The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 22, 2020 — 9:15 a.m. to 12:15 p.m.

MODEL RESPONSE SET

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25 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

Write a set of three congruency statements that would show ASA congruency for these triangles.

\[
\angle Q \cong \angle M \\
\angle P \cong \angle N \\
\overline{QP} \cong \overline{MN}
\]

**Score 2:** The student gave a complete and correct response.
25 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

Write a set of three congruency statements that would show ASA congruency for these triangles.

$$\angle Q \cong \angle M$$
$$QR \cong ML$$
$$LR \cong L$$

Score 2: The student gave a complete and correct response.
25 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

Write a set of three congruency statements that would show $ASA$ congruency for these triangles.

1. $\angle Q \cong \angle M$
2. $\overline{PR} \cong \overline{NL}$
3. $\angle P \cong \angle N$

**Score 1:** The student wrote congruency statements for $AAS$. 

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Geometry – Jan. '20
25 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

Write a set of three congruency statements that would show ASA congruency for these triangles.

1. $\angle Q \cong \angle M$ because they are both right angles and all right angles are congruent (Angle)
2. Since $\triangle PQR$ is formed by a rigid motion of $\triangle NML$ and rigid motions preserve shape and size, then $\overline{QP} \cong \overline{MN}$, $\overline{QR} \cong \overline{ML}$, $\overline{RP} \cong \overline{LN}$, $\angle QRP \cong \angle LNM$, $\angle QPR \cong \angle MNL$
3. By ASA $\triangle PQR \cong \triangle NML$

Score 1: The student wrote all corresponding congruency statements, but did not specify which congruencies were for ASA.
25 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

Write a set of three congruency statements that would show $ASA$ congruency for these triangles.

\[ \angle R \cong \angle N \]
\[ \angle L \cong \angle P \]
\[ \overline{RP} \cong \overline{LN} \]

**Score 0:** The student stated only one correct corresponding congruency statement, $\overline{RP} \cong \overline{LN}$. 
26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of $38^\circ$ with the vertical post. Determine and state the approximate length of the support beam, $x$, to the nearest inch.

\[
\sin 38^\circ = \frac{24\frac{1}{2}}{x}
\]

\[x = 39.79459651\]

\[x \approx 40 \text{ inches}\]

**Score 2:** The student gave a complete and correct response.
Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of $38^\circ$ with the vertical post. Determine and state the approximate length of the support beam, $x$, to the nearest inch.

\[
\sin 38^\circ = \frac{24.5}{x}
\]

So,

\[
24.5 = \sin 38^\circ \cdot x
\]

\[
x \approx 82.667
\]

Score 1: The student made an error by using $38^\circ$ as a radian measure.
Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of $38^\circ$ with the vertical post. Determine and state the approximate length of the support beam, $x$, to the nearest inch.

\[
\frac{\tan(38^\circ)}{} = \frac{24.5}{x}
\]

\[
\frac{\tan(38^\circ)}{} \cdot x = \frac{24.5}{\tan(38^\circ)}
\]

\[
x = 31.35 \approx 31
\]

**Score 1:** The student used an incorrect trigonometric equation, but solved it correctly.
Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of $38^\circ$ with the vertical post. Determine and state the approximate length of the support beam, $x$, to the nearest inch.

\[ A^2 + B^2 = C^2 \]
\[ 38^2 + 24^2 = C^2 \]
\[ 1444 + 576 = C^2 \]
\[ C \approx 44.94 \]

**Score 0:** The student gave a completely incorrect response.
Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

\[ \frac{43 \text{ lb}}{\text{ft}^3} \quad V = lwh \]

\[ 8 \times 3 \times \frac{1}{12} = z \]

\[ 43 \times 2 = 86 \text{ pounds} \]

Score 2: The student gave a complete and correct response.
27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

Score 2: The student gave a complete and correct response.
A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

\[ V = Bh \]
\[ V = 96 \text{ (I)} \]
\[ V = 96 (36) \]
\[ V = 3456 \text{ in}^3 \]
\[ V = 288 \text{ ft}^3 \]

