The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, August 17, 2022 — 12:30 to 3:30 p.m.

MODEL RESPONSE SET

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Updated 08/19/22 to correct the graphics on pages 59 and 62.
25 On the set of axes below, \( \triangle DOG \equiv \triangle CAT \).

Describe a sequence of transformations that maps \( \triangle DOG \) onto \( \triangle CAT \).

\[
\text{Reflection over the y-axis and a translation of } (0, 5),
\]

\[
\begin{align*}
D(-5, -3) & \xrightarrow{\text{y-axis}} G(5, -3) \\
C(-1, -2) & \xrightarrow{\text{y-axis}} C(1, -2) \\
G(-2, -4) & \xrightarrow{\text{y-axis}} G(2, -4)
\end{align*}
\]

Score 2: The student gave a complete and correct response.
25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.

Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

1. Translate $\triangle DOG$ 5 units up
2. Reflection over y-axis

Score 2: The student gave a complete and correct response.
On the set of axes below, \( \triangle DOG \cong \triangle CAT \).

Describe a sequence of transformations that maps \( \triangle DOG \) onto \( \triangle CAT \).

- Translate \( \triangle DOG \) up 5 and right 1
- Reflect \( \triangle DOG \) over the line \( x = 1 \)

Score 1: The student translated up 5 and right 1 instead of up 5 and right 2.
25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.

Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

Reflection and translation

Score 1: The student identified an appropriate sequence of transformations, but did not describe the specific sequence of transformations.
25 On the set of axes below, $\triangle DOG \equiv \triangle CAT$.

Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

A reflection of $\triangle DOG$ over the y-axis, then a translation up 3 units to map onto $\triangle CAT$.

Score 1: The student gave a partially correct response by stating a correct line of reflection, but the translation was not stated correctly.
On the set of axes below, \( \triangle DOG \equiv \triangle CAT \).

Describe a sequence of transformations that maps \( \triangle DOG \) onto \( \triangle CAT \).

**Step 1**  Reflection over \( y \) axis for \( \triangle CAT 

**Step 2**  Transformation over \( x \) axis for \( \triangle CAT 

Now, \( C \) maps over \( D \)

\( A \) maps over \( O \)

\( T \) maps over \( G \)

**Score 0:** The student incorrectly mapped \( \triangle CAT \) onto \( \triangle DOG \), and incorrectly described the second transformation.
25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.

Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

$\triangle DOG$ is rotated $180^\circ$ around the origin.

Score 0: The student gave a completely incorrect response.
26 In right triangle $MTH$ shown below, $m\angle H = 90^\circ$, $HT = 8$, and $HM = 5$.

Determine and state, to the nearest tenth, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $MH$.

\[
\text{Cone Volume Formula: } V = \frac{1}{3} \pi R^2 H
\]

\[
V = \frac{1}{3} \pi (8)^2 (5) \\
\frac{1}{3} \cdot 320 \pi \\
106.66 \pi \\
335.08
\]

\[V = 335.1\]

**Score 2:** The student gave a complete and correct response.
26 In right triangle $MTH$ shown below, $m\angle H = 90^\circ$, $HT = 8$, and $HM = 5$.

Determine and state, to the nearest tenth, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $MH$.

Score 1: The student used the incorrect radius, $r = 4$, but found an appropriate volume.
26 In right triangle $MTH$ shown below, $m\angle H = 90^\circ$, $HT = 8$, and $HM = 5$.

Determine and state, to the nearest tenth, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $MH$.

Score 1: The student rotated the triangle around the wrong leg, but found an appropriate volume.
26 In right triangle $MTH$ shown below, $m\angle H = 90^\circ$, $HT = 8$, and $HM = 5$.

Determine and state, to the nearest tenth, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $MH$.

\[ V = \frac{1}{3}\pi r^2 h \]
\[ V = \frac{1}{3}\pi 8^2 (5) \]
\[ V = 21.33(\pi) (5) \]
\[ V = 67.23(5) \]
\[ V = \boxed{336.2} \]

Score 1: The student made a computational error when multiplying $21.33(\pi)$. 
26 In right triangle \( MTH \) shown below, \( \angle H = 90^\circ \), \( HT = 8 \), and \( HM = 5 \).

Determine and state, to the nearest tenth, the volume of the three-dimensional solid formed by rotating \( \triangle MTH \) continuously around \( MH \).

\[
V = \frac{1}{3} \pi \sqrt{6} \sqrt{10} \cdot 5 = \pi \cdot \approx 261.97
\]

**Score 0:** The student gave a completely incorrect response.
27 Using a compass and straightedge, dilate triangle $ABC$ by a scale factor of 2 centered at $C$. [Leave all construction marks.]

Score 2: The student gave a complete and correct response.
27 Using a compass and straightedge, dilate triangle $ABC$ by a scale factor of 2 centered at $C$. [Leave all construction marks.]

Score 1: The student made an appropriate construction, but used vertex $A$ as the center of dilation.
27 Using a compass and straightedge, dilate triangle $ABC$ by a scale factor of 2 centered at $C$. [Leave all construction marks.]

Score 0: The student gave a completely incorrect response.
27 Using a compass and straightedge, dilate triangle $ABC$ by a scale factor of 2 centered at $C$. [Leave all construction marks.]

**Score 0:** The student gave a completely incorrect response.
27 Using a compass and straightedge, dilate triangle $ABC$ by a scale factor of 2 centered at $C$. [Leave all construction marks.]

