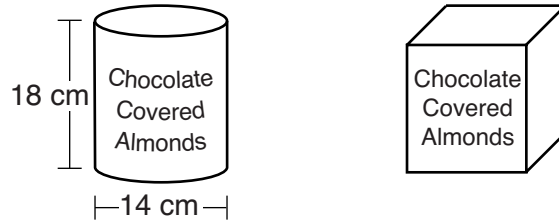
$\frac{60}{13.1} = 4$

The final answer is boxed as 24.

Score 3: The student made one rounding error in determining the side length of the container.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

$V = \pi r^2 h$
 $\pi (7)^2 (18)$
 $2770.88\dots$

$x^2(16) = 2770.8847$
 $\frac{x^2(16)}{16} = \frac{2770.8847}{16}$
 $x^2 = \sqrt{173.1802}$
 $x = 13.2$

The side will be 13.2 centimeter

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

80
 4800
 60

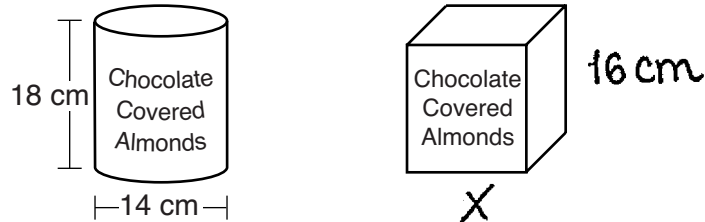
$13.2 \cdot 13.2 =$
 174.24 Square Base
 $4800 \div 174.24 = 27.54$

27 Containers

Score 3: The student found the side length of 13.2, but no further correct work was shown.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

Old Container

$$V = \pi r^2 h$$

$$V = \pi 7^2 \cdot 18$$

$$V = \pi 49 \cdot 18$$

$$V = \pi 882$$

$$V = 882\pi \text{ cm}^3$$

New Container

$x = \text{side length of container}$

$$\frac{882\pi}{16} = \frac{(x^2) 16}{16}$$

$$\sqrt{x^2} = \sqrt{173.180295\dots}$$

$$x \approx 13 \text{ cm}$$

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

$V = 16 \cdot 60 \cdot 80$ shelf

$V = 76,800 \text{ cm}^3$ container

$$V = 16 \cdot 13 \cdot 13 = 2,704 \text{ cm}^3$$

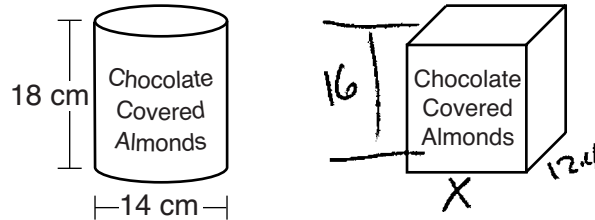
$$\frac{76,800 \text{ cm}^3}{2,704 \text{ cm}^3} \approx 28$$

28 containers

Score 2: The student made a rounding error in finding the side length of the new container. No further correct work was shown.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

$$\frac{16}{18} = \frac{x}{14}$$

$$\frac{18x}{18} = \frac{224}{18}$$

$$x = 12.4 \text{ cm}$$

The side length of the container would be 12.4 cm.

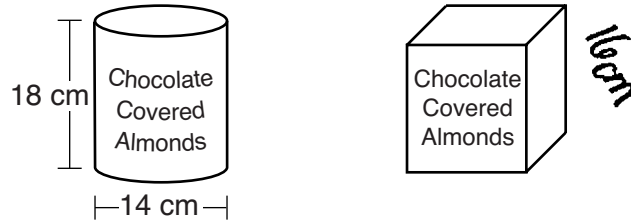
A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

The diagram shows a rectangular shelf with a length of 80 cm and a width of 60 cm. Handwritten calculations show: $60 \div 12.4 = 4.8$, $80 \div 12.4 = 6.5$, and $6(4) = 24$. A handwritten note states: 'The shelf can hold 24 containers of chocolate covered almonds.'

Score 1: The student had a completely incorrect response to find the side length of the new container. The student used the incorrect side length to find an appropriate number of new containers.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

$$V_C = \pi r^2 h$$

$$V_C = \pi (7)^2 (18)$$

$$V_C = 2770.88472 \text{ cm}^3$$

$$V_B = Bh$$

$$2770.88472 = x(16)$$

$$\frac{2770.88472}{16} = \frac{16x}{16}$$

$$x = \frac{173.180295}{2}$$

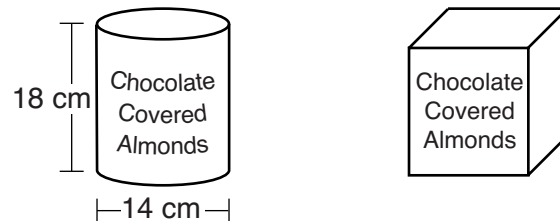
$$x = 86.6 \text{ cm}$$

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 1: The student correctly found the volume of the cylinder, but no further correct work was shown.

Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.



If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

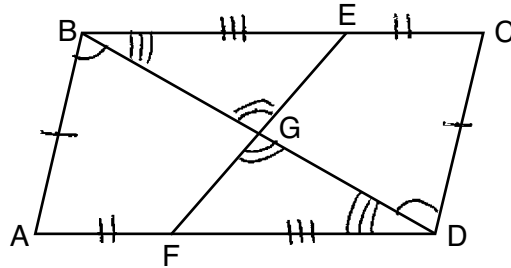
$$V = \pi r^2 h$$
$$V = \pi 14^2 \cdot 18$$
$$V = 11,083.5$$

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 0: The student gave a completely incorrect response.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



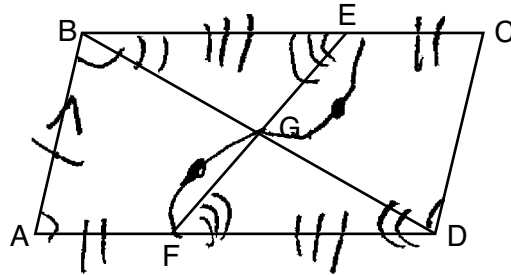
Prove: $\overline{FG} \cong \overline{EG}$

Statement	Reason
① Quad $ABCD$, E and F are points on \overline{BC} and \overline{AD} . and \overline{BGD} and \overline{EGF} are drawn. $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$	① Given
② $\overline{BD} \cong \overline{BD}$	② Reflexive property.
③ $\triangle ABD \cong \triangle CDB$	③ SAS
④ $\overline{BC} \cong \overline{DA}$	④ CPCTC
⑤ $\overline{BE} + \overline{CE} \cong \overline{AF} + \overline{DF}$	⑤ segment addition post.
⑥ $\overline{BE} \cong \overline{DF}$	⑥ subtraction property
⑦ $\angle BGE \cong \angle DGF$	⑦ vertical \angle 's are \cong
⑧ $\angle CBD \cong \angle ADB$	⑧ CPCTC
⑨ $\triangle EBG \cong \triangle FDG$	⑨ AAS
⑩ $\overline{FG} \cong \overline{EG}$	⑩ CPCTC

Score 6: The student gave a complete and correct response.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



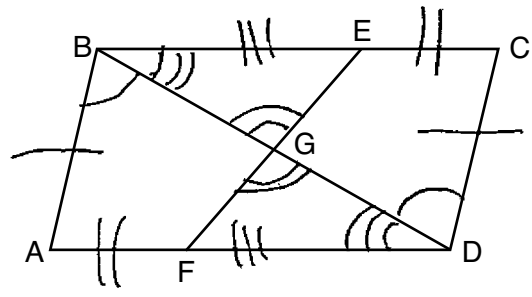
Prove: $\overline{FG} \cong \overline{EG}$

Statements	Reasons
① quad $ABCD$, $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, $\overline{CE} \cong \overline{AF}$	① Given
② $\overline{AB} \parallel \overline{CD}$	② If alt. int. \angle s \cong , then lines are \parallel .
③ $ABCD$ is a \square	③ A quad with a pair of opp. sides that are \cong and \parallel is a \square .
④ $\angle A \cong \angle C$	④ opp. \angle s of a \square are \cong .
⑤ $\triangle ABD \cong \triangle DCB$	⑤ ASA \cong ASA
⑥ $\angle BEG \cong \angle DFG$, $\angle BEG \cong \angle DFG$	⑥ If lines \parallel , ^{then} alt. int. \angle s \cong .
⑦ $\overline{BC} \cong \overline{AD}$	⑦ CPCTC
⑧ $\overline{AD} - \overline{AF} \cong \overline{BC} - \overline{EC}$ $\overline{DF} \cong \overline{BE}$	⑧ Subtraction post.
⑨ $\triangle BEG \cong \triangle DFG$	⑨ ASA \cong ASA
⑩ $\overline{FG} \cong \overline{EG}$	⑩ CPCTC

Score 5: The student did not prove $\overline{AD} \parallel \overline{BC}$ to prove step 6.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



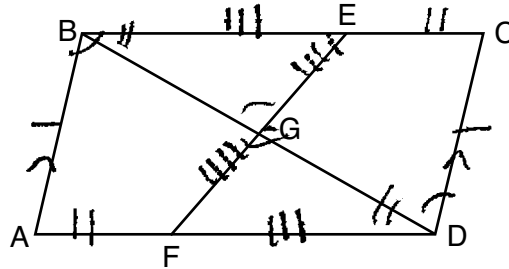
Prove: $\overline{FG} \cong \overline{EG}$