**Score 1:** The student made an error when converting 3456 cubic inches to 288 cubic feet.
27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

\[ V = l \cdot w \cdot h \]

\[ V = 8 \cdot 1 \cdot 3 \]

\[ 24 \text{ feet}^3 \cdot 43 \text{ pounds} \]

\[ 1032 \text{ pounds} \]

**Score 1:** The student did not convert the 1-inch thickness to feet.
Question 27

27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

\[ 1 \text{ inch} = \frac{1}{12} \text{ foot} = 0.083 \]

\[ 8 \times 3 \times 0.083 = 1.992 \]

\[ \times 43 \]

\[ 85.656 \]

Score 1: The student made a rounding error when stating the weight of the table.
27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

\[ V = l \times w \times h \]

\[ V = 8 \times 3 \times \frac{1}{12} \]

\[ V = 2 \, \text{cubic feet} \]

\[ W = 43 \, \text{pounds per cubic foot} \]

\[ W_t = 43 \times 2 = 86 \, \text{pounds} \]

Score 0: The student gave a completely incorrect response.
28 In the diagram below of circle $O$, secant $\overline{ABC}$ and tangent $\overline{AD}$ are drawn.

If $CA = 12.5$ and $CB = 4.5$, determine and state the length of $\overline{DA}$.

\[
\overline{AB} = 8, \quad \overline{AC} = 12.5, \quad \overline{CB} = 4.5
\]

\[
\frac{\overline{CA}}{\overline{CB}} = \frac{12.5}{4.5} = \frac{12.5 - 4.5}{4.5} = \frac{8}{4.5}
\]

\[
\overline{AB} \cdot \overline{AC} = \overline{AD} \cdot \overline{AD}
\]

\[
(8)(12.5) = x \cdot x = 100
\]

\[
x = 10
\]

$\overline{DA} = 10$

**Score 2:** The student gave a complete and correct response.
28 In the diagram below of circle $O$, secant $ABC$ and tangent $AD$ are drawn.

If $CA = 12.5$ and $CB = 4.5$, determine and state the length of $DA$.

\[ DA = 7.5 \]

\[ x^2 = 12.5 \times 4.5 \]

\[ \sqrt{x^2} = \sqrt{56.25} \]

\[ x = 7.5 \]

Score 1: The student made an error by using 4.5 instead of 8.
Question 28

28 In the diagram below of circle $O$, secant $ABC$ and tangent $AD$ are drawn.

If $CA = 12.5$ and $CB = 4.5$, determine and state the length of $DA$.

Score 0: The student made an error with $AC = 12.5 + 4.5$ and made a rounding error to find $DA$. 
Given $MT$ below, use a compass and straightedge to construct a 45° angle whose vertex is at point $M$.[Leave all construction marks.]

Score 2: The student gave a complete and correct response.
29 Given $MT$ below, use a compass and straightedge to construct a $45^\circ$ angle whose vertex is at point $M$.
[Leave all construction marks.]

Score 2: The student gave a complete and correct response.
29 Given $\overline{MT}$ below, use a compass and straightedge to construct a 45° angle whose vertex is at point $M$.
[Leave all construction marks.]

Score 2: The student gave a complete and correct response. The student constructed a 60° angle using an equilateral triangle and then bisected that 60° angle to get a 30° angle. Lastly, the student bisected a 30° angle to combine the other 30° angle with the 15° angle to get a 45° angle.
29 Given $MT$ below, use a compass and straightedge to construct a $45^\circ$ angle whose vertex is at point $M$.
[Leave all construction marks.]

**Score 1:** The student did not construct the line perpendicular to $MT$ through $M$, but correctly bisected the angle.
29 Given $MT$ below, use a compass and straightedge to construct a $45^\circ$ angle whose vertex is at point $M$.
[Leave all construction marks.]

Score 0: The student gave a completely incorrect response.
30 In \( \triangle XYZ \) shown below, medians \( \overline{XE}, \overline{YF}, \) and \( \overline{ZD} \) intersect at \( C \).