Score 0: The student gave a completely incorrect response.
Question 28

28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

\[
\cos 14^\circ = \frac{3.8}{x} \Rightarrow x = 3.92 \text{ m.}
\]

Score 2: The student gave a complete and correct response.
A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

\[
\text{Length} \quad 5 - 1.2 = 3.8
\]

\[
\cos(14^\circ) = \frac{3.8}{x}
\]

\[
x = \frac{3.8}{\cos(14^\circ)}
\]

\[
x \approx 3.92
\]

\[
\text{Length of section of slanted wall is 3.92 meters}
\]

Score 2: The student gave a complete and correct response.
A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

Score 2: The student gave a complete and correct response.
28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

\[ \sin 76^\circ = \frac{3.8}{x} \]
\[ \sin 76^\circ = 3.8 \]
\[ \frac{3.8}{\sin 76^\circ} = x \]
\[ x = 2.9 \text{ m} \]

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

Score 1: The student made a rounding error.
A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work was shown.
Question 28

A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

\[
\cos 14 = \frac{5}{x} \quad x = 5.15 \text{ m}
\]

Score 1: The student used the incorrect height, but found an appropriate hypotenuse length.
28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

Determine and state, to the nearest hundredth, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

\[ \sin(40) \times 3.8 = x \]

\[ 3.8 = x \times \sin(40) \]

\[ x = 3.8 \]

Score 0: The student gave a completely incorrect response.
29 In the diagram below of right triangle $BAL$, altitude $AD$ is drawn to hypotenuse $BDL$. The length of $AD$ is 6.

If the length of $DL$ is four times the length of $BD$, determine and state the length of $BD$.

\[
\frac{6}{x} = \frac{4x}{6} \\
4x^2 = 36 \\
x^2 = 9 \\
x = \pm 3
\]

Reject $x = -3$.

$BD = 3$

Score 2: The student gave a complete and correct response.
29 In the diagram below of right triangle $BAL$, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.

If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

\[
(AB)^2 = x^2 + 36 \\
(AL)^2 = 16x^2 + 36 \\
(BL)^2 = 25x^2
\]

\[
25x^2 = 16x^2 + 36 + x^2 + 36 \\
8x^2 - 72 = 0 \\
8(x^2 - 9) = 0 \\
8 = 0 \\
9x^2 - 9 = 0 \\
(x + 3)(x - 3) = 0 \\
x = -3, x = 3 \\
BD = 3
\]

Score 2: The student gave a complete and correct response.
29 In the diagram below of right triangle $BAL$, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.

If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

Score 1: The student wrote a correct equation to find the length of $\overline{BD}$, but no further correct work was shown.
29 In the diagram below of right triangle $BAL$, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.

If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

\[ BO = 3 \]
\[ DL = 12 \]

Score 1: The student found the length of $\overline{BD}$, but no work was shown.
In the diagram below of right triangle BAL, altitude $AD$ is drawn to hypotenuse $BDL$. The length of $AD$ is 6.

If the length of $DL$ is four times the length of $BD$, determine and state the length of $BD$.

\[
\frac{x}{6} = \frac{14}{x}
\]

$BD = 6$

\[
\begin{align*}
S &= 4 \\
P &= 6 \\
X^2 + 4X - 36 &= 0
\end{align*}
\]

\[
X = -6, 6
\]

Score 0: The student did not show enough correct relevant work to receive any credit.
Question 29

29 In the diagram below of right triangle $BAL$, altitude $AD$ is drawn to hypotenuse $BDL$. The length of $AD$ is 6.

If the length of $DL$ is four times the length of $BD$, determine and state the length of $BD$.

\[
\frac{x}{6} = \frac{4x}{6} \\
\therefore \quad 6x = 24x
\]

Score 0: The student did not show enough correct relevant work to receive any credit.
29 In the diagram below of right triangle $BAL$, altitude $AD$ is drawn to hypotenuse $BDL$. The length of $AD$ is 6.

If the length of $DL$ is four times the length of $BD$, determine and state the length of $BD$.

\[
12 \times 6 = 72
\]

\[
72 \div 3 = 24
\]

$BD \approx 24$

**Score 0:** The student gave a completely incorrect response.
30 Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect $MN$ at $E$, and $AD \cong AE$.

If $\angle DAE = 35^\circ$, $\angle DCE = 25^\circ$, and $\angle NEC = 30^\circ$, determine and state $\angle ABD$.

\[ m \angle ABD = 47.5^\circ \]

**Score 2:** The student gave a complete and correct response.
Question 30

30 Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect at $E$, and $AD \cong AE$.