Statement	Reason
① In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} . \overline{EG} and \overline{FG} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.	① Given
② $\overline{AB} \parallel \overline{CD}$	② If 2 lines are cut by a transversal so that alternate interior angles are \cong , the lines are parallel.
③ Quadrilateral $ABCD$ is a parallelogram	③ If one pair of opposite sides in a quadrilateral are \cong and parallel, the it is a parallelogram.
④ $\angle BEG \cong \angle GDF$	④ Vertical \angle s are \cong .
⑤ $\overline{BC} \parallel \overline{AD}$	⑤ Opposite sides in a parallelogram are parallel.
⑥ $\angle EBG \cong \angle GDF$	⑥ If 2 parallel lines are cut by a transversal, alternate interior angles are congruent.
⑦ $\overline{AD} \cong \overline{BC}$	⑦ Opposite sides in a parallelogram are congruent.
⑧ $\overline{AD} = \overline{AF} + \overline{FD}$ $\overline{BC} = \overline{BE} + \overline{EC}$	⑧ A segment = the sum of its parts.
⑨ $\overline{BE} + \overline{EC} = \overline{AF} + \overline{FD}$	⑨ Substitution property of equality.
⑩ $\overline{BE} \cong \overline{FD}$	⑩ Subtraction property of equality.
⑪ $\triangle BEG \cong \triangle DFG$	⑪ AAS \cong AAS
⑫ $\overline{FG} \cong \overline{EG}$	⑫ Corresponding parts of congruent triangles are congruent.

Score 5: The student incorrectly named the vertical angles in step 4.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



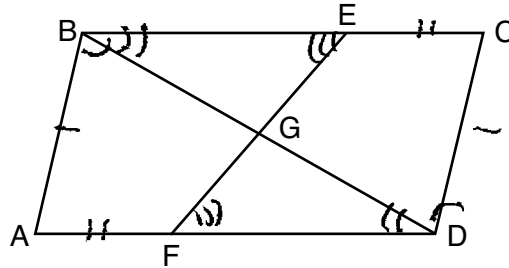
Prove: $\overline{FG} \cong \overline{EG}$

Statement	Reason
1) Quad $ABCD$ $\angle ABG \cong \angle CDG$ $\overline{AB} \cong \overline{CD}$ $\overline{CE} \cong \overline{AF}$	1) Given
2) $\angle BGE \cong \angle DGF$	2) vertical \angle 's are \cong
3) $\overline{BA} \parallel \overline{CD}$	3) lines are \parallel iff they have \cong alt int \angle 's
4) $ABCD$ is a parallelogram	4) $ABCD$ has a pair of sides that are \parallel and \cong
5) $\angle EBG \cong \angle GDF$	5) alt int \angle 's are \cong if lines are \parallel
6) $\overline{BE} \cong \overline{FD}$	6) In a parallelogram opposite sides are \cong and $\overline{EC} \cong \overline{AF}$ so part + part = whole
7) $\triangle BGE \cong \triangle DGF$	7) AAS Thm \cong
8) $\overline{FG} \cong \overline{GE}$	8) CPCTC

Score 4: The student did not prove $\overline{AD} \parallel \overline{BC}$ to prove step 5 and did not show subtraction to prove step 6.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



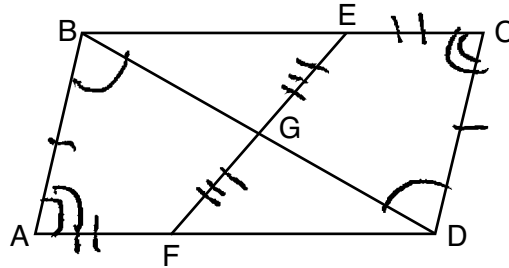
Prove: $\overline{FG} \cong \overline{EG}$

Statement	Reason
1) In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$	1) Given
2) $\angle BEG \cong \angle DFG$, $\angle BGE \cong \angle DGF$	2) parallel lines cut by a transversal form congruent opposite interior angles
3) $\overline{BC} \cong \overline{AD}$	3) opposite sides of a parallelogram are congruent
4) $\overline{BC} \cong \overline{BE} + \overline{EC}$ $\overline{AD} \cong \overline{DF} + \overline{AF}$	4) a segment is congruent to the sum of its parts
5) $\overline{BC} - \overline{EC} \cong \overline{AD} - \overline{AF}$ $\overline{BE} \cong \overline{DF}$	5) subtraction
6) $\triangle FGD \cong \triangle BGE$	6) ASA \cong ASA
7) $\overline{FG} \cong \overline{EG}$	7) CPCTC