If \( CE = 5 \), \( YF = 21 \), and \( XZ = 15 \), determine and state the perimeter of triangle \( CFX \).

\[
\begin{align*}
7 + 7.5 + 10 &= 24.5 \\
\triangle CFX \text{perimeter} &= 24.5
\end{align*}
\]

\[
\begin{align*}
\sqrt{3} &= 7 \\
5 \cdot 2 &= 10 \\
15 / 2 &= 7.5
\end{align*}
\]

Score 2: The student gave a complete and correct response.
30 In \( \triangle XYZ \) shown below, medians \( 
oline{XE}, \oline{YF}, \) and \( \oline{ZD} \) intersect at \( C \).

If \( CE = 5, YF = 21, \) and \( XZ = 15 \), determine and state the perimeter of triangle \( CFX \).

\[
\begin{align*}
15 \div 2 &= 7.5 \\
21 \times \frac{1}{3} &= 7 \\
5 \div \frac{1}{3} &= 15 \\
15 + 7.5 + 7 &= 29.5 \\
\text{Perimeter} &= 29.5 \text{ units}
\end{align*}
\]

Score 1: The student made an error in determining \( XC \).
Question 30

30 In $\triangle XYZ$ shown below, medians $XE$, $YF$, and $ZD$ intersect at $C$.

If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle $CFX$.

$$\text{Perimeter} = 22$$

Score 1: The student found correct lengths for $CF$ and $CX$. 

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In \( \triangle XYZ \) shown below, medians \( \overline{XE}, \overline{YF}, \) and \( \overline{ZD} \) intersect at \( C \).

If \( CE = 5 \), \( YF = 21 \), and \( XZ = 15 \), determine and state the perimeter of triangle \( CFX \).

\[
\frac{15}{7.5} = \frac{x+5}{5} \quad \text{and} \quad \frac{7.5x + 37.5}{9.5} = 7.5
\]

\[
\frac{7.5x}{9.5} = 7.5 - 37.5 \quad \text{and} \quad x = 5
\]

\[7.5 + 5 + 5 = 17.5\]

\[\text{Perimeter of } \triangle \text{CFX} = 17.5\]

Score 0: The student did not show enough correct relevant work to receive any credit.
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point (5,12).

\[
\begin{align*}
5x - 4y &= 10 \\
5x + 10 &= 4y \\
10 - 10 &= 4y \\
\frac{4y}{4} &= \frac{5x - 10}{4} \\
y &= \frac{5}{4}x - \frac{5}{2} \\
n &= \frac{5}{4} \\
m_{\perp} &= -\frac{4}{5} \\
(y - 12) &= -\frac{4}{5}(x - 5)
\end{align*}
\]

Score 2: The student gave a complete and correct response.
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

\[
5x - 4y = 10
\]

\[
\frac{4y}{-4} = -\frac{5x + 10}{-4}
\]

\[
y = \frac{5}{4}x + \frac{-5}{2}
\]

\[
m_1 = -\frac{4}{5}
\]

\[
y = mx + b
\]

\[
12 = (-\frac{4}{5})(5) + b
\]

\[
12 = -4 + b
\]

\[
12 + 4 = b
\]

\[
16 = b
\]

\[
y = -\frac{4}{5}x + 16
\]

**Score 2:** The student gave a complete and correct response.
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point (5,12).

$$5x - 4y = 10$$

$$\frac{5x - 10}{4} = \frac{4y}{4}$$

$$\frac{5}{4}x - 2.5 = y$$

$$m = \frac{5}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 12 = \frac{5}{4}(x - 5) + 12$$

$$y = \frac{5}{4}(x - 5) + 12$$

**Score 1:** The student wrote an equation of the line parallel and passing through (5,12).
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point (5,12).

Score 1: The student made one computational error in determining the $y$-intercept.
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point (5,12).

\[
\frac{4y = 5x - 10}{4y = 5x - 10}
\]

\[
(y - y_1) = m(x - x_1)
\]

\[
(y - 12) = m(x - 5)
\]

\[
y = \frac{-5}{4}x - 2.5
\]

**Score 0:** The student did not show enough correct relevant work to receive any credit.
31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

\[ y = -\frac{1}{5}x + 6 \]

**Score 0:** The student gave a completely incorrect response.
32 Quadrilateral NATS has coordinates N\((-4,-3)\), A\((1,2)\), T\((8,1)\), and S\((3,-4)\).

Prove quadrilateral NATS is a rhombus.