If $\angle DAE = 35^\circ$, $\angle DCE = 25^\circ$, and $\angle NEC = 30^\circ$, determine and state $\angle ABD$.

\[
\begin{array}{c|c|c|c}
180 & 30 & 180 \\
-35 & 125 & \frac{125}{2} \\
145 & 180 & 180 \\
2 & 55 & 55 \\
72.5 & 72.5 & 77.5 \\
+42.5 & +55 & +55 \\
120 & 132.5 & 132.5 \\
\end{array}
\]

\[
\frac{72.5 + 30}{102.5} = \frac{180}{77.5}
\]

Score 2: The student gave a complete and correct response.
30 Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect $MN$ at $E$, and $AD \equiv AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

$\overline{AD} \equiv \overline{AE} \implies \triangle ADE$ is an isosceles triangle

$m\angle DEA = \frac{180^\circ - m\angle DAE}{2} = \frac{180^\circ - 35^\circ}{2} = 72.5^\circ$

$m\angle AED = m\angle EDC + m\angle ECD$

$\implies 72.5^\circ = m\angle EDC + 25^\circ$

$\implies m\angle EDC = 47.5^\circ$

$AB \parallel CD \implies m\angle ABD = m\angle EDC$ (alternate interior angles)

$\implies m\angle ABD = 47.5^\circ$

Score 2: The student gave a complete and correct response.
30 Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect $MN$ at $E$, and $AD = AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

$\angle ABD = 52.5^\circ$

Score 1: The student mislabeled $\angle DAE$ in the diagram, but found an appropriate measure of $\angle ABD$. 
Question 30

30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at $E$, and $AD \cong AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

Score 1: The student appropriately labeled the diagram, but did not state $m\angle ABD$. 
Question 30

30 Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect $MN$ at $E$, and $AD = AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

$m\angle ABD = 22.5^\circ$

Score 1: The student made an error when finding $m\angle DEN$, but an appropriate measure was found for angle $ABD$. The measure of angle $BCE$ is not necessary in finding $m\angle ABD$. 
30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at $E$, and $AD \cong AE$.

If $\angle DAE = 35^\circ$, $\angle DCE = 25^\circ$, and $\angle NEC = 30^\circ$, determine and state $\angle ABD$.

$$180 - 35 = \frac{145}{2} = 72.5$$

Score 1: The student found $\angle ADE$ and $\angle AED$, but $\angle ABD$ was not stated.
30. Trapezoid $ABCD$, where $AB \parallel CD$, is shown below. Diagonals $AC$ and $DB$ intersect $MN$ at $E$, and $AD = AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

$m\angle ABD = 80^\circ$

Score 0: The student did not show enough correct relevant work to receive any credit.
30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at $E$, and $AD \equiv AE$.

If $m\angle DAE = 35^\circ$, $m\angle DCE = 25^\circ$, and $m\angle NEC = 30^\circ$, determine and state $m\angle ABD$.

Score 0: The student gave a completely incorrect response.
31 In the diagram below of circle $O$, the measure of inscribed angle $ABC$ is $36^\circ$ and the length of $OA$ is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

$$A_{\text{shaded}} = \pi r^2 \left(\frac{36}{360}\right)$$

$$A_{\text{shaded}} = \pi (4^2) \left(\frac{36}{360}\right)$$

$$A_{\text{shaded}} = 16\pi \left(\frac{1}{10}\right)$$

$$A_{\text{shaded}} = 1.6\pi$$

$$A_{\text{shaded}} = 5.1$$

$$A_{\text{shaded}} = 5.1\text{ in}^2$$

Score 2: The student gave a complete and correct response.
31 In the diagram below of circle $O$, the measure of inscribed angle $ABC$ is $36^\circ$ and the length of $OA$ is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

\[
A = \pi r^2 \cdot \frac{\theta}{360} = \pi 4^2 \cdot \frac{36}{360} = \pi 16 \cdot \frac{1}{5} = \pi \frac{16}{5}.
\]

\[A = 10.1 \text{ in}^2\]

**Score 2:** The student gave a complete and correct response.
31 In the diagram below of circle $O$, the measure of inscribed angle $ABC$ is $36^\circ$ and the length of $OA$ is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

\[
\frac{72}{360} = \frac{x}{16\pi} \\
\frac{1}{5} = \frac{x}{16\pi} \\
\frac{16\pi}{5} = x \\
x = \frac{16\pi \cdot z}{5} \\
x = 10.1 \text{ in}^2
\]

Score 2: The student gave a complete and correct response.
Question 31

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of OA is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

\[ \text{Area of sector} = \left( \frac{36^\circ}{360^\circ} \right) \pi \cdot 4^2 \]

\[ \text{Area of sector} = \left( \frac{36}{360} \right) \pi \cdot 16 \]

\[ \text{Area of sector} = 5.0 \]

Score 1: The student used an incorrect measure for arc AC.
31 In the diagram below of circle $O$, the measure of inscribed angle $ABC$ is $36^\circ$ and the length of $OA$ is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

Score 1: The student used an incorrect measure for angle $AOC$. 
31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of OA is 4 inches.

\[
\frac{36}{1} = 72
\]

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

Score 0: The student did not show enough correct relevant work to receive any credit.
31 In the diagram below of circle $O$, the measure of inscribed angle $ABC$ is $36^\circ$ and the length of $OA$ is 4 inches.

Determine and state, to the nearest tenth of a square inch, the area of the shaded sector.

\[
A = \pi \times 4^2 \\
= \pi \times 16 \\
= 16\pi \\
A = 50.26548
\]

Score 0: The student did not show enough correct relevant work to receive any credit.
32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, \( T \), is 13.3°. The angle of depression from the top of the building to the bottom of the tree, \( B \), is 22.2°.

\[ \tan 22.2^\circ = \frac{50}{x} \]

\[ \tan 13.3^\circ = \frac{y}{132.5213} \]

\[ x = 132.5213 \]

\[ 28.9628 = y \]

\[ \frac{50 - 28.9628}{21.6372} \]

The tree is 21 meters tall.