Score 3: The student made one conceptual error by not proving $ABCD$ is a parallelogram. The student did not prove $\overline{AD} \parallel \overline{BC}$ to prove step 2.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



Prove: $\overline{FG} \cong \overline{EG}$

① quadrilateral $ABCD$, E + F are points on \overline{BC} and \overline{AD} , \overline{BGD} and \overline{EGF}
 $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, $\overline{CE} \cong \overline{AF}$

② $\overline{AB} \parallel \overline{CD}$

③ parallelogram $ABCD$

④ $\overline{AFD} \cong \overline{CEB}$

⑤ $\angle A \cong \angle C$

⑥ $\triangle BAD \cong \triangle DCB$

⑦ $\overline{FG} \cong \overline{EG}$

① Given

② parallel lines \rightarrow alternate interior angles congruent

③ parallelogram \rightarrow opposite sides parallel and congruent

④ parallelogram have opposite sides congruent

⑤ parallelogram have opposite angles \cong

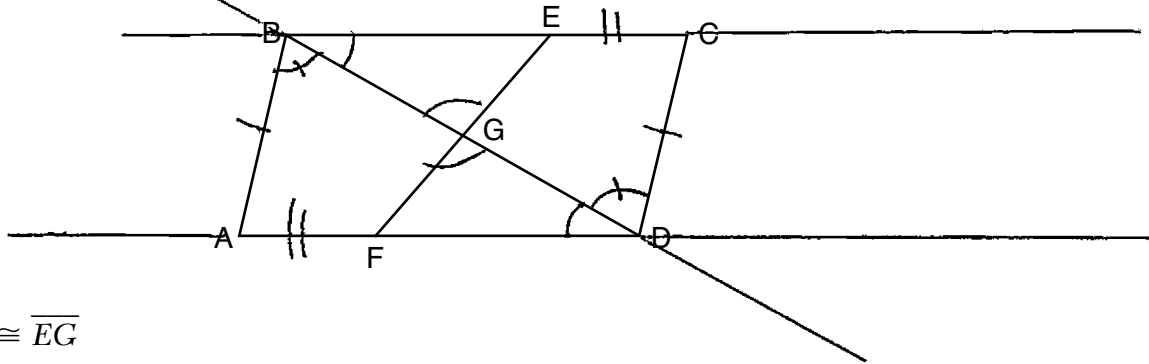
⑥ SAS

⑦ corresponding parts of congruent triangles are congruent

Score 2: The student proved $ABCD$ is a parallelogram and $\triangle ABD \cong \triangle CDB$.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



Prove: $\overline{FG} \cong \overline{EG}$

Statements

- ① quadrilateral $ABCD$,
 E and F are points on \overline{BC} and \overline{AD} ,
 respectively, \overline{BGD} and \overline{EGF} are
 drawn such that $\angle ABG \cong \angle CDG$,
 $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$
- ② $\overline{BC} \parallel \overline{AD}$
- ③ $\angle GBE \cong \angle GDF$
- ④ $\angle BGE \cong \angle DGF$
- ⑤ $\triangle FGD \sim \triangle EGB$
- ⑥ $\overline{BE} \cong \overline{FD}$
- ⑦ $\overline{FG} \cong \overline{EG}$

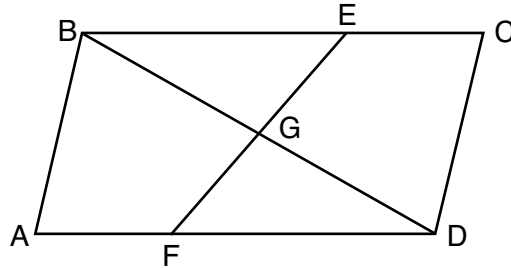
Reasons

- ① Given
- ② A quadrilateral with two congruent
 sides has \parallel lines
- ③ 2 \parallel lines cut by a transversal forms
 Congruent \angle s-
- ④ vertical \angle 's are \cong
- ⑤ AA similarity theorem
- ⑥ CPCTC
- ⑦ G is the md. pt. of \overline{FE}

Score 1: The student had only one correct relevant statement and reason in step 4.

Question 35

35 In quadrilateral $ABCD$, E and F are points on \overline{BC} and \overline{AD} , respectively, and \overline{BGD} and \overline{EGF} are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



Prove: $\overline{FG} \cong \overline{EG}$

Statements

Reasons

1. $\angle ABG \cong \angle CDG$
 $\overline{AB} \cong \overline{CD}$, $\overline{CE} \cong \overline{AF}$

2. $ABCD$ is a parallelogram

3. $\overline{FG} \cong \overline{EG}$

1. Given

2. opp. sides \cong

3. parallelogram's diagonals bisect each other

Score 0: The student gave a completely incorrect response.