[The use of the set of axes below is optional.]

\[ TA = \sqrt{(8-1)^2 + (1-2)^2} = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50} \]

\[ TS = \sqrt{(8-3)^2 + (1-4)^2} = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} \]

\[ NT = \sqrt{(1-8)^2 + (2-1)^2} = \sqrt{7^2 + 1^2} = \sqrt{49 + 1} = \sqrt{50} \]

\[ NS = \sqrt{(3-4)^2 + (4-2)^2} = \sqrt{1^2 + 2^2} \]

\[ NA = \sqrt{(1+4)^2 + (2+3)^2} \]

\[ \text{All sides are equal in the quadrilateral NATS is a rhombus.} \]

\[ \frac{\sqrt{50}}{\sqrt{2}} = AT \]

Score 4: The student gave a complete and correct response.
32 Quadrilateral NATS has coordinates N(−4, −3), A(1, 2), T(8, 1), and S(3, −4).

Prove quadrilateral NATS is a rhombus.
[The use of the set of axes below is optional.]

Score 4: The student gave a complete and correct response.
32 Quadrilateral NATS has coordinates N(−4, −3), A(1,2), T(8,1), and S(3,−4).

Prove quadrilateral NATS is a rhombus.
[The use of the set of axes below is optional.]

Quadrilateral NATS is a parallelogram because it has 2 pairs of opposite sides \( \parallel \). It is a rhombus because it has diagonals that are \( \perp \).

Score 4: The student gave a complete and correct response.
32 Quadrilateral NATS has coordinates N(-4,-3), A(1,2), T(8,1), and S(3,-4).

Prove quadrilateral NATS is a rhombus.

[The use of the set of axes below is optional.]

\[
\begin{align*}
AN &= \sqrt{(1 + 4)^2 + (2 + 3)^2} = \sqrt{25 + 25} = \sqrt{50}, \\
AT &= \sqrt{(8 - 1)^2 + (1 - 2)^2} = \sqrt{49 + 1} = \sqrt{50} \\
SN &= \sqrt{(3 + 4)^2 + (-4 + 3)^2} = \sqrt{49 + 1} = \sqrt{50} \\
TS &= \sqrt{(8 - 3)^2 + (1 + 4)^2} = \sqrt{25 + 25} = \sqrt{50}
\end{align*}
\]

Score 3: The student did not write a concluding statement.
Question 32

32 Quadrilateral NATS has coordinates N(−4,−3), A(1,2), T(8,1), and S(3,−4).

Prove quadrilateral NATS is a rhombus.
[The use of the set of axes below is optional.]

\[ \text{NA} = \sqrt{(2+3)^2 + (1+4)^2} \]
\[ = \sqrt{25 + 25} \]
\[ = 2\sqrt{5} \]
\[ \text{AT} = \sqrt{(1-2)^2 + (8-1)^2} \]
\[ = \sqrt{1 + 49} \]
\[ = \sqrt{50} \]
\[ = 2\sqrt{5} \]
\[ \text{NS} = \sqrt{(2-4)^2 + (3+4)^2} \]
\[ = \sqrt{4 + 49} \]
\[ = \sqrt{53} \]

\[ \frac{\text{NA}}{\text{AT}} \approx \frac{\text{ST}}{\text{NS}} \]

Score 2: The student made a simplification error and did not write a concluding statement.
32 Quadrilateral NATS has coordinates \(N(-4,-3), A(1,2), T(8,1), \) and \(S(3,-4).\)

Prove quadrilateral NATS is a rhombus.
[The use of the set of axes below is optional.]

\[
\begin{align*}
 d &= \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} \\
 d_{AS} &= \sqrt{(2+4)^2+(1+3)^2} \\
 d &= \sqrt{6^2+6^2} \\
 m_{NT} &= \frac{y_1-y_2}{x_1-x_2} \\
 d_{NT} &= \sqrt{(8+4)^2+(1+3)^2} \\
 d &= \sqrt{12^2+4^2} \\
 m_{AS} &= \frac{2+3}{1+4} = \frac{5}{5} = 1 \\
 m_{NT} &= \frac{1+3}{8+4} = \frac{4}{12} = \frac{1}{3}
\end{align*}
\]

\[\text{Quadrilateral NATS is a rhombus because the diagonals are perpendicular to each other.}\]

Score 2: The student made a conceptual error by concluding that a quadrilateral with perpendicular diagonals is a rhombus.
32 Quadrilateral NATS has coordinates N(−4,−3), A(1,2), T(8,1), and S(3,−4).