Score 4: The student gave a complete and correct response.
32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, $T$, is $13.3^\circ$. The angle of depression from the top of the building to the bottom of the tree, $B$, is $22.2^\circ$.

Determine and state, to the nearest meter, the height of the tree.

\[
\tan 22.2^\circ = \frac{50}{x} \quad \Rightarrow \quad x = \frac{50}{\tan 22.2^\circ} \\
x = 122.521
\]

\[
\tan 13.3^\circ = \frac{50}{x+y} \quad \Rightarrow \quad x+y = \frac{50}{\tan 13.3^\circ} \\
x+y = 211.515
\]

\[
122.521 + y = 211.515 \\
y = 88.994
\]

The tree is about 21 m tall.

Score 4: The student gave a complete and correct response.
32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, $T$, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, $B$, is 22.2°.

Determine and state, to the nearest meter, the height of the tree.

\[ \tan 22.2° = \frac{50}{x} \]

\[ \tan 13.3° = \frac{y}{122.52125} \]

\[ x = 122.52125 \]

\[ y = 28 \]

\[ 50 - 28 = 22 \text{ m} \]

**Score 3:** The student made a rounding error.
Score 2: The student correctly found the horizontal distance between the building and the tree, but no further correct work was shown.
32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, $T$, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, $B$, is 22.2°.

Determine and state, to the nearest meter, the height of the tree.

Score 1: The student found the correct height of the tree, but did not show enough work to receive additional credit.
32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, $T$, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, $B$, is 22.2°.

\[ \sin x = \frac{22.2}{50^2} \]
\[ x = 26.3593 \, \text{ft} \]
\[ x = 26.0 \, \text{ft} \]

\[ \sin x = \frac{13.3}{50} \]
\[ x = 15.4263 \, \text{ft} \]
\[ x = 15.0 \, \text{ft} \]

The tree is about 11 feet tall.

**Score 0:** The student did not show enough correct relevant work to receive any credit.
Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, $T$, is $13.3^\circ$. The angle of depression from the top of the building to the bottom of the tree, $B$, is $22.2^\circ$.

Determine and state, to the nearest meter, the height of the tree.

\[
\cos 22.2^\circ \left( \frac{50}{x} \right) = 14.269884935
\]

\[
\cos 13.3^\circ \left( \frac{50}{x} \right) = 8.54907520883
\]

\[5.7 \text{ meters}\]

**Score 0:** The student did not show enough correct relevant work to receive any credit.
Question 32

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, \( T \), is 13.3°. The angle of depression from the top of the building to the bottom of the tree, \( B \), is 22.2°.

Determine and state, to the nearest meter, the height of the tree.

\[
\tan 22.2° = \frac{\text{opp}}{\text{adj}} = \frac{h}{50} \quad \text{and} \quad \tan 13.3° = \frac{\text{opp}}{\text{adj}} = \frac{\text{opp}}{50}
\]

\[
h = 20.40162281 \approx 20.4 \text{ meters}
\]

\[
x = 11.81449975 \approx 11.8 \text{ meters}
\]

Score 0: The student gave a completely incorrect response.
33 The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

\[
\begin{align*}
m_{HV} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-6}{2-(-3)} = \frac{3}{5} \\
m_{EP} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{8-3} = \frac{3}{5} \\
m_{PE} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-9}{-3-8} = \frac{-4}{-11} \\
m_{YP} &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-(-1)}{2-8} = \frac{-10}{-6} = \frac{5}{3}
\end{align*}
\]

$\overline{HV}$ has the same slope as $\overline{EP}$ since they have the same slope.

Quadrilateral $HYPE$ is a parallelogram since both pairs of opposite sides are parallel.

$\overline{HY} \perp \overline{YP}$ since their slopes are opposite reciprocals.

$\angle y$ is a rt. $\angle$ since 1 lines form rt. $\angle$'s.

Quadrilateral $HYPE$ is a rectangle since it is a parallelogram w/a rt. $\angle$.

**Score 4:** The student gave a complete and correct response.
33 The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

$$
\begin{align*}
H \! Y &= \sqrt{(2-(-3))^2 + (9-6)^2} \\
&= \sqrt{25 + 9} \\
&= \sqrt{34} \\
H \! Y &= \sqrt{25 + 9} \\
EP &= \sqrt{(3-(-3))^2 + (-4-6)^2} \\
&= \sqrt{36 + 100} \\
&= \sqrt{136} \\
EP &= \sqrt{136}
\end{align*}
$$

Both pairs of opposite sides are equal so $HYPE$ is a parallelogram.

$$
\begin{align*}
H \! Y &= \sqrt{25 + 9} \\
\ H \! P &= \sqrt{(8-(-3))^2 + (-1-6)^2} \\
&= \sqrt{121 + 49} \\
&= \sqrt{170} \\
Y \! E &= \sqrt{(3-2)^2 + (-4-9)^2} \\
&= \sqrt{1 + 169} \\
&= \sqrt{170}
\end{align*}
$$

The diagonals are equal so when a parallelogram has equal diagonals then it must be a rectangle.

Score 4: The student gave a complete and correct response.
The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

Score 4: The student gave a complete and correct response.
33 The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

Score 3: The student did not write a concluding statement in proving a rectangle.
Question 33

The coordinates of the vertices of quadrilateral \( HYPE \) are \( H(-3,6) \), \( Y(2,9) \), \( P(8,-1) \), and \( E(3,-4) \).

Prove \( HYPE \) is a rectangle. [The use of the set of axes below is optional.]

Score 2: The student proved \( HYPE \) is a parallelogram, but did not prove \( HYPE \) is a rectangle.
33 The coordinates of the vertices of quadrilateral \( HYPE \) are \( H(-3,6), Y(2,9), P(8,-1), \) and \( E(3,-4) \).

Prove \( HYPE \) is a rectangle. [The use of the set of axes below is optional.]

\[
\begin{align*}
\text{Slope of } \overline{HY} &= \frac{3}{5} \quad \text{Slope of } \overline{HE} = \frac{10}{6} \\
\text{Slope of } \overline{EP} &= \frac{3}{5} \quad \text{Slope of } \overline{YP} = \frac{10}{6}
\end{align*}
\]

Quadrilateral \( HYPE \) is a rectangle because opposite sides are parallel, and it has four right angles.

Score 1: The student made a conceptual error in proving a rectangle and a computational error in finding the slopes of \( \overline{HE} \) and \( \overline{YP} \).
33 The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

Score 1: The student proved both pairs of opposite sides parallel, but no further correct work was shown.
The coordinates of the vertices of quadrilateral \( HYPE \) are \( H(-3,6) \), \( Y(2,9) \), \( P(8,-1) \), and \( E(3,-4) \). Prove \( HYPE \) is a rectangle. [The use of the set of axes below is optional.]

\[ \text{Hypo is a rectangle because it has 2 pairs of parallel lines.} \]

**Score 0:** The student did not show enough correct relevant work to receive any credit.
The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3, 6)$, $Y(2, 9)$, $P(8, -1)$, and $E(3, -4)$.

Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

**Statement**

1. $HYPE$ is a \[ \square \]

2. $HY \parallel EP$

3. $HE \parallel YP$

4. $YP \perp PE$ and $HE \perp HY$

5. $HYPE$ is a \[ \square \]

**Reason**

1. Given

2. Same slope

3. Same slope

4. Definition of \[ \perp \]

5. Definition of Rectangle.

**Score 0:** The student did not show enough correct relevant work to receive any credit.
33 The coordinates of the vertices of quadrilateral *HYPE* are \(H(-3,6), Y(2,9), P(8,-1),\) and \(E(3,-4)\).

Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]

\[\overline{HF}\text{ and }\overline{PE}\text{ both have the same slope}\]
\[\overline{YP}\text{ and }\overline{HE}\text{ have the same slope}\]

If two lines have the same slope then they are parallel. Therefore, *HYPE* has 2 pairs of parallel sides. If all sides of a quadrilateral are congruent, then opposite sides are congruent. *HYPE* has 2 pairs of congruent and parallel sides. Therefore *HYPE* is a rectangle.

**Score 0:** The student did not show enough correct relevant work to receive any credit.
34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

Score 4: The student gave a complete and correct response.
A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft $\times$ 1 ft $\times$ 18 in. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

Score 3: The student made an error in finding the number of baseballs.
34 A packing box for baseballs is the shape of a rectangular prism with dimensions of $2 \text{ ft} \times 1 \text{ ft} \times 18 \text{ in}$. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

$$\frac{24}{2.94} \quad \frac{12}{2.94} \quad \frac{18}{2.94}$$

$$8.2 \quad 4.1 \quad 6.1$$

$$8.2 \times 4.1 \times 6.1 = 205.1$$

205 baseballs can fit in the box

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

$$V = \frac{4}{3} \pi r^3$$
$$V = \frac{4}{3} \pi (1.47)^3$$
$$V = 13.3$$

$$13.3 \times 0.025 = 0.3325$$

$$0.3325 \times 205 = 68 \text{ Pounds}$$

Score 3: The student made an error in finding the number of baseballs.
A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft \times 1 \text{ ft} \times 18 \text{ in}. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

\[
\text{Volume of prism} = Bh = 2 \text{ ft} \times 1 \text{ ft} \times 18 \text{ in} = 32.46 \text{ in}^3
\]

\[
\text{Volume of ball} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(1.47^2 \right) 
\text{approx} = 13.305 \text{ in}^3
\]

\[
\text{Number} = \frac{32.46 \text{ in}^3}{13.305 \text{ in}^3} \approx 2.42 \text{ baseballs per box}
\]

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

\[
\text{Weight of ball} = 0.025 \times 13.305 = 0.3326 \text{ pounds/ball}
\]

\[
\text{Total weight} = 2.42 \times 0.3326 \approx 0.79 \text{ pounds per box}
\]

**Score 2:** The student found an appropriate weight of baseballs in a box, but no further correct work was shown.
34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

\[ V = (2)(1)(1.5) \]
\[ V = 3 \text{ ft}^3 = 36 \text{ in}^3 \]

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

Score 1: The student found the volume of one baseball, but no further correct relevant work was shown.
34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.

Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

\[ V = L \cdot W \cdot H \]
\[ V = (24 \text{ in})(12 \text{ in})(18 \text{ in}) \]
\[ V = 5184 \text{ in}^3 \]

The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the nearest pound, the total weight of all the baseballs in the fully packed box.