Prove quadrilateral NATS is a rhombus.

[The use of the set of axes below is optional.]

\[
\begin{align*}
\text{Slope } \overline{NA} & \quad \frac{2 - 3}{1 - (-4)} = \left( \frac{1}{5} \right) \\
\text{Slope } \overline{TS} & \quad \frac{-1 - 1}{2 - 3} = -2 \\
\text{Distance } \overline{NA} & \quad \sqrt{(1 - (-4))^2 + (2 - (-3))^2} = \sqrt{50} = 5\sqrt{2} \\
\text{Distance } \overline{TS} & \quad \sqrt{(3 - 8)^2 + (-1 - 1)^2} = \sqrt{50} = 5\sqrt{2}
\end{align*}
\]

\[\text{Yes. This is a}
\text{rhombus because both sides are congruent.}\]

Score 1: The student had correct work to prove NATS is a parallelogram, but the concluding statement was incorrect.
32 Quadrilateral NATS has coordinates $N(-4,-3)$, $A(1,2)$, $T(8,1)$, and $S(3,-4)$.

Prove quadrilateral NATS is a rhombus.
[The use of the set of axes below is optional.]

\[
\begin{align*}
\text{Step 1:} & \quad m_{12} = \frac{y_2 - y_1}{x_2 - x_1} \\
\text{Step 2:} & \quad m_{12} = \frac{1 - 2}{8 - 1} = -\frac{1}{7} \\
\text{Step 3:} & \quad m_{13} = \frac{-4 - 1}{3 - 8} = \frac{5}{5} = 1 \\
\text{Step 4:} & \quad \text{quad NATS is not a rhombus}
\end{align*}
\]

Score 0: The student did not show enough correct relevant work to receive any credit.
33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[ \tan 56^\circ = \frac{x}{1.3} \]

\[ x = 1.3 \tan 56^\circ \]

\[ x = 3.427329 \]

\[ 1.5 + 3.427329 = 5.627329 \]

\[ \tan \theta = \frac{3.427329}{1.3} \]

\[ \theta = \tan^{-1}(3.427329/1.3) \]

\[ \theta = 69.228395 \]

\[ \cos \theta = \frac{1.3}{y} \]

\[ y = 3.665 \]

\[ 3.7 \]

**Score 4:** The student gave a complete and correct response.
David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David’s eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[
\frac{\tan \theta}{1} = \frac{x}{1.3} \\
x = 1.3(\tan 56^\circ) \\
x = 1.927329259 \\
(3.427329259)^2 + (1.3)^2 = c^2 \\
11.74658585 + 1.69 = c^2 \\
13.43655585 = c^2 \\
3.69954184 = c \\
3.7 \text{ meters}
\]

Score 4: The student gave a complete and correct response.
33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[
\begin{align*}
3.43^2 + 1.3^2 &= x^2 \\
11.7699 + 1.69 &= x^2 \\
13.4599 &= x^2 \\
3.6688 &= x
\end{align*}
\]

Score 3: The student made one rounding error when finding the length of the ladder.
33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

Score 3: The student found the length of the stilt, but no further correct work was shown.
David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[
\tan 56^\circ = \frac{x}{1.3} \\
0.827... = \frac{y}{\tan 56^\circ} \\
y = 0.8257241736 \\

\text{Minimum length} = 1.9 \text{ meters}
\]

**Score 2:** The student found the altitude from the sight line to the top of the ladder, but no further correct work was shown.
Question 33

33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David’s eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[
\tan(56^\circ) = \frac{x}{1.3} \\
x = \tan(56^\circ) \times 1.3 \\
x \approx 3.2
\]

The minimum length of ladder David will need to buy is 3.2 meters of ladder.

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work was shown.
33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David’s eye level is 1.5 meters above the ground.