Score 0: The student did not show enough correct relevant work to receive any credit.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quad $ABCD$, $AC \cap EF$ intersect at $H$. $EF \parallel AD$, $EF \parallel BC$, $AD \cong BC$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $AD \parallel BC$</td>
<td>Transitive Postulate of parallel lines</td>
</tr>
<tr>
<td>3. $ABCD$ is a parallelogram</td>
<td></td>
</tr>
<tr>
<td>4. $\angle 1$ and $\angle 2$ are vertical $\angle$s</td>
<td></td>
</tr>
<tr>
<td>5. $\angle 1 \cong \angle 2$</td>
<td></td>
</tr>
<tr>
<td>6. $AB \parallel CD$</td>
<td></td>
</tr>
<tr>
<td>7. $\angle 3 \cong \angle 4$</td>
<td></td>
</tr>
<tr>
<td>8. $\triangle AHE \sim \triangle CHF$</td>
<td></td>
</tr>
<tr>
<td>9. $\frac{EH}{FH} = \frac{AH}{CH}$</td>
<td></td>
</tr>
<tr>
<td>10. $(EH)(CH) = (FH)(AH)$</td>
<td>In a proportion, the product of the means equals the product of the extremes</td>
</tr>
</tbody>
</table>

Score 6: The student gave a complete and correct response.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

1. Given $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$
2. $AD \parallel BC$
3. $AC \cong AC$
4. $\angle 1 \cong \angle 2$
5. $\triangle ABD \cong \triangle CBA$
6. $\angle 3 \cong \angle 4$
7. $\angle 5 \cong \angle 6$
8. $\triangle HFC \sim \triangle HCA$
9. $\frac{EH}{FH} = \frac{AH}{CH}$
10. $(EH)(CH) = (FH)(AH)$

Score 6: The student gave a complete and correct response.
35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at $H$, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$

Prove: $(EH)(CH) = (FH)(AH)$

- Given quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at $H$, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$.
- Since $\overline{EF} \parallel \overline{BC}$ and $\overline{EF} \parallel \overline{AD}$ then $\overline{AD} \parallel \overline{BC}$ by the transitive property.
- So $\angle 1 \cong \angle 2$ because when two $\parallel$ lines are cut by a transversal, the alternate interior angles are congruent.
- Diagonal $\overline{AC} \cong \overline{AC}$ by reflexive. $\therefore \triangle ABC \cong \triangle CBA$ by SAS.
- $\angle 3 \cong \angle 4$ because corresponding angles of $\cong$ triangles are $\cong$.
- $\angle 5 \cong \angle 6$ because vertical angles are $\cong$.
- So $\triangle FHC \sim \triangle EHA$ by AA and then $\frac{EH}{FH} = \frac{AH}{CH}$ because corresponding sides of similar triangles are proportional.
- Therefore $(EH)(CH) = (FH)(AH)$ because the product of the means equals the product of the extremes.

Score 6: The student gave a complete and correct response.
35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

1. Quadrilateral $ABCD$, $AC$ intersects $EF$ at $H$, $EF \parallel AD$, $EF \parallel BC$, $AD \cong BC$
2. $AD \parallel BC$
3. $ABCD$ is a $\square$
4. $AB \parallel DC$
5. $\triangle AEH \cong \triangle CFH$
6. $\triangle AHE \cong \triangle CHF$
7. $\triangle AHE \sim \triangle CHF$
8. $\frac{EH}{AH} = \frac{FH}{CH}$
9. $(EH)(CH) = (FH)(AH)$

Score 5: The student wrote an incorrect reason in step 9.
Question 35

35  Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Proof Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quadrilateral $ABCD$, $EF \parallel AD$, $EF \parallel BC$, $AD \cong BC$</td>
</tr>
<tr>
<td>2. $\angle AHE$ and $\angle CHF$ are right angles</td>
</tr>
<tr>
<td>3. $\triangle AHE \sim \triangle CHF$</td>
</tr>
<tr>
<td>4. $\angle AHE \cong \angle CHF$</td>
</tr>
<tr>
<td>5. $EF \parallel BC$</td>
</tr>
<tr>
<td>6. $\triangle LEAH \cong \triangle LCHF$</td>
</tr>
<tr>
<td>7. $\triangle AHE \sim \triangle CHF$</td>
</tr>
<tr>
<td>8. $\frac{EH}{FH} = \frac{AH}{CH}$</td>
</tr>
<tr>
<td>9. $(EH)(CH) = (FH)(AH)$</td>
</tr>
</tbody>
</table>

Score 5: The student did not state $AD \parallel BC$ to prove $ABCD$ is a parallelogram.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th><strong>Statements</strong></th>
<th><strong>Reasons</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\text{quad } ABCD$, $AC \cap EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, $AD \cong BC$</td>
<td>1. given</td>
</tr>
<tr>
<td>2. $AB \parallel CD$</td>
<td>2. opp sides of para are $\parallel$</td>
</tr>
<tr>
<td>$\angle 3 \cong \angle 4$</td>
<td>3. int 2 lines $\parallel$ alt int $\angle$'s $\cong$</td>
</tr>
<tr>
<td>$\angle 1 \cong \angle 2$</td>
<td>4. vertical $\angle$'s $\cong$</td>
</tr>
<tr>
<td>5. $\triangle AHE \sim \triangle CHF$</td>
<td>5. $AA$</td>
</tr>
<tr>
<td>6. $\frac{EH}{FH} = \frac{AH}{CH}$</td>
<td>6. corr sides of $\sim$ $\triangle$'s are in proportion</td>
</tr>
<tr>
<td>✓ $\frac{(EH)(CH)}{(FH)(AH)}$</td>
<td>7. prod of means $=$ prod of extremes</td>
</tr>
</tbody>
</table>