Determine and state the minimum length of a ladder, to the nearest tenth of a meter, that David will need to buy for his treehouse.

\[
\cos 56^\circ = \frac{x}{1.3 + 1.5}
\]

\[
x = \frac{1.3}{\cos 56^\circ} + 1.5
\]

\[x = 2.22695\]

the minimum length of a ladder David will have to buy is 2.2 meters

Score 0: The student did not show enough correct relevant work to receive any credit.
34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 4: The student gave a complete and correct response.
34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

\[ V = \pi r^2 h \]

\[ V = \pi (7)^2 (16) \]

\[ V = 770.88472 \]

\[ V = Bh \]

\[ 770.88472 = (16) x^2 \]

\[ 173.2 = x^2 \]

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

\[ \frac{80}{13.1} = 6.1 \]

\[ \frac{60}{13.1} = 4.6 \]

Score 3: The student made one rounding error in determining the side length of the container.
34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 3: The student found the side length of 13.2, but no further correct work was shown.
A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 2: The student made a rounding error in finding the side length of the new container. No further correct work was shown.
34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

\[
\frac{16}{18} = \frac{x}{14}
\]

18x = 224

18

x = 12.4 cm

The side length of the container would be 12.4 cm.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

60 ÷ 12.4 = 4.8

50 ÷ 12.4 = 4.0

50(4) = 24

The shelf can hold 24 containers of chocolate covered almonds.

Score 1: The student had a completely incorrect response to find the side length of the new container. The student used the incorrect side length to find an appropriate number of new containers.
Question 34

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

\[ V_{C} = \pi r^2 h \]
\[ V_{B} = 11(7)^2(18) \]
\[ V_{B} = 2770.88472 \text{ cm}^3 \]

\[ V_{B} = Bh \]
\[ 2770.88472 = x(16) \]
\[ 2770.88472 = 16x \]
\[ \frac{2770.88472}{16} = \frac{16x}{16} \]
\[ x = 173.180295 \]
\[ x = 86.6 \text{ cm} \]

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Score 1: The student correctly found the volume of the cylinder, but no further correct work was shown.
A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

If the new container’s height is 16 cm, determine and state, to the nearest tenth of a centimeter, the side length of the new container if both containers contain the same amount of almonds.

\[ V = \pi r^2 h \]
\[ V = \pi \times 14^2 \times 18 \]
\[ V = 11,083.5 \]

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

**Score 0:** The student gave a completely incorrect response.
35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $\overline{BD}$ and $\overline{EF}$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $\overline{FG} \cong \overline{EG}$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\text{Quad } ABCD$, $E$ and $F$ are points on $BC$ and $AD$, and $\overline{BD}$ and $\overline{EF}$ are drawn. $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $BD \cong BD$</td>
<td>2. Reflexive property</td>
</tr>
<tr>
<td>3. $\Delta ABD \cong \Delta CDB$</td>
<td>3. SAS</td>
</tr>
<tr>
<td>4. $\overline{BC} \cong \overline{DA}$</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>5. $\overline{BE} + \overline{CE} \cong \overline{AF} + \overline{DF}$</td>
<td>5. Segment addition post.</td>
</tr>
<tr>
<td>6. $\overline{BE} \cong \overline{DF}$</td>
<td>6. Subtraction property</td>
</tr>
<tr>
<td>7. $\angle BGE \cong \angle DGF$</td>
<td>7. Vertical $\angle s$ are $\cong$</td>
</tr>
<tr>
<td>8. $\angle CBD \cong \angle ADB$</td>
<td>8. CPCTC</td>
</tr>
<tr>
<td>9. $\Delta EBG \cong \Delta FDG$</td>
<td>9. AAS</td>
</tr>
<tr>
<td>10. $FG \cong EG$</td>
<td>10. CPCTC</td>
</tr>
</tbody>
</table>

Score 6: The student gave a complete and correct response.
Question 35

35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $BDG$ and $EFG$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $FG \cong EG$