Score 4: The student made a conceptual error by not proving $ABCD$ is a parallelogram.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Statements</th>
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</thead>
<tbody>
<tr>
<td>1. Quad $ABCD$</td>
<td>1. Given</td>
</tr>
<tr>
<td>$AC$ and $EF$ intersect at $H$</td>
<td>2. Given</td>
</tr>
<tr>
<td>2. $EF \parallel AD$</td>
<td>3. If two lines are $\parallel$ to the same line, then they are parallel</td>
</tr>
<tr>
<td>$EF \parallel BC$</td>
<td>4. Given</td>
</tr>
<tr>
<td>3. $AD \parallel BC$</td>
<td>5. A quad with one pair of opposite sides that are $\parallel$ and $\perp$ then it is a $\Box$</td>
</tr>
<tr>
<td>4. $AB \parallel DC$</td>
<td>6. def of parallelogram</td>
</tr>
<tr>
<td>5. $\angle HAE = \angle HCF$</td>
<td>7. alternate interior $\angle$s</td>
</tr>
<tr>
<td>6. $\angle EHC$ and $\angle FHC$ are vertical $\angle$s</td>
<td>8. def of vertical $\angle$s</td>
</tr>
<tr>
<td>7. $\angle EHA \cong \angle FHA$</td>
<td>9. vertical $\angle$s are $\cong$</td>
</tr>
<tr>
<td>8. $\triangle AHE \cong \triangle CHF$</td>
<td>10. $\triangle \cong $</td>
</tr>
<tr>
<td>9. $EH = AH$</td>
<td>11. In two $\triangle$s are $\sim$, corresponding sides are in proportion</td>
</tr>
<tr>
<td>$FH = CH$</td>
<td>12. Cross products are equal</td>
</tr>
<tr>
<td>10. $(EH)(CH) = (FH)(AH)$</td>
<td></td>
</tr>
</tbody>
</table>

Score 4: The student gave an incorrect reason in step 7, and stated an incorrect angle in step 9.
Question 35

Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
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<th>Statement</th>
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<tr>
<td>1. Qua $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, $AD \cong BC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Qua $ABCD$ is a parallelogram</td>
<td>2. A parallelogram has one pair of opposite sides congruent and parallel, then $ABCD$ is a parallelogram.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 2$</td>
<td>3. Vertical angles are congruent</td>
</tr>
<tr>
<td>4. $\angle 3 \cong \angle 4$</td>
<td>4. If $\parallel$ lines are cut by a transversal, the alternate int. angles are congruent</td>
</tr>
<tr>
<td>5. $\triangle ADE \sim \triangle CHF$</td>
<td>5. $AA\sim$Thm</td>
</tr>
<tr>
<td>6. $\frac{EH}{FH} = \frac{AH}{CH}$</td>
<td>6. Corresponding sides of a congruent $\triangle$ are in proportion</td>
</tr>
<tr>
<td>7. $EH \cdot CH = FH \cdot AH$</td>
<td>7. The product of the means is equal to the product of the extremes</td>
</tr>
</tbody>
</table>

Score 3: The student did not state $\overline{AD} \parallel \overline{BC}$ to prove $ABCD$ is a parallelogram, did not state $\overline{AB} \parallel \overline{CD}$ to prove $\angle 3 \cong \angle 4$, and incorrectly stated congruent triangles in reason 6.
35 Given: Quadrilateral \(ABCD\), \(AC\) and \(EF\) intersect at \(H\), \(EF \parallel AD\), \(EF \parallel BC\), and \(AD \cong BC\)

Prove: \((EH)(CH) = (FH)(AH)\)

\[
\frac{EH}{CH} = \frac{FH}{AH} \quad EH \cong AH \quad FH \cong CH
\]

Statements

1.) \(EF \parallel AB\), \(EF \parallel BC\)
2.) \(AB \cong BC\)
3.) \(ABCD\) is PARA
4.) \(\triangle AHE \cong \triangle FHC\)
5.) \(\overline{AB} \parallel \overline{CD}\)
6.) \(\triangle BAC \cong \triangle HCF\)
7.) \(\triangle AEH \sim \triangle CFH\)
8.) \((EH)(CH) = (FH)(AH)\)

Reasons

1.) Given
2.) Given
3.) if opp sides \(\cong\), PARA
4.) vert \(\angle\)'s are \(\cong\)
5.) if PARA, opp sides \(\parallel\)
6.) if lines \(\parallel\), alt int \(\angle\)'s \(\cong\)
7.) AA
8.) if \(\triangle\)s \(\sim\), a proportion with sides of \(\sim\) \(\triangle\)s is correct