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<tr>
<td>1. $\text{quad } ABCD \cong \text{quad } CDG$, $AB \cong CD$, $CE \cong AF$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB \parallel CD$</td>
<td>2. If $\angle$, int., $\angle s \cong$, then lines $\parallel$</td>
</tr>
<tr>
<td>3. $ABCD$ is ( \square )</td>
<td>3. A quad with a pair of opp. sides that are $\cong$ and $\parallel$ is a $\square$</td>
</tr>
<tr>
<td>4. $\angle A \cong \angle C$</td>
<td>4. Opp. $\angle s$ of a $\square$ are $\cong$</td>
</tr>
<tr>
<td>5. $\triangle ABD \cong \triangle DCB$</td>
<td>5. ASA $\cong$ ASA</td>
</tr>
<tr>
<td>6. $\angle EBG \cong \angle FDC$, $\angle BEG \cong \angle DFG$</td>
<td>6. If lines $\parallel$, then, int., $\angle s \cong$.</td>
</tr>
<tr>
<td>7. $BC \cong AD$</td>
<td>7. Subtraction post.</td>
</tr>
<tr>
<td>8. $AE \cong BE \cong BC - EC$</td>
<td>8. ASA $\cong$ ASA</td>
</tr>
<tr>
<td>9. $\angle BEG \cong \angle DFG$</td>
<td>9. CPCTC</td>
</tr>
<tr>
<td>10. $FG \cong FG$</td>
<td>10. CPCTC</td>
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Score 5: The student did not prove $AD \parallel BC$ to prove step 6.
Question 35

35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $BGD$ and $EGF$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $FG \cong EG$

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</tr>
<tr>
<td>2. $AB \parallel CD$</td>
<td></td>
</tr>
<tr>
<td>3. Quadrilateral $ABCD$ is a parallelogram</td>
<td></td>
</tr>
<tr>
<td>4. $\angle BEG \cong \angle GDF$</td>
<td></td>
</tr>
<tr>
<td>5. $BC \parallel AD$</td>
<td></td>
</tr>
<tr>
<td>6. $\angle EBG \cong \angle GDF$</td>
<td></td>
</tr>
<tr>
<td>7. $AD \cong BC$</td>
<td></td>
</tr>
<tr>
<td>8. $AD = AF + FD$</td>
<td></td>
</tr>
<tr>
<td>$BC = BE + EC$</td>
<td></td>
</tr>
<tr>
<td>9. $BE + EC = AF + FD$</td>
<td></td>
</tr>
<tr>
<td>10. $BE = FD$</td>
<td></td>
</tr>
<tr>
<td>11. $\triangle BEG \cong \triangle DFG$</td>
<td></td>
</tr>
<tr>
<td>12. $FG \cong EG$</td>
<td></td>
</tr>
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</table>

Score 5: The student incorrectly named the vertical angles in step 4.
Question 35

35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $BGD$ and $EGF$ are drawn such that $\angle ABG \equiv \angle CDG$, $AB \equiv CD$, and $CE \equiv AF$.

**Prove:** $FG \equiv GE$

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<tr>
<td>1) Quadr. $ABCD$</td>
<td>1) Given</td>
</tr>
<tr>
<td>$\angle ABG \equiv \angle CDG$</td>
<td>2) Vertical $\angle$'s are $\equiv$</td>
</tr>
<tr>
<td>$AB \equiv CD$</td>
<td>3) Lines are $\parallel$ iff they have $\equiv$ alt int $\angle$'s</td>
</tr>
<tr>
<td>$CE \equiv AF$</td>
<td>4) $ABCD$ has a pair of sides that are $\parallel$ and $\equiv$</td>
</tr>
<tr>
<td>2) $\angle BGE \equiv \angle FGD$</td>
<td>5) All int $\angle$'s are $\equiv$ if lines are $\parallel$</td>
</tr>
<tr>
<td>3) $EF \parallel CD$</td>
<td>6) In a parallelogram opposite sides are $\equiv$ and $EC \equiv AF$ so part $+$ part $=$ whole</td>
</tr>
<tr>
<td>4) $ABCD$ is a parallelogram</td>
<td>7) $AAS$ Thm $\equiv$</td>
</tr>
<tr>
<td>5) $\angle EBG \equiv \angle GDF$</td>
<td>8) CPCTC</td>
</tr>
<tr>
<td>6) $\overline{BE} \equiv \overline{FD}$</td>
<td></td>
</tr>
<tr>
<td>7) $\triangle BGE \equiv \triangle DGE$</td>
<td></td>
</tr>
<tr>
<td>8) $\overline{FG} \equiv \overline{GE}$</td>
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</table>