Score 3:  The student did not state \(AD \parallel BC\) to prove \(ABCD\) is a parallelogram and gave no correct statements and reasons after step 7.
35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \equiv BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>S</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $EF \parallel AD$, $EF \parallel BC$, $AD \equiv BC$</td>
<td>0. Given</td>
</tr>
<tr>
<td>2. $\angle EAH \equiv \angle HCF$</td>
<td>1. Lines form $\angle A \equiv \angle HCF$</td>
</tr>
<tr>
<td>3. $\angle AHE \equiv \angle FHC$</td>
<td>2. Vertical $\angle$s are $\equiv$</td>
</tr>
<tr>
<td>4. $\triangle AHE \sim \triangle CHF$</td>
<td>3. $AA \text{ similar} \sim$</td>
</tr>
<tr>
<td>5. $\frac{EH}{FH} = \frac{AH}{CH}$</td>
<td>4. Corresponding sides in $\sim$</td>
</tr>
<tr>
<td>6. $(EH)(CH) = (FH)(AH)$</td>
<td>5. $\angle A \equiv \angle HCF$</td>
</tr>
<tr>
<td>6. $\angle A \equiv \angle HCF$</td>
<td>6. $AA \text{ similar} \sim$</td>
</tr>
<tr>
<td>6. Cross multiplying</td>
<td>6. $\angle A \equiv \angle HCF$</td>
</tr>
</tbody>
</table>

**Score 2:** The student made a conceptual error by not proving $ABCD$ is a parallelogram, did not state $AB \parallel CD$ to prove $\angle EAH \equiv \angle FCH$, and wrote an incorrect reason in step 6.
35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

1. $AD \parallel EF$, $EF \parallel BC$
2. $AD \parallel BC$
3. $\angle BAC \cong \angle ADO$
4. $\angle EHA \cong \angle CHF$
5. $\triangle AHE$ is similar to $\triangle CHF$
6. $(EH)(CH) = (FH)(AH)$

1. Given
2. If two lines are parallel to the same line, then they are parallel to each other.
3. Alternate interior angles are congruent to each other.
4. Vertical angles are congruent to each other.
5. AA Similarity
6. Similar triangles are in proportion.

Score 2: The student wrote some correct relevant statements and reasons.
35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

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<tbody>
<tr>
<td>Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$</td>
<td>Given</td>
</tr>
<tr>
<td>Vertical angles are $\cong$</td>
<td>Vertical angles are $\cong$</td>
</tr>
<tr>
<td>$\angle AEH \cong \angle CHF$</td>
<td>$\angle AEH \cong \angle CHF$</td>
</tr>
<tr>
<td>$AB \parallel BC$</td>
<td>Opposite sides of a quadrilateral are parallel</td>
</tr>
<tr>
<td>$\angle AEH \cong \angle BCF$</td>
<td>Two parallel lines cut by a transversal create congruent alternate interior angles</td>
</tr>
<tr>
<td>$\triangle AEH \cong \triangle CBF$</td>
<td>AA</td>
</tr>
<tr>
<td>$(EH)(CH) = (FH)(AH)$</td>
<td>Corresponding parts of congruent triangles are congruent</td>
</tr>
</tbody>
</table>

Score 2: The student made a conceptual error in step 3 and gave no correct statements and reasons after step 5.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle 1, \angle 2$ are vert $\angle$s</td>
<td>2) $\angle$s that form a $\angle$ intersection are vert $\angle$s</td>
</tr>
<tr>
<td>3) $\angle 1 \cong \angle 2$</td>
<td>3) vert $\angle$s are $\cong$</td>
</tr>
<tr>
<td>4) $\overline{AC} \cong \overline{AC}$</td>
<td>4) Reflexive Prop</td>
</tr>
<tr>
<td>5) $\frac{EH}{FH} = \frac{CH}{AH}$</td>
<td>5) C.S.S.T.E</td>
</tr>
<tr>
<td>$\therefore (EH)(CH) = (FH)(AH)$</td>
<td>6) Cross products</td>
</tr>
</tbody>
</table>

Score 1: The student only proved $\angle 1 \cong \angle 2$ correctly, and no further correct relevant work was shown.
Question 35

35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quadrilateral $ABCD$, $AC \cong EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$ and $AD \cong BC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle HEA \cong \angle HFC$</td>
<td>2. $\parallel$ lines create $\cong$ alternate exterior angles</td>
</tr>
<tr>
<td>3. $\angle HEB \cong \angle HFD$</td>
<td>3. $\parallel$ lines create $\cong$ alternate exterior angles</td>
</tr>
<tr>
<td>4. $EH \cong FH$, $AH \cong HC$</td>
<td>4. They are proportional</td>
</tr>
<tr>
<td>5. $\frac{CH}{FH} = \frac{AH}{CH}$</td>
<td>5. proportional</td>
</tr>
</tbody>
</table>

Score 0: The student did not show enough correct relevant work to receive any credit.
35 Given: Quadrilateral $ABCD$, $AC$ and $EF$ intersect at $H$, $EF \parallel AD$, $EF \parallel BC$, and $AD \cong BC$

Prove: $(EH)(CH) = (FH)(AH)$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quadrilateral $ABCD$</td>
<td>1. given</td>
</tr>
<tr>
<td>$EF \parallel AD$</td>
<td>2. They have opposite</td>
</tr>
<tr>
<td>$EF \parallel BC$</td>
<td>parallel sides</td>
</tr>
<tr>
<td>2. $AHFD$ and $EHCB$ are parallelograms</td>
<td>3. Parallelograms have</td>
</tr>
<tr>
<td></td>
<td>opposite parallel sides</td>
</tr>
<tr>
<td>3. $AHFD$ and $EHCB$ have opposite congruent</td>
<td>4. Corresponding parts</td>
</tr>
<tr>
<td>sides</td>
<td>of corresponding figures and</td>
</tr>
<tr>
<td>4. $(EH)(CH) = (FH)(AH)$</td>
<td></td>
</tr>
</tbody>
</table>

Score 0: The student did not show enough correct relevant work to receive any credit.