**Score 4:** The student did not prove $\overline{AD} \parallel \overline{BC}$ to prove step 5 and did not show subtraction to prove step 6.
35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $BDG$ and $EFG$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $FG \cong EG$

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<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle EBG \cong \angle FCG$, $\angle BEG \cong \angle DFG$</td>
<td>2) parallel lines cut by a transversal form congruent opposite interior angles</td>
</tr>
<tr>
<td>3) $EC \cong AD$</td>
<td>3) opposite sides of a parallelogram are congruent</td>
</tr>
<tr>
<td>4) $\overline{BC} - \overline{EC} = \overline{AD} - \overline{AF}$, $\overline{BE} = \overline{OF}$</td>
<td>4) a segment is congruent to the sum of its parts</td>
</tr>
<tr>
<td>5) $\overline{AFCD} \cong \overline{ABGE}$</td>
<td>5) Subtraction</td>
</tr>
<tr>
<td>6) $\overline{AFCD} \cong \overline{ABGE}$</td>
<td>6) $\triangle ASA \cong \triangle ASA$</td>
</tr>
<tr>
<td>7) $\overline{FG} \cong \overline{EG}$</td>
<td>7) CPCTC</td>
</tr>
</tbody>
</table>

Score 3: The student made one conceptual error by not proving $ABCD$ is a parallelogram. The student did not prove $AD \parallel BC$ to prove step 2.
35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $FG \cong EG$

Score 2: The student proved $ABCD$ is a parallelogram and $\triangle ABD \cong \triangle CDB$. 

Geometry – Jan. '20
35 In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $\overline{BGE}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $AB \cong CD$, and $CE \cong AF$.

Prove: $FG \cong EG$

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</tr>
<tr>
<td>2 $\overline{BC} \parallel \overline{AD}$</td>
</tr>
<tr>
<td>3 $\angle GBE \cong \angle GDF$</td>
</tr>
<tr>
<td>4 $\angle BGE \cong \angle DGF$</td>
</tr>
<tr>
<td>5 $\triangle FGD \sim \triangle EGB$</td>
</tr>
<tr>
<td>6 $\overline{BE} \cong \overline{FD}$</td>
</tr>
<tr>
<td>7 $\overline{FG} \cong \overline{EG}$</td>
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<tbody>
<tr>
<td>1 Given</td>
</tr>
<tr>
<td>2 A quadrilateral with two congruent sides has $\parallel$ lines</td>
</tr>
<tr>
<td>3 2 $\parallel$ lines cut by a transversal forms congruent $\angle$s</td>
</tr>
<tr>
<td>4 Vertical $\angle$s are $\cong$</td>
</tr>
<tr>
<td>5 AA similarity theorem</td>
</tr>
<tr>
<td>6 CPCTC</td>
</tr>
<tr>
<td>7 $G$ is the md. pt. of $\overline{FE}$</td>
</tr>
</tbody>
</table>

Score 1: The student had only one correct relevant statement and reason in step 4.
Question 35

In quadrilateral $ABCD$, $E$ and $F$ are points on $BC$ and $AD$, respectively, and $BGD$ and $EGF$ are drawn such that $\angle ABG \equiv \angle CDG$, $AB \equiv CD$, and $CE \equiv AF$.

![Diagram of quadrilateral with points E and F on BC and AD, respectively, and lines BG, BG, and GF drawn]

Prove: $FG \equiv EG$

<table>
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<tbody>
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<td>1. $\angle ABG \equiv \angle CDG$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$AB \equiv CD$, $CE \equiv AF$</td>
<td>2. Opp. Sides $\equiv$</td>
</tr>
<tr>
<td>2. $ABCD$ is parallelogram</td>
<td>3. Parallelogram’s diagonals bisect each other</td>
</tr>
<tr>
<td>3. $FG \equiv EG$</td>
<td></td>
</tr>
</tbody>
</table>

Score 0: The student gave a completely incorrect response.